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Institut für Experimentelle Kernphysik

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A SIMPLE THEORY OF PROTON-PROTON FINAL STATE INTERACTION IN THE REACTION $d+p \rightarrow p+p+n$

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Zusammenfassung

Die Amplitude für den Aufbruch von Deuteronen durch Protonen wird unter Einschluß der Coulomb-Endzustandswechselwirkung in allen Partialwellen analytisch berechnet. Die Rechnung basiert auf der Annahme einer Zweistufenreaktion, wobei der Aufbruchsmechanismus mit einer zero range-Näherung erfasst wird. Bei der Berechnung der "nuklearen Komponente" der Endzustandswechselwirkung wird unter Verwendung der gleichen Näherung nur der s-Zustand berücksichtigt. Der Vergleich mit den experimentellen Ergebnissen zeigt, daß das Modell dem ursprünglichen Watson Migdal-Ansatz klar überlegen ist.

Abstract

The amplitude for deuteron break-up in deuteron-proton-collision including Coulomb final state interaction (in all partial waves) is derived analytically. The calculation is based on the assumption of a two-step reaction with break-up mechanism of zero range. Nuclear final state interaction is taken into account under the same approximations in s-waves. Comparison with experiments indicates a clear improvement compared to the Watson-Migdal theory.

1. Introduction

It is well known that in consequence of the long range of the Coulomb potential low energy proton-proton scattering cannot be described suitably by a partial wave expansion. As the pure Coulomb scattering amplitude can be evaluated analytically, the appropriate method is a partial wave expansion of the difference of exact and pure Coulomb scattering amplitudes |1|. This difference is conventionally referred to as "nuclear" scattering amplitude.

The relative proton-proton state that occurs after a deuteron break-up induced by a deuteron-proton-collision contains much fewer higher partial waves than a plane wave. The subsequent proton-proton scattering (proton-proton final state interaction) at low relative energies can therefore be described in fair approximation by pure s-wave scattering |2|. However, scattering in higher partial waves is apparently more important than for the analogous neutron-neutron or proton-neutron final state interactions. Deviations of the experimental cross section |3| from the results of pure s-wave final state scattering calculations |4| indicate a non-negligible Coulomb scattering in higher partial waves.

It is demonstrated below that the amplitude for deuteron break-up by protons with pure Coulomb final state interaction - in analogy to Rutherford scattering - can be calculated analytically if

a) the break-up is assumed to be a two-particle mechanism of zero range,

b) Coulomb interaction is neglected in the incoming channel, and

c) the deuteron wave function is described by a Hulthén-type wave function.

The "nuclear" final state scattering amplitude then can be expanded again by partial waves.

The results describe a slightly asymmetric final state peak structure as well as a proton-spectator peak. They have therefore a much wider kinematical range of applicability than the pure s-wave (Watson-Migdal) theory. Nonetheless, they are still analytical.

2. Method of Calculation

Specific reaction mechanisms for three particle reactions are known to be dominant in different kinematical regions. Fig. 1 for instance illustrates the mechanism responsible for deuteron break-up in the reaction $d+p \rightarrow p+p+n$ in regions of large neutron angles and large relative neutron-proton final state momenta. This diagram is defined as representing a matrix element of the following type.

$$\delta(\vec{P}-\vec{Q}-\vec{p}_n)M(\vec{P},\vec{p}_n,\vec{k}) = \delta(\vec{P}-\vec{Q}-\vec{p}_n)\langle \vec{p}_n,\vec{k}\,|T_{23}|\vec{P}\ \rangle =$$

$$= \int d^3r_1\, d^3r_2\, d^3r_3\, e^{-i\vec{p}_n\vec{r}_2}\, e^{-i\vec{Q}(\vec{r}_1+\vec{r}_3)/2}\, \chi_{\vec{k}}^{*(-)}(\vec{r}_1-\vec{r}_3)\times \qquad (1)$$

$$\times\, T_{23}\, \phi_d\, (\vec{r}_1-\vec{r}_2)\, e^{i\vec{P}(\vec{r}_1+\vec{r}_2)/2}$$

The matrix element is given here in the rest system of the free proton before scattering. $\vec{P}$, $\vec{Q}$, $\vec{p}_n$ and $\vec{k}$ are the wave numbers of the incoming deuteron, the final proton-proton center of mass, the outgoing neutron and the final proton-proton subsystem. ϕ_d is the deuteron wave function. $\chi_{\vec{k}}^{(-)}$ denotes the proton-proton scattering wave. T_{23} is the proton-neutron scattering operator.

The graph given by fig. 1 includes both mechanisms conventionally referred to as quasifree scattering (QFS) and final state interaction (FSI) by assuming the reaction to proceed via two steps. The validity of such a two step mechanism has been verified experimentally for a limited kinematical region |5|.

The special kinematical conditions quoted above are expected to guarantee that there is no considerable contribution from p-p QFS and n-p FSI. Of course, the applicability of the model is not restricted to this special case. Similar calculations may be performed in all cases where complementary conditions are met.

