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KARLSRUHE

September 1973 KFK 1840

Institut für Experimentelle Kernphysik

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The $0^+ - 0^+$ superallowed beta transitions and the weak vector coupling constant

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Abstract:

In the nuclear beta decay the weak vector coupling constant $G_\beta^{(V)}$ can only be determined from the $0^+ - 0^+$ superallowed beta transitions. Because of the great importance of $G_\beta^{(V)}$ for all the theories of weak interactions a big effort has been directed in the last years to the experimental and theoretical investigation of the ft-values of these $0^+ - 0^+$ superallowed transitions.

In this paper a compilation of the experimental results for the ft-values, and a discussion of the theoretical attempts to calculate the necessary corrections is given. Nevertheless, some existing discrepancies between experimentally measured ft-values of different nuclei can not be explained. Finally a value for $G_\beta^{(V)}$ will be derived and compared with the weak vector coupling constants taken from other weak decay modes. This offers the possibility to discuss the concept of universality of the weak interaction theory.

Zusammenfassung:

Die Vektor-Kopplungskonstante $G_\beta^{(V)}$ der schwachen Wechselwirkung für den β-Zerfall kann nur aus den supererlaubten $0^+ - 0^+$ Übergängen bestimmt werden. Wegen der großen Bedeutung von $G_\beta^{(V)}$ für alle Theorien der schwachen Wechselwirkung wurden in den letzten Jahren große experimentelle und theoretische Anstrengungen unternommen, um die ft-Werte der supererlaubten $0^+ - 0^+$ Übergänge zu untersuchen.
In folgendem Artikel werden alle experimentellen Resultate für die ft-Werte zusammenfassend dargestellt und die zur Berechnung der notwenidgen Korrekturen unternommenen Versuche diskutiert. Trotzdem können einige zwischen experimentell gemessenen ft-Werten verschiedener Kerne auftretenden Diskrepanzen nicht erklärt werden.

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eingereicht am: 18. Juni 1973

Zum Schluß wird ein Wert für $G_{\beta}^{(V)}$ aus den Daten abgeleitet und mit Vektor-Kopplungskonstanten verglichen, die aus anderen schwachen Zerfällen bestimmt wurden. Dieser Vergleich gibt die Möglichkeit, das Konzept der Universalität der schwachen Wechselwirkung zu diskutieren.

C o n t e n t s

1. Introduction

Nuclear beta-decay is one form of the weak interaction involving all elementary particles. The following article begins with a short consideration of the present weak interaction theory, together with a discussion of the concept universality of the weak interactions. Concerning the universality of the weak interaction, one essential point is the accurate determination of the vector coupling constant in the nuclear beta-decay. This coupling constant can only be deduced from experimental and theoretical investigations of the $0^+ - 0^+$ superallowed beta-transitions. The main object of this paper is to give a survey of the present state and the unanswered questions concerning the superallowed $0^+ - 0^+$ beta-transitions. Finally, a comparison of the weak vector coupling constants taken from different weak decay modes and a discussion of the universality of the weak interaction is given.

2. The phenomenological theory of weak interactions

We distinguish essentially between three classes of weak interaction processes. This is shown in tab. 1. Firstly, we have the purely leptonic decays, where leptons, i.e. electron, muon, their associated neutrinos and the corresponding antiparticles, interact only. Since leptons are not subject to strong interactions, we have no influence of the strong interactions on these decays. Secondly we have the semi leptonic decays, where leptons together with mesons or baryons are involved. Since mesons and baryons also interact strongly we expect that the weak interaction will be modified in those decays, or in other words, the coupling constant will be renormalized. In the case of the semi leptonic decays we distinguish between the hypercharge* conserving decays

*According to the so called Gell-Mann-Nakano-Nishijima (GNN) relation, the hypercharge Y is related to the charge Q and the third component of the isospin I_3 by: $Q = I_3 + Y/2$

Type	Examples
Purely leptonic	$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$
Semileptonic (a) $\Delta Y = 0$	$n \longrightarrow p + e^- + \bar{\nu}_e$ $\pi^+ \longrightarrow \mu^+ + \nu_\mu$
Semileptonic (b) $\lvert\Delta Y\rvert = 1$	$\Lambda^0 \longrightarrow p + e^- + \bar{\nu}_e$ $K^+ \longrightarrow \mu^+ + \nu_\mu$
Hadronic	$K^0 \longrightarrow \pi^+ + \pi^-$ $\Lambda^0 \longrightarrow p + \pi^-$

Tab. 1 WEAK INTERACTION PROCESSES

(ΔY = 0, Y = hypercharge) and the hypercharge nonconserving decays ($|\Delta Y|$ = 1). This distinction is obvious as the strength of $|\Delta Y|$ = 1 decays in one order of magnitude weaker than those of ΔY = 0 decays. Finally we have the nonleptonic or hadronic processes where the applicability of the present weak interaction theory is until now completely open.

It is usually assumed (see for example [1], [2]) that all weak interaction processes can be described by the phenomenological hamiltonian (current current interaction)*:

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* $A_\lambda^+ = A_\lambda^+$ for λ = 1, 2, 3 and $A_\lambda^+ = -A_\lambda^+$ for λ = 4

$$H_W = \frac{G}{\sqrt{2}} L_\lambda L_\lambda^+ + \frac{G^{(o)}}{\sqrt{2}} (J_\lambda^o L_\lambda^+ + J_\lambda^{o+} L_\lambda)$$

$$+ \frac{G^{(1)}}{\sqrt{2}} (J_\lambda^1 L_\lambda^+ + J_\lambda^{1+} L_\lambda) + \frac{G^{(2)}}{\sqrt{2}} J_\lambda^o J_\lambda^{o+} \qquad (1)$$

$$+ \frac{G^{(3)}}{\sqrt{2}} (J_\lambda^1 J_\lambda^{o+} + J_\lambda^{1+} J_\lambda^o) + \frac{G^{(4)}}{\sqrt{2}} J_\lambda^1 J_\lambda^{1+}$$

The quantities G, $G^{(o)}$, $G^{(1)}$, $G^{(2)}$, $G^{(3)}$ and $G^{(4)}$ are the coupling constants. J_λ^o and J_λ^1 denote the $\Delta Y = 0$ and $|\Delta Y| = 1$ hadronic currents, respectively. L_λ is the leptonic current, which is given by*

$$L_\lambda(x) = i \sum_\ell \bar{\psi}_{\nu_\ell}(x) \gamma_\lambda (1 + \gamma_5) \psi_\ell \qquad (2)$$

$$\ell = e, \mu$$

Because of complications induced by the strong interactions, it is not possible to express the hadronic currents so simply in terms of field operators as in the case of the leptonic currents.

Another general remark to be made about J_λ^o and J_λ^1 is that these currents are composed of two components, a vector and axial vector part. In particular we can write:

$$J_\lambda = V_\lambda + A_\lambda \qquad (3)$$

where V_λ is a vector and A_λ an axial vector operator.

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* The field operators in eq. 2 are given by

$$\psi(x) = \frac{1}{\sqrt{V}} \sum_{q,r} \{e^{iqx} a_r(q) u_r(q) + b_r^+(q) v_r(q) e^{-iqx}\}$$

$a_r(q)$ is the annihilation operator for a fermion of momentum q and spin r and $b_r^+(q)$ the corresponding creation operator for the antiparticles. The $u_r(q)$ and $v_r(q)$ are both the free particle Dirac spinors.

	$L_\lambda^{e^+}$	$L_\lambda^{\mu^+}$	$J_\lambda^{0^+}$	$J_\lambda^{1^+}$
L_λ^e	$\nu_e + e^- \rightarrow \nu_e + e^-$			
L_λ^μ	$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$	$\nu_\mu + \mu^- \rightarrow \nu_\mu + \mu^-$		
J_λ^0	$n \rightarrow p + e^- + \bar{\nu}_e$ $\Sigma^- \rightarrow \Lambda + e^- + \bar{\nu}_e$ $\pi^- \rightarrow e^- + \bar{\nu}_e$ $\pi^- \rightarrow \pi^0 + e^- + \bar{\nu}_e$	$\mu^- + p \rightarrow n + \nu_\mu$ $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$	$n + p \rightarrow n + p$	
J_λ^1	$\Lambda \rightarrow p + e^- + \bar{\nu}_e$ $\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$ $\Xi \rightarrow \Lambda + e^- + \bar{\nu}_e$ $K^- \rightarrow e^- + \bar{\nu}_e$ $K^- \rightarrow \pi^0 + e^- + \bar{\nu}_e$ $K^- \rightarrow \pi^+ + \pi^- + e^- + \bar{\nu}_e$	$\Lambda \rightarrow p + \mu^- + \bar{\nu}_\mu$ $\Sigma^- \rightarrow n + \mu^- + \bar{\nu}_\mu$ $K^- \rightarrow \mu^- + \bar{\nu}_\mu$ $K^- \rightarrow \pi^0 + \mu^- + \bar{\nu}_\mu$ $K^- \rightarrow \pi^+ + \pi^- + \mu^- + \bar{\nu}_\mu$	$\Lambda \rightarrow p + \pi^-$ $\Sigma^- \rightarrow n + \pi^-$ $\Omega^- \rightarrow \Lambda + K^-$ $\Xi \rightarrow \Lambda + \pi^-$ $K \rightarrow 2\pi$ $K \rightarrow 3\pi$	$\Lambda + p \rightarrow \Lambda + p$

Tab. 2 WEAK PROCESSES ACCORDING TO CURRENT-CURRENT THEORY

In eq. 1 the term $G/\sqrt{2}\ L_\lambda L_\lambda^+$ is responsible for the purely leptonic weak processes, for example the term $G/\sqrt{2}\ L_\lambda^\mu L_\lambda^{\dagger e}$ especially for the muon-decay.
$J_\lambda^o L_\lambda^+ + J_\lambda^+ L_\lambda$ and $J_\lambda^1 L_\lambda^+ + J_\lambda^\ddagger L_\lambda$ are the corresponding terms for the $\Delta Y = 0$ and $|\Delta Y| = 1$ semi-leptonic weak processes, respectively, and $J_\lambda^o J_\lambda^{o+}$ etc. those for the hadronic weak processes.

