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Application of Criteria in a Safeguards System

R. Avenhaus, D. Gupta, R. Kraemer



GESELLSCHAFT FUR KERNFORSCHUNG M.B.H.

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### Zusammenfassung

Eines der wichtigen Gebiete der systemanalytischen Untersuchungen im Rahmen der Spaltstoffüberwachung ist die Quantifizierung der überwachungsrelevanten Begriffe und Zusammenhänge. In der vorliegenden Arbeit wird ein Versuch unternommen, eine Methode zu entwickeln, die es ermöglichen könnte, einige Kriterien zur Verringerung des maximalen Routine-Inspektionsaufwandes der IAEO in quantifizierbarer Weise zu verwenden. Diese maximalen Aufwände für verschiedene Kategorien von Kernanlagen, sowie die Kriterien für die Festlegung der wirklichen Routine-Inspektionsaufwände für solche Anlagen sind in Artikel 80 und 81 des IAEO-Dokumentes INFCIRC/153 festgelegt.

Die hier entwickelte Methode basiert auf einem mit Wichtungsfaktoren versehenen Punktverteilungssystem, das häufig für ähnliche Entscheidungsprozesse angewendet wird. Einige hypothetische Beispiele zeigen die Verwendung dieser Methode.

# Abstract

One of the important aspects of systems analytical investigations in connection with safeguards is the quantification of safeguards terms and relations. In the present paper an effort has been made to develop a method which can be used to apply some criteria in a quantifiable manner for the reduction of maximum routine inspection efforts by the IAEA. These maximum efforts for different categories of nuclear facilities and the criteria to determine the actual routine inspection efforts for such facilities are laid down in the articles 80 and 81 of the Document INFCIRC/153 published by the IAEA.

The method developed is based on a point allocation system with weightage factores which is in common use in other decision making processes. The use of this method is illustrated with some hypothetical examples.

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# Application of Criteria in a Safeguards System

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## 1. Introduction

One of the important aspects of systems analytical investigations in connection with safeguards, is the quantification of safeguards terms and relations  $\int 1_{-}^{-1}$ . In a number of areas classical scientific methods may not be applied successfully for this purpose. Typical examples are the fixation of the errors first and second kind in the preparation of statements on a possible diversion, quantification of the influence of the safeguards measures containment and surveillance, establishment of relation between the plant, national (regional) and international safeguards systems, and so on.

If quantification of such quantities becomes essential, new methods have to be developed or the existing methods modified or the methods from other disciplines used.

# 2. Criteria to be used to reduce routine inspection efforts

A typical problem of quantification arises in connection with the application of Article 81 of the document INFCIRC/153  $\int 2 \sqrt{2}$ . In this article a number of criteria have been enumerated which can be used for the reduction of maximum routine inspection efforts by the IAEA as specified in Article 80 of the same document. For ready reference Article 81 is quoted below  $\sqrt{2}$ .

"81. Subject to paragraphs 78-80 above the criteria to be used for determining the actual number, intensity, duration, timing and mode of routine inspections of any facility shall include:

(a) The form of nuclear material, in particular, whether the material is in bulk form or contained in a number of separate items; its chemical composition and, in the case of uranium, whether it is of low or high enrichment; and its accessibility;

- (b) The effectiveness of the State's accounting and control system, including the extent to which the operators of facilities are functionally independent of the State's accounting and control system; the extent to which the measures specified in paragraph 32 above have been implemented by the State; the promptness of reports submitted to the Agency; their consistency with the Agency's independent verification; and the amount and accuracy of the material unaccounted for, as verified by the Agency;
- (c) Characteristics of the State's nuclear fuel cycle, in particular, the number and types of facilities containing nuclear material subject to safeguards, the characteristics of such facilities relevant to safeguards, notably the degree of containment; the extent to which the design of such facilities facilitates verification of the flow and inventory of nuclear material; and the extent to which information from different material balance areas can be correlated;
- (d) International interdependence, in particular, the extent to which nuclear material is received from or sent to other States for use or processing; any verification activity by the Agency in connection therewith; and the extent to which the State's nuclear activities are interrelated with those of other States; and
- (e) Technical developments in the field of safeguards, including the use of statistical techniques and random sampling in evaluating the flow of nuclear material."

In the present report a method has been discussed which could be used for the application of the above mentioned criteria in a quantifiable manner.

For this purpose these criteria can be divided into two main categories, namely:

- 2.1 Criteria based on the characteristics of the material handled;
  a) form of material handled (81a), b) characteristics of the State's nuclear fuel cycle (81c) and c) international interdependence (81d).
- 2.2 Criteria based on the effectiveness of the State's accounting and control system (81b).

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The criteria 81e, namely the technical development need not be considered separately as it inherently causes a change in the verification efforts.

It is to be noted that the criteria under 2.1 on the characteristics of the material are universal in nature, independent of any specific type of safeguards system.

# 3. Relations; Relative weightage of criteria; Rest verification efforts

# .3.1 Development of a relation

The criteria based on the characteristics of the material handled may be taken to be useful to assess the <u>non-divertibility</u> of the material. If this non-divertibility of the material be denoted by  $\mu$  and the effectiveness of the State's (regional) system as  $\kappa$ , then the simplest possible relation (i.e. linear) between the Agency verification effort denoted by  $\lambda$  and these two variables is given by

$$\lambda = 1 - \frac{\kappa + \mu}{2} \tag{1}$$

where all the values of  $\lambda$ ,  $\kappa$ ,  $\mu$  have been normalised and therefore, have to fulfill the condition

# $0 \leq \lambda \leq 1$ ; $0 \leq \kappa \leq 1$ ; $0 \leq \mu \leq 1$

Eqn. (1) implies that if the total verification effort of the IAEA verification system (equiv. to maximum routine inspection effort) be assumed to be 1, the actual verification effort can be calculated by subtracting the sum of the effectiveness of the national system and the non-divertibility of the material. This sum has to be divided by 2 to fulfill the condition  $0 \le \lambda \le 1$ .

