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Optimization of Reactor Thermal Design by Statistical Hot Spot Analysis

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A b s t r a c t

The influence of temperature and power flattening on the hot channel factor is examined for uniform slab and cylindrical cores. It is shown that, at constant hot channel probability, the maximum available power and maximum average coolant temperature do not coincide with the condition of complete flattening; this is due to the consequent increase in the overall hot channel factor. However the difference between optimum condition and complete flattening is small.

This result is applied to the optimization of the distribution of the coolant flow rate among the several subassemblies of a sodium-cooled fast reactor. By a comparison between different design criteria, it is shown that, for given power distribution and inlet temperature, distributing the coolant in such a way, that the probability of hot spots is constant in each subassembly, results in a higher average coolant temperature at core outlet, that is in a better reactor efficiency.

However the advantage of this criterion is in general small, it increases if the uncertainties are not constant along the core radius.

Z u s a m m e n f a s s u n g

Der Einfluß der Abflachung der Leistungs- und Temperaturprofile auf den Heißkanalfaktor wird bei gleichförmigen "slab"- und zylindrischen Reaktorkernen untersucht. Es wird gezeigt, daß bei konstanter Heißkanalwahrscheinlichkeit die maximale Leistung und die maximale mittlere Kühlmitteltemperatur nicht bei vollständiger Abflachung erreicht werden; das ergibt sich aus der entsprechenden Zunahme des Heißkanalfaktors. Der Unterschied zwischen Optimalbedingung und vollständiger Abflachung ist jedoch klein.

Dieses Ergebnis wird auf der Optimierung der Durchsatzverteilung zwischen den verschiedenen Brennelementen eines schnellen natriumgekühlten Reaktors angewendet. Ein Vergleich zwischen verschiedenen Auslegungskriterien zeigt, daß bei vorgegebener Leistungsverteilung und Eintrittstemperatur die mittlere Kühlmitteltemperatur am Reaktoraustritt einen Maximalwert erreicht, wenn die Drosselung so ausgelegt ist, daß die Heißstellenwahrscheinlichkeit in jedem Brennelement konstant ist.

Der durch die Anwendung dieses Kriteriums erreichte Vorteil ist jedoch im allgemeinen klein, er wächst, wenn die Unsicherheiten im Kern nicht konstant sind.

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1. Introduction

To increase the power output and the plant efficiency particular expedients are studied for power reactors in order to flatten the power and temperature profile. For instance the core of a fast reactor is subdivided in radial zones of different enrichment fuel to compensate the decrease of the neutron flux by a higher reaction rate; and the coolant flow rate is calibrated by an orifice at subassembly inlet with the aim of flattening the profile of the coolant outlet temperature.

However flattening the power and temperature profile results in an increase of the overall hot spot factors, since the number of fuel pins operating at higher temperature increases; therefore the possible advantage is reduced in part by the larger margin, which must be maintained against the allowable maximum temperatures in the core. Moreover beyond a certain degree of flattening the increase in the hot spot factors might prevail, so that no further increase in the power might be allowed. About this point some discussions arose in the literature [1, 2, 3, 4].

This paper presents an analysis of the influence of flattening on the hot channel factors for uniform slab and cylindrical cores. The results are then applied to the optimization of the distribution of the coolant flow rate in a fast reactor.

2. Flattening of power and coolant temperature profiles

Let us consider a uniform core with a radial power distribution defined by

$$p(u) = p_{max}\ f(u) \tag{1}$$

where $u = \frac{r}{R}$ (radial abscissa. R = core radius)

$$p_{max} = p(o) = \text{maximum local power output}$$

$$f(u) \begin{cases} = 1 & \text{for } u = 0 \\ = 0 & \text{" } u = 1 \\ < 1 & \text{" } 0 < u < 1 \end{cases} \quad . \qquad (2)$$

Assuming a constant specific heat (c_p) for the coolant, if the coolant flow rate is constant along r $\left(q(u) = q_{max}\right)$ and no mixing occurs, the profile of the temperature span across the core is given by

$$\Delta\vartheta(u) = \Delta\vartheta_{max}\, f(u) \qquad (3)$$

with

$$\Delta\vartheta_{max} = \frac{p_{max}}{c_p\, q_{max}} \quad . \qquad (4)$$

Let N be the total number of channels in the core, and n(u)du the number of channels at abscissa u:

$$N = \int_u n(u)\,du \quad . \qquad (5)$$

We shall examine separately the effects of flattening the power p(u) at constant flow rate, and of flattening the temperature span $\Delta\vartheta(u)$ at constant power. Independently of the physical possibility of achieving complete flattening, suppose to flatten the power profile at constant flow rate modifying by opportune means the flux distribution in such a way that the local power is given by

$$p'(u) = p_{max}\, f\left(\frac{u}{h}\right) \quad . \qquad (6)$$

If in Eq (6) $h \to \infty$, $p'(u)$ tends to be constant and equal to p_{max}.

