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ASSESSMENT OF MATERIAL UNACCOUNTED FOR (MUF) AND INSPECTION EFFORTS IN A CENTRIFUGE PLANT [x)]

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Zusammenfassung

In dieser Arbeit wird das Problem der Überwachung des spaltbaren Materials in einer Zentrifugenanlage mit Hilfe der Materialbilanzierung analysiert. Es wird eine Referenzanlage mit einer Trennarbeitskapazität von 600 t U/Jahr und einem Materialfluß-Schema, wie man es sich heute vorstellt, betrachtet. Im ersten Teil der Arbeit wird die Frage der Genauigkeit der Materialbilanz mit Hilfe von neu entwickelten statistischen Methoden untersucht: Die auf ein Jahr bezogene Entdeckungswahrscheinlichkeit wird als Funktion des Fehlers 1. Art, der Zahl der Inventuren und des pro Jahr als entwendet angenommenen Betrages spaltbaren Materials berechnet. Im zweiten Teil werden verschiedene Möglichkeiten der Verifikation von Betreiberdaten betrachtet. Es wird gezeigt, daß es genügt, die Brutto-Gewichte der UF_6-Zylinder mit einer groben Waage unter Verwendung der Methode der Vorzeichentests sowie die Anreicherung der Produkt- und Tails-Ströme zu prüfen. Es ist festzuhalten, daß die in dieser Arbeit niedergelegten Gedanken noch nicht als endgültig anzusehen sind; weitere Arbeiten werden folgen.

Abstract

In this paper the problem of safeguarding the nuclear material in a centrifuge plant with the help of material accountancy is analysed. A reference plant with a separative work of 600 t U/yr and a material flow scheme as it is seen today is considered. In the first part the question of the accuracy of the material balance is analysed with the help of recently developed techniques: the probability of detection per year is determined as a function of the error of the first kind, the number of inventories and the amount assumed to be diverted per year. In the second part different possibilities for the verification of the operator's reported data are considered. It is shown that it is sufficient to check the gross weights of the UF_6 cylinders with a rough balance by using the sign test method and furthermore to check the enrichment of the product and tail streams on a random sampling basis either. It is to be noted that the ideas presented in this paper are not yet fully developed; further work is in progress.

Assessment of Material Unaccounted For (MUF) and Inspection Efforts in a Centrifuge Plant [1)]

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1. Introduction

With centrifuge plant for uranium enrichment attracting commercial interest, various safeguards problems considered to be associated with such plants are expected to be examined from all sides for some time to come. For an international safeguards system with material accounting as the main safeguards measure, the Material Unaccounted For (MUF: difference between book and physical inventory) has been recognized to be the primary technical safeguards indicator /1/. An attempt has been made in this paper to analyse the various components of MUF in a centrifuge plant. The influence of measurement uncertainties and the number of physical inventories per year on the amounts of material which can be declared as missing or diverted for a given set of α and β error, has been discussed on the basis of data developed for a reference centrifuge plant. Some preliminary thoughts have been examined on the possible extent of actual inspection efforts in the reference plant. It is to be noted that the ideas expressed here are at an initial stage of development and further work is in progress.

[1)] Paper presented at the IAEA Working Group Meeting on Safeguards procedures for Isotopic Enrichment Facilities, 12.- 16.6.72 in Vienna.

2. Reference plant

The same reference plant used as a basis for the development of a material accountancy structure /2/ forms the basis in this case also. All relevant data on this plant are presented in Table 1. The flow scheme is reproduced in Fig. 1 from /2/.

The input to the plant has been assumed to be 10 t UF_6 cylinders as a batch with natural uranium concentration. The cylinders are transferred to the feed station, weighed, heated up, sampled for chemical and isotopic analysis if required and condensed into the desublimators over the vacuum system to remove traces of HF and then fed in gaseous form into the cascade by-passing the desublimators. A small fraction of HF/UF_6 mixture over the desublimators goes over to the vacuum traps and is removed periodically. The amounts held up in these traps are not known accurately. They go to the decontamination system from time to time and leave the plant in the form of water slurries.The total amount of such wastes is not expected to be more than 0.5 % of the feed and will also have natural concentration. Batch size for the stream has been assumed to be 1.5 tons. The total amount and the U-235 concentration are expected to be measured for each batch.

After enrichment, the product and tail streams are condensed in the respective desublimators before being transferred to the output cylinders at the product and the tails stations respectively. There they are weighed and sampled for chemical and isotopic analysis when necessary, and transferred to storage. The product is expected to be stored in 2 ton cylinders for UF_6, the tail in 10 ton emptied feed cylinders. Each of these make a batch.

The reference plant has been divided into two Material Balance Areas (MBA) for safeguards accountancy purposes namely, the storage and the process MBA. They are reproduced along with the Key Measurement Points from /2/ in Fig. 2. The dotted area around the cascade is assumed to be inaccessible for inspection and houses the separating centrifuges in the cascade and the maintenance and test areas for centrifuges.

It is to be noted that all the intermediate storage vessels like sublimators, traps etc., expected to have any significant uranium hold-up are located in the accessible part of the process MBA and are subject to inspections.

3. Analysis of the MUF components

In an analysis of the available data on MUF for facilities in the open part of the fuel cycle /3,4/, it was shown that both the measurement errors for throughput and inventory measurements as well as process parameters can contribute to the variance and the expectation value of the MUF. All relevant data which influence the establishment of material balance and therefore the MUF in the reference centrifuge plant, are presented in Table 2 for throughput measurements and in Table 3 for inventory determinations. The values of measurement errors in Table 2 are partly from /5/ and partly based on actual performances of a test cascade. Only the values on waste measurements are pure estimates as no actual measurement experience exists at present. They may change as experience grows. The data on inventories in Table 3 are mostly estimates based on pilot plant experience. They may also be altered in the course of further experience.

3.1 Contribution to the variance of MUF

With the conditions laid down in Table 2 and 3 it can be seen that the following components contribute to the variance of MUF.

a) The relative standard deviations (RSD) of random components of measurements for all the input and output batches (Table 2).
However, because of the large number of batches measured for the feed, product and tail streams and the low values of the respective RSDs, their contribution to the total variance may be small. On the other hand, the random error contribution from the waste stream measurements can be high because of fairly small number of batches and normally high value of RSD for this error.

b) The RSD for systematic error for all the different throughput measurements (Table 2).
The contribution of these types of errors may be reduced or eliminated if same measurement instruments and standards are used for the input and output measurements. For the reference plant the systematic errors in the chemical analysis are eliminated in this manner since the same measuring system has been assumed to be used for the feed, product and tail streams and the systematic error of measurement has been assumed to be proportional

to the measured amounts. Similarly, any systematic error in weighing is also eliminated+). However, in the mass-spectrometric measurements, the systematic error associated with the measurement standards has not been eliminated since different mass-spectrometers using different standards are assumed to be used for different streams to avoid cross contamination and memory effects. Also, the systematic error in the measurement of waste streams cannot be eliminated.

c) The RSD for systematic errors for all inventory amounts (Table 3).
Under normal conditions two types of inventory taking have been considered for the reference plant. The first type consists of an accurate measurement of the process inventory and supposed to be carried out once a year. The desublimators are emptied and the only inventories remaining in the process MBA are those in the feed, product and tail cylinders flanged at the respective stations, the gas phase inventory in the cascade, the uranium inventory in the vacuum traps and the hidden inventory in form of solid uranium deposits on various parts of the process area. Excepting the last, all the other inventory items are assumed to be measured with RSDs of systematic error components as shown in Table 3. The contribution of these errors (because of the small amounts involved and relatively good accuracy) to the total standard deviation of MUF is negligible.

The second type of inventory taking consists of an estimation of the desublimator contents at more frequent intervals (1-7 times/yr in addition to the once-a-year accurate inventory taking). As shown later, such a procedure causes a marked reduction in the amount of material which will be denoted by M in the following for which a statement for diversion (or loss) can be made with a given set of α and β errors.++) For this inventory taking, the time is so chosen that the content of the flanged cylinders at the input and output stations can be added to the throughput measurements (instead of to the inventory as in the case of the first type of inventory taking). The process inventories under such a condition are those in the desublimators, the gas phase inventory in the cascade, the uranium inventory in the vacuum traps and the solid phase hidden inventory. With the assumed RSDs for systematic errors for this type of inventories, the standard deviations for the inventory are approximately of the same order of magnitude (particularly for inventories of 4 or more/yr.) as that obtained for the throughput measurement for the same period.

+) For a detailed explanation, see the comment in section 5 of Annex 1.
++) The definitionsof α and β are given in the Annexes.

Both the inventories are expected to be taken without stopping the operation of the plant. The contribution of the standard deviation of measurement for the UF_6 stored in the three storage areas has not been considered as their influence cancels on balancing since it is assumed that the same amounts are kept in the storage at the beginning and end of a balancing period.

3.2 Contribution to the expectation value of MUF

All contributions to the expectation value of MUF (EMUF) in the reference centrifuge plant are expected to be from the process parameters. They are

a) heels in feed cylinders
b) gas phase inventory in the cascade (if inventory is taken during the operation of a plant)
c) solid phase hidden inventory in the process area
d) hold up of vacuum and other traps under equilibrium conditions.

The heel losses have been assumed to occur mainly at the feed end as the cleaning of cylinders are expected to take place at the shipper's facility. Whatever the magnitude of these losses may be, since they are known they need not cause any concern for safeguards. The gas phase inventory, taken to be 120 kgs of UF_6 for the reference plant, is very low compared to the throughput (feed 1700 t UF_6/yr) and is expected to be fairly constant ($\pm$ 12 kgs UF_6) during the normal operation of the plant. It is also fairly unimportant from the point of view of safeguards.

The solid phase hidden inventory has been taken to be slightly more than twice the gas phase hidden inventory in the cascade and is based on small scale test cascade performance. The major fraction of the solid phase non-separating inventory is expected to be built up during the initial time of the cascade operation and the build up is expected to slow down considerably for the rest of the plant operation time being mainly governed by the leakrate for the whole plant. Since a high in-leakage of air and heavy deposition of uranium in the centrifuges, reduce the efficiency of a centrifuge in a significant manner /2/, the solid phase, non-separating hidden inventory cannot be permitted to increase continuously to high values. A proper analysis of this category is however necessary, on the basis of historical data and plant performance, to assess its contribution to the EMUF and the safeguards activity.