Eqn. (1) indicates that under the boundary condition  $\kappa = \mu = 1$ , the IAEA routine inspection effort becomes 0. Since in that case the IAEA cannot verify, the eqn. (1) has to be modified to permit a rest routine inspection effort  $\gamma$ . Besides, in this equation, both the factors  $\kappa$  and  $\mu$  have the same weightage. This would mean that in the complete absence of a national system ( $\kappa=0$ ), with the highest value of non-divertibility factor of  $\mu = 1$  alone, the actual routine inspection effort would be reduced significantly i.e. by a factor of 2 or 50 %. In practice the actual inspection effort should be influenced mainly by the effectiveness of a State's system, as this is the more important of the two variables. This can be ensured by introducing a weightage factor m between  $\kappa$  and  $\mu$ .

These two requirements can be met by modifying eqn. (1) in the following manner:

$$A = 1 - \frac{1 - \gamma}{1 + m} (\kappa + m\mu)$$
(2)

where  $\gamma$  = rest verification effort in case  $\kappa = \mu = 1$ ; (0< $\gamma$ <1) m = weightage factor of  $\mu$  with respect to  $\kappa$ ; (0<m<1)

Eqn. (2) is one of a number of possibilities of reducing the verification efforts by routine inspection by using the criteria of  $\sqrt{2}$ , in a semiquantifiable manner. It is fairly simple, based on addition or subtraction and is linear in  $\kappa$  and  $\mu$ .

Other relations of the following types can also be considered for the application of the criteria in a semiquantifiable manner:

a) 
$$\lambda = (1-\kappa)(1-\mu)$$

b) 
$$\lambda = 1 - \kappa \mu$$

Eqn. b) does not appear to be suitable as in case of either  $\kappa$  or  $\mu$  being 0 the product becomes 0. Eqn. a) has been analysed in Annex 1. Besides the fact that the relation is non-linear in  $\kappa$  and  $\mu$ , the equation is fairly complex and gives (after modifying it with respect to  $\gamma$  and m) consistantly slightly lower values of  $\lambda$  than those obtained by eqn. (2) for the regions considered. Eqn. (2) has been used in this paper for illustration purposes.

# 3.2 Values of m and $\gamma$

The numerical values of the weightage factor m and the rest verification factor  $\gamma$  can be assessed only with the help of actual examples of fuel cycles as discussed later (Table 1) on in  $\sqrt{3}$ . Some numerical examples have been given in Annex 1 to indicate the method of estimating these values. The results indicate that for the cases considered, it will be fairly reasonable to assume the values of both m and  $\gamma$  to be in the range of 0.1. Eqn. (2) can then be written in the form

$$\lambda = 1 - 0.82 \kappa - 0.082 \mu$$

(3)

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# 4. Quantification of the criteria

The most difficult and important aspect of quantification in this connection is the method of assigning numerical values to  $\kappa$  and  $\mu$ . The method suggested and discussed below is based on a decision matrix point allocation system. Similar systems are used quite often in other fields of practical science /4/.

# 4.1 The value of $\mu$

The three categories of criteria under 2.1, characterising the material handled and indicating its non-divertibility may be grouped in the following manner.

- (a) International interdependence (I): This criterion, indicating the extent to which nuclear material is received or sent to other States for storage, use or processing, any Agency verification activities associated therewith and the extent to which the States nuclear activities are interrelated with those of other States, is the most powerful index of the materials non-divertibility. The information generated for safeguards for these materials can be manipulated only with difficulty by a single State.
- (b) Correlation in the cycle (K): This includes the criteria characterising the fuel cycle in which the material is handled, namely, the number and type of facilities in the cycle; the degree of containment; the extent to which the design of such facilities facilitates verification; and the extent to which information from different material balance areas can be correlated. For example, in a hypothetical cycle consisting of a reprocessing and a Pu-fabrication plant, in which Pu from the reprocessing is sent to the Pu-fabrication plant, the information on plutonium produced and processed are highly correlated. Besides, if a diversion is planned, operators of both the facilities have to cover each other. These two characteristics increase the non-divertibility of the plutonium produced in this cycle. The weightage of these criteria (K) is however, lower than that for (I).
- (c) Accessibility (V): This group contains criteria characterising the accessibility of the material handled. They include information on whether the material is in bulk form or contained in separate items; on its chemical composition and in the case of uranium, its enrichment. To make this set of criteria (V) function in the same direction as the other two to indicate the non-divertibility, the relation (1-V) has to be used. Its weightage is also less than that of I.

