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NEGATIVE EDDY DIFFUSIVITIES FOR ASYMMETRIC TURBULENT VELOCITY PROFILES?

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Abstract—It appears from the experimental results obtained by quite a number of authors that for asymmetric turbulent velocity profiles the locations of maximum velocity and zero shear stress are not identical. This report deals with the importance of this effect for the calculation of momentum, heat, and mass transfer in non-circular channels as well as for the discussion on universal velocity profiles. The report is providing a survey of the works referring to this effect. These informations are supplemented by unpublished results derived from the critical interpretation of measurements carried out by several authors.

An empirical relation is indicated which allows to estimate the magnitude of the effect studied.

NOMENCLATURE

b, channel width;
b_1, distance between the wall and the position of U_{max};
b_2, distance between the wall and the position of U_{max};
b'_1, distance between the wall and the position of $\tau = 0$;
b'_2, distance between the wall and the position of $\tau = 0$;
c_p, specific heat;
h, height of roughness;
h^+, height of roughness, dimensionless, $\rho h u^*/\mu$;
k, thermal conductivity;
q, heat flux;
$\Delta P/\Delta x$, pressure drop;
$R(h^+)$, roughness function equation (10);
r, radius;
u, time average velocity in x-direction;
u^+, dimensionless velocity, u/u^*;
u', instantaneous velocity fluctuation in x-direction;
v', instantaneous velocity fluctuation in y-direction;
u_s^*, friction velocity (smooth wall);
u_r^*, friction velocity (rough wall);
x, direction of flow;
y, distance from wall;
y^+, distance from wall, dimensionless, $\rho y u^*/\mu$.

Greek letters

α, annulus parameter, r_1/r_2;
β_0, annulus parameter, r_0/r_2;
β_{max}, annulus parameter, r_{max}/r_2;
ε_H, eddy diffusivity for heat;
ε_M, eddy diffusivity for momentum;
ϑ, temperature;
μ, viscosity;
ρ, density;
τ, shear stress.

Subscripts

0, line of zero shear;
1, inner tube;
2, outer tube;
max, line of maximum velocity.

1. INTRODUCTION

TURBULENT flows in annuli with very small diameter ratios, in annuli roughened on one side or between parallel plates roughened on one side are characterized by asymmetric velocity distributions, which is shown schematically in Fig. 1. Contrary to the results obtained from the investigation of turbulent flows in circular pipes we observe in these cases that the locations of maximum velocity and zero shear stress do not coincide. This effect deserves attention, since, under this condition, it is no longer reasonable to apply the generally used method of calculating velocity and temperature distributions in non-circular channels by using "eddy diffusivities". Besides, this effect is very important with respect to the frequently discussed problem of "universal velocity laws" in non-circular channels.

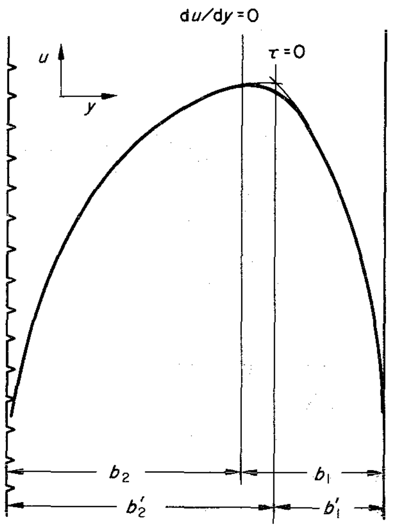

FIG. 1. Asymmetric turbulent velocity profile.

This article indicates the experimental results obtained by several authors in the direct measurement of this effect. Moreover, an appropriate analysis of experimental investigations allows to obtain results which confirm also this effect.

