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Interpolation of Germanium Thermometer Characteristics

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Abstract

A widely used method for the representation of germanium resistance thermometer characteristics consists in a least square fit of the calibration points with high-degree polynomials over the entire working range. Instead of this expensive method a strict interpolation of the calibration points is proposed, using simple formulas with two or four parameters, e.g. spline polynomials. The accuracy of the interpolated values were improved by an additive parabolic correction.

The two parameter calculations can be made with the aid of a desk calculator. The method is applicable to other interpolation problems, e.g. the representation of platinum thermometer characteristics.

Zusammenfassung

Zur Darstellung der Kennlinien von Germanium-Widerstandsthermometern werden üblicherweise Fitpolynome benutzt, die an die Eichpunkte im gesamten Arbeitsbereich der Thermometer angepaßt werden. Stattdessen wird vorgeschlagen, die Eichpunkte in kleineren Bereichen mittels zwei- oder vierparametriger Interpolationsfunktionen, z.B. mittels Splinepolynomen, zu interpolieren. Bei dem gängigen Abstand der Eichpunkte ist die Genauigkeit der Interpolationsmethode mit der der Ausgleichsmethode vergleichbar, insbesondere nach Reduzierung der Modellfehler durch parabelförmige Korrekturfunktionen.

Das beschriebene Verfahren ist auch für andere Interpolationsaufgaben, z.B. zur Darstellung der Kennlinien von Platin-Widerstandsthermometern, geeignet.

Contents

1. Introduction

One of the most favorable instruments allowing to measure temperatures in the range from 1 to 100 deg. Kelvin is the germanium thermometer. This is due to its sensitivity as well as its reproducibility of some millidegrees, even after repeated cycling between working and room temperatures. Furthermore, both thermal capacity and response time of germanium sensors are small according to their small dimensions (Co63).
Unfortunately, the relation between the electrical resistance of doped germanium and temperature cannot be expressed accurately by an analytical formula. Therefore, the characteristics of such thermometers have to be determined by calibration points not too widely separated. Moreover, fitting or interpolation must be performed with special care. In literature a variety of approximation formulas is given which are based on up to ten or fifteen parameters. The idea underlying all these efforts is to cover a temperature range as wide as possible so that the calibration points can be fitted by a single formula. As an example we mention the proposal of Blakemore et al.(Bl70) who recommended a polynomial

$$\log R = \sum_{j=0}^{n} A_j * (\log T)^j, \; n = 6 \ldots 9, \qquad (1)$$

which represents the calibration over the entire 1 - 100 K range with an accuracy of about 0.3 %; by splitting the whole working range into two fit intervals - the first having an upper limit at about 20 K and the other a lower limit at about 15 K - the relative deviations $\frac{\Delta T}{T}$ are reduced by a factor of ten.

In general, any fit function oscillates around the real characteristic defined through point-by-point calibration (Fig. 6); the sign of the deviations alternates as often as indicated by the degree of the fit polynomial or even more

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frequently. The accuracy can neither be improved by doubling the numerical precision nor by reducing the gaps between the calibration points. When the order of the polynomial is increased, the least sums of squared differences reach a minimum and soon diverge, i.e., there is an optimum degree of the fit polynomial. In order to overcome some of the inconveniencies mentioned above, we propose to substitute the commonly used approximation method, which may be justified in the presence of marked statistical errors, by a simple but somewhat refined interpolation method. The deviations can then be reduced below any limit provided that the distances between the calibration points are reduced adequately.

2. Choice of Interpolation Functions

In order to interpolate an unknown value between two neighbouring values of a given tabulated function, an interpolation function must be defined which reproduces exactly the neighbouring values. The number of parameters on which the interpolation formula is based must then be equal to the number of base points assumed. We now postulate that the number of base points on the left side should be the same as the number of base points on the right side. Therefore, we shall look for interpolation formulas having two or four parameters only. Besides, we restrict ourselves to the relation T(R) and not to the reciprocal one: R(T).

To check the interpolation method proposed we examined a germanium thermometer*) calibrated by the manufacturer between 4.25 and 100 K at 26 temperatures; see Table 1, cols. 1 and 2, and Fig. 1. Part of the calculations were performed at the IBM installation, Karlsruhe Nuclear Research Center, using the fitting subroutines VAØ1A (Sc69) and RATFIT (Fi72).

As outlined above, the base points - apart from rounding errors - are reproduced exactly by every interpolation function. Therefore, the interpolation deviations appear only when part of the calibration points are used as control points. For com-

*) CryoCal Inc., Riviera Beach, Florida, fabrication number 2790

parison of various interpolation formulas every second calibration point only will be taken as a base point. By this method the deviations would evidently be greater than with the use of the full information. To evaluate also the deviations for this latter case, we will investigate first how much the deviations increase by doubling once or twice the distances between the base points. Then the deviations at reduced distances may be estimated by extrapolation.

