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Institut für Angewandte Systemtechnik und Reaktorphysik
Projekt Schneller Brüter

Adjustment of Group Cross Sections for Fast Reactor Calculations Using Integral Data from Critical Assemblies

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Anpassung von Gruppenquerschnitten für die Berechnung schneller Reaktoren an integrale Messungen von kritischen Anordnungen

Zusammenfassung

Eine Reihe von integralen Messungen an schnellen kritischen Nullenergieanlagen wurden dazu verwendet, Wirkungsquerschnitte nach der Methode der kleinsten Quadrate anzupassen. Als Ausgangsquerschnitte wurden die Gruppenkonstanten des gut dokumentierten Karlsruher NAPPMB-Satzes verwendet. Die integralen Messungen wurden mit den Methoden nachgerechnet, die in der Analyse kritischer Experimente üblich sind. Dann wurden nach der Methode der kleinsten Quadrate Querschnittsänderungen errechnet, für die die Fehler in den integralen Messungen plus die Querschnittsfehler ein Minimum werden. Das Verfahren wurde für zwei Fälle durchgeführt, nämlich für Messungen des Spaltratenverhältnisses F8/F5 mit Folien, und mit Kammern. Die Ergebnisse werden insbesondere dazu benutzt, Aussagen über die Konsistenz integraler Messungen zu gewinnen.

Adjustment of Group Cross Sections for Fast Reactor Calculations Using Integral Data from Critical Assemblies

Abstract

A series of integral measurements on fast critical assemblies was used to adjust group cross sections by a least-squares procedure. The initial cross sections were those of the well-documented Karlsruhe group constant set NAPPMB. The integral data were calculated by the same methods which are used in the routine analysis of critical experiments. Then, adjustments to these cross sections were obtained by a least-squares procedure, which minimizes the errors on the integral data, plus the errors on the cross sections. The procedure was carried out for two cases, using foil measurements, and fission chamber measurements of the fission ratio F8/F5. The results are discussed, and used to judge the consistency of integral measurements.

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1. Introduction

It is generally recognized that critical experiments are still needed to provide a reliable basis for the physics design of a fast breeder reactor.

This is partly due to uncertainties in the calculational methods, but largely also to deficiencies in the basic cross section data, which inspite of significant progress during the last few years, are not yet accurate enough to allow a calculation of the important reactor parameters with the desired accuracy.

There are, in principle, two ways to improve the cross section data available for the design of a fast breeder. One way is to update group cross sections mainly by evaluation of differential measurements, using the integral data only as a check. The second way is to adjust, by a suitable mathematical procedure, the cross sections to give agreement with measured integral data. At Karlsruhe, a major effort, was spent on the improvement of cross sections by the first way. This work, which was done by Kiefhaber and others, has led to the preparation of the cross section sets MOXTOT /1/ and KFKINR /2/. On a much smaller scale, work along the second path was also carried out. The results will be described in this report.

Mathematical adjustment procedures were used to produce group cross sections at several laboratories; the best known are quoted in Ref. /3/ and /4/. On the other hand, criticism on these procedures arose frequently /5/, and therefore, it seems to be in order to make a few comments on their merits, and on their shortcomings.

1. It was pointed out by Rowlands /4/ that the adjustment does not, in general, give additional information on the single cross sections. In most cases, the changes to the cross sections are within one standard deviation, and frequently the adjusted cross sections have no better accuracy than the original ones. However, the adjusted cross sections provide a much better prediction of integral parameters and are, therefore, a suitable tool for reactor design calculations. Thus, the merit of the method is that it uses the information contained both in differential and in integral experiments in a systematic way to improve the prediction of integral reactor parameters.

2. The adjustment procedure is useful for comparing integral experiments and their analysis, and in spotting integral measurements where errors are present either in the experiment, or in the method of analysis.

3. It is admitted that the uncritical use of adjusted cross sections can lead to large errors, and to erroneous conclusions. Therefore, one must be careful that the system to be calculated is in the range of compositions, and spectra, covered by integral experiments.

2. Description of the Calculational Procedure

2.1 The Method of Least Squares

The procedure consists in applying a least-squares fit to the measurements of the cross sections, and of the integral parameters. The procedure was described in the literature /3,4/; however, for convenience, it will be outlined here.

Nomenclature:

σ_g group cross section, obtained from differential measurements

$\Delta\sigma_g$ standard deviation (s.d.) of the group cross section

$\delta\sigma_g$ adjustment to the group cross section

X_m measured integral parameter

ΔX_m standard deviation of X_m

X_m^c calculated value for X_m

X_m^a adjusted value for X_m

The minimum value of the expression

$$E^2 = \sum_{g=1}^{G} \left(\frac{\delta\sigma_g}{\Delta\sigma_g}\right)^2 + \sum_{m=1}^{M} \left(\frac{X_m - X_m^a}{\Delta X_m}\right)^2 \qquad (1)$$

will be sought. The "adjusted" value X_m^a is, in linear approximation in the adjustments, given by

$$X_m^a = X_m^c \left[1 + \sum_g \alpha_{mg} \frac{\delta\sigma_g}{\sigma_g}\right] \qquad (2)$$

The sensitivities $\alpha_{mg} = \partial X_m/\partial\sigma_g$ of the integral parameter X_m to a change in σ_g must be known.