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To maintain a linearity in eqn. (2), the sum of these characteristics may be considered to define the non-divertibility of the material denoted by µ. As mentioned earlier a point allocation system has been used to obtain a quantifiable value. According to this system, each of the characteristics I, K and V are given 10 points. It was discussed above that all the three characteristics do not contribute equally to this non-divertibility and therefore, can be given different weightage factors. In our consideration, I has been given a weight factor of 1, K 0.5 and (10 - V) 0.8. Eighteen possible and realistic combinations of nuclear facilities in a partial or complete fuel cycle in increasing complexities are listed in Table 1 and the values of I, K and (10 - V)estimated according to the point system discussed here. The sum of these points is shown in the last but one column of the table. Full non-divertibility (i.e.  $\mu = 1$ ) is obtained for the total number of points of 23. The  $\mu$ -values for the various combinations obtained by dividing the sum by 23, are given in the last column. It is to be seen that the value of µ reduces as the fuel cycle becomes complete. This is also understandable as both I and (10 - V) tend to become zero.

A number of cases in Table 1 are discussed below to indicate the mode of allocation.

- i) Systems 1 and 2: Only conversion plant in the cycle. In 1, source of U is not known (for example obtained from a non signatory state), in this case I = 0. K = 2; although no correlation exists between different material balance areas (as only one MBA) each category of facilities has been given 2 correlation points with regard to operators collaboration for a diversion. For a single facility for example, the plant management and the plant operators (2 distinct judicial bodies) have to collaborate for a planned diversion. V = 0; because of natural concentration and absence of enrichment plant, 10 - V = 10. Total =  $0 + 0.5 \cdot 2 + 0.8 \cdot 10 = 9$ ;  $\mu = 9/23 = 0.39$ . In 2, source of U known (for example from a signatory state). I = 10; Rest same as in 1.  $\mu = 19/23 = 0.82$ .
- ii) Systems 7 and 8: The cycle consists of conversion and fabrication plants and heavy water type reactors (HWR). Plutonium can be stored in an uncontrollable manner. In 7, only U-stream considered; source of U known. I = 5; full international dependence value of 10 points cannot be given as an unknown part of uranium can be used to produce Pu in the reactor. K = 2; although three different categories of facilities in the cycle are present only the reactor operator knows about plutonium and could divert

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it without a collaboration of the other two facilities. V = 0 as only natural uranium is available. In 8 the plutonium cycle is considered. The source of U is unknown. I = 0; K = 2 for the same reason as in 7. V = 10, as plutonium is fully accessible to the reactor operator.

iii) System 18: The fuel cycle consists of all the five categories of facilities i.e. conversion plant, light water type reactors, fabrication, reprocessing and isotope separation facilities. Source of U is known. I = 0, for self sufficiency. K = 10; both isotopic correlations (fabrication, reactors, reprocessing) are fully existent and a full collaboration of a number of facility operators is required for a diversion. V = 10; both plutonium and high enriched uranium are accessible and available.

It may again be noted that a linear relation has been assumed for all the values of  $\mu$  in the range of  $0 < \mu \neq 1$ .

# 4.2 The value of K

A similar point allocation system has been used to allocate a numerical value to the effectiveness  $\kappa$  of the State system. Some of the measures which may be considered for assessing the effectiveness of a State's system are listed in Article 32 of  $\sqrt{2}$ . They are reproduced below for ready reference:

"32. The Agreement should provide that the State's system of accounting for and control of all nuclear material subject to safeguards under the Agreement shall be based on a structure of material balance areas, and shall make provision as appropriate and specified in the Subsidiary Arrangements for the establishment of such measures as:

- (a) A measurement system for the determination of the quantities of nuclear material received, produced, shipped, lost or otherwise removed from inventory, and the quantities on inventory;
- (b) The evaluation of precision and accuracy of measurements and the estimation of measurement uncertainty;
- (c) Procedures for identifying, reviewing and evaluating differences in shipper/receiver measurements;
- (d) Procedures for taking a physical inventory;

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- (e) Procedures for the evaluation of accumulations of unmeasured inventory and unmeasured losses;
- (f) A system of records and reports showing, for each material balance area, the inventory of nuclear material and the changes in that inventory including receipts into and transfers out of the material balance area;
- (g) Provisions to ensure that the accounting procedures and arrangements are being operated correctly; and
- (h) Procedures for the submission of reports to the Agency.

It is to be noted that the measures relevant to IAEA verification, which can be executed by the State's system are not limited to the 8 categories of Article  $32 \int 2 \int .$  It would have been possible to take only these 8 categories and proceed with the allocation system as in the case of non-divertibility ( $\mu$ ) of the material. In this report a further subdivision of each of these categories has been considered as each category of measures has different, specific characteristics which should be considered separately in assessing the overall effectiveness. Such refined and graded evaluation can make the quantification more representative of the actual characteristics of the State's system.

Keeping this in mind, the relevant measures by a State's system have been regrouped below. The numbers and comments in brackets are the assumed subgroups for the point system.

- 1. Record system: (1a: completeness of data; 1b: Clarity of presentation; 1c: ease of availability) - corresponding to 32 (f) / 2\_7
- 2. Report system: (2a: agreement between records and reports;
  2b: promptness of submission) 32 (f,h) / 2\_7
- 3. Measurement system: (3a: accuracy of operators material balance;
  3b: clarity, simplicity, tamperresistance; 3c: calibration possibilities)
   30(a,b) / 2/
- 4. Establishment of precedures: (4a: for physical inventory taking and material balance; 4b: for MUF including S/R difference) 32 (c,d,e) / 2\_/
- 5. Containment and surveillance measures: (5a: features of the plant; 5b: additional measures which may be considered by the State's system)
- 6. Inspection by the State's system: (6a: observation or independent analysis of throughput and inventory measurements; 6b: Execution of containment and surveillance measures; 6c: verification of consistency of reports and records; 6d: analysis of MUF; 6e: calibration of instruments) 32 (b,d,e,g) <u>7</u>