With the distances between the base points fixed, the deviations depend on the special interpolation formula chosen. For our test thermometer a linear function

$$T = A + B * R \tag{2}$$

and even a cubic polynomial

$$T = A + B * R + C * R^2 + D * R^3 \tag{3}$$

(Fig.2a) were found to give relative high deviations ΔT. In searching more suitable formulas only heuristical principles will apply. For instance, the observation (Fig.1) that the relation between log R and log T is nearly linear led us to the two-parametric formula

$$T = A * R^B \tag{4}$$

(Fig. 2b). We attempted to improve the formula (4) by introducing two additional parameters which either take into account the curvature of the log T (log R)-relation, Fig. 1, or allow a translation of both the log R-axis and the log T-axis, but frequently it happened that the iterative search for the four free parameters failed to converge. An essential improvement was achieved by using a polynomial with at least one negative exponent

$$T = A * R^{-1} + B + C * R + D * R^2 \tag{5}$$

(Fig. 2c). Such a formula was inspired by the hyperbolic-like dependence of T on R (Fig. 1).

Besides this method of stepwise interpolation, using formulas (1) to (5), we also tested the capacity of a different technique known as spline interpolation (Bu68). Once more, the given base points are represented by third order polynomials. An additional condition, however, is that at each base point the two adjacent polynomials should have the same first and second derivatives.For the calculation of the coefficients (Sp68), the entire set of the base points is required and not only the immediate neighbouring ones, as in the case of the interpolation formulas cited so far.The result is shown in Figs. (2d) and (2e). Compared with Figs. (2a) to (2c), the deviations are smaller here. This is a consequence of the continuous differentiability of the spline function. Indeed, below 80 K, the deviations do not reflect an inaccuracy of the spline interpolation but of the experimental calibration, as it will be shown in the next section.

In spite of the almost satisfactory results achieved by formula (5) and by spline interpolation, we shall now attempt how to remove the remaining deviations at all. In order to show the principle of our procedure which holds for every interpolation function, the following discussions are limited to the two-parametric formula (4) which can be treated easily with the aid of a desk calculator.

As stated above, the amplitudes of the deviations depend strongly on the distances of the base points. As shown in Fig.3, they are always reduced by a factor of about four when the distances of the base points are divided by two. Extrapolating these findings, we expect the relative deviations due to formula (4) to be lowerthan $5 \cdot 10^{-4}$ over the whole working range of the test thermometer, provided the complete set of calibration points is used.

3. Smoothing of Statistical Fluctuations

The differences between the interpolated points and the control points, as shown in Figs. 2 and 3, are affected by experimental and/or rounding errors occuring in the determination of the calibration points. The experimental errors may be of a systematic nature - and under this condition they cannot be recognized in the diagrams referred to above - or of a statistical nature that means independent of point to point. The interpolation method would proof itself to be good if it would be possible, as in fitting procedures, to handle such statistical errors more quantitatively and eliminate them.

For test purposes we started to generate a second series of deviation amplitudes ΔT_{max} by exchanging the meaning of base points and control points. For two different temperature ranges of equidistant divisioning both series are plotted in Fig. 4a that shows the statistical fluctuations more clearly than Fig. 2. After fitting the data points $\Delta T_{max}(T)$ of Fig. 4a by a third degree polynomial the original (experimental) T-values are corrected by adding half the differences between the deviation amplitudes before and after fitting. Half the differences and not the entire ones are taken, since any correction of a single calibration temperature would affect not only the deviation amplitude at this point but also, with reversed sign and reduced amount, the deviation amplitudes at the two neighbouring points. If neccessary, this procedure may be repeated. In Table 1, col. 3, the first order corrected temperatures are noted. Fig. 4b shows the corresponding deviations. As it can be shown, the interpolation parameters A and B of formula (4) involve similar fluctuations to that of the deviation amplitudes. Their plots (not presented) may be used to establish a single curve to fit the deviation amplitudes over the entire working range of the thermometer, regardless of the varying distribution of the calibration points.

4. Reduction of Formula-Dependent Deviations

After smoothing of statistical fluctuations the deviations (Fig. 4b) consist of systematic errors and of formula-dependent deviations. According to Fig. 3, this latter component behaves like parabolas suspended between successive base points. When all calibration points are supposedto be base points, the formula-dependent deviations are minimized. However, their values are unknown and therefore not available for further correction. For this reason we have resumed the idea to admit only every second calibration point as a base point. The deviations at the intermediate control points were used to determine the parameters of parabolas which were subtracted from the interpolation curves for correction. To begin with, Fig. 5 shows the remaining deviations a) for every fourth and b) for every eighth calibration point taken as a base point. This method is applicable, even if the control point which defines a parabola lies eccentrically between the neighbouring base points. In general, the amplitudes of the remaining deviations are five to ten times smaller than the deviation amplitudes which would result from dividing into halves the distances between the base points (Fig. 3). Being comparable to those obtained with formula (5), Fig. 2c, they fluctuate unsystematically due to remaining statistical errors.