2. THE IMPORTANCE OF NON-COINCIDENCE OF $\tau = 0$ AND $\mathrm{d}u/\mathrm{d}y = 0$

Figure 1 is a schematic representation of an asymmetric turbulent velocity profile occurring, e.g. between parallel plates provided that one side is rough and the other smooth. The difference of locations of $\mathrm{d}u/\mathrm{d}y = 0$ and $\tau = 0$ appearing from the figure is such that $\tau = 0$, at a certain distance from $\mathrm{d}u/\mathrm{d}y = 0$, is situated on the narrower side of the velocity profile:

$$\frac{b_1}{b_2} > \frac{b_1'}{b_2'}. \tag{1}$$

Usually, the turbulent momentum transport is related to the velocity gradient through the eddy diffusivity ε_M defined in equation (2):

$$\tau = \mu\frac{\mathrm{d}u}{\mathrm{d}y} + \rho\overline{u'v'} = (\mu + \rho\varepsilon_M)\frac{\mathrm{d}u}{\mathrm{d}y}. \tag{2}$$

It is noted that in the example represented in Fig. 1 the eddy diffusivity would become negative between the locations of $\mathrm{d}u/\mathrm{d}y = 0$ and $\tau = 0$; hence, in this case, this method of describing the transfer of momentum is no longer reasonably applicable.

Mathieu [1] was probably the first to measure in a direct way the difference of locations $\mathrm{d}u/\mathrm{d}y = 0$ and $\tau = 0$ when measuring the distribution of velocities and shear stresses in an inclined jet impinging on a plate. Mathieu recognizes the problems arising from this phenomenon for the calculation of the heat transport in equation (3), where for $Pr = 1$ it is often assumed that $\varepsilon_H = \varepsilon_M$.

$$q = (k + \rho c_p \varepsilon_H)\frac{\mathrm{d}\vartheta}{\mathrm{d}y}. \tag{3}$$

Another reason calling for the study of the magnitude of this effect is its impact on the results obtained by measurements of the velocity profile and the discussion on universal velocity profiles, at least in the vicinity of the wall. Brighton and Jones [2] refer to the publication of Mathieu [1] and Eskinazy [3]; they measure both the velocity distribution and the distribution of shear stress in smooth annuli down to very small diameter ratios which account for a very asymmetric velocity profile. The measured values of shear stress distribution obtained with a hotwire anemometer are scattered to such an extent that neither coincidence, as stated in [2], nor non-coincidence of the locations of u_{max} and $\tau = 0$ can be deduced. Since Brighton and Jones [2] assume coincidence, they calculate the wall shear stresses from the measured location of u_{max}. Due to the considerable change of wall shear stress at the inner pipe with little difference only of the locations of u_{max} and $\tau = 0$ Brighton and Jones obtain velocity distributions for the inner zone of the annuli investigated which differ appreciably from the law suggested by Nikuradse [4] for the circular pipe and expressed by equation (4).

$$u^+ = 2{\cdot}5 \ln y^+ + 5{\cdot}5. \tag{4}$$

The theoretical models for the calculation of velocity distribution in annuli established by Eifler [5] and Levy [6] are mainly derived from the measurements made by Brighton and Jones and therefore these authors obtain also velocity profiles for the inner zone of annuli which become flatter with decreasing diameter ratio. On the other hand, Quarmby [7] measures the wall shear stress directly using Preston tubes [8] and he finds but insignificant deviations from equation (4); the same is found by Smith, Lawn and Hamlin [9].

Quarmby [7] supposes to have determined the "line of maximum velocity"—he actually determines $\tau = 0$—and the differences he finds in comparison with the results of Brighton and Jones lead him to the conclusion that the measurements performed by Brighton and Jones (double Pitot tube to determine u_{max}) are not correct.

All these examples demonstrate how much also the discussion on "universal velocity profiles" is influenced by experimental results the interpretation of which depends on the assumed location of $\tau = 0$, provided that this location is different from that of u_{max}.