Tab. 2 gives a more explicit survey of how all the weak decays* (and scattering processes) can be explained by the current current interaction [1]. If the postulate, that

$$\begin{aligned} G^{(o)} &= G \cos\theta \\ G^{(1)} &= G \sin\theta \\ G^{(2)} &= G \cos^2\theta \\ G^{(3)} &= G \cos\theta \sin\theta \\ G^{(4)} &= G \sin^2\theta \end{aligned} \qquad (4)$$

is fulfilled one speaks of the Cabibbo form of universality of the weak interaction. θ is the Cabibbo angle. Then we have, for instance,

$$G_\beta^{(V)} = G \cos\theta \quad \text{for the beta-decay} \qquad (5a)$$

$$G_\mu = G \quad \text{for the muon-decay} \qquad (5b)$$

(V means the vector part of the interaction)

3. Transition probabilities and ft-values of allowed beta-transitions

3.1 Allowed transitions in general

For a β^-- or β^+-transition between two nuclear states $|i>$ and $|f>$, we obtained the well-known expression for the spectrum

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* The corresponding transition matrix element is, for instance, in the case of the beta-decay

$$n \to p + e^- + \bar{\nu}$$

$$M_{\beta^-} = < p\ e^-\ \bar{\nu}\ | \int H_W(x)\ dx^4 | n\ >$$

where $H_W(x) = G_\beta/\sqrt{2}\ \{V_\lambda^o + A_\lambda^o\} L_\lambda^{\dagger e}$

form using the theory discussed previously together with the standard perturbation theory * (see for example [2], [3]),

$$N(p)dp = \frac{\ln 2}{K} \{G_V^2 M_F^2 + G_A^2 M_A^2\} \cdot \tilde{F}(Z,W)\, p^2 (W_o - W)^2 dp \qquad (6)$$

where

$N(p)dp$ = number of electrons or positrons in the momentum interval between p and p + dp

p = electron (positron) momentum in units of $m_o c$

W = total energy of the electron (positron) in units of $m_o c^2$

W_o = total end point energy of the electron (positron)

$M_F = \langle f | \sum_i t_{\pm}^{(i)} | i \rangle$

$= \langle f | T_{\pm} | i \rangle$ (Fermi matrix element)

$\vec{M}_A = \langle f | \sum_i t_{\pm}^{(i)} \vec{\sigma}^{(i)} | i \rangle$ (Gamov-Teller matrix element**)

$\tilde{F}(Z,W) = \frac{1}{2p^2} \{g^2_{-1}(0) + f^2_{+1}(0)\}$ (Fermi function)

$g_{-1}(r)$ and $f_{+1}(r)$ are the electron radial wave functions. Contrary to $F(Z,W)$ which is evaluated at $r = R$ (the nuclear radius) $\tilde{F}(Z,W)$ is evaluated at $r = 0$ (the center of the nucleus) [4], [5]. So for allowed transitions, less ambiguity and greater accuracy is achieved.

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* In the following we use natural units ($h = m_e = c = 1$). Then $K = 2\pi^3 (\ln 2)$

** $t_{\pm}$ are the usual charge lowering and raising isospin operators

$$t_{\pm} = t_1 \pm i\, t_2$$

We have $t_+ \chi_p = \chi_n$ and $t_- \chi_n = \chi_p$

$\vec{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$

σ_1, σ_2 and σ_3 are the usual Pauli matrices.

If we intend to experimentally determine the coupling constants G_V and G_A, we see from eq. 6 that the Fermi and the Gamov-Teller matrix element should be known. Let us therefore discuss more extensively the selection rules for these matrix elements. We have (see for example [2], [3]:

$$M_F \neq 0 \quad \text{only for} \quad \begin{aligned} \Delta J &= |J_i - J_f| = 0 \\ \Delta T &= |T_i - T_f| = 0 \end{aligned} \qquad (7)$$

$$\pi_i \, \pi_f = +1$$

$$M_A \neq 0 \quad \text{only for} \quad \begin{aligned} \Delta J &= |J_i - J_f| = 0,1 \quad \text{but not } 0 - 0 \\ \Delta T &= |T_i - T_f| = 0,1 \end{aligned} \qquad (8)$$

$$\pi_i \, \pi_f = +1$$

where the quantities J, T and π are the spins, isospins and parities of the initial and final nuclear states, respectively. If we assume the isospin to be a good quantum number, the Fermi matrix element is only different from 0 for transitions taking place between component states of an isospin multiplet (i.e. between analogue states). Such transitions are called superallowed transitions. Since in general the Gamov-Teller matrix element depends on the details of the nuclear structure (with exception of the neutron decay), for our purpose the decays of the type $J_i = 0^+ \rightarrow J_f = 0^+$ are of particular interest, for which the Gamov-Teller matrix element vanishes.

3.2 Superallowed $0^+ - 0^+$ transitions

The Fermi matrix element remains to be discussed in more detail. For a $0^+ - 0^+$ beta transition between members of an isospin multiplet, we may limit ourselves to positron transitions. (β^--transitions are energy forbidden because of the Coulomb energy differences between members of an isospin multiplet).

We have:

$$M_F = \langle T, T_3 + 1 \,|T_+|\, T, T_3 \rangle = \sqrt{(T - T_3)(T + T_3 + 1)} \tag{9}$$

This result follows from the general angular momentum theory which can also be applied, as is well-known, to isospin formalism.

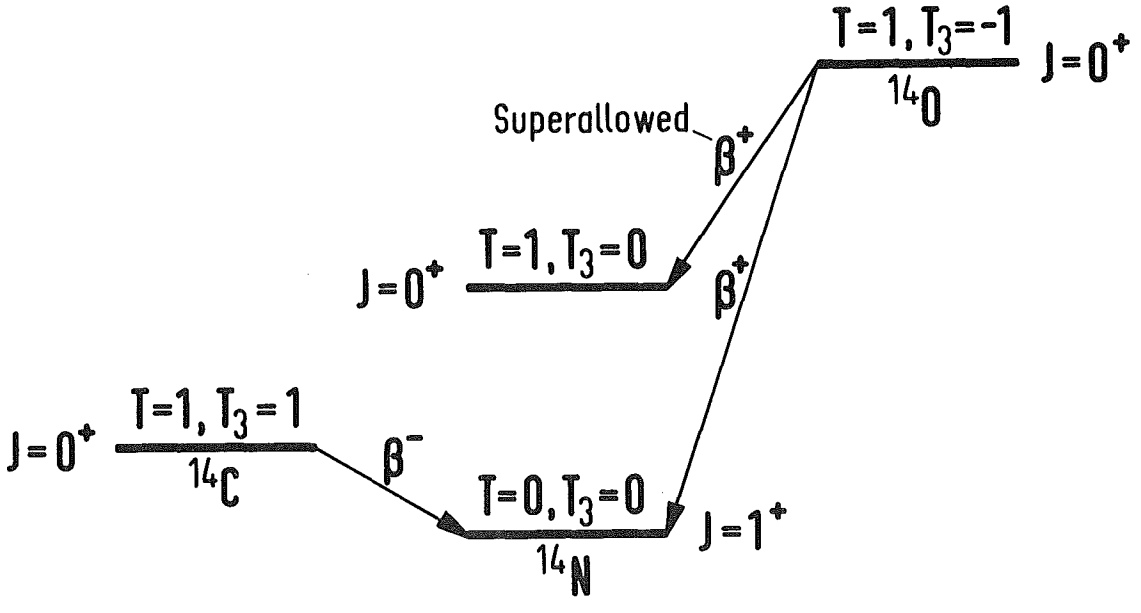

Fig. 1 Superallowed β^+ decay of ^{14}O

The essential point is that eq. 9 is completely independent of the details of the nuclear structure. So we are able to calculate the Fermi matrix element for the superallowed $0^+ - 0^+$ transitions without any ambiguity. This statement is only based on the simple assumption that we have pure isospin states in the nucleus. All the $0^+ - 0^+$ superallowed beta transitions experimentally investigated up to now, take place between member states of an isospin triplet (T = 1). An example is shown in fig. 1. In this case we obtained from eq. 9

$$M_F = \langle T = 1, T_3 = {}^{0}_{+1} \, |T_+| \, T = 1, T_3 = {}^{-1}_{0} \rangle = \sqrt{2} \tag{10}$$

We get the half t for a $0^+ - 0^+$ transition by simply integrating eq. 6

$$\tilde{ft} = \frac{K}{G_V^2 \, M_F^2} \tag{11}$$

where

$$\tilde{f} = \int_1^{p_o} \tilde{F}(Z,W) \; p^2 (W_o - W)^2 dp \quad \text{(integrated Fermi function)}$$

and

$$K = \frac{2\pi^3 (\ln 2) \, \hbar^7}{m_e^5 \; c^4}$$

$$= 2\pi^3 \ln 2 \text{ (natural units)}$$

$$= 1.230627 \cdot 10^{-94} \text{erg}^2 \text{cm}^6 \text{sec (cgs units)}$$

This means that we have to investigate experimentally the ft-values of the superallowed $0^+ - 0^+$ beta transitions if we aim to determine the vector coupling constant of the nuclear beta decay. For this purpose we have to measure the half lives (tha partial half lives) and end point energies of these types of beta transitions.