For comparison with one of the commonly used approximation methods we have fitted the logarithms of the calibration values of our test thermometer (Table 1) by a fifth degree polynomial in the two temperature ranges from 4.5 to 20 and from 15 to 100 K (Fig. 6). The deviations in Figs. 5a and 6 are comparable. However, it should be realized that the remaining interpolation deviations would be reduced strongly by reducing the distances of the base points. By use of every second calibration point instead of every fourth they would probably become smaller by a factor of three or even more than those indicated in Fig. 5a. Evidently, a further improvement can be achieved by starting with a more suitable interpolation function than (4). This finally means that the accuracy obtained by this method would be better than the experimental accuracy given.

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Table 1

Calibration Points for the Germanium Thermometer CryoCal No.2790

T [K]		R [Ohms]		$T_{corrected}$ [K]
4.25	± 0.005	859.3	± 0.01 %	
4.5		764.9		
4.75		684.4		
5.0		615.1		
5.5	0.01	502.6		
6.0		415.9		
6.5		348.0		
7.0		294.1		
7.5		250.9		
8.0		216.0		
8.5		187.6		
9.0		164.4		
9.5		145.2		
10.0		129.3		
11.0		104.76		
12.0		87.16		
13.0		74.17		
14.0	0.01	64.32		
15.0		56.68		
16.0		50.60		15.999
17.0		45.66		
18.0		41.57		18.001
19.0		38.13		
20.0		35.19		20.001
22.0	0.04	30.43		21.998
24.0		26.72		24.003
26.0		23.77		25.996
28.0		21.34		28.001
30.0		19.32		30.004
32.0		17.63		31.996
34.0		16.18		33.999
36.0		14.93		36.003
38.0		13.85		37.999
40.0		12.90		
45.0	0.1	11.003		44.998
50.0		9.588		50.002
55.0		8.506		55.002
60.0		7.661		59.999
65.0		6.988		65.001
70.0		6.445		69.999
75.0		6.001		74.994
80.0		5.633		80.006
85.0		5.328		84.999
90.0		5.072		90.002
95.0		4.856		94.998
100.0		4.672		

