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Quantification of Containment Measures Used in Safeguards

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Zusammenfassung

In dieser Arbeit wird der Versuch gemacht, den Einfluß von Maßnahmen der dichten Umhüllung (containment) im Rahmen von allgemeinen Überwachungsmaßnahmen quantitativ zu formulieren, sowie die optimale Kombination von Maßnahmen der Materialbilanz und der dichten Umhüllung herauszufinden.

In einem ersten Teil wird in allgemeiner Form erklärt, was unter dem Begriff 'Quantifizierung' zu verstehen ist. Es werden Kriterien entwickelt, und es wird der Nutzen der Maßnahmen der dichten Umhüllung bezüglich dieser Kriterien diskutiert. Als Anwendungsbeispiel wird das Absender-Empfänger-Problem detailliert behandelt.

Im zweiten Teil wird das Problem der Überwachung eines Plutonium-Lagers mit Hilfe eines allgemeinen, früher entwickelten spieltheoretischen Modelles quantitativ analysiert. Es wird eine Kostenoptimierung bei fest vorgegebener Entdeckungswahrscheinlichkeit als Randbedingung durchgeführt, die Variablen sind die Zahl der Inspektionen pro Jahr und die Zahl der Siegel, die pro Inspektion zu prüfen sind. Die Ergebnisse, die von den verschiedenen Modellparametern wie Güte und Kosten der Siegel, Inspektor-Manntage-Kosten etc. abhängen, werden diskutiert.

Im letzten Teil werden mögliche Modellerweiterungen, praktische Anwendungen sowie Experimente auf diesem Gebiet angesprochen.

Abstract

In this paper an attempt has been made to formulate quantitatively the effect of containment measures in the framework of a general safeguards system and furthermore, to find out an optimal combination of containment and material accountancy measures.

In a first part it is explained in general terms what is meant by 'quantification'. Criteria are developed, the usefulness of containment measures with respect to these criteria is discussed. As an application the shipper receiver problem is considered in some detail.

In the second part the problem of safeguarding a Plutonium storage is analyzed quantitatively with the help of a general game theoretical model developed earlier. A cost optimization with a fixed probability of detection as a boundary condition is carried through. Here, the variables are the number of inspections per year and the number of seals to be identified per inspection. The results which depend on several parameters of the model, e.g. quality and costs of seals, inspection manday costs, are discussed.

In the conclusion, possible further extensions, practical applications and experimental tests in this field are considered.

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1. Introduction

It is now a generally accepted fact that a safeguards system can be based on the three basic measures material accountancy, containment and surveillance /1, 2, 3, 4/. The first of these has been recognized to be a measure of fundamental importance with containment and surveillance as important complementary measures. As opposed to material accountancy, the other two measures are generally considered to be qualitative in nature /5/. They can normally be used to prevent a diversion or indicate actions which may lead to the detection of a diversion, but seldom to indicate the actual amount diverted. Measures based on material accountancy on the other hand, enable a safeguards organisation to establish the result of a diversion, namely, the amount, in a direct and quantitative manner.

The important role played by the containment measures has been recognized at an early stage /6/ and their implications have been analysed in some detail for a number of cases /3, 7/. Such measures, particularly the use of seals for containers storing nuclear materials, have been shown to cause a significant reduction in safeguards efforts /8/. Some work has also been reported on the categorization and development of the different qualitative measures /5, 9, 10/. However, no effort appears to have been spent on a possible quantification of containment measures.

The present paper discusses the possibility of a quantification of some of the characteristics of containment measures namely, the use of seals. The paper has been divided broadly into two parts. In the first part, it is discussed in general terms what is meant by 'quantification of safeguards measures' furthermore, the types of containment measures considered and their usefulness with respect to the different relevant safeguards parameters is described. In a specific example, the safeguards region exit reprocessing plant to entrance fabrication in a nuclear fuel cycle the use of seals is considered as a complementary measure to material accountancy. In the second part, the application of containment measures in a sealed Plutonium storage facility has been considered in a quantitative manner on the basis of a general game theoretical model developed earlier. In this model the containment measures applied have been considered to be the only safeguards measure. The paper ends with a summary of the main results obtained and some general conclusions.

2. Analysis of Possibilities of Quantification of Containment Measures

2.1 Categorization of Containment Measures

In this paper, a containment measure is understood to consist of two parts:

- a device or system which encloses nuclear material or information to be safeguarded against diversion or falsification

- a device or system which allows to recognize any attempted intrusion.

A categorization of the different containment measures in question has been given in Ref. /10/. According to this categorization one has

- containment measures which enclose nuclear material and information as well as those persons who are in the position to divert material (fences, walls etc.)

- containment measures which enclose only nuclear material and information (sealed containers, lines etc.).

Additionally, one can categorize the containment measures according to whether they permit an immediate recognition of an attempted intrusion (alarm systems) or not, i.e. they permit such a recognition only with a certain delay of time (e.g. seals which are controlled periodically). It is the latter category which is to be discussed here.

2.2 Basic Considerations

2.2.1 Objective of Safeguards

Since the term 'quantification of a safeguards measure' may be interpreted in several ways, it is necessary to clarify the use of this term in the context of the present paper.

The objective of safeguards as specified in Ref. /4/ is 'the timely detection of diversion of significant quantities of nuclear material ....'.

The usefulness of a safeguards measure can be assessed by the extent to which it enables a safeguards organization to attain this objective in as unequivocal a manner as possible, namely, whenever possible, by a set of numbers. It is to be noted that basically two elements are involved in the safeguards objective: the time and the amount of material.

2.2.2 Amount of Material

For material accountancy measures an idea on the amount of material in the case of a diversion, can be obtained in the following way: As long as no physical inventory has been taken the amount M assumed to be diverted may be at maximum the book inventory BI:

$$0 \leq M \leq BI$$

(In reality the upper limit for M will be much smaller for plant operation reasons.) After a physical inventory taking and the establishment of a material balance over a given period of time in a facility the amount M assumed to be diverted can be estimated by the difference between the book inventory and the physical inventory. However, so long as the material balance in a facility is established with the help of measured values which are associated with measurement errors and so long as process losses and other factors influencing the material balance /11/ are subject to random variations, the amount of diversion estimated with the help of the difference between the book and the physical inventory, can not be known with certainty. Therefore, any statement on a diversion based on material accountancy measures can only be of a probabilistic nature associated with the statistical errors of the first kind α (false alarm rate) and of the second kind β (one minus the probability of detection). In the case of a confidence interval statement on the diverted amount M one obtains thus, a statement of the following form

$$BI - PI - x(\alpha) \leq M \leq BI - PI + x(\alpha)$$

Considering a containment which serves to safeguard the amount I of material, one can state that no material has been diverted, so long as the containment is intact. In case of a broken containment (e.g. a birdcage with a manipulated seal) the statement on the amount M of material assumed to be diverted will be

