A. C. Losses in Superconducting Synchrotron Magnets

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by

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*Paper presented at the U.S. Natl. Particle Accelerator Conference
March 1 - 3, 1971, Chicago, Ill.
Summary

A.C. Losses of composite conductors and synchrotron magnets are given. A new equation based on a skin-effect assumption, is presented, which shows, that the generally accepted self field calculations yield too high values. Results from loss equations are compared to a few tests on small solenoids.

Losses in superconducting coils, when exposed to time varying magnetic fields have several origins: Eddy current losses in the conductor matrix, self field, hysteretic and auxiliary losses in the superconducting coils and composite conductors, hysteretic and eddy current losses in the iron return path and eddy current losses in the metallic support structure, reinforcements and containers. As all losses including heat conduction and radiation have to be removed by the coolant, the major effort will be to limit these losses to manageable low values.

Losses in superconductors and composites have been treated by Bean\textsuperscript{1}, Hancox\textsuperscript{2}, Wilson et al.\textsuperscript{3} and many others. Due to discrepancies between theory and experimental data specifically losses due to transport current (self field), the theory of loss generation is reexamined by Ries and Brechna\textsuperscript{4}. This paper summarizes the results of new investigations.
Zusammenfassung


Verluste in Supraleitern und Composites wurden von Bean\(^1\), Hancox\(^2\), Wilson et al.\(^3\) und vielen anderen behandelt. Aufgrund von Abweichungen zwischen Theorie und experimentellen Daten besonders bei Verlusten durch den Transportstrom (Self-Field) wurde die Theorie der Wechselstromverluste von Ries und Brechna\(^4\) neu überarbeitet. Diese Arbeit faßt die Ergebnisse der neuen Untersuchungen zusammen.
I. EDDY CURRENT LOSSES IN COMPOSITS

In a cylindrical shaped composite conductor (single-strand), having a large number of filaments it can be shown that for $l_p \leq l_c$, the eddy current losses are given by

$$\rho_{el} = \frac{1}{\rho_c} \left( \frac{\dot{B}l_p}{2\pi} \right)^2 \text{ (W/cm}^3\text{)}$$

(1)

where $l_p$ the twist pitch length, $\dot{B}$ the rate of field variation and $\rho_c$ the corrected resistivity of the matrix:

$$\rho_c = \rho_{cu} \left( \frac{w}{w-d} \right) \text{ for } d>w .$$

It is assumed that the eddy currents flow only through the matrix material and not across superconducting filaments due to the relatively high interface layer resistance between the matrix and the superconductor.

$\rho_{cu}$ is the resistivity of copper, including magneto-resistance and cold work; $d$ is the filament diameter, $w$ the distance between the centres of adjacent filaments.

II. SELF FIELD LOSSES

Based on the skin-effect assumption that transport-currents penetrate from the outer filament circle, inwards towards the inner filament circles, self field losses per cycle can be calculated by integrating the Pointing vector over the surface of the strand:
In (2) the critical current $I_c$ of a strand is assumed to be independent of the imposed transverse field. $\Delta I$ is change in the transport current from the lowest to the peak value. Equation (2) is only valid for a small twist pitch where additional induced screening currents can be ignored. In equation (2), the product $\Delta I \cdot L$ (operating current multiplied by the total strand length) for a coil of given dimensions is constant, the self field losses are proportional to $\Delta I$ and thus to $D^2$, with $D$ the strand diameter.

It can be seen that the self field losses according to (2) in a strand compared to hysteretic losses is small and the strand diameter can be increased significantly from the presently generally accepted specifications. This results in a fewer number of strands in a cable for a given current. The number of filaments in a strand have to be increased accordingly. A cable for 2000 Amp with 24 strands, each strand having 1000 filaments with a filament-diameter of 9 µm has past manufacturing stage, and is being tested.

The reduction of the number of strands has the advantage of simplicity of cable manufacturing and elimination of strand breakage during strand transposition and cable manufacturing.

III. HYSTERETIC LOSSES

Hysteretic losses are by far the highest in a coil. These losses were derived by Wilson et al.\textsuperscript{3} and are given in the form:

$$P_{sf} = \Delta I^2 \cdot L \mu_0 \left[ \frac{1}{2 \cdot 3} \left( \frac{\Delta I}{2 I_c} \right) + \frac{1}{3 \cdot 4} \left( \frac{\Delta I}{2 I_c} \right)^2 + \frac{1}{4 \cdot 5} \left( \frac{\Delta I}{2 I_c} \right)^3 + \cdots \right]$$

(Ws/cycle) \hspace{1cm} (2)
P_G = 0.5Vd(J_oH_o)\ln \left( \frac{H_{\text{max}} + H_o}{H_{\text{min}} + H_o} \right) (\text{Ws/cycle})

(3)

with V the volume of the superconductor and d the superconducting filament diameter. \(H_{\text{max}}\) and \(H_{\text{min}}\) are the maximum, resp. minimum field amplitudes, averaged over the coil cross section. The current field relation is assumed to be hyperbolic:

\[ J_o(H) = \frac{J_oH_o}{H+H_o} \]

\(J_o\) and \(H_o\) are constants.

The linear dependency of the hysteretic losses from the filament diameter has been experimentally verified down to 4 \(\mu\text{m}\) diameter.