As in the case of the evaluation of  $\mu$ , each of the groups has been allocated 10 points, but they need not have the same weightage for the estimation of the effectiveness. The records (1), measurement system (3) and the inspections (6) may be taken to have the same weightage as they are used to register, generate and verify the primary information for safeguards. The reporting system (2) can be given a slightly lower weightage as it is used mainly for corroboration of data and for planning of inspections. This is followed by containment and surveillance measures (5) as they do not lead to any quantifiable conclusion. The lowest weightage is given to the establishment of procedures (4). The actual weightage factors used for the different points are:

Points	e produktion in the state of the	Weightage factor
1,3,6		1
2 <sup>1</sup> 1		0.8
5		0.5
4		0.2

In Table 2 fifteen cases of State's systems have been considered and the sum of the points for each system estimated. The value of  $\kappa$  for these systems is given in the last column.

# 4.2.1 Comments on groups and subgroups

Some comments on the different groups and subgroups listed in Table 2, are required to explain the basic philosophy underlying the allocation system.

a) On the different groups:

- i) The record, report and measurement systems (groups 1,2,3) are essential in the framework of NPT-safeguards. They must form the basic components of the State's system to enable the Agency to verify. They cannot have therefore, zero points in any States system.
- ii) The group on procedures (4) has been given a very low weightage as it can be prepared by any State fairly easily (particularly because it will be similar for similar categories of plants and measurement methods). It has 2 points in a total of 45. Therefore, no gradual variation is made but either 0 or 2 points have been allocated to this group.

- iii) Although containment and surveillance measures have not been included in Article 32 / 2 /, they have been taken up under group 5 in this paper. It has been shown elsewhere  $\sqrt{3,5}$  that particularly these two measures can contribute significantly to the effectiveness of a safeguards system. However, it must be clearly stated that these measures have nothing to do with the physical protection measures. Physical protection measures are not a subject matter of Safeguards Agreement between the IAEA and a State, and therefore, cannot be considered by the IAEA in evaluating the effectiveness of a State's system in the framework of such an agreement. The types of factors which may be considered under this group are for example, the features of nuclear facilities (glove-boxes in a Pu-fabrication plant, Pu-birdcages, reactor vessels etc., i.e. those inherent featurs of a plant which may be used as complementary measures to material balance for ensuring the correctness of the data) and the measures which the State's system may take on the basis of available conditions in a plant (for example sealing).
- iv) Two types of inspections are carried out by the State's system:
  They are (a) those which are not carried out by the Agency inspectors
  e.g. ensuring that records are kept; preparation and dispatching of
  reports; testing of measurement, system of the operator, at the beginning and in the event of a change, and (b) those which are carried
  out by the Agency inspectors also and are listed under group 6.
  Only these points are considered for effectiveness, as the activities
  under a) are not directly verifiable by the Agency.
  - v) A number of factors in a State's system is in practice in the domain of the facility operator. They are 1b,c (1a being essential and therefore not variable), 3a,b,c and 5a. The rest of the factors has been considered to be under the responsibility of the State's system. In allocating points to different variations of a State's system, a differentiation has therefore, been made between the operator's and the State's system.

# b) On the different subgroups

i) The subgroup 1a, completeness of data in a record system is a basic prerequisit of Agency verification. Therefore, it is given 5 points i.e. 50 % of the weightage in this group. It is further assumed that in all the possible variations of a State's system, it must be available fully i.e. all the 5 points will have to be given.

 ii) For the subgroup 5b, i.e. additional containment and surveillance measures by the State's system, the number of points can either be 5 or 0, as no gradual quantification of these measures is possible.

#### 4.2.2 Comments on the different State's systems

Starting with the three basic groups 1,2,3 for a State's system, other groups are gradually introduced into the system to make it complete. The variations presented in Table 2 do not correspond to any existing system and are not necessarily complete. The basic characteristics of the 15 systems are indicated below:

System I: 1,2,3 fully complete; rest absent

II: 1,2,3; rest absent; State's system: good; operator's system: Measurement system good, recording system poor

III: 1,2,3; rest absent; good State's system; poor operator's system

- IV: 1,2,3; rest absent; poor State's system; poor operator's system
- V: 1,2,3; rest absent; both the State's and the operator's system poor

VI: System I + 5; rest absent; I + full containment

VII: I + 1/2 containment; rest absent

- VIII: V + full containment; rest absent
  - IX: V + 1/2 containment; rest absent
  - X: Fully effective State's safeguards system
  - XI: X + 1/2 containment
- XII: State's system: All measures are good (1a,2a,b; 4a,b; 5b; 6a,b,c,d,e); Operator's system: All measures are poor (1b,c; 3a,b,c; 5a)
- XIII: States system: All measures are poor; Operator's system: All measures are good
  - XIV: State's system: All measures are good; Operator's system: Records are good; measurement and containment system are poor
    - XV: State's system: Reporting and procedures are poor; containment and inspections are good. Operator's system: All measures are good.

# 5. Comments on quantification results

# 5.1 Non-divertibility factor µ

An analysis of the non-divertibility factor  $\mu$  in Table 1 reveals a number of points. They are commented below. It is to be noted that the comments are limited only to the cases considered in this report.