$$0 \leq M \leq I.$$

2.2.3 Detection Time and Critical Time

The element of 'time' can be associated with the 'detection time'(T_D) for a diversion. For material accountancy measures, the time between two consecutive physical inventory takings is an upper limit for the detection time. During the early days of investigation on a safeguards system in conformity with the Non-Proliferation Treaty /12/, the concept of 'critical time' (T_C) was introduced. This time was taken to be the time required to transform diverted material into a useable military device. T_C was taken to be related to the 'accessibility' of nuclear material and was assumed to vary between about a week for Plutonium and highly enriched Uranium and a few months for low enriched Uranium and irradiated materials. From the point of view of safeguards the detection time T_D has to be set into relation with the critical time T_C since T_C together with the amount M of diversion reflects the hazard in the sense of the Non-Proliferation-Treaty. However, if just for arguments sake and as an extreme case $T_D = T_C$, for the material accountancy measures it would imply the taking of physical inventories every week in a fabrication plant for Plutonium or highly enriched Uranium and in a reprocessing plant. Since physical inventory taking in such plants is normally manpower intensive and highly intrusive it was recognized at an early date that such frequent inventory taking for safeguards purposes would not be feasible in practice. As a working hypothesis the frequency of physical inventory taking for such plants was assumed to synchronize with those taken by the plant operators /13/, namely once or twice a year. Thus, one obtains detection times T_D in the region of six to twelve months although the critical times T_C are assumed to be much shorter as mentioned earlier. The facit assumption behind this procedure is that between two inventories the materials in such plants are made 'inaccessible' through suitable containment or surveillance measures.

2.2.4 Safeguards Costs

On the basis of the foregoing discussions quantification of a particular safeguards measure may be taken to be an effort to express the results of that measure in terms of the relevant parameters, time and amount, with a set of numbers. Since a safeguards system has always to be as economic as practicable the costs K for carrying out a given safeguards measure have to be taken up along with the other parameters.

Note: In addition to the safeguards parameters discussed up to now often the tamperresistance of a specific measure is mentioned. In fact, it is obvious that material accountancy statements are only valid if the data processed and reported by the operator are not falsified. Therefore, one has to guarantee that no data falsification is possible or one has to take into account explicitly diversion strategies on the basis of data falsification (see, e.g. Ref. /16/).

The concept of tamperresistance plays only a major role in case the inspection authority has to rely to a large degree on stored data, but in principle no safeguards measure can be fully tamperproof if the effort spent is limited; the possibility of a successful falsification must be taken into account. However, as can be concluded from the discussion up to now, tamperresistance can be expressed by the terms probability of detection and costs (the higher the costs or the effort in general, the higher the probability of detection, i.e. the higher the tamperresistance). Therefore, tamperresistance will not be considered here as an independent safeguards parameter.

2.2.5 Conclusion

Concluding this general discussion, and in conformity with the investigations in the case of material accountancy /14, 15/, one may state that quantification of safeguards measures means the establishment of quantitative relations between the following parameters

- The amount assumed to be diverted M
- The probability of detection $p = 1-\beta$
- The error of the first kind α
- The detection time T_D

It is to be noted that the error of the first kind α (the probability of comitting a false alarm) is quantitatively different from the other parameters (it could be treated as a determinant /16/). Since in the models considered in the following this error appears to be extremely low, this parameter has not been considered in the rest of this paper.

2.4 Specific Example

In this section an example for a containment measure which supports the material accountancy shall be analyzed. With the help of this example it shall be illustrated in which way the considerations given above can be applied in a semi-quantitative manner.

2.4.1 Formulation of the Problem

Let A be the product station of a reprocessing plant where from the timepoint t_0 on Plutonium-nitrate is bottled from a storage tank and furthermore, let B be the input station of a fabrication plant where the bottles arrive from the timepoint t_1 on (more generally t_1 is the timepoint from which on the operator of B takes the responsibility for the material) and where the Plutonium-nitrate is processed from the timepoint t_2 on.

It is assumed that the operator A (shipper) as well as the operator B (receiver) determine the Plutonium content of each bottle quantitatively by determining

(i) the net weight of each bottle and

(ii) the chemical composition of the content of each bottle.

Let $\tau_1 := t_1 - t_o$ be the time interval between the measurement at A and the arrival at B, let furthermore $\tau_2 := t_2 - t_1$ be the time interval between the arrival at B and the measurement at B, let $\tau := t_2 - t_o = \tau_1 + \tau_2$ be the total time interval between the measurements of a single bottle at A and B.

It is assumed that the inspector who has to safeguard this procedure in case there is no limitation in the effort available proceeds as follows:

(1) Checking of the operator's data at A by means of independent measurements

(2) Sealing of the bottles at A after operator's measurements and identification of the seals before the measurements at B

(3) Checking of the operator's data at B by means of independent measurements.

Additionally, the inspector compares all the data available.

These safeguards measures represent counter-strategies against the following possible diversion strategies (here, one has to make a difference whether the operators A and B cooperate for the purpose of diversion or not):

(1) Falsification of data at A

(2) Diversion of material after the measurement at A, before t_1

(3) Diversion of material after t_1, before the measurement at B

(4) Falsification of data at B.

All the (pure) strategies considered here are listed in Table 1. It is to be noted that only strategies of data falsifications are taken into account - the possibility that the operators divert material within the limits of the material balance is excluded as the relevant amounts of material are assumed to be too small.

The problem now is to determine for a finite effort available the optimal combination of measurements and sealing and identification measures.

Here, the criteria for optimization are functions of

- the detection time
- the probability of detection as a function of the amount assumed to be diverted.

Contrary to normal optimization procedures the criterion for optimization is not a skalar but a vector with two components. The question whether it is possible to project this vector to a scalar by means of an appropriate utility function or not shall not be discussed here.

2.4.2 Game Theoretical Analysis of this Problem for Specific Assumptions

The case is considered now that both 'players', inspector and operators, choose only one of the strategies listed above. A two-person zero-sum game is considered now in which the payoff to the operator is a vector the two components of which are the negative probability of detection and the detection time.

> Note: With respect to the probability of detection a justification for this procedure has been given already earlier /17/. If one assumes furthermore, that the gain (in any utility units) of the operators is proportional to the detection time then in a way analogeous to that detailed in /17/ one can take the critical time itself as payoff so long as one is interested mainly in the optimal strategies.

The conflict situation described above can therefore, be represented by a 'payoff-matrix' given in Table 2.

It is to be seen from the matrix that the general case that both operators act illegal is worse from the inspector's point of view (the modifications of cases 1.1-1 and 1.4-3 are not considered to be important in this connection). Therefore, only this general case is considered in the following. Additionally, one sees that strategy 2.3 of the operators is worse than strategy 2.2 from the point of view of the operators thus, it is neglected.

Note: As one can take from Table 2, strategy 1 of the inspector (which in many cases is normal practice today) is not necessarily the best strategy in the framework of these considerations.