Let the relative change

$$\frac{\delta\sigma_g}{\sigma_g} = f_g$$

then one obtains by differentiating Eq. (1) with respect to f_g, and using Eq. (2)

$$\sum_{g'} f_{g'} \left[\delta_{gg'} \left(\frac{\sigma_g}{\Delta\sigma_g}\right)^2 + \sum_m \alpha_{mg} \alpha_{mg'} \left(\frac{X_m^c}{\Delta X_m}\right)^2 \right] = \sum_m \alpha_{mg} \frac{X_m^c (X_m - X_m^c)}{(\Delta X_m)^2} \tag{3}$$

This is a system of linear equations with the matrix of coefficients

$$C_{gg'} = \delta_{gg'} \left(\frac{\sigma_g}{\Delta\sigma_g}\right)^2 + \sum_m \alpha_{mg} \alpha_{mg'} \left(\frac{X_m^c}{\Delta X_m}\right)^2 \tag{4}$$

and the inhomogeneous vector

$$G_g = \sum_m \alpha_{mg} \frac{X_m^c (X_m - X_m^c)}{(\Delta X_m)^2} \tag{5}$$

Thus, the system (3) reads

$$\sum_{g'} C_{gg'} f_{g'} = G_g \tag{6}$$

Following the arguments in /4/, we find that the standard deviations of the adjusted cross sections are

$$\frac{\Delta\sigma_g^a}{\sigma_g} = \sqrt{(C^{-1})_{gg}} \tag{7}$$

whereas the standard deviations of the integral parameters, using adjusted cross sections, are

$$\left(\frac{\Delta X_m^a}{X_m}\right)^2 = \sum_{gg'} \alpha_{mg}\, \alpha_{mg'}\, (C^{-1})_{gg'} \tag{8}$$

In principle, the meaning of the s.d. (standard deviation) given by Eq. (8) is the following: X_m^a, is the best estimate for the integral parameter m, energing from the adjustment procedure using the information from both integral measurements, and cross section measurements. This best estimate has the s.d. ΔX_m^a. However, only a limited number of cross sections is included in the adjustment. For those which are not included, Eq. (8) gives the contribution

$$\sum_{g''} \alpha_{mg''} \left(\frac{\Delta\sigma_{g''}}{\sigma_{g''}}\right)^2 \tag{8a}$$

to the variance, which is, of course, determined by the error propagation law. In the practical work, this contribution is

usually neglected. In other words, both the estimate X_m^a and the s.d. (8) are valid only under the assumption that the contributions (8a) are negligible. Or, putting it still differently, if the measured value X_m and the adjusted value X_m^a are not consistent with ΔX_m^a, one can assume that either a significant error is introduced from a cross section which is not included in the adjustment; or else, there is an error in the experiment, or method of analysis.

2.2 Calculation of the Sensitivities

Two computer routines have been written to calculate the sensitivities. They both use the 26 energy groups scheme which is standard at Karlsruhe /6/. The program SENSIT calculated reactor parameters in zero dimension. Then, some of the cross sections are modified (usually by 10%), and the program calculates the relative changes in the reactor parameters, and punches them on cards. The cards are used as input for the least-squares program. Most sensitivities can be obtained accurately enough by this zero dimensional calculation. However, the sensitivities of k_{eff} to cross sections of ^{238}U are influenced, to some extent, by the properties of the reflector. Therefore, they were calculated by diffusion theory and perturbation theory in a one-dimensional spherical model. A program QERMOD was written, which modifies the cross sections to be used in the diffusion calculation.

3. Cross Section Set Used in the Adjustment

The cross section set used for the adjustment is the 26 group set H2OPMB for assemblies containing polyethylene, and the set NAPPMB for all other assemblies. The two sets differ only in the weighting spectrum used to produce the elastic moderation cross sections. Both sets were developed at Karlsruhe.

These sets were chosen for the following reasons: The NAPPMB set was defined as the reference set for the SNR design; also, both sets are well documented /6/; furthermore, they represent two sets with different weighting spectra, but on the same data base. No other such couple of sets is available.

At the time these cross section sets were chosen for use in the adjustment, it was known that they did not represent the latest state of the art. However, it was believed that this fact would not impair their use in the adjustment, provided that two types of cross section, for which drastic changes were necessary, were brought up to date. It was known that the low Pu-α data and the capture cross section of ^{240}Pu in NAPPMB were truly obsolete. Therefore, these cross sections were modified, prior to the adjustment procedure, in order to improve the data basis for the procedure, and to avoid large adjustments, which would lead to non-linear behaviour. Therefore, following Kiefhaber /1/, σ_{c9} was increased by the following factors in the energy groups 12 to 15

Energy group	12	13	14	15
factor for σ_{c9}	1.23	1.68	1.51	1.32

Furthermore, σ_c of ^{240}Pu was reduced by the factors

Energy group	5	6	7	8	9	10	11
factor for σ_{c40}	0.42	0.39	0.35	0.33	0.40	0.47	0.46

The factors are essentially those obtained by Broeders /7/.

The cross sections to be adjusted are essentially those of the main fuel isotopes, plus iron, above 1 keV which influence strongly the integral data. In particular, the following cross sections are adjusted.

^{235}U	σ_f, σ_c, ν
^{238}U	σ_f, σ_c, ν, σ_{in}, σ_{tr}
^{239}Pu	σ_f, σ_c, ν
Fe	σ_c, σ_{tr}

The adjustment is carried out in a coarse 4 group scheme, where the groups are defined by

Coarse group	ABN-groups
1	1 - 5
2	6 - 8
3	9 - 11
4	12 - 14

4. Integral Measurements Used in the Adjustment

4.1 Selection of the Critical Assemblies and Integral Measurements

Integral measurements carried out on critical facilities are suitable for testing, or adjusting, cross sections only if they fulfill the following requirements: A series of measurements must be consistent, sensitive to the cross sections to be adjusted, and carried out with

good precision. Also, they must be amenable to calculation by standard, non-sophisticated methods, in order to avoid large errors due to the calculational procedure.

The assemblies to be used in the adjustment were selected essentially from Argonne and SNEAK assemblies so as to cover a fairly large range of spectra.