Table 3a: Experimental data for the half-lives of the superallowed transitions

Nucleus	t_o(sec)	Branching ratio %	EC/β^+-ratio (%) (according to ref. [5])	t (sec)	Ref.
C^{10}	19.42 ± 0.04	1.465 ± 0.014	0.3122	1330 ± 13	[7]
O^{14}	70.58 ± 0.035	99.328 ± 0.012	0.0914	71.122 ± 0.036	[9, 10]
Ne^{18}	1.655 ± 0.025	7.65 ± 0.26	0.0833	21.65 ± 0.79	[13]
Mg^{22}	3.857 ± 0.009	54.9 ± 1.4	0.0700	7.03 ± 0.18	[13]
Al^{26m}	6.346 ± 0.005	100.	0.0837	6.351 ± 0.005	[14, 16]
Si^{26}	2.202 ± 0.023	74.9 ± 0.9	0.0646	2.942 ± 0.046	[13]
S^{30}	1.21 ± 0.02	77.5 ± 1.0	0.0659	1.562 ± 0.036	[17]
Cl^{34}	1.526 ± 0.002	100.	0.0805	1.527 ± 0.002	[16, 12]
Ar^{34}	0.839 ± 0.01	94.3 ± 0.25	0.0694	0.890 ± 0.011	[13]
K^{38m}	0.9292± 0.0035	100.	0.0828	0.9300± 0.0035	[16]
Ca^{38}	0.439 ± 0.012	74. ± 4.	0.0722	0.594 ± 0.035	[19]
Sc^{42}	0.6837± 0.0009	100.	0.0968	0.6843± 0.0009	[16, 18]
Ti^{42}	0.202 ± 0.005	44.1 ± 1.2	0.0856	0.458 ± 0.017	[19]
V^{46}	0.4259± 0.0008	100.	0.0979	0.4263± 0.0008	[18]
Mn^{50}	0.2857± 0.0006	100.	0.1026	0.2860± 0.0006	[18]
Co^{54}	0.1937± 0.001	100.	0.1060	0.1939± 0.001	[18]

Table 3b: Experimental data for the end-point energies and ft-values (with screening) of super-allowed Fermi transitions

Nucleus	T_{3i}	E_o(keV)	Ref.	r_o(fm) according to ref. [20, 21]	ft(sec)
C^{10}	- 1	888.1 ± 1.8	[6]	1.47	3092. ± 40
O^{14}	- 1	1809.1 ± 1.5	[8]	1.32	3043. ± 11
Ne^{18}	- 1	2383. ± 5.	[11]	1.36	2931. ± 107
Mg^{22}	- 1	3120. ± 10.	[11]	1.33	3021. ± 88.
Al^{26m}	0	3210.6 ± 1.0	[14, 15]	1.28	3042. ± 5.
Si^{26}	- 1	3817. ± 4.	[13]	1.28	3023. ± 50.
S^{30}	- 1	4439. ± 11.	[11]	1.28	3085. ± 80.
Cl^{34}	0	4467.4 ± 1.9	[18, 12]	1.33	3048. ± 7.
Ar^{34}	- 1	5021. ± 12.	[11]	1.33	2993. ± 51.
K^{38m}	0	5027. ± 10	[11]	1.32	3089. ± 31.
Ca^{38}	- 1	5620. ± 19	[11]	1.32	3254. ± 200.
Sc^{42}	0	5409. ± 2.3	[18]	1.31	3086. ± 7
Ti^{42}	- 1	5964. ± 7.	[11]	1.31	3206. ± 122
V^{46}	0	6032.1 ± 2.2	[18]	1.29	3093. ± 8
Mn^{50}	0	6609. ± 2.6	[18]	1.25	3085. ± 9
Co^{54}	0	7227.7 ± 3.8	[18]	1.27	3089. ± 16.

A number of precise experiments to determine t and W_o has been carried out in the last years. In these experiments the end point energy has always been derived from the threshold of suitable reactions, for instance (p,n) reactions.

In table 3b a compilation of the experimental results for the $0^+ - 0^+$ superallowed transitions is given (see also [28]). It can be seen that the various ft-values agree with each other within 2% as is predicted by eq. 11. But if we consider the compilation more accurately we see that there is no exact agreement within the experimental error, especially between the most accurately measured values of O^{14}, Al^{26m} and Cl^{34}, and the other accurate cases of Sc^{42}, V^{46}, Mn^{50} and Co^{54}, where the error is smaller than ±16.

Looking at table 3b we see also that in four cases, both possible transitions in the isospin triplet have been measured for A = 26, A = 34, A = 38 and for A = 42. In all four cases, these ft-values agree within the margin of the experimental error.

The question arises, whether of not we can explain the small differences between the ft-values. To discuss this problem we must consider various small effects hitherto neglected. This is also necessary because we want to derive a reliable value for G_V. The experimental accuracy is of the order of a few tenths of a percent, so we have to take into account all corrections of this order of magnitude.

4. Corrections

Here we have two kinds of corrections, first the uncertainties in the calculation of the integrated Fermi function f, and second, terms and effects which we have neglected in eq. 6 and eq. 11, respectively.

4.1. Uncertainties in the integrated Fermi function

Let us begin with the uncertainties in the value of f. For the calculation of the integrated Fermi funtion f we need the electron radial wave functions $g_{-1}(r)$ and $f_{+1}(r)$ evaluated at the center of the nucleus r = 0 [5, 22] . The electron radial wave functions are solutions of the Dirac radial equations with a potential corresponding to the extended nuclear charge distribution of radius R and the charge distribution of the electron cloud (screening). There are a number of detailed calculations and tabulations of $\tilde{F}(Z,W)$ and $\tilde{f}(Z,W_o)$, respectively [5, 22, 23, 24, 25, 26, 27, 28]. The calculations carried out after 1966 are all consistent with one another, although different models for the screening by the atomic electrons have been used. It should be noted, however, that ft-values derived earlier from various available tables of Fermi and related functions show some discrepancies compared with the newer results (see the discussion in [3] and [5]). The reasons for this fact are some inconsistencies and errors in the older calculations [24, 29, 30] .

A common factor of nearly all newer calculations is that they have been carried out by solving the Dirac equation for the model of the uniform charge distribution of the nucleus (an exception is ref [28]). The question is now, whether or not there is a relevant change in the ft-values, if instead of the uniform charge distribution of the nucleus a more realistic one, like a Fermi or Gaussian distribution for instance, is used. In order to decide this, the modified Gaussian distribution has been taken [31]

$$\rho(r) = N_o \{1 + A(\frac{r}{R})^2 \} e^{-(\frac{r}{a})^2} \qquad (12)$$

where

$$N_o = \frac{8}{(2 + 3A)\, a^3 \sqrt{\pi}}$$

Since the nuclear radius R is a well-known quantity for each nucleus, only the comparison charge distributions all having the same nuclear radius have been considered. Therefore the modified Gaussian distribution effectively contains only

one free parameter A, while the other a is related to the nuclear radius R by

$$a = R \sqrt{\frac{2(2 + 3A)}{5(2 + 5A)}} \qquad (13)$$

For most of the nuclei, especially the heavier, the Fermi distribution is more appropriate than the modified Gaussian. The former distribution is much closer to the uniform distribution than the latter, and it has been shown by detailed investigations that the same is true for the values of the Fermi functions [33]. So by choosing the modified Gaussian distribution we get an upper limit for the influence of realistic charge distributions to the integrate Fermi function f. Calculated ft-values (without screening) for the uniform and the modified Gaussian distribution are shown in table 4 [31].

Table 4: $\tilde{ft}$-values (without screening) of superallowed Fermi transitions for several different nuclear charge distributions (from [31]) and $r_o = 1.2$ fm ($R = r_o A^{1/3}$)

nucleus	uniform distribution	modified Gaussian distribution		
		A = 0	A = 1	A = 2
^{14}O	3039	3039	3039	3039
$^{26}Al^m$	3037	3038	3038	3037
^{34}Cl	3043	3045	3044	3044
^{42}Sc	3080	3083	3082	3082
^{46}V	3088	3092	3091	3090
^{50}Mn	3082	3087	3086	3085
^{54}Co	3087	3093	3091	3090

For light nuclei up to Cl^{34} the difference are seen to be completely negligible. For Co^{54} in the most unfavourable case A = 0 there is a difference of 0.2%. Since the Fermi distribution is an overestimation. Therefore the true difference in the ft-value, due to the deviation of the charge distribution from the uniform, is expected to be smaller than 0.1% and is negligible (see also [32]). In addition it should be mentioned, that the dependence of f on the nuclear radius R is also small, however, a nearly correct value should be used here [22, 31], as has been done in calculating the ft-values of table 3.

4.2. Other corrections

Now we come to the terms and effects which we have neglected in eq. 6 and eq. 11. If we want to have a more accurate formula for the spectrum of a Fermi transition, eq. 6 has to be modified in some points. We obtain then [3, 34]

$$N(p)dp = \frac{1}{2\pi^3} G_V'^2 M_F'^2 \{1 + \delta_R(W,Z)\} \cdot C(W) \tilde{F}(Z,W) p^2 (W_o - W)^2 dp \qquad (14)$$

where

$$G_V'^2 = G_V^2 (1 + \Delta_R^{(V)})$$

$$M_F'^2 = 2(1 - \delta_c)$$

$\Delta_R^{(V)}$ and $\delta_R(W,z)$ are the model dependent and model independent electromagnetic radiative corrections, respectively. $\Delta_R^{(V)}$ depends on the details of the strong and weak interaction theories, while for $\delta_R(W,Z)$ this is not the case [35, 36, 37, 38] δ_c takes into account the modification of the Fermi matrix element due to isospin impurities of the initial and final nuclear states [3]. C(W) is the shape factor which contains the so-called second forbidden terms [22, 31].