- a) For those variations of a State's system which assume an unknown uranium source (systems 1,4,6,8,13) the µ values are extremely low and practically do not contribute to a reduction in the verification efforts of the IAEA by routine inspections. In reality, such systems are highly improbable and therefore, have not been considered any further.
- b) The µ values for uranium cycles are fairly high and vary in the range of 0.5-0.9 excepting in the case of 17 (a self-containing fuel cycle including an enrichment facility). In this case the non-divertibility reduces to 0.22 and becomes equivalent to Pu-cycles. This appears to be fairly reasonable.
- c) The µ values for Pu-cycles are consist ntly low and vary in the range of 0.2-0.3 and almost independent of the complexity or completeness of the cycle. This fact appears to be consist nt with the actual conditions which may exist in reality.

### 5.2 Effectiveness of a State's system

The effectiveness  $(\kappa)$  values of the different variations of a State's system considered in this report, also indicate some trends. It should be mentioned once more that none of the variations refer to any existing State's system.

a) κ values vary between 0.2-0.94 (system V and IX respectively). For the lowest value, both the State's and the operator's system have been assumed to be insufficient besides the fact that the State's system does not have any inspection activities. In the case of the highest activities only the containment of the plant has been assumed to be insufficient. In practice, the effectiveness value of a particular State's system is expected to remain below the maximum value given in Table 2.

- b) Since 5 points under the group for records and 3 points under the group of measurements are kept constant (basic requirement for verification), the κ value cannot go below 0.18. A rest value for κ appears to be justifiable as any State, ratifying the NPT, will have the obligation to set up an accounting and control system.
- c) The group for inspection activities has the maximum possibilities of variations. This has been done particularly in view of the fact that this group influences the IAEA verification efforts in a most direct manner.

# 6. Implications

# 6.1 Reduced verification efforts

The maximum routine inspection efforts for different categories of facilities as laid down in Article 80 / 2 / can be interpreted to ensure a full coverageby an inspection regime in establishing material balances for different material balance areas, in a timely manner. The uncertainty in such a material balance will be caused only by the standard deviations in measurements. This uncertainty is a measure of the capability of an inspection regime with full coverage. This uncertainty leads to a minimum divertible threshold amount whichcan be detected by the inspection regime giving the full coverage, with a givenprobability.

In the case of a reduction in the IAEA verification effort for the same material balance areas, the IAEA inspectors alone cannot give a full coverage in establishing the material balances. However, it is to be taken that a full coverage will be given in any case by the State's system. Therefore, although IAEA gives a lower coverage, the joint coverage by the IAEA and the State's system gives rise to the same minimum divertible thresholds amounts for the material balances as in the case of a full coverage.

With reduced Agency verification efforts, the threshold values of diverted amounts above which the IAEA <u>alone</u> could detect a diversion with the same probability of detection, would increase. However, since these values remain at the minimum divertible amounts for the sum of the inspection regime, the increased threshold values of IAEA alone, are a measure of the credibility attributed by the IAEA to the State's system giving the full coverage. Therefore, the increased divertible amounts associated with the reduced verification efforts of the IAEA, may be regarded as "credible amounts" corresponding to a given State's system.

# 6.2 Numerical values of $\lambda$

Some values of the reduced IAEA verification efforts  $\lambda$  for different values of  $\mu$  and  $\kappa$  from Tables 1 and 2 are given below. They are obtained by introducing the  $\mu$  and  $\kappa$  values in eqn. 3, which means that for these concentrations, the values of  $\gamma$  and m have been kept at 0.1.

μ	κ	λ	
 0.82	0.81	0.26	
0.82	0.21	0.76	
0.52	0.81	0,29	
0.52	0.21	0.79	
0.22	0.81	0.31	
0.22	0.21	0.81	

It is to be noted that the IAEA verification effort  $\lambda$  varies in the range of 0.26 to 0.81 depending on the values of  $\kappa$  and  $\mu$ . For the same  $\kappa$  value of 0.81 an increase in the  $\mu$  value from 0.22 to 0.81 reduces the  $\lambda$  from 0.31 to 0.26 only. The  $\lambda$  is much more sensitive to the  $\kappa$  values.

#### 6.3 Credible amounts

The credible amounts for a given IAEA verification effort  $\lambda$ , can be calculated only for a specific fuel cycle. The procedure for estimating these credible amounts is fairly complicated.

Six steps are required for this estimation. They are indicated below:

- Step I: Laying down of base fuel cycle data including possible divertible amount/safeguarded unit in the case of reduced verification efforts
- Step II: Fixing data for all safeguards measures for full coverage of the fuel cycle by the State system
- Step III: Estimation of uncertainties in the material balance through full coverage, caused by measurement errors alone
- Step IV: Estimation of reduced verification efforts by the application of criteria
- Step V: Distribution of reduced verification efforts in different nuclear facilities in the cycle
- Step VI: Estimation of credible amounts in material balance for the reduced IAEA verification efforts

## 7. Conclusions

The criteria laid down in Article 81 of the IAEA Document INFCIRC/153  $\int 2_{-}^{-}$  can be applied in a rational manner to reduce the maximum IAEA routine inspection efforts, only if a method is available to quantify these criteria. The work in the present report is an effort in this direction. In spite of the preliminary nature of the study and the limitations imposed by the types of systems considered, a number of generalized conclusions may be made.

7.1 A method based on decision matrix point allocation system has been used to quantify the above mentioned criteria. It should be mentioned that a point allocation system is not fully objective. Subjective moments enter into the picture by the estimation of weightage factors, allocation of points, and by the fixation of the number of groups and subgroups. These moments can be minimized by making different independent groups of persons to allocate points for the same system.

