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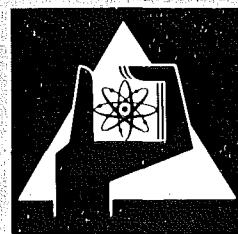
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## Deformation of $^{56}\text{Fe}$ from 104 MeV $\alpha$ -Particle Scattering

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Deformation of  $^{56}\text{Fe}$  from 104 MeV

$\alpha$ -Particle Scattering

von

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## Abstract

Differential cross sections have been measured for the elastic and inelastic scattering of  $10^4$  MeV  $\alpha$ -particles from  $^{56}\text{Fe}$ . The angular distributions are analyzed in terms of coupled channels on the basis of a symmetric and asymmetric rotator model. The analyses use a semimicroscopic folding procedure /1/ and additionally in some cases the conventional extended optical model. Deformation parameters, rms-radii, multipole moments of nuclear matter, and transition probabilities have been extracted.

## Untersuchung der Deformation von $^{56}\text{Fe}$ mit $10^4$ MeV $\alpha$ -Teilchen Streuung

## Zusammenfassung

Am Karlsruher Isochronzyklotron wurden differentielle Wirkungsquerschnitte der elastischen und inelastischen Streuung von  $10^4$  MeV  $\alpha$ -Teilchen an  $^{56}\text{Fe}$  gemessen. Die experimentellen Daten wurden durch "coupled-channel" Rechnungen auf der Basis eines symmetrischen und eines dreiachsial-asymmetrischen Rotationsmodells analysiert. Dabei wurde ein halbmikroskopisches Faltungsverfahren benutzt, das eine deformierte Nukleonendichtheverteilung zugrunde legt. Es werden mittlere quadratische Radien und innere Multipolmomente der Nukleonenverteilung berechnet und Übergangswahrscheinlichkeiten angegeben.

## 1. Introduction

Level schemes and E2-properties of nuclei in the 1f-2p-shell indicate collective features intermediate between harmonic vibrations and rigid rotations (cf Lesser et al. 1972, Towsley et al. 1972, Parikh 1972, Chandra and Rustgi 1972, Rebel et al. 1972a, 1973a, 1973b). The magnitude of  $B(E2)$ -values and the static quadrupole moments  $Q_{2+}$  of the first excited  $2^+$  states provide a sensitive measure for the deformation of the nuclear surface. According to the usual rotational model, the sign of  $Q_{2+}$  distinguishes between prolate and oblate shape of the nucleus. Electromagnetic transition probabilities and static quadrupole moments have been measured by nuclear resonance fluorescence (Kelly and Beard 1961, Metzger 1961) and Coulomb excitation experiments (Andreyev et al. 1960, Lesser et al. 1972, Towsley et al. 1972), respectively. In particular the large quadrupole moment and the E2-properties of  $^{56}\text{Fe}$  exhibit this nucleus to be a strongly deformed almost pure prolate rotator. The experimental  $B(E2; 0^+ \rightarrow 2^+)$ ,  $B(E2; 2^+ \rightarrow 4^+)$  and  $Q_{2+}$  values correspond to intrinsic quadrupole moments  $Q_0$  of  $93 \pm 8$  (Metzger 1961),  $98 \pm 1$ ,  $99 \pm 20$  and  $+87 \pm 20 \text{ e fm}^2$  (Lesser et al. 1972), derived on the basis of a symmetric rotator. Assuming a triaxial asymmetric nucleus with an asymmetry angle  $\gamma = 20^\circ$  the measured probabilities are also consistent with  $Q_0 = 102 \text{ e fm}^2$ . Davydov and Chaban (1960) explained the level scheme of  $^{56}\text{Fe}$  in the framework of an asymmetric rotator model with  $\beta$ -vibrations resulting in  $\gamma = 17^\circ$  and a softness parameter  $\mu = 0.61$ . Recently, an analysis (Rebel et al. 1974) of the level scheme and  $B(E2)$ -values of  $^{56}\text{Fe}$  was performed on the basis of the

generalized collective model worked out by Gneuss and Greiner (cf Gneuss and Greiner 1971, Gneuss et al. 1969, 1970). The resulting collective energy surface representing the potential energy  $V(\beta, \gamma)$  in an intrinsic frame and in terms of the usual shape parameters  $\beta$  and  $\gamma$  shows two minima in the range of the zero-point oscillation: one potential energy minimum for a prolate shape, the other representing an asymmetrically deformed shape.

In this paper, we report investigations of the collective behavior of  $^{56}\text{Fe}$  which use the scattering of  $10^4$  MeV  $\alpha$ -particles. First results of this experiment have been already published elsewhere (Gils et al. 1974a). The experiment aimed at specific information on the shape and intrinsic mass quadrupole moment of the  $^{56}\text{Fe}$  nucleus. Because of the strong absorption in nuclear matter medium energy  $\alpha$ -particle scattering is a surface reaction and therefore an excellent tool for the study of nuclear deformations. Previous studies of the elastic and inelastic  $\alpha$ -particle scattering (Rebel et al. 1972a, 1973b) have shown that these experiments are not only sensitive to higher order deformations such as hexadecapole components in nuclear shape but also sensitive to the sign of the deformation parameters (Rebel 1972). This is due to the interference of single- and higher-order excitation processes which influence the observed distinct diffraction patterns of the differential cross sections and which allow discrimination between prolate and oblate nuclear shapes by a coupled channel analysis.

For  $10^4$  MeV  $\alpha$ -particles the target-projectile interaction is mainly the nuclear interaction, which is conventionally parameterized as an optical potential. Though very successful in

describing the scattering cross sections such a scattering model introduces an unavoidable model dependence into the extracted information. Furthermore, the relation between spatial size and deformation of the optical potential on the one side and of the nuclear matter density distribution on the other side is not quite clear and of complex nature. However, there is considerable evidence (Rebel 1974) that a simple procedure generating the real part of the optical potential by folding the projectile-bound nucleon interaction over the nucleon distribution provides an adequate way to relate size and shape of the nucleus to the experimental cross sections, at least in the diffraction region. The results for the rms-radius  $\langle r^2 \rangle^{1/2}$  and for the intrinsic quadrupole moment  $Q_0$  quoted in this paper are extracted by such a folding model analysis of the experimental data.

## 2. Experimental procedure and results

The present ( $\alpha, \alpha'$ ) scattering experiments continue similar studies (Rebel et al. 1972a, 1973b) of the stable even Ni- and Ti-isotopes. The measurements used the scattering facilities at the 104 MeV  $\alpha$ -particle beam of the Karlsruhe Isochronous Cyclotron. The target was a self supporting  $^{56}\text{Fe}$  foil (99.5 % enrichment) of  $2.1 \pm 0.1 \text{ mg/cm}^2$  thickness. In a first run the  $\alpha$ -particle beam was not energy-analyzed in order to avoid background from slit scattering and to extend the measurements of the elastic and  $2_1^+$  inelastic cross sections on to very forward angles ( $\approx 6^\circ$  lab). For a further independent run the  $\alpha$ -particle beam was energy-analyzed to about 60 keV. The relatively large scattering chamber (130 cm diameter), a small beam spot ( $\sim 1 \text{ mm width}, \sim 2 \text{ mm height}$ )

and a beam divergence of about  $\pm 0.1^\circ$  enabled a good angular accuracy ( $\Delta\theta(\text{FWHM}) \approx 0.25^\circ$ ) of the angular distributions. The absolute zero of the angular scale has been determined by measuring the sharp diffraction minima on both sides of the incident beam. Beam currents were varied from a few nA to the order of 200 nA. The scattered  $\alpha$ -particles were detected by four telescopes of two 2 mm transmission mounted silicon surface barrier detectors. The four telescopes were rigidly mounted to the same movable arm, with a fixed angle of  $1.5^\circ$  between them.

The set-up of the electronics is shown in fig. 1. It consists of a conventional pre- and main-amplifying system for each detector branch. The biased amplifiers select and stretch a small energy interval between 0 and about 5 MeV excitation energy out of the whole  $\alpha$ -particle spectrum. The router unit sets additional bits - characterizing the origin of the converted pulse - to the on-line computer. The computer is programmed as a multichannel pulse height analyzer, and it stores the events in four 1K-memories one for each detector. The overall energy resolution was about 120-200 keV FWHM.

Particle identification was found not to be necessary because the maximum energy loss of protons, deuterons, tritons and  $^3\text{He}$  particles in 4 mm silicon is 26, 37, 43 and 93 MeV, respectively. So the ( $Z=1$ )-particles can only cause pulses in a region of the spectrum which is cut off by the biased amplifier. Due to the large difference in binding energies between  $^3\text{He}$  and  $^4\text{He}$  the maximum energy of  $^3\text{He}$  particles produced is considerably lower than that of the elastically scattered  $\alpha$ -particles.

Accidental coincidences between two or more detectors which may cause a wrong pulse height in the ADC were found to be negligible at the beam currents used. Therefore, it was not necessary to use a special set-up of the routing unit which could suppress such pile-up events but which is more difficult to handle and to adjust. The measurements at very forward scattering angles were carried out with one detector only. More details of the experimental arrangement and the data handling have been described in previous papers (Rebel et al. 1973b, 1972a,b).

As an example fig. 2 shows an energy spectrum of the outgoing  $\alpha$ -particles. For forward angles there are also contamination peaks from  $^{12}\text{C}$  and  $^{16}\text{O}$  which walk out of the region of interest (up to about 2.7 MeV) beyond  $25^\circ$ . In a detailed analysis of the energy spectra the continuous background was subtracted and the line shapes were fitted by an asymmetric Gaussian form thus separating in most cases the contribution of interfering contamination peaks. In fig. 2 the dashed line indicates the contributions of single peaks to the total spectrum.

The measured differential cross sections for the elastic and inelastic scattering to the first  $2^+$  and  $4^+$  states and to the second  $2^+$ -state are shown in fig. 3. The data were taken in  $0.5^\circ$  steps up to about  $50^\circ$  (lab) where the oscillation pattern is clearly damped. The quoted error bars include the uncertainty due to finite angular acceptance which has been transformed into a cross section error. Especially in the region of the deep minima this "angular uncertainty" is dominating when compared to the statistical error which is smaller than 1 % for the elastic cross

section at  $\theta < 30^\circ$  even at the minima. From purely experimental considerations (foil thickness, detector solid angle, beam current) the absolute scale of the cross section is determined to within ca. 10 %. Finally we have normalized our data at most forward angles to the optical model results of elastic scattering as optical model calculations show very little variation in magnitude at small angles for any reasonable description of the elastic scattering cross section.