The last category of inventory consisting of equilibrium hold-up in vacuum and other traps does not represent an amount which remains permanently unknown and unregistered as in the case of the solid phase hidden inventory, but appears as waste at some time of operation of the plant. However, the simultaneous dumping of all the traps in the plant cannot normally be synchronized with a physical inventory taking so that an equilibrium amount always remains in the plant at the time of establishing the MUF. As in the case of the hidden inventory, historical data are required to assess its magnitude and the range of fluctuations.

It is to be noted that all the categories considered here give a total amount of 860 kgs of UF_6 out of which 480 kgs may be considered to be known fairly accurately. The rest of 400 kgs (hidden inventory + hold-up in traps) corresponds to less than 0.05 eff. kgs (assuming natural concentration). An increase of these values by a factor of 2 or 3 will not cause a significant hazard to safeguards even if a part of these amounts is assumed to be convertible to 90 % U-235 as shown in /5/.

4. Material balance and critical amounts

It is necessary to have some numerical examples for material balance to assess the influence of the various SDs for throughput and inventory measurements on the standard deviation of MUF (σ_{MUF}). Another important parameter for safeguards is the amount of material M which can still be detected, in case the amount is lost or diverted, for a given set of α and β errors. Error propagation calculations have been made for the establishment of material balance for a one year reference campaign with the base data from Tables 1, 2 and 3. The amounts of material M (in kg UF_6) have been calculated for the given set of α and β values for different numbers of physical inventory (PI) taking in a year. The results are presented in Tables 4 and 5 and Figs. 3 and 4.

It should be noted that in this case it is assumed that the inspector verifies each step necessary for the establishment of the material balance (i.e. full coverage) that means that he gives no credit at all to the data reported by the operator. In the next chapter the possibility of random sampling from the side of the inspector has been analysed. In such a case the random sampling by the inspector is associated with a broadening of the uncertainty in the material balance which is equivalent to a larger amount M for a given set of α and β values. The broadening of uncertainties means that because of the sampling plan, the operator will be in a position to falsify the measured data. Such a falsification cannot be detected by establishing the material balance on the basis of the reported data alone.

Before examining the results presented in Figs. 3 and 4 it is necessary to lay down the assumptions made for the calculations. The theoretical basis of these calculations with an example is given in Appendix 1.

4.1 Assumptions for the calculation of σ_{MUF} and critical amounts for different frequencies of physical inventory taking in the reference centrifuge plant

a) The batch data and RSD's for throughput and inventory measurements are given in Tables 2 and 3, unless otherwise stated, with conditions governing the propagation of systematic errors as given in 3.1 of this paper.

b) Number of accurate physical inventory (PI) taking is one per year. Number of estimated physical inventory taking varies between 1-7 per year: e.g. 4 PI/yr means one accurate and 3 estimated PI's per year.

c) The starting inventory for an intermediate time period has been calculated on the basis of maximum-likelihood method (Appendix 1).

d) $\alpha = \beta = 0.05$ is valid for a period of one year. If M is the amount of material to be removed from the process over one year and n is the number of inventories per year, then M/n is the amount to be removed per inventory period, i.e. M is equally distributed over all the inventory periods.

e) Tail cylinders which have been measured in previous inventory periods and are incidentally stored at the facility under sealed conditions, have not been considered for inventory taking.

4.2 Contribution of measurement errors to σ_{MUF}

The different contributions of the variances of throughput (σ_D^2) and inventory (σ_I^2) to the σ_{MUF} are listed in Tables 4 and 5. Following points are of interest. Some of these have already been discussed in a qualitative manner under 3.1.

a) The σ_I^2 for accurate PI is about 200 times less than the σ_D^2 for one year (Table 4). In general its contribution to the total σ_{MUF} is negligibly small. Therefore, the RSDs for the gas phase inventory and the hold-ups in traps, which form only a part of the total σ_I^2 can also be considered to be negligible.

b) The σ_I^2s for the estimated inventories are considerably larger than that for the accurately measured inventory and their contribution to the σ_{MUF} increases with increasing number of PI's/yr (Table 4). The major contribution among the various components of the estimated inventory comes from the variances of the 3 sublimator estimates. Although the actual amounts of UF_6 in each of the sublimators are ten times less than that in the feed or the tail cylinders, the RSD of the systematic measurement error is about 75 times worse than those for the feed and the tail cylinders. Since both the RSD of measurement and the amounts for sublimators are based on only rough estimations at present, they are expected to improve with experience. In the case of estimated inventory taking also, the contribution of RSD of systematic errors of measurement for both the gas phase and the trap inventories are negligible.

c) The RSDs for waste measurement contribute significantly both to σ_D and σ_{MUF} (Table 5). For one PI/yr, σ_D and σ_{MUF} increase by a factor of 2.3, from the contribution of RSDs for waste measurements alone. For 4 PI/yr (for which the contribution of σ_Is from estimated inventories plays an important role) the waste contribution is still significant; i.e. in the range of 25 %. Because of the small number of batches assumed for the measurement of the waste stream the contribution of random error is also high for this measurement.

d) If for arguments sake the contribution of the standard deviation of the waste measurement error is eliminated, the total σ_{MUF} is reduced for a given number of PIs/yr. The immediate result is a marked increase in the probability of detection P_D for a given critical amount M (Table 5). This is equivalent to the statement that for a given P_D, the amount of material which can be detected if diverted or lost reduces markedly.

4.3. Number of inventories and critical amounts

The critical amounts M which can be detected with $\alpha = \beta = 0.05$ over one year for different number of PIs/yr, are shown in Figs. 3 and 4. A number of interesting points may be noted:

a) For a given M (in the range of 500-1300 kgs UF_6/yr) the probability of detection P_D increases with increasing number of inventory taking n per year (Fig. 3). Qualitatively this is the same trend as was seen earlier for the improvement of RSD for the waste measurement.

b) For $\alpha = \beta = 0.05$ the critical amount M decreases with increasing n (Fig. 4). This means that the effectiveness of safeguards efforts with regard to the detection of a critical amount can be improved by taking 2-5 additional estimated, less accurate PIs over and above the one-a-year accurate PI determination.

c) However, if the critical amount M is set at around 1300 kgs UF_6/yr, corresponding to a maximum of 1.3 effective kgs/yr, increase in the number of PIs/yr will not bring any significant improvement in P_D (Fig. 4).

5. Estimates of inspection efforts

Because of a number of inherent characteristics of a centrifuge plant, the verification of material balance information for safeguards accountancy purposes may not involve highly intensified routine inspection efforts from an international safeguards authority. For ready reference they are summarized below:

a) Inventory changes to and from a MBA take place in discrete steps (feed, product and tail cylinders, waste containers). All throughput measurements are also in discrete batches.

b) The gas phase inventory in the cascade is negligibly small and can be considered to be constant during the operation of a centrifuge plant. Physical inventories can be made during the operation of the plant and by proper timing, can be made very small compared to the total throughput for the same inventory period.

c) The RSDs of measurements for the main process streams are some of the best attainable at present under normal plant operating conditions in a nuclear fuel cycle.

d) The contribution to the σ_{MUF} and EMUF from the various plant parameters (Table 3) are extremely low and expected to remain well within the RSD of measurement errors.

Some very rough estimates for the extent of routine inspection efforts required in the reference plant, have been presented in Table 9. The activities which form the basis of these estimates are summarized in Table 6. Furthermore, the following assumptions have been made in arriving at these estimates.

i) The reference plant operates on the basis of monthly transports and supply, i.e. on the average 12 shipments each of feed and product materials take place per year. The tail cylinders are supposed to be stored in the storage area under sealed conditions.

ii) All the cylinders in the storage area are weighed with a rough weighing device by the inspection personnel. The RSD of the weighing is 20 kgs per weighing (Table 7).

iii) All the cylinders in the storage MBAs are tested by the inspectors with an enrichment meter for a rough estimation of the U-235 concentration. This is carried out not to check the material balance but to ensure that firstly, no tail cylinders are inadvertently declared as a feed cylinder and secondly, the product and tail concentrations do correspond approximately to the declared values.

iv) The chemical concentration of UF_6 will not be verified by the inspectors since the operationally tolerable gaseous impurities are so low that stoichiometric ratios can be assumed for inspection purposes.

v) The conditions for random sampling exist for the product and the tail streams (explained in Annex 3), so that the inspectors can test the declared data of the plant operator in a random manner. This fact will be utilized by the inspectors to test the operators' data on isotopic composition of the product and (if required) the tail cylinders.

vi) Since the number of waste containers are expected to be small, the inspectors will measure the uranium amounts in each container with the same accuracy as that obtained for plant operation.

With these assumptions and the type of activities mentioned in Table 6, 100-120 inspection mandays appear to be sufficient for routine inspection activities.

5.1 Sampling effort and critical amounts

It is to be noted that the plant operator establishes the material balance in the reference plant with the fairly high accuracies mentioned in Tables 2 and 3. In the suggested inspection procedures, the inspector measures the weight of each of the cylinders independently, however with a lesser accuracy than that by the operator. Besides, he takes random samples to determine the concentrations in the product and the tail streams. He also measures the waste streams with the same accuracy as that obtained by the plant operator.

In this scheme of inspection, the plant operator would have two additional possibilities of diversion besides that obtained by the measurement accuracy of the material balance. These are given by the

a) larger measurement uncertainties of weighing by the inspectors

b) extent of random sampling carried out by the inspector.

and can be summarized as strategies of diversion by means of falsification of reported data. In the framework of the first possibility, the operator would be in a position to declare systematically lower amounts of UF_6 in feed cylinders and higher amounts of UF_6 in the product and tail cylinders but still remain within the uncertainty of weighing by the inspector. The difference in the amounts of UF_6 thus gained, could be used to produce high enriched (e.g. 90 % U-235) uranium.

Similarly, if for example, the tolerances for the U-235 concentrations in the product and the tail streams were to be given to be 1 % and 5 % respectively, the plant operator would have the possibility to declare the concentrations at the lower limit of the tolerance values and use the difference in a similar manner as in the case of the first possibility.

It was therefore, thought desirable to estimate the possible extent and magnitude of the significant amounts which could be diverted in this manner.

A few preliminary cases were considered. The input data for this consideration are presented in Table 7. It was assumed that after withdrawal, the UF_6 would be enriched to 90 % U-235. The amounts which could thus be obtained were taken from Table 4 of /5/. The method used to determine the critical amounts M for the first possibility is the 'sign test method' and is discussed in Annex 2. The other method used to determine the optimum sampling plan for the second possibility is based on the application of the theory of games and is shortly sketched in Annex 3.

It is to be emphasized once more that the results are of a very preliminary nature and at the present stage they illustrate more the application of the method than the possibility of assessing the inspection effort or of estimating the effectiveness of a safeguards system.