7.2 Fairly simple linear relations can be used to formalize the relation between the components relevant to verification and the reduced verification efforts. In the present study four components have been used to develop the relation. They are the rest verification effort  $\gamma$ , weightage factor m (between  $\kappa$  and  $\mu$ ) and the two criteria non-divertibility of material  $\mu$  and the effectiveness of the State's system  $\kappa$ . For the limited number of cases considered and the fuel cycle analysed, the values of  $\gamma$  and m were found to be in the neighbourhood of 0.1 and both the values of  $\kappa$  and  $\mu$  were found to vary in the range of 0.2 to 0.8. Because of reduced influence of m, the IAEA verification effort  $\lambda$  was found to be mainly influenced by the  $\kappa$  and varied between 0.26-0.81.

7.3 The main implication of a reduced IAEA verification effort is an increased threshold divertible amount in the establishment of material balances by the IAEA alone. These increased threshold divertible amounts can be considered to be a measure of the credibility which the IAEA system can allocate to the State's system. These amounts can therefore be considered as credible amounts for the system for which the reduced IAEA verification efforts have been assumed. These credible amounts can be estimated provided adequate information on the fuel cycle and on safeguards measures are available.

# Acknowledgement

The authors would like to thank W. Häfele for his support and interest as well as for valuable discussions during the course of this work.

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Table 1: Characteristics of fissile material influencing its non-divertibility (μ) (In the case of two numbers for the same cycle, 1 is for U and 2 for Pu.) Excepting system 1,2

Point System	Conv.	Reactor	Fabr.	Repr.	Enrich.	I (0 <b>≝</b> I≰10	K (0∕=K≦10)	10-V (0≤10-V≤10)	I	0.5 K	0.8(10-V)	Σ	μ
0 1 2	x* x					10 0 10	10 2 2	10 10 10	10 0 10	5 1 1	8 8 8	23 9 19	1 0.39 0.82
3 4		X (HWR) X <sup>+</sup>				5 0	2 2	10 0	5 0	1	8 0	14 1	0.61 0.043
5		X (LWR)				10	2	0	10	1	0	1.1	0.48
6	x+	ant was many galant for the forward and a fing and and a short of	Sought of Galaxie Strangers and an other states of the		X	0	4.	0	2	0	2	2	0.086
7 8	X X+	X (HWR)	x			5 0	2	10 0	5 0	1	8 0	14 1	0.61 0.043
9 10	x	X (LWR)	X		and an all a second	10	6 2	10 0	10 5	3 1	8 0	21 6	0.91
11 12	X	X (LWR)	X		X	10	4 2	5 0	10 5	2	4 0	16 6	0.69
13 14	X+	X (LWR)	Х		X	0	4 2	0 O	0 5	2 1	0 0	2 6	0.086 0.26
15 16	X	X (LWR)	X	na secondo esta constante	X	10 2	8 8	70 0	10 2	4 4	8 0	12 6	0.52
17 18	x	X((LWR)	Х	x	х	0 0	10 10	0	0	5	0	5	0.22

X<sup>+</sup> Supplied or recovered amounts of U not known.

X Supplied or recovered amounts of U known independently.

I = International interdependence

K = Correlation in the cycle

V = Accessibility of material

 $\mu = Non-divertibility$ 

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   | 1c  
   
  | 28   | 2b  | 38  | 3b  | 3c  | 4a  | 4b  | 58  | 5b  | 6 <b>a</b>                    
   | 6b  | 6c  | 6d  
   | 6e  | 1•(1)   | 0.8•(2)   | 1•(3)  | 0.2.(4)   | 0.5 (5)  
  | 1.(6)                                 | Σ  | к° 100   |
| 5   | 3  
   
   | 2   
   
  | 5  | 5   | 4   | 3   | 3   | 6   | 4   | 5   | 5   | 4                             
   | 1   | 1   | 2   
   | 2   | 10 .  | 8   | 10   | .2  | 5  
  | 10                                    | 45   | 100  |
| 5   | 3  
   
   | 2   
   
  | 5  | 5   | 4   | 3   | 3   | 0   | 0   | 0   | 0   | 0                             
   | 0   | 0   | 0   
   | 0   | 10  | 8   | 10   | .0  | 0  
  | 0                                     | 28   | <b>62.</b> 2   |
| 5   | 0  
   
   | 0   
   
  | 5  | 5   | 4   | 3   | 3   | 0   | 0   | 0   | 0   | 0                             
   | 0   | 0   | 0   
   | 0   | 5   | 8   | 10   | 0   | 0  
  | 0                                     | 23   | 51   |
| 5   | 0  
   
   | 0   
   
  | 5  | 5   | 1   | 1   | 1   | 0   | 0   | 0   | 0   | 0                             
   | 0   | 0   | 0   
   | 0   | 5   | 8   | 3  | 0   | 0  
  | 0                                     | 16   | 35.6   |
| 5   | 3.   
   