2.4.3 Analysis of the Effect of the Sealing Procedure

In order to be able to analyze the value of the sealing procedure it is assumed that for the total safeguards procedure the effort C is available. The two following cases are considered now.

Case I: The inspector checks the data at A and B with the help of independent measurements; this leads to a probability of detection $P_C(M)$ in case an amount M of nuclear material is diverted.

Case II: The inspector checks the data at A and B with the help of independent measurements and seals and identifies all containers; as for the latter procedure a certain effort is necessary the probability of detection with respect to the data falsification strategy will be $\tilde{P}_C(M) < P_C(M)$.

Thus, from the payoff matrix given in Table 2 one deduces a payoff matrix given in Table 3.

As the operators strategies 2.1 and 2.4 are equivalent only the first one will be taken into account in the following. Qualitatively one can draw the following two conclusions from Table 3:

(i) If the sealing procedure is very cheap compared to the checking of measurements, $\tilde{P}_C(M)$ will not be significantly smaller than $P_C(M)$ thus, strategy II is better from the operator's point of view.

(ii) If the sealing procedure is very expensive the inspector will choose strategy case I as on the average he comes out with a higher probability of detection (with respect to the critical times there is no difference between the two strategies of the inspector).

For the quantitative analysis it is useful to consider separately the game the payoff of which is given by the first component of the payoff vector ('P-game',Table 4a) and thereafter the game the payoff of which is given by the detection time (τ-game).

In the P-game strategy 2.1 of the operators dominates strategy 2.2. If one considers the game thus reduced one sees that strategy (1+3) of the inspector dominates strategy (1+2+3). Therefore, the strategies 2.1 and (1+3) are optimal for the two players.

In the τ-game strategy 2.2 of the operators dominates strategy 2.1. After the appropriate reduction one sees that the two inspector strategies are equivalent. The inspector however, will choose strategy (1+2+3) as that strategy guarantees a better probability of detection.

One sees that the result of the optimization changes completely if one changes the criteria for optimization. As has been mentioned already earlier it would be necessary to construct an appropriate scalar utility function from the two payoff-functions if one would be obliged to give a unique solution of the problem.

3. Analysis of the Problem of Safeguarding a Plutonium-Storage by Means of Sealing and Identification Measures

3.1 Formulation of the Problem, Strategies, Costs

A Plutonium storage is considered in which Plutonium is stored during a reference time interval (0, T). It is assumed that the Plutonium is contained in n_2 birdcages and that these n_2 birdcages are stored in n_1 containers. All the birdcages as well as the containers are sealed with seals of the same type. The probability to identify a falsified seal is given by π_1. (It is assumed in this context that this probability is independent of the time the operator spends for the falsification of the seal.)

The inspector visits the storage on the average J times during the reference time interval. The safeguards procedure during a visit consists in the identification and changing of all the container seals and in the identification and changing of a representative sample of k birdcage seals. It is to be noted that this safeguards procedure primarily leads to the detection of a falsification of seals, not to the detection of a diversion of material. Therefore, in the following the amount M of material assumed to be diverted does not occur. This procedure has to be understood in such a way that the detection of a falsification will lead to a physical inventory and thus, to the detection of a diversion of material. However, this problem of second action levels will not be detailed in this paper.

The problem treated here[1)] is the establishment of quantitative relations between the relevant parameters. Specifically it consists in the determination of the necessary number of inspections per year as well as the necessary sample size k of seal identifications per visit for a given overall probability of detection. This means that contrary to the considerations in the foregoing example the detection time is a determinant, not a criterion of optimization.

The time schedule for an operator who has the intention to divert material from the storage is as follows:

[1)] Preliminary results are already given in Ref. /18/.

t_o: Taking of the characteristics of the container seal

(t_o,t_1): Production of two copies of the container seal in question ($\tau_1 = t_1-t_o$)

t_1: Opening of the container seal: taking of the characteristics of the birdcage seal, replacement of the original container seal by the copy produced before

(t_1,t_2): Production of a copy of the birdcage seal ($\tau_2 = t_2-t_1$)

t_2: Diversion of the Plutonium of the birdcage in question, replacing of the birdcage and container seals by the copies produced before

(t_2,t_3): Time necessary in order to use the Plutonium diverted in an appropriate way ($\tau_3 = t_3-t_2$).

As a consequence, the following situations result for the inspector:

Visit during

(t_o,t_1): The inspector states no diversion however, he changes the container seal. This means that the time spent by the operator for the production of a copy of the container seal has been spent in vain.

(t_1,t_2): The inspector states with probability Π_1 a falsification of the container seal however, no diversion has yet been taken place.

(t_2,t_3): The inspector states with probability Π_1 a falsification of the container seal and with probability $\frac{k}{n_2} \cdot \Pi_1$ a falsification of the birdcage seal in case the falsification of the container seal has not been recognized.

If the Plutonium has already been diverted and the falsification of the container seal is detected then the diversion of the Plutonium will be detected in any case as then all the birdcages are controlled. Therefore, the probability Π that a diversion is detected in case the inspector comes in the time period (t_2,t_3) is

$$\Pi = 1\cdot\Pi_1 + \frac{k}{n_2} \cdot \Pi_1(1-\Pi_1) = \Pi_1(1+ \frac{k}{n_2} (1-\Pi_1)) \qquad (1)$$

It is to be noted that in the framework of this model the critical time for the operator is $\tau_1+\tau_2+\tau_3$ whereas the critical time τ from the point of view of the inspector is $\tau = \tau_2+\tau_3$. The average detection time is $\frac{T}{J}$.

Generally it is assumed that the operator has the gain + d if the diversion is not detected during the critical time τ[1]. With respect to the loss of the operator in case of a timely detection of the diversion two different cases have to be considered

(i) In case of an inspection during (t_0,t_1) the operator has only a small loss which consists in the equivalent of the time spent up to the inspection for the production of the copies of the container seal.

(ii) In case of an inspection during (t_1,t_3) and detection the operator has the large loss C. It is assumed that from the point of view of the operator it is equally damaging whether the detection of the falsification takes place in the time interval (t_1,t_2) or in (t_2,t_3).

Note: In the case $\Pi_1 = 1$ there is no advantage for the operator to falsify the seals according to a time schedule described above as a falsification will be detected in any case if the falsified seal is

[1] In any case the diversion is detected after the reference time T if the material is to be used again and the receiver of the material does not collaborate with the operator of the storage however, this detection is considered here to be too late.

checked. Therefore, in this case the operator can follow only a simple time schedule:

t_o: Diversion of Plutonium, replacement of the original seals by roughly falsified seals (in order that it cannot be seen at the first sight that something is wrong).

(t_o, t_1): Time necessary for the use of the Plutonium diverted ($\tau = t_1 - t_o$).

In this case the payoff to the operator is the same as that in the case $\Pi_1 < 1$ with the exception that it refers to a shorter critical time.