In Table 8 a few measurement results are given for the sign test. The explanations for the assumptions and numbers are presented in Annex 2 and Table 7. It is to be noted for example that if more than 55 % of the information submitted by the operator for the three streams were found to be positive by the sign test, a statement can be made to the effect that the plant operator has removed with almost certainty (P_D=1) a critical amount corresponding to at least 32.7 kgs of 90 % enriched U-235, using the strategy of removing from each cylinder $\sigma/2$ (where σ is the SD of gross weighing measurement by the inspector and equals 20 kgs of UF_6). For a P_D of 0.95, the critical amount will be reduced markedly.

The plant operator may not however, use the assumed strategy but some other one; in that case the significant amount will change. Further work is however, necessary to find out the optimum significant amounts for a given β.

In Fig. 5, the probability of detection P_D (= $1-\beta$) has been shown as a function of the number of mass spectrometric samples analysed (expressed in terms of manhours required for analysis; two manhours per sample) with the critical amount M expressed in U-235 as parameter[1].The continuous lines correspond to sampling plan from both product and tail streams whereas the

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[1] see comment 5 table 7

dotted line shows the sampling plan for the product stream only (there is some justification for this assumption on account of the reduced separative work required for 90 % U-235 starting from 3 %). According to the dotted line, about 25 samples (in a total of 130) will have to be analysed by the inspector to detect an assumed diversion of an amount equivalent to 10 kgs of U-235 with a probability of detection of 95 %.

Because of the various possibilities which exist, it is difficult to assess at this stage, the significant amount, the detection of which can be ensured by the routine inspection efforts given in Table 9. However, further work is in progress in this direction.

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Table 1

Relevant Data for the Reference Centrifuge Plant

(all units of amounts are expressed in t UF_6 unless stated otherwise)

Items	Numbers
Reference plant	
Separative work [t U/yr]	600
Feed [t UF_6/yr, natural uranium]	1705
Product [t UF_6/yr, 3 % U-235]	275.5
Tails [t UF_6/yr, 0.3 %-U-235]	1439
Waste [t UF_6/yr, natural U]	8.5
Batch size	
Feed cylinder (transport cylinder)	10
Product cylinder	2
Tail cylinder	10
Waste container	1.5
Inventory	
Input	
Flanged feed cylinder to the feed station	10
Desublimators	1
Cascade	
Gas phase inventory	0.12
Hidden inventory	
Deposites in centrifuges and pipelines etc.	0.3
Content of vacuum traps	0.1

continued

Table 1 continued

Items	Numbers
Product output	
Flanged product cylinder in the product station	2
Desublimators	1
Tail output	
Flanged tail cylinder at the output station	10
Desublimators	1
Storage corresponds to one month production capacity for the three streams	

Table 2

Relevant Data on Batch Sizes and Measurement Errors for Throughput Measurements

Items	Batches/yr	t UF_6/batch	SD for weighing[1)] [kg UF_6/batch]	MSP		Chemical Analysis	
				RSD (in %) of random errors	RSD (in %) of systematic errors	RSD (in %) of random errors	RSD (in %) of systematic errors
Feed	171	10					
Product	129	2	1	0.15	0.1	0.12	0.15
Tails	144	10					
Wastes	6	1.5	RSD (in %) of the total measurement random: 10; systematic: 1				

1) Because of weighing by difference the systematic errors are eliminated. The SD given varies randomly from cylinder to cylinder.

Table 3 **Relevant Data on Batch Sizes, Measurement Errors and Contributions to EMUF**

Plant areas	Contribution to EMUF	Contribution to SD of MUF [kg UF_6/inventory] (RSD of syst.error)
Storage	-	-
Feed station		
heels in cylinders (2 kgs UF_6/batch)	342 kgs UF_6/a	-
flanged feed cylinder	-	16 (0.15 % chem.analys. + 1 kg weighing)
desublimators		
accurate inventory	-	1
estimated inventory	-	100 (10 %)
Cascade and traps		
gas phase inventory	120 kgs UF_6	12 (10 %)
hidden inventory	300 kgs UF_6	-
hold-up in traps	100 kgs UF_6	10 (10 %)
Product-station		
flanged product cylinder	-	4 (0.15 % chem.analys. + 1 kg weighing)
desublimators		
accurate inventory	-	1
estimated inventory	-	100 (10 %)
Tail-station		
flanged tail cylinder	-	16 (0.15 % chem.analys. + 1 kg weighing)
desublimators		
accurate inventory	-	1
estimated inventory		100 (10 %)

<u>Table 4</u> Contribution of Variances of Throughput (σ_D^2) and Inventory (σ_I^2) Measurements to the Standard Deviation of MUF (σ_{MUF}) for Different Frequencies of Physical Inventory Taking per Year

Item	Physical inventories/yr 1	2	4	8
σ_D^2(x 10^4) [kg UF_6^2]	18.96	8.9	4.59	2.84
σ_I^2(x 10^4) [kg UF_6^2]	0.08	1. 3.06 2. 2.3	1. 3.06 2. 4.84 3. 5.07 4. 2.11	1. 3.06 2. 3.48 3. 4.79 4. 4.84 5. 4.85 6. 4.85 7. 4.85 8. 1.87
σ_{MUF} [kg UF_6]	437	1. 346 2. 335	1. 277 2. 307 3. 311 4. 259	1. 243 2. 271 3. 276 4. 277 5. 277 6. 277 7. 277 8. 217

<u>Table 5</u> Influence of σ for Waste Measurements (σ_W) on σ of Throughput Measurements (σ_D) and σ_{MUF}

Number of physical inventories/yr	σ_D [kg UF_6]		σ_{MUF} [kg UF_6]		Detection probability P_D for M = 500 kg UF_6	
	with σ_W	without σ_W	with σ_W	without σ_W	with σ_W	without σ_W
1	435	215	437	217	0.31	0.74
4	214	108	σ_1 = 277 σ_2 = 307 σ_3 = 311 σ_4 = 259	206 224 232 159	0.537	0.891

Table 6

Verification Measures and Activities for Routine Inspection Efforts

1. Input-stream

   Control of seals

   Weighing of each feed cylinder with a rough weighing device

   Rough enrichment measurement for each feed cylinder with an enrichment meter

2. Product-and tail-streams

   Control of seals

   Mass-spectrometric analysis of concentration by random sampling

   Estimation of concentrations in each cylinder by enrichment meter

   Putting of seals

   Weighing of each cylinder with a rough weighing device

3. Waste-stream

   Estimation of amount of UF_6 in each of the waste containers with the same measurement accuracy as those obtained for operation

4. Physical inventory determination

   Presence of inspector during the determination of accurate inventory or of inventory by estimates

5. Further verification activities

   Tag-inventory taking of cylinders in the storage

   Verification of records

   Calibration of apparatus and instruments

   Containment and surveillance activities

Table 7 Input Data for Estimation of Significant Amounts for Diversion on the Basis of Sign Test for Weighing and Sampling Plant for Isotopic Compositions

1. Verification of gross weight for cylinders

SD[1] of rough weighing (performed by inspector)		20 kgs UF_6
Amount assumed to be withdrawable by the plant operator per cylinder		10 kgs UF_6
correspond to 90 % U-235[2] in		
feed cylinder		0.045 kgs U
product cylinder		0.3 kgs U
tail cylinder		0.01 kgs U

2. Random sampling plan

SD as in the case of operator, i.e.
RSD random error 0.15 %, RSD systematic error 0.1 %

Amount assumed to be withdrawable by the operator		
product : 1 % of enrichment [3]		0.4 kgs U-235 [5]
	≘	0.59 kgs UF_6
Amount withdrawable by the operator from		
tail : 5 % of concentration [4]		1 kg U-235 [5]
	≙	1.47 kgs UF_6
Time required for each MSP-measurement [h]		2

Comments to Table 7

1. The SD of the rough weighing by the inspector has been assumed to be such that it remains constant for a given cylinder but varies in a random fashion from cylinder to cylinder.

2. Reference /5/, Table 4.

3. The withdrawable amount results from the specification given by the fuel element manufacturers.

4. In principle the operator could reduce the U-235 concentration in tails to very low values so that the withdrawable amount could range between approximately 0 - 0.3 %. However, such heavy withdrawals could be detected by simple material balance unless of course the extracted U-235 amounts were concentrated to very high values. This would on the other hand require fairly large amount of separative work. Therefore, a range of ± 5 % has been assumed for withdrawal from tail cylinders.

5. This withdrawal of U-235 has been assumed to be equal to approximately 90 % UF_6. The values of the amounts of U-235 diverted were not transferred to amounts of 90 % U-235 however, as this procedure would result in a different net amount of diverted U-235 than assumed before in the framework of falsification if isotopic compositions.

Table 8 Sign Test for the Gross Weights of UF_6 Cylinders. Examples of Some Numerical Results.[1)]

	n	$z_c(\alpha = 0.05)$	$1-\beta(n,0)$	M [kg U 90 %]	$1-\beta(5,n-5)$	M [kg U 90 %]
Feed	171	97	0.9998	5.2	0.07	0.9
Product	129	74	0.998	26.5	0.092	6.0
Tails	144	82	0.9992	1.0	0.087	0.2
Sum		$(\alpha = 0.14)$	~ 1	32.7	0.23	7.1

[1)]Explanation of the numbers and symbols are given in Annex 3.

Table 9 Routine Inspection Effort for the Reference Centrifuge Plant

Note: On the average the reference plant is expected to operate on a monthly transport and supply basis (i.e. 12 shipments each for product and tail cylinders per year).

		IMD/yr
1.	Activities for the process MBA (once a month)	
	Verification of operator's measures e.g.	
	Operator's weighing, concentration and isotopic measurements	
	Rough checking of the gross weights of the UF_6 cylinders	
	Estimation of concentrations	
	Tag-inventory	
	Calibration	
	Sampling	
	Analysis of waste concentrations	
	3 IMD per inspection; 12 per year	36
2.	Physical inventory taking	
	Accurately estimated inventory (once per year)	20
	Estimated inventory (4-times a year, 4 IMD per inventory)	16
3.	Control of records	10
4.	Additional inspection effort for random sampling	
	(1 IMD per sampling)	15
5.	Waiting time	15
		100 - 122 [1]

[1] Since a part of the inspection for inventory estimates and random sampling can be carried out during the monthly routine inspections, some of the periods are expected to overlap.

