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Institut für Angewandte Systemtechnik und Reaktorphysik
Projekt Schneller Brüter

Interpretation of Substitution Experiments Performed in the Fast Critical Facilities MASURCA and SNEAK

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Interpretation of substitution experiments performed in the fast critical facilities MASURCA and SNEAK

by

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Interpretation der in den schnellen kritischen Anordnungen MASURCA und SNEAK durchgeführten Substitutionsexperimente

Zusammenfassung

Verschiedene Substitutionsexperimente, die in den schnellen kritischen Anordnungen MASURCA und SNEAK durchgeführt worden sind, wurden unter Verwendung der spektralen Synthesemethode analysiert. Die Methode wurde verwendet, um das Zweizonen-Reaktorproblem zu lösen, und die dabei erhaltene kritische Gleichung wurde auf eine geeignete Form gebracht, entsprechend derjenigen, die für thermische Reaktoren abgeleitet worden war. Die Abhängigkeit der Ergebnisse für das radiale Buckling von der Größe der substituierten Zone wurde auch untersucht.

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Interpretation of substitution experiments performed in the fast critical facilities MASURCA and SNEAK

Abstract

Several substitution experiments performed in the fast critical assemblies MASURCA and SNEAK are analyzed by using the spectral-synthesis method. The method has been applied to solve the two region reactor problem, and the critical equation obtained is given in a suitable form, analogous to the one derived for thermal reactors. The dependence of radial buckling values on the size of the substituted region has been investigated.

Contents

1. Introduction

The application of progressive substitution technique, that has been used very successfully in thermal reactor systems for material buckling determinations, makes possible the systematic study of various uranium and plutonium fuels, as well as the extrapolation of results in multizone experiments when only a small amount of the fissile material is available.

The theory, based on the use of the spectral-synthesis method (known also as the space-energy method and the overlapping-group method), for the interpretation of substitution experiments in fast media, has been derived by Storrer and Chaumont /1/, and Naudet /2/. Several substitution experiments performed in the critical assembly MASURCA in Cadarache, have been analyzed by the proposed method /3,4,5/.

In the present report, these experiments are recalculated by the more suitable approach in solving the system of non linear equations. The critical equation has been written in a very convenient form, analogous to the corresponding equation derived for thermal reactors /11/, so that instead of the linearization of a system of equations, the buckling difference between two media can be obtained by the procedure of straight line fitting. Special attention has been given in comparing the results obtained by varying the number of substitution steps in the analysis, and trying to find a criterion for choosing the representative value of the buckling differences.

The analysis of substitution experiments performed in the critical facility SNEAK in Karlsruhe, is also included.

2. Theory

In the spectral-synthesis method it is assumed that the space and energy dependent neutron flux can be approximated by the linear superposition of some specified functions $\varphi_i(E)$, i = 1,2,.....,N,

$$\Phi(\vec{r},E) \simeq \sum_{i=1}^{N} \rho_i(\vec{r})\varphi_i(E). \qquad (2.1)$$

$\varphi_i(E)$ are linearly independent trial functions, and $\rho_i(\vec{r})$ are the combining coefficients that have to be found for a given problem. E represents here the energy and $\vec{r}$ is the position vector.

2.1 General spectral-synthesis equations

The diffusion equation for a given region with a constant composition in a continuous energy notation can be written in operator form as:

$$D(E)\nabla^2\Phi(\vec{r},E) + H\Phi(\vec{r},E) = 0, \qquad (2.2)$$

where:

$$H\Phi(\vec{r},E) \equiv -\sigma_t(E)\Phi(\vec{r},E) + \int_0^\infty \sigma_s(E'\to E)\Phi(\vec{r},E')dE' +$$

$$+ \frac{\chi(E)}{\kappa} \int_0^\infty \nu\sigma_f(E')\Phi(\vec{r},E')dE' ,$$

and:

$$\kappa \equiv k_{eff}.$$

The quantities D, σ_t, σ_s and σ_f represent the diffusion coefficient, total, scattering and fission macroscopic cross sections respectively. χ is the fission spectrum and ν is the mean number of neutrons per fission.

Applying the method of weighted residuals, i.e., substituting the approximate form, Eq. (2.1), into Eq. (2.2), multiplying by arbitrary weight functions $w_j(E)$ and integrating over energy, one obtains

$$\overline{D}\nabla^2\overline{\rho} + \overline{H}\,\overline{\rho} = 0, \qquad (2.3)$$

where

$$\overline{\rho} = \text{col}\{\rho_1, \rho_2, \ldots\ldots, \rho_N\}$$

$$\overline{D} = \{D_{ji}\}$$

$$\overline{H} = \{H_{ji}\}$$

using the inner product notation, matrix elements D_{ji} and H_{ji} are defined by:

$$D_{ji} = \langle w_j \mid D\varphi_i \rangle \tag{2.4}$$

$$H_{ji} = \langle w_j \mid H\varphi_i \rangle \quad , \tag{2.5}$$

where

$$\langle w \mid A\varphi \rangle \equiv \int_0^\infty w(E)A(E)\varphi(E)dE.$$

If the index j takes on N values, and if the weight functions, $w_j(E)$, are linearly independent, Eq. (2.3) represents N coupled partial differential equations for the space functions $\rho_i(\vec{r})$. The boundary conditions for Eq. (2.3) at a material interface are derived from the standard diffusion theory boundary conditions. The continuity of flux and current across an interface specifies that:

$$\bar{\rho}- = \bar{\rho}+ \tag{2.6}$$

$$\bar{D}- \; \nabla\bar{\rho}- = \bar{D}+ \; \nabla\bar{\rho}+ \tag{2.7}$$

where the superscripts "-" and "+" refer to the left and right sides of the material boundary. Thus, the continuity of flux requires that each ρ_i be continuous across an interface, while the current is continuous in a weighted integral sense.

In order that expansion (2.1) be, as much as possible, an accurate description of the system, one has to choose proper trial functions. Series of investigations have been done in choosing various trial functions, and it has shown /6,7/ that for the core region a fundamental mode is an excellent choice. Although, for small reactors with highly enriched fuel, the asymptotic properties are in general not established in any part of the core, so that the use of fundamental modes as trial functions may lead to significant errors, very good results that have been obtained /3,4,5/, justify such a choice by interpretation of substitution experiments.

The choice of weight functions is of lesser importance. However, Kaplan /8/ has found that the variational choice is most likely to lead to the best results; this has been proved in many succeeding reports, e.g. /9,10/. The weight functions are chosen as the adjoints to the trial spectra:

$$w_j(E) = \varphi_j^*(E), \tag{2.8}$$

where the adjoint flux, $\Phi^*(\vec{r},E)$, is approximated in the same way as the neutron flux, and satisfies the adjoint equation, corresponding to Eq. (2.2), with the same boundary conditions.

2.2 Two-region two-trial function problem

It is possible to obtain solutions of Eq. (2.3) for certain simple geometries. Restricting themselves to the problem of a bare reactor consisting of two homogeneous regions, Storrer

and Chaumont /1/ applied the spectral-synthesis method in order to describe such a system with only a few parameters. Their procedure will be briefly summarized her. For two reactor regions, an expansion of the form (2.1) is terminated after the second term:

$$\Phi(\vec{r},E) \simeq \rho_1(\vec{r})\varphi_1(E) + \rho_2(\vec{r})\varphi_2(E). \tag{2.9}$$

The trial functions $\varphi_1(E)$ and $\varphi_2(E)$ are here the asymptotic spectra of the reactor regions.