### 3. Elastic Scattering Analysis

The conventional optical model analysis of the elastic scattering using the standard Saxon-Woods-form (with volume absorption) for the nuclear potential

$$U(r_\alpha) = - V_0 (1 + e^{x_v})^{-1} - i W_0 (1 + e^{x_w})^{-1}$$
$$x_{v,w} = (r_\alpha - r_{v,w} \cdot A^{1/3}) / a_{v,w} \quad (3.1)$$

and a Coulomb potential from a uniformly charged sphere with  $R_c = 1.3 \cdot A^{1/3}$  results in

$$J_R = \frac{V_0 \pi}{A} \int_0^\infty f(r_\alpha) r_\alpha^2 dr = \left| \frac{4}{3} V_0 \pi R_v^3 \left[ 1 + \left( \frac{\pi a_v}{R_v} \right)^2 \right] / 4A \right| = 318.6 \text{ MeV fm}^3$$

based on the parameter set given in tab. 1.

As known from previous studies such a parameter set seems to be favored for higher  $\alpha$ -particle energies. In the present analysis we did not look very systematically for alternative parameter families. In general, studies of the ambiguities require data of a larger angular range exceeding the diffraction region. This was out of the scope of our present investigation.

A more microscopic description of the elastic  $\alpha$ -particle-nucleus scattering starts with the intuitive expression

$$U_R(\vec{r}_\alpha) = \int \rho_m(\vec{r}) V_{\text{eff}}(\vec{r}_\alpha - \vec{r}) d^3r \quad (3.2)$$

for the real central interaction potential, where  $\rho_m(\vec{r})$  is the nucleon (centers) density distribution and  $V_{\text{eff}}(\vec{r}_\alpha - \vec{r})$  an adequate effective  $\alpha$ -particle-bound nucleon interaction. We use the Gaussian interaction proposed by Bernstein (1969) (cf Bernstein and Seidler 1971, 1972)

$$V_{\text{eff}} = \lambda_R V_0 \exp(-|\vec{r}_\alpha - \vec{r}|^2/\mu^2) \quad (3.3)$$

with  $V_0 = 37$  MeV and  $\mu = 2.0$  fm. The parameter  $\lambda_R$  takes into account the "renormalization" due to the presence of the other bound nucleons. Its value is determined phenomenologically thus absorbing some uncertainties. Preceding investigations (Bernstein and Seidler 1971, Lerner et al. 1972) on  $\alpha$ -particle scattering from  $^{40}\text{Ca}$  yielded  $\lambda_R = 0.92 \pm 0.1$  for  $\alpha$ -particles of  $E_{\text{lab}} = 104$  MeV. We repeated such studies of  $\lambda_R$  on the basis of our own data for seven ( $N=Z$ )- nuclei (Gils and Rebel 1974b) resulting in a value  $\lambda_R = 0.86 \pm 0.06$ . However, as discussed by Gils and Rebel (1974b) it seems reasonable to us finally to fit  $\lambda_R$  simultaneously with size and shape parameters for every individual nucleus. For our analysis the nucleon density distribution is assumed to have a Fermi form

$$\rho_m = \rho_0 [1 + \exp((r - c_m)/a_m)]^{-1} \quad (3.4)$$

and to be identical for neutrons and protons ( $\rho_m \equiv \rho_p \equiv \rho_n$ ). The folded potential  $U_R$  (eq. 3.2) replaces the real part of the standard optical potential (eq. 3.1); in most cases the imaginary

potential representation of eq. 3.1 remains unchanged (procedure A). In order to get a feeling of the influence of this representation we used alternatively Bernstein's representation (procedure B)

$$U_I(r_\alpha) = (\lambda_I/\lambda_R) U_R(r_\alpha) \quad (3.5)$$

The results are compiled in tab. 1 and displayed in fig. 4. In the first run, the matter parameters  $c_m$  and  $a_m$  were fixed to corresponding values of the charge distribution. Elton and Swift (1964) investigated the charge density distribution of  $^{56}\text{Fe}$  by electron scattering. Assuming a two-parameter Fermi shape (eq. 3.4) they obtained

$$\langle r^2 \rangle_{ch}^{1/2} = 3.75 \text{ fm} \quad c_{ch} = 4.00 \text{ fm} \quad a_{ch} = 0.57 \text{ fm}$$

These values  $c_{ch}$  and  $a_{ch}$  can be related to the corresponding parameters of a Fermi shaped nucleon (= proton) density distribution in the following way: The rms-radius  $\langle r^2 \rangle_{ch}^{1/2}$  of the charge distribution is converted to the rms-radius of the proton (centers) distribution  $\langle r^2 \rangle_p^{1/2}$  via the relation

$$\langle r^2 \rangle_{ch} = \langle r^2 \rangle_p + \langle r^2 \rangle_{s.p.} \quad (3.6)$$

where  $\langle r^2 \rangle_{s.p.}^{1/2} = 0.8 \text{ fm}$  is the rms-radius of the charge distribution of a single proton. The difference in rms-radiis of the charge distribution and of the distribution of the protons'centers now can be ascribed to a change in the diffuseness value  $a_{ch} + a_m$  setting  $c_m = c_{ch}$ , or to a change of  $c_{ch} + c_m$  fixing  $a_{ch} = a_m$ , alternatively. Using  $a_m = a_{ch} = 0.57 \text{ fm}$  the values extracted from electron scattering correspond to

$$c_m = 3.85 \text{ fm} \quad \text{and} \quad \langle r^2 \rangle_m^{1/2} = 3.66 \text{ fm}$$

With that fixed values the parameters  $W_o$ ,  $r_w$  and  $a_w$  were varied to find a minimum in  $\chi^2$  per degree of freedom.

In the second step we did not restrict us to matter values given but we allowed both  $c_m$  and  $a_m$  (and finally  $\lambda_R$ , too) to vary thus determining these parameters independent from other measurements. Detailed studies showed (Gils et al. 1974a, 1974b) that the differential cross sections are not sensitive to the particular choice of  $c_m$  and  $a_m$ , provided that the combination reproduces the correct value of the rms-radius.

The folding model analysis of elastic scattering results in

$$c_m = 3.84 \pm 0.06 \text{ fm} \quad a_m = 0.57 \pm 0.01 \text{ fm}$$

$$\langle r^2 \rangle_m^{1/2} = 3.64 \pm 0.06 \text{ fm}$$

The value of the rms-radius  $\langle r^2 \rangle_m^{1/2}$  of the nucleon distribution corresponds to  $\langle r^2 \rangle_{nm}^{1/2} = 3.73 \pm 0.06 \text{ fm}$  for the nuclear matter distribution which is the quantity comparable to the charge distribution.

#### 4. Inelastic Scattering

The macroscopic approach to the description of inelastic scattering is based on the assumption that the interaction potential follows the deformation of the nuclear surface. In the traditional "extended optical model" the interaction potential is represented by a deformed complex potential the size and shape of which is parametrized by introducing an expansion in spherical harmonics for the radius parameter ( $r_{v,w}$  in expression (3.1)). Since we are mainly interested in the deformation of the nuclear surface

rather than of the optical potential, we applied the folding concept to the inelastic scattering analysis, too. For this purpose we introduced into the relation (3.2) a deformed nucleon density distribution with an angular dependent nuclear radius. The nucleon density distribution was assumed to have Fermi shape and the Gaussian form of the effective interaction (eq. 3.3) with the parameter values  $\lambda_R$ ,  $V_0$  and  $\mu$  given in sect. 3 was used. For the imaginary part of the optical potential we chose the macroscopic Saxon-Woods form enabling the geometry and depth to be independent of the real potential. The deformation of the imaginary potential was fixed to the values resulting from the calculation using the extended optical potential described above.

The calculation of the differential cross sections for the various states was performed by a coupled channel method. It takes explicitly into account the higher order excitation processes due to the strong coupling among the states. Coulomb excitation is considered by a potential which is deformed corresponding to the nuclear one. The numerical calculations were carried out with a modified version (Schweimer and Raynal 1973) of the code ECIS which includes the fitting procedure.

#### 4.1. Symmetric Rotational Model

The axially symmetric rotator model was applied to the scattering from the  $0^+$  (ground state),  $2_1^+$  and  $4_1^+$  states at 0., 0.847 and 2.09 MeV excitation energy. The half-way radius  $R$  of the complex potential (Saxon-Woods form) or of the nucleon density distribution is parametrized in the body fixed system by

$$R(\omega) = R_0(1 + \beta_2 Y_{20}(\omega) + \beta_4 Y_{40}(\omega)) \quad (4.1)$$

The differential cross sections were simultaneously fitted in a  $0^+_1-2^+_1-4^+$  coupling scheme, starting from the results obtained in the elastic scattering analysis. The result is shown in fig. 5, the final parameter values are quoted in table 2. The results indicate prolate deformation of the  $^{56}\text{Fe}$  nucleus with a very small hexadecapole contribution. As expected the folding model requires smaller values for the size and shape parameters  $c$ ,  $a$  and  $\beta_2$ . A more detailed study on prolate-oblate effects and on the dependence of the deformation parameters  $\beta_2$ ,  $\beta_4$  from adopted size parameters  $c$ ,  $a$  is given in ref. (Gils et al. 1974a). From tab. 2 we extract the rms-radius  $\langle r^2 \rangle_m^{1/2}$  and the intrinsic quadrupole ( $Q_{20}$ ) and hexadecapole ( $Q_{40}$ ) moments calculated for the real potential and the nucleon density distribution.

$$\begin{aligned} \langle r^2 \rangle_{\text{Pot}}^{1/2} &= 4.81 \pm 0.05 \text{ fm} & Q_{20}^{\text{Pot}} &= +116 \pm 5 \text{ e fm}^2 & Q_{40}^{\text{Pot}} &= +570 \pm 50 \text{ e fm}^4 \\ \langle r^2 \rangle_m^{1/2} &= 3.70 \pm 0.04 \text{ fm} & Q_{20}^m &= +95 \pm 4 \text{ e fm}^2 & Q_{40}^m &= +450 \pm 40 \text{ e fm}^4 \end{aligned}$$

In order to compare these results with charge rms-radii and multipole moments we again convert the values to values of a nucleon matter distribution according to eq. (3.6). Now the difference in rms-radii can be ascribed to a change of  $c_m$ ,  $a_m$  or of the deformation parameters  $\beta_1$ . For simplicity and consistency we assume the deformation and diffuseness to be unchanged and change only the half-way radius. Thus we obtain

$$\langle r^2 \rangle_{\text{nm}}^{1/2} = 3.78 \pm 0.04 \text{ fm} \quad Q_{20}^{\text{nm}} = +100 \pm 4 \text{ e fm}^2 \quad Q_{40}^{\text{nm}} = +500 \pm 40 \text{ e fm}^4$$