   | 2   
   
  | 1  | 1   | 4   | 3   | 3   | 0   | 0   | 0   | 0.  | 0                             
   | 0   | 0   | .0  
   | 0   | 10  | 1.6   | 10   | 0   | 0  
  | 0                                     | 21.6   | 48   |
| 5   | 0  
   
   | 0   
   
  | 1  | 1   | 1   | 1   | 1   | 0.  | 0   | 0.  | 0   | 0                             
   | 0   | 0   | 0   
   | 0   | 5   | 1.6   | . 3  | 0   | 0  
  | 0                                     | 9.6  | 21.4   |
| 5   | 3  
   
   | 2   
   
  | 5  | 5   | 4   | 3   | 3   | 0   | 0   | 5   | 5   | 0.                            
   | 0   | 0   | 0.  
   | 0   | 10  | . <b>8</b>  | 10   | .0  | 5  
  | 0                                     | 33   | 73.4   |
| 5   | 3  
   
   | 2   
   
  | 5  | 5   | 4   | 3   | 3.  | 0   | 0   | 5   | 0   | 0                             
   | 0   | 0.  | 0   
   | 0   | 10  | 8   | 10   | 0   | 2.5  
  | 0                                     | 30.5   | 67.8   |
| 5.  | 0  
   
   | 0   
   
  | 1.   | 1   | 1   | 1   | 1   | 0   | 0   | 5   | 5   | 0                             
   | 0   | 0.  | 0   
   | 0   | 5   | 1.6   | . 3.   | 0   | 5  
  | 0.                                    | 14.6   | 32.5   |
| 5   | 0  
   
   | 0   
   
  | 1  | 1   | 1   | 1   | 1   | 0   | 0   | 5   | 0   | 0                             
   | 0   | 0   | 0   
   | 0   | 5   | 1.6   | 3  | 0   | 2.5  
  | 0                                     | 12,1   | 26.8   |
| 5   | 3  
   
   | 2   
   
  | 5  | 5   | 4   | 3   | 3   | 6   | 4   | 5   | 5   | 4                             
   | 1   | 1   | .2  
   | 2   | 10  | 8   | 10   | 2   | 5  
  | 10                                    | 45   | 100  |
| 5   | 3  
   
   | 2.  
   
  | 5  | 5   | 4   | 3   | 3   | 6   | 4   | 0   | 5   | 4                             
   | 1   | 1   | 2   
   | 2   | 10  | 8   | 10   | 2   | 2.5  
  | 10                                    | 42.5   | 94.4   |
| 5   | 0  
   
   | 0   
   
  | 5  | 5   | 1   | 1   | 1   | 6   | 4   | 0   | 5   | 4                             
   | 1   | 1   | 2   
   | 2   | 5   | 8   | 3.   | 2   | 2.5  
  | 10                                    | 30.5   | 67.8   |
| 5   | 3  
   
   | 2   
   
  | 1  | 1   | 4   | 3   | 3   | 0   | 0   | 5   | 0   | 1                             
   | 0   | 1   | 0   
   | 1   | 10  | 1.6   | 10   | 0   | 2.5  
  | 3                                     | 27.1   | 60.2   |
| 5   | 3  
   
   | 2   
   
  | 5  | 5   | 1   | 1   | 1   | 6   | 4   | 0   | 5   | 4                             
   | 1   | 1   | 2   
   | 2   | 10  | 8   | 3  | 2   | 2.5  
  | 10                                    | 35.5   | 79   |
| 5   | 3  
   
   | 2   
   
  | 1  | 1   | 4   | 3   | 3   | 0   | 0   | 5   | 5   | 4                             
   | 1   | 1   | 2   
   | 2   | 10  | 1.6   | 10   | 0   | 5  
  | 10                                    | 36.6   | 81.2   |
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# Table 2: Characteristics of states systems influencing their effectiveness (K)

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Literature

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/ 2 / IAEA Document INFCIRC/153 (to be published)

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<u>/</u>4\_7 Aufbau-Seminar Systemtechnik. Technische Universität Berlin (1969)

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An Optimized Material Accounting System for Safeguards in a Nuclear Fuel Cycle.

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# <u>Annex 1</u>

# Supplementary Information on the Quantification of Criteria

# 1.1 <u>Consideration of other Possibilities for the Quantification</u> of Criteria. Numerical Example

In chapter 3 the following relation for the reduction of the verification efforts with the help of the criteria given in  $\frac{\sqrt{2}}{\sqrt{2}}$  had been suggested

 $\lambda = 1 - \frac{1-\gamma}{1+m} - (\kappa + m\mu)$  (1.1)

Here,  $\kappa$  is the effectivity of the national (regional) system,  $\mu$  the nondivertibility of the material, m the relative weightage of effectivity and non-divertibility and  $\gamma$  the rest verification effort in the case  $\kappa=\mu=1$ . The main characteristics of formula (1.1) is its simplicity: It is linear in  $\kappa$ and  $\mu$ .

An alternative formula can be given by starting with the expression

$$\overline{\overline{\lambda}}_{1} = (1-\kappa)(1-\mu) \qquad (1.2)$$

Introduction of the relative weightage m leads to

$$\overline{\lambda}_{1} = (1-\kappa)(1-m\mu)$$
(1.3)  
for  $0 \le m \le 1$ 

If furthermore the rest verification effort  $\gamma$  is introduced, it leads to the following alternative formula

$$\lambda_{1} = (1-\gamma)(1-\kappa)(1-m\mu)+\gamma$$
 (1.4)

Compared to formula (1.1)it can be seen, that it is more complicated with respect to  $\kappa$  and  $\mu$ , and is not linear in  $\kappa$  and  $\mu$ . An advantage of formula (1.4) might be that it contains no denominator as it is the case in formula (1.1).

For the case

$$\gamma = m = 0.1; \kappa = 0.81; \mu = 0.22$$
 (1.5)

(which appears to be representative for a large number of cases) the values of  $\lambda$  obtained with the two different formulas (1.1) and (1.4), do not differ very much. To test this, deviations of the  $\kappa$  and  $\mu$  values, as shown in Fig. 1.1 have been considered.