Taking into account spins and antisymmetrization leads to the following expression for the differential cross section.

$$\frac{d^3\sigma}{d\Omega_p\, d\Omega_n\, dE_p} = A\cdot \sum_{Ss}(2S+1)(2s+1)\left|\sum_\sigma(-1)^\sigma(2\sigma+1)W(\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2}\,S;\sigma 1)\,\times\right.$$

$$\left.\times\; W(\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2}\,S\,;\sigma s)M_{s\sigma}(\vec{P},\vec{p}_n,\vec{k})\right|^2 \qquad (2)$$

Here, A is a pure kinematical factor. S, s and σ are total spin, and the spins of the final state proton-proton system and proton-neutron scattering (2-3)-system, respectively. The matrix elements $M_{s\sigma}$ depend on σ, because of the spin dependence of the scattering operator T_{23}^σ, and on s by the antisymmetrization:

$$M_{s\sigma}(\vec{P},\vec{p}_n,\vec{k}) = \frac{1}{\sqrt{2}}\left\{M_\sigma(\vec{P},\vec{p}_n,\vec{k})+(-1)^s M_\sigma(\vec{P},\vec{p}_n,\vec{k})\right\} =$$

$$\frac{1}{\sqrt{2}}\left\{\langle\vec{p}_n,\vec{k}|T_{23}^\sigma|\vec{P}\rangle + (-1)^s\langle\vec{p}_n,-\vec{k}\,|T_{23}^\sigma|\vec{P}\rangle\right\} \qquad (3)$$

Written explicitely, eq. (2) becomes

$$\frac{d^3\sigma}{d\Omega_p\, d\Omega_n\, dE_p} = A\cdot\left\{\frac{9}{8}\,|M_{oo} + M_{o1}|^2 + \frac{3}{8}\,|3M_{1o} - M_{11}|^2 + 12|M_{11}|^2\right\} \qquad (4)$$

The proton-proton scattering wave function $\chi_{\vec{k}}(\vec{r})$ can be written as the sum of a regular Coulomb wave function $\chi_{\vec{k}}^{Cb}$ and a correction term $\chi_{\vec{k}}^{Nuc}$ due to nuclear interaction

$$\chi_{\vec{k}}(\vec{r}) = \chi_{\vec{k}}^{Cb}(\vec{r}) + \chi_{\vec{k}}^{Nuc}(\vec{r}) \qquad (5)$$

$$\chi_{\vec{k}}^{Cb}(\vec{r}) = e^{-\frac{\pi\eta}{2}}\,\Gamma(1+i\eta)e^{i\vec{k}\vec{r}}\,F\left[-i\eta\,|1|\,i(kr - \vec{k}\vec{r})\right]$$

The parameter η is defined by $\eta=(2kR)^{-1}$ with R=28.8 Fm, the proton Bohr radius. For further definitions concerning the Coulomb wave functions see |6|. Outside the range of nuclear forces the correction can be expanded

$$\chi_{\vec{k}}^{Nuc}(\vec{r}) = - \frac{1}{2kr} \sum_{0}^{\infty} (2\ell+1) i^{\ell+1} e^{i\sigma_\ell} (e^{2i\delta_\ell} - 1) \times \tag{6}$$

$$\times (G_\ell(kr) + iF_\ell(kr)) P_\ell(\cos \Theta_{\vec{k}\,\vec{r}})$$

Here δ_ℓ and σ_ℓ are the "nuclear" and the pure Coulomb phase shifts, respectively.

According to eq. (5), also the matrix elements $M_{s\sigma}$ can be decomposed into "Coulomb" and "nuclear" parts $M_{s\sigma}^{Cb}$ and $M_{s\sigma}^{Nuc}$.

$$M_{s\sigma}(\vec{P},\vec{p}_n,\vec{k}) = M_{s\sigma}^{Cb}(\vec{P},\vec{p}_n,\vec{k}) + M_{s\sigma}^{Nuc}(\vec{P},\vec{p}_n,\vec{k}) \tag{7}$$

More precisely, M^{Cb} describes contributions without final state interaction (i.e., essentially the spectator mechanism) and with pure Coulomb final state interaction whereas M^{Nuc} contains nuclear and mixed final state interactions. The "Coulomb" matrix element M^{Cb} does not vanish for vanishing Coulomb coupling and therefore will more appropiately be called "spectator" matrix element M^{Sp} hereafter.

This spectator matrix element can be evaluated analytically if T_{23}^{σ} is replaced by a δ-function.

$$\begin{aligned} T_{23}^{0} &= V_0 \delta(\vec{r}_1-\vec{r}_2) \\ T_{23}^{1} &= \alpha \cdot V_0 \cdot \delta(\vec{r}_1-\vec{r}_2) \end{aligned} \tag{8}$$

and if the deuteron wave function is represented by a Hulthén-type function.

$$\phi_d(\vec{r}) = N \cdot \frac{e^{-\beta_1 r} - e^{-\beta_2 r}}{r} \tag{9}$$

with the normalization constant N given by

$$N = \left[\frac{\beta_1\ \beta_2\ (\beta_1 + \beta_2)}{2\pi\ (\beta_1 - \beta_2)^2}\right]^{\frac{1}{2}}$$

This procedure is in the spirit of both the Watson-Migdal theory and the usual impulse approximation, which assume that the momentum dependence of the matrix element is essentially due to the final state interaction or to the momentum distribution of the deuteron respectively. As the results of this paper will be applied to a larger kinematical range than the final state interaction region, the validity of the zero range approximation will of course have to be tested anew. It is expected to hold in a limited range of medium energies.

