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New Definition of Reliability, Continuous Lifetime Prediction, and Learning Processes

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NEW DEFINITION OF RELIABILITY, CONTINUOUS LIFETIME PREDICTION, AND LEARNING PROCESSES+)

by

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NEW DEFINITION OF RELIABILITY, CONTINUOUS LIFETIME PREDICTION, AND LEARNING PROCESSES

Abstract

Various methods to improve the reliability of redundant systems are discussed. It is demonstrated that the additional benefit obtained by adding a redundant component to a system decreases with the total number of components.

It is shown that if one is able to predict with high precision the time of failure of each component, the mean time to failure of the system may be significantly improved by replacing (or repairing) the components before they fail (preventative maintenance).

A general theory of the reliability of a device as a function of its characteristics, past history and expected operating conditions is introduced, which leads to a new definition of the concept of reliability.

The concept of "continuous lifetime prediction" (CLP) based on the comprehensive theory above is introduced and its basic principles are discussed.

The method of CLP consists in recording during operation of a device the stresses applied to it, in monitoring any useful and significant quantity (for instance vibrations, noise etc.), and in processing these data and those obtained from eventual tests of the device (during its operation and during its downtime) to predict continuously (or at regular time intervals) the remaining lifetime of the device.

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It is demonstrated that this technique would significantly improve the reliability and availability of devices and systems by making preventative maintenance more effective, as well as giving the benefit of greater knowledge.

Finally the adoption of the CLP method on a large scale gives the possibility of organizing learning processes where the knowledge of the manufacturer and user of a given device merge together. In these processes, which may be called "integrated learning processes", the data are produced in the user's plant and in the laboratory for lifetests and are stored and processed in the bank, which is a computer.

In the case of replaceable devices, new (not yet used) devices will be tested only initially in order to provide sufficient knowledge to render the devices operationable. Later, since the preventative maintenance policy will be currently adopted, lifetests will be carried out preferably on used devices, which were dismissed before failure after having been used for the allowed length of time. It turns out in this case that the learning process takes the form of a cycle. The cycle begins when a new device starts operation in the user's plant, and ends when the information gained from the lifetests, carried out in the laboratory on the same device (now called "used device"), reaches the user through the "bank".

NEUE DEFINITION DER ZUVERLÄSSIGKEIT, KONTINUIERLICHE LEBENSDAUERVORHERSAGE UND LERNPROZESSE

Zusammenfassung

Es werden mehrere Methoden zur Verbesserung der Zuverlässigkeit redundanter Systeme diskutiert, und es wird gezeigt, daß der durch Hinzufügen einer redundanten Komponente zu einem System erzielte zusätzliche Nutzen mit der Gesamtzahl der Komponenten abnimmt.

Außerdem wird gezeigt, daß die durchschnittliche Zeit bis zum Ausfall des Systems durch Austausch (oder Reparieren) der Komponenten vor dem Ausfall (präventive Wartung) beträchtlich verlängert werden kann, wenn der Zeitpunkt des Ausfalls jeder Komponente mit großer Genauigkeit bestimmt werden kann.

Eine allgemeine Theorie über die Zuverlässigkeit eines Bauelementes in Abhängigkeit von seinen Eigenschaften, seiner Vorgeschichte und den zu erwartenden Betriebsbedingungen wird eingeführt.

Diese Theorie führt zu einer neuen Definition des Begriffes "Zuverlässigkeit".

Das Konzept "kontinuierliche Lebensdauervorhersage", das auf der obengenannten umfassenderen Theorie beruht, wird eingeführt und in seinen Grundzügen erläutert.

Die Methode der kontinuierlichen Lebensdauervorhersage besteht in der Aufzeichnung der Beanspruchungen, denen ein Bauelement während

des Einsatzes ausgesetzt ist, der Überwachung jeder nützlichen und relevanten Größe (beispielsweise Schwingungen, Rauschen, u.s.w.) und der Verarbeitung dieser Daten sowie der Ergebnisse aus eventuellen Tests des Bauelementes im Einsatz oder im Abschaltzustand, um daraus die noch verbleibende Lebensdauer des Bauelementes zu bestimmen.

Es wird gezeigt, daß dieses Verfahren die Zuverlässigkeit und Verfügbarkeit der Bauelemente und Systeme wesentlich verbessern würde, da die präventive Wartung effektiver und der Vorteil erweiterter Kenntnisse dazukommen würde.

Schließlich gibt die Anwendung der Methode der kontinuierlichen Lebensdauervorhersage in großem Maßstab die Möglichkeit, Lernprozesse zu organisieren, in denen die Kenntnisse des Herstellers und des Benutzers über eine gegebene Komponente zusammenfließen. Die Daten werden in der Anlage des Benutzers und in dem Labor für Lebensdauertests erworben und in einer Datenbank, einem Computer, gespeichert.

Bei austauschbaren Komponenten werden neue (und noch nicht benutzte) Komponenten nur am Anfang getestet, um die für die Inbetriebnahme erforderlichen Kenntnisse zu erwerben. In Zukunft, wenn die Methode der präventiven Wartung häufiger angewandt werden wird, wird die Lebensdauer zu einem späteren Zeitpunkt vorzugsweise an gebrauchten Komponenten getestet, die vor dem Ausfall entfernt wurden, nachdem sie während der zulässigen Zeit im Einsatz waren. Es zeigt sich, daß der Lernprozess in diesem Fall wie ein Zyklus abläuft. Der Zyklus beginnt, wenn eine neue Komponente in der Anlage des Benutzers in Betrieb genommen wird, und endet, wenn die Informationen, die aufgrund der im Labor an dieser Komponente (nun "gebrauchte Komponente") durchgeführten Lebensdauertests erworben wurden, den Benutzer über die "Datenbank" erreichen.

Contents

1. Introduction

Modern technology continuously asks for higher and higher degrees of reliability. Clear examples are the nuclear, airplane and rocket industries, where reliability plays a vital role. No one infact would let a company build a nuclear plant, if it is not possible to demonstrate that the probability of an accident is below the threshold of acceptance. No one would fly in an airplane, if they were not sure that the probability of an accident during the trip was so low, that they would be prepared to accept this risk against the advantage of considerable less travel time than by other more traditional means, such as a train or ship.

The request of reliability is also dictated by economical considerations, which are extremely important to modern society. If one thinks for example of the enormous economical loss due to the unplanned shut down of a 1000 MW electric nuclear power plant for only a few hours, one would immediately recognize that the plant must be designed and operated in such a way that the probability of an unwanted shut down is very low.

A common practice used to improve the reliability of a plant is to build in it redundant systems, which operate only if a given number, above a minimum, of their components is able to operate.

Fig. 1 shows a schematic diagram of a redundant system made with "n" similar components, which functions if at least one component is able to operate. When a component fails, it is repaired (or substituted by a new one) and again put into operation.

Fig. 2 shows the mean time to failure (MTTF) of the system of fig. 1 as a function of the number "n" of its components. The curves of fig. 2 are characterized by the parameter "α", defined as the ratio between the mean time to failure (MTTF) of a component and its mean time to repair (MTTR).

For a given "α", the MTTF of the system increases, in general, with the number "n" of the components, but always less and less, so that, above a certain number of components, it remains practically constant. This means that the gain factor of the system MTTF produced by the last added component decreases with "n", as it is shown more clearly by the curves of fig. 3.

For instance, let us consider the case $\alpha = 200$ of fig. 3. The first redundant component (n = 2) increases the MTTF of the system by a factor of 101.5, the second (n = 3) by a factor of 67 and the third (n = 4) by a factor of about 50, and so on. This shows that redundancy is rewarding only if the number of the components is low, especially if one considers that the cost of the last added component is equal (if not higher because of the more complicated installation problems) to that of the other components.

In addition there are engineering problems (like limited available space, greater complications for the connections between the various components, and for the alarm system which indicates the failure of each component), so that redundancy is usually limited to only few components (2 or 3), especially if these are large in size and expensive (like the Diesel generators for an emergency power supply system). If now one wants to further improve the MTTF of the system, one has two choises:

1. To decrease the MTTR, t_r, of each component
2. To increase the MTTF, t_f, of each component

A reduction of t_r increases α , which in turn increases the MTTF of the system as clearly shown in fig. 2. However this method also has limitations, because there are dead times which cannot be eliminated (like the time for the repairing crew to reach the point where the repair has to be carried out, and the time needed to find out the part of the component, which has actually failed), and because it may be physically impossible to further decrease the effective repair time, after the failed part of the component has been identified.

If we indicate with "t_s" the mean time to failure of the system, for large values of "α" and for n << α we have (ref. 1),

$$t_s \cong t_f \frac{\alpha^{n-1}}{n!} = \frac{1}{n!} \frac{t_f^n}{t_r^{n-1}} \qquad (1)$$

If we consider for instance the case n = 3, we get from eq. 1 that if t_r is decreased by a factor of 10, t_s increases by a factor of 100, while if t_f is increased by the same factor of 10, a gain in t_s as large as 1000 results.

This simple numerical example shows that the incentive to increase the MTTF of each component belonging to a system may be even greater than that to decrease the MTTR of the same components. In order to increase the MTTF of a component, one may decide to design a stronger component, but this method may reveal to be either too expensive, or even ineffective if the design has already reached the boundaries of the technology, which is being used. For instance, if one has to design a cylindrical vessel to contain a gas at a given pressure, it can be shown that to increase the wall thickness of the vessel becomes practically ineffective above a certain ratio between the wall thickness and the radius of the vessel. One could of course improve the situation by looking at a better material for the vessel, but this would imply the use of a new technology.

There is however a second method to improve the MTTF of a component, and this consists of predicting the time at which the component is going to fail and in carrying out the necessary repairs, before it fails.
This method is commonly called "preventative maintenance".

Subject of this paper is to show that it is possible to make this preventative maintenance much more effective, by continuously predicting during operation the remaining lifetime of the components.

This method has been called "Continuous Lifetime Prediction" method (CLP), and it is described and discussed in the following sections.

The method of CLP consists in recording during operation the environmental stresses (such as temperature, pressure etc.) applied to a device, in monitoring any useful and significant quantity (for instance vibrations, noise etc.), and in processing continuously the data obtained from these measurements and from eventual tests of the device (during its operation and during its downtime) to predict the remaining lifetime of the device.

Some examples should clarify better the above definition. If we have for instance a mechanical structure which is under creep, we may record continuously the temperature of this structure and the load applied to it. This data may be used as input to a theoretical model, which describes the creep, to predict the remaining lifetime of the structure.

In the same way in the case of the under carriage or of the wings of an airplane, we may predict more precisely their time of failure due to fatigue if we would use the information obtained by recording the stresses applied to them and their vibrations, which are both stochastic in their nature. Another example may be that of the ball bearings of a pump. The acoustic vibrations may be monitored and, if at a certain frequency range the amplitude exceeds a preestablished level, it follows that the bearing is near to failure and is therefore replaced by a new one (ref. 5).

One may also think of testing during operation from time to time a relay belonging to a redundant system of relays, and decide on the basis of the information gained from the test whether or not to replace the relay with a new one.

In all the four above examples a continuous (or semi continuous) estimation of the remaining lifetime of the device is carried out, which serves as basis for the decision whether or not a preventative repair (or replacement) should be carried out.

The theory of "Continuous Lifetime Prediction", developed in this paper, should greatly assist one in making these preventative maintenance decisions. This is true because it provides the mathematical tool and basic insight necessary for making these decisions.

In addition, as we shall see in the following section, the analysis leads us to the conclusion that the reliability of a device depends upon the degree of knowledge that one has of its characteristics, and of the processes which take place during operation. A new definition of reliability is therefore proposed, and it is given in section 2.

Finally it must be pointed out that the CLP method allows one, during operation, to produce additional information about the device lifetime, which has the same value as that produced by laboratory tests. This is true because the operating conditions are continuously recorded, so that at the end one knows them exactly like it happens in the case of the laboratory tests, where these conditions are controlled. After operation, the device can be made to fail, and the information gained in this way can be used to increase the knowledge of the device's characteristics and of the processes which have occurred. This gives the possibility of organizing "learning processes", as it will be shown in section 6.

Before closing this introduction we want to inform the reader that in this paper capital letters have been used to indicate the random variables and small letters to indicate a specific value or a specific realization of the same random variable. We shall indicate for example:

$S(t)$ = random variable function of time

$s(t)$ = a realization of the random variable $S(t)$

The application of the methods suggested in this paper must be considered in a long time perspective. High levels of knowledge are required, which in turn require highly sophisticated procedures for producing, transferring and processing information.

In the case of nuclear power plants the requirements for safety are so stringent that they are becoming to be developed. However a long way is in front of us before the proposed methods can be used on a large industrial scale.

2. Fundamentals and new definition of reliability

"The failure of a device occurs because of natural laws, and not because blind fate randomly chooses a group of devices and orders them to fail.

Nevertheless the field of reliability has developed as an application of the statistics and of the theory of probability.

The characteristics of a device degrade with time because of some basic chemical, physical or metallurgical processes, which have a known or measurable dependence upon stresses such as temperature, pressure, electrical voltage etc." (Ref. 6).
For this reason one may expect to be able to calculate the exact time of failure of a specific device, if he could know exactly the state of the device, its behaviour under given operating conditions and how these conditions would develop in the future. The reliability would be in this case a discontinuous function of the time ("ideal case" in fig. 4) with the discontinuity at the time of failure. This may be called a deterministic model of the time of failure.

