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Institut für Angewandte Reaktorphysik
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Projekt Schneller Brüter

A Three-dimensional Synthesis Method Tested and Applied in Fast Breeders

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GESELLSCHAFT FÜR KERNFORSCHUNG M. B. H.
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A three-dimensional Synthesis Method tested and applied in fast breeders

by

S. Pilate[+], G. Buckel, M. Billaux[+], A. Charlier[+],
P. Mc Grath, F. Plum

[+] S.A. Belgonucleaire, Brüssel

Gesellschaft für Kernforschung mbH., Karlsruhe

Z u s a m m e n f a s s u n g

Das Computerprogramm KASY, das in Karlsruhe für die Rechenanlage IBM 360/50 geschrieben wurde, löst die dreidimensionale Diffusionsgleichung mit einer Synthesemethode, die im voraus erstellte zweidimensionale Versuchsfunktionen benützt.

Der erste Teil dieses Berichts beschreibt den Test der in KASY verwendeten Fluss-Synthesemethode durch einen Vergleich mit einem dreidimensionalen, totalnumerischen Programm. Verglichen werden Grössen wie k_{eff}, Kontrollstabreaktivitäten, sowie Fluss- und Reaktionsraten.

Im zweiten Teil wird die Synthesemethode auf die Auswertung einer Reihe von Kontrollstabexperimenten angewandt. Diese umfassen Einzelstabmessungen sowie Abschattungseffekte mehrerer Stäbe in einem schnellen Brütercore.

S u m m a r y.

The computer program KASY, written at Karlsruhe for the I.B.M. 360/65, solves the three-dimensional diffusion equation by means of a synthesis method using available two-dimensional trial functions.

The first part of this report is devoted to the test of the flux synthesis method used in KASY with respect to results obtained by a direct three-dimensional program. The results considered are : k_{eff}, control rod reactivities, flux and reaction rate maps.

In the second part the KASY synthesis is applied to the evaluation of a diversified series of control rod experiments in a fast breeder core containing one or more control rods.

3. August 1971

Sommaire.

Le programme KASY, écrit à Karlsruhe pour l'ordinateur I.B.M. 360/65, résout l'équation de diffusion à trois dimensions au moyen d'une méthode de synthèse employant des fonctions d'essai à deux dimensions prédéterminées.

La première partie de ce rapport est consacrée au test de la méthode de synthèse des flux, employée dans KASY, par rapport aux résultats obtenus par un programme à trois dimensions. Les résultats considérés sont : k_{eff}, les réactivités de barres de contrôle et les cartes de flux et de taux de réaction.

Dans la seconde partie, la synthèse KASY est appliquée à l'interprétation d'un vaste programme d'expériences de barres de contrôle, dans un coeur de réacteur rapide contenant une ou plusieurs barres de contrôle placées en différentes positions.

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0. Introduction.

Written at Karlsruhe for the I.B.M. 360/65, the computer program KASY [1] solves the three-dimensional diffusion equation by means of a synthesis method using available two-dimensional trial functions. The method of solution is based on the variational process due to Kantorowitch and developed by Kaplan [2].

KASY determines k mixing functions Z_k of the third variable Z and constructs the three-dimensional flux functions $\Phi(XYZ)$ by means of the following linear combination :

$$\Phi(XYZ) = \sum_k Z_k(Z) \cdot H_k(XY)$$

where the k functions H_k are linearly independent trial functions of the two variables X and Y.

The mixing functions are determined as solutions of a system of equations analogous to the classical diffusion equation, in which the coefficients are averaged over the plane of the two variables XY.

These coefficients may be averaged with the adjoint flux H_k^* or the direct flux H_k as weighting function. The second function has been preferred here because of the associated computer time savings (this is referred to as Galerkin's method).

The complete details of the KASY program are given in the report [1]; it includes basic theory, method of solution, possible options, input data and comments on the use of the program.

The three possible geometries in the KASY program are XY/Z, Rθ/Z and RZ/θ. Only the XY/Z geometry is utilized in this report.

In principle, the trial functions H_k may be any mathematical functions of X and Y which obey the boundary conditions of the problem. In practice, however, such a function is a flux distribution ; it is the solution of the diffusion equation in two dimensions obtained by a 2-D diffusion program.

The neutron energy group index has been omitted for simplicity. In this report a 4 - or 5 - energy group scheme is used.

The first chapter of this report presents comparisons of KASY results with those from a direct three-dimensional diffusion program, TRITON [3]. The computational results taken for comparison are k_{eff}, reactivity differences Δk and maps of the fluxes and the fission rates. Cases with and without control rods, fully or partially inserted, in a central or off-center position are considered.

In the second chapter, a typical case of application of KASY is described : the evaluation of a series of control rod experiments performed in the fast critical assembly SNEAK at Karlsruhe [4].

1. Comparison with direct three-dimensional results.

1.1. Problems considered.

1.1.1. The 3-D synthesis performed by means of the KASY program is tested by comparing its results with those obtained by a direct 3-D diffusion program TRITON [3], in its version used at BELGONUCLEAIRE.

1.1.2. The geometrical model represented is XYZ.

1.1.3. Two different cases of application have been selected for consideration. These are the following :

A) insertion of a B_4C control rod (with a sodium-steel follower) at the center of ZPR-III-48, a 370 liter UC - PuC core [6] ;

B) insertion of a tantalum control rod at an off-center position in SNEAK-2C, a 880 liter UO_2 - PuO_2 core [4] .

1.1.4. The following results of calculation are compared : multiplication coefficients k_{eff} with a control rod full in, full out and half inserted, the reactivity changes Δk associated with the insertion of the control rod and the three-dimensional maps of fluxes and fission rates with and without the control rod presence in the core.

1.1.5. In the practical application of a synthesis method the problem arises as to choosing adequately the number and the nature of the trial functions. All trial functions used here are 2-D(XY) flux distributions, in 5 or 4 energy groups, generated by the 2-D diffusion program DIXY of Karlsruhe [5] . The trial functions considered in these tests may be grouped in three distinct categories : core functions, blanket functions and auxiliary functions.

1.1.6. The core functions are the solutions of normal 2-D(XY) problems, that is, flux distributions in a horizontal plane which characterizes one axial slice of the reactor. A core containing a control rod partially inserted may be divided in two axial slices : core containing absorber, core containing follower. These 2-D calculations are performed using transverse buckling values, which were selected as constant (region and energy independent).

1.1.7. The blanket functions are also relative to axial slices, but in the axial blanket section. The transverse buckling is set equal to zero. It is of course possible to improve the blanket trial functions by using space and energy dependent B_Z^2 values extracted from previous (RZ) calculations.

It is also possible to do the same for the core trial functions. However the main goal in these comparisons is to find convenient, quick, and systematical means of synthesis which give easily reproducible results practically independent of any assumption on auxiliary data such as the bucklings.

1.1.8. The auxiliary functions are generated by two-dimensional calculations in which fuel is placed in a particular radial region of the reactor, with the remaining space filled with an absorber. The idea is to generate a quasi step function, which when combined with a reference core function, would allow one to improve the synthesis in the region of interest.

1.1.9. In the first application (ZPR-III-48), core and blanket functions are used , while in the second application the three types of trial functions are used and tested.*

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*Sometimes only one trial function is used by KASY. Although trivial, such a synthesis has first of all the advantage of coherence. In addition the results are generally satisfactory.

The second application represents a rather severe test of the synthesis method because of the off-center position of the control rod and also of the low ratio of core height-to-diameter (< 0.5).

1.2. k_{eff} and reactivity changes with a central control rod.

1.2.1. Table I below gives the k_{eff} calculated by TRITON and KASY for the three cases relative to the insertion of a central control rod in ZPR-III-48. These are full insertion ("rod in"), half insertion ("rod half in") and non insertion ("rod out" or "follower in"). The reactivity changes corresponding to the half and full insertion of the rod are also compared.

Four different trial functions have been used in the synthesis :

1°) core with follower, $B^2 = 8.8\ 10^{-4}\ cm^{-2}$;
2°) core with absorber rod, $B^2 = 8.8\ 10^{-4}\ cm^{-2}$;
3°) blanket with follower, $B^2 = 0$;
4°) blanket with absorber rod, $B^2 = 0$.

The synthesis results are distinguished according to the combinations of trial functions. KASY (core functions) means that the core functions only were used (combinations : (1), (1,2), (2)) and KASY (core and blanket functions) means the combinations (1,3), (1,2,3), (2,4).

Table I. ZPR-III-48, comparison KASY-TRITON, k_{eff} and Δk.

Cases	TRITON	(DIXY)*	KASY (core functions)	KASY (core & blanket functions)
k_{eff}				
rod out	.9889	(.9959)	(1) .9883	(1,3) .9886
rod half in	.9782		(1,2) .9776	(1,2,3) .9779
rod in	.9683	(.9754)	(2) .9677	(2,4) .9680
$\Delta k\ (10^{-4})$				
rod out-rod half in	107		107	107
rod half in - rod in	99		99	99
rod out - rod in	206	(205)	206	206

*2-D results obtained with an estimated B^2.

1.2.2. The computer times needed for the three problems on the I.B.M. 360/65 were the following (the preparation of the cross-sections was not taken into account, common time < 1') :

TRITON : 79'
KASY (including the DIXY calculations) :
core functions : 5'30"
core and blanket functions : 7'40"

1.2.3. As far as k_{eff} is concerned, the synthesis using respectively core functions or core and blanket functions reproduce the TRITON results with systematic deviations :

- $6\ 10^{-4}$ (core functions) ;
- $3\ 10^{-4}$ (core and blanket functions).

1.2.4. As far as reactivity changes Δk are concerned, the KASY results are identical to the TRITON results within the convergence accuracy, but with a factor of 14 (core functions) or 10 (core and blanket functions) in computer time savings.

1.3. Flux and fission rate maps with a central control rod.

1.3.1. Table A.2. in the Appendix 2 gives as an example a general flux map for the ZPR-III-48 "rod out" case. The most typical mesh points have been considered, including points inside of homogeneous zones as well as points on the boundaries. The boundaries between the core, blanket and control rod have been doublelined.

The flux map is normalized to 1000 at its maximum value (group 2 at point (1,1,1)).

One may observe that the spectrum is very hard. In all the regions, groups 1, 2 and 3 (E > 10 keV) represent more than 90 % of the total flux. Spatially, the flux is rapidly decreasing through the blanket, roughly one magnitude for each 25 cm distance.

1.3.2. The flux maps obtained by TRITON and KASY are compared for the case "rod half in" in Tables A.3., A.4. and A.5. of the Appendix 2.

The same normalization as mentioned above was applied to both sets of results. The maximum flux occurs here in group 2 at the point (4,1,11), at the boundary between the core and the follower rod, slightly below the core midplane.

The Tables A.3., A.4. and A.5. give directly the relative deviations, $\frac{\Phi_{KASY} - \Phi_{TRITON}}{\Phi_{TRITON}}$, in percent.

1.3.3. Table A.6. gives, in the core and blanket regions, the relative deviations in the total fission rate. The total fission rate is considered the most interesting reaction rate as far as power reactor design is concerned. The values in Table A.6. are renormalized to the integral of the fission rate over the core volume.

In the control rod, the total capture rate is considered. This capture rate, mostly due to B^{10} in the absorber part, is related to the reactivity value of the control rod.

1.3.4. The examination of the deviations in rates is enhanced when one considers separately the different zones of the reactor, and the boundaries of the zones. It is clear for example, that the plane $Z = 5$, which contains the boundary core-axial blanket, is likely to represent a major difficulty for the synthesis process.

Nine zones or boundaries have been distinguished for this purpose :

1°) core, inner part ;

2°) core, inner and outer radial boundaries ;

3°) core, upper and lower planes ($Z = 5$ and 15) ;

4°) core, "corner points" (13,1,5 and 15) ;

5°) radial blanket, surrounding the core (height of the core) ;

6°) axial blanket, surrounding the core (radius of the core) ;

7°) "corners" of the blanket ;

8°) control rod (absorber or follower) in the core ;

9°) control rod (absorber or follower) in the blanket.

1.3.5. Table II below summarizes the largest deviations for the nine zones or subzones mentioned ; for the core and blanket regions (zones 1 to 7), deviations in the fission rate are presented ; for the control rod (zones 8 and 9), deviations in the capture rate.

Table II. ZPR-III-48 - Rod half in.
Largest deviations in the fission and capture rate :

$$\frac{F_{KASY} - F_{TRITON}}{F_{TRITON}} \text{ (in \%)}$$

Zones or boundaries	Largest deviations (in %)
	In the fission rate
1. Core, inner part	0.9
2. Core, inner and outer radial boundaries	2.4
3. Core, upper and lower planes (Z = 5 and 15)	1.6
4. Core, "corner points" (13,1,5 and 15)	7
5. Radial blanket, surrounding the core	6
6. Axial blanket, surrounding the core	10
7. Blanket corners	66
	In the capture rate
8. Control rod in the core	1
9. Control rod in the blanket	6

1.3.6. In the considered case, the synthesis gives relatively bad results in the blanket corners and at the corner point of the core. If one excludes these zones, the comparison gives satisfactory results for KASY.