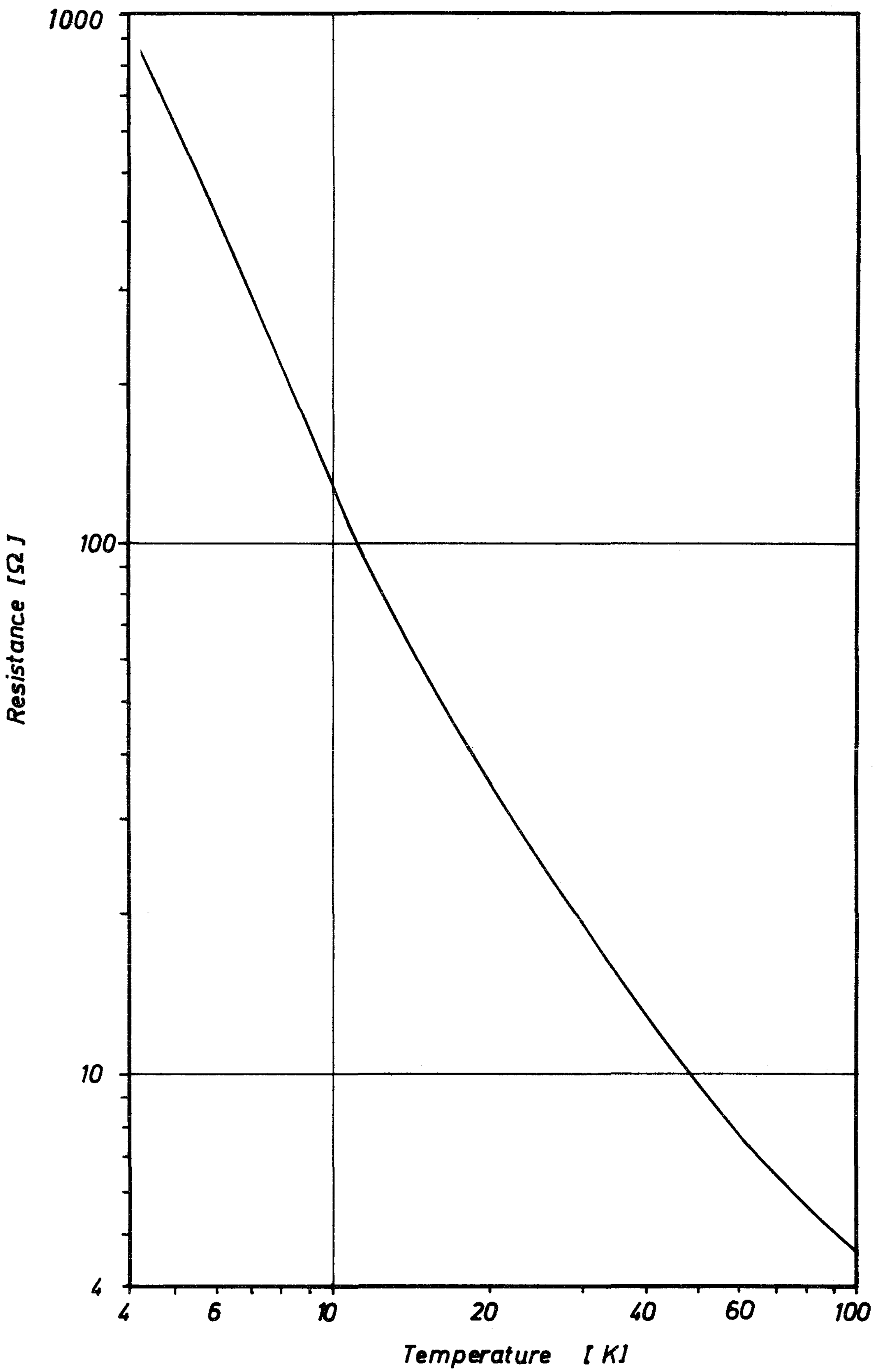

Fig 1: Measured Characteristic of the Germanium Thermometer CryoCal # 2790 (logarithmic scales)

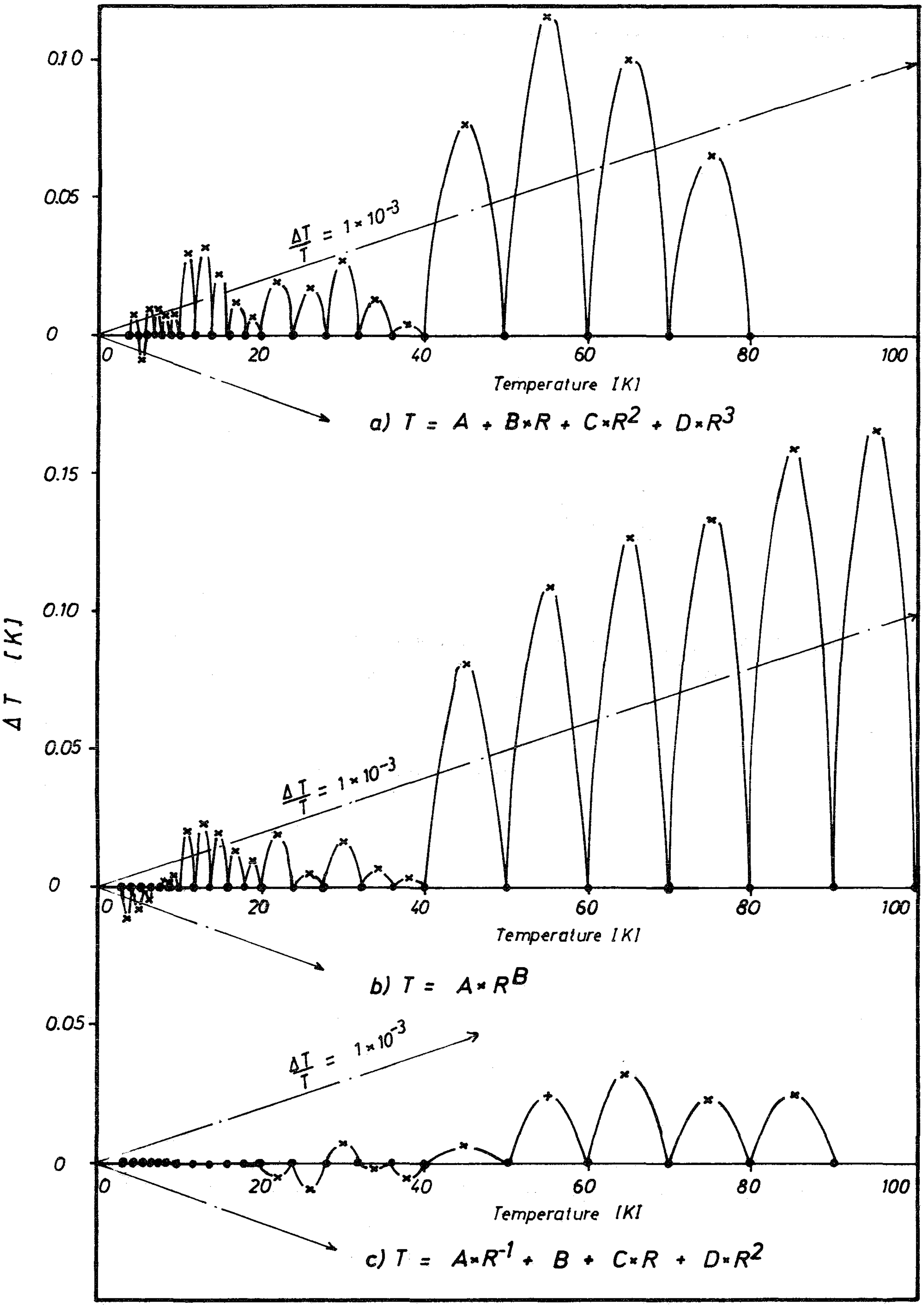

Fig 2: Deviations ΔT vs. Temperature According to Various Interpolation Functions