The following results are obtained for the different values chosen:

- (1)  $\kappa = 0.81$   $\mu = 0.22$  $\lambda = 0.32$   $\lambda_1 = 0.27$
- (2)  $\kappa = 0.7$   $\mu = 0.26$  $\lambda = 0.41$   $\lambda_1 = 0.36$
- (3)  $\kappa = 0.7$   $\mu = 0.18$  $\lambda = 0.41$   $\lambda_1 = 0.37$
- (4)  $\kappa = 0.9$   $\mu = 0.26$  $\lambda = 0.24$   $\lambda_1 = 0.19$
- (5)  $\kappa = 0.9$   $\mu = 0.18$  $\lambda = 0.25$   $\lambda_1 = 0.19$

It is evident, that in the vicinity of the point (0.81, 0.22) in the  $(\kappa,\mu)$ -plane, the values of  $\lambda$  obtained by the two different formulas are not very different.

1.2 Estimation of the Numerical Values of m and  $\gamma$ 

The numerical values of m and  $\gamma$  can be estimated only with the help of special numerical examples.

(a) Numerical value of the relative weightage m of effectivity and nondivertibility.

This value can be fixed with the help of the fuel cycle examples given in table 1. The partial fuel cycle No. 2 consisting of only one conversion plant may be considered. As shown in  $\sqrt{3}$  a full coverage for this facility means independent verification of 60 analyses per year. The maximum number of inspection man days is given according to  $\sqrt{2}$  to be

100 + 0.4 = 94 = 140 Insp.man days/year

as the throughput per year in this plant is 940 t nat. U = 94 eff. kg of U and this amount is greater than the inventory of the plant.

The non-divertibility factor of this plant is  $\mu = 0.82$ .

This is one of the two highest values of  $\mu$  for the fuel cycles considered here. These values of non-divertibilities coupled with a fully effective State's system ( $\kappa$ =1) should therefore, require the minimum possible (not zero) verification effort  $\lambda$ . (The rest verification effort  $\gamma$  does not come into the picture as yet.) For the cases considered here the minimum verification effort may be taken to be the verification of one analysis per year or verification of 1% of the total number of analyses per year, whichever value is the more reasonable.

For the conversion plant considered here, 60 analyses are to be verified per year for full coverage, for which 140 insp. man days are available. Assuming that virtually the whole of these man days will be required for the verification of the different steps involved in the analyses, it can be argued that 140/60≈3 man days will be required per analysis.

The minimum verification effort in this case is the verification of <u>one</u> analysis per year. (1% is less than one analysis hence not reasonable.) Expressed as the fraction of time

$$\lambda_{\min} = \frac{3}{140} = 0.021 \tag{1.6}$$

Therefore, according to formula (1.1), with

a value of

$$\gamma = 0; \kappa = 1; \mu = 0.82,$$
  
 $n = 0.14$ 

is obtained.

Another example is the fuel cycle No. 9 in table 1. This consists of one conversion plant, light water reactors and one fabrication plant for low enriched uranium, and has the highest non-divertibility factor of  $\mu = 0.91$ . Since these non-divertibility factors can be considered facility wise (the maximum routine inspection efforts are calculated per facility), it is justifiable to consider the case of the fabrication plant alone. Here a single analysis per year would not give any reasonable verification as feed and product streams have to be verified individually and the number of items to be measured in each of these streams are very high. Hence verification of 1 % of the total number of analyses has been taken to be the minimum effort required. Therefore

 $\lambda = 0.01$ ; and with  $\kappa = 1$ ;  $\mu = 0.91$ 

according to (1.1)

$$m = 0.12$$
 (1.8)

(1.7)

Having these two estimations of m in mind a value of

$$m = 0.10$$
 (1.9)

has been proposed for use in equ. (1.1). An important consequence of this relatively low value of m is that the influence of the non-divertibility of the material on the actual verification efforts has been reduced considerably, so that, the actual verification efforts are governed mainly by the effectiveness of the States (regional) safeguards system. (b) Numerical value of the rest verification effort  $\gamma$  in case of  $\kappa = \mu = 1$ .

It is assumed that the rest verification effort  $\gamma$  in case of  $\kappa = \mu = 1$  is given by the game theoretical value, that means by that value which induces the operator to act legally  $\int 3_{-}7$ . In this connection it is assumed that the gain of the operator in case of successful diversion is smaller than the loss of the operator in case of a detected diversion. The following numerical results were obtained in Ref.  $\int 3_{-}7$ :

Conversion plant II (natural U)	γ = 0.02
Fabrication plant (3% enriched U)	$\gamma = 0.004 \text{ (input)}^{1)}$ $\gamma = 0.0001 \text{ (output)}^{1)}$
Reprocessing plant	$\gamma = 0.08$ (input)
	$\gamma = 0.03$ (U output)
	$\gamma = 0.099$ (Pu output)
Isotope separation plant	$\gamma = 0.006 (input)^{1}$
·	$\gamma = 0.067 (output)^{1}$

It can be seen that two groups of  $\gamma$ -values are obtained: One group with  $\gamma$ -values between 7 and 10% and one group with  $\gamma$ -values below 2%. As in the first group the Pu-verification is contained and this seems to be the more important of the two, a rest verification effort of

 $\gamma = 0.1$  (1.10)

has been assumed for all types of fuel cycles, to be on the safe side.

<sup>1)</sup>These values differ from those given in Ref. /3/, as there a different definition for effective kg in the case of uranium with an enrichment of 3% and 1% has been used.

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