1.4. k_{eff} and reactivity changes with an off-center control rod.

1.4.1. In this second test (SNEAK-2C), many different trial functions have been utilized in the synthesis. These are :

1°) core with a tantalum rod, $B^2 = 1.16\ 10^{-3}\ cm^{-2}$

2°) a reference core (without control rods), $B^2 = 1.16\ 10^{-3}\ cm^{-2}$

3°) blanket with a tantalum rod, $B^2 = 0$

4°) reference blanket, $B^2 = 0$

5°), 6°), 7°), 8°) auxiliary trial functions, $B^2 = 0$

The auxiliary trial functions correspond to the presence of :

5°) core zone Z_1 (SNEAK) in the rod position ;

6°) core zone Z_1 (SNEAK) in core zone 1;

7°) core zone Z_2 (SNEAK) in core zone 2;

8°) blanket material in radial blanket ;

surrounded each time by an absorber (tantalum) elsewhere.

The purpose of these functions is, as outlined in paragraph 1.1.8., to emphasize in the synthesis a particular region of the horizontal cross-section of the core. These trial functions are also used as weighting functions of the diffusion parameters D and Σ in the synthesis. The coefficients weighted by such functions (and leading to a particular set of mixing functions Z_k) are approximately representative of the region of interest.

Different syntheses have been performed, combining one, two or three trial functions among those available.

1.4.2. Table III below gives the k_{eff} calculated by TRITON and KASY, for the two considered cases : reference and control rod full inserted. The reactivity changes corresponding to the replacement of fuel by absorber (tantalum) are also compared.

The synthesis results are distinguished according to the combinations of trial functions which were used.

Table III. SNEAK-2C, comparison KASY-TRITON, k_{eff} and Δk.

Methods	k_{eff} reference	k_{eff} Rod in	Δk (10^{-4})
TRITON	.9770	.9600	170
(DIXY)*	(.9813)	(.9639)	(174)
KASY**			
1 trial function	(2) .9764	(1) .9594	170
2 trial functions	(2,4) .9768	(1,3) .9598	170
	(2,6) .9768	(1,6) .9598	170
	(2,7) .9767		
	(2,8) .9766		
		(1,5) .9595	
3 trial functions	(2,4,6) .9769	(1,3,6) .9599	170
	(2,4,8) .9768	(1,3,8) .9598	170
		(1,2,3) .9598	

* 2-D results obtained with an estimated B^2.

** The numbers of the trial function used are indicated in brackets.

1.4.3. One observes from table III that in spite of the approximate B^2 values used in the DIXY 2-D calculations, which give a 0.4 % too high k_{eff}, the k_{eff} obtained by synthesis are very good.

As an additional test the constant B^2 of 1.16 10^{-3} cm^{-2} was modified to 1.13 10^{-3} cm^{-2} in the DIXY calculations. With these DIXY fluxes, the synthesis was repeated with one and two trial functions (2,4 and 1,3).

The influence of the 3 % difference in B^2 on the k_{eff} given by synthesis was found to be smaller than 10^{-5} Δk. This confirms a similar conclusion already obtained in [1] for other core configurations.

It is an advantage of the synthesis technique that approximate B^2 values in the 2-D calculations are sufficient.

1.4.4. As far as k_{eff} is concerned, the synthesis using, respectively 1, 2 or 3 trial functions reproduce the TRITON results with systematic deviations :

- 6 10^{-4} (one trial function) ;
- 2 10^{-4} (two trial functions, with the best combination) ;
- 1 10^{-4} (three trial functions, with the best combination).

1.4.5. The best single T.F.* is unambiguosly fixed. It is the core function which corresponds to the considered case (reference or rod in).

The best 2 T.F. combination associates the core function with either the appropriate blanket function (3 or 4), or the auxiliary T.F. Nr 6, which emphasizes the central core zone Z_1. Emphasizing other zones, namely the control rod instead of the central zone, does not give as good results.

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*T.F. = trial function.

The best 3 T.F. combination is the appropriate core and blanket functions and the auxiliary function typical of the central core zone. Substituting the latter by other T.F. gives slightly worse results.

1.4.6. As far as reactivity changes Δk are concerned, KASY reproduces the TRITON results within the convergence accuracy.

1.4.7. Such remarkable results correspond to the following running times :

TRITON (reference and rod in, respectively 8 and 16 iterations) : 90'

KASY (reference and rod in, including the DIXY calculations[**])

one T.F.	5'30"
two T.F.	8'30"
three T.F.	9'

1.5. Flux and fission rate maps with an off-center control rod.

1.5.1. Table A.7. in Appendix gives a general flux map for the SNEAK-2C "rod in" case. The most interesting mesh points are considered. The boundaries between core, blanket and control rod are doublelined.

The flux values are normalized to 1000 at the maximum (group 3 at point 1,22,13).

[**] Detail of the DIXY times : T.F. 1 (18 iterations): 2'20"
T.F. 2 (14 iterations): 1'50"
T.F. 3+4 (8 iterations):1'
T.F. 5+6+7+8 (11 iterations) : 1'20"

Detail of the KASY times : one T.F. (13 iterations) : 40"
two T.F. (19 iterations) : 1'
Three T.F. (22 iterations) : 1'20"

Which respect to the first test (ZPR-III-48) a larger number of points in the blanket are considered, and namely points placed on the most outer boundaries of the system, where the fluxes are relatively small (about a factor 50 smaller than on the outer core boundary).

The neutron spectrum in the core and blanket is approximately as hard as in the preceding test. The groups 1, 2 and 3 ($E > 21.5$ keV) comprise about 90 % or more of the total flux.

1.5.2. The flux maps[*] obtained by TRITON and KASY are compared for the case "rod in" for all the groups (1 to 4) in Tables A.8., A.9., A.10. and A.11. of the Appendix. The values in the tables are the relative deviations $\frac{\Phi_{KASY} - \Phi_{TRITON}}{\Phi_{TRITON}}$, in percent.

1.5.3. Table A.12. of the Appendix gives the relative deviations in the fission rate in core and blanket regions after renormalisation to the integral of the fission rate over the core volume. In the control rod the capture rate is examined.

1.5.4. Three different KASY syntheses are compared with TRITON using respectively, one, two and three trial functions. These are the best combinations of trial functions found as far as k_{eff} and Δk are concerned, that is :

one trial function : (1)
two trial functions : (1,3)
three trial functions : (1,3,6)

1.5.5. As for the first test, the deviations have been examined by subdividing the system into zones and boundaries. The same nine zones or boundaries (see paragraph 1.3.4.) have been distinguished and, in addition, the upper and lower boundaries of the axial blanket (not examined for ZPR), which are outer boundaries for the system itself.

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[*] Here are presented results for the plane Y=22. The perpendicular plane X=14 was also considered in detail ; the deviations were found similar.

1.5.6. Table IV below summarizes the largest deviations in each subzone. In the core and blanket regions deviations in the fission rate are presented. In the control rod, deviations in the capture rate are given.

<u>Table IV</u>. SNEAK-2C, rod in,

Largest deviations in the fission or capture rate : $\frac{F_{KASY} - F_{TRITON}}{F_{TRITON}}$ (in %).

Zones	Largest deviations (in %)		
	1 trial function	2 trial functions	3 trial functions
	On the fission rate		
1. Core, inner part	2.5	2.4	1.0
2. Core, inner and outer radial boundaries	2.9	2.9	1.2
3. Core, upper plane (Z = 6)	4.5	4.6	4.3
4. Core, "corner point" (21,22,6)	6.4	9.4	7.8
5. Radial blanket, surrounding the core	4	8	3
6. Axial blanket, surrounding the core	10	10	11
6. bis. Axial blanket, upper plane	53	59	55
7. Blanket corners	60	68	94
	On the capture rate		
8. Control rod, core section	1.4	1.5	0.8
9. Control rod, blanket section	12	12	14

1.5.7. Here again the synthesis gives relatively bad results in the blanket corners and at the corner point of the core. This is also the case for the upper plane of the axial blanket. Excluding these zones the results are generally satisfactory, as illustrated in Figure 6 which represents, superimposed on the geometrical reactor model, the lines of constant 1 %, 5 %, 20 % and 50 % deviations for the synthesis using three trial functions.

1.5.8. With the combinations of trial functions formed, the use of several trial functions does not result in any substantial improvement in the crucial zones mentioned.

In the core regions, and to a lesser extent in the blanket regions (inner part), one observes, in the considered cases, no real change when passing from one to two trial functions, but a significant improvement with three functions.

2. Application to an experimental evaluation.

2.1. Description of the problem.

2.1.1. The series of SNEAK-2 experiments [4], performed from June to November 1969, was the first simulation in SNEAK of a sodium-cooled two-zone core of the SNR* type. In a portion of this program, SNEAK-2C, reactivity changes and fission rate traverses were measured in the core with various configurations of absorber rods (B_4C,Ta) and/or followers (Na,Al). The traverses were measured in the radial, axial and azimuthal directions. The control rods were located either at the center or near the boundary between two core zones. In the other positions, three rods were present simultaneously in order to study the shielding effects.

The SNEAK-2C core configuration resulted from the substitution of a sector zone, containing plutonium, into a uranium fuelled core (SNEAK-2A) : see Figure 3 for a typical representation. Throughout the SNEAK-2C experiments, 5 different core configurations, differing by the radial size of the core, were loaded. The larger radial core size was necessary in order to compensate for the relatively high negative reactivity of the absorbers used (up to 3 % Δk).

The report [4] describes in detail the principles and purposes of these control rod experiments ; it gives the sequence of the measurements and reports on the complete evaluation.

2.1.2. A portion of the measured quantities could be calculated using a two-dimensional geometrical model. However, the adequate representation of partially inserted control rods requires the three dimensional capabilities of KASY.

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*SNR : Schneller Natriumgekühlter Reaktor.

The synthesis method was applied to the entire series of experiments in SNEAK-2C (see [4]). In this report only three particular cases of application are selected and considered in detail. They are :

- reactivity changes associated with a single Al-Ta control rod in outer position 1 ;
- reactivity changes and fission rate traverses associated with three half inserted B_4C control rods, placed on outer positions 1, 2, 3 ;
- characteristic curve of reactivity versus insertion depth for a single B_4C control rod, in inner (but not central) position 4, when the three positions 1, 2, 3 are occupied by full inserted B_4C.

2.1.3. As mentioned in Chapter 1, all the reactivity changes are calculated to an accuracy of 10^{-4} or greater.

The accuracy of the fission traverses as calculated with KASY is given in Chapter 1.

In the following applications, the simplest means of synthesis was chosen : one trial function (full or no insertion of the control rod) or two trial functions (partial insertion of the control rod).

With respect to a direct three-dimensional calculation, the calculation by KASY of a radial fission rate traverse in the core midplane is accurate to within :

1 % in the central core zone ;
1 % in the outer core zone ;
4 % in the radial blanket,

if the traverse is normalized to a unit core average value. The capture rate in an off-center control rod is reproduced within 1 % in the core section.

An axial traverse in a core zone 2 (see Fig. 2) close to an off-center control rod is calculated to within the same accuracy, if one excludes the boundaries, and to within 4 % on the core-blanket boundary.

An azimuthal traverse, between off-center control rods, is calculated as accurately as a radial traverse.

For all points of the fission traverses located within the core or the control rod, if one renormalizes the axial traverses to their core average value[*], the maximum deviation due to the synthesis approximation is 1 %.

2.1.4. No reference to the corresponding results of measurements will be made here. The aim of this report is to assess the applicability of a mathematical tool used to solve control rod design problems in complex geometries.

Clearly the synthesis calculations reported here are limited by the diffusion approximation on one side, and are affected by the errors of the basic cross-sections on the other side.

A full comparison between calculation and experiment for the SNEAK-2C control rod series may be found in Chapter 7 of the report [4] .

2.2. Case one : reactivity changes with one control rod.

2.2.1. In the core of 494 elements, represented in Figures 3 and 4, one single control rod of 4 SNEAK elements (10.88 x 10.88 cm) is inserted in outer position 1. This is near the boundary between the two radial zones Z_1 and Z_2 of different enrichment.

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[*] Average over the core portion of the traverse.

The absorber consists of tantalum and the follower of aluminium ; see exact compositions in Table A.1. of the Appendix.

The cases calculated are :

1°) 2-dimensions (XY) :
   a) core with Ta in position 1 ;
   b) core with Al in position 1 ;

2°) 3-dimensions (XY/Z) : Ta full inserted, half inserted (designated Al-Ta) and not inserted (designated Al) in position 1.