In this case flattening the power results in a flattening of the coolant temperature also

$$\Delta\vartheta'(u) = \Delta\vartheta_{max} \quad f(u/h) \tag{7}$$

and the average temperature span is given by

$$\Delta\vartheta_{av,p} = \frac{\int_u n(u) \quad \Delta\vartheta'(u)\, du}{\int_u n(u)\, du} \tag{8}$$

since the flow-rate has been assumed to be constant along r.

The other possibility is, at constant power profile, to flatten only the coolant outlet temperature for a better efficiency, and we suppose that it is possible to distribute the flow-rate in such a way that the coolant temperature profile is still given by Eq. (7), while the power profile remains unchanged [Eq.(1)].

This is obtained when the flow-rate is distributed according to the following relation

$$q(u) = \frac{p(u)}{c_p \;\Delta\vartheta'(u)} = \frac{p_{max}}{c_p\; \Delta\vartheta_{max}} \quad \frac{f(u)}{f(\frac{u}{h})} \quad . \tag{9}$$

In this case the average temperature span is given by

$$\Delta\vartheta_{av,c} = \frac{\int_u n(u)q(u)\;\Delta\vartheta'(u)\; du}{\int_u n(u)\; q(u)\; du} \quad . \tag{10}$$

If the probability P_h that no channel exceeds the allowable temperature $\Delta\vartheta_{all}$ must be constant for each h, the maximum temperature span $\Delta\vartheta_{max}$ is a function of h, which is implicitly defined by (see Ref. [5])

$$P_h = \exp \int_u n(u) \log\left[P\left(\frac{\Delta\vartheta_{all} - \Delta\vartheta_{max} f(u/h)}{\sigma \quad \Delta\vartheta_{max} f(u/h)}\right)\right] du = \text{const.} \tag{11}$$

where σ is the relative standard deviation of $\Delta\vartheta_{max}$

and $P\left(\frac{\Delta\vartheta_{all} - \Delta\vartheta_{max} f(u/h)}{\sigma \quad \Delta\vartheta_{max} f(u/h)}\right)$

is the probability that $\Delta\vartheta_{all}$ is not exceeded in a channel at abscissa u.

3. Uniform slab and cylindrical cores

Let us particularize the analysis to uniform slab and cylindrical cores defined by:

Slab

$$n(r)dr = \frac{N}{2R} dr$$

$$p(r) = p_{max} \cos\left(\frac{\pi r}{2 R}\right)$$

Cylinder

$$n(r)dr = \frac{2\ N\ r}{R^2}\ dr$$

$$p(r) = p_{max}\ J\ (2.405\ \frac{r}{R})\ .$$

Particularization of the principal relations at item 2 is given in Appendix.

For these cores, the function $\Delta\vartheta_{max}$ (h) [Eq. (10)] has been calculated by means of a digital computer, varying h between 1 (no flattening) and ∞ (complete flattening) for different values of N, σ and P_h. By Eqs. (8) and (11) the consequent obtainable average temperatures $\Delta\vartheta_{av,\ p}$ (h) and $\Delta\vartheta_{av,c}$ (h) have been calculated.

We remember that $\Delta\vartheta_{av,p}$ (h) is the average temperature span at constant flow rate, which is related to the total power output by the proportionality relation

$$p_{tot}\ (h) = c_p\ q_{max}\ N\ \ \Delta\vartheta_{av,p}\ (h),$$

therefore we can chose $\Delta\vartheta_{av,p}$ (h) as a representative figure for the power achievable by flattening.

$\Delta\vartheta_{av,c}$ (h) is the average temperature span for a given power distribution, it is representative for the coolant temperature at core outlet assuming a constant inlet temperature.

The ratio of $\Delta\vartheta_{av,p}$(h) [or $\Delta\vartheta_{av,c}$ (h)] to $\Delta\vartheta_{max}$ (h) is a measure of the degree of flattening reached for a given h; this ratio tends to unity as h $\rightarrow\infty$.

Fig. 1 shows $\Delta\vartheta_{max}$, $\Delta\vartheta_{av,p}$ and $\Delta\vartheta_{av,c}$ as a function of h, for $P_h = 99.9$ % , $\Delta\vartheta_{all} = 200^o$ C, $\sigma = 0.1$ and N = 10,000. Similar results have been obtained for different values of P_h, σ and N.