#### 4.2. Asymmetric Rotational Model

Suggested by results of the generalized collective model (Rebel et al. 1974), we analyzed the scattering from the  $0^+$ ,  $2_1^+$ ,  $4_1^+$  and  $2_2^+$  states in the framework of a triaxial deformed rotator with pure quadrupole deformation (Davydov and Filippov 1958). The radius parameter  $R$  of the interaction potential or of the nucleon density distribution, respectively, is characterized in the intrinsic frame by

$$R(\theta', \varphi') = R_0 \left\{ 1 + \sum_{\mu=0, \pm 2} a_{2\mu} Y_{2\mu}(\theta', \varphi') \right\} \quad (4.2)$$

Usually one describes the deviation from axial symmetry by the asymmetry angle  $\gamma$  and a deformation parameter  $\beta$  which corresponds to the quadrupole deformation  $\beta_2$  in the limit of vanishing asymmetry. The relations between these parameters are

$$a_{20} = \beta \cos \gamma; a_{22} = a_{2-2} = \frac{1}{\sqrt{2}} \beta \sin \gamma = a_{20} \frac{1}{\sqrt{2}} \tan \gamma \quad (4.3)$$

In this model the collective wave functions for the  $n$ -th state

$$|IM; n\rangle = \sum_{K=\text{even}} A_K^{(n)} \Phi_{MK}^I(\vec{s})$$

are superpositions of states of different  $K$  quantum numbers. The mixing amplitudes  $A_K^{(n)}$  are functions of the asymmetry angle  $\gamma$ . Thus the deviation from axial symmetry enters the scattering both via the ratio  $a_{22}/a_{20}$  and via the amplitudes  $A_K^{(n)}$ . As e.g. shown in ref. (Rebel et al. 1973a) the asymmetry angle  $\gamma$  and the energy scale factor (containing a mass parameter and the deformation parameter  $\beta$ ) can be determined by fitting the level spectrum.

A fit to the states  $2_1^+$ ,  $2_2^+$ ,  $3^+$ , and  $4_1^+$  with equal weight on each state resulted in  $\gamma = 19,6^\circ$ . The experimental and calculated excitation energies and the mixing amplitudes for various

different  $\gamma$ -values are given in tab. 3.

Recently, Abecasis and Hernández (1972) worked out an asymmetric rotator model with a variable moment of inertia. We varied the asymmetry angle  $\gamma$  and the softness parameter  $\rho$  of this model to fit the energy level positions. The fit resulted in nearly the same asymmetry angle ( $\gamma = 19.0^\circ$ ) as in the rigid Davydov-Filippov model. The parameter  $\rho$  has been found to be  $\rho = 6 \cdot 10^{-3}$  indicating the softness of  $^{56}\text{Fe}$ . This corroborates the information extracted from the collective energy surface reported in ref (Rebel et al. 1974).

Fig. 6 shows the measured and calculated angular distributions of the  $0^+$ ,  $2_1^+$ ,  $4_1^+$  and  $2_2^+$  states based on a coupled channel calculation including the folding procedure with parameter values given in line 2 of tab. 4. In some additional calculations, we tried to use various different  $\gamma$ -values between  $10^\circ$  and  $28^\circ$ . In the lower left hand part of fig. 6 the  $\chi^2$ -values of these runs are plotted. The minimum is at  $\gamma = 19^\circ$ . Potential parameters and the deformation  $\beta$  were fixed during the scan. A 6-parameter fit at  $\gamma = 10^\circ$  (tab. 4, line 3) shows that the  $\chi^2$ -value does not change significantly when varying the potential parameters. Looking on the partial  $\chi^2$ -values (tab. 5) concerning the different states, it is obvious that the minimum mainly arises from a better description of the  $2_2^+$ -state. The calculated angular distribution of the  $2_2^+$  state for  $\gamma = 10^\circ$  (dashed line in fig. 6) indicates the sensitivity to the  $\gamma$ -value. From the parameter values given in tab. 4 we quote the following results

$$\beta = 0.241$$

$$\gamma = 19^\circ$$

$$\langle r^2 \rangle_m^{1/2} = 3.77 \pm 0.06 \text{ fm} \quad Q_{20}^m = 101 \pm 2 \text{ e fm}^2 \quad Q_{22}^m = 20.2 \pm 0.4 \text{ e fm}^2$$

$$\langle r^2 \rangle_{nm}^{1/2} = 3.85 \pm 0.06 \text{ fm} \quad Q_{20}^{nm} = 106 \pm 2 \text{ e fm}^2 \quad Q_{22}^{nm} = 21.2 \pm 0.4 \text{ e fm}^2$$

## 5. Summary

Completing recent studies of the collective behavior of even f-p-shell nuclei (Rebel et al. 1972, 1973b) we have experimentally investigated the scattering of 104 MeV  $\alpha$ -particles from  $^{56}\text{Fe}$ . The measured differential cross sections have been analyzed in terms of coupled channels on the basis of a symmetric and a triaxial rotator model. We mainly were interested in the deformation of the nuclear matter density distribution and tried to extract from the data intrinsic multipole moments and values of the transition probabilities for  $^{56}\text{Fe}$ . The traditional way describing inelastic scattering on the basis of a macroscopic, phenomenologically deformed optical potential provides information on the deformation of the interaction potential which is affected by the finite size of the projectile. Indeed, the values of the deformation parameters obtained in the framework of the usual extended optical model are different e.g. for  $\alpha$ -particle and proton scattering. The deformation parameters of the interaction potential reflect only roughly the deformation of the density distribution of the target nucleus. Therefore, we used for our analysis a semi-microscopic folding model which relates the differential cross sections in a more direct way to size and shape of target nucleus. The folding model generates the interaction potential by averaging an effective  $\alpha$ -bound nucleon interaction over a deformed distribution of the target nucleons. Such an interaction is used in the coupled channel analysis fitting the measured cross sections

by adjusting the size and shape parameters of the nucleon distribution. This procedure has been successfully applied in several preceding investigations. The following details of the procedure used in the present paper should be remarked:

- (i) Contrary to other authors (Lerner et al. 1972, Bernstein and Seidler 1971, 1972) we find it reasonable to determine the phenomenological parameter  $\lambda_R$  for every individual target nucleus separately, as  $\lambda_R$  may also reflect some structure effects.
- (ii) Exchange effects which sometimes are discussed (Bimbot et al. 1973) have not been explicitly included in the present analysis. Preliminary calculations showed that they are of minor importance for the diffraction region of the angular distribution and may be already absorbed by the phenomenological adjustment of the effective  $\alpha$ -nucleon interaction.
- (iii) Converting our results of the nucleon distribution to the nuclear matter distribution which is comparable to the charge distribution we use for the rms-radius of a single neutron  $\langle r^2 \rangle_{s.n.}^m$  the proton value  $\langle r^2 \rangle_{s.p.}^m = 0.64$  and ascribe the difference between nucleon and nuclear matter distribution to the half way radius of the underlying functional form of the distribution. In this way rms-radii and intrinsic multipole moments are calculated by numerical integration over the deformed distribution.

We quote as final results:

A) Analysis of the ground-state band: Symmetric Rotator Model

$$Q_{20} = + 100 \pm 4 \text{ e fm}^2 \quad Q_{40} = 500 \pm 40 \text{ e fm}^4$$

$$Q_{2_1^+} = -28.5 \pm 1.1 \text{ e fm}^2$$

$$B(E2; 0^+ \rightarrow 2^+) = 990 \pm 80 \text{ e}^2 \text{ fm}^4$$

$$B(E4; 0^+ \rightarrow 4^+) = (4.4 \pm 0.4) \cdot 10^4 \text{ e}^2 \text{ fm}^8$$

B) Analysis including the second  $2^+$ -state: Asymmetric Rotator Model \*)

$$\beta = 0.241 \quad \beta_{\text{eff}} = 0.216 \quad \gamma = 19^\circ$$

$$Q_{20} = +106 \pm 2 \text{ e fm}^2 \quad Q_{22} = 21.2 \text{ e fm}^2$$

$$Q_0 = +101 \pm 2 \text{ e fm}^2$$

$$Q_{2_1^+} = -28.9 \pm 0.6 \text{ e fm}^2 \quad Q_{2_2^+} = -Q_{2_1^+}$$

$$B(E2; 0^+ \rightarrow 2^+) = 1010 \pm 40 \text{ e}^2 \text{ fm}^4$$

It is conspicuous that the asymmetric rotator model analysis which reveals a triaxial deformation of  $\gamma \approx 20^\circ$  has little effect to the values of quadrupole moment  $Q_{2_1^+}$  extracted by the symmetric rotator model analysis of the ground band.

In tab. 6 the results are compared with other experimental results.

The values extracted from helion and  $\alpha$ -particle scattering in the framework of a deformed optical potential underestimate moments and transition probabilities compared to proton scattering and electromagnetic findings, even when - by the recipe of Bernstein (1969) - the deformation parameters are "renormalized".

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\*)  $\beta_{\text{eff}}$ ,  $Q_0$ , and  $Q_{2_1^+}$  are defined according to Davydov and Filippov (1958)

The value of the asymmetry angle  $\gamma$  which we obtain from our analysis is in good agreement with the results of analyses of the level spectrum of  $^{56}\text{Fe}$ . (Davydov and Chaban 1960, Davydov and Filippov 1958, Abecasis and Hernández 1972). A recent investigation of the collective energy surface of  $^{56}\text{Fe}$  results in effective  $\gamma$ -values for the various states ranging from about  $19^\circ$  to  $21^\circ$ .

However, it should be remarked that there are significant deviations of the experimental  $2_2^+$ -differential cross section from a pure collective description. This has been also found in a recent investigation on the basis of a more generalized collective model (Rebel et al. 1974). We ascribe the deviation to an unknown admixture to the  $2_2^+$ -amplitude which is neglected by the collective model. Though spurious such an amplitude influences the cross section very sensitively.

#### Acknowledgements

We thank Dr. G. Schatz for his encouraging interest in our studies. We take the opportunity to thank Ing. F. Schulz and the cyclotron staff for their continuous effort in preparing the  $\alpha$ -beam with excellent quality, and Miss C. Rämer and Dipl. Ing. W. Karbstein for help in using the scattering and computer facilities of the Karlsruhe Cyclotron Laboratory. We thank Miss W. Nowatzke and Mrs. V. Walsch for their assistance in data handling and preparation of the figures.