2.2.2. Table V below gives the k_{eff} and Δk obtained. The simple difference $\Delta k = k_1 - k_2$ is retained as the reactivity corresponding to the insertion of the absorber.

Table V. SNEAK-2C, first case, k_{eff} and Δk.

k_{eff}			Δk (10^{-5})		
Cases	2-D(XY)	3-D(XY/Z)	Cases	2-D(XY)	3-D(XY/Z)
Al	.98393	.97908	Al → Al-Ta	-	203
Al - Ta	-	.97705	Al-Ta → Ta	-	172
Ta	.98015	.97533	Al → Ta	378	375

2.2.3. From the k_{eff} given in Table V one observes a systematical discrepancy 2-D/3-D of 0.5 % Δk which is due to the low B_Z^2 used in the 2-D calculations. The 3-D k_{eff} values have been checked for a reference case without control rod with a 2-D(RZ) calculation with a correction for cylindrisation, and found to be correct. This confirms the lack of influence of the B_Z^2 value used in the 2-D calculations on the final synthesis result (paragraph 1.4.3.).

2.2.4. The reactivity value Δk calculated by KASY for the half insertion of the absorber is 8 % larger than half the full insertion value. Such a departure from symmetry for the characteristic curve (explainable by the change of spectrum induced by the absorber at its lower end) may not be evidenced of course by simple 2-D(XY) results.

2.2.5. The three-dimensional flux maps from the KASY calculations have been used to evaluate the fission rate traverse measurements. A typical example of such an evaluation is shown in paragraph 2.3.

2.2.6. The running times were the following on the I.B.M. 360/65 :

preparation of cross-sections (with condensation) :	1'
two DIXY runs	12'
three KASY runs	
one using two trial functions	2'20"
two using one trial function	2'20"

The synthesis calculation represents here only one third of the time used for the preparation of cross-sections and for the two-dimensional calculations.

2.3. Case two : reactivities and traverses with three control rods.

2.3.1. In the core of 539 elements represented in Figure 5, three control rods of 4 SNEAK elements each, are placed in outer positions 1, 2 and 3, near the boundary between the two radial core zones Z_1/Z_2. The positions have a distance of 30 cm from each other (center-to-center).

The absorber consists of B_4C and the follower of Na ; see exact compositions in the Appendix, Table A.1.

The control rods in positions 2 and 3 are half inserted (Na-B_4C) while the central one is, respectively, not-, half- and full-inserted (Na, Na-B_4C and B_4C). The corresponding reactivity changes are calculated as well as the map of fission rates for the entire rod bank half inserted.

2.3.2. The necessary 2-D calculations are therefore :

1°) core with B_4C in positions 2 and 3, with alternately :
   a) Na in position 1 ;
   b) B_4C in position 1 ;

2°) core with Na in positions 2 and 3, with alternately :
   a) Na in position 1 ;
   b) B_4C in position 1.

2.3.3. Table VI below gives the reactivity changes Δk calculated by KASY (no corresponding results obtainable by DIXY).

Table VI. SNEAK-2C, case 2, Δk (10^{-5}).

Rods in positions 2,3	Changes in position 1	Δk (10^{-5}) 2-D(XY)	Δk (10^{-5}) 3-D(XY/Z)
Na-B_4C	Na→Na-B_4C	-	234
	Na-B_4C→B_4C	-	188
	Na→B_4C	-	422

2.3.4. One observes again from Table VI the same departure from symmetry of the reactivity versus insertion depth as already found in paragraph 2.2.

The use of a simple 2-D(XY) model is not able to answer such problems.

2.3.5. Fission rate traverses in the radial, azimutal and axial directions were also measured. The evaluation of these measurements with the synthesis method is illustrated in Figure 7.

One sees on this figure the radial distributions of the total fission rate, as calculated for the core midplane (transition plane between the three absorbers and their followers) by KASY, compared with the corresponding distributions through, respectively, the upper and lower core halves (core with B_4C and core with Na in positions 1,2,3) obtained in two-dimensional calculations.

2.4. Case three : characteristic curve of a control rod.

2.4.1. In the core of 539 elements of Figure 5, a complete B_4C characteristic curve of reactivity, $\delta k(h)$, is measured in position 4 an off-center position in the inner core zone Z_1.

The control rod is of the type Na-B_4C, and occupies only one SNEAK element (5.44 x 5.44 cm). The core also contains three B_4C rods fully inserted in positions 1, 2 and 3.

2.4.2. This is a case requiring a three-dimensional geometrical model, and moreover, best suited to a synthesis solution. Only two 2-D calculations are performed with, respectively B_4C and Na in position 4. The two trial functions so obtained are then combined in KASY with the axial position of the interface Na-B_4C as a variable parameter. The total computer time per point of the curve is small.

2.4.3. Table VII gives the reactivity changes calculated by KASY. The zero of the reactivity scale has been fixed at the core midplane which is in the linear portion of the characteristic curve. The curve obtained* is shown in Figure 8.

* On Figure 8, the measurements are also drawn. The reactivity changes are converted from Δk in ¢ using a β_{eff} value of 510 10^{-5} [4].

2.4.4. One interesting synthesis problem is raised here concerning two extreme cases (full B_4C or full Na). It seems consistent to always combine the two trial functions, but the extreme cases require only one trial function (B_4C or Na). The results obtained for the extreme cases with one or two trial functions are not identical ; but the difference, fortunately is small. The use of two functions throughout produces a smoother curve.

Table VII. SNEAK-2C, case three : characteristic curve.

Insertion depth of B_4C (cm) (B_4C is replacing Na downwards)		Δk (10^{-5}) obtained with 2 trial functions ; between brackets with one trial function
Upper blanket region :	0	- 204 (174)
	30.5	- 196 (170)
Core region :	39.545	- 163
	51.605	- 79.6
	60.65	0
	69.695	77.5
	81.755	155
	90.8	185 (199)
Lower blanket region :	121.3	188 (201)

2.4.5. The consistency of the results is noticeable in spite of their small magnitude. They represent very close values of k_{eff}. The accuracy of 10^{-5} has been attained with a limited computer time.

3. Conclusions.

3.1. The first chapter of this report has been devoted to the test of the 3-D flux synthesis method of the KASY program with respect to results obtained by a direct 3-D program TRITON. Both tests considered were representative of fast breeders.

The k_{eff} values obtained by KASY are quite satisfactory. They differ from the TRITON k_{eff} values by less than 10^{-3}, but with an appropriate choice of three trial functions in the synthesis method the difference is reduced to 10^{-4}.

Concerning the reactivity differences due to the insertion, partial or full, of a control rod in a center or off-center position, the Δk obtained by KASY are remarkably identical with the TRITON results within the convergence accuracy of 10^{-4}.

The results of comparison of flux and reaction rate maps indicate that, if one excludes blanket corners, the outer boundary of the axial blanket, or corner points of the core, the agreement is generally satisfactory.

3.2. In the second chapter, it has been shown how readily applicable the KASY synthesis is in the evaluation of a diversified series of control rod experiments, and particularly to reactivity changes and spatial rate traverses in a fast breeder core containing one or several control rods in various positions.

The reactivity changes are calculated very accurately. The portion of the fission traverses located within the core has an error not larger than 1 % due to the synthesis approximation.

The number of mesh points used in the KASY calculations was 7 times too large for the TRITON program in its present version.

KASY proves therefore to be a very flexible tool, requiring a quite acceptable computer time for the nature of the problems involved.

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- APPENDICES -

- A P P E N D I X 1 -

NUMERICAL DATA (COMPOSITION, GEOMETRY, PREPARATION).

A.1. The geometrical models used for the comparison KASY-TRITON (Chapter 1) are described in Table I below and represented in Figure 1 (ZPR-III-48) and Figure 2 (SNEAK-2C).

A.2. The model corresponding to ZPR-III-assembly 48 [6] is a quarter of a transversal section of the core. The entire core height is represented for the case of the control rod half in, and half the height (with a symmetry condition) is represented for the cases rod in and rod out.

The control rod, placed at the center, is relatively small. It occupies 0.18 % of the core section. The core has a volume of 370 liters and is surrounded by a 30-33 cm thick blanket in the radial and axial directions. A 10-15 cm stainless steel reflector was added around the system (with the flux equal to zero at the external boundary) in order to improve the boundary condition in the blanket.

The atom densities are given in table A.1. of the Appendix 2. The core contains UC - PuC fuel, with an enrichment of about 18 % (fissile to total U + Pu), mixed with Na and stainless steel. The blanket is depleted uranium and steel. The control rod contains 90 % enriched B_4C, mixed with Na and steel, with a Na and steel follower.

A.3. The SNEAK-2C model used for the comparison KASY-TRITON (Chapter 1) is a quarter of the core horizontal section. Half the height of the system is represented with a symmetry condition.

Two cases are calculated : a reference case, without control rods and a "rod in" case, with a tantalum control rod placed in an off-center position at the boundary of the two radial core zones of different enrichements. Such a rod, 10.88 cm x 10.88 cm, occupies 0.81 % of the core section. The core has a volume of 880 liter. In the simplified model used here, 60 % of the core are plutonium fuelled and 40 % uranium fuelled. There are two radial zones Z_1 and Z_2 in the plutonium zone, with respective enrichments of about 18 % and 25 % (fissile on total U + Pu). Moreover both plutonium zones consist each of two batches, made, respectively, of rodlets (MASURCA)* and platelets (SNEAK), but the homogenized compositions are approximately the same.

These compositions are given in Table A.1. of the Appendix 2. The core contains UO_2-PuO_2, steel and sodium, the blanket contains only depleted uranium. There is no outer reflector (outer boundary condition : extrapolated flux equal to zero). The absorber rod contains tantalum, mixed with aluminium and steel.

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* On loan from the MASURCA facility, Cadarache Center of the Commissariat à l'Energie Atomique, France.

<u>Table I.</u> Geometrical models
used for the comparison KASY-TRITON (Chapter 1).

A. ZPR-III-48 (fig. 1)

X or Y direction			Axial direction Z (lower half of the core)		
Designation	Outer boundary mesh point	Outer dimension (cm)	Designation	Outer boundary mesh point	Outer dimension (cm)
Control rod	4	2.77	Core	6	38.18
Core around control rod	6	8.31	Blanket	9	68.66
Core	8	13.85	Reflector	10	83.66
Core	13	41.55			
Blanket	18	74.79			
Reflector	19	85.87			

B. SNEAK-2C (fig. 2)

X direction			Y direction			Z direction (upper half of the core)		
Designation	Outer mesh point	Outer dimension (cm)	Designation	Outer mesh point	Outer dimension (cm)	Designation	Outer mesh point	Outer dimension (cm)
Z_1 SNEAK	8	38.08	Blanket	6	27.20	Blanket	6	27.20
Z_1 SNEAK around control rod	10	43.52	R_2	8	38.08	Core	7	30.71
Control rod	18	54.40	Z_2 MASURCA	10	48.96	Core	13	57.35
Z_2 SNEAK around control rod	20	59.84	Z_1 MASURCA	16	81.60			
Z_2 SNEAK	21	65.28	Z_1 SNEAK	22	92.48			
Blanket	26	92.48						

A.4. The preceding SNEAK-2C model was a simplified one, only used for the sake of the comparison KASY-TRITON in Chapter 1. In Chapter 2 one is concerned with two different core configurations effectively realized in the SNEAK-2C series. They consist of, respectively, 494 and 539 SNEAK core elements with equivalent radii of 68.1 and 71.1 cm. These configurations are represented in Figures 3 and 5.

For the sake of the calculations, the exact core configurations are replaced by calculation models symmetrical with respect to the North-South axis. Figure 4 shows the 180° model used to represent the actual configuration of the core with 494 elements as shown in Figure 3. Such a simplification and some other slight departures from reality have a negligible influence on the results of the calculations.

The comments on composition already made for the simplified model apply also to the exact models, except that the latter comprise a 150° plutonium sector in a uranium core, and that a portion of the radial blanket, referred to as a breeder blanket, is constructed of a mixture of UO_2 (depleted), sodium and steel. All control rods worths and traverses are measured in the plutonium sector and the traverses extend through the breeder blanket. The compositions of the control rods (B_4C,Ta) and their respective followers (Na,Al) are given with the core and blanket compositions in Table A.1. of the Appendix 2. The rods simulate safety or shim rods as currently envisaged for fast breeders.

The SNEAK-2C core is 60.3 cm high. The lower and upper axial blankets are 30.5 cm thick. The axial blanket is constructed of normal SNEAK blanket platelets (depleted uranium) except in the positions of the control rods. The absorber or follower portion of the control rods extends out of the core through the upper axial blanket.