Since we have a limited knowledge of all the facts, we are bound to calculate a spectrum of possible times of failure, and to associate with them a probability density distribution, which is the probability of occurence of failure within an infinitesimal time interval.

In this case the reliability would be a continuous function of time ("real case" in fig. 4).

We now want to formulate a mathematical model for calculating the reliability of a device, starting from its characteristics and operating conditions.

For this reason let us first consider the concept of failure.

We shall say that a device has failed, if it no longer can fulfill the task (for which it was built and installed in the plant) with that degree of accuracy which was foreseen when the plant was designed. Failure is here being used in a rather general context which includes, as one special case, the rupture of a pressure vessel where the term is universally accepted. In general we shall say that an electric resistance has failed, if its value exceeds the limits which are not supposed to be exceeded for a correct operation of the electric circuit in which the resistance is operating.

In the same way we may say that a cylindrical tube has failed if its ovality exceeds the limits which are not supposed to be exceeded for a correct operation of the system to which the tube belongs.

This definition of failure entails the concept of the comparison between the capability (strength) of the device to fulfill its task and the minimum (or maximum) value (reference or load) which the strength can take at the failure. In the case of a pressure vessel, the task is to contain the energy released by an explosion, so that we shall say that the vessel has failed when it breaks. In this example "strength" is understood to be the normal mechanical engineering definition, and "reference" is the minimum allowed value of strength before rupture.

In the case of an electric resistance the "strength" is the value of the resistance and the "reference" is, for example, the maximum value which the resistance is allowed to take before an abnormal behaviour in the circuit of the resistance occurs.

The strength of a device will change in general with time, because the stresses due to environmental conditions (such as temperature, pressure, electric voltage etc.) may produce degradation of the device properties, which reduces the value of the strength. We shall call this change "permanent loss of strength". A change of the strength may also occurr which is "not permanent". This means that this change is cancelled out, if the stresses again take their initial values.

For a given device, we may therefore identify the following quantities

M = reference or load

Y = initial strength evaluated at design conditions (that is with all the stresses and the reference taking specific values, which are called design values)

L = permanent loss of strength

C = non permanent change of strength

The quantities "M", "L", and "C" may be predictable or stochastic functions of time, while "Y" does not depend upon time. The device will function correctly as long as

$$Y - L + C - M > 0 \qquad (1)$$

The time of failure is the minimum real and positive value of the roots of the following equation

$$Y - L(t) + C(t) - M(t) = 0 \qquad (2)$$

The quantity Y-L in eq. 2 may be called "strength evaluated at design conditions". It is useful to introduce the ratio "N" between the "non permanent change of strength C" and "Y-L", that is

$$C = N\ (Y-L) \qquad (3)$$

In addition we define the quantity "X"

$$X = \frac{M}{1+N} \qquad (4)$$

which we call "effective reference" or "effective load".

Taking into account eqs. 3 and 4, eq. 2 becomes finally

$$Y - L - X = 0 \qquad (5)$$

Eq. 5 contains three quantities, "Y", "L" and "X".

"Y" is a characteristic of the device, which is constant with time, and it is a function of the fabrication process.

"L" is a quantity which depends upon the device's past history (from the time at which the device is put into operation (t=o) until time "t").

"X" is a quantity which depends almost entirely upon the values of the environmental stresses and the reference at time "t". The statistical properties of Y and L may of course influence X (because of N), but this may be considered as a second order effect.

Returning to eq. 5, we can now point out the following

A) If Y is known exactly (Y=y), and L and X are both predictable functions of time, eq. 5 becomes

$$y - \mathcal{L}(t) - x(t) = 0 \qquad (6)$$

where $\mathcal{L}(t)$ and x(t) indicate predictable functions of time respectively for L and X.
The minimum real and positive value of "t" which solves eq.6, is the exact value of the time of failure "t_f".

B) If Y is not known exactly, and/or L or X are not predictable functions of time, one finds that the solution of eq. 5 is a random value "T_f" characterized by a probability density distribution, and it is not a particular value "t_f".

The most general case will be that in which Y is a random variable and both "L" and "X" are stochastic functions of time.
In general we have to solve the stochastic equation

$$Y - L(t) - X(t) = 0 \qquad (7)$$

One may solve eq. 7, by considering a large number of randomly chosen combinations of realizations y; $\ell(t)$ and x(t) of Y; L and X, and by solving the resultant deterministic equations (which are similar to eq. 6). For each deterministic equation, one could find the exact solution "t_f", to which one can associate the corresponding value of the probability density distribution obtained by calculating the frequency of occurrence of each value of "t_f". From this distribution one can easily calculate the reliability "R(t)" that is the probability that the sum of three random variables, namely Y; L and X, is larger than "0" during the whole time interval between "0" and "t".

$$R(t) = P\{ Z > 0 \quad \text{during the whole time interval until "t"} \} \qquad (8)$$

where

$$Z = Y - L - X = \text{Margin of strength} \qquad (9)$$

and "P {...... } " indicates probability.

We can now discuss the definition of reliability. Other authors (ref. 7) have already pointed out that the usual definitions of reliability are unsatisfactory. Barlow and Proshan (ref. 3) in their book, "Mathematical Theory of Reliability" write that "the definition of reliability given in the literature are sometimes unclear and inexact and vary among different writers". At the end they choose the following definition given by the "Radio Electronics Television Manufacturers Association" in the year 1955.

"Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered".

This definition is incomplete because not all the conditions are specified, and it does not seem adequate because it does not state clearly that the reliability depends upon the knowledge that the estimator has of the phenomena which occur during device operation. In addition the expression "operating conditions encountered" is very vague and may lead to different interpretations.

Let us consider the case in which a population of devices is tested under controlled operating conditions, that is the environmental stresses and loads are predictable functions of time. An estimator, who is supposed to know in advance the strength distribution of the population, by solving the stochastic equation 7, would calculate the exact distribution of the time of failure measured during the experiment.

Let us now consider the case in which the operating conditions are not controlled. The two extreme subcases exist. Each member of the population is tested under the same stochastic stresses and loads (1) or the applied stresses and loads are derived from the same probability distributions but are uncorrelated (2). For a better understanding consider the following example. Electrical capacitors are operated in parallel under a stochastic voltage (stress), which is identical for all capacitors, and which is artificially produced by means of some probability distributions and a sequence of random numbers. This corresponds to subcase 1. Subcase 2, instead, corresponds to the experiment in which each capacitor is operated independently under a separate stochastic voltage. All the voltages have the same probability distributions, but are produced by different random number sequencies. The two distributions of time of failure which result from the two experiments will be in general different. However the estimator at the initial time, by solving the stochastic eq. 7, is able to calculate the distribution of only subcase 2. In fact he cannot make a distinction between the two subcases, because in subcase 1 he does not know yet which one among all the possible realizations of the stress will occurr.

This indicates that in the most general case it is more appropriate to look for a definition of reliability, which is linked to the degree of knowledge that the estimator has of the device's characteristics, its past history and expected operating conditions. Returning to the case of controlled operating conditions, one can consider the specific device as an individual, and say that its characteristics (for instance its strength) are known to him not exactly but with uncertainty. For example one can regard strength as a random variable "Y-L" characterized by a probability density distribution, which was measured during the past by means of lifetime tests carried out on devices very similar to the specific device which is being considered. This function is therefore the knowledge that the estimator has of the device strength, which is used in eq. 7 to calculate the time of failure. The only alternative to this procedure is that of testing the specific device directly. This would produce the exact knowledge of the time of failure of the device. But this procedure would imply the destruction of the device, before it can even be used. This means that the exact knowledge of the device's characteristics prior to the operation of the device is impossible to obtain, and that we are bound to assume that the specific device in question will behave in a way similar to that of the other devices which were similarly fabricated and installed, and which previously failed. This is true as long as no technical method exists to make a distinction between the various devices belonging to a given population, without destroying each member of the population. Non destructive preoperational tests usually will only provide more confidence that the device has the same characteristics of the population which was previously operated or tested. The probability density functions of this population can then be applied to calculate the reliability of the device.

The above way of thinking brings us therefore to the conclusion that reliability must be explicitly defined in terms of the knowledge that the estimator (a person or a machine) has of the phenomena which occur during device operation. The proposed new definition follows.

"The reliability of a device "R(t)" is the probability that the device will perform the required function up to the time "t". This probability is a function of the degree of knowledge of the device's characteristics, of its past history and of the expected operating conditions".

The definition of reliability proposed in this paper seems to be general because it covers all the cases, concise because it is contained in only two short sentences, exact because it is based on a well defined mathematical model, complete because all the conditions are specified so that only one interpretation is possible, and finally clear because it eliminates the misunderstandings between statisticians and engineers, by stating that the probability enters into the picture only as a means to measure the lack of knowledge of the physical processes which are taking place.

Let us now consider the failure rate h(t)

$$h(t) = -\frac{dR/dt}{R(t)} \tag{10}$$

which may also be defined as the following conditional probability

$$h(t) = \lim_{dt \to 0} \frac{1}{dt} P \left\{ Z < 0 \text{ between } t \text{ and } t + dt \;\middle|\; Z > 0 \text{ during the whole time until "}t\text{"} \right\} \tag{11}$$

In eq. 11 the hypothesis is included that the evaluation of "h" is done at the initial time. In general the device will be replaced (or repaired), when the failure rate reaches a given upper level, above which the operation of the device is considered unsafe. Since the calculation of "h(t)" is performed at the initial time, this decision of the time of repair is taken at that time. Now it usually happens that the time, at which this upper level is reached, is much larger than the time Δt needed to carry out the necessary repair (for instance one year against few days). The repair time "Δt" is intended here to also include planning time for maintenance.
One therefore needs to take the decision only at the time "$t - \Delta t$", which is much nearer to "t".
This means that the failure rate may be evaluated at the time "$t - \Delta t$" and it will therefore be a function of both "t" and Δt.

$$h(t-\Delta t;t) \lim_{dt \to 0} \frac{1}{dt} P \left\{ \begin{array}{l} Z < 0 \text{ between "t" and "t+dt"} \,\middle|\, Z > 0 \text{ during the} \\ \text{whole time from } t - \Delta t \text{ until "t" and that the} \\ \text{calculation is carried out at } t - \Delta t \end{array} \right\} \qquad (12)$$

From eq. 12 we get

$$\left[h(t-\Delta t;t) \right]_{\Delta t = t} = h\ (0;t) \qquad (13)$$

which is equal to eq. 11, because it indicates that the evaluation of the failure rate is carried out at the initial time.

The advantage of adopting eq. 12 (CLP method) is that now the estimator can use for its calculations all the additional information obtained from the operating past history until the time $t - \Delta t$. We shall see (sect. 4) that the use of this additional information allows one to increase the operating time of the device.
The adoption of eq. 12 implies the continuous recording of the environmental stresses and of the loads applied to the device, the continuous monitoring of some significant quantities (for instance vibrations, noise etc.), and eventually the carrying out of tests from time to time of the device.
The data obtained in this way must then be continuously processed to calculate at each time the probability density distribution of the remaining lifetime of the device. The failure rate h $(t-\Delta t;\ t)$ can be calculated from this

distribution.
For this reason we have called this method "Continuous Lifetime Prediction". Note that the word "continuous" must not be understood "ad litteram". It may suffice that all these measurements and calculations are carried out at time intervals sufficiently small.
We consider again the example of the electrical capacitors which are all operated under the same stochastic voltage (subcase 1), and we suppose now that two persons, which we call respectively A and B, are requested to predict at each time the number of capacitors which will fail in the next time interval "dt". Estimator "A" is assumed to know only the statistical characteristics of the population of capacitors and the stochastic properties of the voltage. Estimator "B" is assumed to be also continuously informed of the voltage applied to the capacitors. Estimator "A" will make his prediction at the initial time and will never change it up to the time "t", because his knowledge will remain unchanged. Estimator "B", instead, will wait until "t" to make his prediction, because at that time he will know all the past history of the electrical voltage until "t" and will take advantage of this additional information in his calculation. For this reason his prediction will be more precise than that of "A". At the initial time therefore, "B" cannot say what his prediction will be, but he can surely say that his prediction, whatever it will be, will be better than that of "A". This leads us to the definition of the failure rate of a device again in terms of the knowledge that the estimator has of the phenomenona which occurr during device operation. The proposed new definition follows.
"The failure rate of a device is the limit of the ratio between the calculated conditional probability that the device with age "t" will fail in the subsequent time interval "dt" and the same "dt" for "dt" tending to zero. This limit is a function of the degree of knowledge of the device's characteristics, of its past history and of the expected operating conditions."
This definition also takes into account the different predictions about the failure rate at a given time which one gets by carrying out the estimations at different times. These estimations will, in general, be different, because the degrees of knowledge associated with each of them will also, in general, be different.

Before closing this paragraph we want to point out that we have purposely limited ourselves to the case in which only one mode of failure can take place in the device.
Some devices have more than just one mode of failure. One should then write a separate equation (such as eq. 7) for each mode of failure. These equations may be eventually correlated in some manner.
In order to avoid mathematical complications, but without any loss of generality, the case of only one mode of failure is developed in this paper.