Applying the same method of solution as in the two-group, two-region problem, the solutions of Eq. (2.3) can be expressed as a linear combination of two eigenvectors, $F(\vec{r})$ and $G(\vec{r})$, whose eigenvalues, for each of the two zones, are given as (i=1,2):

$$\begin{aligned} \lambda_i'^2 &= B_i^2 \\ \lambda_i''^2 &= -\mu_i^2 , \end{aligned} \tag{2.10}$$

where B_i^2 is the material buckling of the region i, and μ_i's are given by:

$$\mu_i^2 = \frac{1}{B_i^2} \cdot \frac{H_{(12)i}\, H_{(21)i} - H_{(11)i}\, H_{(22)i}}{D_{(12)i}\, D_{(21)i} - D_{(11)i}\, D_{(22)i}} \tag{2.11}$$

The mixing functions $\rho_i(\vec{r})$ can then be expressed in the following way:

$$\rho_1(\vec{r}) = \begin{cases} C_1 F_1(\vec{r}) - C_2 S_1 G_1(\vec{r}) & \text{in zone 1} \\ E_2 G_2(\vec{r}) & \text{in zone 2} \end{cases} \tag{2.12}$$

$$\rho_2(\vec{r}) = \begin{cases} C_2 G_1(\vec{r}) & \text{in zone 1} \\ E_1 F_2(\vec{r}) - E_2 S_2 G_2(\vec{r}) & \text{in zone 2} \end{cases} \tag{2.13}$$

where C_i and E_i are proportionality constants, and the coefficients S_1 and S_2 are given by:

$$S_1 = \frac{D_{(12)1}}{D_{(11)1}}$$

$$S_2 = \frac{D_{(21)2}}{D_{(22)2}} \tag{2.14}$$

These are the equations derived by Storrer and Chaumont. It is seen from Eqs. (2.12) and (2.13) that the flux in each region has two components: the fundamental and a harmonic which decreases with a relaxation length μ^{-1}.

2.3 Critical condition

For the bare, two-zone cylindrical reactor, the eigenfunctions $F_i(\vec{r})$ and $G_i(\vec{r})$ can be separated in r and z directions, where

the radial parts, $f_i(r)$ and $g_i(r)$, satisfy the corresponding eigenvalue equations with eigenvalues β_i^2 and γ_i^2 given by:

$$\begin{aligned} \beta_i^2 &= B_i^2 - \alpha_i^2 \\ -\gamma_i^2 &= -\mu_i^2 - \alpha_i^2 . \end{aligned} \tag{2.15}$$

α_i is the axial buckling of the zone i. The solutions of the eigenvalue equations, omitting the proportionality constants, can be written in terms of Bessel functions as:

$$\begin{aligned} f_1(r) &= J_o(\beta_1 r) \\ g_1(r) &= I_o(\gamma_1 r) \\ f_2(r) &= J_o(\beta_2 r) + \varepsilon Y_o(\beta_2 r) \\ g_2(r) &= I_o(\gamma_2 r) + \eta K_o(\gamma_2 r). \end{aligned}$$

The constants ε and η are determined by the requirement that the flux must vanish at the extrapolated boundary[+):

$$\begin{aligned} \varepsilon &= -\left(\frac{J_o}{Y_o}\right)_{\beta_2 R_e} \\ \eta &= -\left(\frac{I_o}{K_o}\right)_{\gamma_2 R_e}. \end{aligned}$$

+) The notation $(.....)_{\alpha R}$ means that the argument αR is common to all the Bessel functions appearing in the brackets.

Applying the equations of Storrer and Chaumont to the boundary conditions (2.6) and (2.7), at the zone interface R, Naudet /2/ has derived a critical condition of a two-zone reactor system, which can be expressed as:

$$X_2 - X_1 = \frac{S_1 S_2 - 1}{S_1 S_2} \cdot \frac{(X_2 - Y_1)\,(Y_2 - X_1)}{(Y_2 - Y_1)}\,, \qquad (2.16)$$

where the following notations have been used:

$$X_1 = -\frac{m}{\beta_1} \left(\frac{J_o}{J_1}\right)_{\beta_1 R}$$

$$X_2 = -\frac{1}{\beta_2} \left(\frac{J_o + \varepsilon Y_o}{J_1 + \varepsilon Y_1}\right)_{\beta_2 R}$$

$$Y_1 = \frac{m}{\gamma_1} \left(\frac{I_o}{I_1}\right)_{\gamma_1 R}$$

$$Y_2 = \frac{1}{\gamma_2} \left(\frac{I_o + \eta K_o}{I_1 - \eta K_1}\right)_{\gamma_2 R}\,.$$

m is the average value of the weighted ratios of diffusion coefficients, given as:

$$m_{ij} = \frac{D_{(ij)2}}{D_{(ij)1}} , \quad (i,j = 1,2).$$

In many cases diffusion coefficients $D_1(E)$ and $D_2(E)$ have nearly the same variations with energy, so that the ratio D_2/D_1 depends very little on energy, what justifies the substitution of four different coefficients, m_{ij}, by their average value.

With some rearrangements the critical condition (2.16) can be expressed as:

$$u + Sv = 1, \tag{2.17}$$

where the following symbols have been introduced:

$$S \equiv 1 - S_1 S_2$$

$$u \equiv \frac{X_1}{X_2} \tag{2.18}$$

$$v \equiv \frac{(Y_1 - X_1)\ (Y_2 - X_2)}{X_2\ (Y_2 - Y_1)} .$$

This equation, relating the interface radius to the extrapolated radius, depends on six parameters: β_1, β_2, μ_1, μ_2, m, and S. Parameter S gives indication about the migration of neutrons between the two media. According to the expressions (2.14), it is seen that the product $S_1 S_2$ does not depend on the normalization of applied spectra:

$$S_1 S_2 = \frac{\langle \varphi_1, D_1 \varphi_2^* \rangle}{\langle \varphi_1, D_1 \varphi_1^* \rangle} \frac{\langle \varphi_2, D_2 \varphi_1^* \rangle}{\langle \varphi_2, D_2 \varphi_2^* \rangle} .$$

The sign of S depends on whether the product $S_1 S_2$ is smaller or greater than one. The two regions are well adapted if S is small. For S=o, equation (2.17) reduces to $X_1 = X_2$, i.e., to the critical condition obtained by the one-group theory.

3. Application to substitution experiments

The substitution technique consists in replacing the central part of a multiplying medium, taken as a reference, by another medium whose properties are to be determined. The changes in neutron balance, produced by such a procedure, allow the study of some basic properties of the new medium. In general, one is trying to evaluate the material buckling of the substituted medium from the resulting critical configurations.

The form of the critical condition (2.17) is very suitable for the analysis of the two-zone criticals. A similar equation has already been used, very successfully, for the interpretation of substitution experiments in thermal and fast reactors /3,11/.

3.1 Buckling difference determination

Trying to evaluate the radial buckling of the substituted region, β_1, from the difference between the radial bucklings of the two regions:

$$\Delta\beta = \beta_1 - \beta_2 , \qquad (3.1)$$

where β_2 is the known radial buckling of the reference region, all the other parameters that enter into Eq. (2.17) have to be known. The radii R and R_e are known experimental data. The radius of the substituted zone, R, is directly determined by the number of substituted fuel elements N_1, as

$$R = d\sqrt{\frac{N_1}{\pi}} ,$$

where d is the lattice pitch of the reactor core. The extrapolated critical radius of the reactor, R_e, is obtained by measuring the change in reactivity and then finding the corresponding value of the critical radius, or by changing the critical radius itself, in order to reestablish the criticality of the reactor system.

The other four parameters: m, μ_1, μ_2 and S, describing the interface effect in the model, have to be calculated. Since, in most cases, the two media (the reference and the substituted one) differ only in the fissile nuclei, the parameters m_{ij} are close to one, and it is supposed that they could be calculated very precisely. The possible uncertainty in the values of these parameters, as well as their substitution by the average one, has relatively small influence on the value of $\Delta\beta$. Still less influence have the parameters μ_1 and μ_2. The way how these parameters are calculated by a multigroup procedure is described in the next section.

The parameters that plays the most important role in determining the radial buckling difference is the parameter S.

Since it influences very much the value of $\Delta\beta$, it ought to be very well known. Its sensitivity to the interface effect, however, may lead to significant errors in its computation. In thermal reactors one has already been faced with this problem, and the way out has been found in applying the progressive substitution technique. Instead of evaluating only one multizone system, one replaces stepwise the fuel elements in the reference core with new elements, starting from the reactor center. After each substitution, the produced effect is measured, as already explained, and analyzing the results it is possible to determine the buckling difference and the coupling parameters S at the same time.