Table II below describes the geometrical model used in the SNEAK evaluation, core with 494 elements (Chapter 2). The total number of mesh points used in KASY (50 x 32 x 33 = 52,800) greatly exceeds the maximum allowable number in the present version of TRITON (8000).

<u>Table II</u>. Geometrical model
used in the SNEAK evaluation (Chapter 2)
Core with 494 elements (Figure 3).

X direction			Y direction			Z direction		
Designation	Outer mesh point	Outer dimension (cm)	Designation	Outer mesh point	Outer dimension (cm)	Designation	Outer mesh point	Outer dimension (cm)
SNEAK blanket	6	27.20	SNEAK blanket	4	16.32	SNEAK blanket	7	30.5
R_2	10	48.96	SNEAK blanket	8	27.20	Core	27	90.8
R_1	13	65.28	R_2	16	48.96	SNEAK blanket	33	121.3
Z_1 MASURCA	16	81.60	R_1	22	65.28			
Z_1 SNEAK	34	130.56	Z_1 MASURCA	28	81.60			
Z_1 SNEAK Around control rod	36	136.00	Z_1 SNEAK	32	92.48			
Control rod	40	146.88						
Z_2 SNEAK around control rod	42	152.32						
Z_2 SNEAK	44	157.76						
Breeder blanket	50	184.96						

A.5. In the ZPR-III case, the cross-sections used are 5 group condensed cross-sections,* obtained by weighting ABBN [7] 26-group sections with one-dimensional fluxes. The 5 broad groups comprise the following fine groups :

Fine groups	Energy structure
1 to 5	10.5 MeV - 0.8 MeV
6 to 8	800 keV - 100 keV
9 to 11	100 keV - 10 keV
12 to 14	10 keV - 1 keV
15 to 26	1 keV - 0

In the SNEAK-2C case, the cross-sections used are 4 group condensed cross-sections,* obtained by weighting NAP-RMB [8] 26-group cross-sections with one-dimensional fluxes. The 4 broad groups comprise the following fine groups :

Fine groups	Energy structure
1 to 4	10.5 MeV - 1.4 MeV
5 to 6	1.4 MeV - 0.4 MeV
7 to 10	0.4 MeV - 21.5 keV
11 to 26	21.5 keV - 0

A.6. According to recommendations originating from the control rod study reported in [9,10], the control rod (absorber and follower part) was each time surrounded by a fuel zone of influence, of normal core composition but having few group cross-sections condensed using as weighting spectrum that of the considered subzone. Such creation of two different few group section sets for the core, respectively unperturbed and perturbed by the control rod, allows one to minimize the errors due to the group condensation.

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*Few group cross-sections were used in order to limit the computer time. But there is no limitation on the number of groups in KASY.

Another recommendation of [9,10] has been followed but only for the comparison KASY-TRITON (Chapter 1) : the mesh spacing has been refined in the control rod and in its close vicinity.

A.7. In the two-dimensional calculations for the synthesis, the transverse bucklings, energy and region independent, (see paragraph 1.1.7.) were :

ZPR-III-48 (76.36 cm core height) : $8.8\ 10^{-4}\ cm^{-2}$
SNEAK-2C (60.3 cm core height) : $1.16\ 10^{-3} cm^{-2}$

A.8. The convergence criterion used in all the calculations was 10^{-3} on half the difference between the maximum and minimum k_{eff} values. This corresponds to a higher accuracy in k_{eff}, which was the following :

3-D TRITON : $< 10^{-4}$
2-D DIXY : $< 10^{-4}$
(preparation of KASY)
3-D synthesis KASY : $< 10^{-5}$

- A P P E N D I X 2 -

T A B L E S.

List of the Tables of Appendix 2.

TABLE A.1.

Atomic compositions (10^{22} at/cm^3)

Part A : ZPR-III-48					
	1	2	3	4	5
Elements	Absorber rod	Core	Blanket	Reflector	Follower rod
U^{235}		.0016	.0083		
U^{238}		.7427	3.9798		
Pu^{239}		.1645			
Pu^{240}		.0106			
Pu^{241}		.0011			
Cr	.7189	.2681	.1193	.1264	.3512
Fe	2.6775	.9985	.4444	.4710	1.3080
Ni	.3537	.1330	.0587	.0622	.1728
Mo		.0206			
C	.6350	2.0767			
Na	.6263	.6231			1.6701
Al		.0109			
B^{10}	1.5486				
B^{11}	.1584				

TABLE A.1.

Part B : SNEAK-2C						
	1	2	3	4	5	6
Elements	Z_1 SNEAK (Typ 12)	Z_2 SNEAK (Typ 13)	Z_1 MASURCA	Z_2 MASURCA	R_1 (Typ 10)	R_2 (Typ 11)
U^{235}	.0044	.0037	.0019	.0012	.1876	.2533
U^{238}	.6022	.5083	.6469	.5277	.6511	.5850
Pu^{239}	.1276	.1684	.1210	.1614	-	-
Pu^{240}	.0115	.0151	.0112	.0149	-	-
Pu^{241}	.0010	.0014	.0011	.0014	-	-
Pu^{242}	.00005	.00007	.00013	.00018	-	-
Cr[1)]	.3234	.3970	.3442	.3485	.3581	.3181
Fe	1.1960	1.3098	1.2548	1.2786	1.2164	1.2082
Ni[2)]	.2281	.1997	.1749	.1699	.1867	.2360
O	1.2592	1.3949	1.1597	1.1597	.9022	1.0051
C	.0051	.0061	-	-	.3050	.2529
Na	.8975	.7109	.8856	.8856	.8733	.9518
Al[3)]	.0006	.2419	-	-	-	-
Si	.0153	.0192	-	-	.0164	.0147
Ti	.0008	.0021	-	-	.0026	.0016
H	-	-	-	-	.0022	.0018
B^{10}	-	-	-	-	-	-
B^{11}	-	-	-	-	-	-
Ta	-	-	-	-	-	-

1) Mn has been assimilated to Cr

2) Co has been assimilated to Ni

3) Mg has been assimilated to Al

TABLE A.1.

Part B : SNEAK-2C (continued)						
	7	8	9	10	11	12
Elements	B_4C rod	Na follower	Ta rod	Al follower	SNEAK blanket	Breeder blanket (Typ 17-2)
U^{235}	-	-	-	-	.0176	.0075
U^{238}	-	-	-	-	4.1840	1.0297
Pu^{239}	-	-	-	-	-	-
Pu^{240}	-	-	-	-	-	-
Pu^{241}	-	-	-	-	-	-
Pu^{242}	-	-	-	-	-	-
Cr 1)	.2552	.3170	.3505	.3505	.1063	.1824
Fe	.8293	1.0728	1.1789	1.1789	.3957	.6290
Ni 2)	.1176	.1845	.1633	.1633	.1717	.1040
O	-	-	-	-	-	2.0911
C	1.2096	.0043	.0029	.0029	-	.0045
Na	-	1.6611	-	-	-	.6644
Al 3)	.0504	-	2.5868	4.3865	-	.5200
Si	.0172	.0173	.0365	.0505	-	.0108
Ti	-	-	.0117	.0167	-	.0005
H	-	-	-	-	-	.0009
B^{10}	.9449	-	-	-	-	-
B^{11}	3.8762	-	-	-	-	-
Ta	-	-	1.6537	-	-	-

1) Mn has been assimilated to Cr

2) Co has been assimilated to Ni

3) Mg has been assimilated to Al

TABLE A.2.

ZPR-III-48 , rod out, flux map
(For each mesh-point, 5 group fluxes, 1 to 5)

Z \ X	1	4	7	10	12	13	15	17
1	477 1000 539 147 35	481 1000 539 145 36	474 960 519 136 30	363 755 410 106 22	215 526 280 64 12	129 416 207 37 6.5	20 158 68 6.4 0.9	2.9 45 19 1.3 0.1
2	464 973 525 143 35	468 973 525 142 34	461 935 505 132 28	353 734 399 104 21	209 510 273 62 12.	125 404 202 35 6.3	19 153 66 6.2 .8	2.8 43 18 1.3 .1
3	424 897 485 132 32	427 897 485 130 31	421 861 466 122 27	322 676 368 96 20	191 469 251 57 1.1	114 331 185 33 5.8	17 141 61 5.7 .8	2.5 39 17 1.2 .1
4	357 778 421 113 27	360 778 421 112 27	356 745 409 104 22	272 584 319 81 17	161 404 217 49 9.	96 319 160 28 4.9	15 120 52 4.9 .7	2.1 34 15 1.0 .1
5	264 632 338 86 20	267 632 337 65 20	264 603 324 79 16	201 471 254 61 12	118 322 173 37 7.	70 251 125 21 3.6	11 94 41 3.8 .5	1.6 27 12 .8 .1
6	147 483 241 48 10.1	147 484 241 47 9.6	140 466 228 41 7.3	106 361 177 31 5.3	61 239 115 18 3.0	26 177 83 11 1.6	6.4 66 16 2.4 .3	1.1 19 8.6 .6 < .1
7	40 236 109 14.7 2.7	39 234 108 13.8 2.5	31 211 92 9.9 1.5	23 160 70 7.3 1.0	13 105 46 4.0 0.6	8.4 77 34 3.0 0.3	2.3 33 15 .1 .1	0.5 10 4.7 < .1 < .1
8	9.7 89 41 .4 .7	9.4 88 40 .4 .6	6.9 75 39 .3 .3	4.9 56 24 .2 .2	2.9 37 16 .1 .1	2.0 27 12 0.1 0.1	0.7 13 5.8 < .1 < .1	0.2 4.4 2.0 < .1 < .1

Boundaries are doublelined (in fact X = 4 or 13, for example, are on the boundaries)

TABLE A.3.

ZPR-III-48, rod half in, flux of group 1

Deviations KASY - TRITON : $\frac{\Phi_K - \Phi_T}{\Phi_T}$ (in %)

	Z \ X	1	4	7	10	12	13	15	17
	3	- 3	+ 0.9	+ 8	+ 6	+ 0.3	- 8	-58	-75
	4	- 7	- 4	+ 2	- 0.3	+ 0.6	- 4	-45	-63
	5	- 0.3	+ 0.1	+ 0.1	- 0.2	+ 1.4	+ 4.8	-10	-20
	6	+ 0.6	+ 0.4	-	+ 0.3	+ 1.0	+ 1.7	+ 2	+ 1
	7	+ 0.4	+ 0.4	+ 0.2	+ 0.5	+ 0.9	+ 1.3	+ 2	+ 4
	8	+ 0.6	+ 0.4	+ 0.2	+ 0.6	+ 0.9	+ 1.2	+ 2	+ 4
	9	+ 0.8	+ 0.5	+ 0.3	+ 0.6	+ 1.1	+ 1.3	+ 2	+ 4
Core midplane	10	+ 0.2	+ 0.4	+ 0.5	+ 0.5	+ 1.0	+ 1.3	+ 2	+ 5
	11	+ 0.2	+ 0.4	+ 0.7	+ 0.6	+ 0.9	+ 1.5	+ 2	+ 6
	12	+ 0.3	+ 0.5	+ 0.6	-	+ 0.9	+ 1.6	+ 4	+ 7
	13	+ 0.5	+ 0.4	+ 0.5	+ 0.5	+ 1.0	+ 1.5	+ 4	+ 7
	14	+ 0.6	+ 0.4	+ 0.2	+ 0.3	+ 1.2	+ 2.5	+ 4	+ 5
	15	- 1.0	- 0.8	+ 0.6	- 0.2	+ 1.5	+ 5.4	- 8	-16
	16	- 9	- 8	+ 4	+ 0.3	- 0.4	- 4	-43	-60
	17	- 6	- 6	+11	+ 5	+ 0.1	- 9	-57	-77

TABLE A.4.

ZPR-III-48, rod half in, flux of group 2

Deviations KASY - TRITON : $\frac{\Phi_K - \Phi_T}{\Phi_T}$ (in %)

Z \ X	1	4	7	10	12	13	15	17
3	-	+ 9	+ 6	+ 7	+11	+18	- 4	-22
4	+ 5	+ 4	- 1.0	- 1.2	+ 4	+12	- 1	-13
5	+ 7	+ 5	- 1.5	- 1.5	+ 1.8	+ 9.5	+10	+ 5
6	+ 1.1	+ 0.7	- 0.3	- 0.1	+ 1.4	+ 3.0	+ 4	+ 5
7	+ 0.5*	+ 0.4	+ 0.1	+ 0.3	+ 1.1	+ 1.6	+ 2	+ 4
8	+ 0.7	+ 0.4	+ 0.2	+ 0.6	+ 1.0	+ 1.3	+ 2	+ 4
9	+ 1.0	+ 0.7	+ 0.3	+ 0.7	+ 1.0	+ 1.2	+ 2	+ 3
Core midplane 10	- 0.1	+ 0.3	+ 0.5	+ 0.7	+ 0.9	+ 1.2	+ 1	+ 2
11	- 0.4	0*	+ 0.6	+ 0.7	+ 0.8	+ 1.0	+ 1	+ 2
12	- 0.1	-	+ 0.6	+ 0.7	+ 0.9	+ 1.2	+ 1	+ 1
13	+ 0.1	+ 0.1	+ 0.5	+ 0.6	+ 1.1	+ 1.7	+ 1	+ 1
14	+ 0.7	+ 0.4	+ 0.1	+ 0.1	+ 1.4	+ 3.2	+ 3	+ 1
15	+ 5	+ 3	-	- 0.2	+ 1.9	+ 9.6	+ 9	+ 1
16	+ 3	+ 1	+ 0.7	- 0.8	+ 3	+11	- 3	-20
17	+ 9	+ 7	+ 9	+ 7	+10	+16	- 8	- 31

*Normalisation point

TABLE A.5.