By integration we receive then in the place of eq. 11

$$\tilde{ft} \cdot \overline{C(W)}(1 + \overline{\delta_R}) = \frac{K}{G_V'^2 \, M_F'^2} \qquad (15)$$

where the bars in $\overline{C(W)}$ and $\overline{\delta_R}$ denote the appropriate average over the β-spectrum functions.

4.2.1. Radiative corrections

Of the effects listed above, the most important ones are the electromagnetic radiative corrections. The other corrections are propably much smaller. Therefore we first discuss the radiative corrections. If the decay of the nucleus can be considered as the decay of one induvidual proton which is bound in the nucleus, we have for the leading electromagnetic correction terms of order α, $Z\alpha^2$ and $Z^2\,\alpha^3$, the Feynman diagrams* [38, 40, 41] shown in fig. 2. For the model independent part $\delta_R(W,Z)$ of the electromagnetic correction, i.e. the part which depends neither on the details of the strong interaction nor on the existence of an intermediate boson, we may write:

$$\delta_R(W,Z) = \delta_1(W) + \delta_2(W,z) + \delta_3(W,Z) \qquad (16)$$

where δ_1, δ_2, and δ_3 denote the terms of order α, $Z\alpha^2$, and $Z^2\,\alpha^3$, respectively.
$\delta_1(W)$ can be written as [36, 38]

$$\delta_1(W) = \frac{\alpha}{2\pi}\, g(W,W_o) \qquad (17)$$

where $g(W,W_o)$ is a well-known analytic function. The numerical values for the spectrum averaged radiative corrections $\overline{\delta_1}$, $\overline{\delta_2}$ and $\overline{\delta_3}$ are displayed in table 5 [40, 41, 42].

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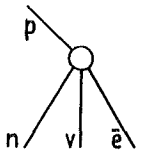
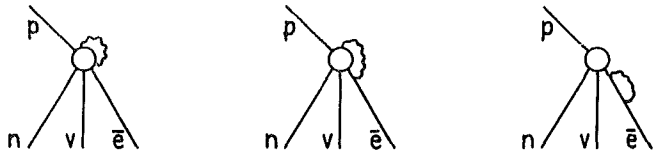
*It has been shown by Beg et al. [39] that the radiative corrections can generally be written in the form

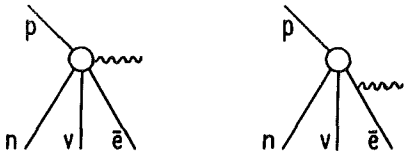
$$\overline{\delta_R} + \Delta_R^{(V)} = \sum_{\substack{m=1 \\ n=0}}^{\infty} a_{mn}\, \alpha^m (Z\alpha)^n$$

i.e. each term contains a higher power of α than of Z.

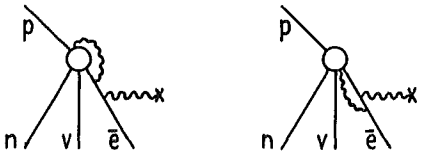


β - decay without radiative corrections

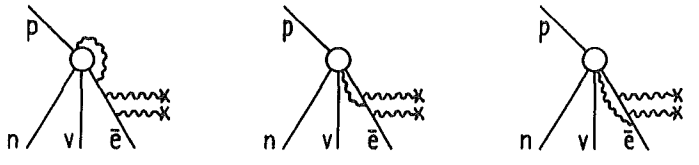




terms of order α



terms of order $Z\alpha^2$



terms of order $Z^2\alpha^3$

Fig. 2

Feynman diagrams for the radiative corrections to $p \longrightarrow n + e^+ + \nu$

Table 5: The model independent radiative corrections $\overline{\delta}_1$, $\overline{\delta}_2$ and $\overline{\delta}_3$ (from [40, 41, 42]).

Nucleus	$\overline{\delta_1}$(%)	$\overline{\delta_2}$(%)	$\overline{\delta_3}$(%)
C^{10}	1.48	0.18	0.01
O^{14}	1.30	0.26	0.02
Ne^{18}	1.22	0.33	0.03
Mg^{22}	1.13	0.41	0.04
Al^{26m}	1.12	0.44	0.05
Si^{26}	1.07	0.49	0.06
S^{30}	1.02	0.56	0.08
Cl^{34}	1.01	0.60	0.09
Ar^{34}	0.98	0.63	0.10
K^{38m}	0.98	0.67	0.12
Ca^{38}	0.94	0.71	0.13
Sc^{42}	0.95	0.75	0.14
Ti^{42}	0.92	0.78	0.16
V^{46}	0.91	0.82	0.17
Mn^{50}	0.88	0.90	0.21
Co^{54}	0.85	0.97	0.24

Up to now, an exact value for the model dependent radiative correction $\Delta_R^{(V)}$ does not exist. When current algebra methods are used, one obtains [35,37,38]

$$\Delta_R^{(V)} = \frac{\alpha}{2\pi} \{3(1 + 2\overline{Q}) \ln(\frac{\Lambda}{M_p}) - 1\} \qquad (18)$$

Then $\Delta_R^{(V)}$ depends mainly on the average charge $\overline{Q}$ of the

fundamental isodoublet underlying current algebra and on the cut-off parameter Λ. If the weak interaction is based on the exchange of an intermediate vector boson, Λ is equal to the mass of the intermediate boson M_W. Otherwise Λ will be usually set equal to the nucleon mass M_p. For the fractional quark model we have $\bar{Q} = -\frac{1}{2}$ and for the integral quark model $\bar{Q} = +\frac{1}{6}$ [38]. Note that $\Delta_R^{(V)}$ is finite for $\bar{Q}=-\frac{1}{2}$, i.e. no cut off Λ is needed. There is an other completely different calculation of $\Delta_R^{(V)}$ carried out by Källen ([44], see also [45]), who has taken into account the effects of the strong interaction through their influence on various nuclear form factors. Values of $\Delta_R^{(V)}$ for different models are given in table 6. In conclusion we can say that there are big differences between the values of $\Delta_R^{(V)}$ according to the different models. Up to now this is one of the open questions in the whole discussion on universality of weak interactions.

Table 6: Model dependent part of the electromagnetic radiative correction

	Λ/M_p	$\Delta_R^{(V)}$ (%)
Källen [44]		0.64 ± 0.26
$\bar{Q} = -\frac{1}{2}$		- 0.12
$\bar{Q} = 0$	0.1	- 0.92
	1.0	- 0.12
	5.0	0.44
	10.0	0.69
	50.0	1.25
	100.0	1.49
	150.0	1.63
	200.0	1.73
	300.0	1.87
$\bar{Q} = \frac{1}{6}$	0.1	- 1.19
	1.0	- 0.12
	5.0	0.63
	10.0	0.95
	50.0	1.70
	100.0	2.02
	150.0	2.21
	200.0	2.35
	300.0	2.53

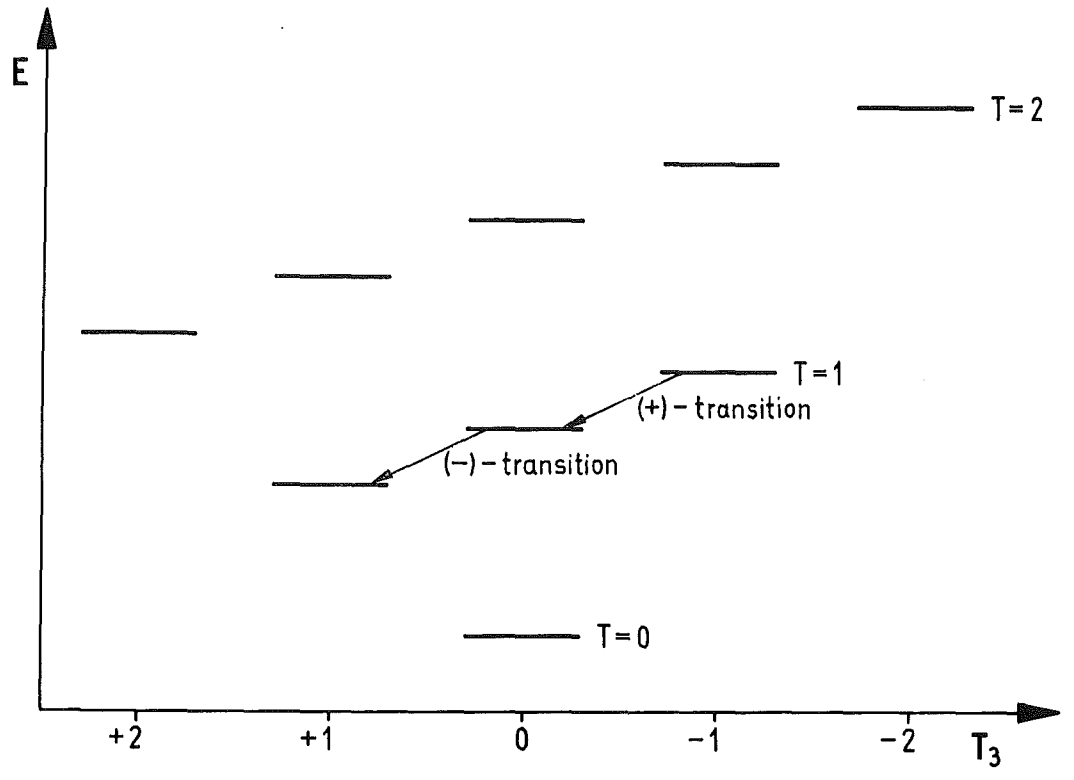


Fig. 3 Schematic representation of the energy levels for different isospin multipletts

4.2.2. Isospin impurity corrections

4.2.2.1. General relations

The next point is to consider the influence of the isospin impurities of the initial and final nuclear states (detailed reviews of this problem are given in references [3, 46, 48] where also older references can be found). Besides a small charge dependence of the nuclear forces, which can be neglected in a first approximation, the Coulomb interaction is the main source of the isospin impurities. The Coulomb potential V_c can be expressed as a sum of a scalar, vector and second rank tensor in isospin space (This is also true for the charge dependent nuclear potential). The isoscalar part of V_c leads to no isospin impurities and therefore is not of importance in this case. In the first order perturbation theory, the isovector and second rank isotensor part of the Coulomb potential is able to admix states with isospin values $T = T_o + 2$, $T_o + 1$, T_o, $T_o - 1$ and $T_o - 2$ into a state with isospin T_o (see fig. 3).

