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# The Numerical Solution of the Three Dimensional Neutron Transport Equation

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# On the Numerical Solution of the Three Dimensional Neutron Transport Equation

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## Abstract

M.R. Wagner, D.A. Sargis and S.C. Cohen describe a method of solving the three-dimensional neutron transport equation by using time-saving difference equations. The discrete angular flux is computed in 14 directions. This paper shows how to use a set of 24 angular directions identical with those proposed by B.G. Carlson and K.D. Lathrop for $S_4$-calculations.

## Zusammenfassung

M.R. Wagner, D.A. Sargis und S.C. Cohen beschreiben eine Methode zur Lösung der dreidimensionalen Neutronen-Transportgleichung mit 14 diskreten Winkelrichtungen. Wie in der vorliegenden Arbeit gezeigt wird, ist es möglich,diese Methode zu kombinieren mit einer diskreten Winkelverteilung, wie sie von B.G. Carlson und K.D. Lathrop für eine $S_4$-Rechnung (24 Richtungen) vorgeschlagen wurde.

## Introduction

The long computing time of two-dimensional SN-programs leads to the assumption that three-dimensional SN-programs working with conventional difference equations are uneffective in time for present day computers.

M.R. Wagner, D.A. Sargis and S.C. Cohen, therefore, proposed a method based on time-saving difference equations ([2], [3]). In connection with this method two problems arise:

1) The existence of only 14 discrete angular directions allows the appearance of the well known Ray-Effect.

2) Six of the fourteen directions are parallel to the boundaries of zones. In most cases the angular flux is not continuous (at least in one derivative) for these directions; consequently these directions are not favorable for a discrete numerical integration.

Subsequently a method is described using the difference equations of [2], [3]. The set of angular directions is identical with that of a conventional $S_4$-calculation with 24 directions, proposed by B.G. Carlson and K.D. Lathrop in [1].

It can be assumed that by using these directions we have a method being effective in computing time and accuracy of angular quadrature.

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## 1. Difference Equations and Spatial Mesh

The transport equation

$$(1) \qquad \Omega \cdot \nabla \cdot \phi + \Sigma_a \cdot \phi = S \qquad \text{(S=source)}$$

can be written as

$$(2) \qquad \frac{dg(t)}{dt} + \Sigma_a \, g(t) = S$$

if $g(t) = \phi(\mathfrak{r}+t\cdot\Omega) = \phi(\mathfrak{r}+ t\Omega, \Omega, E)$

For constant S and $\Sigma_a$ we get $g(t) = g(o)\exp(-\Sigma_a \cdot t) + S/\Sigma_a$

It follows

$$(3) \qquad \phi(\mathfrak{r}+t\cdot\Omega) = \exp(-\Sigma_a \cdot t)\cdot\phi(w) + S/\Sigma_a$$

and (3) is for piece-wise constant S and $\Sigma_a$ a suitable difference equation.

We now consider a rectangular cubic mesh with the constant spacing a. S and $\Sigma_a$ may be constant within a cube. The flux may be defined in the following points: the central point $P_o$ of the cube, the central points of the six cube sides, the eight corner points. The corner points may be defined through $P_1, P_2, \ldots, P_8$ corresponding to the arrangement of figure 1, and the central points of the sides may be defined through $P_{ijkl}$, if $P_i, P_j, P_K, P_l$ are the corner points of the sides.