ZPR-III-48, rod half in, flux of group 3.

Deviations KASY - TRITON : $\frac{\Phi_K - \Phi_T}{\Phi_T}$ (in %)

Z \ X	1	4	7	10	12	13	15	17
3	+ 3	+ 6	+ 6	+ 7	+11	+ 9	-24	-37
4	- 1	+ 1	+ 0.1	+ 2	+ 5	+ 5	-19	-28
5	+ 1.8	+ 2.0	- 1.0	- 0.5	+ 2.8	+ 7.1	+ 1	- 2
6	+ 0.5	+ 0.2	- 0.3	+ 0.2	+ 1.6	+ 2.6	+ 3	+ 4
7	+ 0.4	+ 0.1	- 0.1	+ 0.4	+ 1.3	+ 1.7	+ 2	+ 4
8	+ 0.7	+ 0.3	- 0.1	+ 0.7	+ 1.0	+ 1.4	+ 2	+ 5
9	+ 1.7	+ 0.8	- 0.1	+ 1.2	+ 1.3	+ 1.3	+ 2	+ 4
Core midplane 10	-	+ 0.8	+ 0.5	+ 0.7	+ 0.9	+ 1.1	+ 2	+ 3
11	- 0.1	+ 0.4	+ 1.1	+ 0.6	+ 0.7	+ 0.9	+ 1	+ 2
12	+ 0.4	+ 0.4	+ 0.7	+ 0.5	+ 0.7	+ 1.2	+ 2	+ 2
13	+ 0.5	+ 0.4	+ 0.4	+ 0.5	+ 1.1	+ 1.7	+ 2	+ 3
14	+ 0.8	+ 0.3	- 0.2	+ 0.3	+ 1.8	+ 3.0	+ 3	+ 2
15	+ 1.7	+ 0.5	- 0.9	- 0.3	+ 3.0	+ 7.5	+ 1	- 3
16	+ 0.1	- 2	+ 0.8	+ 2	+ 5	+ 6	-19	-30
17	+ 7	+ 4	+ 7	+ 9	+11	+ 9	-22	-40

TABLE A.6.

ZPR-III-48, rod half in, fission rate (in core and blanket regions) capture rate (in control rod region)

Deviations KASY - TRITON : $\frac{F_K - F_T}{F_T}$ (in %)

Z \ X	1	4	4	7	10	12	13	15	17
3	+ 3	+ 6	+ 2	+ 8	+ 6	+ 2	- 7	-46	-63
4	- 1	+ 1	- 3	+ 2	- 1	+ 2	- 5	-36	-52
5	+ 1.0	+ 1.2	+ 1.5	- 1.6	- 1.5	+ 1.1	+ 6.6	- 8	-16
6	- 0.3	- 0.6	- 0.4	- 0.4	- 0.9	+ 0.5	+ 2.3	+ 2	+ 2
7	- 0.4	- 0.7	- 0.5	- 0.7	- 0.4	+ 0.3	+ 1.0	+ 2	+ 4
8	- 0.1	- 0.5	- 0.4	- 0.7	- 0.2	+ 0.2	+ 0.8	+ 2	+ 4
9	+ 0.9	-	- 0.1	- 0.6	-	+ 0.3	+ 0.8	+ 2	+ 4
Core midplane 10	- 0.8	-	- 0.3	- 0.3	- 0.2	+ 0.1	+ 0.7	+ 2	+ 4
11	- 0.9	- 0.5	- 0.5	-	- 0.2	-	+ 0.6	+ 2	+ 5
12	- 0.6	- 0.5	- 0.5	- 0.2	- 0.4	+ 0.1	+ 0.8	+ 4	+ 6
13	- 0.4	- 0.5	- 0.5	- 0.3	- 0.3	+ 0.3	+ 1.1	+ 4	+ 6
14	- 0.1	- 0.4	- 0.4	- 0.7	- 0.6	+ 0.7	+ 2.4	+ 4	+ 5
15	+ 2.1	+ 0.1	+ 0.2	- 0.9	- 1.0	+ 0.7	+ 7.0	- 6	-13
16	- 2	- 3	- 8	+ 4	-	-	+ 5	-34	-51
17	+ 3	+ 2	- 5	+10	+ 6	+ 2	- 8	-45	-66

TABLE A.7.

SNEAK-2C, rod in, flux map
(4 group fluxes, 1 to 4)

Z \ X	1	5	10	14	20	21	23	26
1	.25 4.2 13 .80	.23 3.9 12 .73	.18 2.8 11 .63	.17 2.4 11 .65	.10 1.6 5.7 .33	.07 1.2 4.2 .25	.03 .57 2.0 .12	.003 .061 .14 .065
3	2.1 28 125 10	2.0 26 115 9.2	1.4 17 87 6.2	1.2 15 80 5.4	.78 10 52 3.9	.55 7.8 39 2.9	.20 3.4 18 1.3	.02 .34 1.3 .72
5	17 100 371 40	16 92 341 37	10 59 243 22	8.4 49 217 19	5.8 36 148 14	3.7 27 112 10	.76 9.9 46.5 3.6	.05 .86 2.9 .17
6	55 168 586 89	51 154 537 82	28 98 373 44	20 77 316 32	19 61 234 29	11 46 180 20	1.4 15 66 5.4	.07 1.2 4.0 .23
8	104 229 756 152	96 210 694 139	56 141 505 74	37 114 432 52	37 88 315 48	23 70 255 33	2.4 23 94 7.9	.10 1.7 5.5 .32
10	138 284 907 194	128 261 833 178	74 177 616 95	49 145 527 67	49 112 387 62	31 90 317 43	3.2 29 117 10	.13 2.2 6.9 .40
13	157 318 1000 217	145 293 919 199	85 199 678 107	56 163 585 75	55 126 430 69	35 102 353 48	3.6 33 132 11	.15 2.5 7.7 .45

TABLE A.8.

SNEAK-2C, rod in, flux of group 1

$$\text{Deviations KASY - TRITON} : \frac{\Phi_K - \Phi_T}{\Phi_T} \text{ (in \%)}$$

for the three synthesis considered, using respectively, 1, 2 and 3 trial functions.

Z \ X	1	5	10	14	20	21	23	26
1	+45 +41 +54	+46 +42 +53	+ 9 + 7 + 7	-22 -24 -28	+30 +40 +27	+13 +39 +29	-72 +13 +22	-89 +96 +121
3	- - + 5	- 2 - 4 + 1	-19 -20 -21	-36 -37 -40	- 3 + 2 - 6	-13 - 1 - 7	-47 -27 -20	-88 +45 +68
5	- 1 - 2 + 2	- 1 - 1 + 1	- 8 - 8 - 9	-27 -27 -29	+ 5 + 6 + 2	+ 3 + 8 + 2	-47 -22 -16	-64 +31 +61
6	- 0.8 - 0.6 + 0.2	- 0.4 - 0.4 + 0.2	+ 5.0 + 5.0 + 4.3	+ 1.0 + 1.0 - 0.5	+ 4.8 + 4.8 + 2.2	+10.8 +10.8 + 8.1	- 9 -14 -12	-23 -51 -29
8	+ 0.5 + 0.5 + 0.2	+ 0.4 + 0.3 -	+ 2.0 + 2.0 + 1.1	+ 3.0 + 3.0 + 1.9	+ 2.2 + 2.2 + 0.5	+ 2.6 + 2.6 + 0.9	+ 2 + 5 + 1	+ 3 +15 + 3
10	+ 0.6 + 0.6 + 0.2	+ 0.2 + 0.1 - 0.3	+ 1.5 + 1.5 + 0.5	+ 1.9 + 1.8 + 0.8	+ 1.8 + 1.6 + 0.4	+ 2.1 + 2.3 + 0.7	+ 2 + 3 + 1	+ 5 +12 + 5
13	+ 0.3 + 0.2 - 0.2	+ 0.7 + 0.3 - 0.2	+ 1.2 + 1.1 + 0.2	+ 1.4 + 1.4 + 0.4	+ 1.6 + 1.6 + 0.4	+ 1.7 + 1.7 + 0.3	+ 3 + 3 + 2	+ 4 + 6 + 3

TABLE A.9.

SNEAK-2C, rod in , flux of group 2

Deviations KASY - TRITON : $\frac{\Phi_K - \Phi_T}{\Phi_T}$ (in %)

for the three syntheses considered using respectively, 1, 2 and 3 trial functions

Z \ X	1	5	10	14	20	21	23	26
1	+52 +50 +63	+51 +49 +59	+44 +43 +41	+38 +37 +31	+61 +63 +50	+74 +80 +66	+19 +38 +41	-15 +24 +50
3	- 1 - 2 + 5	- 1 - 1 + 4	+ 1 + 1 -	- 3 - 3 - 7	+ 8 + 9 + 1	+16 +18 + 9	-14 - 8 - 6	-34 -22 - 4
5	- 2 - 1 + 2	- 2 - 1 + 1	+ 5 + 5 + 4	+ 4 + 4 + 1	+ 9 + 8 + 3	+19 +17 +11	+ 5 - 4 - 2	- 8 -30 -14
6	- 1.7 - 0.8 + 1.4	- 1.2 - 0.5 + 1.2	+ 6.9 + 7.3 + 6.6	+11.1 +11.1 +10.9	+ 8.3 + 7.2 + 3.3	+11.7 +11.4 +11.0	+17 + 3 + 3	+11 -29 -18
8	- 0.7 - 0.5 + 0.1	- 0.2 - 0.1 - 0.1	+ 1.8 + 2.0 + 2.2	+ 3.3 + 3.4 + 2.0	+ 4.1 + 3.9 + 1.5	+ 5.5 + 4.9 + 2.5	+ 7 + 4 + 2	+ 8 - -
10	+ 0.1 + 0.1 + 0.1	+ 0.3 + 0.3 + 0.1	+ 1.4 + 1.4 + 0.2	+ 1.6 + 1.6 + 0.4	+ 2.1 + 2.1 + 0.4	+ 2.7 + 2.6 + 1.0	+ 3 + 3 + 1	+ 5 + 4 + 2
13	+ 0.3 + 0.2 - 0.1	+ 0.3 + 0.2 - 0.2	+ 1.2 + 1.2 + 0.3	- 0.5 - 0.5 + 0.1	+ 1.7 + 1.8 + 0.4	+ 2.3 + 2.3 + 0.8	+ 2 + 3 + 1	+ 3 + 4 + 2

TABLE A.10.

SNEAK-2C, rod in, flux of group 3

Deviations KASY - TRITON : $\frac{\Phi_K - \Phi_T}{\Phi_T}$ (in %)

for the three syntheses considered using respectively, 1, 2 and 3 trial functions

Z \ X	1	5	10	14	20	21	23	26
1	+72 +71 +80	+74 +73 +80	+45 +45 +43	+21 +21 +18	+78 +80 +69	+99 + 102 +90	+56 +69 +70	+31 +57 +75
3	+ 1 - + 5	+ 1 + 1 + 4	- 1 - 1 - 2	- 7 - 7 - 9	+ 7 + 7 + 1	+17 +18 +11	- 6 - 2 - 2	-23 -16 - 5
5	- 1 - 1 + 2	- 1 - 1 + 1	+ 3 + 3 + 2	- - - 2	+ 9 + 8 + 4	+18 +17 +12	+ 6 + 1 + 1	- -12 - 3
6	- 1.4 - 0.6 + 1.2	- 1.2 - 0.2 + 0.9	+ 5.9 + 6.5 + 5.4	+ 8.1 + 8.2 + 6.3	+ 8.1 + 6.8 + 3.5	+15.6 +13.0 + 9.3	+18 + 6 + 6	+15 -12 - 5
8	- 0.8 - 0.3 + 0.6	- 0.4 - + 0.4	+ 1.9 + 2.1 + 1.2	+ 3.1 + 3.1 + 1.7	+ 4.6 + 4.2 + 1.8	+ 6.3 + 5.4 + 2.8	+ 8 + 4 + 3	+ 9 - 1 + 1
10	- 0.2 - 0.1 + 0.1	+ 0.1 + 0.1 + 0.1	+ 1.4 + 1.4 + 0.5	+ 1.9 + 1.9 + 0.7	+ 2.7 + 2.5 + 0.8	+ 3.1 + 2.9 + 0.9	+ 5 + 4 + 2	+ 4 + 2 + 2
13	0* 0 0	+ 0.2 + 0.2 -	+ 1.1 + 1.1 + 0.3	+ 1.5 + 1.5 + 0.3	+ 2.2 + 2.1 - 1.8	+ 2.3 + 2.3 + 0.6	+ 3 + 2 + 1	+ 3 + 3 + 2

* Normalisation point

TABLE A.11.