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$v_o$ [MeV]	$r_v$ [fm]	$a_v$ [fm]	$w_o$ [MeV]	$r_w$ [fm]	$a_w$ [fm]	$\langle r^2 \rangle_{\text{Pot}}^{1/2}$ [fm]	$\chi^2/F$			
$\lambda_R$	$c_m A^{-1/3}$ [fm]	$a_m$ [fm]	$w_o$ [MeV]	$r_w$ [fm]	$a_w$ [fm]	$\langle r^2 \rangle_m^{1/2}$ [fm]	$\chi^2/F$	real para- meters varied	macroscopic optical potential 6 parameters varied	comments
114(1)	1.31(1)	0.71(1)	21.4(4)	1.58(1)	0.61(2)	4.69(6)	5.5			
0.86	1.01	0.57	26(1)	1.54(1)	0.60(2)	3.66	16.0	-		proc. A
0.86	1.037(5)	0.54(1)	26(1)	1.56(2)	0.57(1)	3.67(3)	15.6	$c_m, a_m$		A
0.92	1.02(2)	0.55(2)	24(1)	1.60(1)	0.54(2)	3.65(6)	10.3	$c_m, a_m$		A
0.900(4)	1.00(1)	0.57(2)	22.1(8)	1.60(1)	0.55(2)	3.64(6)	9.0	$\lambda_R, c_m, a_m$		A
$\lambda_I$										
0.86	1.09(1)	0.47(2)	0.494(5)	-	-	3.67(4)	16.3	$c_m, a_m$		B
0.90	1.06(1)	0.48(2)	0.514(6)	-	-	3.61(4)	16.4	$c_m, a_m$		B

Tab. 1: Results of the elastic scattering analysis. Procedure A of the folding model uses a macroscopic imaginary potential the three parameters of which are all varied. The numbers in brackets are the standard errors in the last digit of the quoted values. The procedure of determination of the errors is given in ref. (Rebel et al. 1972b).

$v_o$ [MeV]	$r_v$ [fm]	$a_v$ [fm]	$w_o$ [MeV]	$r_w$ [fm]	$a_w$ [fm]	$\beta_2$	$\beta_4$	$\langle r^2 \rangle_{\text{Pot}}^{1/2}$ [fm]	$\chi^2/F$		
108(2)	1.348(8)	0.71(1)	21.7(7)	1.61(1)	0.50(1)	0.178(2)	0.009(4)	4.81(5)	14.4	extended optical potential	
$\lambda_R$	$c_m A^{-1/3}$ [fm]	$a_m$ [fm]	$w_o$ [MeV]	$r_w$ [fm]	$a_w$ [fm]	$\beta_2$	$\beta_4$	$\langle r^2 \rangle_m^{1/2}$ [fm]	$\chi^2/F$	real parameters varied	comments
0.86	1.01	0.57	18.3(6)	1.60(1)	0.58(2)	0.256(3)	0.02(1)	3.71	24.0		proc.A
0.86	1.23(2)	0.32(4)	28(3)	1.59(1)	0.47(2)	0.208(5)	0.011(6)	3.88(4)	18.4	$c_m, a_m$	A
0.92	1.06(2)	0.51(2)	18.5(9)	1.63(1)	0.52(1)	0.239(4)	0.019(8)	3.71(4)	15.5	$c_m, a_m$	A
0.950(8)	1.04(2)	0.53(2)	18(1)	1.65(1)	0.49(1)	0.238(5)	0.026(9)	3.70(4)	14.3	$\lambda_R, c_m, a_m$	A
$\lambda_I$											
0.95	1.09(1)	0.43(2)	0.454(4)	-	-	0.237(4)	0.036(7)	3.65(4)	19.0	$c_m, a_m$	B

Tab. 2: Results of the symmetric rotator coupled channel analysis ( $0^+ - 2_1^+ - 4^+$  - coupling). In all cases the deformation parameters and the imaginary potential parameters were varied. The numbers in brackets indicate the standard errors in the last digit.

I	$E_n^{\text{exp}}$ [MeV]	$E_n^{\text{th}}$ [MeV]	$A_0^{(n)}$	$A_2^{(n)}$	$A_4^{(n)}$	
$2_1^+$	0.847	0.690	0.9968	0.0797		$\gamma = 19.6^\circ$ obtained by fitting the energy levels
$2_2^+$	2.66	2.70	-0.0797	0.9968		
$3^+$	3.45	3.39	0.0	1.0		
$4_1^+$	2.09	2.17	0.9612	0.2758	0.0083	
$2_1^+$		0.641	0.9993	0.0377		$\gamma = 16^\circ$ fixed
$2_2^+$		3.84	-0.0377	0.9993		
$3^+$		4.48	0.0	1.0		
$4_1^+$		2.09	0.9899	0.1418	0.0020	
$2_1^+$		0.762	0.9821	0.1885		$\gamma = 24^\circ$ fixed
$2_2^+$		1.97	-0.1885	0.9821		
$3^+$		2.73	0.0	1.0		
$4_1^+$		2.21	0.8758	0.4816	0.0331	

Tab. 3: Different sets of mixing amplitudes resulting from the asymmetric rotator description of the  $^{56}\text{Fe}$  level spectrum

$v_o$ [MeV]	$r_w$ [fm]	$a_w$ [fm]	$w_o$ [MeV]	$r_w$ [fm]	$a_w$ [fm]	$a_{20}$	$\gamma$	$\langle r^2 \rangle_{\text{Pot}}^{1/2}$ [fm]	$\chi^2/F$		
107(2)	1.357(8)	0.70(1)	22(1)	1.61(1)	0.51(2)	0.176(2)	19.6	4.82	17.2	extended optical model 7 parameters varied	
$\lambda_R$	$c_m A^{1/3}$ [fm]	$a_m$ [fm]	$w_o$ [MeV]	$r_w$ [fm]	$a_w$ [fm]	$a_{20}$	$\gamma$	$\langle r^2 \rangle_m^{1/2}$ [fm]	$\chi^2/F$	real para- meters varied	comments
0.93(3)	1.16(1)	0.38(2)	28(3)	1.56(2)	0.52(2)	0.227(5)	19.6	3.77(6)	20.6	$\lambda_R, c_m, a_m$	proc. A
0.96(1)	1.09(1)	0.48(3)	18(2)	1.66(2)	0.47(2)	0.227	10°	3.73(6)	25.9	$\lambda_R, c_m, a_m$	A $a_{20}$ fixed
		$\lambda_I$									
0.93	1.10(1)	0.43(2)	0.449(9)	-	-	0.225(4)	19.6	3.69(6)	21.2	$c_m, a_m$	B

Tab. 4: Triaxial rotator analysis ( $0^+ - 2_1^+ - 4^+ - 2_2^+$  - coupling). In all cases the imaginary potential parameters were varied. The numbers in brackets indicate the standard errors in the last digit.

$\gamma$	$\chi^2/F$ (total)	$\chi^2/F$ ( $0^+$ )	$\chi^2/F$ ( $2_1^+$ )	$\chi^2/F$ ( $4_1^+$ )	$\chi^2/F$ ( $2_2^+$ )
10	28.2	19.3	25.5	7.3	79.9
14	23.0	19.7	24.2	7.1	47.9
17	20.5	18.2	24.7	5.5	35.5
18	20.2	18.0	24.9	5.4	33.9
19	20.1	17.8	25.2	5.2	33.4
19.6	20.2	17.7	25.4	5.2	33.6
21	20.4	17.4	25.7	4.9	35.5
24	22.3	16.8	26.2	4.3	48.3
28	28.3	16.2	27.2	3.8	88.0

Tab. 5: Total and partial  $\chi^2$ -values per degree of freedom for different asymmetry angles  $\gamma$ . In all cases the potential parameters were fixed to the values of tab. 4, line 2 and the parameter  $B$  was set to  $B = 0.241$ .

Method	Reference	$\beta_2$	$Q_{20} [e \text{ fm}^2]$	$Q_{21}^+ [e \text{ fm}^2]$	$B(E2)^\dagger [e^2 \text{ fm}^4]$	$B(E2)^\dagger/B(E2)_{s.p.}$
Electromagnetic (average)	Stelson and Grodzins (1965)	0.23	(95)	(27)	$900 \pm 100$	$14.1 \pm 1.6$
Electromagnetic (reorientation)	Lesser et al. 1972		$(+87 \pm 20)$	$-24.9 \pm 5.8$	$970 \pm 20$	$15.2 \pm 0.3$
$p, p'$ (average)	Peterson 1969		(102)	(29)	$1030 \pm 300$	$16.2 \pm 4.7$
$p, p'$ ; DWBA	Main 1971	$0.20 \pm 0.02$	(90)	(26)	$810 \pm 100$	$12.7 \pm 1.6$
$^3\text{He}; \alpha$ (average)	Peterson 1969		(84)	(24)	$700 \pm 70$	$11.0 \pm 1.1$
$^3\text{He}, ^3\text{He}'$ ; CC	Marchese et al. 1973	0.21	(79)	(23)	$623^*)$	9.8
$\alpha, \alpha'$ ; DWBA	Bernstein 1969		$(89 \pm 6)$	$(26 \pm 2)$	$792 \pm 121$	$12.4 \pm 1.9$
$\alpha, \alpha'$ ; CC, Folding, SR	present experiment	0.238	$+100 \pm 4$	$(-28.5 \pm 1.2)$	$(990 \pm 80)$	$15.6 \pm 1.3$
$\alpha, \alpha'$ ; CC, Folding, ASR	present experiment	$\beta = 0.241$	$Q_0 = +101 \pm 2$	$(-28.9 \pm 0.6)$	$(1010 \pm 40)$	$15.9 \pm 0.6$

Tab. 6: Comparison of deformation parameters, quadrupole moments and transition probabilities of  $^{56}\text{Fe}$  obtained by different experimental methods. Values in brackets are deduced from  $B(E2)$ -values given by the relations  $Q_{20} = \sqrt{\frac{16\pi}{5}} B(E2)^\dagger$  and  $Q_{21}^+ = -\frac{2}{7} Q_{20}$ . In the case of the asymmetric rotator analysis (ASR) the parameters  $\beta$  and  $Q_0$  are comparable to  $\beta_2$  and  $Q_{20}$ .

\*) Calculated from  $\beta_2$  and  $R_v$  by the method given by Bernstein (1969).

## Figure Captions

Fig. 1: Set-up of the electronics. The numbers correspond to the producers' type identifications of the NIM-modules used (BA: Canberra; all the others: Ortec).