3. The statistical properties of the initial strength, permanent loss of strength and effective reference

The initial strength "Y" of a device is a random variable, which is characterized by a probability density distribution, $\varphi(y)$, which is measured by testing similar devices produced by means of the same fabrication process. The effective load (or effective reference), X, may be considered as a stationary stochastic function of time.
The permanent loss of strength "L" will be instead a monotonically increasing stochastic function of time.
Let us now consider the rate, V, of the permanent loss of strength

$$V = \frac{dL}{dt} \qquad (1)$$

This rate V will, in general, be a function of time, of some stresses, S_n, and of X;

$$V = V\ (t;\ S_1\ \ldots\ldots;\ S_n;\ X) \qquad (2)$$

Without any loss of generality, we restrict ourselves to the case of only one stress acting on V

$$V = V\ (t;\ S) \qquad (3)$$

The functional link between "V" and the independent variables "t" and "S" is measured by means of laboratory lifetime tests, where the stresses and the load are controlled, that are predictable functions of the time (in most cases constant with time).

In order to evaluate the statistical properties of V (and consequently those of L) we need to know the properties of S. The stress S (like X) may also be considered as a stationary stochastic function of the time. Fig. 5 shows a realization of S as a function of time. We may approximate such a function by means of a sequence of rectangular pulses, each pulse terminating when the next starts (fig. 5). Amplitude "S" and duration "T" of each pulse are random. The following probability density distributions are introduced for S and T;

$\psi(s_1;s)$ = probability density distribution for the transition from the state S = s_1, to the state S = s, when an elementary pulse takes place.

$\lambda(s)\, e^{-\bar{t}\lambda(s)}$ = probability density distribution for the waiting time at the state S = s.

where

$\bar{t}$ = time measured from the beginning of the pulse

$\lambda(s)$ = inverse of the average waiting time at state S = s

For the sake of simplicity, we shall limit ourselves in this paper to the case

$$\psi(s_1;s) = \psi(s) \tag{4}$$

$$\lambda(s) = \lambda = \text{const} \tag{5}$$

The more general case is treated in ref. 2.

The statistical properties of S (and of X), that is " $\psi(s)$" and "λ", can be obtained by analyzing a realization of S, and by making use of the properties of the ergodic functions. This is shown in Appendix 1.

The statistical properties of the permanent loss of strength "L" can now be evaluated from those of the rate "V".
We have from eq. 1

$$L(t) = \int_0^t V(S,t')\,dt' \tag{6}$$

We can develop V(s;t) in a Taylor's series with respect to the time "t"

$$V(S,t) = \sum_{n=0}^{\infty} A_n(S) \cdot t^n \tag{7}$$

For the sake of simplicity we shall limit ourselves in this paper to the case n = o (linear kinetics). The more general case of eq. 2 is treated in ref. 2. We can therefore write

$$V(s;t) = A_o(S) = V(S) \tag{8}$$

We can now calculate the probability density distribution of L. This has been done in Ref. 2. Here we have limited ourselves to calculate only the average value of "L" and its variance (Appendix 3)

$$\bar{L} = \bar{V}t \tag{9}$$

$$\sigma_L^2 = \frac{\sigma_V^2}{\lambda^2}\left(\lambda t + e^{-\lambda t} - 1\right) \tag{10}$$

where

$\bar{L}$ = average value of L

$\bar{V}$ = average value of V = $\int_0^{\infty} V(s)\,\varphi(s)\,ds$ (11)

σ_L^2 = variance of L

σ_V^2 = variance of V = $\int_0^{\infty} [V(s) - \bar{V}]^2 \varphi(s)\,ds$ (12)

Higher moments of "L" can be calculated by procedures similar to that shown in Appendix 3.

However for the sake of simplicity we have assumed in this paper the Gamma distribution as an appropriate approximation of the exact probability density distribution $g(\ell;t)$ of "L" and therefore (ref. 4),

$$g(\ell;t) \cong \frac{1}{\beta^{\alpha+1}\,\Gamma(\alpha+1)}\,\ell^{\alpha} e^{-\ell/\beta} \tag{13}$$

The parameters α and β must be chosen in such a way, that the average value and the variance of the Gamma distribution satisfy respectively eqs. 9 and 10. We have therefore,

$$\beta(\alpha+1) = \bar{V}\cdot t \tag{14}$$

and

$$\beta^2(\alpha+1) = \frac{\sigma_V^2}{\lambda^2}\left(\lambda t + e^{-\lambda t} - 1\right) \tag{15}$$

which can be solved to obtain α (t) and β (t).

Returning to eq. 10, we see that " σ_L^2 " is given by the product between a constant σ_V^2/λ^2 and a function of the time "f(t)"

$$f(t) = \lambda t + e^{-\lambda t} - 1 \tag{16}$$

This function is shown in fig. 6. It starts with the value "O" at time t=0, and it tends to increase first with the square of the time and later linearly for large values of time.

This suggests the following.

If one records the stress S up to the time "$t - \Delta t$" at which the evaluation of "L(t)" is carried out, the function "L" up to this time can be evaluated exactly.
This means that the variance of L at time "$t - \Delta t$" is zero.
In the case of linear kinetics the variance σ_L^2 of $\Delta L = L(t) - L(t-\Delta t)$ will be given by

$$\sigma_L^2 = \frac{\sigma_V^2}{\lambda^2} f(\Delta t) \tag{17}$$

where

$$f(\Delta t) = \lambda \Delta t + e^{-\lambda \Delta t} - 1 \tag{18}$$

This means that, if one records the stress S, one is in a position at time $t - \Delta t$ to reduce the value of σ_L^2 (eq. 17) because one uses the additional information recorded during the time interval between "0" and $t - \Delta t$.

The same reasoning may be applied to the calculation of the higher moments, and we can conclude that the distribution of $\Delta L = L(t) - L(t-\Delta t)$ calculated at the time $t - \Delta t$ is given by

$$g(\ell; \Delta t) \simeq \frac{1}{\beta^{\alpha+1}\,\Gamma(\alpha+1)}\, \ell^{\alpha} e^{-\ell/\beta} \tag{19}$$

with

$$\beta(\alpha+1) = \bar{V}\cdot\Delta t \tag{20}$$

and

$$\beta^2(\alpha+1) = \frac{\sigma_V^2}{\lambda^2}\left(\lambda\,\Delta t + e^{-\lambda\Delta t} - 1\right) \tag{21}$$

Eq. 19 gives the distribution of "ΔL" in the case that the rate V at time $t - \Delta t$ is unknown. If we suppose now that the stress S is known at that time ($S(t - \Delta t) = s$), the rate of loss of strength will be also known ($V(t - \Delta t) = v$), and the conditional density distribution $\rho(\ell;\Delta t; v)$ defined by eq. 22, will be given by eq. 23 (see also Appendix 4)

$$\rho(\ell;\Delta t, v) = P\left\{\Delta L = \ell \ \text{ at } t \,/\, V(t-\Delta t) = v\right\} \tag{22}$$

$$\rho(\ell;\Delta t; v) = e^{-\lambda\cdot\Delta t}\,\delta(\ell - v\Delta t) + \lambda\int_0^{\Delta t} e^{-\lambda t_1}\, g(\ell - v t_1; \Delta t - t_1)\,dt_1 \tag{23}$$

where "δ" indicates the impulse function.

It is interesting to point out the particular case

$$\text{For } \lambda\cdot\Delta t \ll 1 \qquad \rho(\ell;\Delta t; v) \longrightarrow \delta(\ell - v\Delta t) \tag{24}$$

The following equation must be satisfied

$$\int_0^{\infty} \varrho(\ell; \Delta t; v) \cdot q(v) \, dv = g(\ell; \Delta t) \qquad (25)$$

where

q(v) = probability density distribution of V

Eq. 25 is of course satisfied only in the case in which the exact distribution $g(\ell; \Delta t)$ is used in eq. 23. The demonstration is given in ref. 2.

In the case of the CLP method the conditional probability density distribution $\varrho(\ell; \Delta t; v)$ is used to calculate the failure rate. In the case instead "without CLP" the probability density distribution $g(\ell; t)$ must be used, because the rate of the permanent loss of strength is unknown.

4. The particular case of constant reference

In the case of constant reference (X = x = const.), eq. 5 of section 2 is reduced to

$$Z = Y - L - x = 0 \tag{1}$$

Since Y does not depend upon the time and L is a monotonically increasing function of the time, we can write for the failure rate

$$h(t-\Delta t;t) = \lim_{dt \to 0} \frac{1}{dt} P \left\{ \begin{array}{l} Z < 0 \text{ between } t \text{ and } t + dt \,|\, Z > 0 \text{ at } t \text{ with} \\ \text{the calculation being carried out at } t - \Delta t \end{array} \right\} \tag{2}$$

If Y and L are statistically independent, we have in the case of linear kinetics

$$h(t-\Delta t; t) = - \frac{\displaystyle\int_{y_{min}}^{y_{max}} \varphi(y) \cdot \left[\int_0^{y-(\ell+x)} \frac{\partial p(\ell'; \Delta t; v)}{\partial \Delta t} \, d\ell' \right] dy}{\displaystyle\int_{y_{min}}^{y_{max}} \varphi(y) \left[\int_0^{y-(\ell+x)} p(\ell'; \Delta t; v) \, d\ell' \right] dy} \tag{3}$$

where

y_{min} = minimum value of the random variable Y estimated at the time $t - \Delta t$

y_{max} = maximum value of Y

v = value of V estimated at the time $t - \Delta t$

ℓ = value of L estimated at $t - \Delta t$

Let us look at fig. 7. At the time $t - \Delta t$, if the device has not yet failed, we must have

$$Y > y_{min} \tag{4}$$

with

$$y_{min}(t-\Delta t) = x + \ell(t-\Delta t) \qquad \text{if } y_{min} > y_{min}(o) = y_o \tag{5}$$

otherwise

$$y_{min} = y_o \tag{6}$$

where $\ell(t-\Delta t)$ is the known permanent loss of strength calculated at "$t-\Delta t$".

This explains why the density distribution of Y at the time $t-\Delta t$ is given by

$$\frac{\varphi(y)}{\int_{y_{min}}^{y_{max}} \varphi(y)\,dy} \tag{7}$$

We introduce now in eq. 3 for $\rho(\ell;\Delta t;v)$ the expression given by eq. 23 of section 3. This has been done in the Appendix 5.

In this section we consider the expressions of only two particular cases

1st Case Without CLP

$$\bar{k}(0;t) = \frac{\int_{y_o}^{y_{max}} \varphi(y)\left[\int_0^{y-X} \frac{\partial g(\ell;t)}{\partial t}\,d\ell\right]dy}{\int_{y_o}^{y_{max}} \varphi(y)\left[\int_0^{y-X} g(\ell;t)\,d\ell\right]dy} \tag{8}$$

2nd Case With CLP and $\Delta t \ll \frac{1}{\lambda}$

$$k(t-\Delta t;t) = \frac{\varphi(y_{min}+v\cdot\Delta t)}{\int_{y_{min}+v\Delta t}^{y_{max}} \varphi(y)\,dy}\, v \tag{9}$$

We want now to compare the results obtainable by applying eq. 8 with those obtainable by applying eq. 9.

Since the values which "y_{min}" and "v" will take at the time "t" are not known at the initial time, the failure rate defined by eq. 9 is stochastic. This means that the minimum time, at which this failure rate reaches a preestablished value h_1 satisfies the stochastic equation

$$V \frac{\varphi\left(Y_{min} + V \cdot \Delta t\right)}{\int_{Y_{min}}^{y_{max}} \varphi(y)\, dy} = h_1 \qquad (10)$$

We shall therefore speak of a distribution of the minimum time at which the level h_1 is reached.

If we choose $y_o = x$, eq. 6 will always apply.

Eq. 10 has been calculated in the special case $\Delta t = o$. The numerical values of the parameters are given in Appendix 6. The results are shown in fig. 8 (curve 2, with CLP). Here the time in abscissa is the expected time (probabilistic) at which the level "h" of failure rate is reached for the first time.

The case "without CLP" given by eq. 8 is also shown in fig. 8 (curve 1). In this case the time on the abscissa is the deterministic time. In fig. 8 the time is measured in absolute units.

We set now the maximum acceptable level of the failure rate at 10^{-7} per unit of time, because the operation above that level is considered to be unsafe. Referring to the curve 1 (without CLP), we say that we shall replace the device at time 0.43, because initially we calculate a failure rate which at time 0.43 will reach the level of 10^{-7} per unit of time.

Referring to the curve 2 (with CLP) we say instead that we shall probably replace the device at time 0.78, because initially we expect that we shall calculate at that time a failure rate which reaches the level of 10^{-7} per unit of time.

We conclude therefore that the time of replacement is deterministic in the case "without CLP" and probabilistic in the case "with CLP".

On the other hand we shall have instead a probabilistic loss of strength (degree of wearout) at the time of replacement in the case "without CLP", because we calculate only once and initially the expected loss of strength at the time of replacement.
In the case "with CLP" we shall have instead a deterministic degree of wear-out, because we shall calculate at each time the exact loss of strength which has already occurred.