SNEAK-2C, rod in, flux of group 4

Deviations KASY - TRITON : $\frac{\Phi_K - \Phi_T}{\Phi_T}$ (in %)

for the three syntheses considered using respectively, 1, 2 and 3 trial functions

Z \ X	1	5	10	14	20	21	23	26
1	-81 -82 -82	-80 -81 -81	+23 +24 +23	-15 -10 -11	+69 +72 +70	+41 +78 +74	-31 +64 +57	-48 + 113 + 101
3	+ 8 + 3 + 3	+ 9 + 4 + 5	-13 -12 -13	-29 -25 -26	- 9 - - 1	-15 + 4 + 1	-56 - 4 - 7	-68 +18 +13
5	+ 3 + 1 + 1	+ 4 + 1 + 1	- 6 - 6 - 7	-21 -18 -19	- 2 + 3 + 2	- 6 + 4 + 2	-39 - 4 - 8	-47 +19 +13
6	+ 1.2 + 0.7 + 0.4	+ 1.6 + 1.1 + 0.8	+ 3.0 + 3.2 + 2.5	- 1.9 - 1.1 - 2.0	+ 1.8 + 3.2 + 2.0	+ 3.3 + 6.2 + 4.7	-11 + 1 - 2	-16 +12 + 5
8	- 0.2 - 0.2 - 0.2	+ 0.3 + 0.3 + 0.2	+ 1.5 + 1.6 + 0.8	+ 2.1 + 2.2 + 1.1	+ 2.4 + 2.7 + 1.1	+ 2.6 + 3.0 + 1.4	+ 2 + 4 + 2	+ 2 + 7 + 4
10	- 0.1 - 0.1 - 0.1	+ 0.1 + 0.1 - 0.1	+ 1.2 + 1.2 + 0.4	+ 1.7 + 1.7 + 0.6	+ 2.1 + 2.1 + 0.6	+ 2.5 + 2.6 + 1.0	+ 3 + 3 + 1	+ 4 + 5 + 3
13	- - - 0.1	+ 0.2 + 0.2 - 0.1	+ 0.8 + 0.8 -	+ 1.4 + 1.4 + 0.3	+ 1.8 + 1.8 + 0.4	+ 2.0 + 2.1 + 0.6	+ 3 + 3 + 1	+ 4 + 4 + 2

TABLE A.12.

SNEAK-2C, rod in, fission rate (in core and blanket regions)
Capture rate (in control rod region)

$$\text{Deviations KASY - TRITON} : \frac{F_K - F_T}{F_T} \text{ (in \%)}$$

for the three syntheses considered using respectively, 1, 2 and 3 trial functions.

Z \ X	1	5	10	10	14	20	21	23	26
1	+47 +44 +55	+48 +45 +54	+27 +26 +25	+38 +36 +35	+12 +13 +15	+53 +59 +47	+52 +68 +57	-13 +39 +43	-36 +34 +94
3	+ 1 - 1 + 5	- 2 - 1 + 3	-10 -10 -11	-12 -12 -13	-10 -11 -14	- 2 + 6 - 2	+10 + 8 + 2	-30 -14 -11	-57 +13 +30
5	- 1 - 1 + 2	- 1 - 1 + 1	- 2 - 2 - 3	+ 1 - 1 - 3	- 6 - 5 - 8	+ 6 + 7 + 3	+10 +12 + 7	-21 -11 - 8	-33 + 7 +26
6	- 2.5 - 1.9 + 0.1	- 1.7 - 1.5 - 0.1	+ 4.3 + 4.6 + 4.3	+ 4.0 + 4.6 + 3.6	+ 2.5 + 2.3 + 3.7	+ 4.5 + 4.3 + 2.2	+ 6.4 + 9.4 + 7.8	+ 9 - 6 - 5	- 9 -35 -20
8	- 2.0 - 1.6 - 0.6	- 1.7 - 1.4 - 0.7	+ 0.3 + 0.5 + 0.2	+ 0.3 + 0.5 + 0.2	+ 1.4 + 1.5 + 0.8	+ 2.5 + 2.4 + 0.5	+ 2.9 + 2.9 + 1.2	+ 3 + 4 + 1	+ 4 + 8 + 1
10	- 1.3 - 1.4 - 0.8	- 1.4 - 1.4 - 1.0	- 0.2 - 0.1 - 0.4	- 0.2 - 0.1 - 0.4	+ 0.2 + 0.3 + 0.2	+ 0.8 + 0.7 - 0.3	+ 1.2 + 1.2 -	+ 2 + 2 + 1	+ 4 + 7 + 3
13	- 1.5 - 1.4 - 1.0	- 1.3 - 1.3 - 1.0	- 0.6 - 0.5 - 0.7	- 0.5 - 0.4 - 0.7	- 0.6 - 0.3 - 0.7	+ 0.4 + 0.4 - 0.4	+ 0.5 + 0.6 - 0.3	+ 2 + 2 + 1	+ 3 + 4 + 2

List of figures.

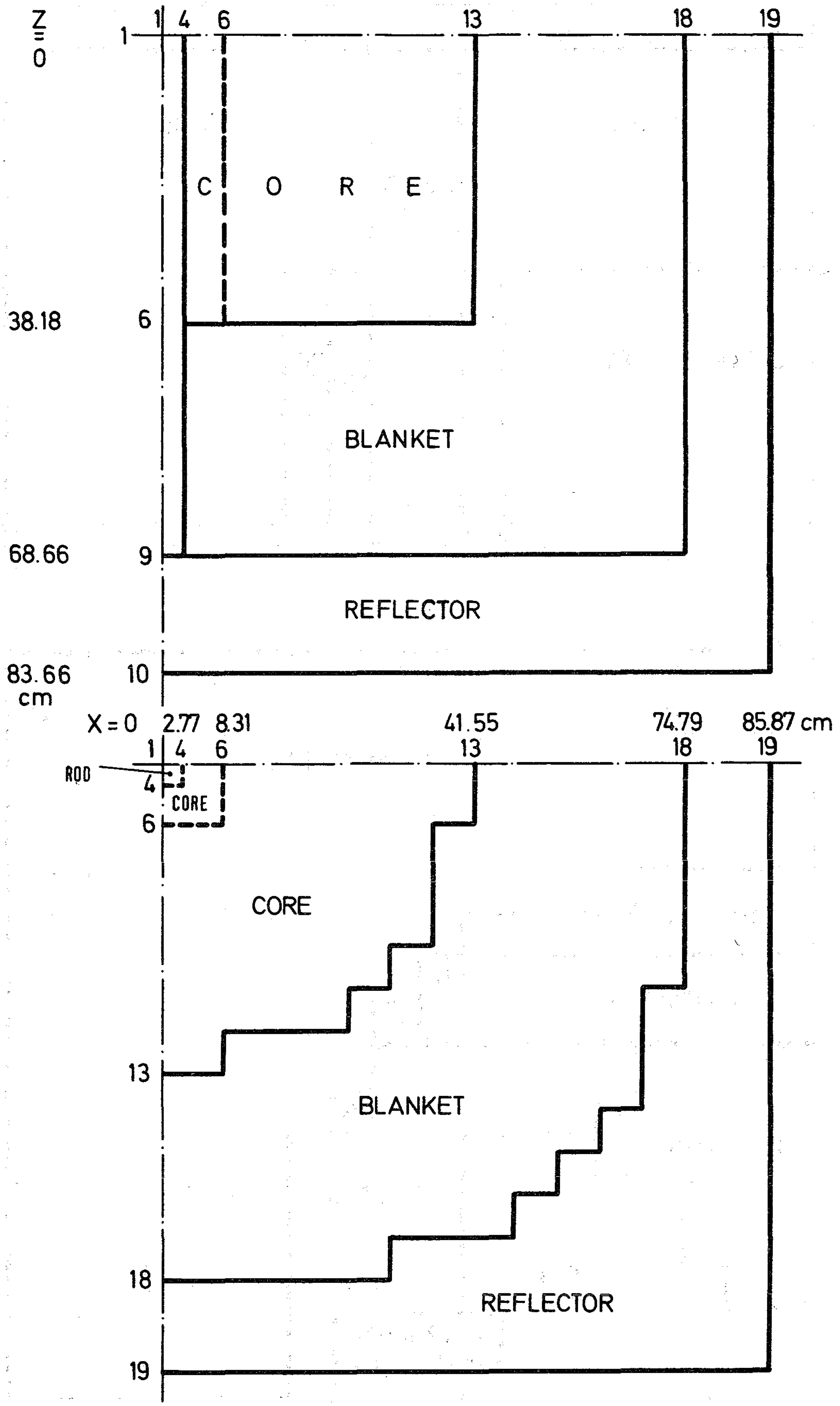


Fig.1 Geometrical model ZPR-III-48
used for the comparison KASY - TRITON

above : transversal section in the core midplane (Z =1 or 10)
below : axial section in the plane Y = 1