For several substitution steps, one measures the critical extrapolated radius, R_{el}, for each radius R_e of the substituted zone. From the system of n critical equations (l = 1,2,....,n), it is possible to find the best solutions for $\Delta\beta$ and S, applying the method of least squares. The radial buckling of the substituted medium enters into the critical equation implicitly. In order to obtain its value by that equation, it is necessary to specify an initial value ${}^{o}\beta_1$, or ${}^{o}\Delta\beta$, and to apply the iterative procedure. However, the critical equation will not be satisfied in that core. It may then be represented as:

$$u + Sv = \rho \,, \tag{3.2}$$

where ρ differs from unity, so much as the initial value ${}^{o}\beta_1$ differs from the exact one. The calculation procedure follows than in this way: for each substitution step one measures the extrapolated radius and calculates the quantities u and v according to the Eqs. (2.18) with the initial value for buckling difference, ${}^{o}\Delta\beta$. Since u and v contain measured radii, the system of critical equations can be represented as:

$$U_1 + SV_1 = \rho + \varepsilon_1, \qquad (1=1,2,.....,n) \tag{3.3}$$

where U_1 and V_1 are calculated values of u and v, for the substitution step 1. Applying the method of least squares, i.e., minimizing the sum of squared residuals ε_1, one is able to find the best solutions for ρ and S. With the obtained value for ρ, after each iteration step k, the new input value for the buckling difference is formed in the following way:

$$^{k+1}\Delta\beta = \begin{cases} k_\rho \, {}^{k}\Delta\beta & , \text{ for } \Delta\beta > o \\ \frac{1}{k_\rho} \, {}^{k}\Delta\beta & , \text{ for } \Delta\beta < o \end{cases} \tag{3.4}$$

The iterations are stopped when the value of ρ becomes close enough to unity. The fitting procedure itself, consists in minimizing the weighted squares of residuals:

$$\Omega = \sum_{1=1}^{n} \omega_1 \varepsilon_1^{\,2} = \sum_{1=1}^{n} \omega_1 \left[(u_1-U_1)^2 + (v_1-V_1)^2 \right]. \tag{3.5}$$

The choice of the weights ω_1 is in a way arbitrary. One has to deal not only with experimental errors by determining the extrapolated radii R_{e1}, but also with errors that are the

product of the applied evaluation model, and errors arising from calculating various parameters entering the critical equation. All these errors can hardly be estimated, and in a simplified manner if can be assumed that those weights could be interpreted as differences in critical radii:

$$\omega_1 \sim | R_{e1} - R_e |, \tag{3.6}$$

where R_e is the extrapolated critical radius of the reference core. Since each measurement of the radius is done with the same absolute precision, the points with smaller change in the critical radius will thus have a smaller weight.

The standard deviations σ_ρ and σ_s of the parameters ρ and S are obtained in the usual way by the fitting procedure. It remains to determine the error of the buckling difference, $\Delta\beta$. Again, taking into account only pure mathematical grounds, since the parameter ρ was obtained by varying $\Delta\beta$, one can write:

$$\sigma_{\Delta\beta} \cdot \frac{\partial F}{\partial(\Delta\beta)} = \sigma\rho \cdot \frac{\partial F}{\partial \rho},$$

where F is the implicit form of the critical equation (2.17),

$$F\ (\beta_1,\ \beta_2,\ \mu_1,\ \mu_2,\ R,\ R_e,\ m,\ S) = 0.$$

For the series of substitution steps, one has:

$$\sigma^2_{\Delta\beta} = \sigma^2_{\rho} \cdot \frac{n}{\sum_{1=1}^{n} \left(\frac{\partial F}{\partial \beta_1}\right)^2_1} \cdot \tag{3.7}$$

From the definition of symbols X_i and Y_i, entering into the critical equation, it can be seen that only the term X_1 depends on β_1. Therefore, $\partial F/\partial \beta_1$ is given as:

$$\frac{\partial F}{\partial \beta_1} = \frac{\partial F}{\partial X_1} \cdot \frac{\partial X_1}{\partial \beta_1} ,$$

where:

$$\frac{\partial F}{\partial X_1} = \frac{1}{X_2} \left(1 - S \frac{Y_2 - X_2}{Y_2 - Y_1} \right)$$

$$\frac{\partial X_1}{\partial \beta_1} = m \cdot \frac{R}{\beta_1} \left[1 + \left(\frac{J_o}{J_1}\right)^2_{\beta_1 R} \right].$$

Denoting with:

$$X_{1\beta} \equiv \frac{1}{X_2} \cdot \frac{\partial X_1}{\partial \beta_1} ,$$

the standard deviation of the buckling difference can be written as:

$$\sigma^2_{\Delta\beta} = \sigma^2_{\rho} \frac{n}{\sum_{1=1}^{n} \left[X_{1\beta,1} \left(1 - S \frac{Y_{2,1} - X_{2,1}}{Y_{2,1} - Y_{1,1}}\right) \right]^2} \tag{3.8}$$

3.2 Reflector savings calculation

Until now, all the equations have been derived assuming the two-zone reactor, consisting of reference and substituted medium. However, fast reactor assemblies are generally always reflected by a blanket region. In principle, the described method could be easily extended to take into account more than only two regions. However, the spectrum in the blanket region is largely influenced by the leakage of neutrons from the reactor core, and this effect cannot be accounted for by the infinite-medium spectrum. Calculating blanket trial functions for the physically more realistic situation, i.e., taking the blanket center spectrum from a spatial diffusion calculation with the core leakage into the blanket, leads to complex bucklings /6/. Since such a solution in cylindrical geometry involves an infinite series of Bessel functions, instead of the two-term expression for real bucklings, it would not be possible to obtain the critical equation of such a system in a closed analytical form. Therefore, it is more practical to use two-region theory also in the case of reflected systems; one then must properly take into account the changes in reflector savings during substitutions.

For each substitution step 1 (see Fig. 1), the reflector saving δR_1, defined as:

$$\delta R_1 = R_{e1} - R_{21},$$

must be found. Applying the multigroup calculation to the given reactor system, this difference can be calculated, and it may be assumed that it can fairly well represent the experimental value:

$$(R_{e1} - R_{21})_{calc} = (R_{e1} - R_{21})_{meas}. \qquad (3.9)$$

It follows that

$$R_{el} = R_{21} + (\delta R_2)_{calc}, \qquad (3.10)$$

where R_{21} are measured critical radii of the reference zone during substitutions. These radii are obtained by addition or substraction of fuel elements at the core-reflector boundary. In order to avoid any discrepancies between the experimental and calculated values of the critical radii, one has to correlate the experimental and calculated data. This correction procedure, as pointed out in /3/, consists in adjustments of:

1) the reference medium in such a way that the calculated radial buckling be identical with the measured one. This is done by changing the mean number of neutrons per fission of the reference medium.

2) the blanket medium, so that the measured reflector saving of the reference core be equal to the calculated one (by changing the diffusion coefficient of the blanket).

3) the substituted medium for each substitution step, so that the calculated critical radius R_{21} be equal to the measured one (by changing the mean number of neutrons per fission of the substituted medium).

After these adjustments have been done, the two-zone calculation of the bare reactor gives the calculated extrapolated radii R_{el}, i.e., the calculated reflector savings δR_1. According to the Eq. (3.10), the corresponding experimental values of the extrapolated radii during substitutions are obtained.

Besides the radial reflector, the fast critical assemblies have usually an axial one, too. The method applied in the interpretation of substitution experiments is based on the assumption that the radial bucklings of the substituted and reference region remain constant during substitutions. The axial reflector savings are different for the two media taken isolately. Since the physical height of the both regions is equal and remains constant during progressive substitution procedure, the axial curvature of the neutron flux in the central region varies: at the beginning it is approximately equal to the curvature of the reference region, and it approaches the value of the substituted region alone, as the radius of the central zone becomes big enough. Since the material buckling of the region is a constant, i.e., the sum of the axial and radial buckling must remain constant during substitutions (one deals always with a critical reactor), the radial buckling of the substituted region should vary too.