● Base Points
× Control Points

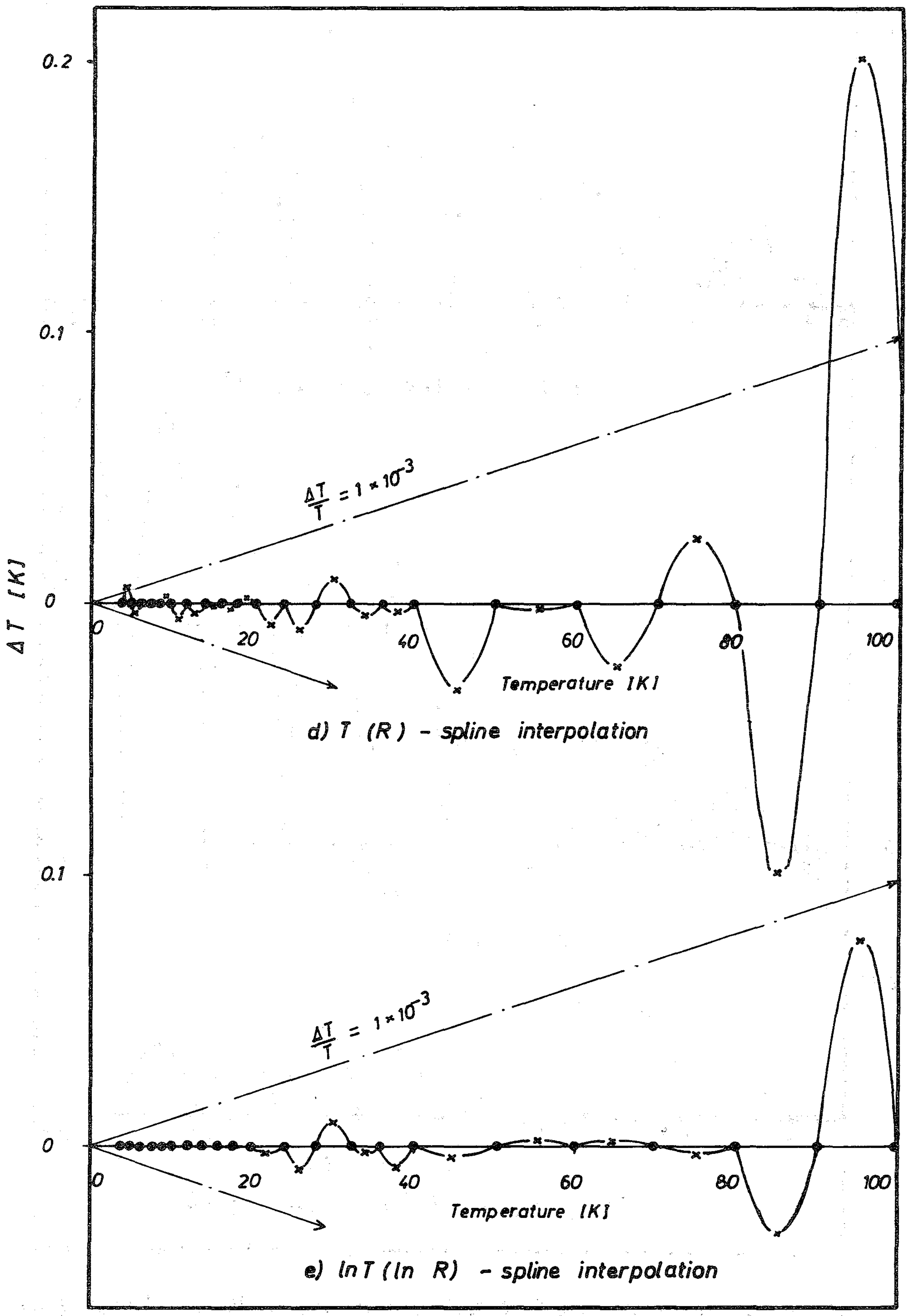

Fig 2 (continued)