From these curves it can be noted that the maximum average outlet temperature and the maximum power do not coincide with the condition of complete flattening.

In particular beyond h = 6 no further increase in the outlet temperature can be obtained. For h = 6 the ratio of the average to maximum temperature span is equal to 0.993 for a slab reactor, whereas this ratio is equal to 0.987 for a cylindrical core.

Analogously, beyond h = 9 no further increase in the total power outlet can be obtained. In this case the ratio of average to maximum power is equal to 0.995 for a slab reactor, whereas this ratio is equal to 0.991 for a cylindrical core.

It can be observed that these limits are beyond the physical possibility of achievable flattening and therefore at the actual stage improving flattening still results in a improved reactor performance. Moreover the statement of Judge and Bohl [1] is confirmed from these results: the difference between optimum condition and complete flattening is in fact very small.

Fig. 1 shows that the decrease in the advantage of the flattening due to the increase in the hot channel factor is larger for the cylindrical core than for the slab one. For the cylinder $\Delta\vartheta_{max}$ goes from 142^o C for h = 1 to 131.4^o C. This decrease is only due to the increase in the hot channel factor; it corresponds to a reduction in the available power of 7.5 % for the cylinder and of 3.7 % for the slab reactor. For h = 3 the corresponding values are 4 % and 1.5 % respectievely.