The total costs K of the inspection in the reference interval of time consist of

Costs K_1 for seals

Costs K_2 for the inspector.

In total n_1+k seals are replaced during one inspection. Let b be the costs per birdcage seal and b' be the costs per container seal. (It is assumed that the costs of container and birdcage seals may be different because of different modes of fixing even in case the seals have the same quality expressed by Π_1). Then one obtains

$$K_1 = J(b' \cdot n_1 + b \cdot k) \qquad (2a)$$

The costs for one inspection consist of the travelling time (2 days per inspection) and time for identification and replacement of seals (time necessary per seal x inspector mandays). Let e be the costs per inspector manday. Then one has

$$K_2 = J \cdot e \cdot (2 + x(n_1 + k)) \qquad (2b)$$

Thus, the total safeguards costs per reference time are

$$K = J\cdot\left[b'\cdot n_1+b\cdot k+e(2+x\cdot(n_1+k))\right] \quad (3)$$

One can conceive a different cost model where the time necessary for one inspection is independent of the number of seals checked during the inspection. In this case one would get instead of (3)

$$K = J\cdot(b'\cdot n_1+b\cdot k+c) \quad (3')$$

where c are the costs for one inspector's visit plus identification of the seals checked.

3.2 Game Theoretical Model; Mathematical Formulation of the Optimization Problem

The problem formulated above can be analyzed with the help of a game theoretical model developed earlier /19/; it describes in general terms situations like that formulated in the preceding section. According to this model the probability p to detect timely a falsification during the reference time T is given by

$$p = 1-(1-\Pi)^{\xi}\cdot\left(1-\frac{\eta\cdot\Pi}{g}\right) \quad (4a)$$

Here, Π is given by eq. (1), g is the greatest integer smaller than $\frac{T-1}{\tau}$, τ is the critical time, the integers ξ and η are defined by the division of J with respect to the integer g:

$$J = \xi\cdot g+\eta$$

$$0\leq\eta<g;\ 0\leq\xi \quad (4b)$$

For the sake of clarity the three different probabilities of detection occuring in the framework of this theory are collected once more: p has been defined above, Π is the probability to detect a falsification of a seal during a visit and Π_1 is the probability to identify a falsified seal as a falsified seal.

The analysis of the problem of safeguarding a Plutonium storage by means of sealing and identification therefore, according to the considerations of the foregoing section results in the determination of the values of J and k and can be formulated in the following way:

> The total costs K have to be minimized with respect to the variables J and k under the boundary condition of a fixed total probability p of detection.

It has to be noted that the boundary condition that a given probability of detection has to be guaranteed could be changed into an equivalent boundary condition that the operator has to be induced to act legally (see Ref. /17/). The boundary condition used here has the advantage that the payoff parameters do not occur explicitly.

3.3 Discussion of the Probability of Detection

In this section the probability of detection p, eq. (4), is discussed as it constitutes the most important boundary condition for the cost optimization.

For this purpose eq. (4) is written in the following form

$$1-p = (1-\Pi)^{\xi}\cdot(1-\Pi\cdot\frac{n}{g}) \qquad (5a)$$

$$\frac{J}{g} = \xi + \frac{n}{g} \qquad (5b)$$

$$0 \leq \frac{n}{g} < 1 \qquad (5c)$$

$$0 \leq \xi,\ \xi \text{ integer} \qquad (5d)$$

Here, Π is the probability of detection for a single inspection in case this inspection falls into the critical time after the diversion, defined by eq. (1), and g is the greatest integer smaller than $\frac{T-1}{\tau}$, i.e. the maximum number of critical time intervals τ during the reference time interval T.

It is the aim of this section to study the interdependence of J and for a given total probability of detection p. For this purpose three different cases are considered:

(i) $\xi = 0$

In this case one obtains from eq. (5b)

$$\frac{J}{g} = \frac{\eta}{g} < 1 \qquad (6a)$$

and therefore from eq. (5a)

$$p = \Pi \cdot \frac{J}{g} \qquad (6b)$$

This means that in this case the average time interval between two inspections may be smaller than the critical time; the ratio of these two time intervals is determined by p and Π (eq. 6b).

(ii) $\xi = 1$

In this case one obtains from eq. (5b)

$$\frac{J}{g} = 1 + \frac{\eta}{g} \qquad (7a)$$

and therefore, from eq. (5a)

$$1-p = (1-\Pi)\cdot(1-\Pi\ (\frac{J}{g} - 1)) \qquad (7b)$$

For $\eta = 0$ one obtains $\frac{J}{g} = 1$ and therefore,

$$p = \Pi \qquad (8)$$

This result can be interpreted in the following way:
In case the probability Π of detection for a single inspection is smaller (greater) than the postulated overall probability p of detection, the average time interval between two inspections has to be greater (smaller) than the critical time.

It is to be noted that the function $\frac{J}{g}(\Pi)$ with p as a parameter is not differentiable in the point $\Pi = p$. In fact one obtains from (6b) and (7b) the following relations:

$$\left.\frac{d(\frac{J}{g})}{d\Pi}\right|_{\Pi \searrow p} = -\frac{1}{p} \; ; \; \left.\frac{d(\frac{J}{g})}{d\Pi}\right|_{\Pi \nearrow p} = -\frac{1}{p(1-p)}$$

As one sees the relative difference between the two derivations becomes small if p goes to zero.

(iii) $\xi \geq 2$

In this case one obtains from eq. (5b)

$$\frac{J}{g} = \xi + \frac{n}{g} \quad ; \quad \xi = 2, 3, \ldots \tag{9a}$$

and therefore, from eq. (5a)

$$1-p = (1-\Pi)^{\xi} \cdot (1-\Pi(\frac{J}{g} - \xi)); \; \xi = 2, 3, \ldots \tag{9b}$$

The relation between J and Π is represented graphically in Fig. 1 for different values of p. As one can see, a small probability Π of detection always can be balanced by a large J in order to guarantee the postulated p however, if J is smaller than $g \cdot p$, even with $\Pi = 1$ the postulated p cannot be guaranteed.

3.4 Cost Optimization

The costs as a function of J and k are given by eq. (3):

$$K = J \cdot \left[k(b+e \cdot x)+b' \cdot n_1+e \cdot (2+x \cdot n_1)\right] \tag{10}$$

If one replaces k by Π with the help of eq. (1), one obtains

$$K = J \cdot \left[\frac{(\Pi-\Pi_1)}{\Pi_1(1-\Pi_1)} \cdot n_2 \cdot (b+e \cdot x)+b' \cdot n_1+e \cdot (2+x \cdot n_1)\right] \tag{11}$$

In order to be able to proceed further one has to discern between the two cases $\frac{J}{g} < 1$ and $\frac{J}{g} \geq 1$.