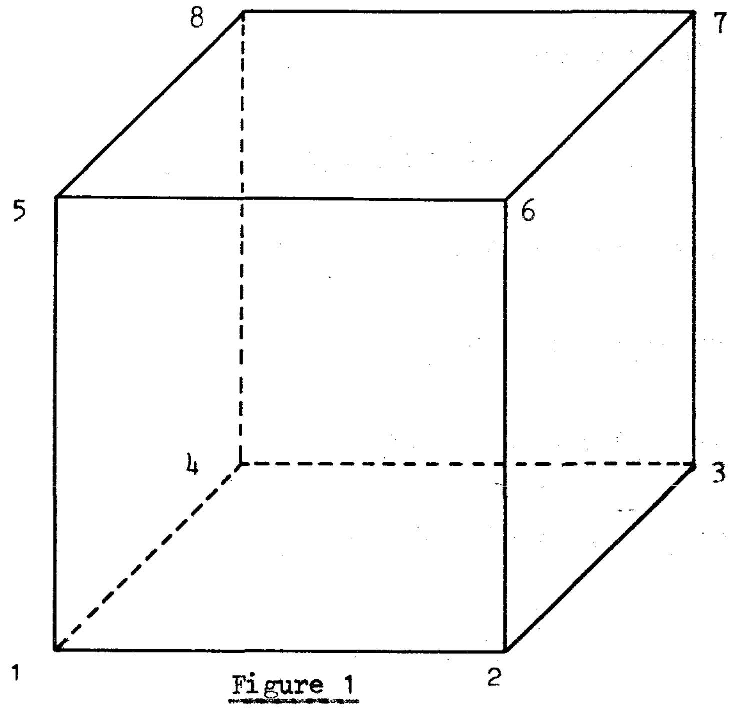


Figure 1

We now choose as discrete directions $\Omega_j$ the directions of the straight lines which connect the corner points with one of the central points of the three opposite sides. So we have, for instance,

$$\Omega_1 : P_1 \rightarrow P_{2367}$$
$$\Omega_2 : P_1 \rightarrow P_{3478}$$
$$\Omega_3 : P_1 \rightarrow P_{5678}$$

Let us now consider $\Omega_1$. Formula (3) enables us to compute the angular flux of this direction by the following equations:

$$\phi^1_{2367} = \exp(-\Sigma_a \cdot l) \cdot \phi^1_1 + S/\Sigma_a \tag{4}$$
$$\phi^1_7 = \exp(-\Sigma_a \cdot l) \cdot \phi^1_{1458} + S/\Sigma_a$$

where $\phi^m_{ijke}$ denotes the flux with direction $\Omega_m$ at the point $P_{ijke}$, $\phi^m_q$ is the flux in $P_q$ and $l$ means the length of the line $\overline{P_1 P_{2367}}$. The angular flux in $P_o$ is obtained from

$$\phi_o^1 = \frac{1}{4} \cdot \left[ \phi_{2367}^1 + \phi_1^1 + \phi_7^1 + \phi_{1458}^1 \right]$$

In the same way we have for $\Omega_2$:

$$\phi_{3478}^2 = \exp(-\Sigma_a \cdot 1) \cdot \phi_1^2 + S/\Sigma_a$$

$$\phi_7^2 = \exp(-\Sigma_a \cdot 1) \cdot \phi_{1256}^2 + S/\Sigma_a$$

$$\phi_o^2 = \frac{1}{4} \cdot \left[ \phi_{3478}^2 + \phi_1^2 + \phi_7^2 + \phi_{1256}^2 \right]$$

etc.

After computing all the angular dependent fluxes $\phi_o^j$ $(j=1,2,\ldots,24)$ we get the total flux $\hat{\phi}_o$ in $P_o$ by

$$\hat{\phi}_o = \sum_{j=1}^{24} w_j \, \phi_o^j$$

with suitable weights $w_j$.

## 3. Weights and Directions

As a result of the distribution of the angular directions described in chapter 2 we have 24 directions or 24 points on the unit sphere. There are three points per octant and the set of points is invariant under all 90-degree rotations around one axis. Thus, the description of one octant is sufficient to describe the arrangement of all points.

The direction $\Omega_j$ may have the coordinates ($\mu_j$, $\eta_j$, $\zeta_j$) with

$$\mu_j^2 + \eta_j^2 + \zeta_j^2 = 1$$

Figure 2 shows the directions of one octant as a function of their coordinates ($A = \sqrt{1/6}$)

| | $\Omega_1$ | $\Omega_2$ | $\Omega_3$ |
|---|---|---|---|
| $\mu$ | 2A | A | A |
| $\eta$ | A | 2A | A |
| $\zeta$ | A | A | 2A |