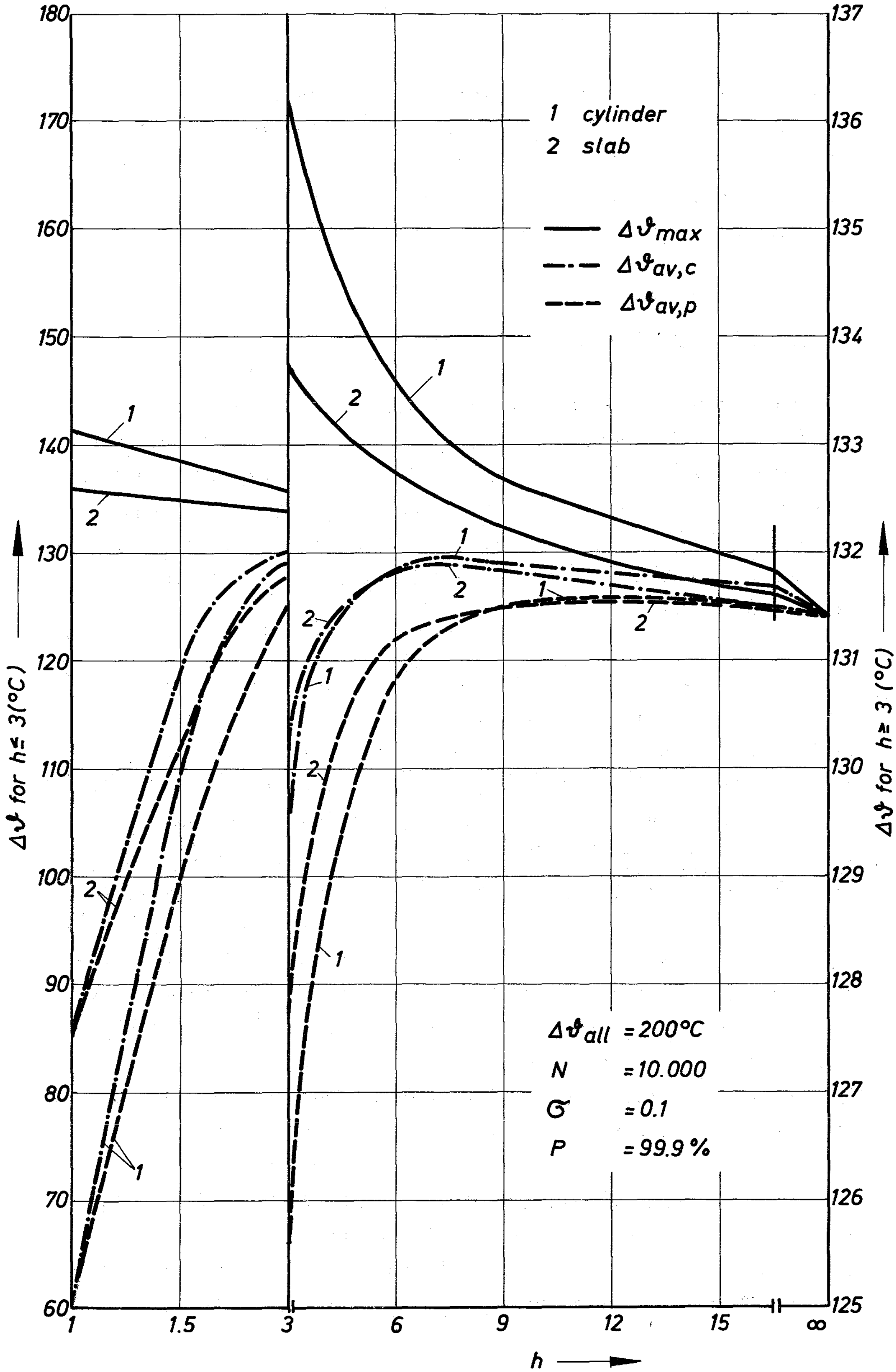

Fig. 1 - Maximum and average coolant temperature span at constant hot channel probability as a function of attained flattening.

This behavior is due to the fact that the "equivalent number of channels" for $h = 1$ is much smaller for the cylindrical core than for the slab one [5], whereas it tends to be the same and equal to the number of channels actually present in the core as $h \rightarrow \infty$.

4. Fast reactor optimization

Power distribution optimization is beyond the limits of the present work since it, involves a large number of different parameters such as burn-up, fuel cycle etc. It is sufficient here to observe that the designer dealed with such problems should not neglect the influence of the power distribution on the hot spot factors. Optimization is here limited to the distribution of the coolant flow rate.

The problem can be formulated as follows: for given power distribution and inlet temperature, which coolant flow rate distribution, at a preassigned confidence level, allows the maximum coolant outlet temperature, e. g. the best reactor efficiency? From Fig. 1 it was derived, that complete flattening does not lead to the maximum, but this condition does not differ significantly from the optimum condition. Complete flattening means that all the channels have the same probability of beeing "hot". Now in a fast reactor it is not possible to distribute the flow rate in a continuous way, since we can act only on the orifice at subassembly inlet but not on the channels of a subassembly. Therefore the problem is reduced to the optimization of the coolant distribution among the subassemblies. By the previous consideration it can be expected that distributing the flow rate in such a way that the probability of hot spot is constant in each subassembly, should lead to a near-to-optimum condition. This criterion is compared in the following with other usual criteria of coolant flow rate distribution in the case of the sodium cooled fast reactor Na-2. [6]

5. Criteria of orifice calibration

As in Ref. [7], the Na-2 core has been divided into 7 concentrical zones of equal power subassemblies. The zone are indicated by the index i.

The flow rate distribution considered are the following:

I. criterion: the average temperature span ($\Delta \vartheta_{av,i}$) of the coolant is the same for all the subassemblies.

II. criterion: the maximum temperature ($\Delta \hat{\vartheta}_{c,i}$) of the coolant is the same for all the subassemblies.

III. criterion: the maximum temperature ($\hat{\vartheta}_{cl,i}$) of the cladding is the same for all the subassemblies.

IV. criterion: the cladding hot spot probability is the same in each subassemblies.

The first three criteria take into account the nominal temperatures only, the last one takes into account the uncertainties also.

The mathematical formulation of criterion IV is the following:

$$\hat{\vartheta}_{cl,i} + m^{eq}_{s,i} + \lambda \sqrt{(\sigma^{eq}_{s,i})^2 + (\sigma_{z,i})^2} = \text{const.} \qquad (13)$$

where

$\hat{\vartheta}_{cl,i}$ is the nominal maximum temperature of the cladding

$m^{eq}_{s,i}$ and $\sigma^{eq}_{s,i}$ are the mean and the standard deviation of the equivalent subassembly distribution (see Ref. [7], we remember here that this distribution takes already into account the local, channel and subassembly uncertainties, the number of spots in a pin, the number of pins in a subassembly, and the radial and axial temperature profiles in a subassembly).

$\sigma_{z,i}$ is the standard deviation of the zone uncertainties.

λ is a factor depending upon the desired confidence level.

If the uncertainties are the same for each subassembly in the core, the criterion IV differs from the III one only because in the subassemblies with greater power gradients the number of limiting pins is smaller than in the central subassemblies. This means that higher nominal maximum temperatures of the cladding are allowable in the subassemblies near to the core boundary. The advantage of criterion IV cannot be great in this case. However a reactor is constituted of subassemblies with fuel at different burn-up and therefore with different uncertainties; moreover some systematic deviations, such as due to the position of the control rods for instance, are different from zone to zone. In this cases greater advantages can be expected. Therefore we considered three cores different only in the uncertainties.

Core 1: No systematic deviations, constant uncertainties;

Core 2: No systematic deviations, different uncertainties;

Core 3: Different systematic deviations and different uncertainties. This core is the same considered in Ref. [7] .