3. EXPERIMENTAL RESULTS DERIVED FROM LITERATURE

Brighton and Jones [2] also refer to the results suggested by Eskinazy [3] who observes the non-coincidence of $\overline{u'v'} = 0$ and $du/dy = 0$. However, the effect was measured in a curved channel, i.e. for a three-dimensional flow, whilst in our case the occurrence of this effect is discussed for one-dimensional flows.

A significant difference of the locations $\tau = 0$ and $du/dy = 0$ was measured directly by Kjellström and Hedberg [10] for concentric annuli with rough inner tube. Also, in [10] results are reported of similar measurements made in smooth annuli. Here a significant difference of locations could not be measured; however, the diameter ratio was $\alpha = 0{\cdot}446$ and, consequently, the velocity profile was only slightly asymmetric. However, for small diameter ratios we have found an indirect proof by comparison of results indicated by Barthels [11] with the results of other authors, that the effect discussed here occurs also in smooth annuli. Barthels determined the wall shear stresses with Preston tubes and found the following empirical relation for the location $\tau = 0$:

$$\frac{\beta_0^2 - \alpha^2}{\alpha(1 - \beta_0^2)} = \left(\frac{1 - \beta_0}{\beta_0 - \alpha}\right)^{0{\cdot}2}. \tag{5}$$

Kays and Leung [12] suggest the following empirical expression for the location $du/dy = 0$:

$$\frac{\beta_{max} - \alpha}{1 - \beta_{max}} = \alpha^{0{\cdot}343}. \tag{6}$$

When comparing the β_0-values calculated from (5) with the β_{max}-values calculated from (6) increasing deviations are observed with decreasing α which can be explained by the growing asymmetry (Table 1).

Asymmetric velocity profiles were found also by Wilkie *et al.* [13] when measuring between rough parallel plates. Wilkie measured pressure drop and velocity distribution and determined the wall shear stress from the measured location of u_{max}. He found a discrepancy in the absolute

Table 1. *Comparison for smooth annuli*

α	β_0 equation (5)	β_{max} equation (6)
0·001	0·0432	0·0865
0·01	0·1223	0·1792
0·02	0·1670	0·2231
0·04	0·2278	0·2790
0·1	0·3430	0·3810
0·2	0·4681	0·4923
0·4	0·6430	0·6532
0·8	0·8954	0·8962

value of the calculated friction factors, depending on whether a smoother or rougher wall is opposite the tested rough surface. Wilkie concludes that the location of u_{max} cannot agree with that of $\tau = 0$.

Lawn and Hamlin [14] measure also significant differences for internally roughened annuli. Besides, Smith, Lawn and Hamlin [9] find the non-coincidence of $\tau = 0$ and $du/dy = 0$ when measuring in a smooth annulus with $\alpha = 0·088$. The wall shear stress is measured here directly using a method which is independent of the universal velocity profile.

4. INTERPRETATION OF THE VALUES MEASURED BY SCHLICHTING

Schlichting [15] investigated a very great number of dissimilar roughnesses in a channel which was roughened on one side. He measured velocity distributions with particular regard to the effect exerted by roughness on the parameters of the velocity profiles. In the following paragraphs the experimental results of Schlichting are examined under the aspect discussed in this article. The measure of asymmetry will be assumed to be the ratio of distances of u_{max} and $\tau = 0$, respectively, from the channel walls.

Schlichting obtains the friction velocity u_s^* on the smooth wall by plotting the measured velocity distribution and assuming that the dimensionless velocity profile [equation (4)] has the slope 2·5. This assumption is certainly correct since it is confirmed by a sufficient number of measured results for a plane wall. From the experimental results indicated by Schlichting ([15], Table 3) the location $\tau = 0$ can be calculated as follows with u_s^* and the pressure gradient

$$b_1' = \frac{u_s^{*2}}{(1/\rho)\,\Delta P/\Delta x}. \tag{7}$$

In this calculation neither a universal profile with the slope 2·5 is assumed on the rough wall nor is a statement made on the location of $y = 0$ on the rough wall. The location of $\tau = 0$ calculated with equation (7) was determined for all experimental results. In Table 2 the values for $b_1'/b_2'(\tau = 0)$ and $b_1/b_2(u_{max})$ are listed, which are averaged for each roughness.