With the above assumptions the (unsymmetrized) spectator matrix element assumes the form

$$M^{Sp}_{\sigma=o} = M^{Cb}_{\sigma=o} = (2\pi)^3 V_o N\ e^{-\frac{\pi\eta}{2}} \Gamma(1+i\eta) \int d^3r\ \frac{e^{i(\vec{q}-\vec{k})\vec{r}}}{r} \times$$

$$\times\ (e^{-\beta_1 r} - e^{-\beta_2 r})\ F|-i\eta\ |1|\ i(kr + \vec{k}\vec{r})| \tag{10}$$

where $\vec{q}$ is defined by $\vec{q}=\vec{p}_n/2$

Here the time reversal relation $\chi^{(-)}_{\vec{k}}(\vec{r}) = \overset{*}{\chi}_{-\vec{k}}(\vec{r})$ has been used.

The integral can be evaluated by applying the method of Sommerfeld |7| or Morinigo |8|

$$\int d^3r\ \frac{1}{r}\ e^{+i(\vec{q}-\vec{k})\vec{r}-\beta r} F(-i\eta|1|i(kr+\vec{k}\vec{r}))=$$

$$\frac{4\pi}{(\vec{q}-\vec{k})^2+\beta^2} \left\{ \frac{\beta^2+q^2-k^2-2ik\beta}{(\vec{q}-\vec{k})^2+\beta^2} \right\}^{i\eta} \tag{11}$$

This leads to the following expression for the spectator matrix element.

$$M^{Sp}_{\sigma=0} = \bar{M}^{Sp}_{\sigma=0}(\beta_1) - \bar{M}^{Sp}_{\sigma=0}(\beta_2) \tag{12 a}$$

with

$$\bar{M}^{Sp}_{\sigma=0}(\beta) = 2(2\pi)^4 V_o N\, e^{i\sigma_o} C_o(\eta)\left[\frac{1}{(\vec{k}-\vec{q})^2 + \beta^2}\right]\times \tag{12 b}$$

$$\times\left[\frac{\beta^2 + (\vec{k}-\vec{q})^2 - 2\vec{k}(\vec{k}-\vec{q}) - 2ik\beta}{(\vec{k}-\vec{q})^2 + \beta^2}\right]^{i\eta}$$

where the relation

$$e^{-\pi\eta/2}\Gamma(1+i\eta) = C_o(\eta)e^{i\sigma_o}$$

has been used. The "Coulomb penetration factor" C_o is defined by $C_o^2=2\pi\eta/(\exp(2\pi\eta)-1)$

According to eq. (8) one obtains

$$M^{Sp}_{\sigma=1} = \alpha\cdot M^{Sp}_{\sigma=0} \tag{12 c}$$

This matrix element describes a spectator peak at $\vec{k}=\vec{q}$ modified by Coulomb interaction. In particular it becomes zero for vanishing relative proton-proton momentum. It should be noted, that the angular dependence does not vanish for $k\to 0$ ($\eta\to\infty$), as

$$\lim_{k\to o}\left[\frac{q^2-k^2+\beta^2-2ik\beta}{(\vec{q}-\vec{k})^2+\beta^2}\right]^{i\eta} = \exp\left[2\eta\,\frac{k\beta+\vec{q}\vec{k}}{q^2+\beta^2}\right] \tag{13}$$

For $\eta \to o$ (k fixed) one obtains the well known expression valid for neutron-deuteron scattering.

Nuclear final state interaction will be assumed here to be relevant only in s-waves. We did not succeed in finding in analogy to Equ. 11 a closed analytic expression for the radial integral of

$$M^{Nuc}_{\sigma=o} = (2\pi)^3 V_o N \int d^3r\, e^{i\vec{q}\vec{r}}\, \chi^{Nuc}_{-\vec{k}}(\vec{r})\, \frac{e^{-\beta_1 r} - e^{-\beta_2 r}}{r}$$

$$= -i(2\pi)^4 V_o \frac{N}{k} e^{i\sigma_o}(e^{2i\delta_o}-1) \times \tag{14}$$

$$\times \int dr\, j_o(qr)(G_o(kr)+iF_o(kr))(e^{-\beta_1 r} - e^{-\beta_2 r})$$

This expression remains after substituting the lowest term of expansion (6) into eq. (1) The power expansion of the spherical Coulomb functions leads to a series converging for $k < \sqrt{q^2 + \beta^2}$. For $k > \sqrt{q^2 + \beta^2}$ the approximation $(G_o+iF_o) \approx e^{ikr}$ may be used, because in this region a) the parameter η is small, b) the spectator matrix element predominates. One obtains in this case