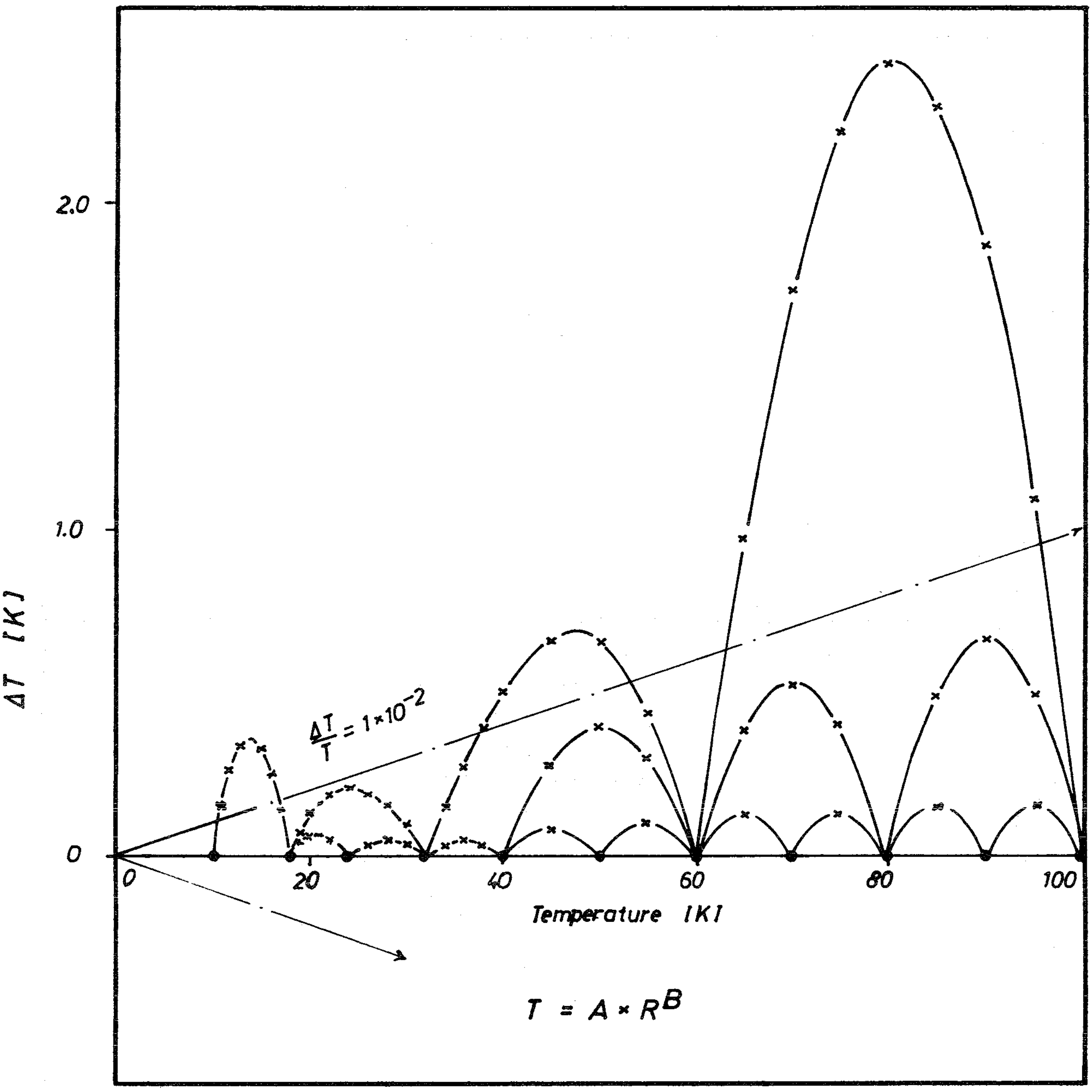

Fig 3 : *Deviations ΔT vs. Temperature with different Distances between Base Points*