On the other hand, assemblies which require special methods of analysis were not included in this study. Thus, only cores were used which are large enough so that diffusion theory gives a valid approximation, with transport effects to be considered as a correction. This excludes small, highly enriched assemblies. On the other end of the line, cores with very soft spectra, which are difficult to calculate because of large heterogeneity effects, were also left out. Furthermore, cores which contain large amounts of non-breeder materials were also not included. The list of assemblies which were selected contains 22 critical experiments. The integral parameters used in the study are critical mass, central fission ratios, and ratios of central reactivity worths. The atom densities for these assemblies are given in Tables 14, 15, and 16.

4.2 Analysis of the Criticality Measurements

4.2.1 Experimental Criticality Data

The criticality of assemblies in the facilities ZPR-3, ZPR-6, and SNEAK was analyzed. The experimental criticality data are compiled in Table 1. The references for all the experimental data are compiled in Table 13.

For the older ZPR-3 assemblies, the critical masses were converted to spherical geometry, by Davey , using the "Shape Factor" method;

the data are published in /8/. A slightly more comprehensive collection of critical masses was given by Baker /9/. The "benchmark series" in ZPR-3, Assembly 48 etc, were analyzed in cylindrical geometry; the critical dimensions were taken from Till et al /10/. The critical dimensions of the SNEAK-3 cores are from different reports. The core $3B2_{ex}$ is a fictitions Pu-core, whose critical dimensions were obtained by the method of progressive substitution, inserting a Pu-loaded zone into a U-loaded reference composition. In some of the more recent SNEAK assemblies, the material buckling of the core composition was measured by fission rate traverses. The method is described, for example, in /11/. In these cases, an analysis by zero dimensional calculation is possible.

4.2.2 Calculation of the Criticality Parameter k_{eff}

To obtain the k_{eff} values, calculations in diffusion theory were carried out, either in spherical geometry, or, for the cylinders, by the "buckling iteration method", iterating between axial and radial calculations. In two cases calculations in 2-dimensional R-Z-geometry with the code DIXY were carried out. In those cases where measured bucklings are available, the basic calculation is simply a zero dimensional one, using the experimental buckling.

The results of the diffusion calculations were corrected for heterogeneity, and for transport effects. In addition, an improved calculation of the elastic slowing down cross section, based on the spectrum of the particular assembly, rather than on a standard spectrum, was performed. This procedure is called REMO (from elastic removal), and was described earlier /6/; it leads to an additional correction to the calculated k_{eff}. It was confirmed that the REMO-corrected k_{eff} were identical, whether the NAPPMB set, or the H2OPMB set was used originally. Both sets differ only by the weighting spectrum used to obtained the elastic removal cross section.

The results obtained with the cross section set NAPPMB or H2OPMB, are given in Table 2. The heterogeneity corrections were obtained with the ZERA code /12/. Only for the "benchmark series" ZPR-3/48, etc., the heterogeneity corrections are fairly large. Note that they differ from values published earlier by Davey /13/. Most of the S_n corrections were taken from a paper by Baker /9/; spot checks made with the code DTK confirmed these results very well.

4.3 Analysis of the Central Ratios of Reaction Rates

The central fission ratios F8/F5 and F9/F5 were used, and, in addition, the ratio C8/F5 of capture in ^{238}U to fission in ^{235}U.

The measurements of F8/F5 need some discussion. In most assemblies, this fission ratio was measured with fission counters; in addition, foil measurements are available for some assemblies (Table 3). It is known that fission counter measurements are influenced by the neutron degradation in the chamber walls. Most published results, however, are corrected for this effect /8/. Furthermore, the counter averages over the space taken by several plates, which form the reactor core, and it was claimed in the literature /8/ that these measurements are representative for the fission ratio in the equivalent homogeneous mixture.

It is easily seen from Table 3 that the foil measurements are, on the average, about 7% higher than the counter measurements. If the counter sees the spectrum of the homogeneous mixture, this difference can only be due to heterogeneity effects inherent in the plate structure. In order to check this hypothesis, the fission ratios for both the heterogeneous and homogeneous case were calculated for the pertinent assemblies. The results are also given in Table 3. It is seen that the value for the "heterogeneous" case is at most 1%

higher, in many cases even lower than the one for the "homogeneous" case. Thus, the difference which appears between foil and counter measurements cannot be explained by heterogeneity. It is probably due to degradation of neutrons in the subassembly wall (or drawer material). Thus, the foil measurements are certainly more reliable.

It is unfortunate that foil measurements are available only for some assemblies. For reasons of consistency, one cannot combine the existing foil measurements with counter measurements in all the other assemblies. It was, therefore, decided to do 2 runs in the adjustment procedure. The first one uses only counter measurements (except for SNEAK-7A and 7B, where they are not available). In the second run, the existing foil measurements, and for all other assemblies, the counter measurements increased by 7% were used, assuming that this figure is representative for the difference. The results will be discussed later.

There seems to be no problem for the fission ratio F9/F5. However, the experimental capture ratios C8/F5 had to be corrected to the homogeneous case, because the adjustment was carried out using values corrected for a homogeneous model. The corrections are fairly large for ZPR-3/48 and ZPR-6/7 (Table 4). In those cases where they are less than 1%, they are neglected.

4.4 Ratios of Central Reactivity Worths

Central reactivity worth measurements of absorbing materials can give important information, provided that the following two problems can be solved: The uncertainty in the reactivity scale must be removed, and the geometry of the reactivity worth sample and the surrounding core must be accessable to calculation.

The first problem may be solved, for the purpose of the present analysis, by using ratios of reactivity worths, rather than absolute values. Following a common practice, the worths were normalized to the worth of ^{235}U.

It was attempted to use the worths of ^{239}Pu, and ^{238}U, normalized to ^{235}U, in the adjustment procedure. For the fissile isotopes, effects due to sample size and environment are small. They were first studied, and generally understood, in assembly 48 in ZPR-3, and in SNEAK. Therefore, the results from ANL (starting with ZPR-3/48) and SNEAK were used in the adjustment. However, fairly large sample size effects appear in the measurements with ^{238}U. For the ZPR assemblies, the suggested procedure is to extrapolate worths measured with different samples to an infinitely thin sample.