We have seen from the curves of fig. 8 that, for a given level of the failure rate, the use of the CLP method may increase considerably the operating time of the device (from 0.43 to 0.78). This is due only to the fact that with this method one has a better knowledge of the state of the device and one can therefore more completely utilize the device's strength.

In order to better understand this point, let us look at fig. 9. At time t=0 we have a density distribution $\varphi(y)$ with an average value $\overline{Y}$ and a variance σ_y^2. If we now consider the case without CLP, the distribution of Y-L at time $t > 0$ will be brooder than $\varphi(y)$, because the variance now includes that due to L, which, as seen in section 3 (fig. 6), increases with time.
In the case "with CLP", the variance instead remains constant, because the variance due to "L" is equal to zero. In addition the information is used of the rate "V" of the permanent loss of strength, which is calculated from the stress S, which is measured.

5. The CLP method as a policy for preventative maintenance and as a means to detect correlations among failures of different devices

We have seen in section 4 that, for a maximum acceptable failure rate level, the use of the CLP method may allow the operation of the same device for a time longer than it would be allowed in the case "without CLP".

We now want to compare the different types of maintenance policies. For this reason let us refer to the table of fig. 10. In this table "Δt_m" indicates the time interval between the time at which the decision to carry out the replacement is taken, and the time at which the replacement is effectively carried out.

One may define three types of maintenance policies, namely

1. Normal Maintenance
2. Preventative Maintenance without CLP
3. Preventative Maintenance with CLP

In the case of "normal maintenance", the device is replaced when it has failed. This means that its degree of wearout is complete (because it has failed) and deterministic (because the failed state is a very determined state). The time of replacement is probabilistic, because we don't know the exact time at which the device will fail. The time interval Δt_m is equal to zero, because failure, decision and replacement all occur at the same time. The basic parameter is the estimated time of failure, because it allows one to calculate the expected number of devices which shall be replaced in a given operating time interval.

In the case of preventative maintenance without CLP, the device is replaced before failure. This means that its degree of wearout is less than complete. The time of replacement is deterministic, because it is planned in advance.

This in turn gives a probabilistic degree of device wearout, because the wearout is not known exactly at the time of replacement. The time interval Δt_m between decision and replacement is equal to the whole maintenance interval. The basic parameter is the failure rate at the time of replacement, because the time of replacement is chosen as the time at which the failure rate reaches a preestablished level.

In the case of preventative maintenance with CLP, the device is replaced before failure, but at a time which is probabilistic in its nature. The degree of wearout is less than complete, and deterministic. This in turn gives a probabilistic time of replacement. The time interval Δt_m is equal to zero, because decision and replacement occurr both at the same time, which is the time at which the degree of wearout reaches a level above which the operation of the device is considered to be unsafe at the operating conditions encountered. The basic parameter is the estimated time at which the failure rate reaches a preestablished level, because this is the probable time at which the replacement will be carried out.
It is interesting to point out that this type of maintenance is preventative, because the device is replaced before failure, but it is very similar to the normal maintenance because in both cases the time of replacement is probabilistic and the degree of wearout deterministic.

The CLP method gives one also the possibility of detecting in a complex plant the correlation between the failure or malfunction (cause) of a device and the change (effect) of the stress applied to another device belonging to the plant. The cause may also be a change of the environmental conditions external to the plant.

It has been often observed that a device shows during operation a failure rate higher than that measured during lifetests carried out with the same environmental conditions as the ones the device experiences during normal operation. It may infact happen that, due to the failure (or malfunction) of another component in the plant, the device in question is exposed for some time to stresses and/or loads which are higher than those which were foreseen.
Consider, for instance, the case of an engine of a motorcar where the lubrication oil is cooled by water which in turn is cooled by means of an air fan.

If the belt (which links the main shaft to that of the fan) fails, the oil temperature (stress) will raise and the bearing of the main shaft will suffer a higher rate of wearout. If now the driver does not notice the failure of the belt and continues to drive the car, the bearing will also fail and the engine will stop. The driver will examine the engine and will easily discover the correlation between the failure of the belt and that of the bearing. However, it may also happen that the driver notices the failure of the belt before the bearing fails, and stops the engine. In this case the bearing will suffer, during the time in which the car is driven with the belt broken,

partial damage larger than it would have suffered if the belt would have not failed. The driver will replace the belt and start the engine again. It may now happen that the bearing, due to the higher partial damage already suffered, will fail before it is expected to fail. The driver will conclude that the failure rate of the bearing is higher than that obtained during the laboratory lifetests, but in general he will not be able to correlate the increased failure rate of the bearing to the failure rate of the belt, especially if the engine is very complex, and the failure of many components may effect the failure rate of the bearing.
If the engine is now provided with two recorders which record respectively the time of failure of the belt and the oil temperature (as is the CLP method), the driver, by analyzing both the records, will find out the correlation between the increase of the bearing failure rate and the failure rate of the belt.

Analysis using traditional fault trees is unsuitable for studying such correlations, because the failures of two (or more) devices are supposed either to be uncorrelated (independent) or completely correlated (dependent), that is if the first device fails the second fails too.

For this reason a new technique which can analyze this type of correlation must be developed, in which each device should be described by its margin of strength (eq. 9 of section 2).

6."Integrated Learning Processes"

The adoption of the CLP method on a large scale gives the possibility of organizing learning processes where the knowledge of the manufacturer and user of a given device merge together. In these processes, which may be called "integrated learning processes", the data are produced in the laboratory for life-tests and in the user's plant and are stored in the information bank, which is a computer.
In the case of replaceable devices, new (not yet used) devices will be tested only initially in order to provide sufficient knowledge to render the devices operationable Later, since the preventative maintenance policy will be currently adopted, life-tests will be carried out preferably on used devices, which were dismissed before failure after having been used for the allowed length of time. It turns out in this case that the learning process takes the form of a cycle. The cycle begins when a new device starts operation in the user's plant, and ends when the information gained from the lifetests, carried out in the laboratory on the same device (now called "used device"), reaches the user through the "bank". Information will be produced continuously, and the speed of learning will be proportional to the flow of devices in the cycle, that is to the ratio between the number of devices present in the cycle and their residence time in the cycle.

Fig. 11 shows a schematic diagram of an integrated learning process. The manufacturer may be for example a firm which produces ball bearings and the user may be an air company (like Lufthansa) or an electricity producer (like the R.W.E.). Solid lines in the diagram indicate flows of materials, while the dotted lines indicate flows of information. The manufacturer "A" gives a sample of new devices to the laboratory for lifetime tests. The information gained from these tests together with that coming from the manufacturer is stored in the "bank", where is processed and made available to the user "B", who buys the device from A. The user B operates the device for a length of time, which is determined by the value of the reliability imposed upon him, and then replaces the device with a new one, before it fails. The used device is then given to the laboratory, while the information on the operating experience of the device is given to the bank. The laboratory will perform a lifetime test of the used device and will give the information gained from these tests to the bank. Information about devices which eventually fail during operation in the user's plant is also given to the bank. All the information stored in the bank about a given device is available to the laboratory, to A and to B.

It is important to point out that with the system of Fig. 11 only a limited amount of new devices must be sacrificed initially in order to get an initial amount of knowledge about the characteristics of the device. Later, further new device tests are not needed. The used devices will be tested, and the information gained from these is as good as that obtained from the new one, because, due to the application of the CLP method, the full operating history of the used devices is made available from the user to the bank.

It follows (Fig. 11) that the integrated learning process takes the form of a cycle. The new devices enter the cycle at "M", where they start to be operated in the plant owened by the user. After being operated, the devices are given to the laboratory, where they are brought to failure, in order to produce information. The information is given from the bank to the user, which will use it to decide the operating time of the new devices arriving at M. A new cycle characterized by a higher level of knowledge starts, and the process can be repeated continuously and indefinitely.
A secong path is possible, with which the user is by-passed. This is the path LN, where the new devices are given directly to the laboratory, which will be especially used at the beginning to obtain initial information.

In addition the "bank" will provide a means to quickly diagnose devices that need improvement, and provide information for their redesign.

Fig. 12 qualitatively shows various paths of a learning process by giving the allowed operating time of a device as a function of the time.
A path which has been considered is that indicated with OBDE. This would correspond to the case in which the operation of the devices is started after one has obtained their reliability by means of lifetests performed on new devices over the time interval OB. At the time corresponding to the point B the knowledge has been gained which would allow one to operate the device in the user's plant up to the final value of its operating time with the associated maximum allowed value of the failure rate. This would correspond in Fig. 11 to the case in which only the path LN is used. This learning process is very safe, but it may also be very expensive.

If instead, one makes use of the path LN (Fig. 11) only at the beginning to acquire an initial knowledge and then takes advantage of the cycle (Fig. 11) by making lifetests on the used devices, one would get a path in Fig. 12 of the type OFGCE. New devices will be tested (path LN in Fig. 11) until the time corresponding to the point F is reached. At this time, for a given maximum value of the failure rate, the allowed operating time of each device is lower than its final value.

The allowed operating time now increases with time (GC in Fig. 12) because of the knowledge continuously gained by means of the lifetests on the used devices (cycle in Fig. 11).
For a given maximum failure rate there is a family of learning paths of the type OFGCE, which may be obtained by properly choosing the stress levels in the laboratory for lifetests (accelerated tests). The learning path also depends upon the number of devices which are put into operation. Among all possible paths belonging to a given family, the most economical path should be chosen.

7. Conclusions

In ref. 2 the author has developed a more complete theory, which includes also the case where the reference X is a stochastic function of the time. However, for the sake of simplicity, we have limited ourselves here to a very simple case. The treatment of more general cases would not have added anything new to the concept of preventative maintenance with CLP (section 5) to the discussion on the "integrated learning processes" (section 6) and to the new definition of reliability (section 2), but the additional mathematical complexity would have distracted the reader. For this reason, if the reader is interested in the details of the general theory of reliability calculation from the properties of the device and from its operating conditions, he is referred to reference 2.

We can now close our paper by listing the advantages which derive from the use of the CLP method and of the "integrated learning processes" and the problems which arise by their adoption.

A) Advantages

1. A more precise estimate of the lifetime of devices and systems is possible. Therefore either their reliability and availability are improved, or, for a given reliability, their operating time is increased.

2. The possibility exists of detecting in a complex plant the correlation between the failure or malfunction (cause) of a device and the change (effect) of the stress applied to another device belonging to the plant. The cause may also be a change of the environmental conditions external to the plant.

3. The learning process is rationalized by ensuring a preestablished maximum allowed failure rate during the whole learning process, and by providing a means to quickly diagnose devices that need improvement and the information for their redesign.

B) Problems

1. The ability to closely reproduce the device's characteristics is required, which in turn asks for high standardization of the fabrication processes.

2. Standardization of the installation methods and of the procedures for repair is also required.

3. Lifetests must be carried out, which allow the measurement of the important parameters. Interpretation of the tests must be done through correct theoretical models.

4. Diagnostic tests during device operation must be developed, because these too may improve the knowledge of the state of the device at a specific time.

5. Development of special instrumentation and special methods for the transmission of the measurements may be needed.

6. The automatic continuous recording and processing of an enourmous quantity of data is also required. Criteria must be developed to decide what should be recorded, what should be recorded and afterward discarded, and how data should be processed.

The higher the degree of reproduceability of a device, the greater the incentive to use the CLP method. Since the fabrication processes continuously improve their degree of reproduceability and since more and more new diagnostic tests are found, one should expect that the CLP method will become in the long range the most powerful tool to improve the reliability and availability of devices and systems.

The application of the "integrated learning processes" will be a revolution in the fields of design, production and operation of technical devices. All the existing human scientific and technical knowledge must be organized in the bank. But more information must be produced at high rates to make these learning processes effective.

To reach the stage, where the application of these integrated learning processes is possible, is a gigantic task, which will require a tremendous coordinated effort in all fields of scientific and technical research.

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10 Appendix 1

Evaluation of the statistical properties of a stationary and ergodic stochastic function of the time

Let us consider the stationary and ergodic stochastic function of the time S(t).

Fig. 5 shows a realization of "S" as a function of time. We may approximate such a function by means of a sequence of rectangular pulses, each pulse terminating when the next starts (fig. 5). Amplitude "S" and duration T of each pulse are random. The following definitions are introduced;

$\psi(s)$ = probability density distribution that the state S=s will occurr, when a pulse takes place

$\lambda e^{-\bar{t}\lambda}$ = probability density distribution for the waiting time at the state S=s

where

$\bar{t}$ = time measured from the beginning of the pulse

λ = inverse of the average waiting time at any state S = s

The average value $\bar{S}$ can be evaluated by averaging the recorded S(t) over a sufficiently long period of time

$$\int_0^{\infty} s\,\psi(s)\,ds = \lim_{t\to\infty} \frac{1}{t}\int_0^{t} S(t')\,dt' \qquad (1)$$

The term on the left side of eq. 2 is called the "ensemble average" while that on the right side is called the "time average".

They are equal only if the process is ergodic.

For a generic moment of order "n", we can write the equation

$$\int_0^{\infty}(s-\bar{S})^n \psi(s)\,ds = \lim_{t\to\infty} \frac{1}{t}\int_0^t (S(t')-\bar{S})^n dt' \qquad (2)$$

Again the term on the left side is called "ensemble average" and that on the right side "time average".