The axial and radial bucklings of the reference region are supposed to remain constant, because the reference region should always be thick enough in order to essentially suppress the influence of the central region on the flux curvature. Otherwise, the applied method, taking this effect not into consideration, would lead to wrong results.

The numerical experiment performed /5/, has shown that in spite of small changes in axial buckling, the radial buckling may still be assumed to be constant during substitutions. No obserable effect on β_1 could be found. This conclusion was well supported by the excellent agreement between the directly measured values of radial bucklings and those obtained by the substitution procedure.

3.3 Multigroup calculations

After determinating the critical extrapolated radii, R_{el}, one has to calculate the values of parameters m, μ_1 and μ_2 that enter into the critical equation. As already said, the radii of the substituted zone, R_{11}, as well as the material buckling of the reference core are known quantities, and β_1 and S are the parameters to be determined.

The parameters m, μ_1 and μ_2 are expressed in terms of coefficients D_{ji} and H_{ji} given by Eqs. (2.4) and (2.5). These coefficients have to be calculated by the available multigroup cross-section data, and can be represented as:

$$D_{(ji)z} = \left\langle \varphi_j^{*g} , D_z^g \varphi_i^g \right\rangle$$

$$H_{(ji)z} = \left\langle \varphi_j^{*g} , H_z \varphi_i^g \right\rangle \qquad (i,j,z = 1,2),$$

where index z stands for the substituted and reference zone, and g denotes the energy group. Taking into account the definition of the operator H, and proceeding the calculation in G energy groups, one can write:

$$D_{(ji)z} \simeq \sum_{g=1}^{G} D_z^g \varphi_j^{*g} \varphi_i^g$$

$$H_{(ji)z} \simeq \sum_{g=1}^{G} \varphi_j^{*g} \left(-\sigma_{t,z}^g \varphi_i^g + \sum_{g'=1}^{g} \sigma_{s,z}^{g'\to g} \varphi_i^{g'} + \chi_z^g \sum_{g'=1}^{G} \overline{\nu}\sigma_{f,z}^{g'\to g} \varphi_i^{g'}\right). \qquad (3.11)$$

Introducing the concept of a "removal" cross-section:

$$\sigma^{g}_{rem} \equiv \sigma^{g}_{t} - \sigma^{g\to g}_{s} ,$$

and writing $\bar{\nu}\sigma^{g'}_{f} \equiv \bar{\nu}\sigma^{g'\to g}_{f}$, the equation for the coefficient $H_{(ji)z}$ can be written as:

$$H_{(ji)z} = \sum_{g=1}^{G} \varphi^{*g}_{j}\left(-\sigma^{g}_{rem,z}\varphi^{g}_{i} + \sum_{g'=1}^{g-1}\sigma^{g'\to g}_{s,z}\varphi^{g'}_{i} + \chi^{g}_{z}\sum_{g'=1}^{G}\bar{\nu}\sigma^{g'}_{f,z}\varphi^{g'}_{i}\right). \qquad (3.12)$$

3.4 Adjustment of the model

The applied spectral-synthesis method is based on the assumption that the spatially dependent neutron spectrum could be adequately represented by a linear overlapping combination of a few infinite medium spectra that are characteristics of the subregions of the reactor system. This assumption can hardly be justified in small systems, where generally no asymptotic properties are present in any of the regions. On the other hand, the approximation of four different m_{ij} parameters with a single value m, that was introduced in the derivation of the critical equation, may also influence the obtained results if the energetic dependence of diffusion coefficients in the two media is not nearly the same.

Calculating the changes in reflector savings during substitutions, the group constants of the reactor regions are adjusted, so that the multigroup theoretical model describes the critical system accurately. Numerical solutions of the multigroup diffusion equations can then be considered exact with respect to the applied synthesis approximation. An adjustment of the model to the multigroup calculations is thus necessary.

As already pointed out in /2/ and /12/, parameters μ, as well as parameter S, cannot be calculated accurately, since the interface effects are represented in a very simplified way. However, the buckling difference is much less sensitive to μ than to S. The parameter μ can then be chosen as an adjustment parameter.

Adjustment of the substitution region, as described previously, gives values for radial bucklings of the substituted zone for each substitution step. Critical equations, corresponding to each step of the substitution, can then be considered as equations with two unknowns: μ_1 and μ_2 (actually γ_1 and γ_2), where other parameters are obtained by multigroup calculations. Trying to solve them, an infinite set of solutions for γ_1 and γ_2 would be obtained. Indetermination can be eliminated by adding a supplementary condition:

$$\left(\frac{\mu_1}{\mu_2}\right)_{\text{adjusted}} = \left(\frac{\mu_1}{\mu_2}\right)_{\text{calculated}}, \tag{3.13}$$

as given in /3/, where the calculated values are obtained according to Eq. (2.11). Finally, the system of critical equations is to be solved for β_1 and S, applying the procedure described in Section 3.1.

4. Results of the experiments

Six different substitution experiments performed in the fast reactor assemblies MASURCA in Cadarache, and SNEAK in Karlsruhe, were analyzed by the described method. The necessary

multigroup calculations for evaluation of parameters D_{ij} and H_{ij}, as well as reflector savings during substitutions, were done using the existing NUSYS-program system with KFKINR cross-section set /13/, developed in Kernforschungszentrum Karlsruhe.

26-group diffusion theory calculations in one dimension were applied, expressing the axial leakage through the measured axial buckling of the reference core. Calculations of the various parameters entering the critical equation, adjustment of the equation to the multigroup calculation through parameters μ_1 and μ_2, and the evaluation of experiments according to the procedure described in the Section 3.1, were done with a FORTRAN-IV program SUBSTI that was written for these purposes.

Besides the progressive substitution experiments, in the assembly MASURCA, the critical experiments with core regions consisting of substituted medium only, were also made. Thus, the values obtained by the application of synthesis approximation in the treatment of substitution experiments, were tested with the directly measured ones.

In all the experiments, the substituted zone extended axially throughout the whole core.

4.1 Experiments performed in the critical facility MASURCA

The following MASURCA substitution experiments were evaluated:

1) 1B/1A', where the uranium fuel (30% ^{235}U, 70% ^{238}U) in the reference core, 1B, was replaced by the plutonium fuel (25% Pu, 74% ^{238}U, 1% Fe) giving the substituted core 1A';

2) R2/Z2, where also the uranium fuel of the reference core, R2, was replaced by the plutonium fuel, forming the substituted core Z2;

3) R2/R2 $\frac{1}{2}$ Na, where one half of the sodium present in the reactor cell was simply taken out, forming thus a void in the substituted core, R2 $\frac{1}{2}$ Na.

All the experiments are described in detail in /3/, /4/ and /5/, respectively. The experimental data for critical radii during progressive substitutions, as well as the calculated corresponding extrapolated radii, are given in Tables 3, 4 and 5. It should be noted that the data for the experiment 1B/1A' were obtained using the Karlsruhe cross-section sets, while for the other two, the results obtained in Cadarache are presented. The composition of the reactor cores 1B and 1A', used in multigroup calculations, is given in Table 1. The measured values of radial bucklings for all the investigated assemblies, by means of the fission rate traverses /12/, are given in Table 2. Expressing the results of these experiments in terms of radial buckling differences, $\Delta\beta$ (in cm^{-1}), one obtains:

$$\Delta\beta \text{ (1B/1A')} = -0.00198 \pm 0.00014$$

$$\Delta\beta \text{ (R2/Z2)} = 0.00274 \pm 0.00016$$

$$\Delta\beta \text{ (R2/R2 } \tfrac{1}{2} \text{ Na)} = -0.00233 \pm 0.00015 ,$$

where the value for the reactor system 1B/1A' is taken from /12/. These values, denoted as "measured", are to be compared with those obtained by substitution experiments and interpreted by the described spectral-synthesis approach.