However, for small samples, the relative statistical error becomes rather large leading to a large uncertainty in the extrapolation. Therefore, those measurements could not be used in this work.

For the SNEAK assemblies, the difference "worth of heterogeneous sample minus homogeneous worth" was calculated with one of the programs described in /11/ and /18/. This procedure is certainly well defined, and definitely preferable to extrapolation of experimental data points. Of course, there are also uncertainties in the calculated correction, but they are of the same degree of reliability as calculated heterogeneity effects in k. Thus, the SNEAK measurements were used in the adjustment.

5. Results of the Cross Section Adjustment

It is important for the following presentation, and discussion of the results to have a clear definition in mind of the errors in-

volved. Any disagreement between an integral measurement, and a calculated value can be due to a) an error in the experiment, b) an error due to the method of analysis, including inadequacy of the geometrical model, and mathematical approximations, c) errors in the cross sections.

The adjustment procedure should eliminate, to a large extent, the error c); thus, the results allow to conclude whether the experiments are consistent within estimated errors a) and b). Therefore, the errors attached to the integral measurements are estimates of these errors a) and b). It is recognized that they are based on judgement.

As mentioned earlier, results for two cases of adjustment will be presented: The reference case (Case 1), where F8/F5 was taken from foil measurements, where available. If only chamber measurements were carried out, they were increased by 7%, which is the average difference between the two experimental methods. In Case 2, chamber measurements were used, as far as available.

Table 5 shows estimated uncertainties of the cross sections (Case 1) before adjustment, and also the errors after adjustment, which are obtained from the least-squares procedure using Eq. (7). As mentioned in the introduction, the adjustment usually does not improve the uncertainties of the single cross sections, and indead, the errors are, in general, not significantly reduced by the procedure.

The resulting adjustments are listed in Table 6. They are never significantly larger than the input error. Note that σ_{in8} is significantly reduced in both Case 1 and Case 2, and so is σ_{c8}. The only important difference between the two cases is in the changes of σ_{f8}, and σ_{c8}. Case 2 requires a reduction of σ_{f8}, and a larger reduction of σ_{c8} than Case 1. To the extent to which these changes are meaningful, Case 1 seems to give the more reasonable results.

Tables 7 - 12 show the comparison of integral experiments with calculations. As one excepts from such a procedure, the agreement with integral data is considerably better with the adjusted values. Thus, as a first result, the prediction of reactor parameters can be considerably improved if cross section adjustment is used, provided the reactor is in the general range of compositions and spectra where integral measurements are available.

The next point is to examine the consistency of integral measurements, by comparing the difference (C-E)/E after adjustment with the standard deviation. One finds that the k_{eff} are very well consistent. Only for SNEAK-7A and ZEBRA-6A, the difference reaches or exceeds two standard deviations. In the case of SNEAK-7A, the measured buckling may be slightly too low. As the critical mass can be easily measured, and the method of analysis is well developed, it is not surprising that consistent results are obtained. Note that the results are equally good for both Case 1 and Case 2.

The results for the fission ratios F9/F5 are also very satisfactory indicating that this ratio can be measured with confidence. Larger deviations exist for the two assemblies on ZPR-6.

For the fission ratio F8/F5 (Table 9), the results in both cases are, in general, compatible with the standard deviations. Two large discrepancies exist, for ZPR-6/5, and for SNEAK-7A. In the latter case, it is suspected that the difference is due to errors in the analysis, or in cross sections, which are not adjusted, because the experimental result is rather well established. No comment can be given to the ZPR-6/5 result. Note that only foil measurements were included for SNEAK-7A and 7B, which explains the larger difference in Case 2.

The results of Case 1 and Case 2 demonstrate that the foil measurements are consistent, among themselves, and so are the fission

chamber measurements. The typical average difference between the two sets is 7%. The cross section adjustment is possible, and consistent, for each case. However, from the geometry of the experiments, it is clear that foil measurements should be preferred.

Table 10 shows that large discrepancies exist between the measured ratios C8/F5. Three measurements are too high, and as they are rather old ones, one might suspect that they are in error. The other five ones are at least consistent within three s.d. It is gratifying to see that, although these capture measurements have larger errors than the fission ratios, the more recent data are compatible within reasonable limits.

The results for the reactivity worth ratios $^{239}Pu/^{235}U$ are shown in Table 11. In four out of 16 cases, the difference is outside three s.d. The most likely reason is that experimental details, and corrections for higher Pu isotopes are not well known for some of these measurements.

The reactivity worth ratios $^{238}U/^{235}U$ are listed in Table 12. Only SNEAK measurements, for which the geometry is well known, were used. The calculated values are greatly improved by the adjustment procedure, though the remaining differences, and errors, are still larger than for the worth ratio $^{239}Pu/^{235}U$. The fact that the KFKINR set /11/ predicts $^{238}U/^{235}U$ very well for SNEAK-7A and 7B indicates that these differences are, at least in part, due to the shortcomings of the coarse group structure. It is indead not surprising that the coarse four-group structure used in the adjustment is not adequate to analyze the worth of ^{238}U, which is rather sensitive to the flux and adjoint spectrum.

6. Conclusions

The cross section adjustment described in this report shows that suitable cross section changes, which are mostly within error limits, can greatly improve the agreement of calculated and measured integral parameters. The compatibility of integral measurements was examined by comparison of measured and adjusted values. While the k_{eff}, and the fission ratios are well consistent, the compatibility is not as good for the reaction ratio C8/F5, and for the reactivity worth ratios. It was further observed that a consistent difference of about 7% exists between foil measurements, and chamber measurements of the fission ratio F8/F5. Adjustments based on either techniques of measurement lead to similar, and equally consistent, results, except that the chamber values lead to a decrease of 7% in σ_{f8}, while the foil values do not. Also, the resulting worth ratios $^{238}U/^{235}U$ are slightly better if the foil values are used. Thus, it has been proved that the adjustment procedure is a useful tool, both for improving predictions, and for the diagnosis of integral measurements.