A significant difference of the two locations is observed, which increases with growing asymmetry of velocity profiles. For comparison the location of $\tau = 0$ which was taken as the basis of calculations made by Schlichting is listed

Table 2. *Interpretation of Schlichting's results*

Plate No.	$R(h^+)$	b_1/b_2 (u_{max})	b (cm)	Equation (7) b_1'/b_2' $(\tau = 0)$	Equation (8) b_1'/b_2' $(\tau = 0)$
XII	12·18	0·6063	3·99	0·4651	0·4674
III	8·93	0·4500	3·99	0·3782	0·3512
I	5·68	0·3237	3·96	0·2022	0·2011
II	5·15	0·3079	3·88	0·1866	0·1675
V	9·65	0·4603	3·68	0·4195	0·4066
VI	8·98	0·5133	3·99	0·4207	0·4112
IV	5·27	0·3699	3·97	0·2480	0·2368
XIII	13·83	0·7260	3·99	0·7283	0·7055
XIV	12·70	0·6681	3·99	0·7285	0·6626
XV	9·89	0·6542	3·98	0·4411	0·4233
XIX	7·64	0·4620	3·85	0·3454	0·3407
XXIII	13·07	0·7213	3·99	0·6094	0·5780
XXIV	10·55	0·6016	3·98	0·4529	0·4138
XXV	8·50	0·5173	3·95	0·3290	0·3055
XVI	8·57	0·5686	4·0	0·3190	0·3079
XVIII	6·67	0·4378	4·0	0·2008	0·2004
XVII	4·53	0·3711	3·99	0·1613	0·1581
XX	4·18	0·2365	3·90	0·0877	0·0954
XXI	2·28	0·1906	3·96	0·0793	0·0785
XXII	2·33	0·1952	3·96	0·0631	0·0630
IX	7·22	0·4567	3·87	0·4129	0·3950

also in Table 2. This result is obtained by the following relations

$$\frac{b_1'}{b_2'} = \left(\frac{u_s^*}{u_r^*}\right)^2 \tag{8}$$

and

$$b_1' + b_2' = b \tag{9}$$

the width b being the distance of the smooth wall from a fictitious rough wall obtained by evening out the roughnesses through melting. The comparison of results obtained by equations (7) and (8) yields but small differences. This result confirms the assumptions made by Schlichting with respect to the channel width b and the slope 2·5 of a universal velocity profile over the rough wall too [equation (10)].

$$u^+ = 2{\cdot}5 \ln (y/h) + R(h^+). \tag{10}$$

Figure 2 contains all individual results derived from the investigations of Schlichting. It is clearly evident that with a strongly asymmetric velocity profile (b_1/b_2 small) the location of the line $\tau = 0$ is still more asymmetric ($b_1'/b_2' < b_1/b_2$). Only 11 of the 118 measured points show an inverse effect the majority of them for only very slightly asymmetric velocity profiles where this effect becomes very small and where it is difficult to determine it precisely.