4.2 Experiments performed in the critical facility SNEAK

A detailed description of the experiments performed in the assemblies SNEAK-3A and 3B is given in /15/ and /16/. In the reference uranium core 3A2, the plutonium zone was introduced giving the core 3B2. The final plutonium zone radius reached 29.9 cm, so that the 3B2 core consisted of two zones (the uranium zone radius extended to 44.9 cm). The reflector saving of the reference core 3A2 was estimated to be 13.53 cm /17/, and the radial buckling was (in cm^{-1}):

$$\beta_{ref} = 0.04133 \pm 0.00020.$$

Although these data are not known accurately, it can be expected that the buckling differences obtained by the substitution technique are well determined.

The two-zone core 3B2 served as a reference core to the partial replacement of the steel by nickel and molybdenum for the simulation of Inconel. The substitution was performed in 5 steps with a final radius of 20.59 cm. Since the evaluation method cannot be applied to multi-zone cores, it was assumed that the two-zone treatment will provide good results as long as the substituted zone does not significantly approach the plutonium-uranium interface. The whole reference core was treated as consisting of Pu-zone only, with the radial buckling of (in cm^{-1})

$$\beta_{ref} = 0.04084 \pm 0.00026 ,$$

obtained by the 3A2/3B2 substitution experiment.

The radius of the cylindrized blanket zone was in both cases 80.86 cm, and the axial buckling according to the measurements /16/ was estimated to be 0.02864 cm^{-1}. The material composition of these assemblies is given in Table 6, and the experimental data for both substitution experiments are given in Tables 7 and 8.

In the assembly SNEAK-7B /14/, here denoted as a reference core 7B0, the uranium-oxide was replaced by plutonium-uranium-oxide fuel (3% Pu), forming the substituted zone 7B1. The substitutions were performed in five successive steps with 2, 4, 6, 8 and 16 elements replaced. The cross-section of the core for the largest substituted region is given in Fig. 2. The material compositions of both core media and a surrounding reflector (radial blanket) are given in Table 9. Since no control and no shim rods were present in the central substituted region, the compositions of the two cores has been assumed as consisting of normal cell platelets only.

The effect measured after each substitution was the reactivity change of the reactor, combined with the change of the radius of the reference core. The data, obtained by the cylindrization of regions, are given in Table 10. The measurements on the reference core alone have given the radial reflector saving of 12.0 cm, and a radial buckling (in cm^{-1}):

$$\beta_{ref} = 0.04825 \pm 0.00010.$$

Two more data used in the analysis were the cylindrized radius of the radial blanket - 67.84 cm, and the measured axial buckling of the reference core - 0.03386 cm^{-1}.

The final substituted zone radius was 12.28 cm, which is approximately one third of the radius of the reference core

of 37.84 cm. Although the substituted zone was very small, the measured reactivity changes were very marked, as can be seen in Table 10.

4.3 Analysis of the results and conclusions

The results obtained for the synthesis parameters entering the critical equation, are given in Tables 11 and 12. The complete calculations were done for the SNEAK substitution experiments and the MASURCA 1B/1A' substitution. All cases show a very good spectral matching between the substituted and reference media, leading to small values of the parameter S, and very similar values of different diffusion coefficients, qualify ing their representation through the average one.

In Tables 13 to 18 are given the results of radial buckling differences obtained by varying the number of substitution steps in the analysis. Starting from the smallest number of the first three substitution steps, it can be seen that the results obtained show a tendency of having a minimum value if the substituted zone was large enough. The inconsistency in the results may be attributed to the following reasons:

1) the theory itself, since for the small test regions the choice of the asymptotic trial function is a poor one. On the other hand, for large test regions, the interaction between the test and reflector zones becomes significant, which has been neglected in the analysis.

2) approximate treatment of the radial reflector and the changes in the material composition during substitutions (because of the presence of control and shim rods).

The comparison between the values obtained for radial buckling differences and the corresponding volumes of the substituted zone, shows that the more or less stationary values of $\Delta\beta$ coincide with the replacement of about 25% to 30% of the volume of the reference core. These values show, at the same time, the best agreement with the directly measured ones, as can be seen in Table 19, where $V_{2,ref}$ stands for the volume of reference core. The ratio $(\Delta\beta)_{sub}/(\Delta\beta)_{meas}$ for various substitution steps that were taken into account, is shown in Fig. 3.

The results obtained for substitution experiments performed in MASURCA indicate a certain unreliability for the buckling difference in the SNEAK-7B experiment, since the substituted region was too small. Only 1/10 of the reference core has been replaced by the new fuel elements. It might be expected that the real value for $\Delta\beta$ should be smaller. In addition, the material buckling difference was calculated by a one-dimensional diffusion program with 26 energy groups giving

$$B_{sub} - B_{ref} = 0.00670 \text{ cm}^{-1} .$$

The asymptotic properties of the cores 3A2 and 3B2 were very similar, and the changes in the critical radius during substitutions were small. Inconsistency of the results, when only a small number of fuel elements were replaced, as can be seen in Table 16, is due to numerical difficulties arising in such cases.

From the results obtained for the 3B2/3B2-In substitution experiment, it may be concluded that there were no interactions between the substituted inconel zone and the uranium

zone. The volume of the substituted zone was below the optimum one, but the consistency of the results for the last two substitution steps, as shown in Table 17, gives a certain confidence in the value of the buckling difference.

All the results have been obtained by applying the weights defined by Eq. (3.6) in the least squares method. If these weights were omitted (ascribing thus the same weight to each measurement), one gets results which differ up to about 10% compared with those given in Table 19. Another way of finding the solutions for β_1 and S from the system of n non-linear equations by their linearization /4,5/, where no weight factors can be used, seems to be less precise.

The results given in Table 19 show a very good agreement between the directly measured bucklings, by a fission chambers technique, and those obtained by substitution experiments. The standard error of the obtained radial bucklings is smaller than 1%. As long as the spectra of the two media compared are not too different, and if the substituted to reference volume ratio has a value of about 1/4 to 1/3, the two-trial function spectral-synthesis approximation can be very successfully applied for the interpretation of substitution experiments in fast media.

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Table 1 Composition of the MASURCA core (atoms/cm^3 x 10^{24})

Element	1A'	1B	Reflector
^{235}U	$0.125020 \cdot 10^{-4}$	$0.215993 \cdot 10^{-2}$	$0.176406 \cdot 10^{-3}$
^{238}U	$0.554809 \cdot 10^{-2}$	$0.580557 \cdot 10^{-2}$	$0.418977 \cdot 10^{-1}$
^{239}Pu	$0.169669 \cdot 10^{-2}$		
^{240}Pu	$0.156480 \cdot 10^{-3}$		
^{241}Pu	$0.148408 \cdot 10^{-4}$		
^{242}Pu	$0.184742 \cdot 10^{-5}$		
Fe	$0.489409 \cdot 10^{-2}$	$0.389184 \cdot 10^{-2}$	$0.389184 \cdot 10^{-2}$
Cr	$0.122814 \cdot 10^{-2}$	$0.104502 \cdot 10^{-2}$	$0.104502 \cdot 10^{-2}$
Ni	$0.604271 \cdot 10^{-3}$	$0.877353 \cdot 10^{-3}$	$0.170971 \cdot 10^{-2}$
C	$0.564682 \cdot 10^{-1}$	$0.564682 \cdot 10^{-1}$	

Table 2 Measured radial bucklings in MASURCA cores

Core type	β (cm^{-1})
1A'	0.05039 ± 0.00033
1B	0.05237 ± 0.00037
R2	0.03520 ± 0.00009
R2 $\frac{1}{2}$ Na	0.03287 ± 0.00015
Z2	0.03794 ± 0.00009