$$M^{Nuc}_{\sigma=o} = M^{Nuc}_{\sigma=o}(\beta_1) - M^{Nuc}_{\sigma=o}(\beta_2)$$

$$\tag{15}$$

$$M^{Nuc}_{\sigma=o}(\beta) = -i(2\pi)^4 V_o N\, e^{i\sigma_o} \left(\frac{e^{2i\delta_o} - 1}{C_o k}\right) \frac{1}{2q}\left[\left\{ \operatorname{arctg} \frac{q+k}{\beta} + \operatorname{arctg} \frac{q-k}{\beta}\right\} + \right.$$

$$\left. + \frac{i}{2} \ln \frac{(q+k)^2+\beta^2}{(q-k)^2-\beta^2} \right]$$

This approximation is valid for $\eta \ll 1$ and used for $k > \sqrt{q^2 + \beta^2}$.

Otherwise the power expansion |9|

$$G_o(kr) + iF_o(kr) = C_o^{-1} \sum_{n=o}^{\infty} \left\{ 2\eta\ (2\gamma - 1 + h(\eta) + \frac{iC_o^2}{2\eta}) A_n + a_n \right\} (kr)^n + \\ + 2\eta\ A_n (kr)^n \ln(2\eta kr) \qquad (16)$$

with the subsequent simple recursion relations can be used.

$$A_n = \frac{2\eta\ A_{n-1} - A_{n-2}}{n(n-1)}$$

$$a_n = \frac{2\eta\ a_{n-1} - a_{n-2} - 2\eta(2n-1)A_n}{n(n-1)} \qquad (16\ b)$$

$$A_o = 0,\ A_1 = 1,\ a_o = 1 \text{ and } a_1 = 0$$

The parameter γ is Euler's constant. The special function $h(\eta)$ is defined as

$$h(\eta) = \mathrm{Re}\ [\Gamma'(-i\eta)/\Gamma(-i\eta)| - \ell n\ \eta = -\ell n\ \eta - \gamma + \eta^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+\eta^2)}$$

The expansion (16) leads to the following integrals.

$$I_n^{(1)} = \int dr\ \frac{\sin qr}{qr}\ e^{-\beta r} (kr)^n \qquad (17\ a)$$

for $n \geq 0$

$$I_n^{(2)} = \int dr\ \frac{\sin qr}{qr}\ e^{-\beta r} (kr)^n \ln(2\eta kr)$$

for $n > 0$

(17 b)

They can be evaluated according to |10|

$$I_n^{(1)}(\beta) = \begin{cases} \frac{1}{q}\,\mathrm{arctg}(\frac{q}{\beta}) & \text{for } n = 0 \\ \\ \frac{(n-1)!}{q}\,\mathrm{Im}\left[\frac{k}{\beta - iq}\right]^n & \text{for } n \geq 1 \end{cases} \tag{18 a}$$

$$I_n^{(2)}(\beta) = \frac{(n-1)!}{(2\eta)^n q}\left[\left\{-\gamma + \sum_{m=1}^{n-1}\frac{1}{m} - \frac{1}{2}\ln\frac{\beta^2+q^2}{4\eta^2k^2}\right\}\mathrm{Im}\left(\frac{2\eta k}{\beta - iq}\right)^n + \mathrm{arctg}\left(\frac{q}{\beta}\right)\mathrm{Re}\left(\frac{2\eta k}{\beta - iq}\right)^n\right.$$

$$\text{for } n \geq 1 \tag{18 b}$$

For the "nuclear" matrix element one obtains

$$M_{\sigma=o}^{Nuc} = M_{\sigma=o}^{Nuc}(\beta_1) - M_{\sigma=o}^{Nuc}(\beta_2)$$

with
$$M_{\sigma=o}^{Nuc}(\beta) = -i(2\pi)^4 V_o N\, e^{i\sigma_o}\frac{(e^{2i\delta_o} - 1)}{C_o k} \times$$

$$\times \sum_{n=o}^{\infty}\{2\eta(2\gamma - 1 + h(n) + \frac{iC_o^2}{2\eta})A_n + a_n\}\, I_n^{(1)}(\beta) + 2\eta\, A_n\, I_n^{(2)}(\beta)$$

(19 b)