The time averages can be evaluated from a record of S(t). The distribution ψ (s) can be evaluated by means of the eq. 2 and the series of eqs. 3. An alternative method of evaluating ψ(s) from a record of S(t) may be the following. Let us consider the time intervals t_m during which $S \geqslant s$ (fig. 13).

Clearly for an ergodic process we can write the equation

$$\int_s^{\infty} \psi(s)\,ds = \lim_{t\to\infty} \frac{\sum_{m=1}^{\infty} t_m}{t} \qquad (3)$$

which allows us to evaluate the function ψ(s).
Finally the constant "λ" can be calculated by equating the autocorrelation function evaluated theoretically (ensemble average) to that evaluated experimentally from a record of S(t) (time average) (see Appendix 2).

$$\left[\,\overline{S^2} - (\bar{S})^2\right] e^{-\lambda\Delta t} = \lim_{t\to\infty}\frac{1}{t}\int_0^t [S(t')-\bar{S}][S(t'+\Delta t)-\bar{S}]\,dt' \qquad (4)$$

11. Appendix 2

Calculation of the autocorrelation function

Let us consider a stationary stochastic function S(t), which is approximated by a sequence of uncorrelated elementary pulses characterized by an amplitude and a duration (fig. 5).

Let us indicate with $\psi(s)$ the probability density function of the amplitude S. The duration of the pulses are assumed to be exponentally distributed with an average value $1/\lambda$ with λ = const.

Let us consider now the joint probability density distribution $\chi(s_1;\ t_1$ and $s_2;\ t_2)$ of the event

$$S = s_1 \quad \text{at} \quad t = t_1 \tag{1}$$

$$S = s_2 \quad \text{at} \quad t = t_2 \tag{2}$$

$$\text{with} \quad t_2 > t_1 \tag{3}$$

where

t = time

t_1 and t_2 being two particular values of the time and s_1 and s_2 being two particular values of the amplitude.

We have

$$\chi\,(s_1;t_1 \text{ and } s_2;t_2) = \psi(s_1)\, e^{-\lambda(t_2-t_1)}\delta(s_2-s_1) + \left[1-e^{-\lambda(t_2-t_1)}\right]\psi(s_1)\psi(s_2) \tag{4}$$

where

$$\delta(s_2-s_1) = \infty \qquad \text{for } s_2-s_1 = 0 \tag{5}$$

and

$$\delta(s_2-s_1) = 0 \quad \text{for} \quad s_2 - s_1 \neq 0 \tag{6}$$

The autocorrelation function $A(t_2-t_1)$ is given by

$$A(t_2-t_1) = E\left\{ \left[S(t_1)-\bar{S}\right]\left[S(t_2)-\bar{S}\right] \right\} =$$

$$= \iint_0^\infty \chi\,(s_1\,;\,t_1 \text{ and } s_2\,;\,t_2)\left[s_1-\bar{S}\right]\left[s_2-\bar{S}\right] ds_1\,ds_2 \tag{7}$$

where "E" indicates expectation and

$$\bar{S} = \int_0^\infty s\cdot\psi(s)\,ds \tag{8}$$

Taking into account eq. 4, eq. 7 gives finally

$$A(t_2-t_1) = \left[\,\overline{S^2} - (\bar{S})^2\right] e^{-\lambda(t_2-t_1)} \tag{9}$$

where

$$\overline{S^2} = \int_0^\infty s^2\psi(s)\,ds \tag{10}$$

12 . Appendix 3

Calculation of the variance of the permanent loss of strength

Let us indicate with V(S) the rate of permanent loss of strength. We have

$$L(t) = \int_0^t V(S(t'))\,dt' \qquad (1)$$

where "S" is a stochastic stress applied to the device. If σ_V^2 is the variance of V and "λ" is the average frequency of occurrence of the elementary pulses which approximate S, the auto-correlation function "A" is given by (Appendix 2)

$$A(t_2 - t_1) = \sigma_V^2\, e^{-\lambda (t_2 - t_1)} \qquad (2)$$

with

$$t_2 > t_1 \qquad (3)$$

The variance σ_L^2 is given by

$$\sigma_L^2 = E\left\{ \int_0^t \int_0^{t_2} [V(S(t_1)) - \overline{V}][V(S(t_2)) - \overline{V}]\,dt_1\,dt_2 \right\} \qquad (4)$$

where

$\overline{V}$ = average value of V

Eq. 4 can be written as follows

$$\sigma_L^2 = \int_0^t \int_0^{t_2} E\left\{\left[V(S(t_1)) - \bar{V}\right]\left[V(S(t_2)) - \bar{V}\right]\right\} dt_1\, dt_2 = \int_0^t \int_0^{t_2} A(t_2 - t_1)\, dt_1\, dt_2 \tag{5}$$

From eq. 5, taking into account eq. 2, we get finally

$$\sigma_L^2 = \frac{\sigma_V^2}{\lambda^2}\left(\lambda t + e^{-\lambda t} - 1\right) \tag{6}$$

The function

$$f(\lambda t) = \lambda t + e^{-\lambda t} - 1 \tag{7}$$

is shown in fig. 6.

13. Appendix 4

Calculation of the conditional probability density distribution of the permanent loss of strength

Let us indicate with $\wp(\ell;\Delta t;\, v)$ the conditional probability density distribution of the event

$$\Delta L = L(t) - L(t-\Delta t) = \ell \quad \text{at "}t\text{"} \tag{1}$$

under the condition

$$V(t-\Delta t) = v \tag{2}$$

where

- L = stochastic permanent loss of strength
- V = stochastic rate of the permanent loss of strength
- t = time
- $t-\Delta t$ = time at which the evaluation is carried out

We have

$$\Delta L = v \cdot T + \left[L(t) - L(t-\Delta t + T) \right] \tag{3}$$

where

T = duration of the first elementary pulse (belonging to the sequence of elementary pulses which approximate V) in the time interval between $t-\Delta t$ and t.

By applying the basic theorem of the sum of the probabilities of the mutually exclusive events, we get

$$\rho(\ell;\Delta t;v) = P\left\{\Delta L = \ell \text{ at } t \;\middle|\; V(t-\Delta t) = v\right\} = P_1 + P_2 \tag{3}$$

where

$$P_1 = P\left\{\Delta L = \ell \quad \text{at } t \text{ and } T \geq \Delta t \;\middle|\; V(t-\Delta t) = v\right\} \tag{4}$$

$$P_2 = P\left\{\Delta L = \ell \text{ at } t \text{ and } T < \Delta t \;\middle|\; V(t-\Delta t) = v\right\} \tag{5}$$

Since T is supposed to be exponentially distributed with average value $1/\lambda$, we obtain easily

$$P_1 = e^{-\lambda\Delta t}\,\delta(\ell - v\Delta t) \tag{6}$$

and

$$P_2 = \lambda \int_0^{\Delta t} e^{-\lambda t_1}\, g(\ell - v t_1;\, \Delta t - t_1)\, dt_1 \tag{7}$$

where $g(\ell,\Delta t)$ is the probability density distribution of ΔL and "δ" indicates the impulse function.

14 . Appendix 5

Calculation of the failure rate

Let us start from eq. 3 of section 4 and eq. 23 of section 3

$$\ell(t-\Delta t;t) = - \frac{\int_{y_{min}}^{y_{max}} \varphi(y)\left[\int_0^{y-(\ell+x)} \frac{\partial \rho(\ell';\Delta t;v)}{\partial \Delta t}\, d\ell'\right] dy}{\int_{y_{min}}^{y_{max}} \varphi(y)\left[\int_0^{y-(\ell+x)} \rho(\ell';\Delta t;v)\, d\ell'\right] dy} \qquad (1)$$

$$\rho(\ell';\Delta t;v) = e^{-\lambda\Delta t}\,\delta(\ell'-v\Delta t) + \lambda\int_0^{\Delta t} e^{-\lambda t_1} g(\ell'-v t_1;\Delta t - t_1)\, dt_1 \qquad (2)$$

where

y_{min} = minimum value of the random variable Y calculated at the time $t - \Delta t$

v = value of V calculated at "$t - \Delta t$"

δ = impulse function

ℓ = value of L calculated at "$t - \Delta t$"

If we put eq. 2 into 1, we obtain

$$h(t-\Delta t;t) = - \frac{v e^{-\lambda \Delta t} \varphi(y_{min} + v\Delta t) - \lambda \int_{y_{min}}^{y_{max}} \int_0^{\Delta t} \int_0^{y-(x+\ell+vt_1)} \varphi(y)\, e^{-\lambda t_1} \frac{\partial g(\ell';\Delta t - t_1)}{\partial \Delta t}\, d\ell'\, dt_1\, dy}{e^{-\lambda \Delta t} \int_{y_{min}+v\Delta t}^{y_{max}} \varphi(y)\, dy + \lambda \int_{y_{min}}^{y_{max}} \int_0^{\Delta t} \int_0^{y-(x+\ell+vt_1)} \varphi(y)\, e^{-\lambda t_1} g(\ell';\Delta t - t_1)\, d\ell'\, dt_1\, dy} \quad (3)$$

In the particular case

$$\lambda \Delta t \ll 1 \quad (4)$$

we have simply from eq. 3

$$h(t;t) = v \frac{\varphi(y_{min} + v\Delta t)}{\int_{y_{min}+v\Delta t}^{y_{max}} \varphi(y)\, dy} \quad (5)$$

In the case "without CLP", the density distribution of "L" is given by the function g (ℓ;t), and we have for "h" the following expression

$$\bar{h}(0;t) = \frac{\int_{y_0}^{y_{max}} \varphi(y) \left[\int_0^{y-X} \frac{\partial g(\ell;t)}{\partial t}\, d\ell \right] dy}{\int_{y_0}^{y_{max}} \varphi(y) \left[\int_0^{y-X} g(\ell,t)\, d\ell \right] dy} \quad (6)$$

15. Appendix 6

Numerical example

The numerical example given in this paper is characterized by the following values of the parameters

$\varphi(y)$ = truncated normal density distribution

$$y_o = 0.1\ \bar{Y} \qquad (1)$$

$$y_{max} = 2,2\ \bar{Y} \qquad (2)$$

$$\sigma_y = 0.04\ \bar{Y} \qquad (3)$$

$$\bar{Y} = 1 \qquad (4)$$

$$x = y_o \qquad (5)$$

The time scale has been normalized by means of a factor

$$\tau = t\ \frac{\bar{V}}{\bar{Y}-x} \qquad (6)$$

where

τ = dimensionless time (absolute units)