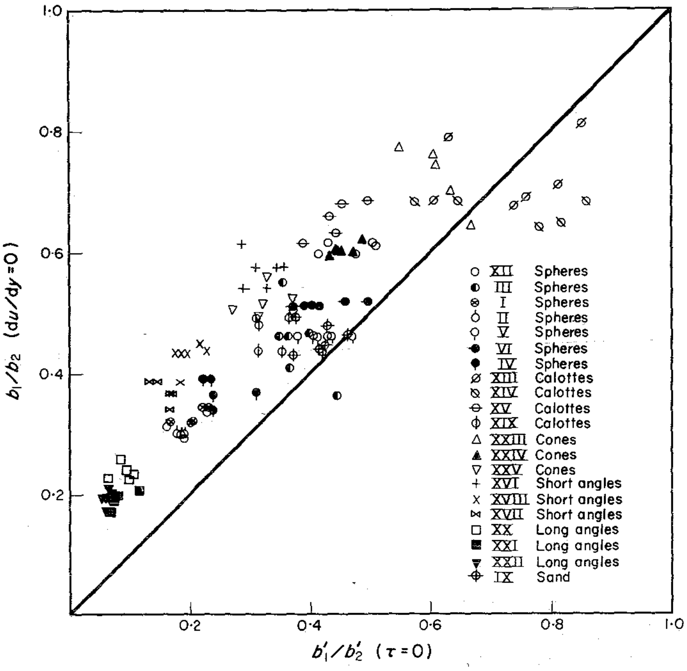

FIG. 2. Interpretation of Schlichting's results [15].

There is a relatively wide scattering of measured results. One reason may be seen in the difficulty to measure shear stresses and velocity maxima. Besides, the difference between the locations is often very small and, consequently, incertainties in the measurement will have a considerable impact. The Reynolds number will also have a slight influence on this effect. Considering the various kinds of roughnesses

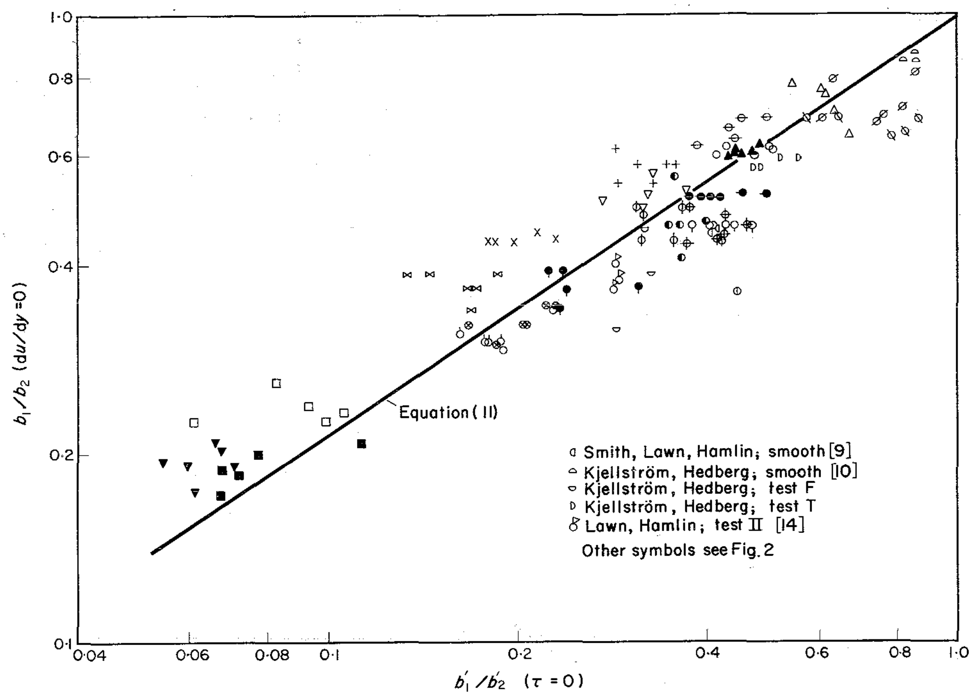

FIG. 3. Compilation of measured results discussed.

represented in Fig. 2 it appears that cones and "short angles" behave in a different way compared to spheres. It can be supposed that not only the degree of asymmetry but also the form of roughness will exert a noticeable influence on this phenomenon.

In order to find a relationship serving as an orientation with respect to the magnitude of the described effect, Fig. 3 shows the measured results mentioned above for smooth and rough annuli in addition to the results found by Schlichting.