- ● *Base Points*
- × *Control Points*

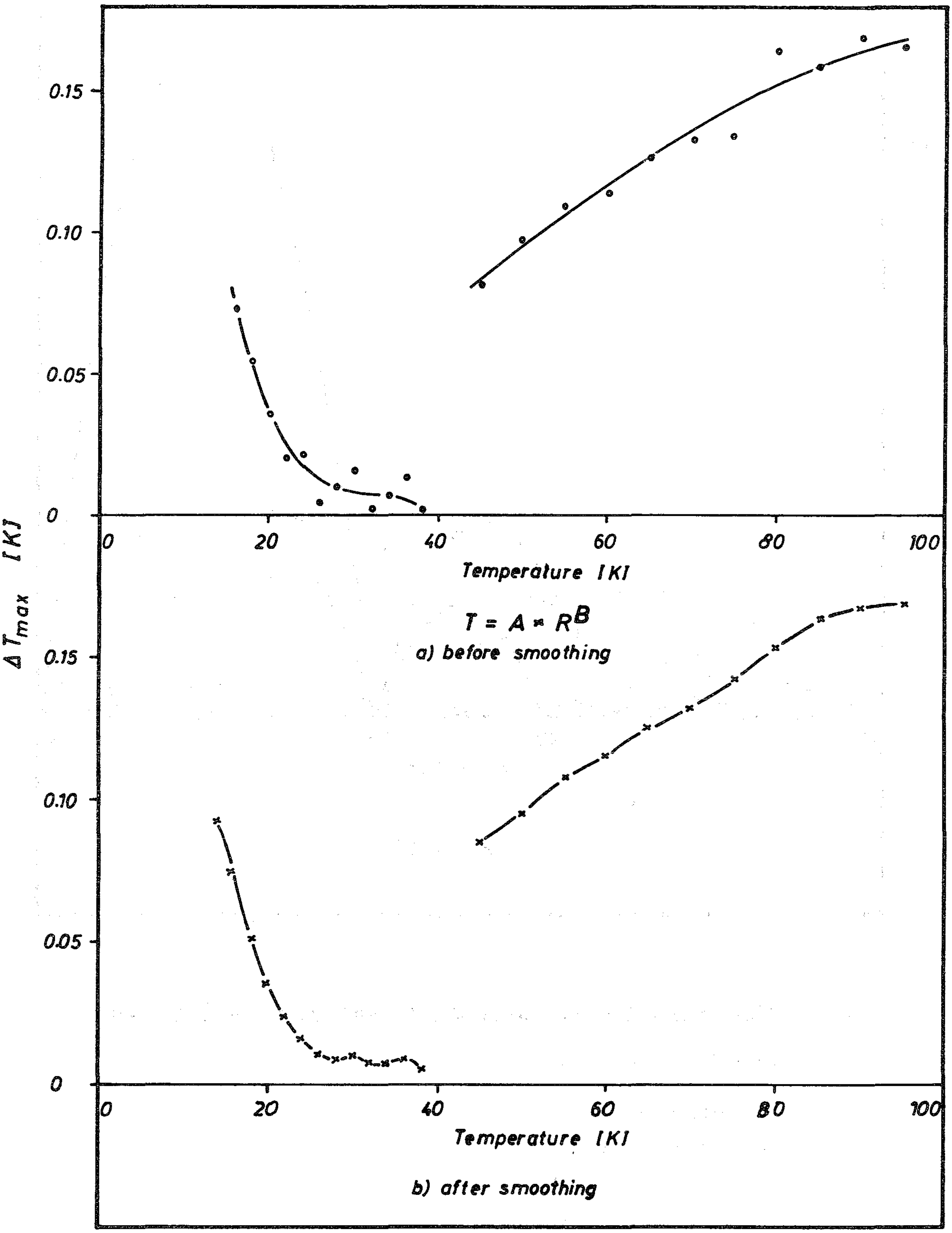

Fig 4 : Deviation Amplitudes ΔT_{max} vs. Temperature

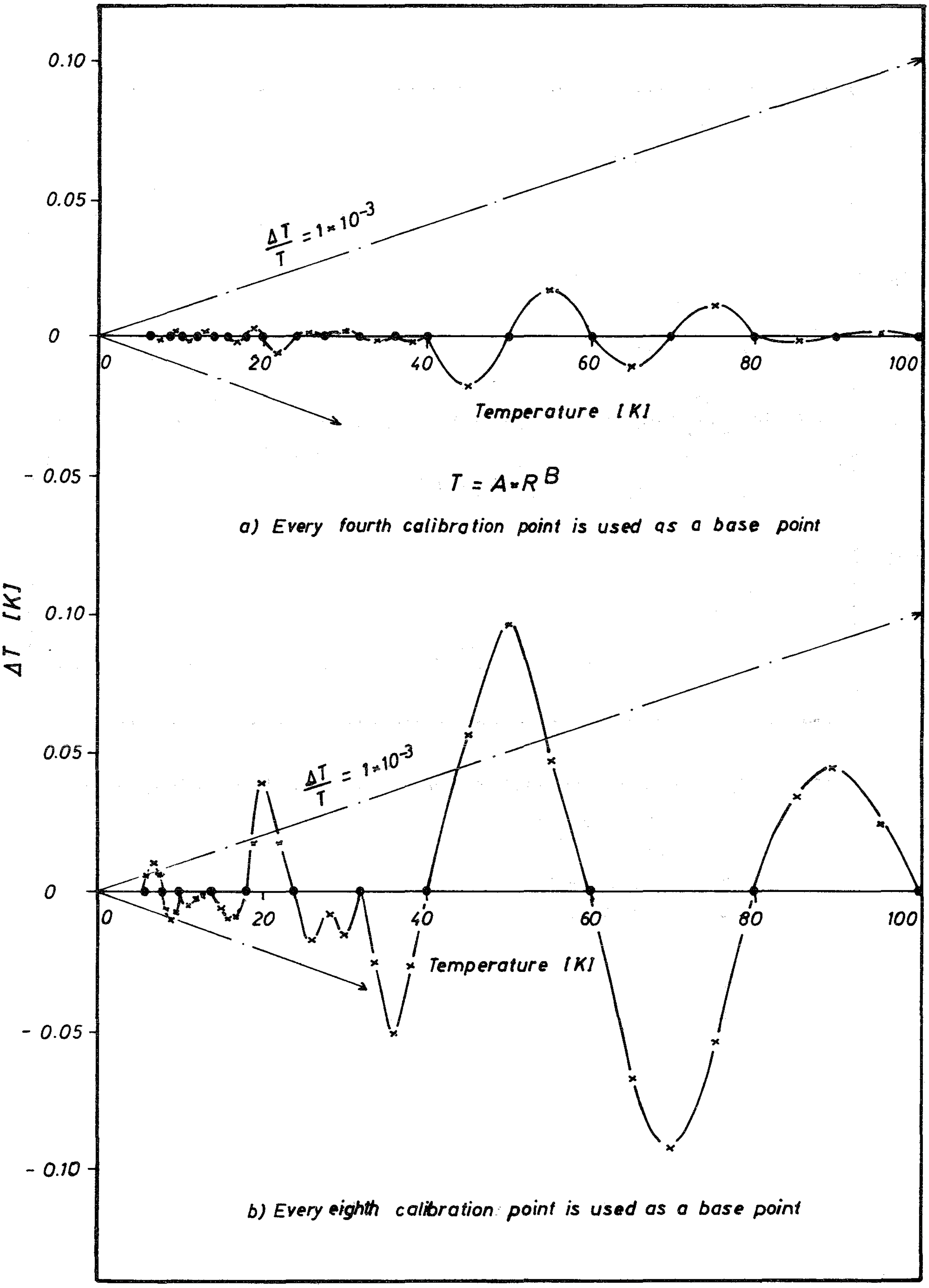

Fig 5: Remaining Deviations ΔT vs. Temperature after Smoothing of Statistical Fluctuations and