This is valid for $k < \sqrt{\beta^2+q^2}$. In analogy to eq. (12c) the matrix-elements for the intermediate interaction occuring in the n-p triplet and singlet state, respectively, are related by

$$M^{Nuc}_{\sigma=1} = \alpha\, M^{Nuc}_{\sigma=o} \tag{19 c}$$

For small values of k the lowest order

$$M^{Nuc}_{\sigma=o} = 2(2\pi)^4 V_o N\, e^{i\sigma_o}\, e^{i\delta_o} \left(\frac{\sin\delta_o}{C_o k}\right)\frac{1}{q}\left[\mathrm{arctg}\left(\frac{q}{\beta_1}\right) - \mathrm{arctg}\left(\frac{q}{\beta_2}\right)\right] \tag{20}$$

closely related to the original Watson-Migdal-Ansatz is sufficient. Comparison with eq. (15) shows that the simple expression

$$M^{Nuc}_{\sigma=o}(\beta) = -i(2\pi)^4 V_o N\, e^{i\sigma_o} \left(\frac{e^{2i\delta_o} - 1}{C_o k}\right) \frac{1}{2q}\left[\left\{ \mathrm{arctg}\,\frac{q+k}{\beta} + \mathrm{arctg}\,\frac{q-k}{\beta}\right\} + \right.$$
$$\left. + \frac{i}{2} \ln \frac{(q+k)^2+\beta^2}{(q-k)^2-\beta^2} \right] \tag{21}$$

is valid as well for small as for large values of k.

It is noteworthy that the method developed here can also be applied easily to a deuteron-deuteron double break-up reaction with two-fold final state interaction. Results which should be particularly helpful for determining the neutron-neutron scattering length from the reaction d+d→p+p+n+n will be published by H. Thies.

3. Results and discussion

Fig. 2 shows the theoretical prediction for a kinematically complete experiment. The deuteron bombarding energy of 52.3 MeV and the detector angles of $\theta_p=22.4^{\circ}$ for the proton- and $\theta_n=48.3^{\circ}$ for the neutron detector are chosen in accordance with an experiment carried out previously at the Karlsruhe Isochronous Cyclotron. The special choice of kinematical parameters should warrant a possible contribution of the neutron spectator mechanism to be very small only.

The differential cross section divided by the phase space factor A is shown as a function of the proton energy E_p. Because for each proton energy the energy of the coincident neutron may assume two different values, the spectrum splits up into two parts. One part is dominated by the p-p FSI whereas the other part shows the typical shape of the spectator peak. The contributions arising from the pure "spectator"- and the pure"nuclear" matrix elements are shown separately. Apparently the spectator amplitude is very important even in the FSI-region, which is characterized by a small relative momentum in the p-p subsystem. Because of the strong interference of the "nuclear"- and the "spectator" amplitude in this region the value of the p-p scattering length a_{pp} extracted from experimental data by using this theory must be expected to differ considerably from those obtained by a Watson-Migdal fit. Vice versa the "nuclear" matrix element is still large at the spectator maximum although its influence is small because of "accidential" neutron interference behaviour.

For the calculation of the matrix elements the standard parameters $\beta_1=0.232\ \text{fm}^{-1}$ and $\beta_2=1.202\ \text{fm}^{-1}$ were used in the Hulthên function. For the"nuclear" phase shift δ_o the well known shape independent effective range approximation

$$k \cdot \text{ctg}\ \delta_o = \frac{1}{C_o^2(\eta)} \left[- \frac{1}{a_{pp}} + \frac{1}{2} r_o k^2 - \frac{h(\eta)}{R}\right]$$

with scattering length $a_{pp}=-7.66$ fm and the effective range parameter $r_o=2.62$ fm derived from p-p scattering data have been used.

In order to derive conclusions from the experimental results it is of particular interest to understand the dependence of the theoretical results on these basic parameters. Fig. 3 demonstrates how the spectrum given by fig. 2 is affected by varying the value of the scattering length or the effective range respectively. For comparison also the result of calculation without Coulomb forces but with fixed scattering length is shown. As a consequence the characteristic p-p FSI minimum at zero relative energy is replaced by a maximum. On the other hand all these variations of basic scattering parameter and the electromagnetic coupling constant have only a small influence in the region of the spectator peak.

Deviations of this theory from Watson Migdal are characterized by fig. 4. It shows the spectra for some pairs of angles as a function of the p-p subsystem wave number k together with the prediction of the Watson-Migdal theory. In contrast to the Watson-Migdal theory the spectra show an asymmetric shape with respect to the origin of the relative momentum axis. This effect is demonstrated for the spectrum at $\theta_p=22.4^o$ and $\theta_n=48.3^o$ by mapping one FSI-peak onto the other.

The superiority of this theory compared to the original Watson-Migdal-ansatz is demonstrated by fig. 5. Theoretical results obtained after a scattering length fit are compared with experimental data at scattering angles $\theta_p=22.4^o$, $\theta_n=48.3^o$. The theory leads to a scattering length of $a_{pp}=-7.7$ fm whereas from a slightly modified Watson Migdal fit |3| a value of $a_{pp}=-8.8$ fm was obtained. These values have to be compared with a two particle value of $a_{pp}=-7.66$ fm. Former discrepancies between the two- and three particle values derived from Watson Migdal fits therefore seem to be resolved by the introduction of Coulomb-effects. While on the one hand good agreement is observed in FSI region, the agreement in the spectator region is a qualitative one. In fact there remains a small deviation if the experimental and the theoretical spectra are adjusted in the FSI region. These discrepances may be removed only by more sophisticated calculations including Coulomb effects.