The measured points are fitted by a curve which satisfies the following equation:

$$\frac{b_1'}{b_2'} = \left(\frac{b_1}{b_2}\right)^{1\cdot 5}. \tag{11}$$

Using this equation the difference of the locations of u_{max} and $\tau = 0$ can be estimated for asymmetric velocity profiles.

5. CONCLUSIONS

It follows as a conclusion from the experimental results of many works that $\tau = 0$ and $du/dy = 0$ do not coincide in cases where asymmetric velocity profiles are obtained for turbulent flows as a result of the channel form or roughnesses on one side. In these cases, the eddy diffusivity will adopt a negative value, provided the commonly used concept, which had been developed from measurements of symmetric velocity profiles, will be applied to describe the turbulent momentum transport.

Since negative eddy diffusivities are physically absurd, a model must be developed which will meet with this condition. Further experiments should be carried out because of the fundamental importance of this question with respect to the calculation of velocity and temperature distributions in turbulent flows, the interpretation of experiments and the question of a universal velocity profile. Only additional and more accurate measured results will allow to describe satisfactorily this effect.

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DIFFUSIVITÉS NÉGATIVES PAR TURBULENCE POUR DES PROFILS DE VITESSES DISSYMÉTRIQUES

Résumé—Les résultats expérimentaux obtenus par un assez grand nombre d'auteurs montrent que pour des profils de vitesse turbulente dissymétriques, le maximum de vitesse et le zéro de contrainte tangentielle ne se produisent pas au même point. Cet article traite de l'importance de cet effet pour le calcul du transfert de quantité de mouvement, de chaleur et de masse dans des canaux non circulaires aussi bien que pour la discussion sur les profils de vitesse universels. Il analyse les travaux se référant à cet effet. Ces informations sont complétées par des résultats non publiés dérivés de l'interprétation critique des mesures menées par plusieurs auteurs. On indique une relation empirique qui permet d'estimer la grandeur de l'effet étudié.

NEGATIVE WIRBELAUSBREITUNG FÜR UNSYMMETRISCHE TURBULENTE GESCHWINDIGKEITSPROFILE

Zusammenfassung—Aus experimentellen Ergebnissen einer Anzahl von Autoren wird ersichtlich, dass bei unsymmetrischen turbulenten Geschwindigkeitsprofilen der Ort der maximalen Geschwindigkeit und der Scherspannung null nicht identisch sind. Dieser Bericht behandelt die Wichtigkeit dieses Einflusses auf die Berechnung von Impuls-, Wärme- und Stofftransport in nicht kreisförmigen Kanälen als auch auf die Diskussion über universelle Geschwindigkeitsprofile.

Der Bericht bringt einen Überblick der Arbeiten, die sich mit diesem Einfluss beschäftigen. Diese Informationen werden ergänzt durch unveröffentlichte Ergebnisse, die aus kritischen Betrachtungen der von mehreren Autoren ausgeführten Messungen abgeleitet sind. Es wird eine Beziehung angegeben, die es erlaubt, die Grössenordnung des untersuchten Einflusses abzuschätzen.

ОТРИЦАТЕЛЬНЫЕ КОЭФФИЦИЕНТЫ ВИХРЕВОЙ ДИФФУЗИИ ДЛЯ АСИММЕТРИЧНЫХ ПРОФИЛЕЙ ТУРБУЛЕНТНОЙ СКОРОСТИ

Аннотация—Из экспериментальных результатов, полученных рядом авторов, следует, что для асимметричных профилей турбулентной скорости местоположения максимальной скорости и нулевого напряжения сдвига не идентичны.

В настоящей работе рассматривается применение этого эффекта для расчета переноса количества движения, тепла и массы в некольцевых каналах, а текже для рассмотрения универсальных профилей скорости.

В статье приводится обзор работ, в которых рассматривается данный эффект. Эти сведения дополнены еще не опубликованными результатами критического анализа измерений, выполненных несколькими авторами. Указано эмпирическое соотношение, позволяющее рассчитать величину исследуемого эффекта.