Table 3 Experimental critical radii for MASURCA 1B/1A' substitution

Number of substituted fuel elements	Test zone radius R_1 (cm)	Critical reference zone radius R_2 (cm)	Reflector saving δR (cm)	Critical extrapolated radius R_e (cm)
0	0.	33.09	12.83	45.92
16	5.98	33.11	12.84	45.95
52	10.78	33.28	12.83	46.11
64	11.96	33.32	12.83	26.16
112	15.82	33.52	12.83	26.35
144	17.94	33.65	12.81	26.47
192	20.72	33.89	12.78	46.68
256	23.92	34.13	12.74	46.90
320	26.75	34.36	12.67	47.08

Table 4 Experimental critical radii for MASURCA R2/Z2 substitution

Number of substituted fuel elements	Test zone radius R_1 (cm)	Critical reference zone radius R_2 (cm)	Reflector saving δR (cm)	Critical extrapolated radius R_e (cm)
0	0.	47.86	20.46	68.32
16	5.98	47.58	20.46	68.04
52	10.78	47.14	20.46	67.60
144	17.94	45.98	20.46	66.44
192	20.72	45.60	20.46	66.06
256	23.92	45.05	20.43	65.48
320	26.75	44.61	20.38	64.99
548	35.00	43.78	20.00	63.78

Table 5 Experimental critical radii for MASURCA R2/R2 $\frac{1}{2}$ Na substitution

Number of substituted fuel elements	Test zone radius R_1 (cm)	Critical reference zone radius R_2 (cm)	Reflector saving δR (cm)	Critical extrapolated radius R_e (cm)
0	0.	47.77	20.55	68.32
64	11.96	47.96	20.53	68.49
112	15.82	48.10	20.52	68.62
192	20.72	48.34	20.51	68.85
256	23.92	48.50	20.51	69.01
320	26.75	48.65	20.50	69.15
548	35.00	49.22	20.50	69.72

Table 6 Composition of the SNEAK cores 3A2/3B2/3B2-In (atoms/cm^3 x 10^{24})

Element	3B2	3B2-In	3A2	Reflector
Al	$0.1274 \cdot 10^{-1}$	$0.1274 \cdot 10^{-1}$	$0.1291 \cdot 10^{-1}$	-
C	$0.956 \cdot 10^{-3}$	$0.956 \cdot 10^{-3}$	$0.932 \cdot 10^{-3}$	$0.14 \cdot 10^{-4}$
Cr	$0.3595 \cdot 10^{-2}$	$0.1888 \cdot 10^{-2}$	$0.3647 \cdot 10^{-2}$	$0.1267 \cdot 10^{-2}$
Fe	$0.1197 \cdot 10^{-1}$	$0.6199 \cdot 10^{-2}$	$0.1218 \cdot 10^{-1}$	$0.3955 \cdot 10^{-2}$
H	$0.1849 \cdot 10^{-2}$	$0.1849 \cdot 10^{-2}$	$0.1792 \cdot 10^{-2}$	-
Mg	$0.131 \cdot 10^{-3}$	$0.133 \cdot 10^{-3}$	$0.64 \cdot 10^{-4}$	-
Mo	$0.29 \cdot 10^{-4}$	$0.885 \cdot 10^{-3}$	$0.39 \cdot 10^{-4}$	$0.19 \cdot 10^{-4}$
Nb	-	$0.5 \cdot 10^{-5}$	-	-
Ni	$0.1755 \cdot 10^{-2}$	$0.9500 \cdot 10^{-2}$	$0.1854 \cdot 10^{-2}$	$0.984 \cdot 10^{-3}$
O	$0.1222 \cdot 10^{-4}$	$0.1222 \cdot 10^{-1}$	$0.1453 \cdot 10^{-1}$	-
Si	$0.254 \cdot 10^{-3}$	$0.179 \cdot 10^{-3}$	$0.188 \cdot 10^{-3}$	$0.46 \cdot 10^{-4}$
Ti	$0.38 \cdot 10^{-4}$	-	$0.40 \cdot 10^{-4}$	-
^{239}Pu	$0.1476 \cdot 10^{-2}$	$0.1476 \cdot 10^{-2}$	-	-
^{240}Pu	$0.133 \cdot 10^{-3}$	$0.133 \cdot 10^{-3}$	-	-
^{241}Pu	$0.11 \cdot 10^{-4}$	$0.11 \cdot 10^{-4}$	-	-
^{242}Pu	$0.6 \cdot 10^{-5}$	$0.6 \cdot 10^{-5}$	-	-
^{235}U	$0.56 \cdot 10^{-4}$	$0.56 \cdot 10^{-4}$	$0.2031 \cdot 10^{-2}$	$0.1625 \cdot 10^{-3}$
^{238}U	$0.8186 \cdot 10^{-2}$	$0.8186 \cdot 10^{-2}$	$0.8104 \cdot 10^{-2}$	$0.399414 \cdot 10^{-1}$

Table 7 Experimental critical radii for SNEAK 3A2/3B2 substitution

Number of substituted fuel elements	Test zone radius R_1 (cm)	Critical reference zone radius R_2 (cm)	Reflector saving δR (cm)	Critical extrapolated radius R_e (cm)
0	0.	44.66	13.53	58.19
1	3.07	44.64	13.54	58.18
5	6.86	44.59	13.54	58.13
9	9.21	44.52	13.55	58.07
21	14.06	44.47	13.55	58.02
37	18.67	44.40	13.55	57.95
57	23.17	44.34	13.54	57.88
69	25.49	44.29	13.54	57.83
71	25.86	44.29	13.54	57.83
93	29.60	44.33	13.53	57.86
95	29.91	44.33	13.53	57.86

Table 8 Experimental critical radii for SNEAK 3B2/3B2-In substitution

Number of substituted fuel elements	Test zone radius R_1 (cm)	Critical reference zone radius R_2 (cm)	Reflector saving δR (cm)	Critical extrapolated radius R_e (cm)
0	0.	44.33	14.54	58.87
1	3.07	44.36	14.55	58.91
5	6.86	44.48	14.55	59.03
9	9.21	44.59	14.55	59.14
25	15.35	45.01	14.54	59.55
45	20.59	45.43	14.54	59.97

Table 9 Composition of the SNEAK-7B core (atoms/cm^3 x 10^{24})

Element	7B1	7BO	Reflector
Al	$0.603026 \cdot 10^{-5}$	$0.120403 \cdot 10^{-4}$	-
C	$0.169621 \cdot 10^{-3}$	$0.659598 \cdot 10^{-4}$	$0.13559 \cdot 10^{-4}$
Cr	$0.293490 \cdot 10^{-2}$	$0.286026 \cdot 10^{-2}$	$0.11955 \cdot 10^{-2}$
Fe	$0.999023 \cdot 10^{-2}$	$0.995765 \cdot 10^{-2}$	$0.39549 \cdot 10^{-2}$
H	-	$0.759470 \cdot 10^{-5}$	-
Mg	$0.267494 \cdot 10^{-5}$	$0.534040 \cdot 10^{-5}$	-
Mo	$0.230847 \cdot 10^{-4}$	$0.197950 \cdot 10^{-4}$	$0.9970 \cdot 10^{-5}$
Nb	$0.930045 \cdot 10^{-5}$	$0.902870 \cdot 10^{-5}$	$0.8544 \cdot 10^{-5}$
Ni	$0.149339 \cdot 10^{-2}$	$0.146020 \cdot 10^{-2}$	$0.9845 \cdot 10^{-3}$
O	$0.332400 \cdot 10^{-1}$	$0.338377 \cdot 10^{-1}$	-
^{239}Pu	$0.218166 \cdot 10^{-2}$	$0.197406 \cdot 10^{-2}$	-
^{240}Pu	$0.212618 \cdot 10^{-3}$	$0.177338 \cdot 10^{-3}$	-
^{241}Pu	$0.182663 \cdot 10^{-4}$	$0.161162 \cdot 10^{-4}$	-
^{242}Pu	$0.152830 \cdot 10^{-5}$	$0.809970 \cdot 10^{-6}$	-
Si	$0.820402 \cdot 10^{-4}$	$0.119667 \cdot 10^{-3}$	$0.4532 \cdot 10^{-4}$
^{235}U	$0.103978 \cdot 10^{-3}$	$0.106317 \cdot 10^{-3}$	$0.162451 \cdot 10^{-3}$
^{238}U	$0.141194 \cdot 10^{-1}$	$0.145684 \cdot 10^{-1}$	$0.399401 \cdot 10^{-1}$