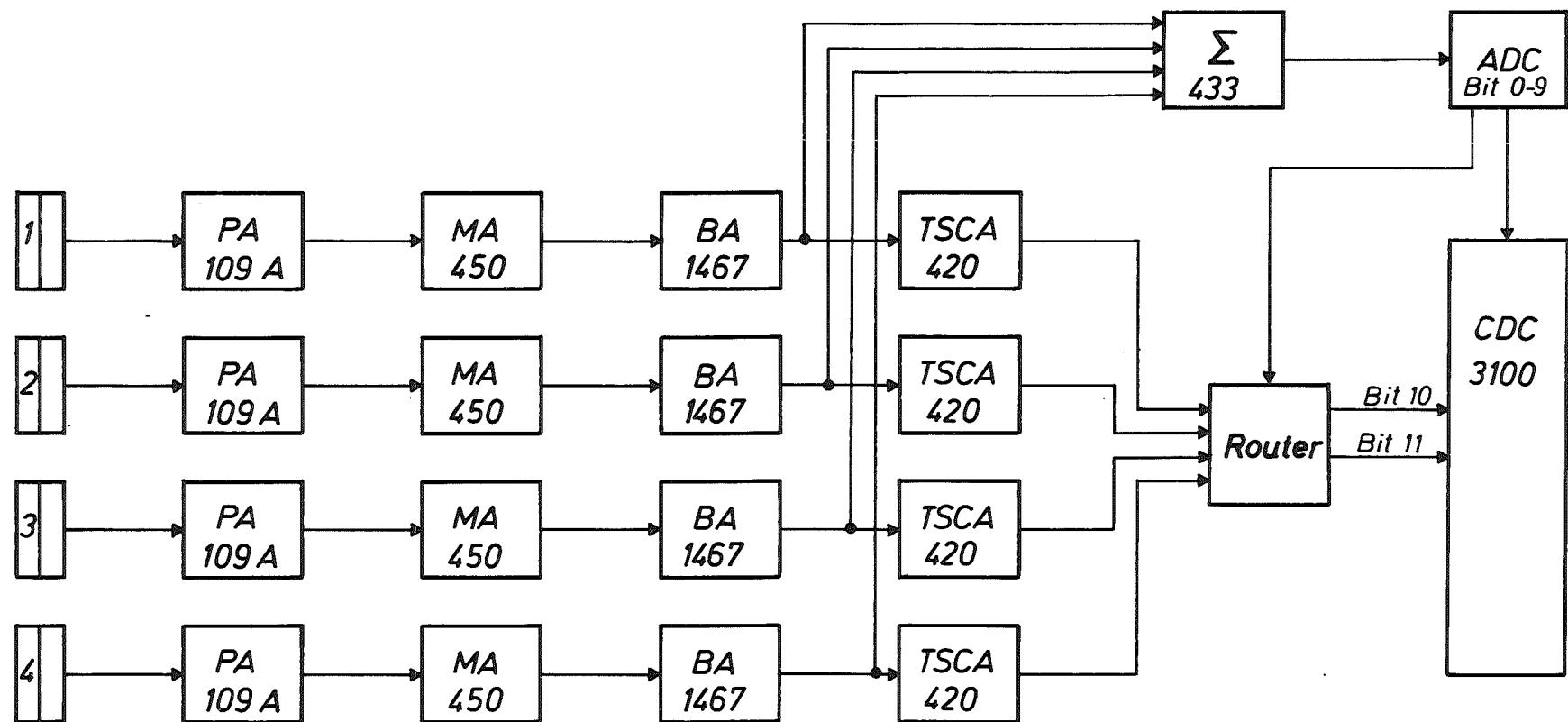
Fig. 2: Energy spectrum of the outgoing  $\alpha$ -particles from  $^{56}\text{Fe}(\alpha, \alpha')^{56}\text{Fe}$ . The dashed line indicates the contribution of a single peak to the whole spectrum. The states at -2.96, -3.125 and -3.37 MeV are only analyzed for background considerations.

Fig. 3: Experimental differential cross sections:  $^{56}\text{Fe}(\alpha, \alpha')^{56}\text{Fe}$ . The theoretical curve represents the result of the conventional optical model analysis of elastic scattering. This paper is not engaged in the angular distribution of the  $3^-$  state shown.

Fig. 4: Folding model analysis of elastic scattering. In the inserted  $\chi^2$ -contour plot over the  $c_m$ - $a_m$ -plane the correlation between these two parameters is demonstrated. The full line shows the corresponding values of  $c_m$  and  $a_m$  for  $\chi^2 = 1.5 \cdot \chi^2_{\min}$ . The dashed straight lines mark constant rms-radii and the bend dashed line follows the valley of minimal  $\chi^2/F$  for given  $a_m$  shown in the second inserted graph.

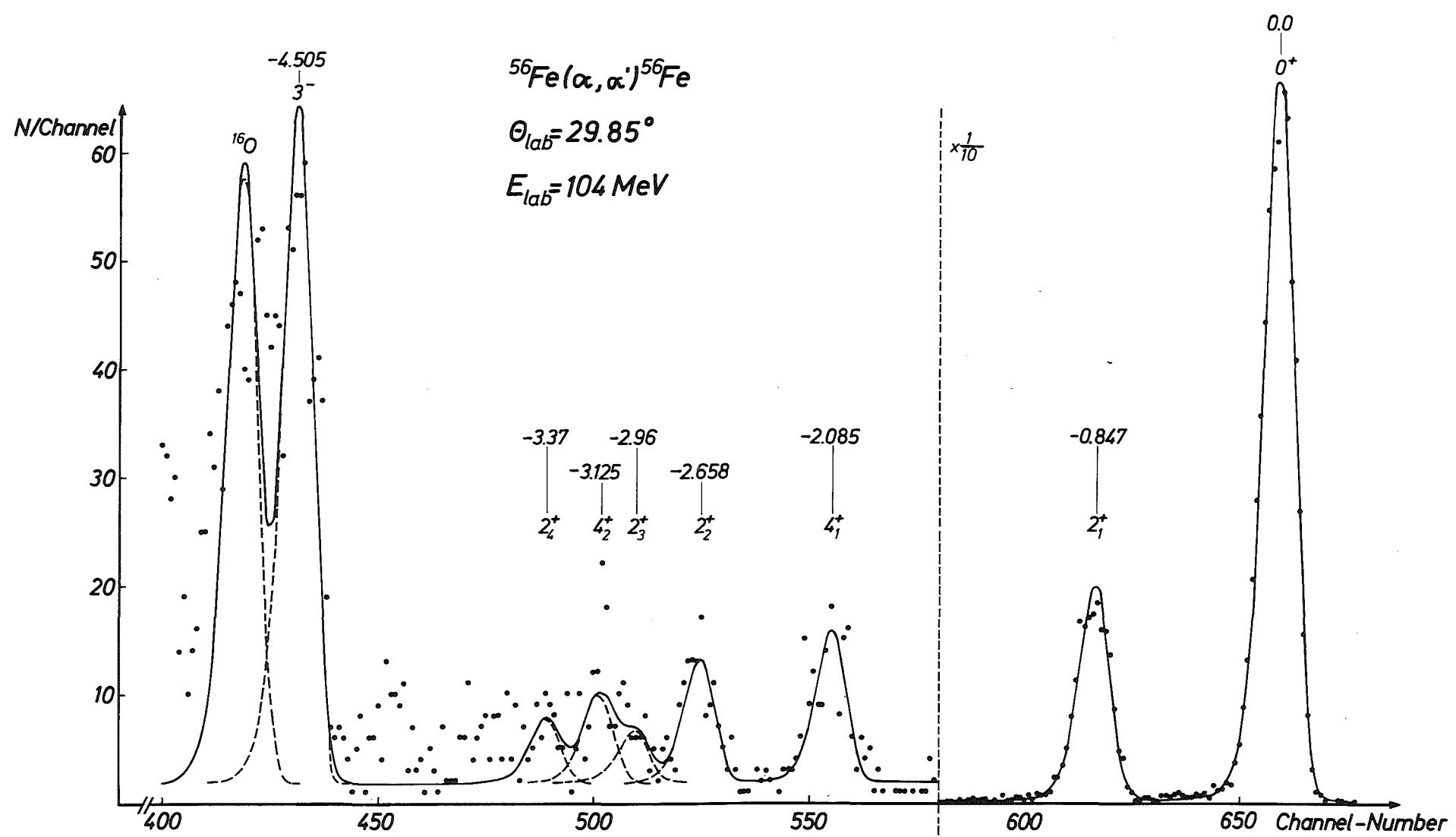
Fig. 5: Symmetric rotator model analysis of inelastic scattering using the folding procedure. The parameters of the interaction potentials are given in tab. 2 (line5).

Fig. 6: Triaxial rotator model analysis of inelastic scattering using the folding procedure. The dashed  $2_2^+$  angular distribution corresponds to  $\gamma = 10^\circ$ . Interaction potential parameters are given in tab. 4 (line2). The inserted graph shows the curve of the  $\chi^2$ -values per degree of freedom over the asymmetry angle  $\gamma$ .