t = real time

In addition we have

$$\lambda = 10.5\ \frac{\bar{V}}{\bar{Y}-x} \qquad (7)$$

and

$$\sigma_V^{\ 2} = 0.1\ (\bar{V})^2 \qquad (8)$$

$$\bar{V} = 0.86 \qquad (9)$$

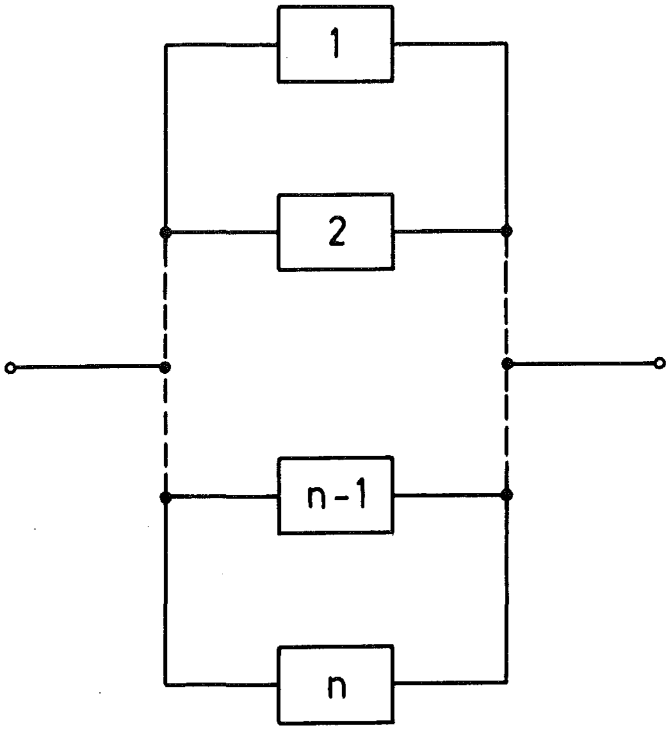

Fig.1 Schematic diagram of a redundant system made of "n" components

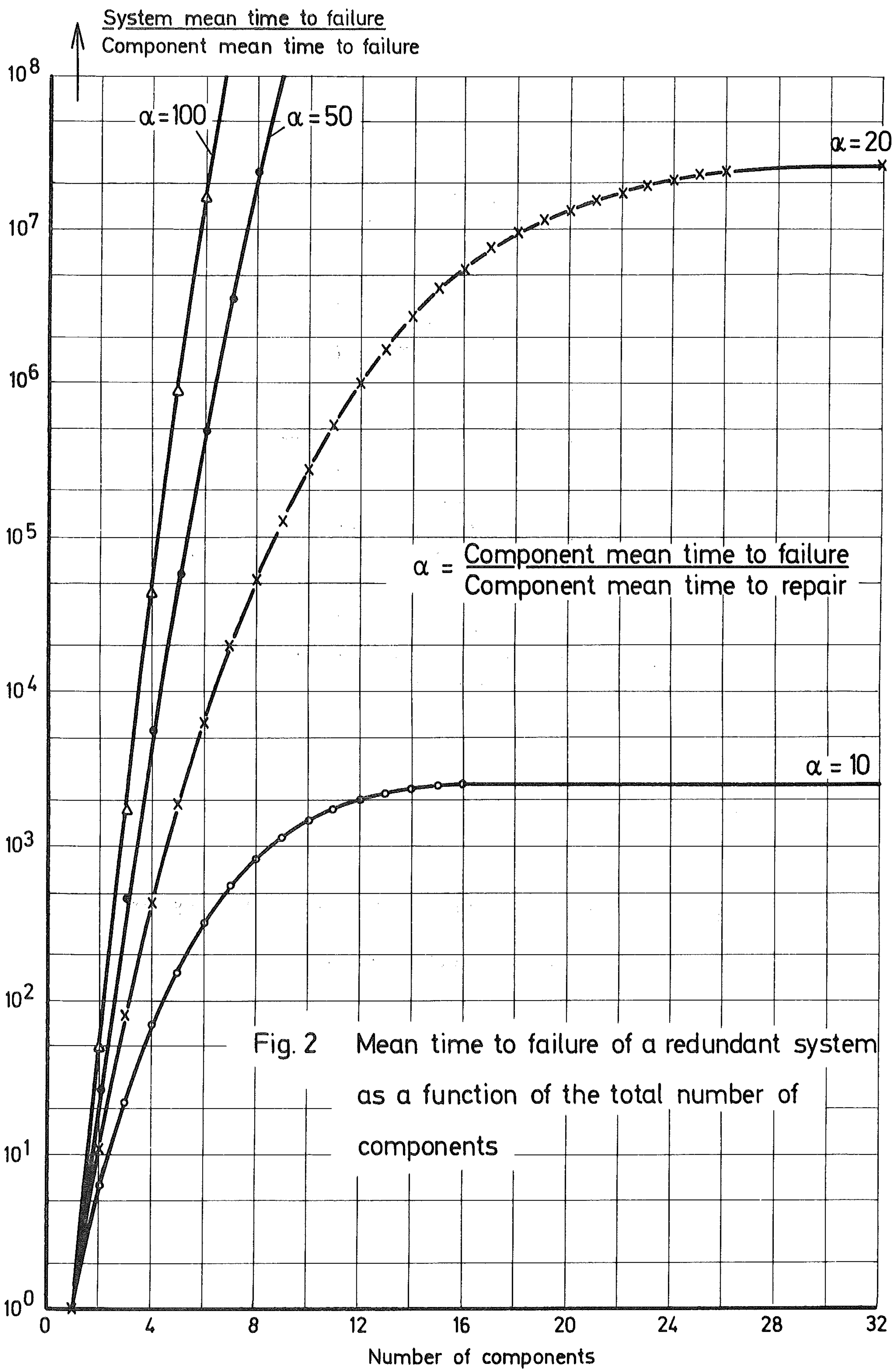

Fig. 2 Mean time to failure of a redundant system as a function of the total number of components

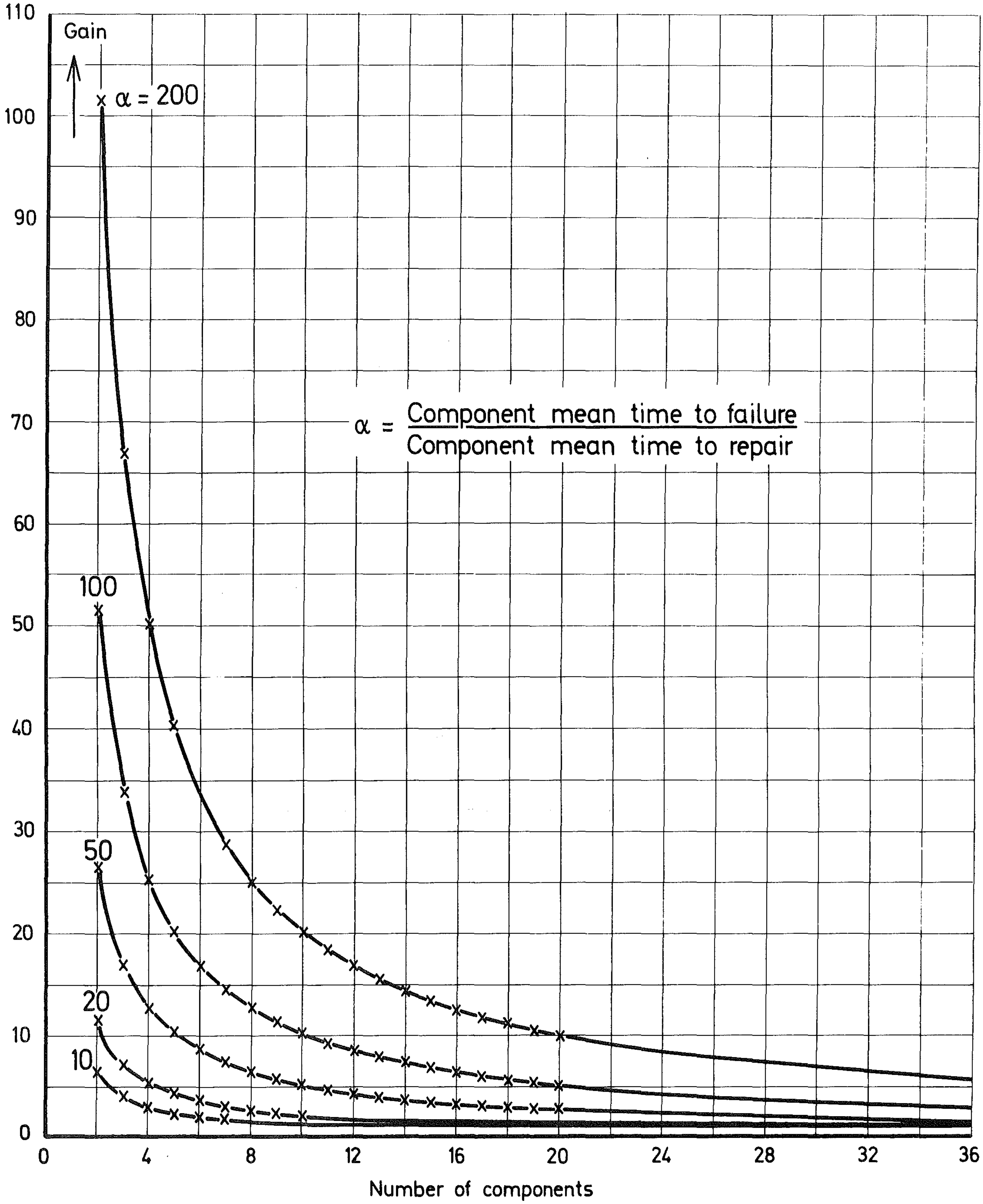

Fig. 3 System lifetime gain produced by the last added redundant component as a function of the total number of components

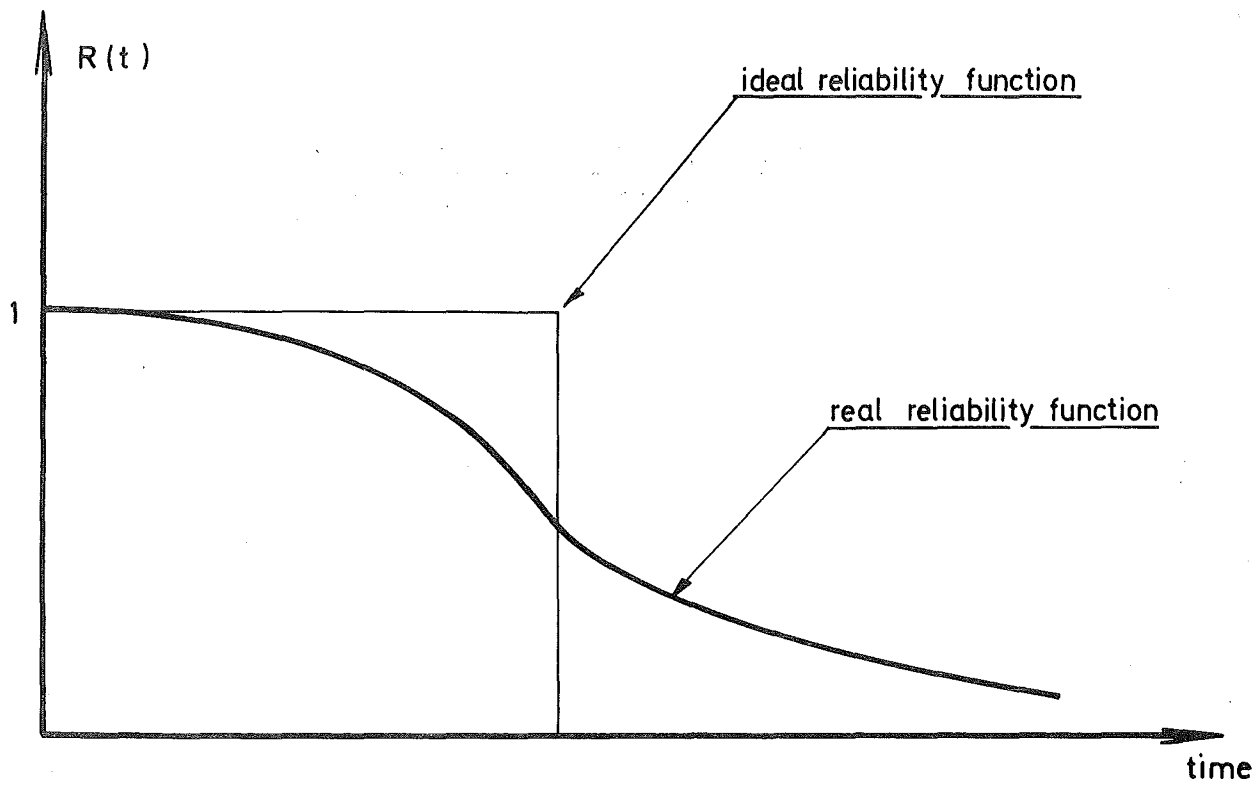

Fig. 4 Real and ideal reliability

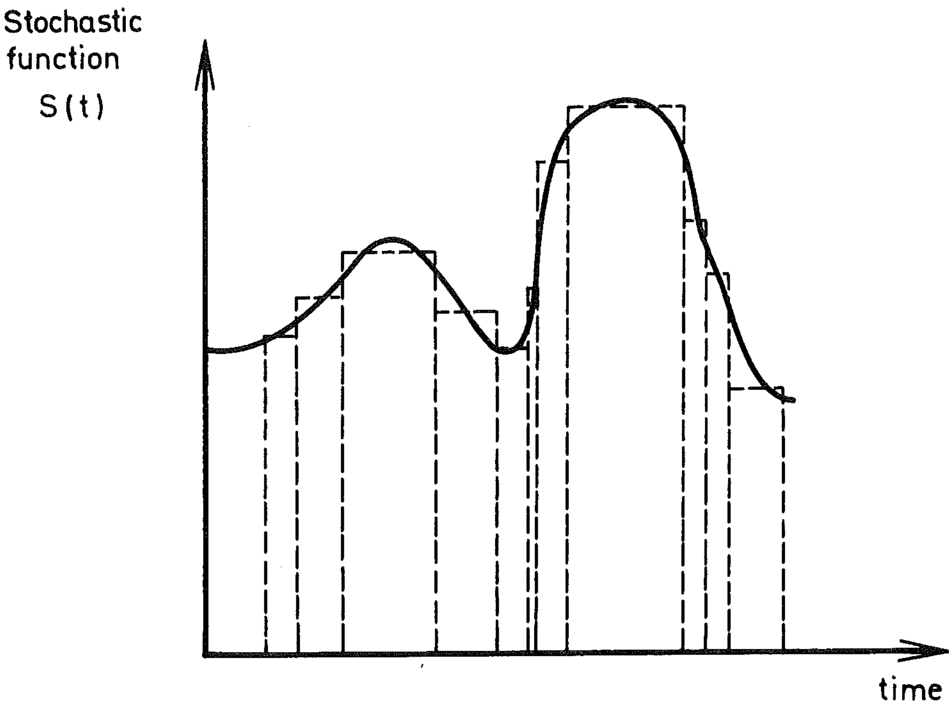

Fig.5 Approximation of a stochastic function by means of a sequence of rectangular pulses