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Figure Captions

Fig. 1 Graph corresponding to the matrix element defined by eq. (1)

Fig. 2 Theoretical prediction for a kinematically complete experiment with E_d=52.3 MeV, Θ_p=22.4°, θ_n=48.3°. The cross section devided by phase space is given as function of proton energy E_p. Contribution of "spectator"- and "nuclear" matrix element are shown separately.

Fig. 3 Influence of basic scattering parameters on the shape of spectrum.

Fig. 4 Comparision of theoretical prediction (this work: full curves) for different angles with the prediction of Watson Migdal-theory (hatched curve on the right sid) being independent of angle. The hatched curve on the left side is obtained by mapping the right hand side FSI-peak onto the left side in order to demonstrate asymmetry.

Fig. 5 Comparison with experimental results in FSI- and QFS-region. Experimental points are shown only for that part of the spectrum, which can be projected onto the E_p axis. The differential cross section is shown in relative units.

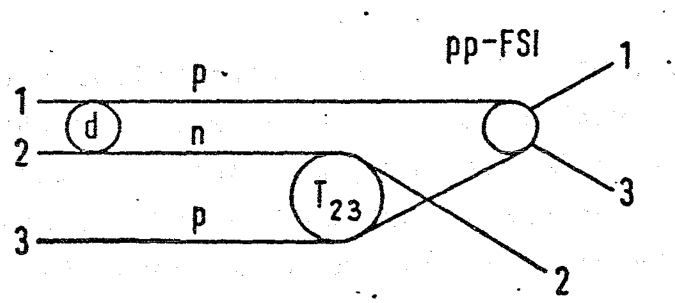
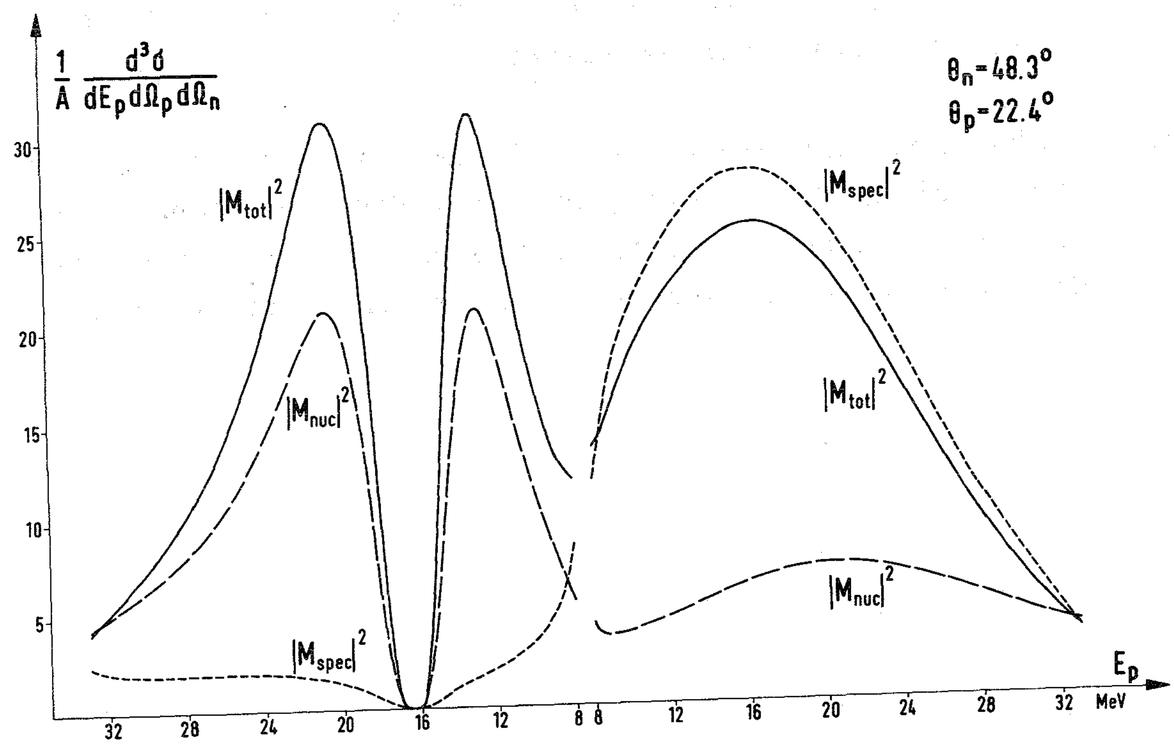