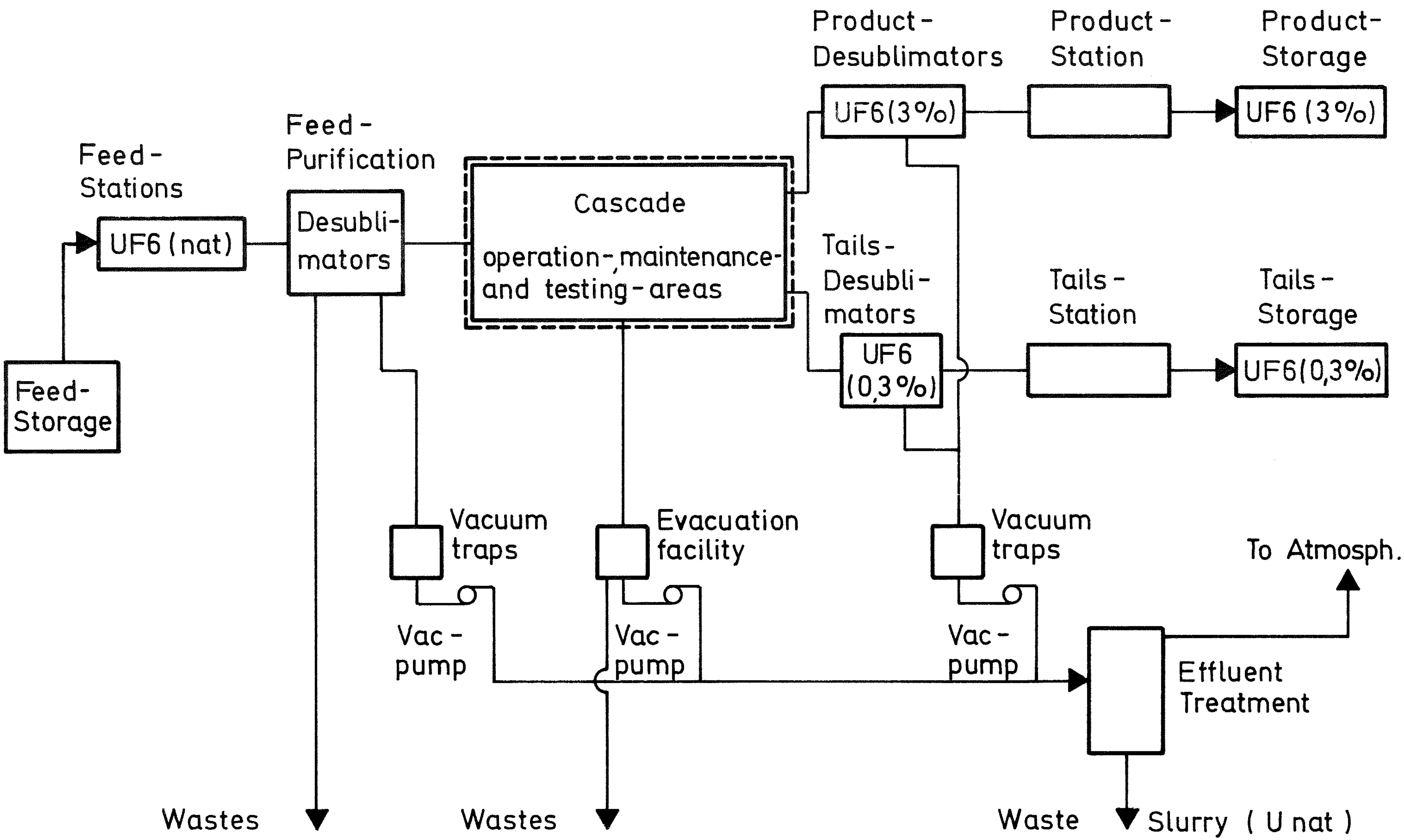

Fig. 1: Schematic flow-sheet for the reference centrifuge plant

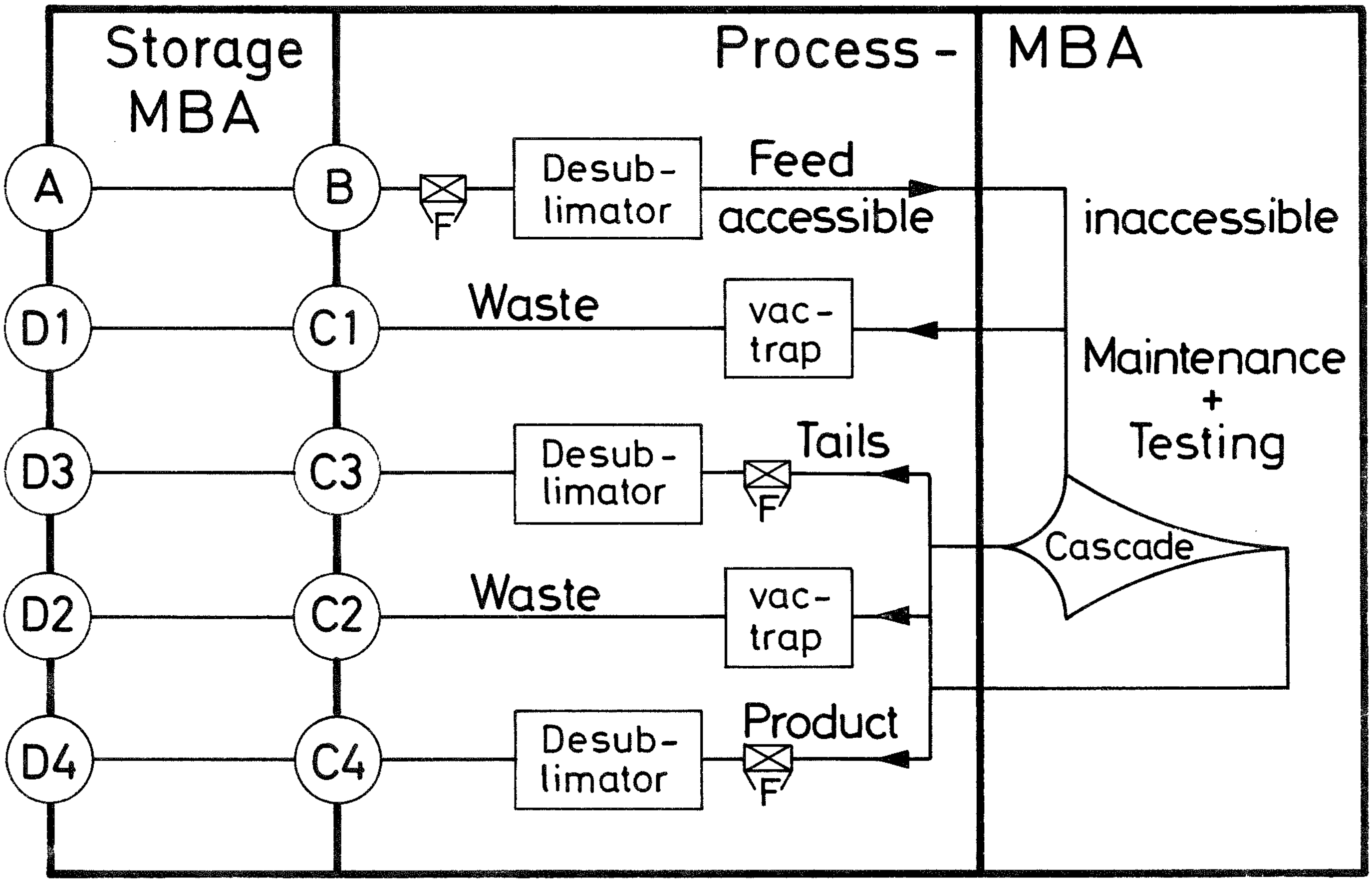

Fig. 2 MBAs and Location of KMPs in the reference centrifuge plant

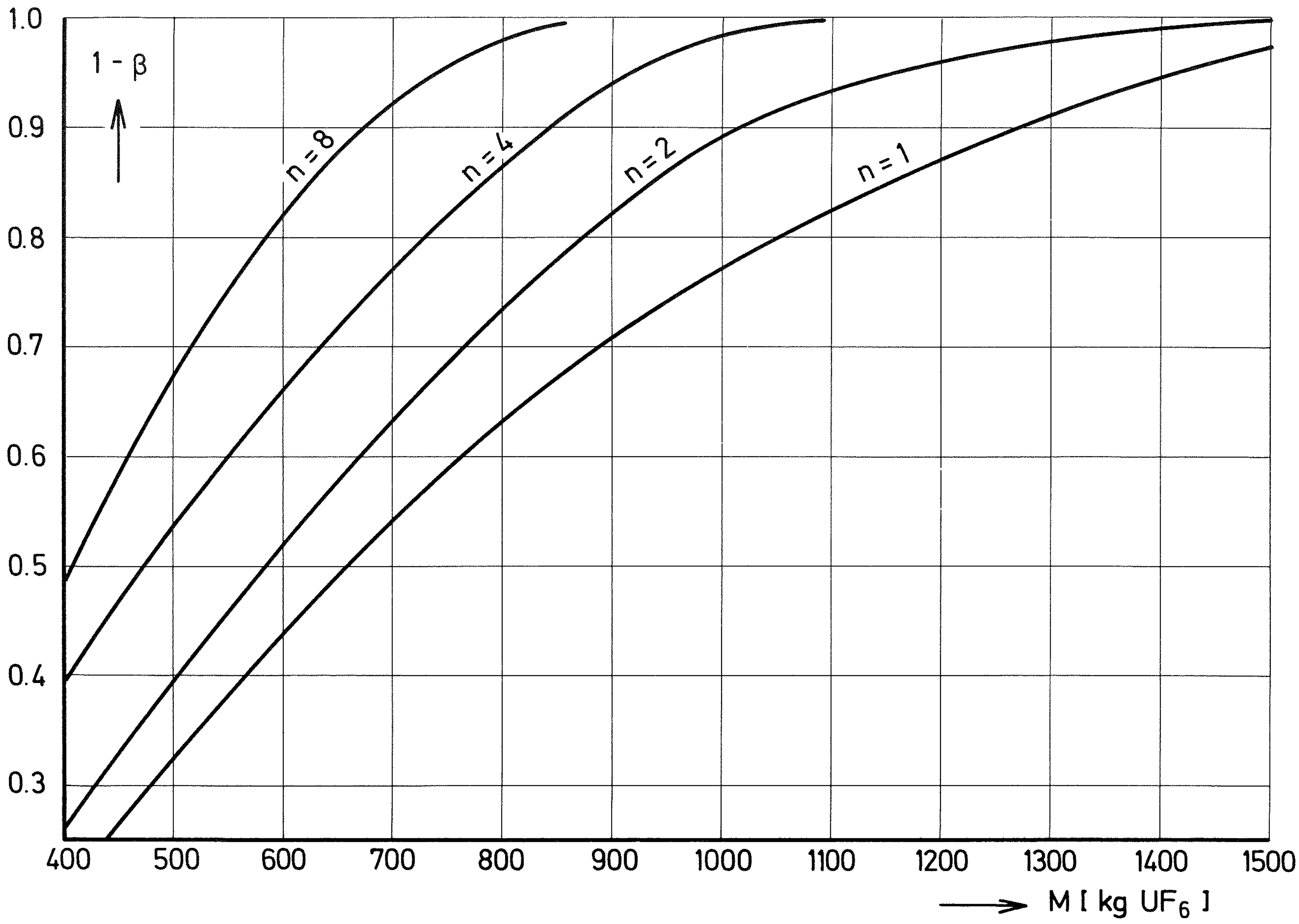

Fig. 3 Probability of detection 1- β as a function of critical amount M with number n of inventories per year as parameter