6. Numerical results for the Na-2 reactor

The design data of the Na-2 reactor are reparted in Table 1. This table shows the radial power profile: this profile has been assumed to be linear within a subassembly. Moreover it has been assumed that, in a subassembly the power profile is the same as the coolant profile (no mixing). The axial power profile has been assumed to be of cosine shape.

Table 1 Na-2 core

Total power	300 MW(e)
Maximum specific power	420 W/cm
Critical cladding temperature	700 °C
Nominal sodium inlet temperature	380 °C
Average/maximum axial power	0.8
Active length	95 cm
Hot spot length (pellet length)	1 cm
Number of pins in a subassembly	169
Maximum temperature drop cladding-coolant	59 °C

Zone	1	2	3	4	5	6	7
Number of subassemblies	6	12	12	24	30	24	42
Ratio of max. power in a subassembly to max. power in the core	0.965	0.934	0.895	0.834	1	0.894	0.715
Ratio of average to max. power in a subassembly	0.985	0.971	0.961	0.952	0.918	0.865	0.807

Table 2 reports the uncertainties and systematic factors for the cores 1, 2 and 3.

Table 2 Statistical and systematic deviations

Core	1	2		3						
Zone	1.2.3.4.5.6.7	1.2.3.4	5.6.7	1	2	3	4	5	6	7
local % σ_l	1.1	1.1	1.4	1.1	1.1	1.1	1.1	1.1	1.1	1.1
channel % σ_{ch}	2.7	2.7	3.2	2.7	2.7	2.7	2.7	2.7	2.7	2.7
subassembly % σ_s	2.9	2.9	4.5	2.9	2.9	3.7	3.7	2.9	3.2	3.7
zone % σ_z	2.5	2.5	4.3	2.5	2.5	2.5	2.5	2.5	2.5	3.0
systematic deviations %	0.0	0.0	0.0	10.0	7.0	4.0	4.0	7.0	9.0	12.0
core % σ_c	3.8	3.8		3.8						
All deviations are reported in % of the temperature difference between cladding and inlet coolant.										

For the design of the core a 99 % (2.4σ) confidence level was assumed. (1 % probability of exceeding the critical cladding temperature 7oo° C). In order to obtain an overall confidence level of 2.4σ for the core, each subassembly should have larger not exceeding probability; therefore, for the individual subassemblies, λ in Eq. (13) was chosen equal to 3.5. *

By iterative application of the SHØSPA code (Ref. [7]), the allowable coolant temperature span was calculated for the first core according to the previously exposed criteria. The average temperature span in the core is given by:

$$\Delta\vartheta_{av} = \frac{\sum_i N_{si}\, q_i\, c_{pi}\, \Delta\vartheta_{av,i}}{\sum_i N_{si}\, q_i\, c_{pi}} = \sum_i \Delta\vartheta_{av,i} \cdot W_i \qquad (14)$$

Where

$$W_i \simeq \frac{N_{si}\, q_i}{\sum_i N_{si}\, q_i}$$

N_{si} = number of subassemblies in the zone i

q_i = sodium flow rate in a subassembly in the zone i

c_p = specific heat (approximatively constant. In the temperature range 5oo - 7oo° C [8])

$\Delta\vartheta_{av,i}$ = average temperature span in a subassembly of the zone i.

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* There are 15o subassemblies in the core. The subassembly uncertainties are sampled 15o times: for these uncertainties an overall confidence level of 2.4σ means a confidence level of $\sim 4\sigma$ for each subassembly. The zone uncertainties are sampled however 7 times, in this case an overall confidence level of 2.4σ implies a confidence level of $\sim 2.8\sigma$. $\lambda = 3.5$ is an intermediate value between these limits. In any case varying λ between 3 and 4, no significant variation in the results was found.

The results of the calculations are reported in Table 3 for the first core (no systematic deviations, constant uncertainties).