Case 1: $\frac{J}{g} < 1$

If one replaces in this case J by Π with the help of eq. (6b) one obtains the total costs K as a function of Π alone:

$$\frac{K}{g} = \frac{p}{\Pi} \cdot \left[\frac{(\Pi - \Pi_1)}{\Pi_1(1-\Pi_1)} \cdot n_2 \cdot (b+e \cdot x) + b' \cdot n_1 + e \cdot (2+x \cdot n_1) \right]$$

This can be written in the following form

$$\frac{K}{g} = p \cdot a_1 \cdot \left[1 + \frac{1}{\Pi} (-\Pi_1 + \frac{a_2}{a_1}) \right] \tag{12a}$$

where

$$a_1 = \frac{n_2 \cdot (b+e \cdot x)}{\Pi_1 \cdot (1-\Pi_1)} \tag{12b}$$

$$a_2 = b' \cdot n_1 + e \cdot (2+x \cdot n_1) \tag{12c}$$

As Π is a monotonely increasing function of k, the minimum of the costs with respect to varying k is given at k_o, where

$$k_o = \begin{cases} n_2 & \text{for} \quad -\Pi_1 \cdot a_1 + a_2 > 0 \\ \text{arbitrary} & \text{for} \quad -\Pi_1 \cdot a_1 + a_2 = 0 \\ 0 & \text{for} \quad -\Pi_1 \cdot a_1 + a_2 < 0 \end{cases} \tag{13}$$

As it will be shown in the next section in all cases considered one has $k_o = 0$. This leads to the following minimum costs ($\Pi(k_o = 0) = \Pi_1$):

$$\left(\frac{K}{g}\right)_{min} = \frac{p}{\Pi_1} \cdot (b' \cdot n_1 + e \cdot (2+x \cdot n_1)) \tag{14}$$

Case 2: $\frac{J}{g} \geq 1$

In this case it is not possible to eliminate J in a simple analytical way as it could be achieved in case 1. Thus, it is best to start with eq. (11) which is written in the following form:

$$\frac{K}{g} = \frac{J}{g} \cdot \left[\Pi \cdot \frac{n_2(b+ex)}{\Pi_1(1-\Pi_1)} - \Pi_1 \frac{n_2(b+ex)}{\Pi_1(1-\Pi_1)} + b'n_1 + e(2+xn_1)\right] \quad (15)$$

Using the notations (12b) and (12c) one obtains

$$\frac{K}{g} = \frac{J}{g} \cdot a_1 (\Pi - \Pi_1 + \frac{a_2}{a_1}) \quad (16)$$

For the optimization procedure one has to choose for a given p a certain value of k and take from Fig. 1 the corresponding value of $\frac{J}{g}$.

Note: For $\frac{n}{g} = 0$ ($\frac{J}{g}$ is an integer ξ) one obtains from

eq. (4a)

$$\frac{J}{g} = \frac{\ln(1-p)}{\ln(1-\Pi)}$$

(This relation one also obtains for $\frac{n}{g} = 0$ with the help of the approximation

$$1-\Pi \cdot \frac{n}{g} \approx (1-\Pi)^{\frac{n}{g}}$$

which is valid for $\Pi \ll 1$.)

In this case one obtains from eq. (16)

$$\frac{K}{g} = a_1 \cdot \ln(1-p) \left[- \frac{\Pi}{\ln(1-\Pi)} + \frac{1}{\ln(1-\Pi)} \cdot (\frac{a_2}{a_1} - \Pi_1)\right] \quad (17)$$

The relevant functions

$$f(\Pi) = - \frac{\Pi}{\ln(1-\Pi)} \qquad g(\Pi) = - \frac{1}{\ln(1-\Pi)}$$

are presented graphically in Fig. 2a.

In order to decide which of the two cases, $\frac{J}{g} < 1$ or $\frac{J}{g} \geq 1$, is valid one has to consider the allowed region for Π. From eq. (1) one can take that for varying number of controlled birdcages Π varies in the following limits

$$\Pi_1 \leq \Pi \leq \Pi_1 \cdot (2-\Pi_1)$$

According to the discussion in the foregoing section one can conclude that

$$\text{Case I (II) holds if } \Pi_{1} \binom{>}{\leq} p$$

Otherwise case I as well as case II is possible.

3.5 Numerical Example

The numerical examples considered in this section are summarized in Table 5. It is to be noted that one must not specify numerical values of the reference time T and the critical time τ as long as one expresses all relevant quantities, i.e. J and K in terms of $g = \frac{T-1}{\tau}$.

First, the question is analyzed whether or not one has $\frac{J}{g} < 1$. The result is shown in Table 6.

Second, it is analyzed whether or not the quantity

$$A = -\Pi_1 + \frac{a_2}{a_1}$$

is negative for the different values of the parameters n_1, n_2, b, b', x,e . The result is shown in Table 7. From this table one can take that A is negative in all numerical cases considered.

For the case $\frac{J}{g} < 1$ this means according to the relations (13) that it is optimal for the inspector not to check birdcage seals at all.

For the case $\frac{J}{g} \geq 1$ this means according to eq. (17) that one has to check whether or not the function

$$f(\Pi) + A \cdot g(\Pi)$$

decreases with increasing Π. From Fig. 2b one can take that the function in all cases considered has its minimum for the smallest Π possible, this means that also in this case it is optimal for the inspector not to check birdcage seals at all.

In Table 8 the results of the optimization are collected. In all cases one has $k_o = 0$; for $\frac{J}{g} < 1$ one obtains from eq. (6b)

$$(\frac{J}{g})_o = \frac{p}{\Pi 1}$$

and $(\frac{K}{g})_{min}$ is given by eq. (14). For $\frac{J}{g} \geq 1$ one can take $(\frac{J}{g})_o$ from Fig. 1 for $\Pi = \Pi_1$, $(\frac{K}{g})_{min}$ is given by eq. (17) for $\Pi = \Pi_1$.

It is to be noted that in the case $p = 0.9$, $\Pi_1 = 0.8$ the result is only approximately true as in this case the approximation used $(1- \frac{n}{g} \cdot \Pi \approx (1-\Pi)^{\frac{n}{g}})$ does not work very well.

Up to now the optimal number of birdcage seals to be checked always was zero. This must not lead to the conclusion that this is true in any case. If one takes for example the cost model given by eq. (3') for b = b'

$$K = J \cdot (b(n_1+k)+c)$$

and takes b = 5 DM (Euratom seals) and c = 3000 DM (four days per visit plus 1000 DM for the evaluation of the seal characteristics in a central laboratory) one obtains for $n_1 = 3$, $n_2 = 150$, $\Pi_1 = 0.7, p<\Pi_1$ that the minimum costs are reached in case all birdcage seals are checked.