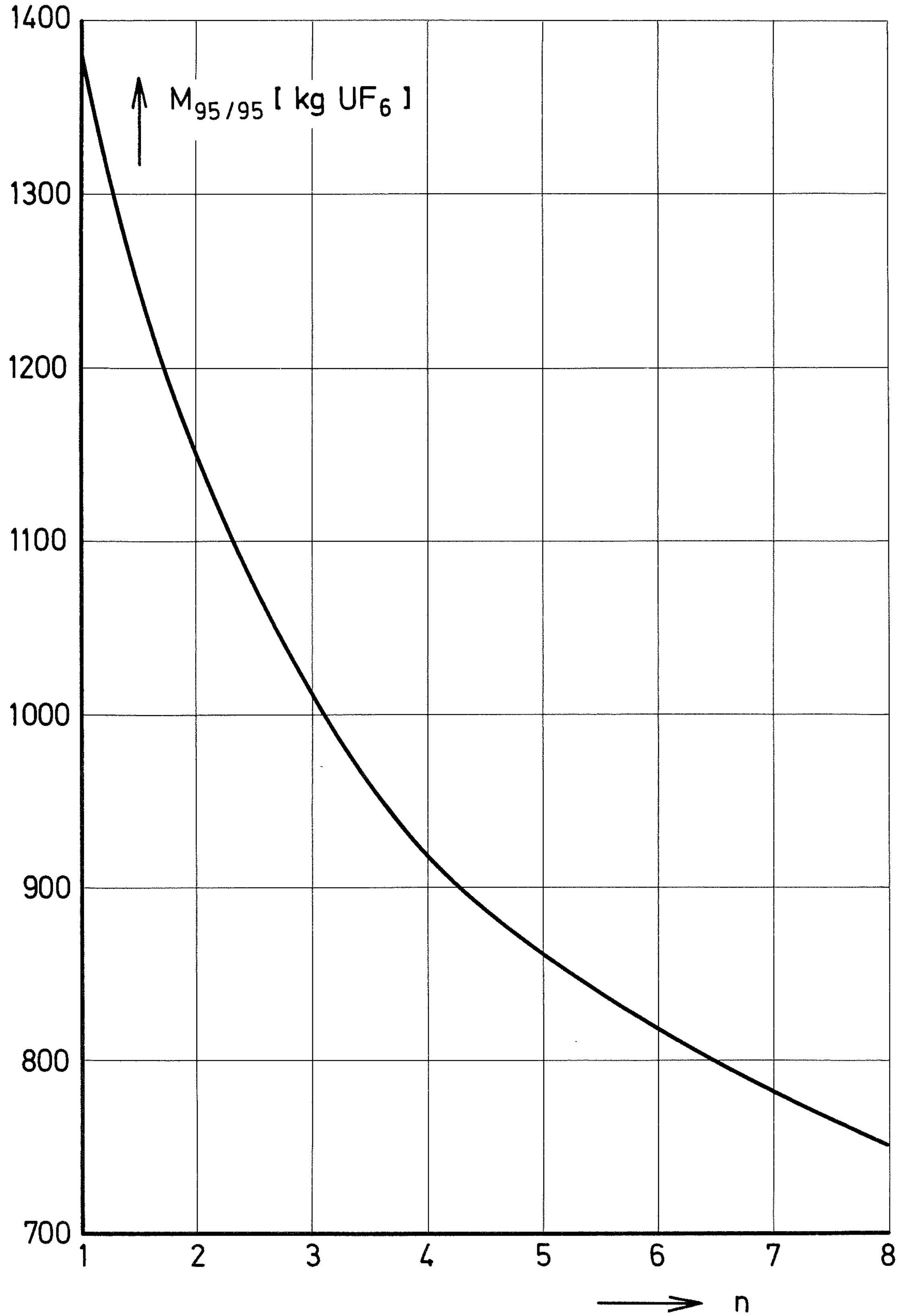

Fig. 4 Critical amount M for $\alpha = \beta = 0.05$ as a function of the number n of inventories per year

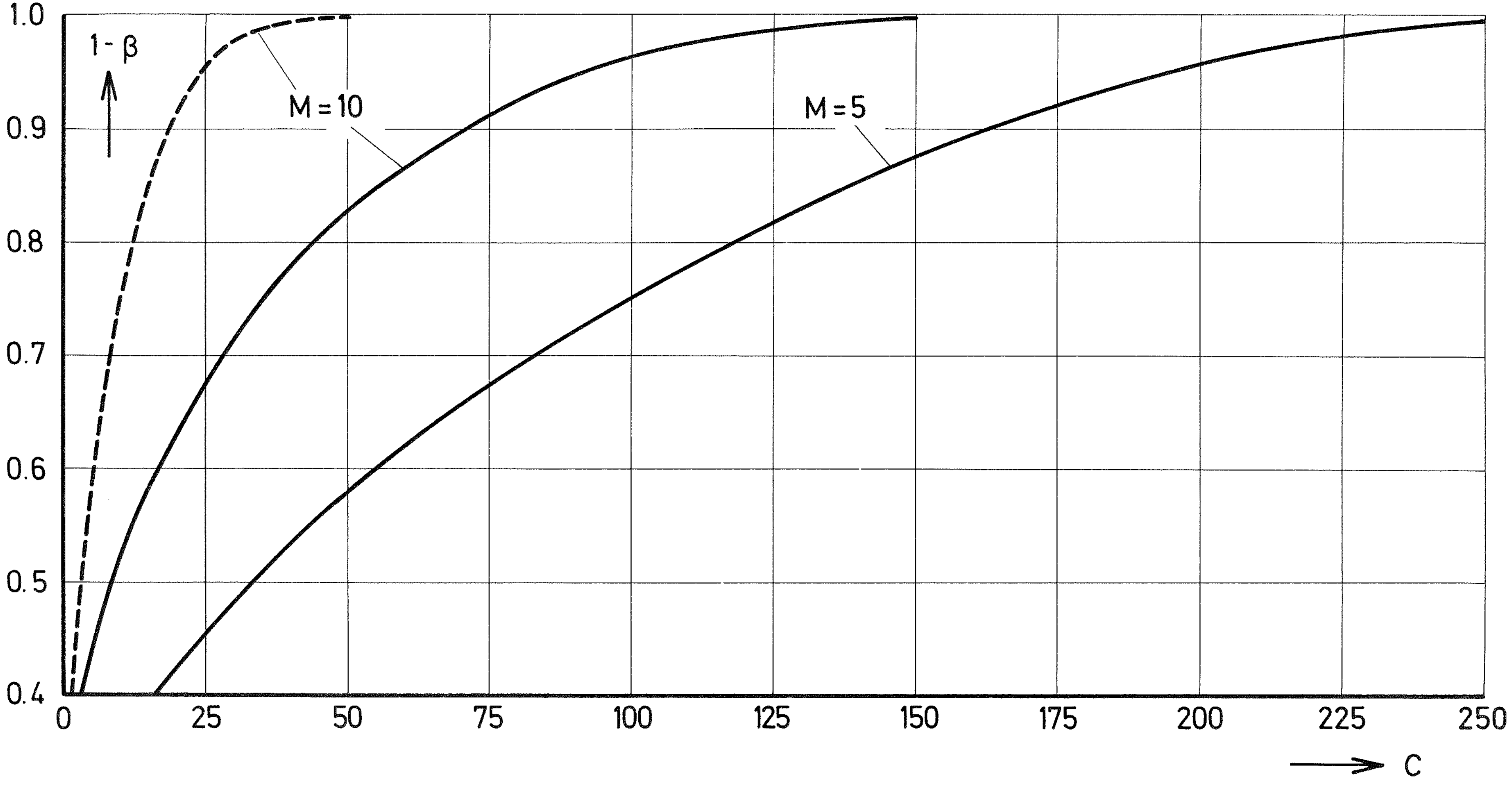

Fig. 5 Verification of the isotopic composition of product and tails cylinders : probability of detection 1 - β as a function of number of analyses/yr with critical amount M [kg U-235] as parameter. The dotted line corresponds to the verification of only product cylinders.

Annex 1: Evaluation of MUF, σ_{MUF} and Different Probabilities of Detection for the Reference Centrifuge Plant

1. Definition of MUF

For the considered reference centrifuge plant the MUF (Material Unaccounted For) is defined as follows:

$$MUF = I_o + D - I_1 \qquad (A1-1)$$
$$= I_o + (F-P-T-W)-I_1$$

I = Inventory
F = Feed
P = Product
T = Tails
W = Wastes

Due to measurement errors the MUF is a random variable. Under the assumption that the values are independent from each other (that means neglecting covariance terms), one obtains for one inventory period the variance var (MUF) from equation (A1-1) in the following way:

$$\text{var (MUF)} = \text{var}(I_o) + \text{var (F)} + \text{var (P)} + \text{var (T)} + \text{var (W)} + \text{var}(I_1) \qquad (A1-2)$$

2. Evaluation of var F_1 var P, var T and var W

The safeguarded material is measured batchwise, that means each batch is weighed and sampled. From the gross- and tare-weight one obtains the netweight M. From the sample one gets the chemical factor C and the enrichment N.