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Table 1 Experimental Criticality Data

a) Assemblies Analyzed as a Homogeneous Sphere

	Measured Critical Mass, kg	Crit. Mass of Homogeneous Cylinder, kg	Shape Factor f	Crit. Mass of Homogeneous Sphere, kg	Blanket Type
ZPR-3/24	460.7	490.0	0.93	456	A
25	581.6	612.9	0.92	564	A
32	227.5	234.0	0.91	213	A
33	238.0	244.5	0.92	225	A
35	505.4	532.5	0.91	485	B
36	242.7	249.9	0.87	217	A
ZPR-6/7	Inner Core Radius 24.34 cm, Outer Core Radius 92.01 cm				C

b) Assemblies Analyzed as a Heterogeneous Cylinder

	Dimensions of the Critical Cylinder			Method of Analysis	Blanket
	Height, cm	Radius, cm	Radius of the Inner Core Zone		
ZPR-3/48	76.4	41.6	-	BJ	D
49	76.4	43.3	-	BJ	D
50	76.4	37.8	-	BJ	D
53	61.0	34.4	-	BJ	D
ZEBRA-6A	60.16	36.15		BJ	E
ZPR-6/5	142.2	78.4	-	2D	G
6	152.4	90.9	85.5	2D	H
SNEAK-3A1	80.3	51.2	-	BJ	F
3A2	80.3	44.66	-	BJ	F
$3B2_{ex}$	80.3	44.97	-	BJ	F

BJ = Buckling Iteration
2D = R-Z-Calculation with the Code DIXY

c) Assemblies Analyzed with Experimental Buckling

	Measured B^2, m^{-2}
SNEAK-2A	16.20
SNEAK-6A	15.42
SNEAK-7A	59.68
SNEAK-7B	34.74

Table 2 Calculated k_{eff} Values (Original NAPPMB or H2OPMB Set)

Assembly	$\frac{N_8}{N_{fiss}}$	k_{eff} Hom	$\frac{\Delta k}{(Het-Hom)}$	k_{eff} Het	Correction		k_{eff}
					REMO	S_n	
Uranium-Assemblies							
ZPR-3/24	9.6	0.966	--	--	0.001	0.003	0.970
25	10.4	0.964	--	--	0.001	0.002	0.967
32	0.07	0.982	--	--	-0.007	0.009	0.984
33	0.07	0.986	--	--	-0.007	0.010	0.989
35	0.07	0.991	--	--	-0.006	0.009	0.994
36	5.3	0.967	--	--	0	0.008	0.975
ZPR-6/5	6.8	0.967	0.001	0.968	0.007	0	0.975
6	5.0	0.969	0.002	0.971	0.010	0	0.981
SNEAK-3A1	4.0	0.984	0.003	0.987	-0.001	0.003	0.989
3A2	4.0	0.980	0.004	0.984	-0.001	0.004	0.987
SNEAK-2A	3.5	0.997	0.003	1.000	0.001	--	1.001
Pu-Assemblies							
ZPR-3/48	4.5	0.963	0.015	0.978	0.011	0.006	0.995
49	4.5	0.965	0.011	0.975	0.008	0.007	0.990
50	4.5	0.957	0.020	0.977	0.015	0.005	0.997
53	1.6	0.977	0.018	0.995	0.009	0.008	1.012
ZPR-6/7	6.3	0.970	--	--	0.020	0	0.990
SNEAK-3B2	5.5	0.984	0.002	0.986	0.003	0.004	0.993
SNEAK-6A	5.8	0.980	0.001	0.981	0.011	--	0.992
SNEAK-7A	2.9	1.011	0.001	1.012	0.001	--	1.013
7B	7.0	0.986	0.001	0.987	0.001	--	0.988
ZEBRA-6A	3.4	0.965	--	--	0.010	0.009	0.984

Table 3 Fission Ratios F8/F5 for some Assemblies, Measured with Chambers and Foils

	σ_{f8}/σ_{f5}				
	Measured		Calculated		
	Chambers	Foils	Homogeneous	Heterogeneous	
ZPR-6/6	--	0.0229	0.0220	0.0219	a
7	0.0205	0.0220	0.0210	0.0208	a
SNEAK-3A2	0.0273	0.0300	0.0294	0.0293	b
ZPR-3/48	0.0307	0.0326	0.0319	0.0314	a
			0.0314	0.0317	b
SNEAK-7A		0.0448	0.0374	0.0377	b
7B	0.0308	0.0328	0.0303	0.0303	b

Table 4 Ratio C8/F5

	Measured	Calculated		
		Homogeneous	Heterogeneous	
ZPR-3/48	0.137	0.1359	0.1285	a
		0.1418	0.1353	b
ZPR-6/6	0.1344	0.1434	0.1417	a
7	0.132	0.1432	0.1373	a
SNEAK-3A1	0.142			
3A2	0.130	0.1373	0.1384	b
SNEAK-7A	0.138	0.1488	0.1489	b
7B	0.132	0.1482	0.1490	b

a) From Zolotar et al., Ref. 26

b) Calculation with original NAPPMB Set

Table 5 Standard Deviations of the Cross Sections, %
(Input / Adjusted, Case 1)