pp-FSI
1
2
3
d
n
p
p
T_{23}
1
3
2

Fig. 1


$\frac{1}{A}\frac{d^3\sigma}{dE_p d\Omega_p d\Omega_n}$
$\theta_n = 48.3°$
$\theta_p = 22.4°$
$|M_{tot}|^2$
$|M_{nuc}|^2$
$|M_{spec}|^2$
$|M_{spec}|^2$
$|M_{tot}|^2$
$|M_{nuc}|^2$
E_p
MeV
30
25
20
15
10
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32
28
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20
16
12
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16
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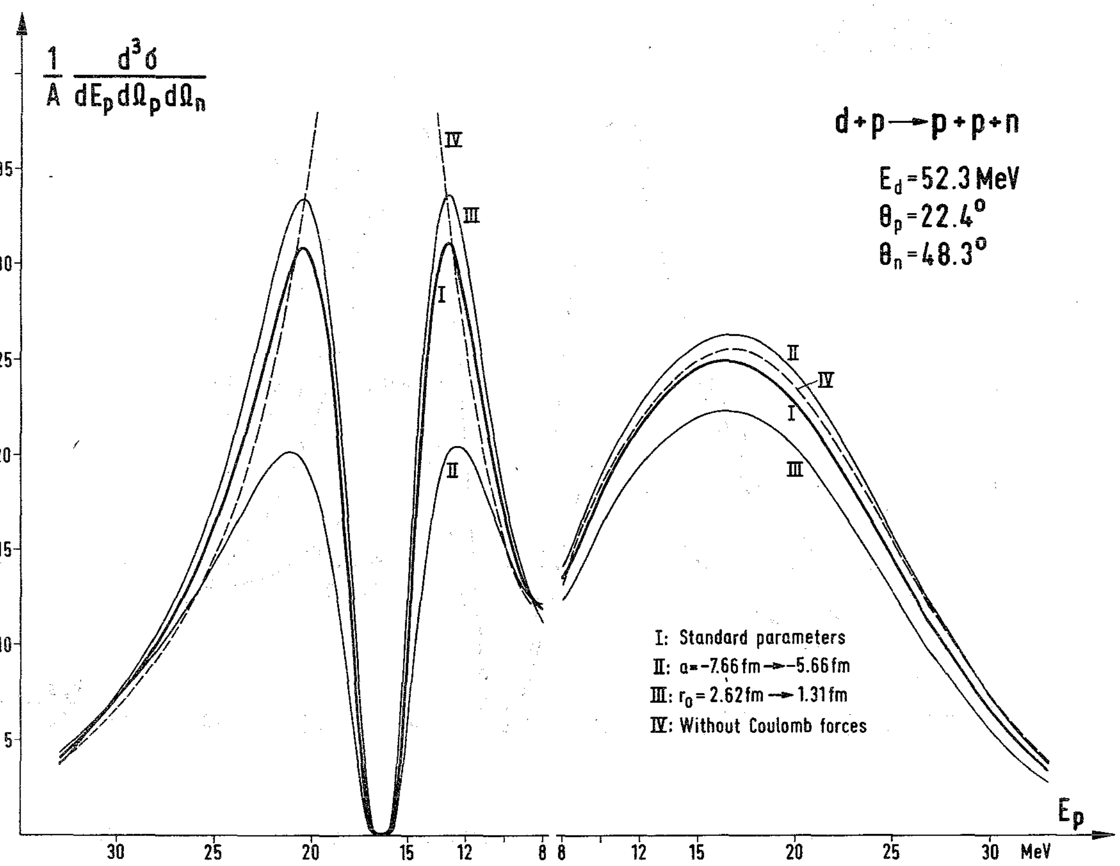
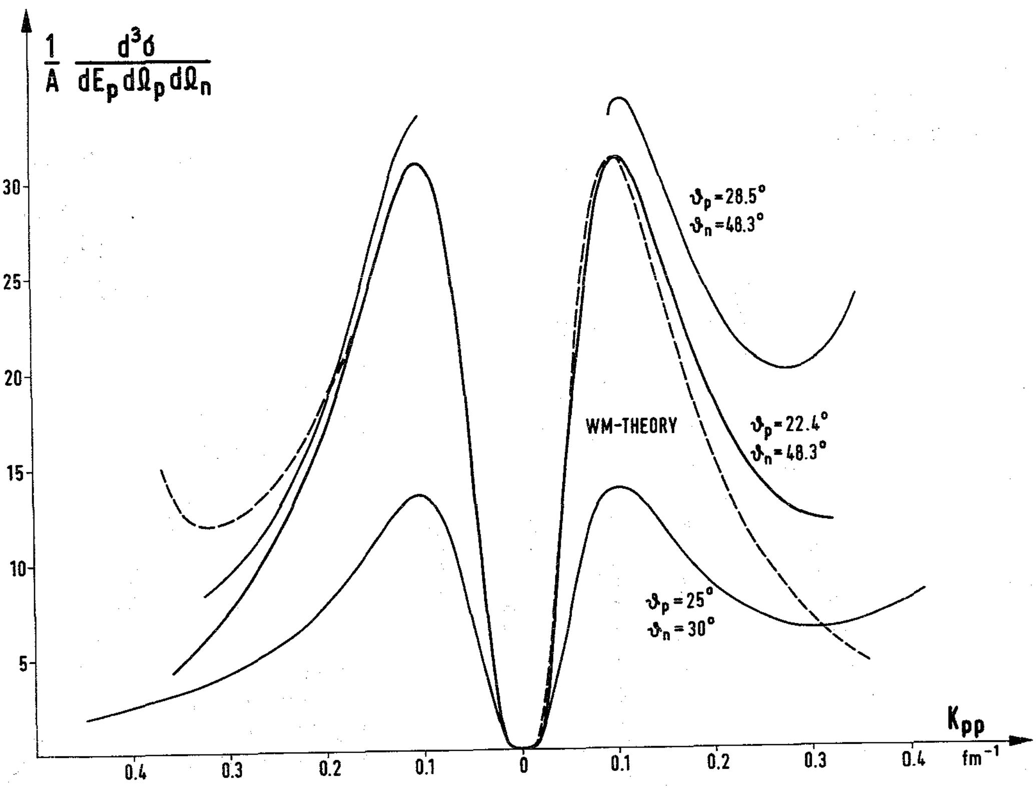
28
32