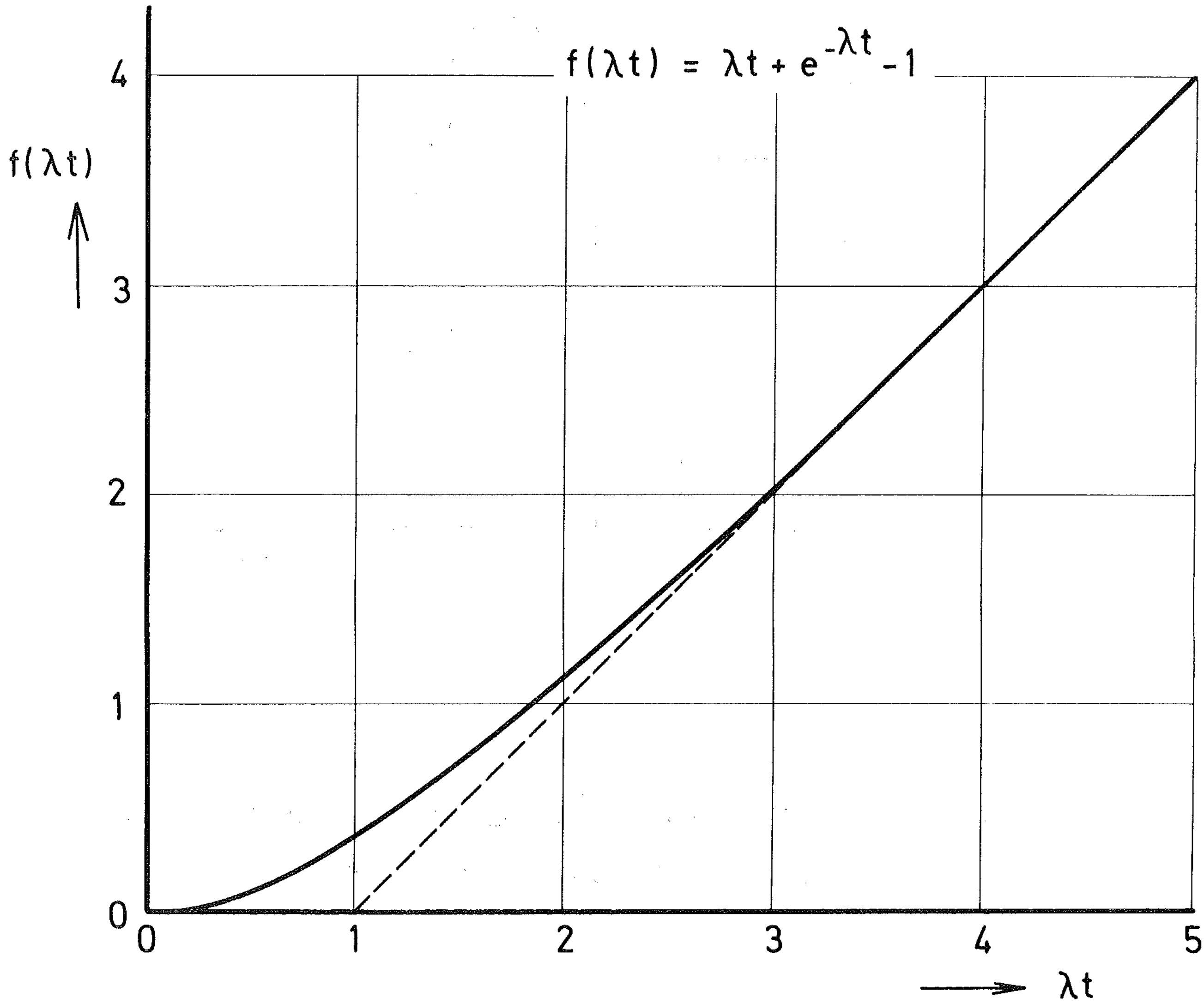

Fig. 6 Dependence of the normalized variance $f(\lambda t)$ of the permanent loss of strength upon time

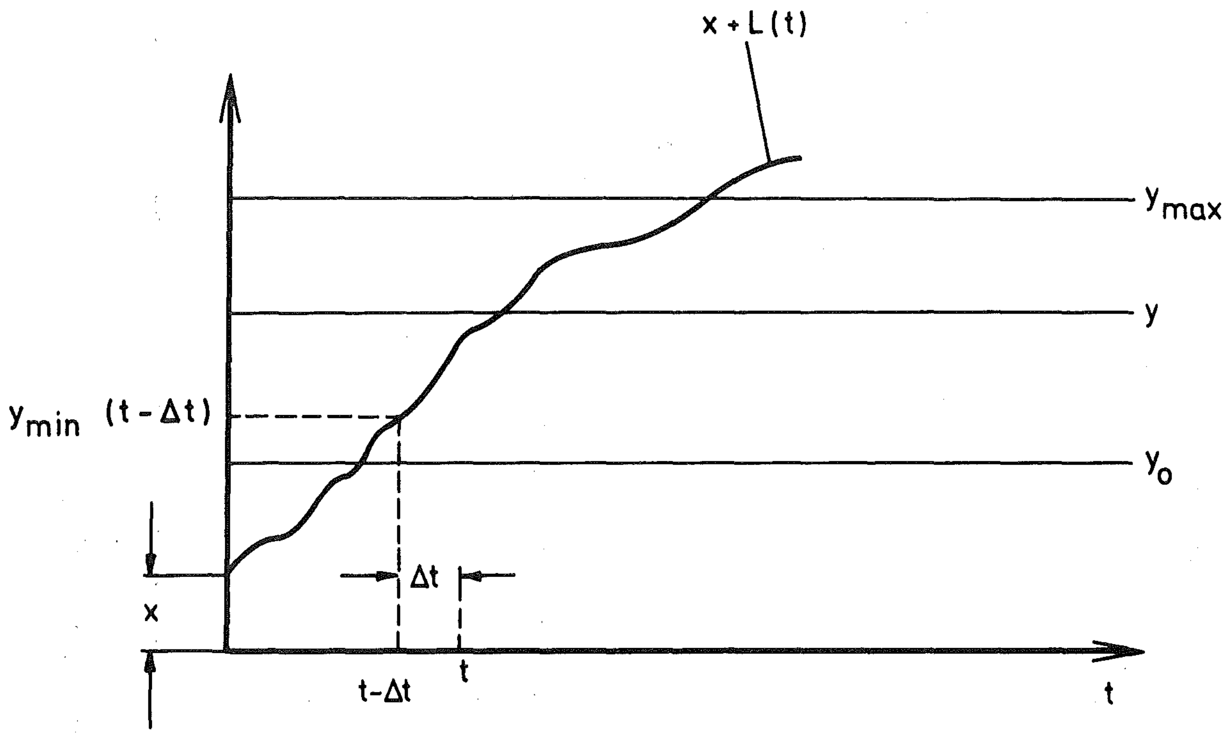

Fig. 7 The loss of strength as function of time

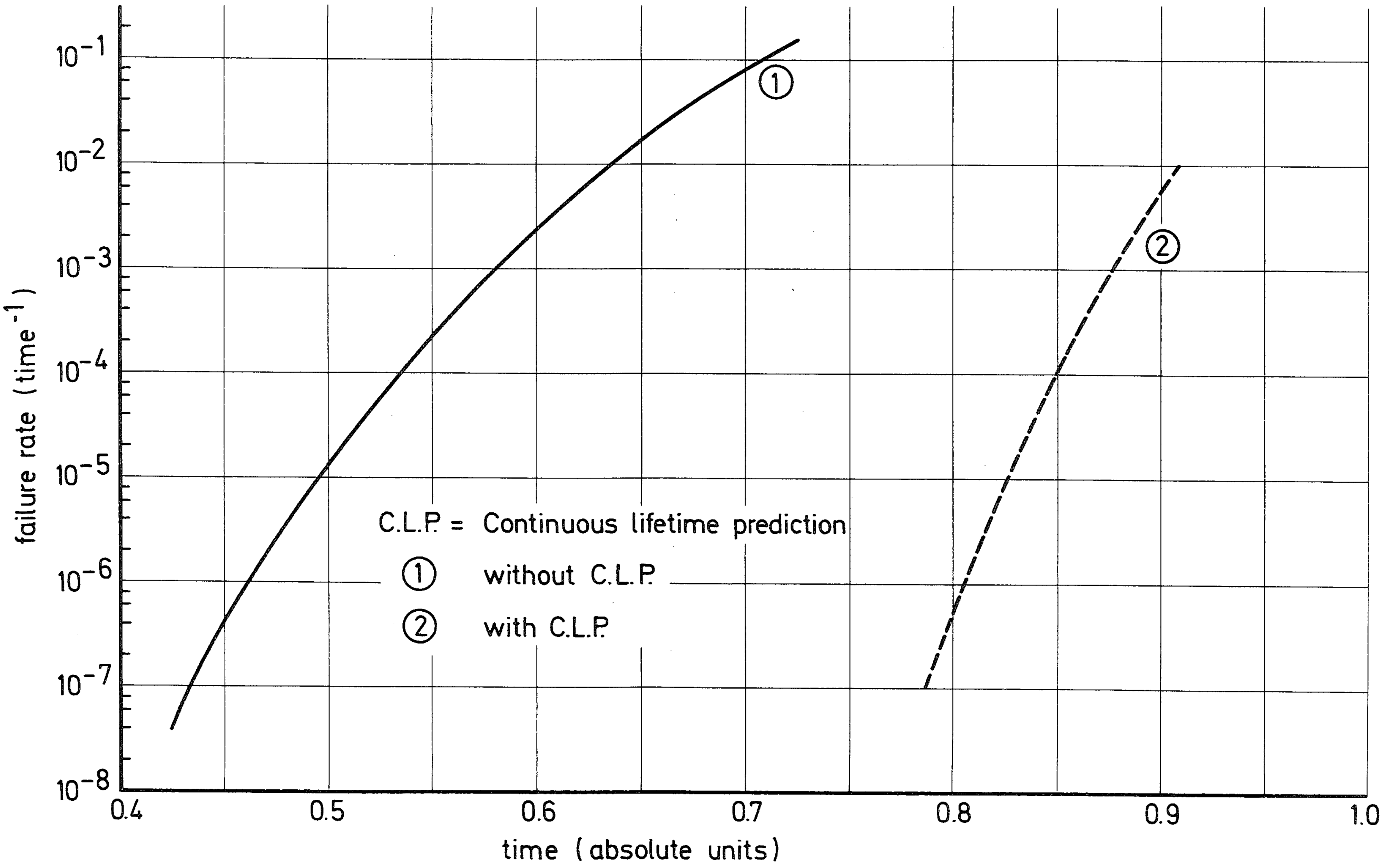

Fig. 8 Failure rates as functions of the time

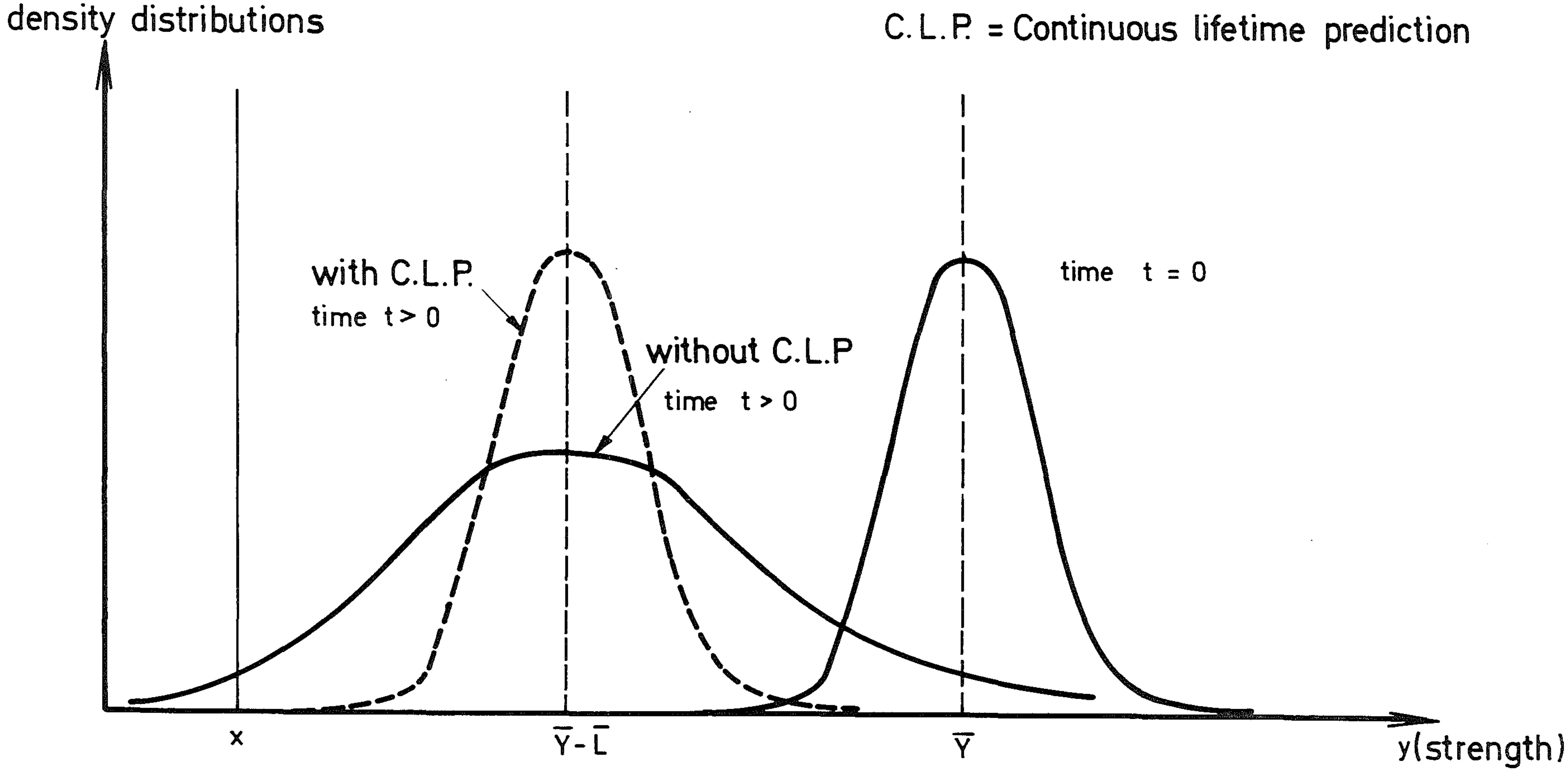

Fig. 9 Strength distributions at different times

Type of maintenance		Time of replacement	Degree of device wearout	Time interval, Δt_m, between the time at which the decision to carry out the replacement is taken and the time at which the replacement is effectively carried out	Basic parameter
1	Normal	Probabilistic	Complete and deterministic	None	E { time of failure }
2	Preventative	Deterministic	Less than complete and probabilistic	Whole maintenance time interval	failure rate at the time of replacement
3	Preventative with CLP	Probabilistic	Less than complete and deterministic	None	E { time at which a given failure rate is reached }