Zykl. 30.10.74

Fig. 1



Zykl. 31.10.74

Fig. 2

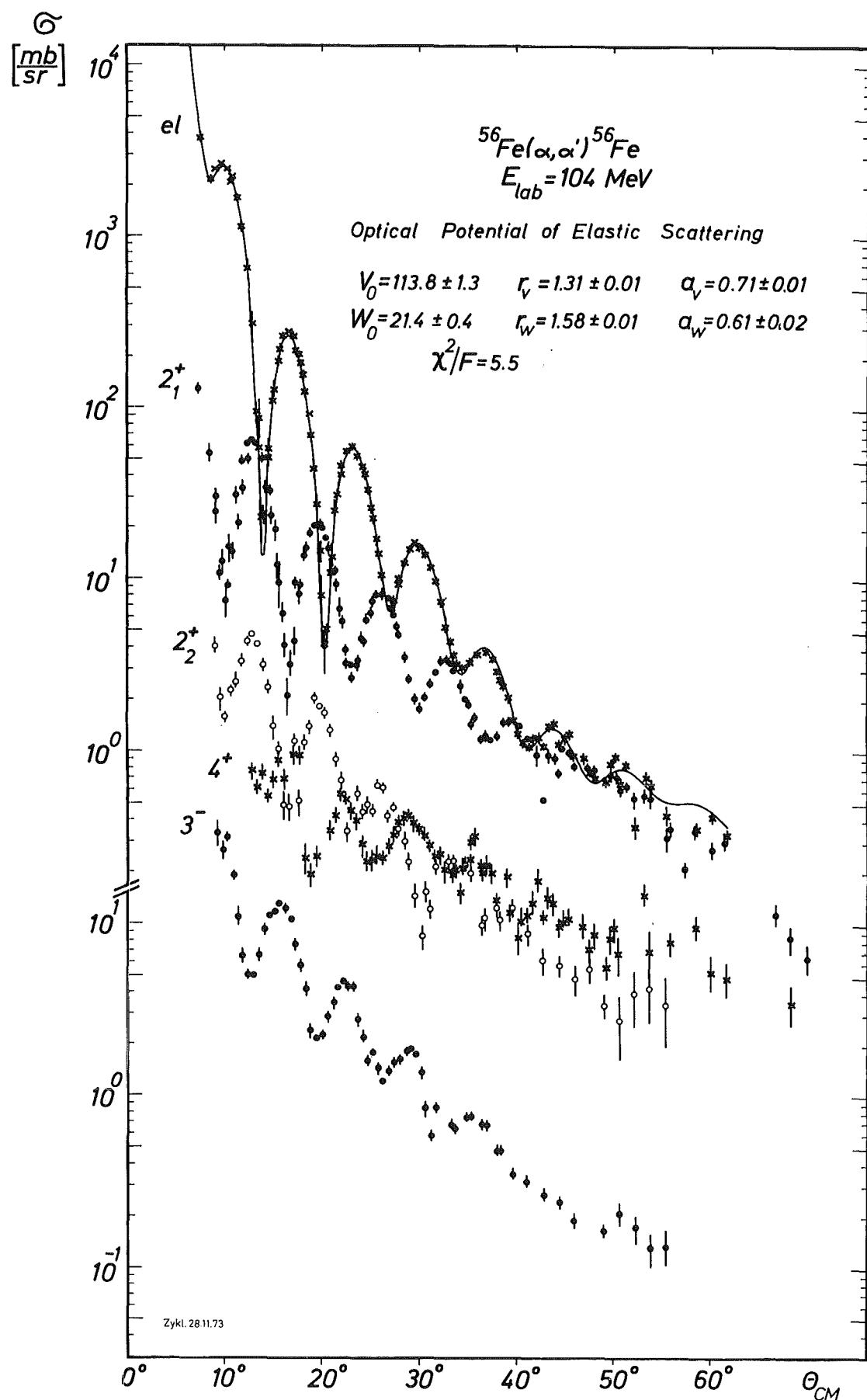


Fig. 3

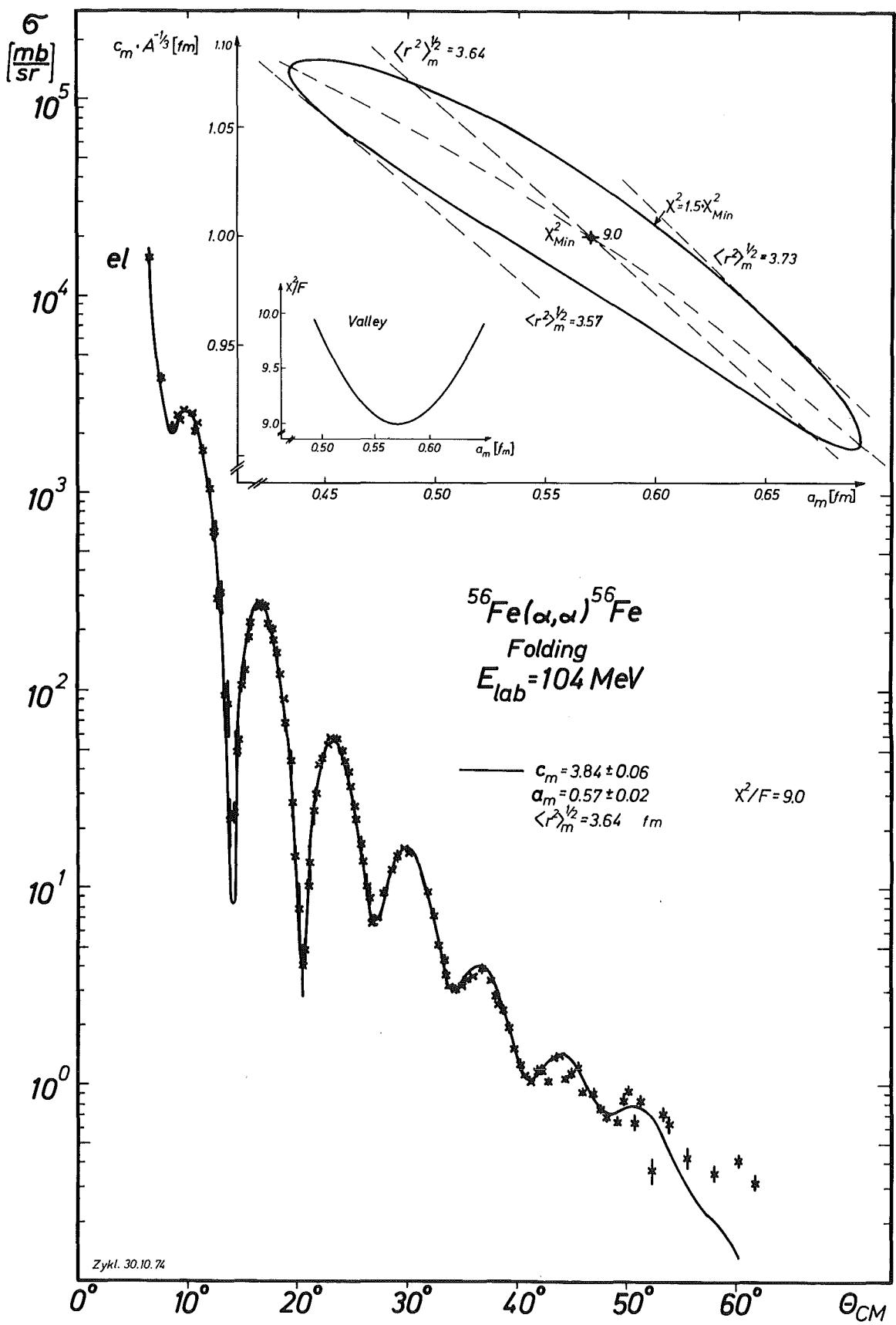


Fig. 4

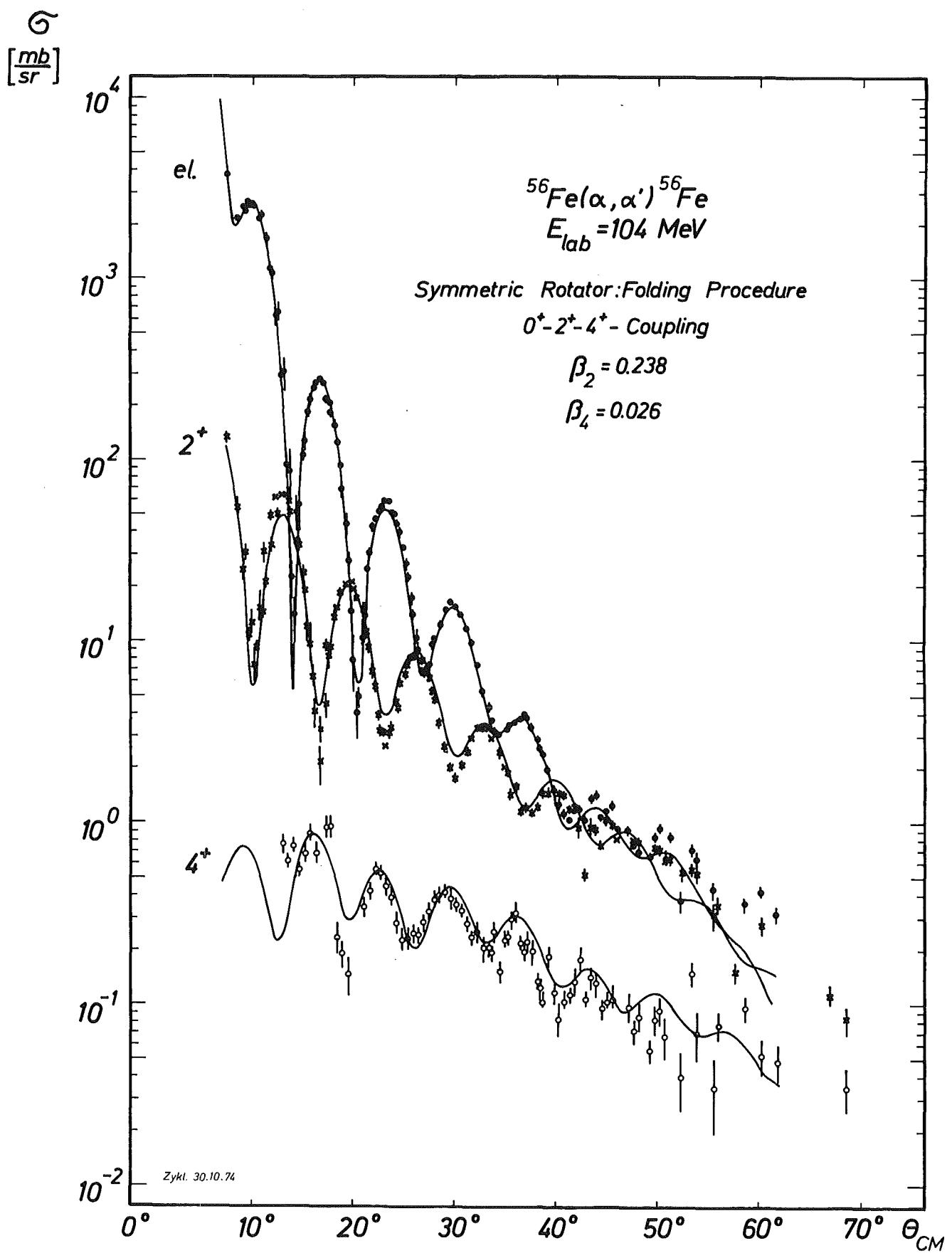


Fig. 5

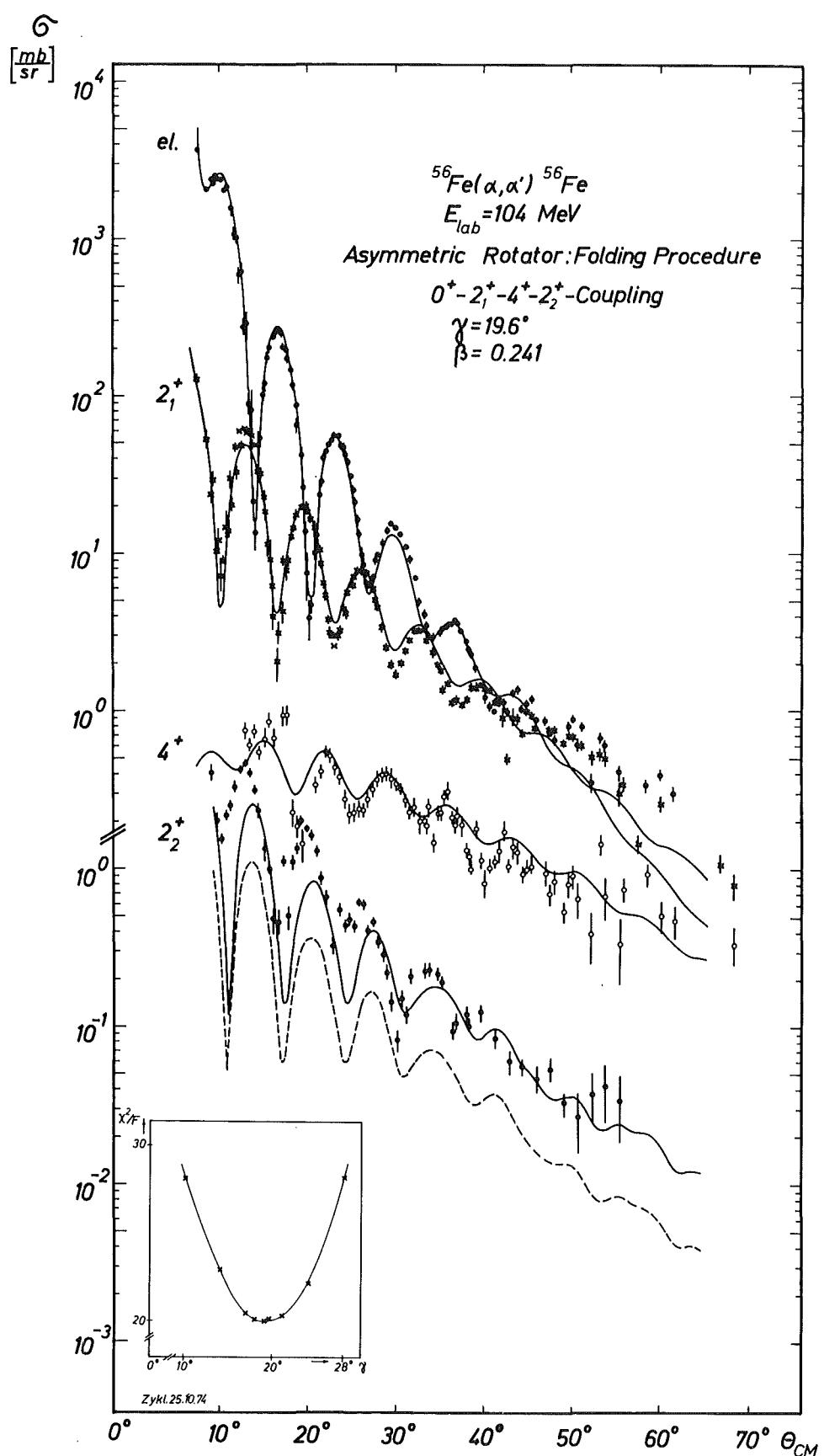


Fig. 6

Appendix

Experimental cross sections: The quoted errors of the experimental cross sections include the error arising from the finite angular acceptance which is converted into a cross section error.

This paper is not engaged in the cross sections of the  $\bar{3}$  state given.





SCATTERING OF 104 MEV ALPHAPARTICLES ON 56 FE

Q = -2.085 MEV

I = 4+

ECM = 94.979 MEV K = 4.1198/FERMI ETA = 1.62381

LABORATORY DATA			CM DATA		
THETA DEGREE	SIGMA MB/SR	DSIGMA %	THETA DEGREE	SIGMA MB/SR	DSIGMA MB/SR
12.15	8.811E-01	11.4	13.04	7.662E-01	8.757E-02
12.65	7.182E-01	7.5	13.58	6.247E-01	4.686E-02
13.15	8.691E-01	9.3	14.12	7.562E-01	7.030E-02
13.65	6.362E-01	9.2	14.65	5.537E-01	5.067E-02
14.15	7.940E-01	11.5	15.19	6.912E-01	7.948E-02
14.65	1.029E 00	9.4	15.72	8.961E-01	8.409E-02
15.15	7.936E-01	13.5	16.26	6.913E-01	9.330E-02
16.15	1.085E 00	12.9	17.33	9.458E-01	1.219E-01
16.65	1.097E 00	12.8	17.87	9.566E-01	1.229E-01
17.15	2.726E-01	21.0	18.40	2.378E-01	4.999E-02
17.65	2.186E-01	14.9	18.94	1.908E-01	2.842E-02
18.15	2.841E-01	14.1	19.47	2.480E-01	3.489E-02
19.65	3.981E-01	12.5	21.08	3.480E-01	4.336E-02
20.15	4.808E-01	10.0	21.61	4.204E-01	4.213E-02
20.65	6.430E-01	8.3	22.15	5.625E-01	4.665E-02
21.15	6.065E-01	9.1	22.68	5.308E-01	4.841E-02
21.65	5.154E-01	10.1	23.22	4.513E-01	4.543E-02
22.15	4.549E-01	11.3	23.75	3.985E-01	4.487E-02
22.65	3.234E-01	13.7	24.28	2.834E-01	3.891E-02
23.15	2.617E-01	13.7	24.82	2.295E-01	3.134E-02
23.65	2.659E-01	11.9	25.35	2.333E-01	2.771E-02
24.15	2.826E-01	10.4	25.89	2.481E-01	2.579E-02
24.65	2.751E-01	8.2	26.42	2.416E-01	1.974E-02
25.15	3.217E-01	10.6	26.95	2.827E-01	2.994E-02
25.65	3.715E-01	10.3	27.49	3.266E-01	3.379E-02
26.15	4.324E-01	11.5	28.02	3.804E-01	4.367E-02
26.65	4.582E-01	9.3	28.55	4.033E-01	3.765E-02
27.15	4.765E-01	7.0	29.09	4.196E-01	2.921E-02
27.65	4.304E-01	11.5	29.62	3.792E-01	4.370E-02
28.15	3.971E-01	9.0	30.15	3.501E-01	3.143E-02
28.65	3.727E-01	8.3	30.69	3.288E-01	2.719E-02
29.15	3.147E-01	7.9	31.22	2.778E-01	2.201E-02
29.65	2.688E-01	8.3	31.75	2.374E-01	1.979E-02
30.15	2.842E-01	10.9	32.28	2.512E-01	2.731E-02