ABN-group	1-5	6-8	9-11	12-14
^{238}U σ_f	10/0.7	-	-	-
σ_c	10/1.0	10/11.2	10/9.7	10/13.7
σ_{in}	10/2.5	10/20.2 (group 6-11)		-
σ_{tr}	15/19.3	10/14.0 (group 6-11)		-
ν	-	1/3.6 (group 1-16)		-
^{235}U σ_f	5/6.0	5/3.6	5/3.7	5/4.5
σ_c	-	15/29 (group 6-11)		15/29 (group 12-16)
ν	-	1/2.2 (group 1-16)		-
^{239}Pu σ_f	10/9.6	10/5.5	10/7.7	10/8.0 (group 12-16)
σ_c	-	10/35 (group 6-11)		15/22 (group 12-16)
ν	-	1/2.0 (group 1-16)		-
Fe σ_c	-	-	10/38	10/38
σ_{in}	10/1.5	-	-	-
σ_{tr}	20/45	10/13	-	-

Table 6 Adjustments to the Cross Sections, %

Case 1 / Case 2

ABN-group	1-5	6-8	9-11	12-14
^{238}U σ_f	-1.3/-7.1	-	-	-
σ_c	+1.8/+2.1	-6.4/-11.0	-13.9/-12.4	+2.0/+1.8
σ_{in}	-13.4/-13.4	+0.9/+1.8 (group 6-11)		-
σ_{tr}	-8.4/-8.8	-2.7/+0.5 (group 6-11)		-
ν	-	+0.5/+1.0 (group 1-16)		-
^{235}U σ_f	+2.7/+2.4	+4.1/+4.1	-4.5/-2.9	-0.9/-0.4
σ_c	-	+18.6/+19.0 (group 6-11)		-8.7/-8.6 (group 12-16)
ν	-	-0.07/-0.10 (group 1-16)		-
^{239}Pu σ_f	-6.0/-7.7	+6.4/+7.4	-7.1/-5.9	+10.3/+11.1 (group 12-16)
σ_c	-	+10.6/+12.5 (group 6-11)		+5.3/+6.0 (group 12-16)
ν	-	-0.14/+0.10 (group 1-16)		-
Fe σ_c	-	-	+0.2/+0.6	-1.4/-1.1
σ_{in}	+2.0/+2.8	-	-	-
σ_{tr}	+3.8/-2.1	+7.2/+8.7	-	-

Table 7 k_{eff} before and after Adjustment

(input s.d. of all experimental values: 1%)

	k_{eff}				s.d. of adjusted value, %
	Original NAPPMB Set	Corrected for High α_9 and low σ_{c40}	Adjusted Case 1	Adjusted Case 2	
ZPR-3/24	0.970		0.999	0.999	1.0
25	0.967		1.003	1.004	0.8
32	0.984		0.997	0.997	0.6
33	0.989		1.002	1.001	0.6
35	0.994		0.996	0.997	0.5
36	0.975		0.996	0.996	0.7
ZPR-6/5	0.975		0.993	0.992	0.5
6	0.981		0.991	0.991	0.5
SNEAK-3A1	0.989		0.999	0.999	0.4
3A2	0.987		0.997	0.996	0.5
SNEAK-2A	1.001		1.006	1.006	0.4
ZPR-3/48	0.995	0.989	1.005	1.004	0.4
49	0.990	0.985	1.000	0.999	0.5
50	0.997	0.984	1.000	1.000	0.5
53	1.012	0.993	1.003	1.003	0.6
ZPR-6/7	0.990	0.988	1.007	1.008	0.6
SNEAK-3B2	0.993	0.985	1.004	1.003	0.5
SNEAK-6A	0.992	0.984	0.996	0.997	0.5
SNEAK-7A	1.013	1.007	1.010	1.010	0.5
7B	0.988	0.987	1.005	1.005	0.5
ZEBRA-6A	0.984	0.977	0.987	0.987	0.5

Table 8 F9/F5 before and after Adjustment

Assumed s.d. of the experiment: 3%

	Experiment	(C-E)/E, %				s.d. of adjusted value, %
		Original NAPPMB Set	Corrected for High α_9 and Low σ_{c40}	Adjusted Case 1	Adjusted Case 2	
ZPR-3/24	1.16	+1.9	+1.9	+1.7	+1.8	1.4
25	1.17	+0.2	+0.2	-0.4	+0.1	1.4
32	1.20	+1.4	+1.4	+0.5	+0.6	1.3
33	1.21	+1.0	+1.0	-0.1	-0.1	1.4
35	1.09	-1.5	-1.5	-1.1	-1.1	0.8
36	1.19	+1.7	+1.7	+1.3	+1.4	1.3
ZPR-6/5	0.966	+2.8	+2.8	+3.6	+3.6	0.9
SNEAK-3A1	1.03	-3.1	-3.1	-1.9	-1.9	0.8
3A2	1.01	-0.5	-0.5	+1.5	+1.5	1.1
3A0	1.03	+0.2	+0.2	+0.4	+0.4	1.1
ZPR-3/48	0.976	-2.9	-2.2	-0.7	-0.7	0.9
49	0.986	-0.6	-0.1	+1.0	+1.0	0.9
50	0.903	-5.0	-4.3	-1.3	-1.3	1.6
ZPR-6/7	0.955	-6.3	-6.0	-4.2	-4.1	1.1
SNEAK-7A	0.977	-4.3	-3.8	-2.3	-2.3	1.1
7B	0.973	+0.2	+0.5	+1.1	+1.1	1.1
ZEBRA-6A	0.961	-2.0	-1.4	+0.4	+0.3	1.1