Table 10 Experimental critical radii for SNEAK 7B0/7B1 substitution

Number of substituted fuel elements	Test zone radius R_1 (cm)	Critical reference zone radius R_2 (cm)	Reflector saving δR (cm)	Critical extrapolated radius R_e (cm)
0	0.	37.84	12.00	49.84
2	4.34	37.62	12.04	49.66
4	6.14	37.39	12.05	49.44
6	7.52	37.14	12.06	49.20
8	8.68	36.91	12.05	48.96
16	12.28	36.04	12.05	48.09

Table 11 Calculated values of synthesis parameters for the MASURCA cores

Parameter	Substitution experiment		
	1B/1A'	R2/Z2	R2/R2 $\frac{1}{2}$ Na
S_1	1.024		
S_2	0.979		
S	-0.0024	-0.0056	0.00094
m_{11}	0.9805		
m_{12}	0.9815		
m_{21}	0.9800		
m_{22}	0.9812		
m	0.98085	0.9786	0.916
μ_1 (cm^{-1})	0.26663	0.1218	0.0517
μ_2 (cm^{-1})	0.33055	0.1579	0.0771

Table 12 Calculated values of synthesis parameters for the SNEAK cores

Parameters	Substitution experiment		
	3A2/3B2	3B2/3B2-In	7B0/7B1
S_1	0.9730	1.0643	0.8164
S_2	1.0282	0.9395	1.2255
S	-0.00047	0.000084	-0.00052
m_{11}	0.9529	1.0752	0.9909
m_{12}	0.9530	1.0753	0.9910
m_{21}	0.9528	1.0757	0.9910
m_{22}	0.9529	1.0757	0.9910
m	0.9529	1.0755	0.9910
μ_1 (cm^{-1})	0.5574	0.2262	0.1740
μ_2 (cm^{-1})	0.5970	0.0809	0.1644

Table 13 Results of the synthesis calculations.
Experiment MASURCA 1B/1A'

Number of substitution steps taken into account	Volume ratio V_1/V_2, ref	$\Delta\beta$ (cm^{-1})
3	0.131	-0.00286
4	0.229	-0.00208
5	0.294	-0.00200
6	0.392	-0.00209
7	0.523	-0.00214
8	0.654	-0.00215

Table 14 Results of the synthesis calculations.
Experiment MASURCA R2/Z2

Number of substitution steps taken into account	Volume ratio V_1/V_2, ref	$\Delta\beta$ (cm^{-1})
3	0.141	0.00285
4	0.187	0.00270
5	0.250	0.00274
6	0.312	0.00279
7	0.535	0.00287

Table 15 Results of the synthesis calculations.
Experiment MASURCA R2/R2 $\frac{1}{2}$ Na

Number of substitution steps taken into account	Volume ratio V_1/V_2, ref	$\Delta\beta$ (cm^{-1})
3	0.188	-0.00244
4	0.251	-0.00237
5	0.314	-0.00229
6	0.537	-0.00228

Table 16 Results of the synthesis calculations.
Experiment SNEAK 3A2/3B2

Number of substitution steps taken into account	Volume ratio V_1/V_2, ref	$\Delta\beta$ (cm^{-1})
3	0.043	0.000229
4	0.099	-0.000352
5	0.175	-0.000470
6	0.269	-0.000480
7	0.326	-0.000457
8	0.335	-0.000449
9	0.439	-0.000461
10	0.449	-0.000466

Table 17 Results of the synthesis calculations.
Experiment SNEAK 3B2/3B2-In

Number of substitution steps taken into account	Volume ratio $V_1/V_{2,\ ref}$	$\Delta\beta$ (cm^{-1})
3	0.043	-0.000325
4	0.118	-0.000612
5	0.213	-0.000683

Table 18 Results of the synthesis calculations.
Experiment SNEAK 7B0/7B1

Number of substitution steps taken into account	Volume ratio V_1/V_2, ref	$\Delta\beta$ (cm^{-1})
3	0.040	0.00944
4	0.053	0.00926
5	0.105	0.00819

Table 19 Comparison of the direct measured radial bucklings with those obtained by substitution experiments

Reactor system	$\Delta\beta_{dir}$ (cm^{-1})	$\Delta\beta_{sub}$ (cm^{-1})	$\beta_{1_{dir}}$ (cm^{-1})	$\beta_{1_{sub}}$ (cm^{-1})
MASURCA 1B/1A'	-0.00198±0.00014	-0.00200±0.00014	0.05039±0.00033	0.05037±0.00040
MASURCA R2/Z2	0.00274±0.00016	0.00274±0.00034	0.03794±0.00009	0.03794±0.00035
MASURCA R2/R2$\frac{1}{2}$Na	-0.00233±0.00015	-0.00229±0.00009	0.03287±0.00015	0.03292±0.00013
SNEAK 3A2/3B2	-	-0.00048±0.00017	-	0.04085±0.00026
SNEAK 3B2/3B2-In	-	-0.00068±0.00011	-	0.04017±0.00028
SNEAK-7B0/7B1	-	0.00819±0.00056	-	0.05644±0.00057

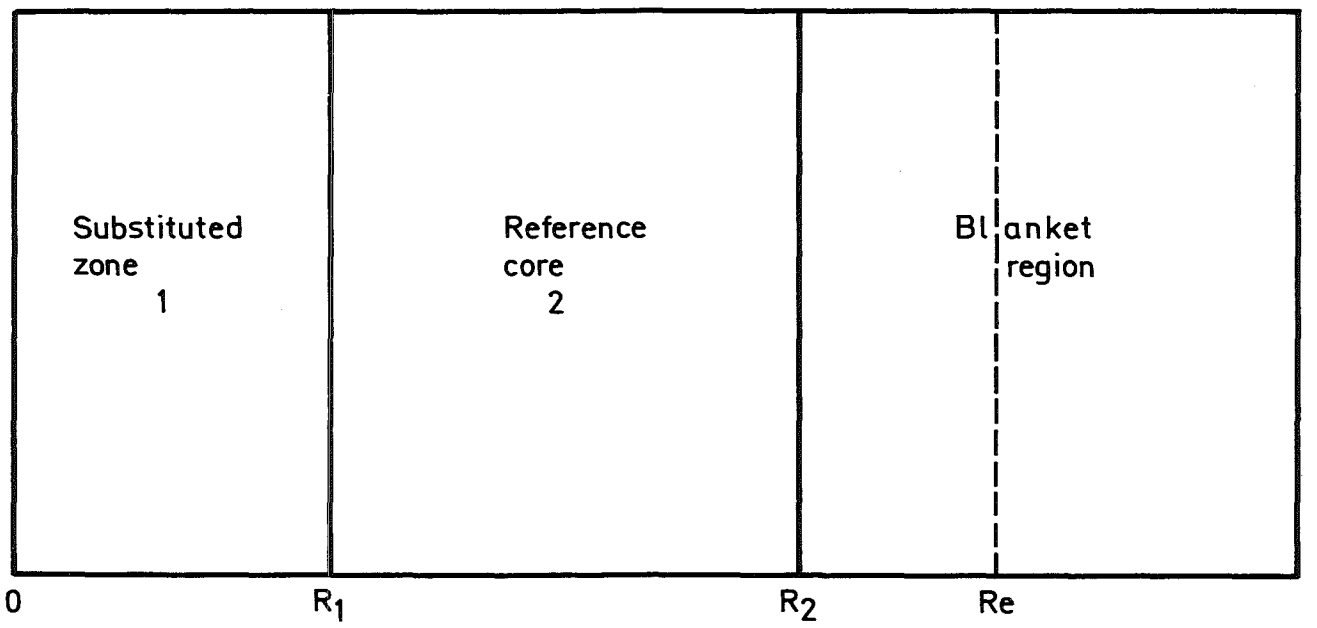

Fig.1 Geometrical Representation of the Core and Blanket for the Evaluation of Substitution Experiments