The U-235-content of the batch i is therefore:

$$U\text{-}235_i = M_i \cdot N_i \cdot C_i \qquad (A1-3)$$

M = netweight
N = enrichment U-235/(U-235+U-238)
C = chemical factor $UF_6/(UF_6+\text{impurities}) \cdot 0.68$

Because of measurement-errors the errors for a single measurement are given as follows:

$$M_i = WM_i + d_M + e_{Mi} \qquad (A1\text{-}4)$$

$$N_i = WN_i + d_N + e_{Ni}$$

$$C_i = WC_i + d_c + e_{ci}$$

W = true value

d = systematic error

e_i = random error

Equation (A1-3) together with equation (A1-4) leads to

$$U\text{-}235_i = (WM_i + d_M + e_{Mi})\,(WN_i + d_N + e_{Ni})\,(WC_i + d_c + e_{ci}) \qquad (A1\text{-}5)$$

The neglection of terms which are small of second order gives

$$\begin{aligned} U\text{-}235_i = {} & WN_i \cdot WM_i \cdot WC_i \\ & + WN_i \cdot WC_i (d_M + e_{Mi}) \\ & + WM_i \cdot WC_i (d_N + e_{Ni}) \\ & + WM_i \cdot WN_i (d_c + e_{ci}) \end{aligned} \qquad (A1\text{-}6)$$

If one adds all the estimates of the U-235-contents of the single feed-batches corresponding to eq. (A1-6) one obtains the following estimate of the total amount of U-235-feed for the considered inventory period (it is assumed that $WM_i = WM$, $WN_i = WN$, $WC_i = WC$ for all $i = 1 \ldots n$):

$$\begin{aligned} \text{Feed} = {} & \sum_i U\text{-}235_i \\ = {} & n \cdot WM \cdot WN \cdot WC + \\ & + WN \cdot WC\,(n \cdot d_M + \Sigma e_{Mi}) \\ & + WM \cdot WC\,(n \cdot d_N + \Sigma e_{Ni}) \\ & + WM \cdot WN\,(n \cdot d_c + \Sigma e_{ci}) \end{aligned}$$

$i = 1 \ldots . n$

n = number of feed batches

The variance var (Feed) is given by the following expression:

$$\begin{aligned} \text{var (Feed)} = \; & WN^2 \cdot WC^2 \cdot (n^2 \text{ var } d_M + n \cdot \text{var } e_M) + \\ + \; & WM^2 \cdot WC^2 \cdot (n^2 \text{ var } d_N + n \cdot \text{var } e_N) + \\ + \; & WM^2 \cdot WN^2 \cdot (n^2 \cdot \text{var } d_c + n \cdot \text{var } e_c) \end{aligned} \qquad \text{(A1-7)}$$

To show the different influences of the number of batches on the random and the systematic errors eq. (A1-7) may be written in the following form:

$$\frac{\text{var (Feed)}}{(n \cdot WM \cdot WN \cdot WC)^2} = \frac{1}{n}\,\delta^2_{rM} + \delta^2_{sM} + \frac{1}{n}\,\delta^2_{rN} + \delta^2_{sN} + \frac{1}{n}\,\delta^2_{rC} + \delta^2_{sC} \qquad \text{(A1-8)}$$

δ_s = rel. standard deviation (RSD) fo the systematic error

δr = RSD of the random error

Eq.A1-8 shows, that the variances of the random errors decrease with increasing number of batches, while the variances of the systematic errors are independent of the number of batches.

For the uranium feed eq. (A1-8) simplifies to

$$\frac{\text{var (Feed)}}{(n \cdot WM \cdot WC)^2} = \frac{1}{n}\,\delta^2_{rM} + \delta^2_{sM} + \frac{1}{n}\,\delta^2_{rC} + \delta^2_{sC} \qquad \text{(A1-9)}$$

Eq. (A1-8) and eq. (A1-9) are equivalent for product and tails.

Theoretically for the wastes the same way of establishing the variance is possible as for feed, product and tails. Because of lack of data concerning the measurement of wastes for the following numcerical examples the variance var (W) is written in the following way:

$$\frac{\text{var } W}{(n \cdot W)^2} = \frac{1}{n} \cdot \delta^2_{rW} + \delta^2_{sW} \qquad \text{(A1-10)}$$

δ_{rW} = RSD of the overall random error of the waste measurement

δ_{sW} = RSD of the overall systematic error of the waste measurement

3. Evaluation of var I

Two different cases are considered:

a) the physical inventory of the plant is measured accurately (dumping of desublimators, emptying vaccuum-traps, etc.)

b) the physical inventory is estimated on operational data.

For the first case the variance var I can be determined with Eq. (A1-8) because the inventory is measured batchwise.

For the case b the inventory is estimated on the basis of flow rates. This procedure leads to an accuracy of ca. 10 %. The contents of the transport-containers flanged at the input and output stations are not added to the inventory-estimate, they are added to the book inventory.

4. Probability of detection for a potential diversion of nuclear material based on var (MUF)

4.1 The case of one inventory period

The probability of detection for the potential diversion M of nuclear material is a function of the error of the first kind α, the variance var (MUF) and the amount M. The probability of detection p (which is one minus the error of the second kind probability β) is given by

$$p(M) = \Phi\left(\frac{M}{\sigma_{MUF}} - U_{1-\alpha}\right) \qquad (A1-11)$$

$$U_{1-\alpha} = \Phi^{-1}(1-\alpha)$$

Here, Φ^{-1} is the inverse function of the Gaussian distribution function. σ_{MUF} is given by

$$\sigma_{MUF} = \sqrt{var(MUF)} = \sqrt{var\ I_o + var\ D + var\ I_1} \qquad (A1-12)$$

4.2 Sequence of inventory periods

For a sequence of inventory periods the problem arises to choice the value for the starting inventory for the next time interval. Three theoretical possibilities exist for the estimation of the starting inventory: The so called book inventory, the physical inventory or a combination of both can be taken as the estimator [1]. For the following numerical examples the maximum likelihood estimate has been chosen for reasons discussed in ref. /1-2/ and /1-3/. It has been assumed that the physical inventory is measured accurately at the beginning and the end of the year, while the inventories within the year are based on estimated values, as pointed out in 3.

The probability of detection p for the total amount M to be diverted is given by

$$1-p = \beta = \prod_{i=1}^{n} \left[U_{\sqrt[n]{1-\alpha}} - \frac{i \cdot M}{n \cdot \sigma_{MUF}} \right] \qquad \text{(A1-13)}$$

Here, n is the number of inventories per year. It has been assumed that the operator will divert in each inventory period the same fraction $\frac{M}{n}$ of the total amount M.

The overall error of the first kind α (i.e. the probability to cause a false alarm) is connected with the errors of the first kind α_i for the single inventory periods by the following relation

$$1-\alpha = \prod_{i=1}^{n} (1-\alpha_i) \qquad \text{(A1-13a)}$$

For $\alpha_1 = \ldots = \alpha_n =: \alpha_o$ one obtains

$$1-\alpha = (1-\alpha_o)^n \qquad \text{(A1-13b)}$$

σ_{MUF_i} is given by:

$$\sigma_{MUF_i} = \sqrt{\text{var (MUF)}_i} = \sqrt{\text{var } I_{o,i} + \text{var } D_i + \text{var } I_{1,i}} \qquad \text{(A1-14)}$$

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[1] For further details see ref. /1-1/.

5. Numerical examples for the accuracy of the material balance for the uranium accountancy

The following assumptions have been made to obtain the resulting values:

a) The systematic error of weighing has been eliminated, due to the facts, that first the gross- and tare-weight of each cylinder is assumed to be taken on the same scales and second the systematic error of weighing is independent of the weight.

b) The systematic error of the chemical analysis has been neglected because it has been assumed that the input- and output-streams will be measured in the same laboratory with the same systematic error.

Comment to a) and b)

The net weight M of a UF_6 cylinder is determined by the difference of the gross weight A and the tare weight B. If WA and WB are the true values, e_A and e_B are the corresponding random errors and d the systematic error (independent of the true value) one has

$$M = (WA + e_A + d) - (WB + e_B + d)$$

$$= WA - WB + e_A - e_B$$

therefore

$$\text{var } M = \text{var } e_A + \text{var } e_B$$

This can be shown also by formally introducing a covariance term:

$$\text{var } M = \text{var } e_A + \text{var } e_B + 2 \text{ var } d - 2 \text{ cov } (A, B)$$

As one has

$$\text{cov } (A, B) = E\left[(d + e_A)(d + e_B)\right] = Ed^2 = \text{var } d$$

one obtains the same result as before.

In the case of the chemical analyses one has systematic errors which are proportional to the true values:

$$d_c = WC \cdot \tilde{d}_c; \quad \tilde{d}_c \text{ independent of WC}$$

Thus, the variance of the difference between feed (F) on one side and product (P) and tails (T) on the other side is according to eq. (A1-6) given by

$$\mathrm{var}\Big[n\cdot WM_F\cdot WN_F\cdot WC_F + WN_F\cdot WC_F\ (n.d_M + \sum_i^n e_{Mi}) +$$

$$+ WM_F\cdot WC_F\ (n\cdot d_N + \sum_i^n e_{Ni}) + WM_F\cdot WN_F\ (n\cdot WC_F\cdot \tilde{d}_C + \sum_i^n e_{Ci}) +$$

$$- m\cdot WM_P\cdot WN_P\cdot WC_P - WN_P\cdot WC_P\ (m_{dM} + \sum_j^m e_{Mj}) +$$

$$- WM_P\cdot WC_P\ (m\cdot d_N + \sum_j^m e_{Nj}) - WM_P\cdot WN_P\ (m\cdot WC_P\cdot \tilde{d}_C + \sum_j^m e_{Cj}) +$$

$$- k\cdot WM_T\cdot WN_T\cdot WC_T - WN_T\cdot WC_T\ (k_{dM} + \sum_l^k e_{Ml}) +$$

$$- WM_T\cdot WC_T\ (k\cdot d_N + \sum_l^m e_{Nl}) - WM_T\cdot WN_T\ (k\cdot WC_T\cdot \tilde{d}_C + \sum_l^k e_{Cl}) \Big]$$

Here, n, m, k are the total numbers of feed-, product and tails batches for the inventory period considered. Collection of the $\tilde{d}_c$ terms gives

$$(n\cdot WM_F\cdot WN_F\cdot WC_F - m\cdot WM_P\cdot WN_P\cdot WC_P - k\cdot WM_T\cdot WN_T\cdot WC_T)\cdot \tilde{d}_c$$

If one neglects the waste and the physical inventory in this connection one sees that this term is zero because of material balance reasons. This could have been shown again by formally introducing covariance terms.

c) For one inventory period per year the physical inventory will be measured exactly at the beginning and at the end of the year. For n inventory periods per year the physical inventory will be measured exactly at the beginning as well as at the end of the year however, within the year the physical inventory will be estimated.

d) The variance of the estimate σ_s^2 for the starting inventory is given by /1/

$$\frac{1}{\sigma_S^2} = \frac{1}{\sigma_B^2} + \frac{1}{\sigma_I^2}$$

σ_B^2 = variance of book inventory of the foregoing inventory period

σ_I^2 = variance of the estimated physical inventory at the end of the foregoing inventory period

e) The error of first kind α has been taken constant for one year independent of the number of inventories per year, which means that for more than one inventory period per year the error of first kind α_i for the considered inventory period i is given by (see formula (A1-13b))

$$1-\alpha_i = \sqrt[n]{1-\alpha} \quad \text{for } i = 1....n$$

For the following calculations a value of α = 0.05 has been chosen.