3.6 Discussion, Interpretation

A model for safeguarding a Plutonium storage has been analyzed; the problem was to find the optimal combination between number of inspections J per reference time and number of seals k to be identified and changed. The result was that in all numerical cases considered on the basis of the cost model given by eq. (3) the number of seals to be verified was zero. This can be explained with the help of the cost relation (12a) for the case $\frac{J}{g} < 1$: The costs for identifying and changing the birdcage seals are always higher than the costs for travelling and identifying and changing the container seals. Only in case the latter costs are higher than the former the result would be different. The same argument holds for the case $\frac{J}{g} \geq 1$.

In the case $\frac{J}{g} < 1$ (i.e. for $\Pi_1 > p$ according to section 3.3) and under the assumption that the optimal number of birdcage seals to be verified is zero one obtains the following relation between probability of detection p, number of inspections J per reference time T, critical time τ and probability for the identification of a falsified seal Π_1:

$$p = \left[\frac{\tau}{T-T}\right] \cdot J \cdot \Pi_1 \qquad (18)$$

This relation can be interpreted in the following way:

For a postulated probability of detection p with respect to the reference time T, a critical time τ and a quality of the seals expressed by Π_1, the necessary number of inspections per year is J, given by eq. (18).

However, under the circumstances given the critical time is not known and furthermore, the budget is given which determines the number of inspections per reference time in the framework of this model. Under these circumstances one may interprete relation (18) in the following alternative way:

If the inspection authority fixes a certain value for the probability of detection with respect to a reference time and furthermore, the number of inspections per year as well as the quality of the seals then the inspection authority behaves as if the critical time has a value given by relation (18).

This interpretation may be used to judge which type amongst a number of types of seals available, differing in quality and costs, should be used for safeguards purposes: If one considers a specific type of seals the costs per seal and the quality Π_1 are given therefore, for a given budget for the reference time T the number of inspections J is given. For a fixed probability of detection thus, one obtains a critical time in the sense discussed above. As a result of this consideration one should use that type of seals which leads to the shortest critical time (pessimistic behaviour).

4. Summary and Conclusions

In this paper an attempt has been made to quantify the relevant characteristics of containment measures in a similar manner used for material accountancy measures. It is shown that the parameters: assumed degree of falsification, probability of detection, average detection time and safeguards costs can be set into relation with each other. Two game theoretical models have been considered to illustrate the quantification and optimization procedures.

In the case of a sealed Plutonium storage, it appears that such quantification is possible if sealed barriers are assumed to be present in stages, e.g. first a sealed container and then a number of sealed birdcages inside the container. In such a system, the storage operator if he plans a diversion, has to falsify the seals in a sequential manner.

The idea of critical time particular in the case of a sealed storage has to be introduced for the quantification of the concept of 'timely detection', accepted as a part of the objective of international safeguards. In the context of the quantification of the characteristics of sealing measures, the critical time means the time required to falsify the sealing system in addition to the time required to convert nuclear material into a nuclear explosive device after the diversion. The frequency of inspections which determines the average detection time; should be put into relation to the critical time, unless additional containment or surveillance measures are introduced to ensure that the material remains unaccessible for diversion in the intervals between two inspections.

In a large number of cases investigated in the paper for the sealed storage, the optimum strategy of inspections has been found to consist of checking <u>only</u> the container seals and not checking the birdcage seals at all. This corresponds to the basic philosophy that more frequent and superficial inspections are more effective than less frequent and thorough inspections. For sealed storages with Plutonium, frequent inspection visits may not be hampering. However, such basic trends have been found to be dependent on the assumptions made for the model and cost structure assumed.

It appears useful to devote further effort in this area of quantification with particular attention to the possibility of establishing inherent relations between the critical and detection times. The measurement uncertainties, process variations and the inspection sampling procedures give information on the amount of diversion which a safeguards system is capable of detecting. The frequencies of inspection and physical inventory takings, on the other hand, give information on the capability of a safeguards organization for a timely detection. Since the latter actions may be often manpower intensive and intrusive, a proper utilization of containment measures may lead to less intrusive and intensive activities related to safeguards.

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Table 1: Strategies of the Operators and the Inspector in the Shipper-Receiver-Situation

(A: Shipper Station resp. Shipper; B: Receiver Station resp. Receiver; from the point of time t_1 on Receiver is responsible for the material)

Strategies of the Inspector:

1. Checking of the operator's data at A
2. Sealing of the bottles at A after operator's measurements at A, identification of seals at B before operator's measurements
3. Checking of the operator's data at B

Strategies of the Operators:

	No Diversion Cooperation between A and B	Diversion Cooperation between A and B
Falsification of data at A	1.1	2.1
Diversion after measurement at A, before t_1	1.2	2.2
Diversion after t_1, before measurement at B	1.3	2.3
Falsification of data at B	1.4	2.4

Table 2: Payoff Matrix for Single Inspection and Single Diversion Strategies

($P_c(M)$): probability of detection; τ : critical time

Operators / Inspector	No Diversion Cooperation				Diversion Cooperation			
	1.1	1.2	1.3	1.4	2.1	2.2	2.3	2.4
1	$(-1,\tau)$[1] $(-P_c(M),0)$	$(-1,\tau)$	$(-1,\tau_2)$	$(-1,0)$[2]	$(-P_c(M),0)$	$(0,\infty)$	$(0,\infty)$	$(0,\infty)$
2	$(-1,\tau)$	$(-1,\tau)$	$(-1,\tau_2)$	$(-1,0)$	$(0,\infty)$	$(-1,\tau)$	$(-1,\tau_2)$	$(0,\infty)$
3	$(-1,\tau)$	$(-1,\tau)$	$(-1,\tau_2)$	$(-1,0)$ $(-P_c(M),0)$	$(0,\infty)$[4]	$(0,\infty)$	$(0,\infty)$	$(-P_c(M),0)$

[1] If the inspector would check no measurements at all, the operator B would state after the time τ that operator A had falsified data. If the inspector checks the measurements at A he will state at once - dependent of his effort C - with a certain probability $P_M(C)$ that the data reported are falsified.

[2] With the help of a comparison of the reported data of the operators A and B the inspector will detect at once that the data are falsified.

[3] It is assumed here that operator B knows that the inspector checks only the measurements at A. Thus, all the falsifications will be done at B. (In case the inspector would check measurements at A and B, one could imagine that a distribution of the falsifications to the places A and B would be better from the side of the operators.)

[4] It is assumed that the operator A knows that the inspector checks only the measurements at B. Thus, all the falsifications will be done at A.