Fig. 2



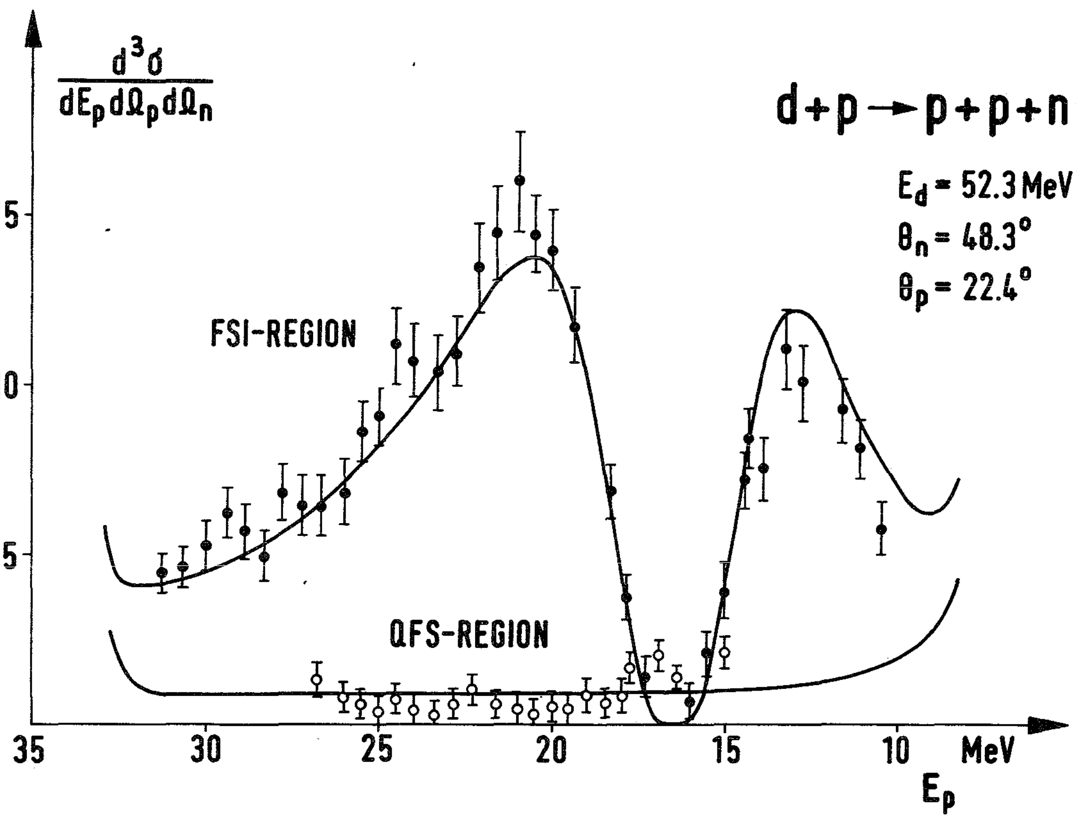
$\frac{1}{A}\frac{d^3\sigma}{dE_p d\Omega_p d\Omega_n}$
d+p→p+p+n
E_d=52.3 MeV
θ_p=22.4°
θ_n=48.3°
I: Standard parameters
II: a=-7.66 fm→-5.66 fm
III: r_0=2.62 fm→1.31 fm
IV: Without Coulomb forces
E_p
MeV

Fig. 3



$\frac{1}{A}\frac{d^3\sigma}{dE_p d\Omega_p d\Omega_n}$
30
25
20
15
10
5
$\vartheta_p = 28.5°$
$\vartheta_n = 48.3°$
WM-THEORY
$\vartheta_p = 22.4°$
$\vartheta_n = 48.3°$
$\vartheta_p = 25°$
$\vartheta_n = 30°$
K_{pp}
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
fm^{-1}

Fig. 4



$\frac{d^3\sigma}{dE_p d\Omega_p d\Omega_n}$
$d+p \rightarrow p+p+n$
$E_d = 52.3$ MeV
$\theta_n = 48.3°$
$\theta_p = 22.4°$
FSI-REGION
QFS-REGION
15
10
5
35
30
25
20
15
10
MeV
E_p

Fig. 5