Table 3 Core 1 - Comparison among the different criteria

Zone	1	2	3	4	5	6	7	Δ_{av} (Core)	Criterion
$\hat{\vartheta}_{cl,i}$	599.2	601.2	602.2	602.7	614.4	624.4	635.6		I Constant coolant average temperature
$\Delta\hat{\vartheta}_{c,i}$	195.1	198.0	200.0	202.0	209.4	222.2	238.1		
$\Delta\vartheta_{av,i}$	192.3	192.3	192.3	192.3	192.3	192.3	192.3	192.3	
w_i									
$\hat{\vartheta}_{cl,i}$	629.1	628.3	627.3	625.7	630.0	627.3	622.7		II Constant coolant max. temperature
$\Delta\hat{\vartheta}_{c,i}$	225.0	225.0	225.0	225.0	225.0	225.0	225.0		
$\Delta\vartheta_{av,i}$	221.7	218.4	216.3	214.1	206.5	194.6	181.6	202.9	
W_i	0.046	0.088	0.083	0.154	0.251	0.165	0.213		
$\hat{\vartheta}_{cl,i}$	628.1	628.1	628.1	628.1	628.1	628.1	628.1		III Constant clad max. temp.
$\Delta\vartheta_{c,i}$	224.1	224.9	225.8	227.4	223.3	225.7	230.4		
$\Delta\vartheta_{av,i}$	220.8	218.4	217.1	216.4	205.0	195.3	186.0	203.8	
W_i	0.046	0.088	0.083	0.153	0.254	0.166	0.210		
$\hat{\vartheta}_{cl,i}$	625.4	626.9	627.5	628.1	628.0	629.6	631.1		IV Constant hot spot prob.
$\Delta\hat{\vartheta}_{c,i}$	221.2	223.4	225.0	227.3	222.9	227.2	233.3		
$\Delta\vartheta_{av,i}$	218.0	216.9	216.4	216.3	204.6	196.6	188.4	204.4	
W_i	0.047	0.089	0.083	0.154	0.256	0.163	0.207		

All temperatures are expressed in $^{\circ}$ C.

Table 3 shows that criterion I offers the greatest disadvantage. Between the other criteria the differences are smaller. As expected criterion IV leads to the maximum outlet temperature, but its advantage is very little.

It might be asked whether the advantage for other values of the uncertainties can be greater; even if we examined only a case it is possible to respond negatively to this question. In fact the smaller the local and channel uncertainties are, the greater is the influence of the other types of uncertainties, for which only the nominally hottest pins in a subassembly are important, therefore for σ_{ch}, $\sigma_l \rightarrow 0$ criterion IV coincides with the III one. Moreover, if σ_{ch}, $\sigma_l \rightarrow \infty$ the equivalent number of channels tends to the actual number of channels /‾5_7, therefore criterion IV tends to coincide again with the III one.

Relatively greater advantages were obtained for the cores 2 and 3. The results are summarized in Table 4.

Table 4 Average coolant temperature span for the different criteria and cores.

Criterion	I $\Delta\vartheta_{av,i}$ = const.	II $\Delta\hat{\vartheta}_{c,i}$ = const.	III $\hat{\vartheta}_{cl,i}$ = const.	IV Const. Hot spot prob.	Core
$\Delta\vartheta_{av}$ ($^\circ$ C)	192.3	202.9	203.8	204.4	1
	-	-	189.4	191.2	2
Conf. Level 99 %	-	182.3	-	185.1	3

From this table it can be observed that an increase of $\sim 3^{o}C$ in the outlet temperature is offered by criterion IV for the Na-2 core considered at Ref. 7 in respect to the original design (criterion II, Ref. /‾6_7). A graphycal representation of the thermal design is given for this case in Fig. 2.

7. Conclusions

Even if the advantage is not great, distributing the flow rate in such a way that each subassembly has the same hot spot probability results in a better efficiency of the reactor plant. This criterion offers greater advantages when the uncertainties or the systematic deviations are not constant along the core. In this case it has also a more precise physical significance than other criteria which consider only the nominal temperatures and not the uncertainties.

8. References

/‾1_7 F. D. Judge and L. S. Bohl, "Effective Hot-Channels Factors for "Flat" Power Reactors", Nucl. Sci. Eng. 28,296 (1967)

/‾2_7 H. Fenech and H. M. Guéron, "The Synthesis Method of Uncertainty Analysis in Nuclear Reactor Thermal Design", Nucl. Sci. Eng. 31, 5o5 (1968)

/‾3_7 F. D. Judge and C. S. Bohl, Letter to the Editor, Nucl. Sci. Eng., 33, 146 (1968)

/‾4_7 H. M. Guéron, Letter to the Editor, Nuc. Sci. Eng. 33,146 (1968)

/‾5_7 A. Amendola, "Effective Equally Limiting Hot Channels in Reactor Thermal Design", Nucl. Sci. Eng. 39,121 (197o)

/‾6_/ K. Gast, E. G. Schlechtendahl, "Schneller Natriumgekühlter Reaktor Na 2", KFK 66o, EUR 3706d, Kernforschungszentrum Karlsruhe (1967)

/‾7_/ A. Amendola, "Advanced Statistical Hot Spot Analysis", KFK 1134, EUR 3687e, Kernforschungszentrum Karlsruhe (197o)

/‾8_/ A. Pee, "Stoffdaten von Natrium", KFK 924, EUR 4168d, Kernforschungszentrum Karlsruhe (1969)