Table 3: Payoff matrix for Composed Inspection and Single Diversion Strategies in Case of Diversion Cooperation of the Operators

	2.1	2.2	2.4
1+3	$(-P_c(M),0)$	$(-P_c(M),\tau)$	$(-P_c(M),0)$
1+2+3	$(-\tilde{P}_c(M),0)$	$(-1,\tau)$	$(-\tilde{P}_c(M),0)$

Table 4a: Reduced P-Game $(P_c(M) > \tilde{P}_c(M))$

$$\begin{pmatrix} -P_c(M) & -P_c(M) \\ -\tilde{P}_c(M) & -1 \end{pmatrix}$$

Table 4b: Reduced τ-Game

$$\begin{pmatrix} 0 & \tau \\ 0 & \tau \end{pmatrix}$$

Table 5: <u>Numerical Values for the Parameters of the Plutonium Storage Safeguards Model</u>

$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$		$\begin{pmatrix} 3 \\ 150 \end{pmatrix}$; $\begin{pmatrix} 30 \\ 1500 \end{pmatrix}$
p		0.8 ; 0.2
Π_1		0.5 ; 0.9
x	[mandays]	$\frac{1}{30}$
$\begin{pmatrix} b \\ b' \end{pmatrix}$	[DM] [DM]	$\begin{pmatrix} 20 \\ 20 \end{pmatrix}$; $\begin{pmatrix} 20 \\ 200 \end{pmatrix}$
e	[DM]	400

Table 6: <u>Analysis of Cases I and II</u>

Π_1 \ p	0.5	0.9
0.2 $0.2 \leq \Pi \leq 0.36$	$\frac{J}{g} > 1$	$\frac{J}{g} > 1$
0.8 $0.8 \leq \Pi \leq 0.96$	$\frac{J}{g} < 1$	$\frac{J}{g} \overset{>}{(<)} 1$

Table 7: Numerical Values of $A = -\Pi_1 + \frac{a_2}{a_1}$

$\binom{b}{b'}$ \ $\binom{n_1}{n_2}$	$\binom{3}{150}$	$\binom{30}{1500}$
$\binom{20}{20}$	-0.171 for Π_1 = 0.2 -0.771 for Π_1 = 0.8	-0.194 for Π_1 = 0.2 -0.794 for Π_1 = 0.8
$\binom{20}{200}$	-0.154 for Π_1 = 0.2 -0.754 for Π_1 = 0.8	-0.177 for Π_1 = 0.2 -0.777 for Π_1 = 0.8

Table 8: Results of Optimization: Optimal Number of Inspections $(\frac{J}{g})_o$ per Reference Time and Minimum Costs $(\frac{K}{g})_{min}$

		p = 0.5		p = 0.9	
		Π = 0.2 $(\frac{J}{g})_o$ = 3.1	0.8 0.62	0.2 10.327	0.8 1.63
$\binom{n_1}{n_2}$ = $\binom{3}{150}$	$\binom{b}{b'}$ = $\binom{20}{20}$	2820.1	562.5	9371.3	1310.4
	$\binom{20}{200}$	4482.4	900	14895.0	2070.1
$\binom{30}{1500}$	$\binom{20}{20}$	5846.6	1124.9	19428.3	2825.2
	$\binom{20}{200}$	22375.6	4500	74354.2	10422.2

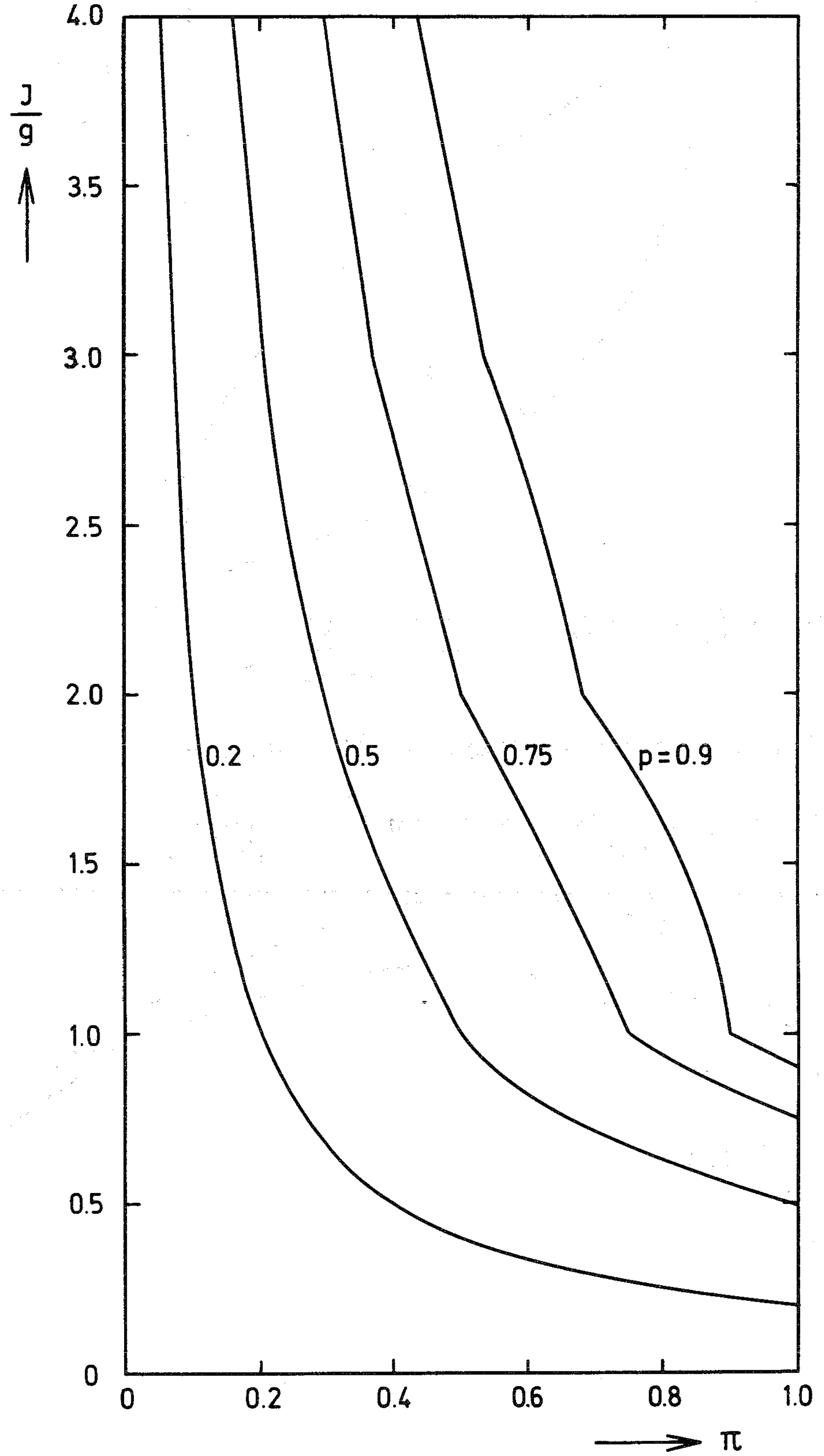

Fig.1 Graphical representation of the relation

$$1-p = (1-\pi)^{\xi}\left(1-\pi\cdot\frac{\eta}{g}\right);$$

$$\frac{J}{g} = \xi + \frac{\eta}{g};\ \xi \text{ integer};\ 0 \le \frac{\eta}{g} < 1$$