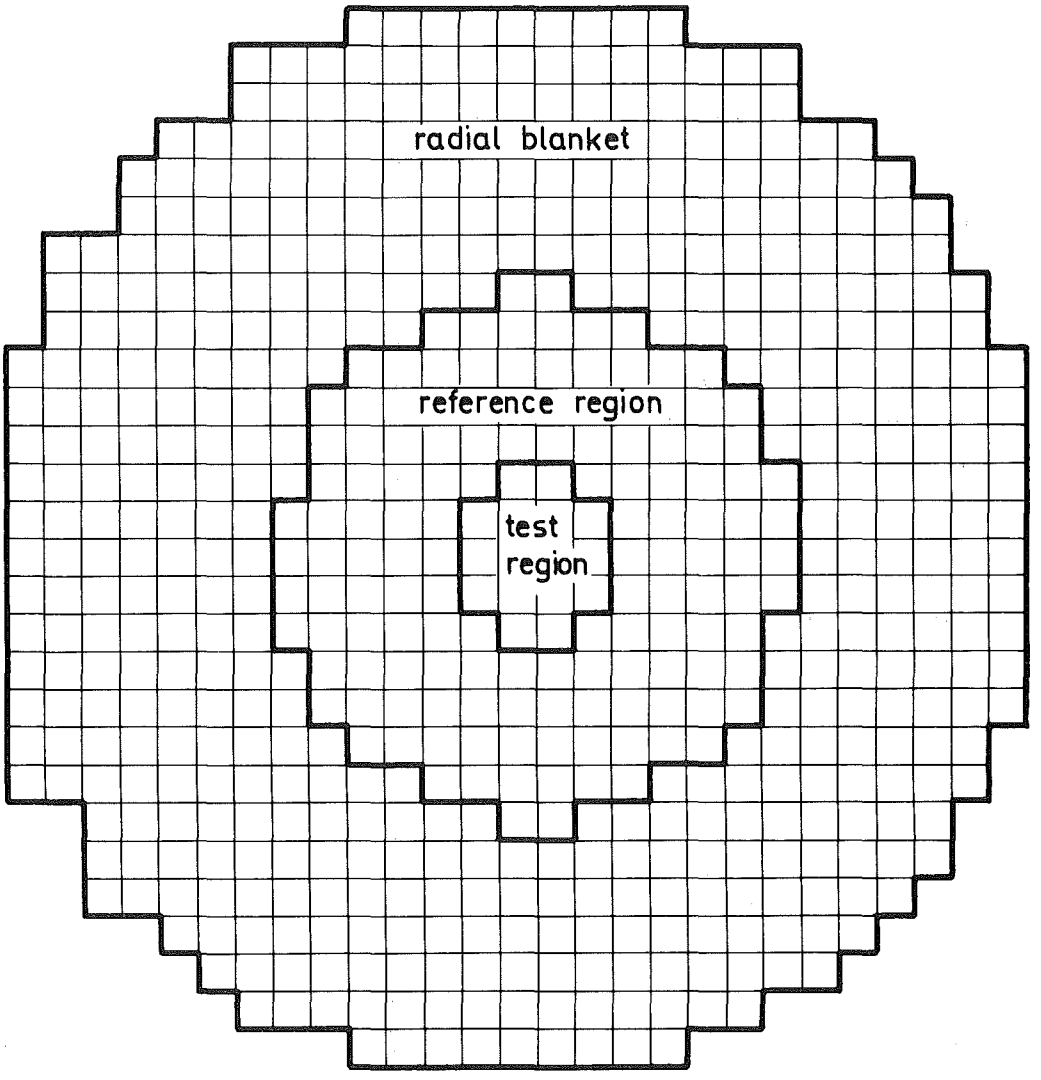

Fig. 2 Cross Section of the SNEAK-7B Core for the 16-Elements Substitution

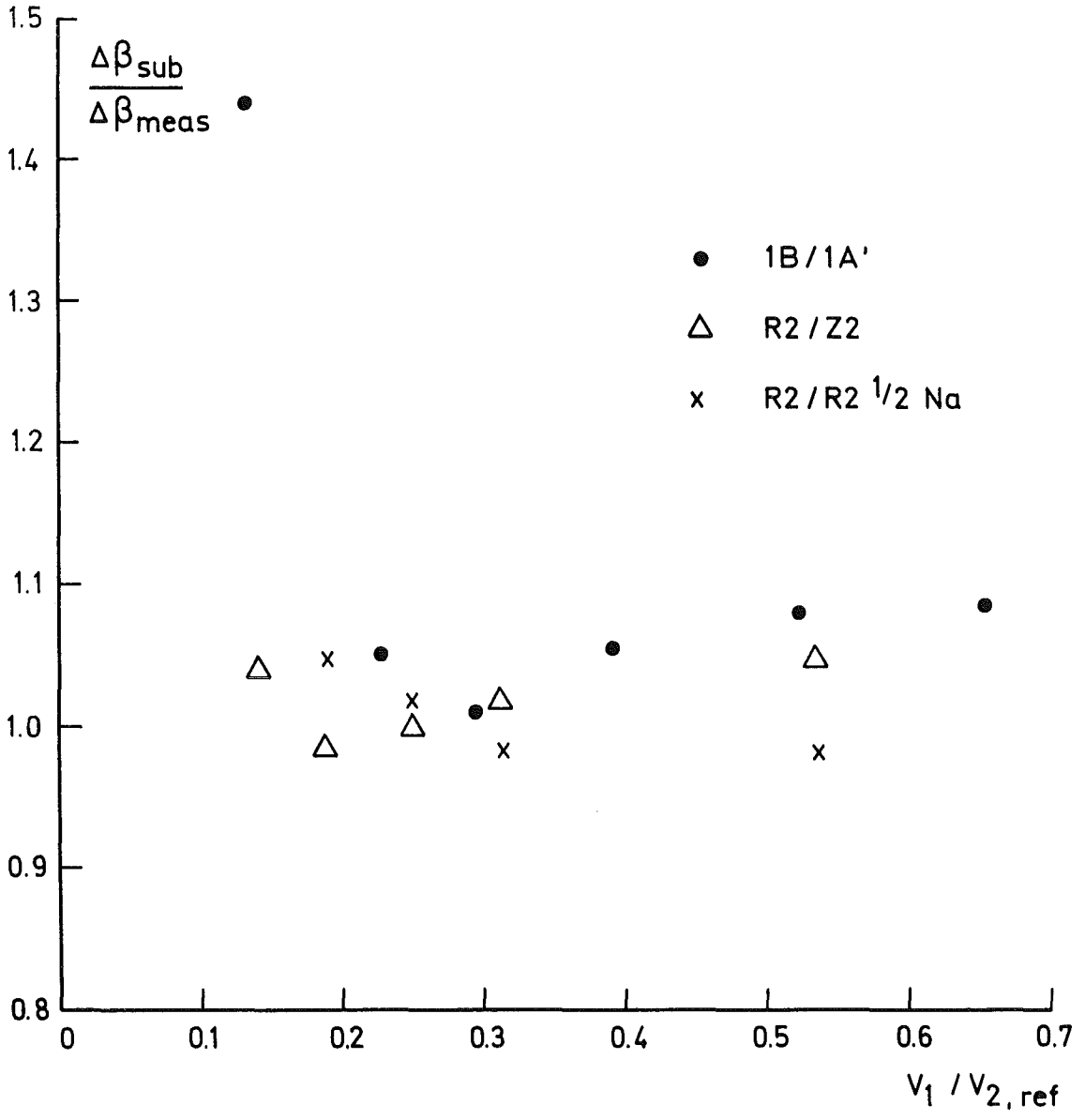

Fig.3 $(\Delta\beta_{sub} / \Delta\beta_{meas})$ Ratios for MASURCA Experiments as a Function of the Volume of the Substituted Region