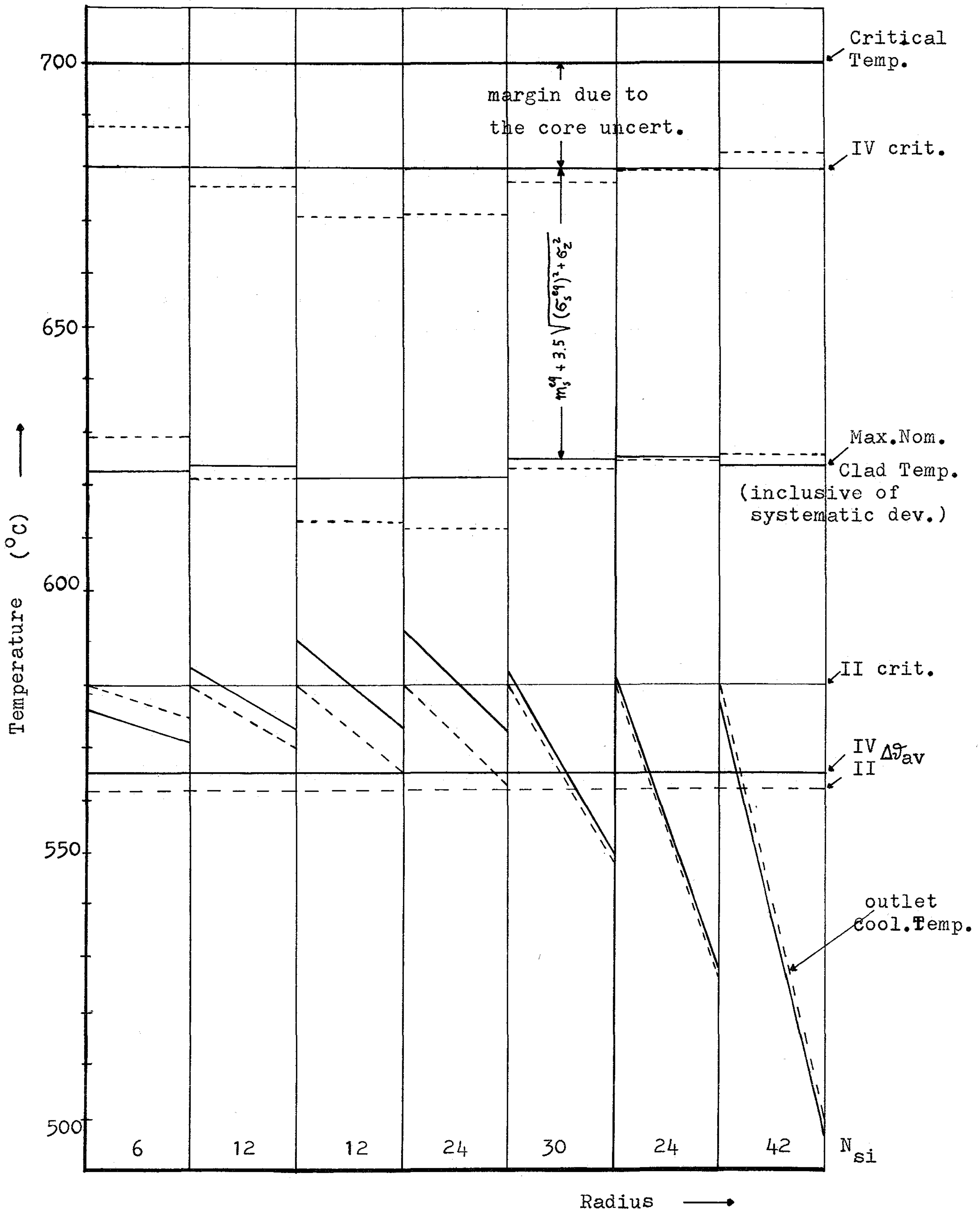

Fig.2 - Graphical representation of thermal design according to criteria II and IV