Expanding the initial and final nuclear states in eigenstates of the charge independent part of the nuclear hamiltonian, we can write for a Fermi transition from $T_i = 1$, $T_{3i} = -1$ to $T_f = 1$, $T_{3f} = 0$ (see references [3, 47])

$$\psi_i = b_o\psi_o(1, -1) + \sum_\nu \{b_\nu^{(1)}\ \psi_\nu(1, -1) + b_\nu^{(2)}\ \psi_\nu(2, -1) + b_\nu^{(3)}\ \psi_\nu(3, -1)\} \tag{19}$$

$$\psi_f = a_o\psi_o(1, 0) + \sum_\nu \{a_\nu^{(o)}\ \psi_\nu(0, 0) + a_\nu^{(1)}\ \psi_\nu(1, 0) + a_\nu^{(2)}\ \psi_\nu(2, 0) + a_\nu^{(3)}\ \psi_\nu(3, 0)\} \tag{20}$$

where in first order perturbation theory

$$a_\nu^{(T)} = \frac{< \psi_\nu\ (T, 0)|V_c|\ \psi_o(1, 0) >}{E_\nu(T) - E_o(1)} \tag{21}$$

$$b_\nu^{(T)} = \frac{< \psi_\nu(T, -1)|V_c|\psi_o(1, -1) >}{E_\nu(T) - E_o(1)} \tag{22}$$

It is now straightforward to calculate the square of the Fermi matrix element. Taking the condition that ψ_i and ψ_f are correctly normalized we get

$$M_F^{\ 2} = 2(1 - \delta_c) \tag{23}$$

where

$$\delta_c = \sum_\nu \Big|(a_\nu^{(o)})^2 + (a_\nu^{(1)} - b_\nu^{(1)})^2 + \{(a_\nu^{(2)})^2 - 2\sqrt{3}\ a_\nu^{(2)}b_\nu^{(2)} + (b_\nu^{(2)})^2\} + \{(a_\nu^{(3)})^2 - 2\sqrt{6}\ a_\nu^{(3)}b_\nu^{(3)} + (b_\nu^{(3)})^2\}\Big|$$

Thus the problem of determing δ_c is reduced to the calculation of $a_\nu^{(T)}$ and $b_\nu^{(T)}$.

Let us first consider the admixing of T = 2,3 states. From the investigation of isospin forbidden ($\Delta T = 1$ transitions) Fermi matrix elements we know that $< T + 1, T_3 | V_c | T, T_3 >$ is smaller than 60 keV [3, 34, 49]. Further, for light nuclei it is known that $\Delta E = E_\nu(2) - E_o(1) \geq 5$ MeV [50]. Combining these two estimates we get the upper limit

$$|a_\nu^{(2)}| \approx |b_\nu^{(2)}| \lesssim 10^{-2}$$

In addition, it should also be noted that we can assume

$$|a_\nu^{(3)}| < |a_\nu^{(2)}| \quad \text{and} \quad |b_\nu^{(3)}| < |b_\nu^{(2)}|$$

Thus we expect that the overall contribution to δ_c of the T = 2,3 states can be neglected (see also the discussion of the core excited states).

By looking at eq. 23 we see firstly that if only T = 0 and T = 1 admixtures contribute, δ_c is positiv. Secondly there is an effect of admixing other T = 1 states into the zero order states, if the $a_\nu^{(1)}$ are different from the $b_\nu^{(1)}$, i.e. if other T = 1 states are mixed in the initial and final nuclear states with different strength. This latter effect is called dynamic distortion.

Eg. 23 has been derived for the transition $T_{3i} = -1$ to $T_{3f} = 0$ which we denote with +, so we have now to discuss the transition $T_{3i} = 0$ to $T_{3f} = +1$, which we denote with -.
By application of the Wigner-Eckart theorem we can easily show that [13, 28]

$$(a_\nu^{(o)})_+^2 = (b_\nu^{(o)})_-^2 \qquad (24)$$

and

$$(a_\nu^{(1)} - b_\nu^{(1)})_\pm = - C_1 \pm 3C_2 \qquad (25)$$

where

$$C_1 = - \frac{\langle \psi_\nu(1) || V_c^{(1)} || \psi_o(1) \rangle}{\sqrt{6}\ |E_\nu(1) - E_o(1)|}$$

$$C_2 = \frac{\langle \psi_\nu(1) || V_c^{(2)} || \psi_o(1) \rangle}{\sqrt{30}\ |E_\nu(1) - E_o(1)|}$$

We are able to determine experimentally the difference

$$\delta_c^+ - \delta_c^- = 12 C_1\ C_2 \qquad (26)$$

between the two possible Fermi transitions in the T = 1 triplet by measuring the difference $ft^+ - ft^-$. In this way we can get an order of magnitude estimate of δ_c. Many authors have tried to calculate δ_c. By using different models the following results have been obtained:

4.2.2.2 Fermi gas model

In this model, which has been introduced by Mc Donald to estimate isospin impurities, the nucleus is approximated by an impenetrable box containing non interacting nucleons. Using this model, values of δ_c have been obtained which lie between 0.26% and 0.66% for the decays of O^{14} to Co^{54} [3, 48]. These results should be considered more as upper limits as the real values of δ_c.

4.2.2.3 Collective model

As usual, the nucleus is treated as a core plus two or more valence nucleons. Thus we have first to consider the influence of the core excited states to the isospin impurities. The following types of core excited states are of interest in our case which can be described macroscopically within the hydrodynamical model or microscopically by excited particle hole states.

(i) A 0^+ monopole state with isospin $\tau = 1$ (polarization mode) which can couple to the 0^+ $T = 1$ state of the valence nucleons and build up states with $T = 0$, 1, and 2. In the hydrodynamical model this state corresponds to a mode, where neutrons and protons move with respect to each other leaving the total density constant [51, 52].

Within the hydrodynamical model, one obtains for the excitation energy of the polarization mode [51]

$$E_o = 169\ A^{-\frac{1}{3}}\ \text{MeV}.$$

Within the one-particle-one-hole- picture by using realistic residual interactions [52], lower values up to a factor two have been obtained. As Bohr et al. [51] have shown, the polarization monopole excitation causes an isospin impurity correction δ_c, which is different from zero only, since the energy of the $T = 0$, 1 and 2 states resulting from the coupling of $T_{valence}$ and $T_{monopole}$ are split by the symmetry potential V_1/A. They find for the correction δ_c to the Fermi matrix element [51]

$$\delta_c = +\ 4\ \alpha^2(T = 1)\ \frac{V_1}{E_o A} = 2.4\ \frac{\alpha^2(T = 1)}{A^{2/3}} \qquad (27)$$

where a value of $V_1 = 100$ MeV has been used. In the past many attempts have been undertaken to calculate the admixing $\alpha^2(T_o+1)$ of the polarization monopole state in nuclear ground states with isospin T_o. To do this, different methods and models have been used, the one-particle-one-hole picture [51, 53, 54, 55] the two fluid 46 and three fluid [55] hydrodynamical model and sum rule methods [56].