Reduction of Formula-Dependent Deviations

• Base Points
× Control Points

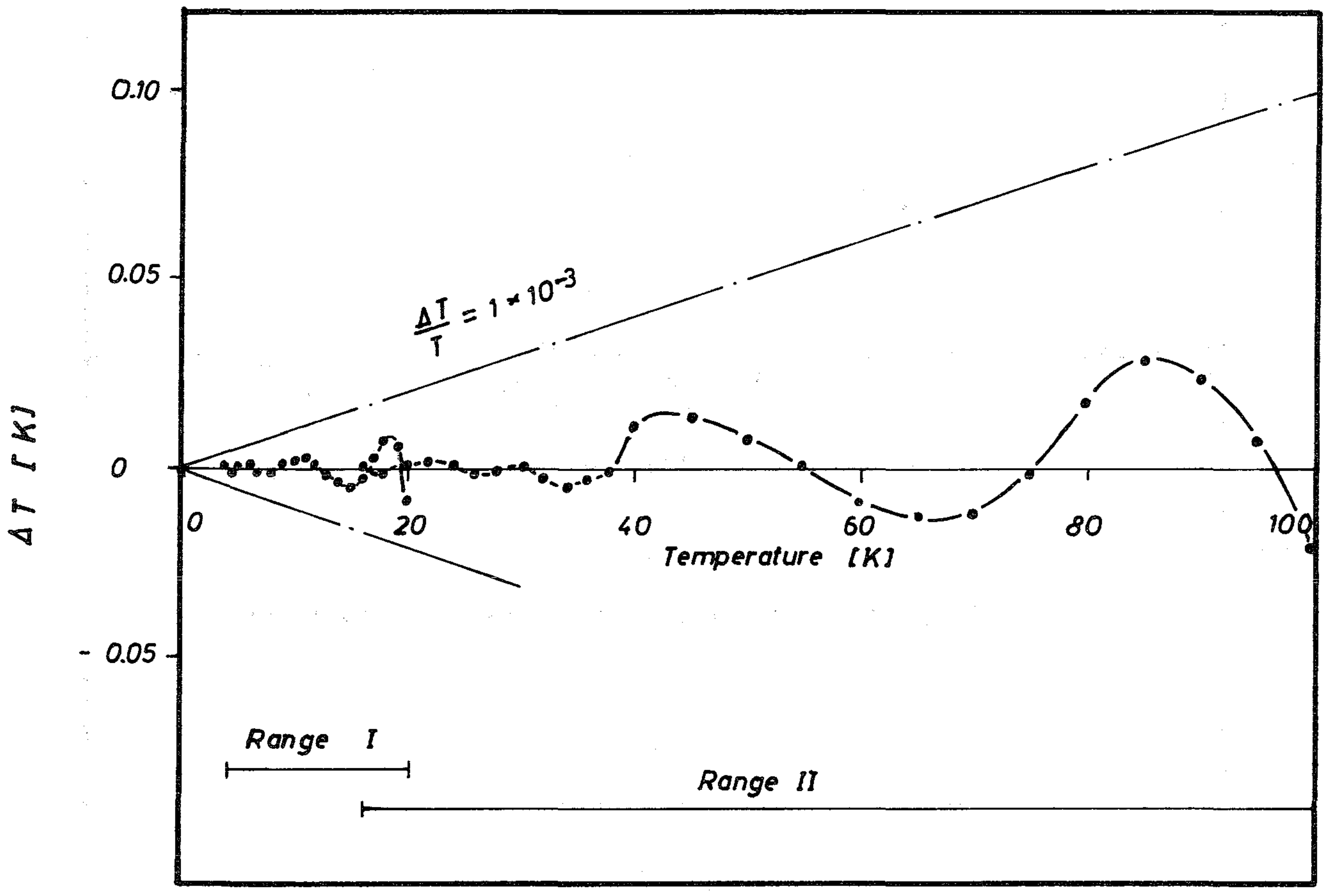

Fig 6: Deviations ΔT vs. Temperature Resulting from Fitting with $\log R = \sum_{j=0}^{5} A_j * (\log T)^j$

- ∘ Base Points Range I
- • Base Points Range II