9. Appendix

Particularization of the equations at item 2 for slab and cylindrical cores

Slab

$$p(u) = p_{max} \cos(\frac{\pi}{2}u) \qquad (1\text{-s})$$

$$n(u)du = N/2 \; du \qquad (5\text{-s})$$

$$p'(u) = p_{max} \cos(\frac{\pi u}{2h}) \qquad (6\text{-s})$$

$$\Delta\vartheta'(u) = \Delta\vartheta_{max} \cos(\frac{\pi u}{2h}) \qquad (7\text{-s})$$

$$\Delta\vartheta_{av,p} = \int_0^1 \Delta\vartheta_{max} \cos(\frac{\pi u}{2h}) \, du = \frac{2h}{\pi} \sin(\frac{\pi}{2h}) \Delta\vartheta_{max} \qquad (8\text{-s})$$

$$q(u) = \frac{p_{max}}{c_p \Delta\vartheta_{max}} \cos(\frac{\pi}{2}u) / \cos(\frac{\pi u}{2h}) \qquad (9\text{-s})$$

$$\Delta\vartheta_{av,c} = \frac{\Delta\vartheta_{max} \; 2/\pi}{\int_0^1 \frac{\cos(\frac{\pi}{2}u)\,du}{\cos(\frac{\pi u}{2h})}} \qquad (10\text{-s})$$

$$P_h = \exp \int_{-1}^1 N/2 \log\left[P\left(\frac{\Delta\vartheta_{all} - \Delta\vartheta_{max} \cos(\frac{\pi u}{2h})}{\sigma \; \Delta\vartheta_{max} \; \cos(\frac{\pi u}{2h})} \right)\right] du = \text{const.} \qquad (11\text{-s})$$

Cylinder

$$p(u) = p_{max} \, J_o(2.405 \, u) \qquad (1\text{-c})$$

$$n(u)du = 2N \, u \, du \qquad (5\text{-c})$$

$$p'(u) = p_{max}\, J_o(2.405\; u/h) \tag{6-c}$$

$$\Delta\vartheta'(u) = \Delta\vartheta_{max}\, J_o(2.405\; u/h) \tag{7-c}$$

$$q(u) = \frac{p_{max}}{c_p\, \Delta\vartheta_{max}} \quad \frac{J_o(2.405\; u)}{J_o(2.405\; u/h)} \tag{8-c}$$

$$\Delta\vartheta_{av,p} = 2\, \Delta\vartheta_{max} \int_0^1 u\; J_o(2.405\; u/h)\; du \tag{9-c}$$

$$\Delta\vartheta_{av,c} = \Delta\vartheta_{max} \frac{\int_0^1 u\; J_o(2.405\; u)\; du}{\int_0^1 u\, \frac{J_o(2.405\; u)}{J_o(2.405\; u/h)}\; du} \tag{10-c}$$

$$P_h = \exp \int_0^1 2Nu\; \log\Big[P\Big(\frac{\Delta\vartheta_{all} - \Delta\vartheta_{max}\, J_o(2.405\; u/h)}{\sigma\;\; \Delta\vartheta_{max}\, J_o(2.405\; u/h)} \quad du = \text{const.} \tag{11-c}$$

10. List of Symbols

c_p	= specific heat of the coolant
$f(u)$	= radial power profile (≤ 1)
$f(\frac{u}{h})$	= flattened radial profile
h	= flattening parameter
i	= index describing a zone
m_s^{eq}	= mean of the equivalent subassembly distribution
$n(u)$	= frequency function oftheradial distribution of the channels
N	= total number of channels in a core
N_{si}	= number of subassemblies in a zone
$p(u)$	= power distribution
$p'(u)$	= flattened power distribution
p_{max}	= maximum power output
p_{tot}	= total power output of the core
P_h	= confidence level, probability that no channel exceeds a certain critical temperature
$P_{(x)}$	= probability of not exceeding the deviation x in a normal distribution
$q(u)$	= radial flow rate distribution
q_{max}	= maximum flow rate in a channel
q_i	= flow rate in a subassembly
r	= radial abscissa between 0 and R
R	= core radius
u	= normalized radial abscissa ($u = r/R$)
W_i	= weight to give to the average temperature of a zone in order to calculate the average temperature of the core

$\Delta\vartheta(u)$	=	radial profile of the coolant temperature span
$\Delta\vartheta'(u)$	=	flattened profile of the coolant temperature span
$\Delta\vartheta_{max}$	=	maximum temperature span in the radial profile
$\Delta\vartheta_{av,p}^{(h)}$	=	average temperature span for flattened power
$\Delta\vartheta_{av,c}^{(h)}$	=	average temperature span for flattened coolant temperature
$\Delta\vartheta_{all}$	=	allowable temperature span
$\Delta\vartheta_{av,i}$	=	average temperature span in a subassembly
$\Delta\vartheta_{av}$	=	average temperature span in the core
$\Delta\hat{\vartheta}_{c,i}$	=	maximum temperature span in a subassembly
$\hat{\vartheta}_{cl,i}$	=	maximum cladding temperature in a subassembly
λ	=	ratio of a deviation to the standard deviation in a normal distribution
σ	=	standard deviation
$\sigma_s eq$	=	standard deviation of equivalent subassembly distribution
σ_z	=	standard deviation of the zone uncertainties