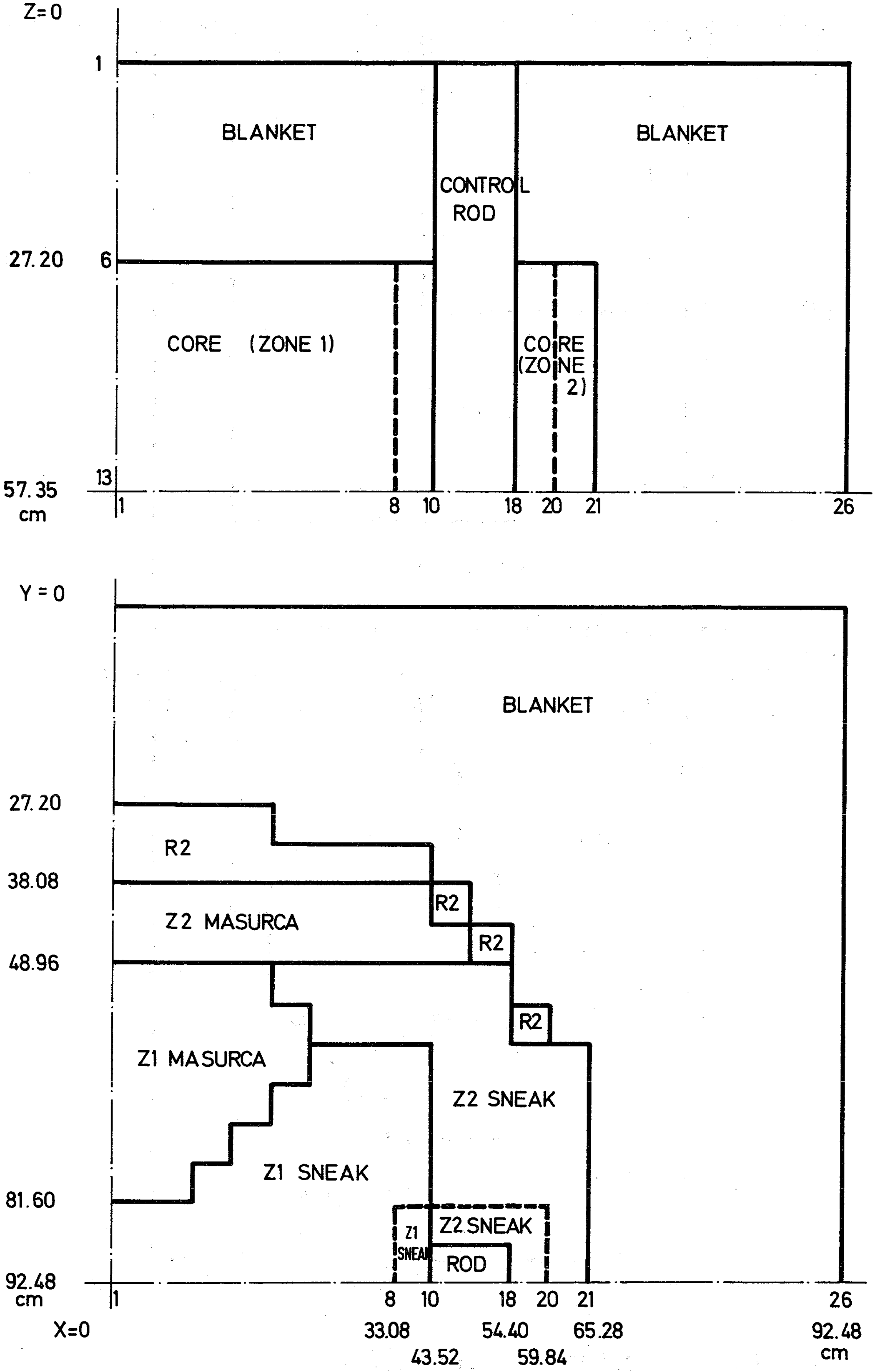
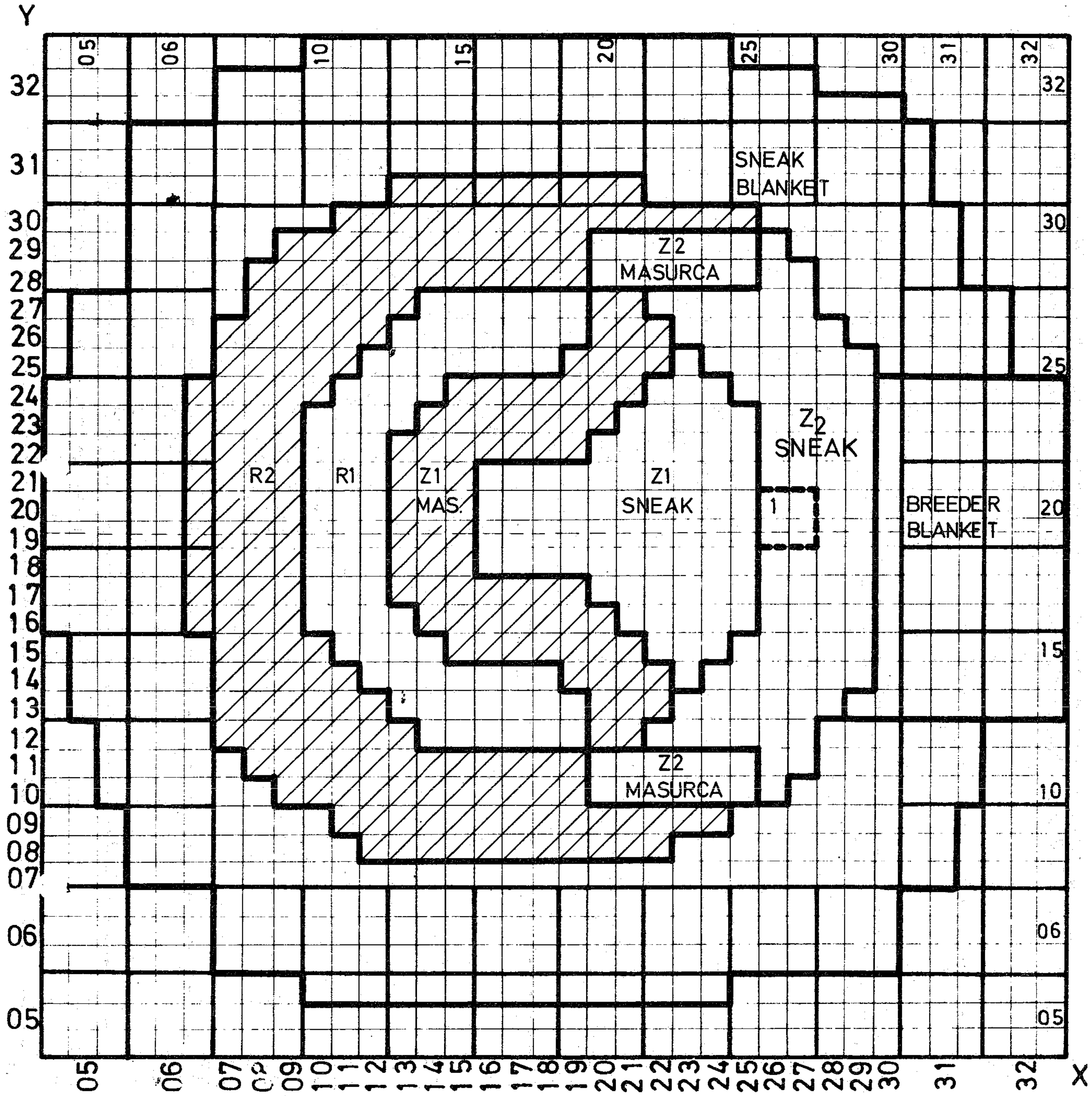


Fig. 2 Geometrical model SNEAK-2C (simplified) used for the comparison KASY - TRITON

above : horizontal section in the core midplane (Z = 13)

below : vertical section in the plane Y = 22



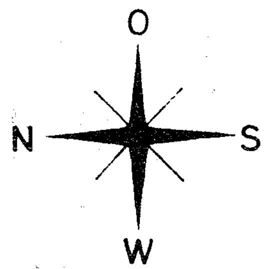


Fig. 3 SNEAK-2C Control Rod Experiments

Core with 494 elements

[1] = Position 1 for absorber or follower rod

Maßstab 1:10

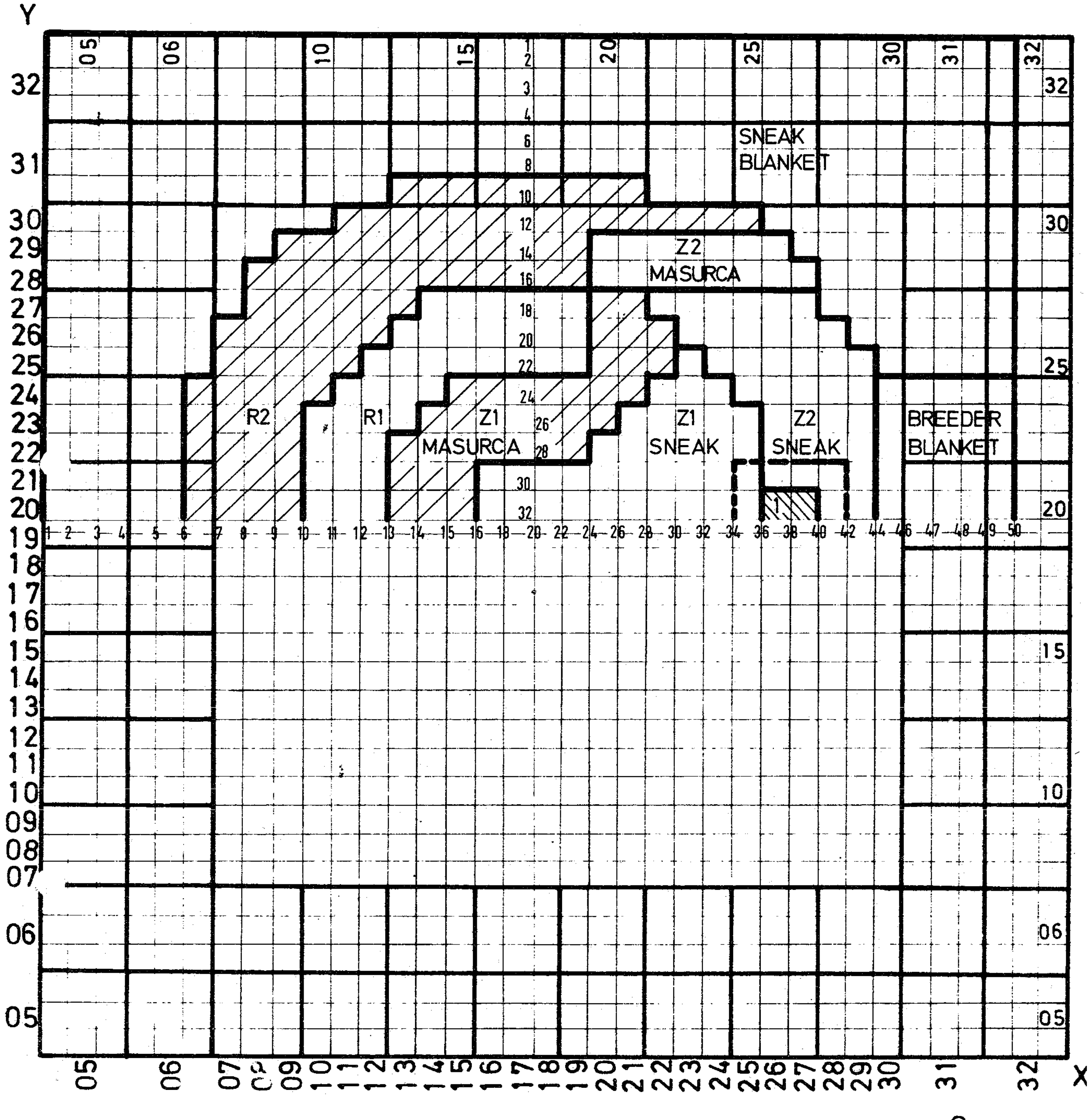


Fig. 4 . SNEAK-2C Control Rod Experiments

Core with 494 elements, calculational model

1, 6, 10.... numeration of the mesh points (1 SNEAK - Element = 5.44 x 5.44 cm)

Zone of influence of the control rod

Maßstab 1: 10

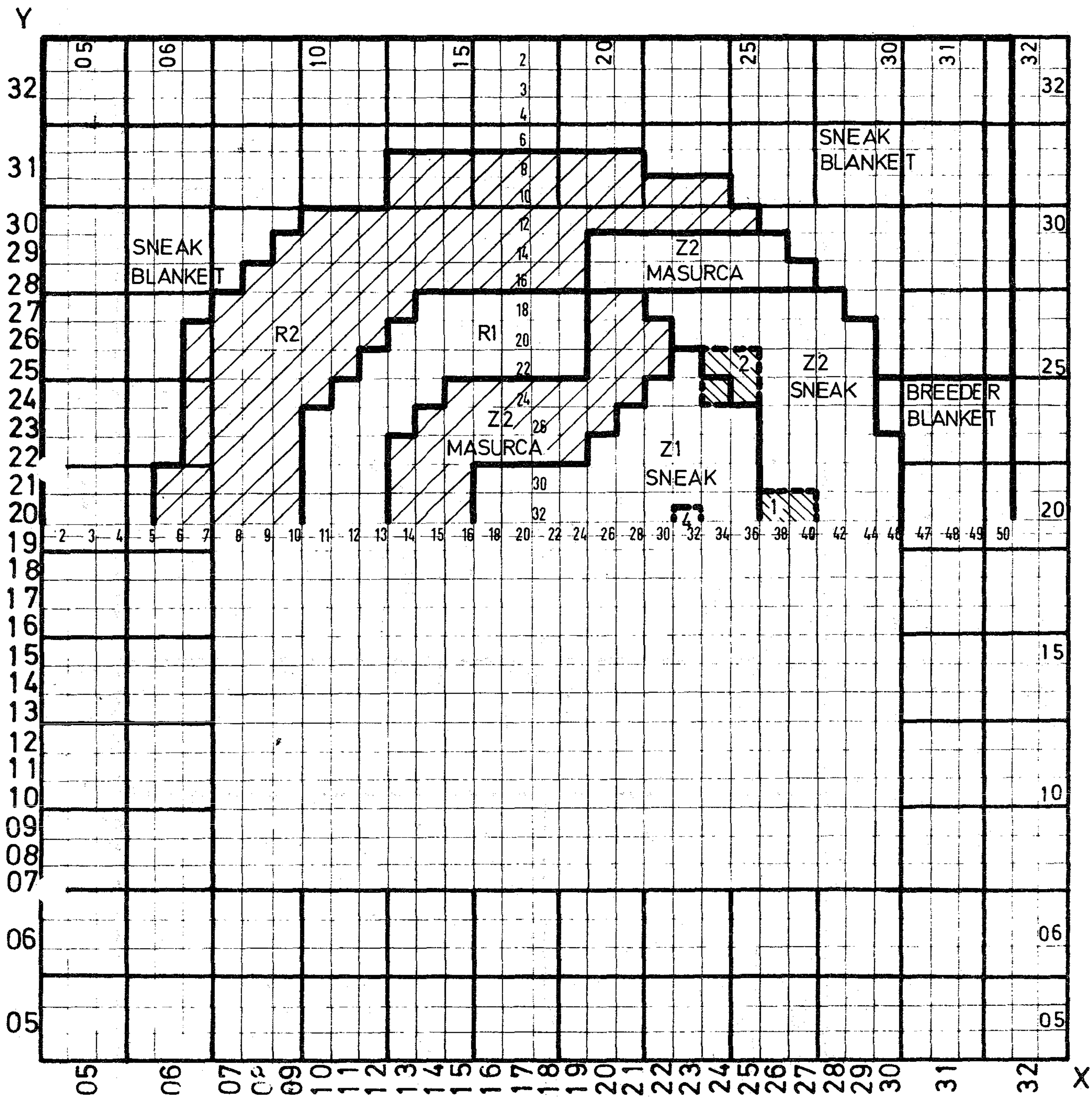
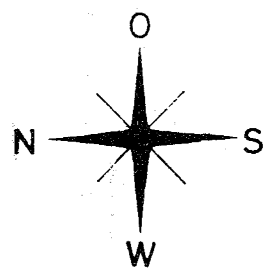