5.1 Evaluation of σ_D, σ_I and σ_{MUF} for the reference centrifuge plant

The variance var D of the throughput measurement is with the assumptions made above given by

$$\text{var } D = (n_1+n_2+n_3)\cdot\sigma_{rM}^2 + \qquad \text{weighing}$$

$$+ M_1^2(n_1\cdot\delta_{rC}^2) \qquad \text{feed, chemical analysis}$$

$$+ M_2^2(n_2\cdot\delta_{rC}^2) \qquad \text{product, chemical analysis}$$

$$+ M_3^2(n_3\cdot\delta_{rC}^2) \qquad \text{tails, chemical analysis}$$

$$+ W^2(n_W\cdot\delta_{rW}^2 + n_W^2\cdot\delta_{sW}^2) \qquad \text{waste-measurement}$$

(A1-17)

a) The case of one inventory period per year

The variance var D of the sum of all throughput measurements in the year is

$$\text{var } D = 444\cdot 1^2$$
$$+ 10^8(171\cdot 0.0012^2)$$
$$+ 4\cdot 10^6(129\cdot 0.0012^2)$$
$$+ 10^8(144\cdot 0.0012^2)$$
$$+ 1.5^2\cdot 10^6(6\cdot 0.1^2+36\cdot 0.01^2)$$
$$= 18.96\cdot 10^4(\text{kg } UF_6)^2$$

(for the corresponding data see Table 1 and Table 2 of the main text)

$$\sigma_D = 435 \text{ kg } UF_6$$

The variance of physical inventory is with the data of Table 3 of the main text given by

$$\text{var } I = 16^2+1^2+12^2+10^2+4^2+1^2+16^2+1^2$$

$$= 0.078 \cdot 10^4 \ (\text{kg } UF_6)^2$$

$$\sigma_I = 28 \text{ kg } UF_6$$

Therefore, the variance var(MUF) is given by

$$\text{var MUF} = \text{var } D + 2 \cdot \text{var } I$$

$$= 17.5 \cdot 10^4 + 2 \cdot 0.078 \cdot 10^4$$

$$= 19.11 \cdot 10^4$$

$$\sigma_{MUF} = 437 \text{ kg } UF_6$$

b) The case of four inventory periods per year

In this case the variance var D of the sum of all throughput measurements in one inventory period is

$$\text{var } D = 111 \cdot 1^2$$

$$+ 10^8(43 \cdot 0.0012^2)$$

$$+ 4 \cdot 10^6(33 \cdot 0.0012^2)$$

$$+ 10^8(36 \cdot 0.0012^2)$$

$$+ 2.25 \cdot 10^6(1.5 \cdot 0.1^2 + 2.25 \cdot 0.01^2)$$

$$= 4.59 \cdot 10^4 \quad \text{kg } \ UF_6^2$$

$$\sigma_D = 214 \text{ kg } UF_6$$

The variance of the measured physical inventory is the same as in the case of one inventory period

$$\text{var } I_{\text{measured}} = 0.078 \cdot 10^4 \quad (\text{kg } UF_6)^2$$

The variance of the estimated physical inventory is (see Table 3)

$$\text{var } I_{\text{estimated}} = 100^2+12^2+10^2+100^2+100^2$$

$$= 3.02 \cdot 10^4 \quad (\text{kg } UF_6)^2$$

In the following the variances of the MUF's for the different inventory periods are given.

First period:

$$\text{var } I_{0,1} = 0.078 \cdot 10^4$$

$$\text{var } I_{1,1} = 3.02 \cdot 10^4$$

$$\text{var MUF}_1 = (0.078+4.59+3.02) \cdot 10^4$$

$$= 7.69 \cdot 10^4$$

$$\sigma_{\text{MUF}_1} = 277 \text{ kg } UF_6$$

Second period:

$$\text{var } I_{0,2} = \frac{10^4}{\frac{1}{4.59} + \frac{1}{3.02}} = 1.82 \cdot 10^4$$

$$\text{var MUF}_2 = (1.82+4.59+3.02) \cdot 10^4$$

$$= 9.43 \cdot 10^4$$

$$\sigma_{\text{MUF}_2} = 307 \text{ kg } UF_6$$

Third period:

$$\text{var } I_{0,3} = 2.05 \cdot 10^4$$

$$\text{var MUF}_3 = (2.05+4.59+3.02) \cdot 10^4$$

$$= 9.66 \cdot 10^4$$

$$\sigma_{\text{MUF}_3} = 311 \text{ kg } UF_6$$

Fourth period:

$$\text{var } I_{0,4} = 2.07 \cdot 10^4$$

$$\text{var } MUF_4 = (2.07+4.59+0.078) \cdot 10^4$$

$$\sigma_{MUF_4} = 259 \text{ kg } UF_6$$

5.2 Probability of detection p for $M = 1000$ kg UF_6

a) One inventory period per year

The probability of detection is given by formula (A1-11)

$$1-p = \Phi(U_{1-\alpha} - \frac{M}{\sigma_{MUF}})$$

For $\alpha = 0.05$ one obtains

$$1-p = \Phi(1.65 - \frac{M}{\sigma_{MUF}})$$

$$= \Phi(1.65 - \frac{1000}{437}) = 0.26$$

This is equivalent to

$$p = 0.74$$

b) Four inventory periods per year

The probability of detection is given by formula (A1-13)

$$1-p = \Phi(U_{4\sqrt{1-\alpha}} - \frac{M}{4\sigma_1}) \cdot \Phi(U_{4\sqrt{1-\alpha}} - \frac{2 \cdot M}{4 \cdot \sigma_2}) \cdot$$

$$\cdot \Phi(U_{4\sqrt{1-\alpha}} - \frac{3 \cdot M}{4\sigma_3}) \cdot \Phi(U_{4\sqrt{1-\alpha}} - \frac{M}{\sigma_4})$$

$$= 0.907 \cdot 0.725 \cdot 0.429 \cdot 0.051 = 0.014$$

This is equivalent to

$$p = 0.986$$

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to be published

Annex 2: Sign Test for the Verification of the Gross Weights of UF_6 Cylinders

1. Principle of the method

It is assumed that the operator[1)] reports the values of the gross weights of the UF_6 cylinders and that the inspector checks all these reported data by independent measurements with the help of an own balance. The inspector thus, can check the reported data within the accuracy of his own measurement. However, as the measurement of the operator is much more accurate, one can imagine that the operator falsifies the data by small amounts which are covered by the measurement error of the inspector's measurement. In order to be able to detect even such small falsifications the inspector performs a so-called sign test: He forms the differences between the single reported data and his own independent measurements. If the two measurements are unbiased and if there exists no falsification the true values of the differences should be zero. Therefore, the differences between the measured data should be positive and negative with equal probability. On the basis of this assumption, the inspector establishes a critical value of the 'positives': if the number of positive differences is larger than this critical value (which depends on the chosen error of the first kind) the inspector states that the number of positives is 'significant' and calls for a second action level.

Remark: It is important to note that in this connection a one sided test is considered. There is a difference between the input and the output of the enrichment plant as one has to assume that the operator (in order to gain material) at the input may report a value which is smaller than in reality whereas at the output he may report a value which

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1) At the input the operator of the enrichment plant weighs the cylinders only immediately before the cylinders go into the process. Thus, the inspector cannot check the weights after they have been reported by the operator. Here, the inspector can take the reported data of the shipper of the conversion plant.

is greater than in reality. As a result of this consideration the inspector at the input has to test the differences $y_i - x_i$ (here, y_i is the inspector's value, x_i is the operator's reported value) whereas at the output he has to test the differences $x_i - y_i$.

In the following, first the test, i.e. the critical value for the number of positives as a function of the total number of items and the error of the first kind α is constructed. Thereafter, the error of the second kind for arbitrary falsification strategies of the operator is derived and special cases are considered. The results are illustrated by some numerical calculations.

2. Significance threshold of the test as a function of the total number of items and error of the first kind α

The probability that out of n differences in total at least z differences happen to be positive if the probability that a single difference happens to be positive is p, is given by

$$F_a\,(z;\,p,n) = \sum_{v=0}^{z} \binom{n}{v} p^v (1-p)^{n-v} \qquad \text{(A2-1)}$$

According to the theorem of De Moivre and Laplace, this probability is approximately given by

$$F_a\,(z;\,p,n) = \Phi\,\left(\frac{z - n\cdot p}{\sqrt{n\cdot p(1-p)}}\right) \qquad \text{(A2-2)}$$

Here, Φ is the Gaussian distribution function. The zero hypothesis is given by $p_o = 0.5$; thus, the error of the first kind α and the significance threshold z_c, the critical number of positives, is given by

$$\alpha = \sum_{v=z_c}^{n} \binom{n}{v} p_o^{\,v} (1-p_o)^{n-v} = 1-F_a(z_c;\,p_o,\,n) \qquad \text{(A2-3)}$$

With the help of (A2-2) one obtains

$$z_c = \frac{n}{2} + \frac{\sqrt{n}}{2} \cdot U_{1-\alpha} \qquad \text{(A2-4)}$$

where $U_{1-\alpha}$ is the 1-α quantile of the Gaussian distribution.

3. <u>Conditioned probability of detection p as a function of the amount falsified per cylinder</u>

Let the variance of the differences D of the operator's reported data and the inspector's own measurement be σ^2. (If the variance of the inspector's measurement is much worse than that of the operator, the variance of the difference is essentially given by the variance of the inspector's measurement.) Then the probability p that the difference between the operator's and the inspector's measurement is positive, is in the case that the operator falsifies his measurement by an amount ρ given by

$$p = \text{prob}\ \{D>0/ED = \rho,\ \text{var}\ D = \sigma^2\} = \Phi\left(\frac{\rho}{\sigma}\right) \qquad \text{(A2-5)}$$

One obtains from (A2-5)

$$p = \begin{cases} \Phi(0.5) = 0.69 \\ \\ \Phi(2) \quad = 0.98 \end{cases} \quad \text{for } \rho = \begin{cases} \frac{\sigma}{2} \\ \\ 2\sigma \end{cases} \qquad \text{(A2-6)}$$

4. <u>Error of the second kind β for arbitrary falsification strategies; special cases</u>.