Figure 2

The geometrical arrangement on one octant is

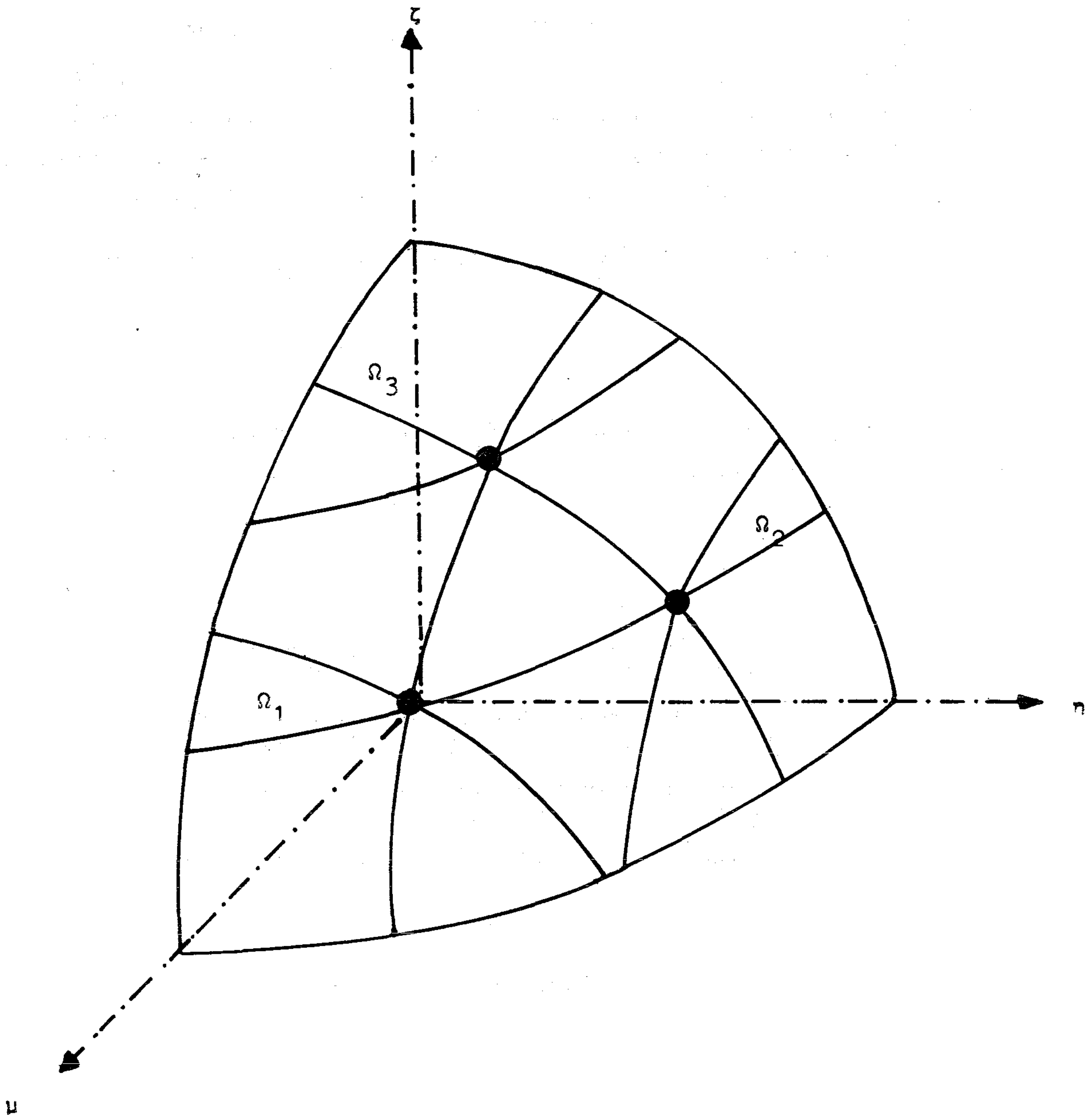


Figure 3

shown in figure 3. The directions satisfy the requirements given by B.G. Carlson and K.D. Lathrop for suitable $S_4$-directions. They can be calculated by formula (4) in /‾3_7.

The weights $w_j$ for numerical integration must be identical with those contained in /‾3_7. We, therefore, choose $w_j = w = 1/24$ for all j. Then we habe

$$(5) \qquad \frac{1}{2}\int_{-1}^{1} \mu^k \, d\mu = \sum_{j=1}^{24} \mu_j^k \cdot w = \begin{cases} 0 & (k \text{ odd}) \\ \frac{1}{k+1} & (k \text{ even}) \end{cases}$$

for k=0,1,2,3. For k=2 (5) is the well known "diffusion theory condition".

A code based on the theory described should be effective in computing-time and accuracy of the angular integration.

Literature

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