Table 9 F8/F5 before and after Adjustment

Assumed s.d. of the experiment: 5%

	Experiment			(C-E)/E, % (Case 1)				s.d. of adjusted value, %
	Case 1		Case 2	Original NAPPMB Set	Corrected for High α_9 and Low σ_{c40}	Adjusted Case 1	Adjusted Case 2	
ZPR-3/24	0.0353	a	0.0330	-8.2	-8.2	-0.6	+0.9	2.6
25	0.0337	a	0.0315	-8.3	-8.3	-0.8	-0.1	2.7
32	0.0521	a	0.0487	+1.3	+1.3	-0.1	+0.2	2.3
33	0.0556	a	0.0520	+5.4	+5.4	+4.2	+4.5	2.3
35	0.0343	a	0.0320	+1.5	+1.5	0	+0.7	2.1
36	0.0474	a	0.0443	-4.2	-4.2	+3.0	+2.5	2.1
ZPR-6/5	0.0253	a	0.0236	+4.4	+4.4	+8.2	+9.6	1.5
6	0.0229	b	0.0229	+1.7	+1.7	+3.6	-2.3	1.3
SNEAK-3A1	0.0318	a	0.0297	-2.5	-2.5	+0.2	+1.2	1.3
3A2	0.0300	b	0.0273	-1.3	-1.3	+1.2	+4.9	1.3
3A0	0.0331	a	0.0309	-1.5	-1.5	+1.4	+2.5	1.3
ZPR-3/48	0.0326	b	0.0307	-3.7	-2.5	+0.3	+0.5	1.4
49	0.0370	a	0.0345	-5.7	-4.6	-1.6	-0.4	1.4
50	0.0268	a	0.0251	-1.5	-0.7	-3.3	+4.2	1.5
ZPR-6/7	0.0220	b	0.0205	-6.4	-5.5	-3.8	+1.5	1.9
SNEAK-7A	0.0448	b	0.0448	-16.5	-15.4	-13.3	-18.0	1.5
7B	0.0328	b	0.0328	-8.2	-7.6	-3.9	-9.1	1.5
ZEBRA-6A	0.0390	a	0.0364	-5.6	-4.1	-1.5	-0.3	1.5

a) Measurement with fission chambers, increased by 7%
b) Measurement with foils

Table 10 C8/F5 before and after Adjustment

Assumed s.d. of the experiment: 5%

	Experiment	(C-E)/E, %				s.d. of adjusted value, %	
		Original NAPPMB Set	Corrected for High α_9 and Low σ_{c40}	Adjusted Case 1	Adjusted Case 2		
ZPR-6/6	0.1360	+11	+11	+3.6	+3.0	1.5	
SNEAK-3A1	0.142	0	0	-6.8	-7.5	1.6	
3A2	0.130	+4.6	+4.6	-1.1	-1.7	1.7	
ZPR-3/48	0.1436	-0.9	-0.4	-7.4	-8.0	1.5	a)
SNEAK-7A	0.1375	+8.2	+8.6	+0.7	+0.1	1.6	
7B	0.1312	+13.0	+13.3	+4.5	+3.9	1.8	
ZPR-6/7	0.1377	+6.7	+7.0	-0.1	-0.5	1.5	a)
ZEBRA-6A	0.140	+1.0	+1.4	-5.8	-6.3	1.7	

a) Corrected to the homogeneous case

Table 11 Ratio of Reactivity Worths $^{239}Pu/^{235}U$ before and after Adjustment

Assumed s.d. of the experiment: 5%

	Experiment	(C-E)/E, %				s.d. adjusted value, % (Case 1)
		Original NAPPMB Set	Corrected for High α_9 and Low σ_{c40}	Adjusted		
				Case 1	Case 2	
ZPR-6/5	1.45	-4.2	-5.5	-3.9	-3.8	1.0
6	1.37	-2.4	-4.4	-2.4	-2.4	1.2
SNEAK-3A1	1.29	+2.6	+0.4	+2.2	+2.2	1.0
3A2	1.295	+2.7	-0.4	+1.4	+1.4	1.1
3A0	1.53	-7.0	-7.8	-6.3	-6.3	1.4
SNEAK-2A	1.58	-5.3	-6.7	-5.1	-5.0	1.2
ZPR-3/48	1.32	+1.0	-1.5	+1.0	+1.0	1.0
49	1.45	-6.0	-7.8	-5.8	-5.8	1.0
50	1.21	+2.5	-2.0	+1.3	+1.3	1.5
53	1.27	+1.4	-5.5	-2.5	-2.5	2.4
ZPR-6/7	1.17	+2.5	+0.3	+3.2	+3.3	1.3
SNEAK-3B2	1.18	+1.2	-2.0	+0.8	+0.9	1.6
SNEAK-6A	1.38	-2.5	-4.1	-2.0	-1.9	1.1
SNEAK-7A	1.352	+1.2	-1.3	+1.1	+1.1	1.1
7B	1.34	-0.9	-2.1	-0.3	-0.2	1.3
ZEBRA-6A	1.395	-1.8	-4.7	-2.4	-2.4	1.1

Table 12 Ratio of Reactivity Worths $^{238}U/^{235}U$ before and after Adjustment (SNEAK Measurements). Assumed s.d. of the measurement: 10%

	Experiment	(C-E)/E, %				s.d. of adjusted value, %
		Original NAPPMB Set	Corrected for High α_9 and Low σ_{c40}	Adjusted Case 1	Adjusted Case 2	
SNEAK-3A1	-0.0675	+19	+19	+5.9	+7.4	2.4
3A2	-0.0634	+20	+20	+9.1	+10.5	3.0
3A0	-0.0643	+15.5	+15.5	-1.6	0	3.3
SNEAK-2A	-0.0640	+13.1	+13.1	-1.5	0	2.6
SNEAK-3B2	-0.0672	+ 8.3	+ 3.7	-5.2	-4.2	2.6
SNEAK-6A	-0.0607	+21	+17.3	+0.5	0	8.0
SNEAK-7A	-0.0515	+40	+29	+9.3	+10.8	3.7
7B	-0.0556	+25	+21	+8.5	+10.0	2.3