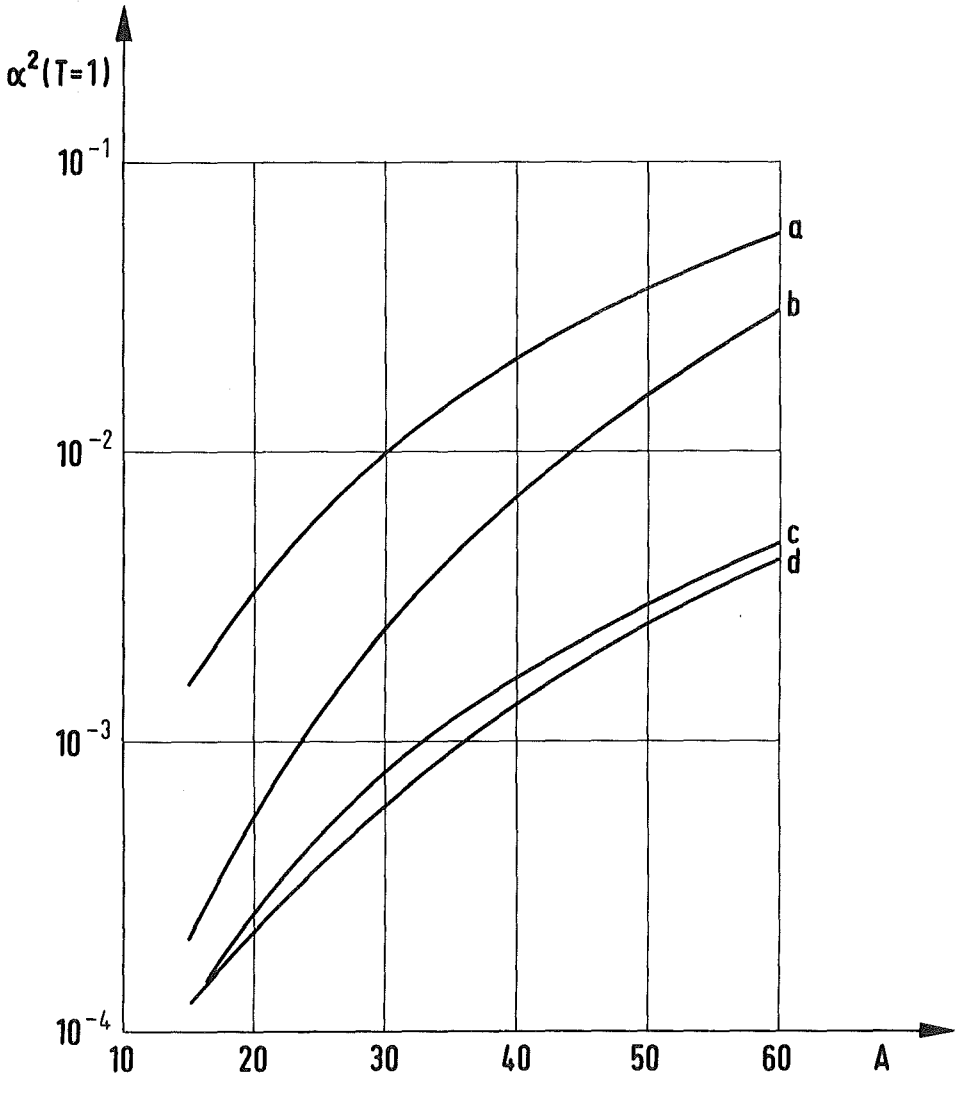


Fig. 4: Ground-states isospin impurities of N = Z nuclei in

a) the shell model without residual interactions [55]

b) the case that the mixing is entirely through the giant monopole state (sum rules techniques have been applied) [56]

c) the hydrodynamical model (two-fluid model) [51]

d) the shell model including residual interactions. [55]

The results of the most recent calculations [55, 56] are shown in fig. 4. From fig. 4 the necessity to include the residual interaction in shell model calculations is evident, otherwise the impurities would be too large by a factor of ten. Applying eq. 27 we obtain

$$\delta_c \leq 0.06\%$$

for the considered superallowed transitions from C^{10} to Co^{54}. So the admixing of the core excited polarization mode on the Fermi matrix element can be completely neglected.

(ii) A 0^+ $\tau = 0$ monopole state (breathing mode) which cannot give rise to isospin impurities, but to a dynamic distortion if it is admixed into initial and final states in different amounts i. e. $(a_\nu^{(1)} - b_\nu^{(1)})$ is different from zero.

In the hydrodynamical model this state corresponds to compressionaldilatational vibration with an excitation energy of $E_o = 75\ A^{-1/3}$ MeV [52]. The excitation energy estimation of the breathing mode however, is much more uncertain than those of the polarization mode [52, 57]. Also it is until now not possible to identify an excited 0^+ state, known from experiment, with the breathing mode. Using the breathing mode wave functions from [57] Damgaard [58] has calculated $(a_\nu^{(1)} - b_\nu^{(1)})^2$ and shown, that

$$\delta_c < 0.03\%$$

i. e. the influence of this mode is negligible.

(iii) Two-particle-two-hole (2P - 2H) or four-particle-four-hole (4P - 4H) states, respectively. The 0^+ core excited states of this type have a much lower energy than the excited 0^+ one particle one hole states considered before. We will discuss these states and their admixing on the example of the T = 1 triplet Ca^{42}, Sc^{42}, Ti^{42}, where we have two valence nucleons outside the doubly magic core of Ca^{40} (a very similar case is the triplet O^{18}, F^{18}, Ne^{18}). Detailed theoretical and experimental investigations of these nuclei [59, 60] have shown, that the second excited 0^+ T = 1 state (in Ca^{42} 1.83 MeV) has probably mainly (4P - 2H) character. Assuming that the lowest 0^+ states are a mixture of the simplest possible configurations $(1f_{7/2})^2$ and $(1f_{7/2})^4(1d_{3/2})^{-2}$ Towner [61] has derived for the dynamic distortion $(a_\nu^{(1)} - b_\nu^{(1)})^2$ a value of

$$\delta_c = 0.18\%$$

Garvey et al. [62], as well as Kennedy and McCullen [63] have tried to test this prediction experimentally by measuring the branching ratio of the β^+-decay of the Sc^{42} ground state to the 1.83 MeV state in Ca^{42}.
The latter authors reported as an upper limit for the branching ratio $R < 1.2 \cdot 10^{-4}$. This means we have

$$\delta_c < 0.1\%$$

Thus this example suggests that the isospin mixing of this type of states can be neglected. Nevertheless this aspect should be investigated more carefully in the future.

4.2.2.4 Shell model

The valence nucleons which carry the nuclear isospin require special treatment. The influence of the Coulomb interaction can be divided into two types:

(i) Configurations of the same major shell (with T = 1) are generally mixed due to the residual nuclear forces. For nuclei with two nucleons outside a doubly magic core, which we also have taken as an example before, the wave function $\psi(J,T)$ looks like [59, 60, 64, 65]

$$\left.\begin{matrix} O^{18} \\ F^{18} \\ Ne^{18} \end{matrix}\right\} \quad \psi(0,1) = a_1(1d_{5/2})^2 + a_2(2s_{1/2})^2 + a_3(1d_{3/2})^2 \qquad (28)$$

$$\left.\begin{matrix} Ca^{42} \\ Sc^{42} \\ Ti^{42} \end{matrix}\right\} \quad \psi(0,1) = a_1(1f_{7/2})^2 + a_2(2p_{3/2})^2 + a_3(2p_{1/2})^2 + a_4(1f_{5/2})^2$$

where $\sum a_\nu^2 = 1$.

The Coulomb interaction causes an additional mixing of these states.

Since we have always in the initial state one proton more than in the final the Coulomb potential and therefore also the mixing coefficients are different in the initial and final state of the $0^+ - 0^+$ β-transition [28, 66]. Or in other words, the two or three higher J = 0, T = 1 states, which can be built up from these configurations, are mixed in

the lowest state with different amounts for the three members of the isospin triplet. So we have a dynamic distortion. In a very recent and detailed calculation using the Rochester-Oak-Ridge shell model code Towner and Hardy [28] have estimated the effect on the Fermi matrix element. For δ_c they obtained the values shown in table 7.

(ii) The radial wave functions of the valence nucleons are different for neutrons and protons since the proton wave functions are dilated by the single particle Coulomb potential, i.e. by the average electrostatic field of the other protons in the nucleus. The radial overlap between the initial and final nuclear states is therefore not perfect. So we obtain a decrease in the Fermi matrix element which is equal to the decrease in overlap from 1.

To calculate this effect we can apply alternative methods [28, 46, 58]

a) One particle, for instance, the last proton can be excited by the Coulomb force in a state with the next higher radial quantum number, for example in Ti^{42} or Sc^{42} from the $1f_{7/2}$-shell to the $2f_{7/2}$-shell. The excited states of this type can have T = 1 and T = 0 (in the N = Z nucleus). The admixing ε of this excited states can be, for instance, calculated in first order perturbation theory by using the harmonic oscillator model [46, 58]. According to Damgaard [58] we obtain then

$$\delta_c = \varepsilon^2 = 2.7 \cdot 10^{-5} \frac{Z^2}{A^{2/3}} \{\nu (\nu + \ell + \tfrac{1}{2})\} \qquad (29)$$

where ν is the radial quantum number and ℓ the orbital angular momentum.

b) The radial wave functions of the valence nucleons can be calculated by solving the Schrödinger equation within a potential well, which is different for neutrons and protons because of the Coulomb potential. The remaining potential parameters well-depth, surface thickness, radius and strength of the spin-orbit term should be chosen to be the same for both the proton in the initial and the neutron in the final state.

Otherwise (if we include, for example, a finite symmetry potential) our result would not be correct, since our states then contain spurious isospin impurities, even without the Coulomb interaction [46]. As before the square of the decrease in overlap from 1 gives δ_c [58].

By applying method a (eq. 29) and method b (Woods-Saxon potentials with the parameters from [67, 68]) values for δ_c have been calculated. They are shown in table 7.

4.2.2.5 Other methods

To close the discussion of isospin impurity corrections we should mention two other attempts to estimate this effects. In the one, Jaus [69] has treated the nucleus as an elementary particle and made use partly of the technique of dispersion relations and partly of perturbation theory. He obtains a value of

$$|\delta_c| < 0.08\%$$

with a negative sign.
In the other one, Fayans [70] has used the finite Fermi systems theory and obtained

$$0.03 < \delta_c < 0.4\%$$

for the superallowed $0^+ - 0^+$ transition from O^{14} to Co^{54}.

Table 7: Isospin impurity correction δ_c

a) from charge dependent configuration mixing (from ref. [28])

b) from imperfect overlap calculated by using perturbation methods in the harmonic oscillator model (eq. 29),

c) from imperfect radial overlap calculated by using Saxon-Woods wave functions [67] or Hartree-Fock (density dependent theory) wave functions [68]

Nucleus	$\delta_{c(a)}$ %	$\delta_{c(b)}$ %	$\delta_{c(c)}$ %
C^{10}	0.001	0.036	
O^{14}	0.09	0.056	0.065
Ne^{18}	0.14	0.109	0.318
Mg^{22}	0.07	0.143	
Al^{26m}	0.05	0.152	
Si^{26}	0.04	0.178	
S^{30}	0.26	0.308	
Cl^{34}	0.19	0.226	
Ar^{34}	0.15	0.255	
K^{38m}	0.10	0.265	
Ca^{38}	0.20	0.296	
Sc^{42}	0.18	0.394	0.378
Ti^{42}	0.11	0.434	
V^{46}	0.03	0.448	
Mn^{50}	0.02	0.505	
Co^{54}	0.03	0.563	

From table 3, we have in the A = 26 and A = 34 triplet for both the mirror decays (see eq. 26)

$$A = 26 \qquad \delta_c^+ - \delta_c^- = -(0.6 \pm 3)\%$$

$$A = 34 \qquad \delta_c^+ - \delta_c^- = -(1.8 \pm 1.7)\%$$

This is in agreement with the theoretical estimations. The errors, however, are not small enough to confirm the theoretical calculations experimentally.