It is assumed now that n_1 items (reported gross weights) are falsified by an amount ρ and $n_2 = n-n_1$ items are not falsified. Then the probability of detection (which is one minus the error of the second kind $\beta(n_1,n_2)$) is given by

$$\text{prob}\ \{a_1+a_2>z_c\ |n_1,p;\ n_2,p_o\} =: 1-\beta(n_1,n_2) \qquad \text{(A2-7)}$$

Here, a_1 is the number of positive differences where the operator's reported data are falsified and a_2 is the number of positive differences where no data are falsified.

Remark: According to the terminology of eq. (A2-7) one can write for the error of the first kind

$$\alpha = 1-\beta(0,n)$$

From eq. (A2-7) one obtains

$$1-\beta(n_1,n_2) = \sum_{\mu=\max(0,z_c-n_1)}^{\min(z_c,n_2)} p(a_1>z_c-\mu|n_1,\ p)\cdot p(a_2=\mu|n_2,p_o) \qquad \text{(A2-8)}$$

This gives with the help of eq. (A2-1)

$$1-\beta(n_1,n_2) = \frac{1}{2^{n_2}} \sum_{\mu=\max(0,z_c-n_1)}^{\min(z_c,n_2)} (1-F_{a_1}(z_c-\mu|n_1,p)\binom{n_2}{\mu}) \qquad \text{(A2-9)}$$

or, with the approximation corresponding to eq. (A2-2)

$$1-\beta(n_1,n_2)=\frac{1}{2^{n_2}} \sum_{\mu=\max(0,z_c-n_1)}^{\min(z_c,n_2)} \Phi\left(\frac{n_1p-(z_c-\mu)}{\sqrt{n_1\cdot p(1-p)}}\right)\binom{n_2}{\mu} \qquad \text{(A2-10)}$$

Special cases

(i) In the case $n_1=n$, i.e. in the case the operator falsifies all the data by an amount ρ, one has

$$\max(0,z_c-n_1) = 0,\ \min(z_c,n_2) = n_2 = 0$$

Therefore, one obtains from (A2-10)

$$1-\beta(n,0)=\Phi\left(\frac{np-z_c}{\sqrt{np(1-p)}}\right) \qquad \text{(A2-11)}$$

(ii) In the case the operator falsifies items by very large amounts $\rho(\geq 2\sigma)$ one has $p\simeq 1$ and one obtains from (A2-9)

$$1-\beta(n_1,n_2) = \begin{cases} 1 & \text{for } n_2<z_c<n_1 \\ \frac{1}{2^{n_2}} \sum\limits_{\mu=z_c-n_1}^{n_2} \binom{n_2}{\mu} & \text{for } n_1,n_2<z_c \\ \frac{1}{2^{n_2}} \sum\limits_{\mu=z_c-n_1}^{z_c} \binom{n_2}{\mu} & \text{for } n_1<z_c<n_2 \end{cases} \qquad \text{(A2-12)}$$

(The case $n_1,n_2>z_c$ is not possible as $z_c>\frac{n}{2}$))

Thus, one sees that the operator cannot falsify a great fraction n_1 of the items by large amounts without being detected.

(iii) In the case $n_1<<n$, i.e. in the case the operator falsifies only a small amount of items one has

$$\max(0,z_c-n_1)=z_c-n_1, \quad \min(z_c,n_2) = z_c$$

and from (A2-10) one obtains

$$1-\beta(n_1,n_2) = \frac{1}{2^{n_2}} \sum_{\mu=z_c-n_1}^{z_c} \Phi\left(\frac{n_1p-(z_c-\mu)}{\sqrt{n_1p(1-p)}}\right)\binom{n_2}{\mu} \qquad \text{(A2-13)}$$

5. Numerical calculations

In the following, the feed (1), product (2) and tails (3) streams of an enrichment plant are considered. In Table 8, numerical results are given for the special cases

(i) $n_1 = n$, $n_2 = 0, \rho = \frac{\sigma}{2}$

This means all cylinders are falsified by an amount of $\frac{\sigma}{2}$.

(ii) $n_1 = 5$, $n_2 = n-5$, $\rho = 2\sigma$

This means that only few cylinders are falsified by large amounts compared to the measurement accuracy. (One could imagine that the inspector performs a test of significance for each single cylinder which is verified by him. In that case here, the probability to detect a single falsification would be 0.64 for $\alpha = 0.05$.)

The three streams are considered separately; this means that separate sign tests with separately given α errors are performed, and the β errors are calculated for each sign test separately.

One can however, determine the overall α and β-errors for all three tests together as one has

$$1-\alpha_{total} = (1-\alpha_1)(1-\alpha_2)(1-\alpha_3) = 0.95^3 = 0.86$$

$$\beta_{total} = \beta_1 \cdot \beta_2 \cdot \beta_2$$

The overall values of the α and β-errors are given in the last row of Table 8. One sees: even in the case that the probabilities of detection for the single stream sign test are only in the order of 10 %, the overall probability of detection is 24 %. (In this case, one should take into account additionally the possible detection with the help of the above mentioned single cylinder tests of significance.)

6. Concluding remarks

It has been shown in which way the inspector can verify the operator's reported data of the gross weights of the UF_6 cylinders with the help of statistical sign tests. The critical number of positives has been calculated for a given α error and furthermore, the β error has been calculated for two special falsification strategies.

As it has been mentioned already earlier it is important for the inspector to perform additionally a test of significance for each single cylinder in order to detect a diversion which is based on large amount falsifications in small numbers of cylinders. Thus, the problem of the verification of the gross weights of the UF_6 cylinders of the three streams should be formulated in the following way:

Let α_{1i}, i = 1,2,3 be the error of the first kind of the sign test for the i-th stream, and let α_{2i}, i = 1,2,3 be the error of the first kind of the sum of the single cylinder tests for the i-th stream. Let $\beta_{1i}(M_{1i})$ and $\beta_{2i}(M_{2i})$ be the corresponding errors of the second kind and M_{1i} and M_{2i} the critical amount M for the i-th stream which refer to the corresponding falsification strategies: defeating of the sign test (M_{1i}) and defeating of the single cylinder tests (M_{2i}). The overall α and β errors are then given by

$$1-\alpha = \prod_{i=1}^{3} (1-\alpha_{1i}) \prod_{i=1}^{3} (1-\alpha_{2i})$$

$$\beta(M) = \prod_{i=1}^{3} \beta_{1i}(M_{1i}) \prod_{i=1}^{3} \beta_{2i}(M_{2i})$$

The rigorous treatment would require to minimize β(M) for a given α with respect to the α_{1i} and α_{2i}, i = 1,2,3, and to maximize β(M) with respect to the M_{1i} and M_{2i} for a fixed M where

$$M = \sum_{i=1}^{3} M_{1i} + \sum_{i=1}^{3} M_{2i}$$

In the light of this formulation the numerical calculations given above can only be seen as illustrative examples.

Annex 3: Verification of the Isotopic Composition of the Product and Tails Streams by Means of Random Sampling

1. Principle of the method

It is assumed that the inspector verifies the operator's reported data of the enrichment of the product and tails cylinders by independently taking and analyzing samples. The randomness of the sampling procedure of the inspector can be guaranteed provided that the cylinders remain flanged at the product or the tail station for 4-8 hours after the sampling of the operator. Thus, the inspector has enough time to decide whether he should take a sample or not after he has got the operator's data.

With the help of his independent data the inspector checks by means of the so-called 'D-statistics' whether the reported data are falsified or not: He forms the differences between the single reported data and his independent measurements, extrapolates the sum of all these differences to the hypothetical sum of the differences with respect ot all reported data and performs a test of significance for this sum of differences ('D').

As there exist two different sets of cylinders (product and tails) with different characteristics, the problem arises to distribute the verification effort in an optimal way on the two sets of cylinders. This optimisation procedure has been subject of different papers /3-1, 3-2/ and shall not be repeated here.

In the following the relevant formulae are summarized and some numerical calculations are presented.

2. List of formulae and numerical results

The relation between the basic parameters

- α error of the first kind
- β error of the second kind
- M critical amount
- C total verification effort

is given by the following expression

$$U_{1-\beta} \cdot \sigma_{D/H_1} = M - U_{1-\alpha} \cdot \sigma_{D/H_o} \qquad \text{(A3-1)}$$

Here, U is the quantile of the normal distribution. The variances σ^2_{D/H_o} and $\sigma^2_{D/H}$ of the D-statistics under the zero hypothesis (no diversion) and under the alternative hypothesis (critical amount M) are given by

$$\sigma^2_{D/H_o} = \frac{1}{C} \sum_{i=1}^{2} \sqrt{e_i} N_i \sigma_i)^2 \qquad \text{(A3-2)}$$

$$\sigma^2_{D/H_1} = \frac{1}{C} \left(\sum_{i=1}^{2} \sqrt{e_i} N_i S_i \right)^2 - \sum_{i=1}^{2} N_i p_i (1-p_i) \cdot \mu_i^2 \qquad \text{(A3-3)}$$

where

$$S_i = \sqrt{\sigma_i^2 + p_i(1-p_i)\mu_i^2} \; ; \quad p_i = \frac{r_i}{N_i}$$

Here, the index i refers to the two sets of cylinders (product 1, tails 2). The single parameters have the following meaning:

- σ_i^2 Variance of the difference of the single comparison: operator's reported value-inspector's own finding
- N_i total number of cylinders in a single set
- e_i effort for the verification of a single cylinder [1)]
- r_i number of falsified data
- μ_i falsified amount per cylinder

The optimal number of cylinders to be verified in the i-th set (i=1,2) is given by

$$n_i^{opt} = C \cdot \frac{S_i N_i}{\sqrt{e_i} \sum_{j=1}^{2} \sqrt{e_j} N_j S_j} \qquad i = 1,2 \qquad \text{(A3-4)}$$

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1) Here, it is assumed $e_1 = e_2 = 2h$

The optimal number of cylinders to be falsified in the i-th set is given (to a first approximation, as the exact figure cannot be given analytically)

$$r_i^{opt} = M \cdot \frac{e_i N_i}{\sum_j e_j \mu_j N_j} \qquad \text{(A3-5)}$$

The numerical values of the parameters σ_i, r_i, μ_i are given in Table 7.

The results of the numerical calculations are presented in Fig. 5 for two different values of the critical amount M (in kg U^{235}) and error of the first kind $\alpha = 0.05$ as a function of the total effort C. As e = 2h, $0.5 \cdot C$ is the total sample size for the two sets of cylinders. The dashed line corresponds to the case where only the product cylinders are verified (and, consequently, assumed to be falsified).

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