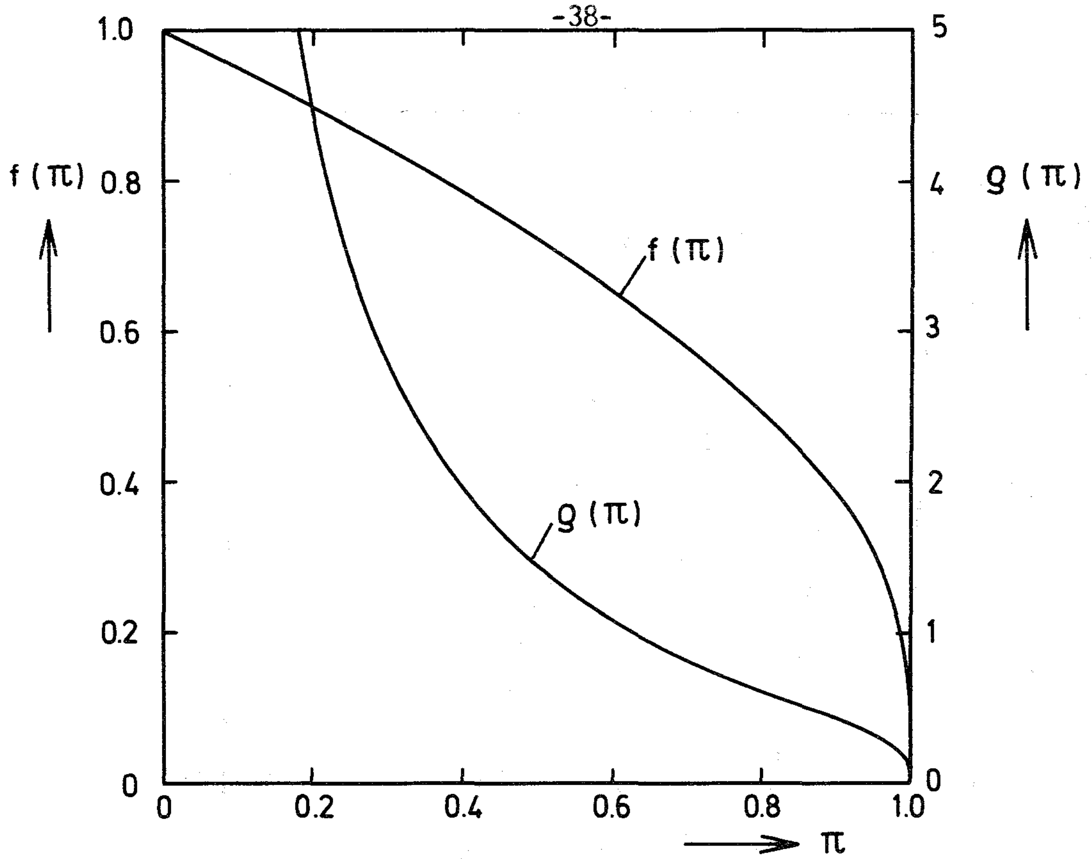

Fig. 2a Graphical representation of the functions

$$f(\pi) := -\frac{\pi}{\ln(1-\pi)} \text{ and } g(\pi) := -\frac{1}{\ln(1-\pi)}$$

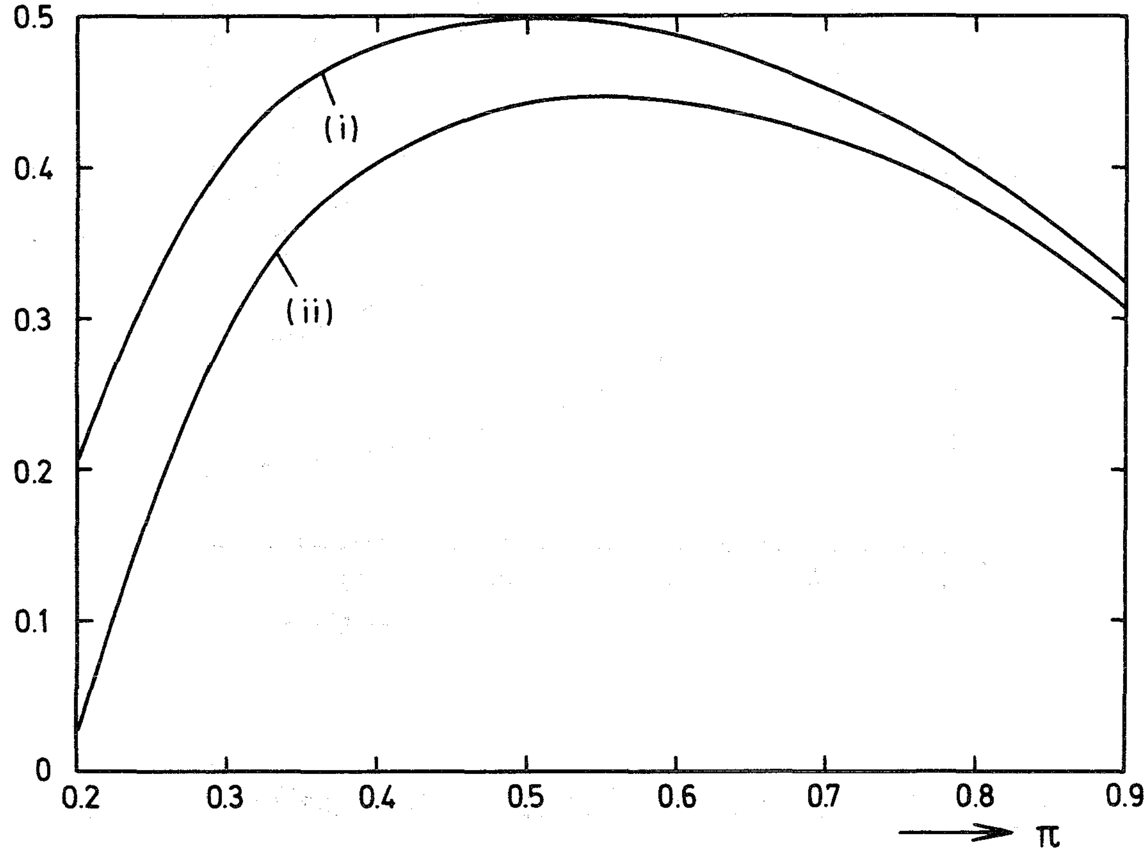

Fig. 2b Graphical representation of the function

$f(\pi) + A \cdot g(\pi)$ for $\pi_1 = 0.2$ and

(i) $n_1 = 3$ $n_2 = 150$ $b = 20$ $b' = 200$ $(A = -0.154)$

(ii) $n_1 = 30$ $n_2 = 1500$ $b = 20$ $b' = 20$ $(A = -0.194)$