4.2.3 Corrections from forbidden contributions

The last correction which we have to consider is produced by the so-called second forbidden terms. These terms are responsible for the energy dependent shape factor C(W). In obtaining the vector part of eq. 6 we have neglected the relativistic parts of the beta decay hamiltonian and the variation of the electron and neutrino radial wave functions over the interior of the nucleus. In fact we have [71]

$$H_\beta^{(\nu)} = -\frac{G_\beta^{(\nu)}}{\sqrt{2}} \int J_\lambda^{o+}(x)\, L_\lambda(x)\, d^3x$$

$$+\frac{G_\beta^{(\nu)}}{\sqrt{2}} \int i(\psi_N^+\, 1\psi_P)\, L_4(x)\, d^3x$$

↘ variation of L(x) over the interior of the nucleus

$$-\int i(\psi_N^+\, \vec{\alpha}\, \psi_P)\, \vec{L}(x)\, d^3x$$

↘ relativistic terms

where $\vec{\alpha}$ is the Dirac operator.

Thus in order to know the energy dependence of C(W) we have to consider the following two types of form factor coefficient or matrix elements, respectively [22, 31, 71]

$$V_{F_{ooo}}^{(N)}(1,\ m,\ n,\ \sigma) = \langle f \left| \sum_i \phi(r_i)\ t_+^{(i)} \right| i \rangle \qquad (31)$$

$$V_{F_{o11}}^{(N)}(1,m,n,\sigma) = \langle f \left| \sum_i \phi(r_i) \frac{\vec{\alpha}^{(i)}\vec{r}_i}{R}\ t_+^{(i)} \right| i \rangle \qquad (32)$$

where

$$\phi(r_i) = \left(\frac{r_i}{R}\right)^{2N} I(1,m,n,\sigma;r_i)$$

The operators should always be applied to the i^{th} nucleon, and a summation of all nucleons must be carried out.
The functions

$$\left(\frac{r_i}{R}\right)^{2N} I(1,m,n,\sigma;r_i)$$

which essentially determine the variation of the electron radial wave function over the region of the nucleus, depend on the shape of the nuclear charge distribution [71]. Since the matrix elements just mentioned do depend on the special nuclear structure it is very important to look for a suitable way to calculate them. Fortunately we are able to simplify this problem by making use of the conserved vector current (CVC) theory. This theory says that the isovector part of the electromagnetic current and the weak current can be considered as different components of one current in the isospin space. So for our case we have [72]

$$O_\beta^{(V)} = -\frac{G_\beta^{(V)}}{e\sqrt{2}} \left[T_+,\ O_{el}\right] \qquad (33)$$

where $O_\beta^{(V)}$ and $O_{el}^{(V)}$ are the corresponding operators for the beta transition and for the electromagnetic case, respectively (e elementary charge).

If we apply equation 30 to our case of a transition between members of a T = 1 triplet we get

$$< f(1,{}^{0}_{1}) \left| O^{(V)}_{\beta} \right| i(1,{}^{-1}_{0}) > = \mp \sqrt{2}\, \frac{G^{(V)}_{\beta}}{e\sqrt{2}} < f(1,\mp 1) | O_{el} | i(1,{}^{-1}_{+1}) > \tag{34}$$

Applying this equation we obtain (see ref. [73])

$$< f(1,{}^{0}_{1}) \left| \sum_i \phi(r_i) t^{(i)}_{+} \right| i(1,{}^{-1}_{0}) > \tag{35}$$

$$= \mp \sqrt{2} < f(1,\mp 1) \left| \sum_i \phi(r_i)\, t^{(i)}_{3} \right| i(1,\mp 1) >$$

$$= \mp \sqrt{2} \{ N < \phi_n(r) > - Z < \phi_p(r) > \}$$

where N and Z are the neutron and proton numbers in the nucleus for $T_3 = \pm 1$, respectively. $< \phi(r) >$ is defined by

$$< \phi_x(r) > = \int_0^\infty \phi(r)\rho(r)\, r^2 dr \tag{36}$$

where $\rho(r)$ is the neutron (x=n) or proton (x=p) distribution normalized to unity in the nucleus. If we assume that the distributions for neutrons and protons are identical we get simply

$$F^{(N)}_{ooo}(1,m,n,\sigma) = \sqrt{2} \int_0^\infty (\frac{r}{R})^{2N}\, I(1,m,n,\sigma;r) \cdot \rho(r)\, r^2 dr \tag{37}$$

Because of $I(1,m,n,0;r) = 1$ this equation contains as a special case the well known result for the Fermi matrix element. We see the form factor coefficient $F^{(N)}_{ooo}(1,m,n,\sigma)$ does only depend on the shape of the nucleon distribution, but not on the special nuclear structure.

The second type of form factor coefficient $F^{(N)}_{o11}(1,m,n,\sigma)$ represents a relativistic matrix element. By using the CVC-theory

as in the forgoing case we can derive the following relation [58, 71, 74, 75, 76]

$$-F_{o11}^{(N-1)}(1,m,n,\sigma) = \langle f \left| \left[H_o, \sum_i \phi'(r_i)\, t_+^{(i)} \right] \right| i \rangle \qquad (38)$$

where

$$\phi'(r_i) = \int_o^r \left(\frac{x}{R}\right)^{2N-1} I(1,m,n,\sigma;x)\, dx$$

H_o is the charge independent part of the nuclear hamiltonian. For a beta transition between different member states of an isospin multiplet we get by application of eq. 38

$$-F_{o11}^{(N-1)}(1,m,n,\sigma) = \underbrace{\{E(T,T_{3f}) - E(T,T_{3i})\}}_{0} \cdot \langle f \left| \sum_i t_+^{(i)}\, \phi'(r_i) \right| i \rangle \qquad (39)$$

$$= 0$$

Even if we take into account that the initial and final nuclear states are not pure isospin states one can show that this matrix element is negligible small [31, 58, 71]. Therefore neglecting the contributions from all relativistic matrix elements and other small terms one finds [31, 71]

$$C(W) = 1 - \left\{\frac{1}{3}(WR)^2 + \frac{1}{3}(qR)^2 + \frac{2}{9}(qR)(WR)\right\}\frac{V_{F_{000}}^{(1)}}{V_{F_{000}}^{(0)}} - \frac{2}{9}(\alpha Z)(qR)\frac{V_{F_{000}}^{(1)}(1,1,1,1)}{V_{F_{000}}^{(0)}} - \frac{2}{3}(\alpha Z)(WR)\frac{V_{F_{000}}^{(1)}(1,2,2,1)}{V_{F_{000}}^{(0)}} - \frac{1}{3}(\alpha Z)^2\frac{V_{F_{000}}^{(1)}(1,2,2,2)}{V_{F_{000}}^{(0)}} \qquad (40)$$

Here $q = W_o - W$ is the neutrino energy in natural units, as usual, and Z the nuclear charge number of the daughter nucleus (for positron decays and negative value of Z has to be inserted in eq. 40). When we assume a uniform nucleon distribution in eq. 37 we have

$$\frac{V_{F_{ooo}}(1)}{V_{F_{ooo}}(o)} = \frac{3}{5}$$

$$\frac{V_{F_{ooo}}(1)\ (1,1,1,1)}{V_{F_{ooo}}(o)} = \frac{27}{35} \qquad (41)$$

$$\frac{V_{F_{ooo}}(1)\ (1,2,2,1)}{V_{F_{ooo}}(o)} = \frac{57}{70}$$

$$\frac{V_{F_{ooo}}(1)\ (1,2,2,2)}{V_{F_{ooo}}(o)} = \frac{233}{210}$$

These matrix elements are the same for all decays under consideration. Inserting them in eq. 40 we get the numerical values of the shape factor $\overline{C(W)}$ averaged over the beta-spectrum. They are listed in table 8. A very similar result has been obtained by Jaus [77] using a completely different method and a little larger result by Fayans [70] applying the finite Fermi systems theory.

Table 8: $\overline{C(W)}$-1 for the superallowed Fermi transitions |according to [31]|

Nucleus	$\overline{C(W)} - 1$ (%)
C^{10}	- 0.0235
O^{14}	- 0.0442
Ne^{18}	- 0.0790
Mg^{22}	- 0.1032
Al^{26m}	- 0.1268
Si^{26}	- 0.1452
S^{30}	- 0.1885
Cl^{34}	- 0.2120
Ar^{34}	- 0.2347
K^{38m}	- 0.2670
Ca^{38}	- 0.2915
Sc^{42}	- 0.3332
Ti^{42}	- 0.3607
V^{46}	- 0.4007
Mn^{50}	- 0.4801
Co^{54}	- 0.5522

5. Conclusions

Our final ft-values which additionally include the 'outer' radiative (to order $Z^2\alpha^3$) and the so called second forbidden corrections are listed in table 9. Corrections for isospin impurities in the initial and final nuclear states have not been applied since, as discussed before, all the calculations contain a large number of approximations and uncertainties. We are able to give the sign and an order of magnitude of δ_c only. All theoretical calculations agree that δ_c will be positive, increase

with the atomic number A and lie between $0 \leq \delta_c \leq 0.5$ %.