Fig. 10 Maintenance policies

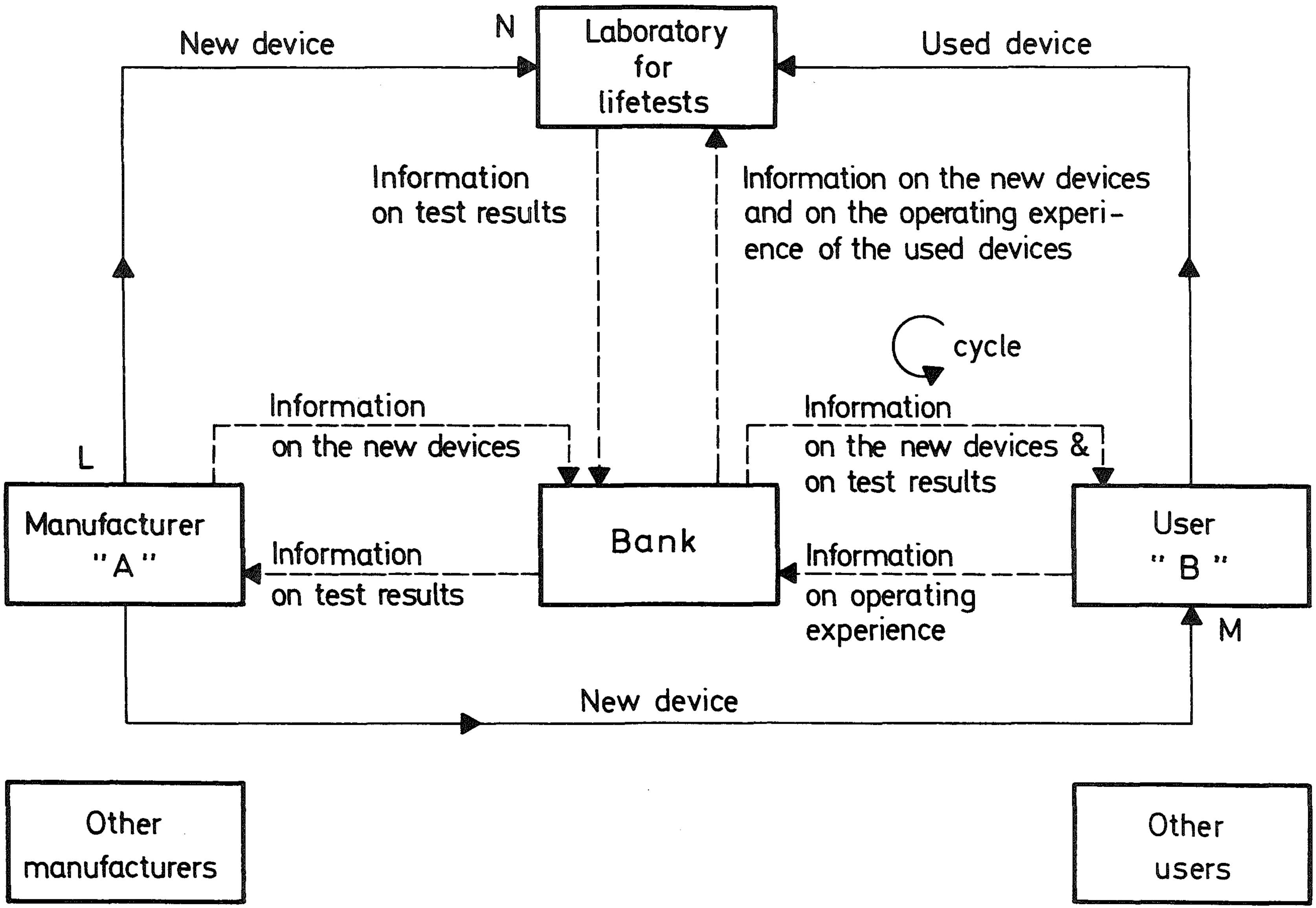

Fig. 11 Schematic diagram of an integrated learning process. Case of replaceable devices.

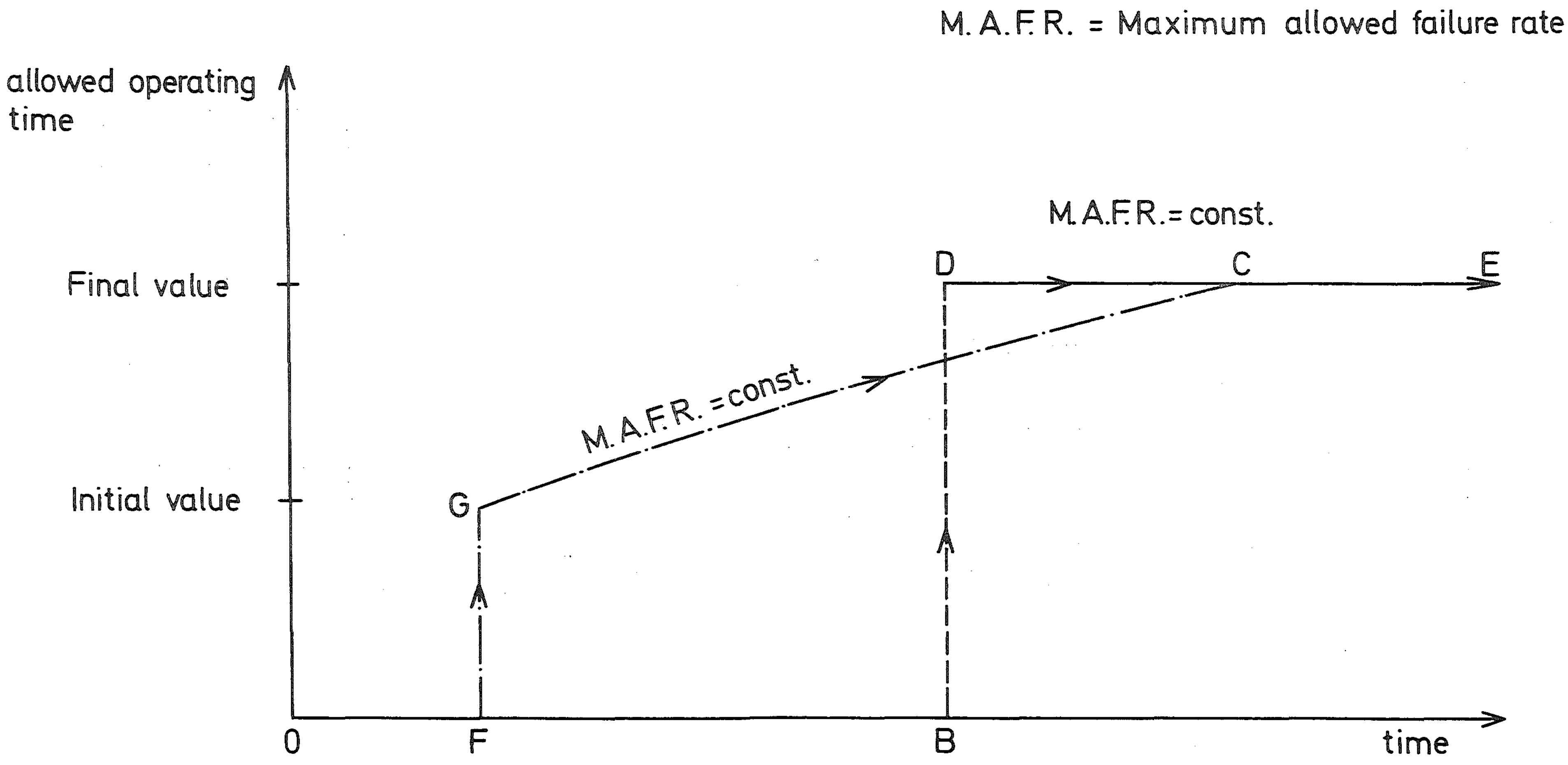

Fig. 12 Various paths of a learning process

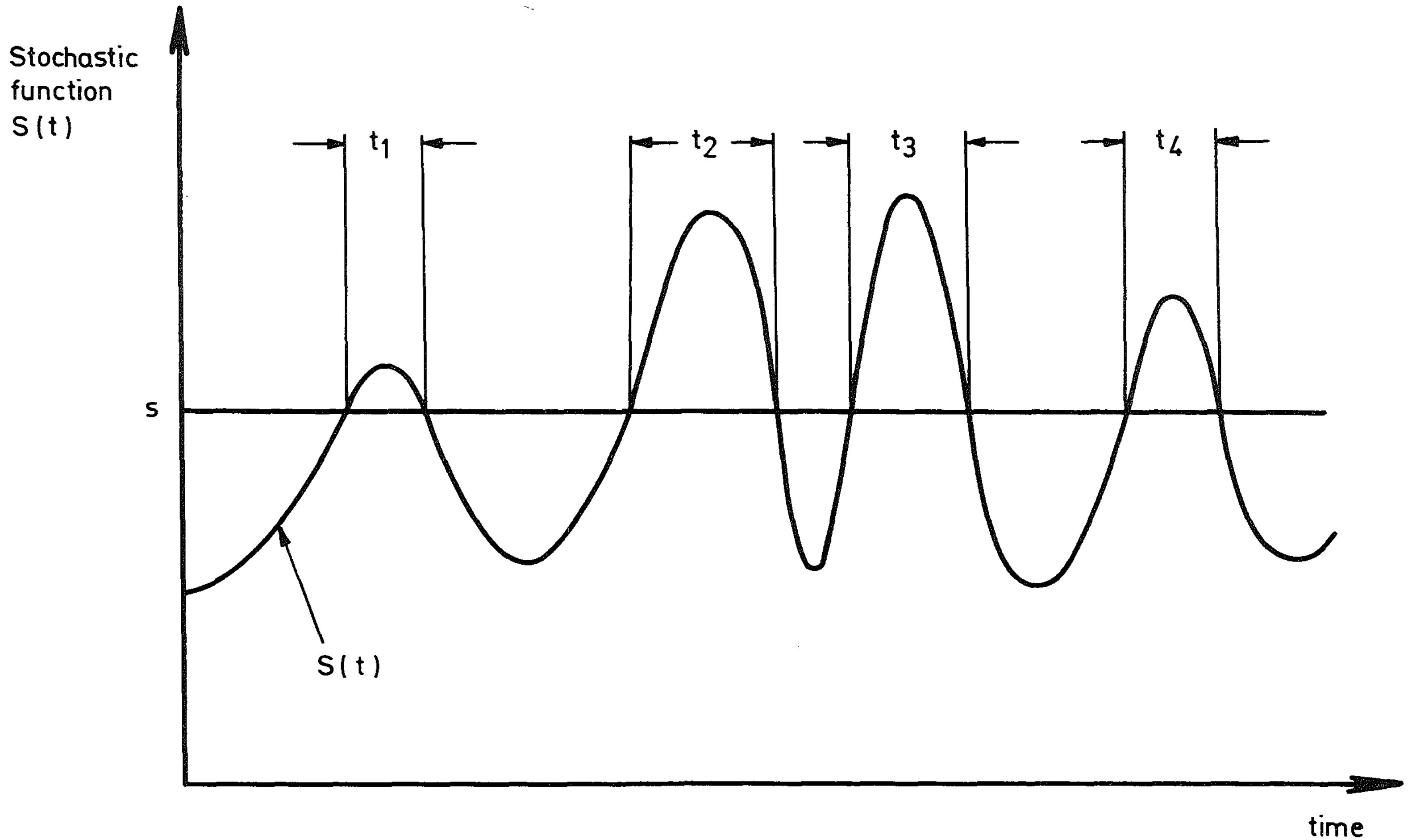

Fig. 13 Evaluation of the time intervals "t_m" during which the stochastic function S(t) exceeds the level s