Table 13 Compilation of References for the Experimental Data

ZPR-3/24, 25, 32, 33, 35, 36	All data (including the critical mass of the equivalent sphere) are from Davey, Ref. 8
ZPR-3/48, 49, 50, 53	Most data are from Till, Ref. 10. The ratios F8/F5 (measured with foils) and C8/F5 in Assembly 48 are from Broomfield, Ref. 14. See also Little, Ref. 27. For reactivity worths, compare Ref. 28.
SNEAK-3A1	Barleon, Ref. 15
3A2, 3A0	Schröder, Ref. 16
3B2	Fischer, Ref. 17
	See also Edelmann, Ref. 18. The measured ratios F8/F5 were reduced to 89%, due to a new calibration, which was taken from Meister, Ref. 19.
SNEAK-2A	Helm, Ref. 20.
SNEAK-6A	Jourdan, Ref. 21. Reactivity worths are from SNEAK-6B.
SNEAK-7A, 7B	Böhme, Ref. 11. The measured ratios C8/F5 were increased by 4.5%, due to a new calibration (W. Scholtyssek, private communication).
ZPR-6/5	The criticality data and the fission ratios are from Karam, Ref. 22. The reactivity worths are from Karam, Ref. 23.
ZPR-6/6	Criticality Data: Karam, Ref. 24
ZPR-6/7	Criticality Data: Ref. 25
	Fission ratios and reactivity worths (for assemblies 6A and 7): Zolotar, Ref. 26.
ZEBRA-6A	All data are from Adamson, Ref. 29.

Table 14 Atom Densities of the Uranium Cores, 10^{20} at/cm^3

	ZPR-3						ZPR-6/5	ZPR-6/6[a]			SNEAK			
	24	25	32	33	35	36		Zone 1	Zone 2	Zone 3	3A-1	3A-2	3A-0	2A
^{235}U	36.34	34.42	44.45	44.50	19.49	44.98	15.4	11.50	10.7	15.6	20.31	20.31	20.31	18.76
^{238}U	349.9	356.0	3.2	3.3	1.4	237.6	105.6	57.96	57.90	57.8	81.02	81.04	81.02	65.57
Cr	14.3	14.0	124.5	97.7	76.1	19.5	23.9	27.88	32.5	31.1	36.6	36.5	36.6	35.6
Fe	56.6	55.5	491.7	386.1	300.9	77.0	90.4	141.0	157.2	152.4	123.1	122.0	123.1	121.1
Ni	8.6	8.4	74.7	58.6	45.6	11.7	11.3	13.65	15.8	15.2	19.0	18.5	19.0	18.6
Al											129.2	129.7	129.2	2.7
C							129.3	91.4	77.9	83.1	4.1	9.3	0	30.0
Na				40.0	78.2	40.1	92.0	146.5	146.5	146.5				85.1
O					39.8						144.7	145.3	144.7	90.4
H											7.40	17.92	0	0.22
Si											1.9	1.9	1.9	1.8
Mo												0.4		

a) Radius of Zone 1: 77.17 cm, Zone 2: 85.5 cm, Zone 3: 90.87 cm

Table 15 Atom Densities of the Pu Cores, 10^{20} at/cm^3

	ZPR-3				ZPR-6/7		SNEAK				ZEBRA
	48	49	50	53	Inner core	Outer core	3B-2	6A	7A	7B	6A
^{235}U	0.16	0.16	0.16	0.06	0.12	0.13	0.56	0.46	0.59	2.66	0.46
^{238}U	74.05	74.06	74.04	26.10	57.75	57.98	81.86	67.89	79.60	145.79	63.53
^{239}Pu	16.45	16.45	16.45	16.61	8.87	8.89	14.76	12.11	26.37	18.31	18.79
^{240}Pu	1.06	1.06	1.06	1.07	1.18	1.18	1.33	1.09	2.37	1.65	1.44
^{241}Pu	0.11	0.11	0.11	0.11	0.13	0.15	0.11	0.10	0.22	0.15	0.16
^{242}Pu	0.004	0.004	0.004	0.004	0.01	0.02	0.006	0.005	0.011	0.007	
Cr + Mn	26.4	26.1	18.9	19.0	29.2	29.1	36.0	32.3	23.5	28.3	12.7
Fe	101.8	100.8	73.0	73.3	130.6	136.0	119.8	119.1	79.7	98.0	45.25
Ni	11.2	11.2	8.0	8.0	11.7	11.7	17.5	22.7	11.7	14.6	4.4
Al	1.1	1.1	1.1	1.11	--	--	126.9	--	--	12.1	25.0
C	207.7	207.7	459.4	558.1	--	--	9.7	0.5	261.0	0.6	295.9
H	--	--	--	--	--	--	18.49	--	--	0.07	--
Na	62.3	--	--	--	92.8	91.5	--	84.8	--	--	44.7
Mo	2.06	2.06	2.06	2.08	2.25	2.27	0.4	--	0.25	0.27	--
O	--	--	--	--	137.8	146.6	122.2	120.0	218.5	331.9	--
Si	1.2	1.2	0.9	--	--	--	2.5	1.5	0.9	1.2	--
Cu	--	--	--	--	--	--	--	--	--	--	8.3

Table 16 Atom Densities of the Blankets, 10^{20} at/cm^3

Blanket type	A	B	C	D	E	F	G	H
^{235}U	0.91	0.43	0.85	0.83	1.94	1.62	0.81	0.83
^{238}U	399.8	190.1	400.5	391.0	268.1	399.4	399.3	400.8
Fe	44.0	114.5	42.21	49.2	32.5	39.6	43.2	42.3
Cr + Mn	11.2	29.0	12.65	12.7	9.1	11.9	11.4	12.15
Ni	6.7	17.3	5.09	5.3	3.2	9.8	5.4	5.7
C	--	--	--	--	234.1	--	--	--
Na	--	71.5	--	--	--	--	--	--
Mo	--	14.0	--	--	--	0.2	--	--
Blanket thickness, cm	30	30	30	30	30	30	30	30