Table 9: ft values of the superallowed Fermi transitions including the outer radiative and the second forbidden corrections

Nucleus	ft-values (sec)
C^{10}	3143 ± 40
O^{14}	3090 ± 11
Ne^{18}	2975 ± 109
Mg^{22}	3066 ± 89
Al^{26m}	3087 ± 5
Si^{26}	3068 ± 51
S^{30}	3130 ± 81
Cl^{34}	3093 ± 7
Ar^{34}	3037 ± 51
K^{38m}	3135 ± 31
Ca^{38}	3302 ± 203
Sc^{42}	3132 ± 7
Ti^{42}	3254 ± 124
V^{46}	3139 ± 8
Mn^{50}	3131 ± 9
Co^{54}	3135 ± 16

It can be seen from table 9 that there is not exact agreement within the experimental error between the different ft-values as predicted by the theory. The last accurate ft-values (Sc^{42}, V^{46}, Mn^{50} and Co^{54}) lie significantly higher (1.5 %) than the first (O^{14}, Al^{26m} Cl^{34}). If this discrepancy can be attributed to an underestimation of the isospin impurity correction δ_c we should prefer the lowest ft-values, since we know δ_c is positive, and also the lightest nuclei, since the lower the atomic number A, the smaller δ_c. Thus for a determination of the vector coupling constant G_V the most reliable ft-values are those of O^{14}, Al^{26m} and Cl^{34}.

For these cases δ_c should be smaller than 0.5 %. If the differences in ft-values are based on experimental faults then again the values for O^{14}, al^{26m} and Cl^{34} should be chosen because these decays have been recently reinvestigated very carefully |8, 12, 14, 15|. By averaging the three ft-values of O^{14}, Al^{26m} and Cl^{34} we obtain

$$ft = 3089.1 \pm 3.8 \text{ secs}$$

By application of eq. 15 this corresponds to an effective coupling constant

$$G_V' = (1.4113 \pm 0.0009) \cdot 10^{-49} \text{ erg cm}^3 \text{ if } \delta_c = 0$$

$$G_V' = (1.4149 \pm 0.0009) \cdot 10^{-49} \text{ erg cm}^3 \text{ if } \delta_c = 0.5\ \%$$

The true effective coupling constant should be between these two limits.

To consider the question of universality of the weak interaction we have, as discussed at the beginning, to relate this value of G_V' to the coupling constant of the muon decay [78]

$$G_\mu = (1.4349 \pm 0.0003) \cdot 10^{-49} \text{ erg cm}^3.$$

These quantities are related by (see eqs. 5a and 5b)

$$\cos \Theta_V = \frac{G_V'}{G_\mu} \frac{1}{\sqrt{1 + \Delta_R^{(V)}}} \qquad (42)$$

From the discussion of radiative corrections we remember that there are different values for $\Delta_R^{(V)}$ (see table 6).
Unfortunately it is therefore not possible to give a single representative value of the Cabibbo angle Θ_V determined from β-decay, but only a relation between Θ_V and the model dependent part of the electromagnetic radiative correction $\Delta_R^{(V)}$. This relation is shown in fig. 5 for the probable range of $\Delta_R^{(V)}$ (see table 6).

This has to be compared with the Cabibbo angle derived from other decays.

As we have seen in chapter II, especially from table 2, the Cabibbo angle θ_V can be obtained by investigating the semileptonic decays of baryons. A fit of the Cabibbo angle θ_V (together with the other parameters of the Cabibbo theory) to all the experimental data has been carried out by Brene et al. [35] and by Ebenhöh et al. [79]. The result of the fit obtained by the latter authors (one angle fit) was

$$\theta_V = 0.239 \pm 0.005 \text{ radians}$$

θ_V can also be determined from the decay $K^+ \to \pi^o e^+ \nu$, but the result depends on the details of the K_{e3} form factor (an extensive discussion of this problem is given in ref [80]). The K_{e3} decay leads to the following value for the Cabibbo angle* [80]

$$\theta_V = 0.214 \pm 0.005 \text{ radians}$$

This value does not include the radiative and SU(3) - symmetry-breaking corrections [34, 80] to the K_{e3} form factor.

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*Fischbach et al. [81] have used a Kemmer equation instead of a Klein Gordon equation in order to describe the pion and kaon. They obtained another value for the K_{e3} form factor and therefore a different Cabibbo angle [82]

$$\theta_V = 0.192 \pm 0.016 \text{ radians.}$$

It is remarkable, that this value of θ_V is in excellent agreement with θ_V derived from nuclear beta decay if we take for the model dependent electromagnetic radiative correction

$$\Delta_R^{(V)} = (0.64 \pm 0.26)\%$$

the value calculated by Källen [43] (see table 6). This author has used a local model of weak interactions and has taken into account the strong interactions by including electromagnetic and weak form factors.

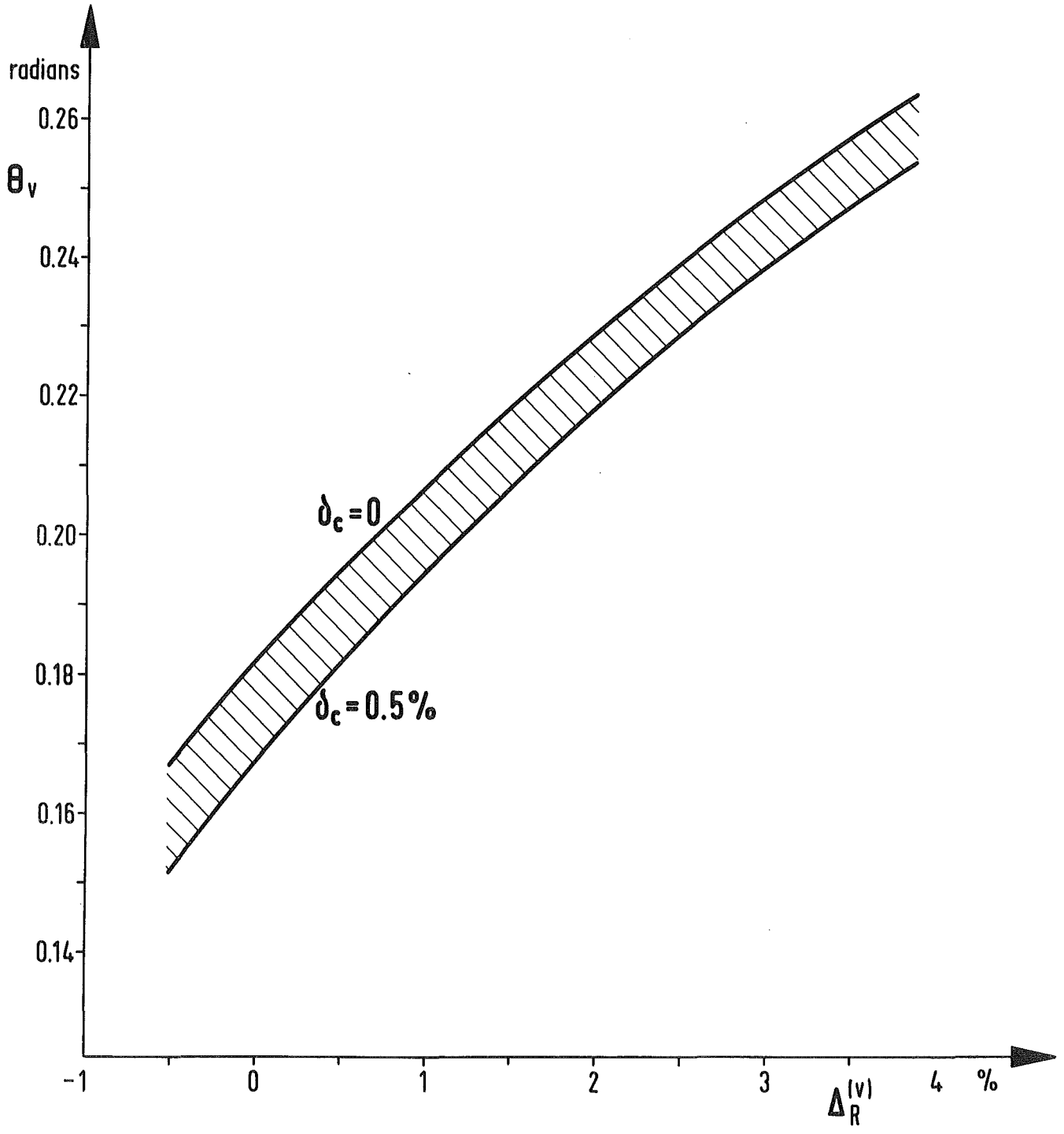

Fig. 5 Plot of the Cabibbo angle θ_V against the model dependent electromagnetic radiative correction $\Delta_R^{(V)}$

Looking at fig. 5 we see that the model dependent part of the electromagnetic radiative correction should have a value of

$$1.5\ \% < \Delta_R^{(V)} < 3.3\ \%.$$

Only in this case the Cabibbo angle Θ_V^β determined from nuclear β-decay agrees with Θ_V obtained from the analysis of the semi-leptonic decays of baryons and of the K_{e3} decay (see also ref. [34]).

As we have discussed in chap. IV (see table 6) such an order of magnitude of $\Delta_R^{(V)}$ can only be obtained by assuming a nonlocal weak interaction Hamiltonian H_W, which requires the existence of an intermediate vector boson W with a mass in the range 50-300 M_p. Thus in the moment it is not possible to test the universality of the weak interaction in the Cabibbo form as long as we have no calculation of the radiative electromagnetic corrections admitting but one interpretation.

Note added in proof:

The half-lives of V^{46}, Mn^{50} and Co^{54} have been recently remeasured by Alburger [83]. The results, which are shown in the following table, agree with previous work.

Nucleus	half-lives (sec)
V^{46}	0.4253 ± 0.002
Mn^{50}	0.2851 ± 0.0009
Co^{54}	0.1931 ± 0.0008

These results, therefore, do not change the conclusions drawn in this paper.

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