30.65	2.273E-01	15.5	32.81	2.010E-01	3.111E-02
31.15	2.358E-01	10.1	33.35	2.087E-01	2.110E-02
31.37	2.196E-01	9.0	33.58	1.944E-01	1.752E-02
31.65	2.844E-01	13.8	33.88	2.518E-01	3.473E-02
32.15	1.697E-01	15.8	34.41	1.504E-01	2.378E-02
32.65	2.619E-01	8.5	34.94	2.322E-01	1.981E-02
32.87	2.597E-01	7.3	35.17	2.303E-01	1.680E-02
33.15	3.351E-01	10.7	35.47	2.973E-01	3.182E-02
33.65	3.596E-01	11.3	36.00	3.193E-01	3.614E-02
34.15	2.436E-01	7.8	36.53	2.165E-01	1.685E-02
34.37	2.194E-01	9.5	36.77	1.950E-01	1.849E-02
34.65	2.474E-01	13.3	37.06	2.200E-01	2.936E-02
35.15	2.199E-01	13.5	37.59	1.957E-01	2.641E-02
35.65	1.519E-01	10.1	38.12	1.353E-01	1.370E-02
35.87	1.390E-01	11.9	38.36	1.238E-01	1.470E-02
36.15	1.123E-01	18.6	38.65	1.001E-01	1.858E-02
36.65	2.072E-01	13.8	39.18	1.848E-01	2.549E-02
37.15	1.309E-01	11.6	39.71	1.168E-01	1.359E-02
37.65	9.357E-02	20.6	40.24	8.357E-02	1.720E-02
38.15	1.173E-01	17.8	40.77	1.048E-01	1.866E-02
38.65	1.267E-01	10.6	41.30	1.133E-01	1.197E-02
39.15	1.499E-01	16.0	41.83	1.342E-01	2.146E-02
39.65	1.991E-01	14.0	42.36	1.784E-01	2.499E-02
40.15	1.199E-01	10.4	42.89	1.075E-01	1.122E-02
40.65	1.564E-01	15.8	43.41	1.404E-01	2.219E-02
41.15	1.457E-01	15.7	43.94	1.309E-01	2.055E-02
41.65	1.064E-01	10.7	44.47	9.565E-02	1.019E-02
42.15	1.114E-01	18.1	45.00	1.002E-01	1.809E-02
42.65	1.168E-01	18.3	45.53	1.052E-01	1.924E-02
43.15	8.493E-01	1.2	46.05	7.654E-01	9.283E-03
44.15	1.079E-01	18.3	47.11	9.741E-02	1.780E-02
44.65	7.816E-02	15.0	47.63	7.062E-02	1.062E-02
45.15	9.411E-02	17.7	48.16	8.511E-02	1.507E-02
46.15	6.163E-02	15.5	49.21	5.584E-02	8.633E-03
46.65	9.055E-02	18.8	49.74	8.212E-02	1.544E-02
47.15	1.023E-01	17.0	50.26	9.286E-02	1.578E-02
47.65	7.296E-02	26.3	50.79	6.629E-02	1.743E-02
48.15	1.354E-02	46.4	51.31	1.231E-02	5.716E-03
49.15	4.327E-02	36.7	52.36	3.942E-02	1.449E-02
50.15	1.610E-01	13.5	53.41	1.470E-01	1.991E-02
50.65	7.643E-02	30.7	53.93	6.984E-02	2.143E-02
51.15	1.653E-02	48.4	54.46	1.512E-02	7.319E-03
52.15	3.748E-02	45.1	55.50	3.435E-02	1.550E-02
52.65	8.389E-02	17.6	56.02	7.696E-02	1.355E-02
54.15	1.735E-02	39.0	57.59	1.597E-02	6.230E-03
55.15	1.029E-01	16.9	58.63	9.489E-02	1.599E-02
56.65	5.619E-02	23.8	60.19	5.198E-02	1.236E-02
58.15	5.179E-02	23.6	61.75	4.806E-02	1.134E-02
63.15	1.172E-02	45.2	66.94	1.100E-02	4.967E-03
64.65	3.600E-02	26.9	68.48	3.390E-02	9.115E-03
66.15	7.950E-03	59.7	70.03	7.512E-03	4.487E-03