Fig. 5 SNEAK-2C Control Rod Experiments
Core with 539 elements, calculational model

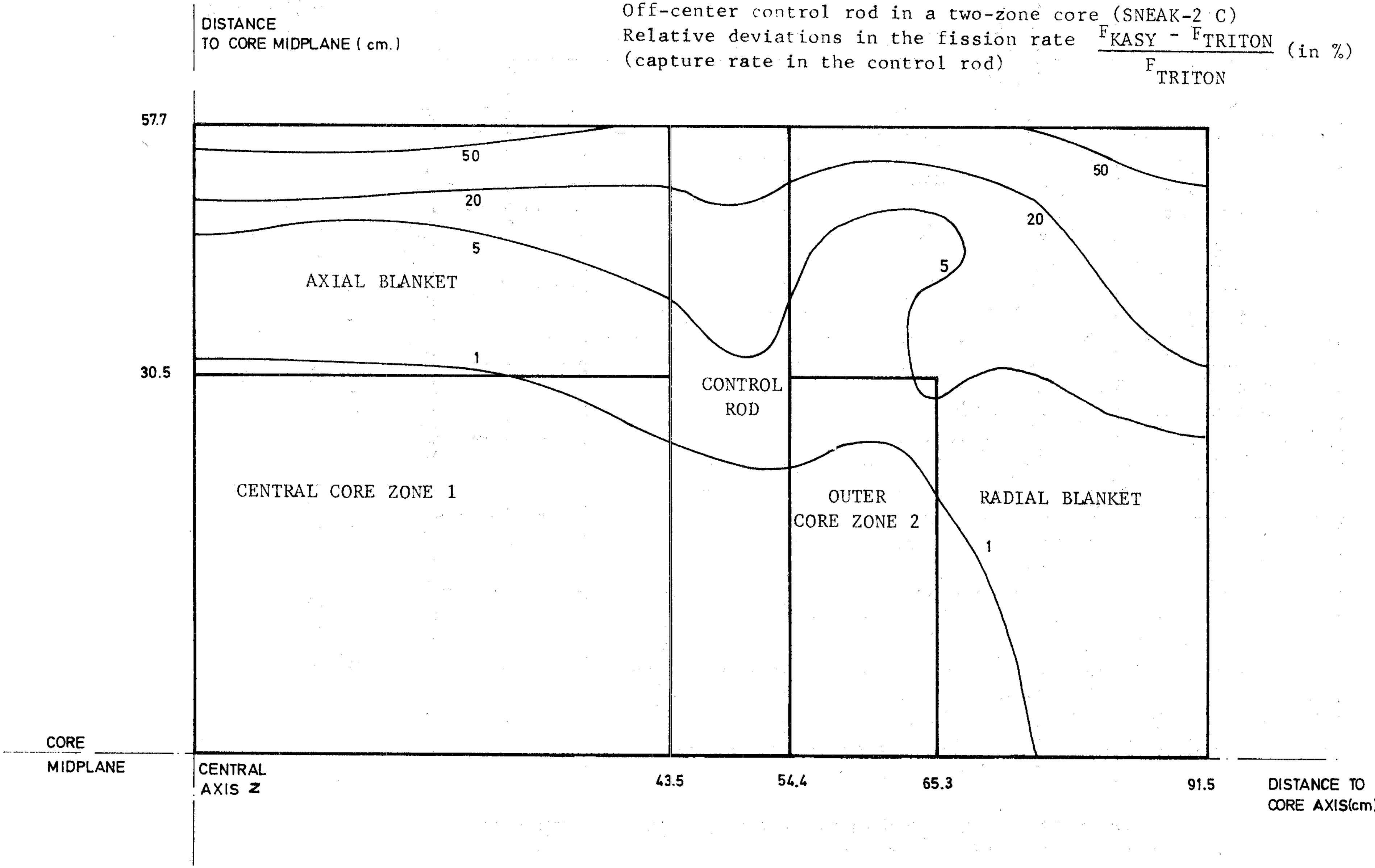
1, 2, 3 ... Numeration of the mesh points



Maßstab 1:10

FIGURE 6

KASY results compared to TRITON results
Off-center control rod in a two-zone core (SNEAK-2 C)
Relative deviations in the fission rate $\frac{F_{KASY} - F_{TRITON}}{F_{TRITON}}$ (in %)
(capture rate in the control rod)



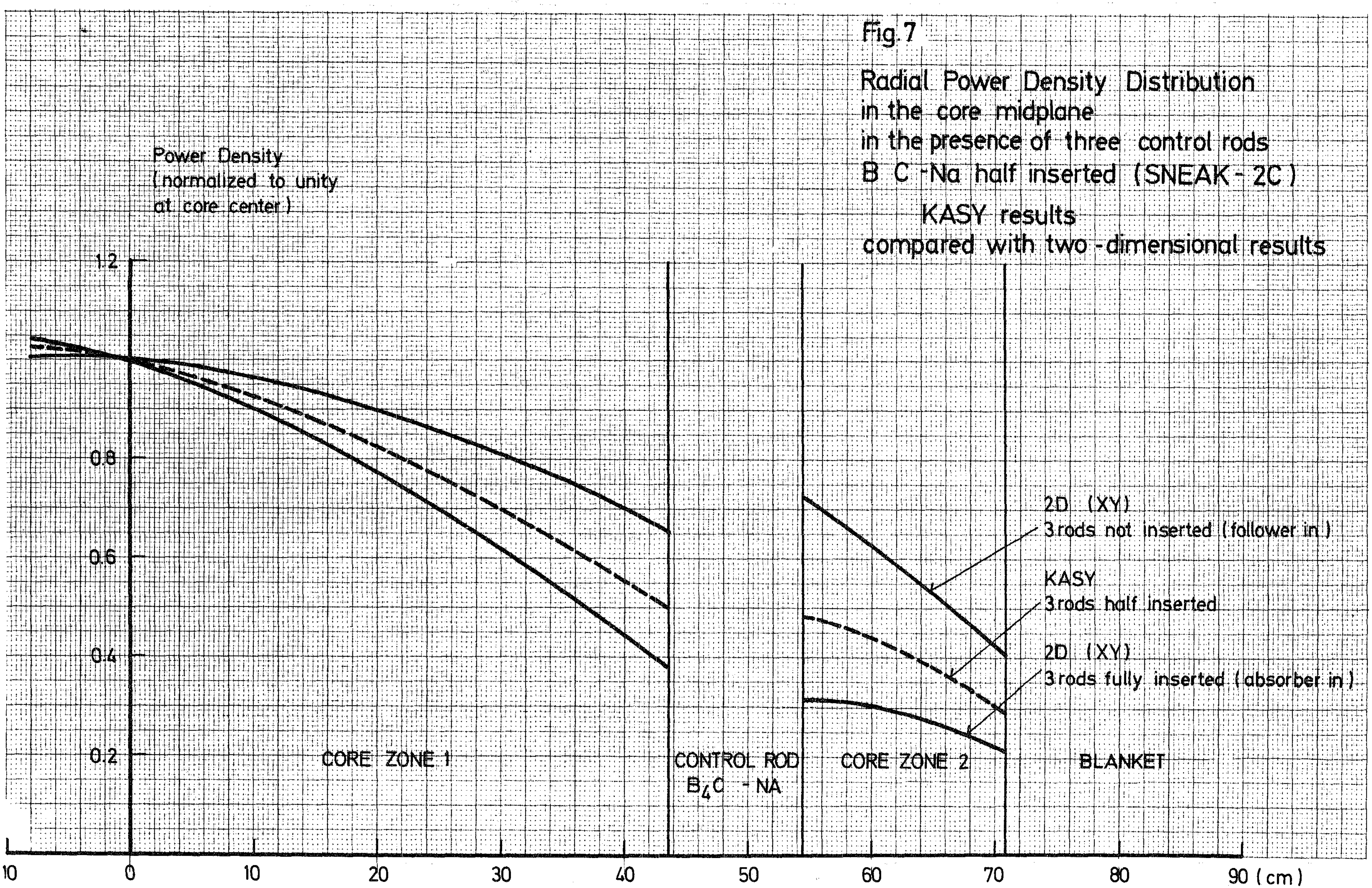


Fig. 7

Radial Power Density Distribution
in the core midplane
in the presence of three control rods
B C -Na half inserted (SNEAK - 2C)

KASY results
compared with two-dimensional results

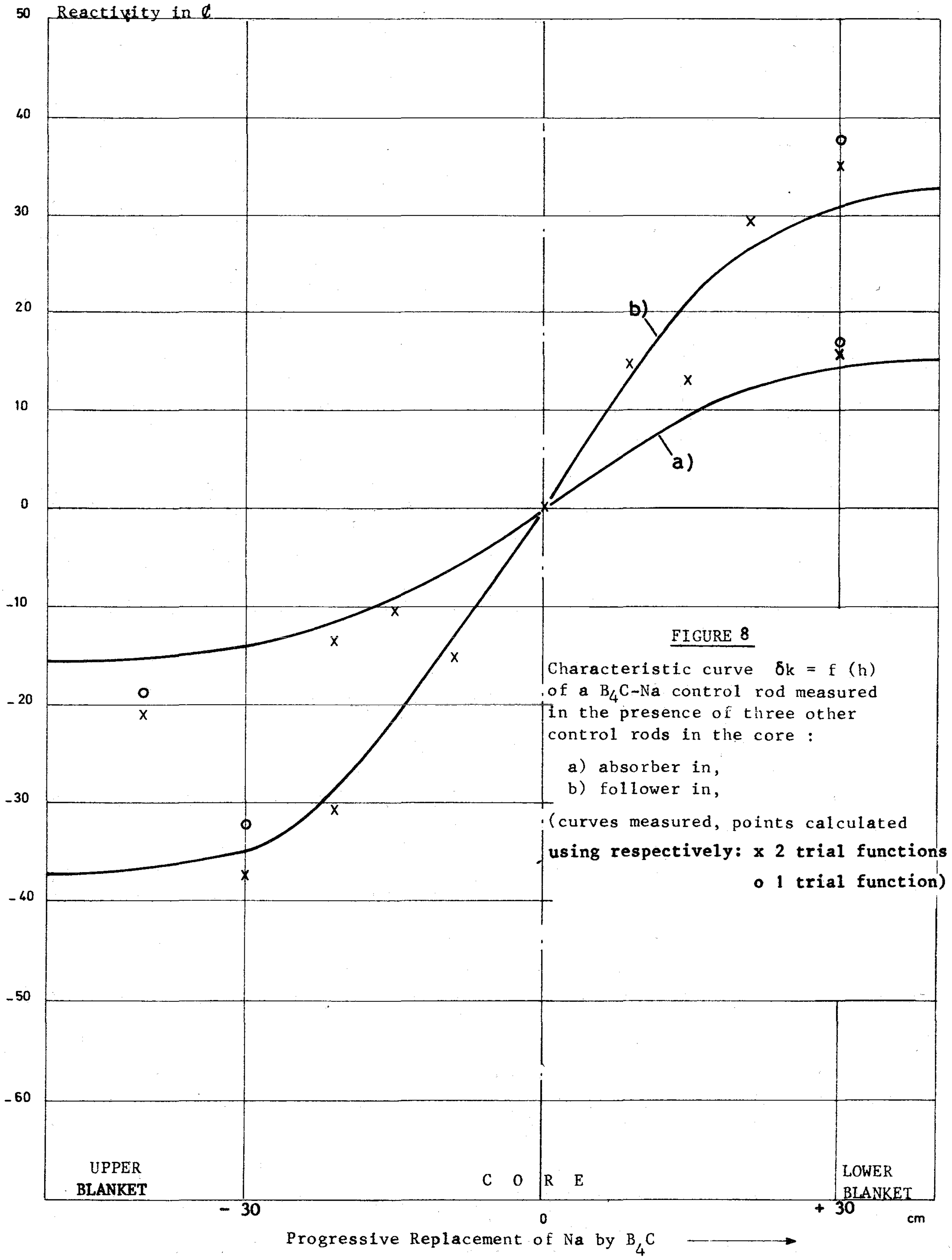


FIGURE 8

Characteristic curve $\delta k = f(h)$ of a B_4C-Na control rod measured in the presence of three other control rods in the core :

a) absorber in,
b) follower in,

(curves measured, points calculated **using respectively: x 2 trial functions**
o 1 trial function)