IV. AUXILIARY LOSSES

The nonuniform field distribution in the coil region causes additional eddy current losses. The exact calculation of these losses is complex, but from skin-effect phenomenon we can calculate the ratio of the conductor ac to dc resistance of rectangular shaped coils:

\[ \frac{R_\text{ac}}{R_\text{dc}} = C_1 \left[ k + \frac{1}{4} \left( \frac{K_{\text{bm}}}{\text{w}} \right)^2 \left( \frac{D_o}{c} \right) \right] \cdot C_2 n^2 \left( \frac{D_s}{D_o} \right)^2 \]

(4)

\(C_1\) and \(C_2\) are functions of the conductor diameter, the electrical conductivity \(k\) and the pulse frequency \(f\). \(C_1\) and \(C_2\) are functions of \(x\) given by:

\[ x = \pi d (2f \cdot 10^{-9} k)^{1/2} \]

For small values of \(x\) in the region of \(0 \leq x \leq 0.4\), \(C_1 = 1\) and \(C_2 = (1/64) \cdot x^4\); \(K\) is a function of the number of strands \(n\) in the conductor. For \(n \leq 20\)

\[ K = 1.5 - 1.9. \]
\(D_s\) is the strand diameter, \(D_o\) the hydraulic diameter of the conductor (cable, or braid), \(c\) is the spacing between the centres of adjacent conductors, \(m\) is the number of layers, \(b\) the coil height parallel to the field direction and \(W\) the total coil width perpendicular to the field. Values of \(K\) for various coil shapes are given in literature \(^4\). For flat coils with \(W/b\geq 5\), calculated values of \(Kb/W\) vary between 1.5 and 4 for \(t/W=0.1 - 0.5\), where \(t\) denotes the width of a layer.

For low frequency pulses with \(f\leq 5\) Hz, \(x\) and thus \(C_2\) are practically zero for copper. In this case additional losses due to field nonuniformity may be ignored.

V. CORE LOSSES

If the iron core is placed in the helium container adjacent to the coil, Sampson et al. \(^5\) have shown that the field enhancement factor at 5 T can approach 40\% of the field produced by the coil only. However, the iron must be cooled to 4.2 K and the iron losses (hysteretic and eddy current losses) must be removed by the liquid helium. At low frequencies the major loss portion are hysteretic and eddy current losses.

The core losses (without endeffects) are given by

\[
P_{cl} = G_{Fe}(\sigma_e f^2 B^2 + \sigma_h f B^h)\quad (W)
\]

\(\sigma_e\) and \(\sigma_h\) are material constants. The exponent \(h\) varies in the range 1.6 - 2; \(G_{Fe}\) is the weight of the laminated core and \(f\) is the applied frequency.
Core endlosses could be reduced by appropriate shaping of the iron at the entrance and exit of the magnet (e.g. slanting the end portions 15 - 30 and 45° with respect to the median plane). If Rogowski type curved endshapes are selected, additional endlosses are negligible. Solutions approximating Rogowski type shapes have been developed such that core endlosses are fractions (<10%) of the losses obtained from (5).

VI. EDDY CURRENT LOSSES IN NONACTIVE METALLIC PARTS

Winding structure and coil reinforcements, if constructed from high strength metals or alloys produce eddy current losses. The magnet dewar with warm bore will contribute to eddy current losses, heat radiation and heat conduction. To eliminate eddy current losses, the coil support structure and the dewar are being designed with glass reinforced epoxy structures. For coil reinforcement unidirectional preimpregnated semi cured glass epoxy tapes are employed.

VII. COMPARISON WITH EXPERIMENTAL RESULTS

Two types of magnets have been investigated:

a) Small Solenoid with an ID of 2.4 cm, OD of 6.5 cm and axial length of 5.3 cm. 6

6.125 turns of a 0.04 cm diameter strand are wound into a solenoid. The composite conductor consists of 61 filaments with 35 μm diameter each. The filaments have a twist pitch of 0.6 cm. The matrix is copper.
The field is changed at a rate of 7 T/s. The current change is $\Delta I = 63 \, A$, $I_c = 65 \, A$ at 5.3 T. The cycle duration is 3.5 s.

Table 1 illustrates the various loss components.

<table>
<thead>
<tr>
<th>Loss Component</th>
<th>Calculated (Watts)</th>
<th>Measured (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy current losses</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>Self field losses</td>
<td>0.0035</td>
<td></td>
</tr>
<tr>
<td>Hysteretic losses</td>
<td>2.14</td>
<td></td>
</tr>
<tr>
<td>Auxiliary losses</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td><strong>Total losses</strong></td>
<td><strong>3.10</strong></td>
<td><strong>2.9</strong></td>
</tr>
</tbody>
</table>

b) One meter long dipole coil with iron shield

The dipole generates a field of 5 T in a warm bore of 6 cm diameter. The coil is composed of two intersecting ellipses and is energized by a current of $I = 2000 \, A$. The coil volume is $3.4 \times 10^{-3} \, m^3$. The strand diameter is 0.038 cm and the filament diameter 5 $\mu$m and 9 $\mu$m resp., for the two cases investigated.

In the first case each strand has 4000 filaments, in the second case 1000 filaments. The copper to superconductor ratio in the strand is 1:1. Each strand is insulated.

26 strands are transposed into a rectangular cable of $2.5 \times 2.3 \, mm^2$ dimensions. The filament twist pitch in each strand is $l_p = 0.15 \, cm$ and 0.2 cm resp. The pulse cycle is 3 s.

The radial thickness of the iron shield is 8 cm. With an inner iron radius of 6 cm field enhancement is 1.36.
Table 2

Calculated Losses of a 1m Long Dipole Coil with Iron Shield

<table>
<thead>
<tr>
<th>Loss Type</th>
<th>9μm filaments (Watts)</th>
<th>5μm filaments (Watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eddy current losses</td>
<td>1.67</td>
<td>0.96</td>
</tr>
<tr>
<td>Self field losses</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>Hysteretic losses</td>
<td>20.0</td>
<td>11.0</td>
</tr>
<tr>
<td>Auxiliary losses</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Core losses</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Dewar static losses</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>30.91</strong></td>
<td><strong>21.19</strong></td>
</tr>
</tbody>
</table>
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