SCATTERING OF 104 MEV ALPHAPARTICLES ON 56 FE

Q = -2.658 MEV

I = 2+

ECM = 94.406 MEV K = 4.1073/FERMI ETA = 1.62873

LABORATORY DATA			CM DATA		
THE TA DEGREE	SIGMA MB/SR	DSIGMA %	THETA DEGREE	SIGMA MB/SR	DSIGMA MB/SR
8.65	4.779E 00	10.7	9.29	4.148E 00	4.434E-01
9.15	2.397E 00	13.5	9.83	2.081E 00	2.806E-01
9.65	1.822E 00	6.3	10.36	1.582E 00	9.993E-02
10.15	2.560E 00	7.0	10.90	2.223E 00	1.562E-01
10.65	2.936E 00	6.8	11.44	2.550E 00	1.736E-01
11.15	3.885E 00	6.8	11.97	3.375E 00	2.306E-01
11.65	4.992E 00	4.4	12.51	4.338E 00	1.907E-01
12.15	5.516E 00	3.8	13.05	4.795E 00	1.809E-01
12.65	4.729E 00	4.9	13.58	4.112E 00	2.034E-01
13.15	3.675E 00	7.0	14.12	3.196E 00	2.247E-01
13.65	2.744E 00	9.5	14.65	2.387E 00	2.258E-01
14.15	1.590E 00	12.3	15.19	1.384E 00	1.703E-01
14.65	1.163E 00	12.1	15.73	1.012E 00	1.221E-01
15.15	5.603E-01	19.5	16.26	4.879E-01	9.500E-02
15.65	5.433E-01	17.1	16.80	4.733E-01	8.106E-02
16.15	1.338E 00	6.9	17.33	1.166E 00	8.059E-02
16.65	5.802E-01	16.0	17.87	5.058E-01	8.109E-02
17.15	1.289E 00	9.9	18.41	1.124E 00	1.114E-01
17.65	1.577E 00	8.9	18.94	1.376E 00	1.221E-01
18.15	2.389E 00	4.9	19.48	2.085E 00	1.027E-01
18.65	2.077E 00	4.7	20.01	1.813E 00	8.470E-02
19.15	1.904E 00	6.0	20.55	1.663E 00	1.005E-01
19.65	1.499E 00	7.7	21.08	1.310E 00	1.007E-01
20.15	1.021E 00	9.4	21.62	8.925E-01	8.350E-02
20.65	7.668E-01	11.5	22.15	6.706E-01	7.717E-02
21.15	3.802E-01	13.1	22.69	3.326E-01	4.359E-02
22.15	6.401E-01	8.8	23.75	5.605E-01	4.934E-02
22.65	5.037E-01	10.1	24.29	4.413E-01	4.442E-02
23.15	5.544E-01	9.5	24.82	4.860E-01	4.595E-02
23.65	5.098E-01	6.6	25.36	4.471E-01	2.970E-02
24.15	7.248E-01	5.5	25.89	6.360E-01	3.517E-02
24.65	6.871E-01	5.3	26.43	6.032E-01	3.789E-02
25.15	4.721E-01	7.1	26.96	4.147E-01	2.930E-02
25.65	5.345E-01	5.2	27.49	4.697E-01	2.465E-02
26.15	4.022E-01	7.6	28.03	3.537E-01	2.695E-02
26.65	3.386E-01	9.4	28.56	2.979E-01	2.799E-02
27.15	2.607E-01	10.6	29.09	2.295E-01	2.442E-02
27.65	1.652E-01	13.3	29.63	1.455E-01	1.940E-02
28.15	9.484E-02	16.2	30.16	8.359E-02	1.355E-02
28.65	1.756E-01	10.6	30.69	1.549E-01	1.635E-02
29.15	1.366E-01	12.6	31.22	1.205E-01	1.519E-02
29.65	2.407E-01	9.8	31.76	2.125E-01	2.078E-02
31.15	2.628E-01	8.6	33.35	2.325E-01	2.009E-02
31.37	2.666E-01	8.2	33.59	2.359E-01	1.929E-02
32.65	2.472E-01	8.9	34.95	2.191E-01	1.955E-02
32.87	2.220E-01	9.1	35.18	1.968E-01	1.792E-02
34.15	1.099E-01	13.1	36.54	9.762E-02	1.276E-02
34.37	1.229E-01	12.4	36.77	1.092E-01	1.359E-02
35.65	1.379E-01	11.8	38.13	1.228E-01	1.446E-02
35.87	1.166E-01	13.1	38.36	1.038E-01	1.359E-02
37.15	1.385E-01	11.9	39.72	1.236E-01	1.468E-02
38.65	9.702E-02	13.8	41.31	8.676E-02	1.200E-02
40.15	6.802E-02	16.6	42.89	6.097E-02	1.013E-02
41.65	6.375E-02	14.1	44.48	5.729E-02	8.066E-03
43.15	5.264E-02	19.3	46.06	4.743E-02	9.136E-03
44.65	6.028E-02	17.3	47.64	5.445E-02	9.404E-03
46.15	3.650E-02	16.4	49.22	3.306E-02	5.427E-03
47.65	2.991E-02	41.0	50.80	2.717E-02	1.114E-02
49.15	4.264E-02	36.0	52.37	3.884E-02	1.399E-02
50.65	4.568E-02	39.7	53.94	4.173E-02	1.655E-02
52.15	3.726E-02	44.7	55.51	3.414E-02	1.527E-02

SCATTERING OF 104 MEV ALPHAPARTICLES ON 56 FE

Q = -4.505 MEV

I = 3-

ECM = 92.559 MEV K = 4.0669/FERMI ETA = 1.64491

LABORATORY DATA			CM DATA		
THETA DEGREE	SIGMA MB/SR	DSIGMA %	THETA DEGREE	SIGMA MB/SR	DSIGMA MB/SR
8.65	3.949E 01	18.5	9.30	3.423E 01	6.319E 00
9.15	3.132E 01	11.4	9.83	2.715E 01	3.104E 00
9.65	3.710E 01	5.1	10.37	3.217E 01	1.648E 00
10.15	2.207E 01	2.5	10.91	1.914E 01	4.753E-01
10.65	1.315E 01	11.2	11.44	1.141E 01	1.276E 00
11.15	7.698E 00	8.0	11.98	6.679E 00	5.373E-01
11.65	5.863E 00	4.2	12.52	5.089E 00	2.150E-01
12.15	5.822E 00	4.7	13.05	5.054E 00	2.373E-01
12.65	7.615E 00	6.9	13.59	6.613E 00	4.552E-01
13.15	1.074E 01	5.6	14.13	9.329E 00	5.264E-01
13.65	1.298E 01	3.3	14.66	1.128E 01	3.672E-01
14.15	1.337E 01	3.0	15.20	1.162E 01	3.451E-01
14.65	1.516E 01	2.2	15.74	1.318E 01	2.939E-01
15.15	1.395E 01	3.9	16.27	1.213E 01	4.715E-01
15.65	1.211E 01	4.9	16.81	1.054E 01	5.137E-01
16.15	8.799E 00	5.9	17.35	7.658E 00	5.265E-01
16.65	6.690E 00	7.6	17.88	5.824E 00	4.410E-01
17.15	4.837E 00	8.7	18.42	4.213E 00	3.654E-01
17.65	2.789E 00	9.4	18.95	2.430E 00	2.272E-01
18.15	2.495E 00	4.4	19.49	2.175E 00	9.622E-02
18.65	2.608E 00	4.8	20.02	2.274E 00	1.100E-01
19.15	3.341E 00	5.8	20.56	2.914E 00	1.680E-01
19.65	4.012E 00	5.1	21.10	3.501E 00	1.785E-01
20.15	4.964E 00	3.8	21.63	4.334E 00	1.666E-01
20.65	5.359E 00	3.0	22.17	4.681E 00	1.401E-01
21.15	5.024E 00	3.2	22.70	4.390E 00	1.392E-01
21.65	5.000E 00	4.8	23.24	4.371E 00	2.109E-01
22.15	3.174E 00	8.6	23.77	2.776E 00	2.379E-01
22.65	2.588E 00	6.7	24.31	2.265E 00	1.510E-01
23.15	1.871E 00	5.8	24.84	1.638E 00	9.420E-02
23.65	2.108E 00	3.0	25.37	1.846E 00	5.544E-02
24.15	1.655E 00	5.4	25.91	1.450E 00	7.800E-02
24.65	1.402E 00	3.6	26.44	1.229E 00	4.398E-02
25.15	1.615E 00	4.3	26.98	1.417E 00	6.060E-02
25.65	1.799E 00	3.0	27.51	1.579E 00	4.744E-02
26.15	1.825E 00	3.0	28.04	1.603E 00	4.882E-02
26.65	2.038E 00	3.8	28.58	1.791E 00	6.799E-02
27.15	2.156E 00	2.9	29.11	1.896E 00	5.480E-02
27.65	2.012E 00	4.2	29.64	1.770E 00	7.352E-02
28.15	1.577E 00	7.8	30.18	1.388E 00	1.084E-01
28.65	9.508E-01	10.5	30.71	8.375E-01	8.765E-02
29.15	6.742E-01	5.3	31.24	5.942E-01	3.132E-02
29.65	9.824E-01	4.9	31.78	8.664E-01	4.261E-02
31.15	7.825E-01	5.4	33.37	6.914E-01	3.753E-02
31.37	7.388E-01	5.0	33.61	6.530E-01	3.243E-02
32.65	8.538E-01	4.7	34.97	7.560E-01	3.551E-02
32.87	8.717E-01	4.0	35.20	7.721E-01	3.073E-02
34.15	7.807E-01	4.9	36.56	6.927E-01	3.415E-02
34.37	7.722E-01	5.0	36.80	6.854E-01	3.435E-02
35.65	5.552E-01	6.0	38.15	4.937E-01	2.986E-02
35.87	5.408E-01	6.1	38.39	4.811E-01	2.927E-02
37.15	3.996E-01	7.2	39.75	3.561E-01	2.561E-02
38.65	3.627E-01	7.1	41.33	3.240E-01	2.300E-02
40.15	3.012E-01	7.9	42.92	2.697E-01	2.136E-02
41.65	2.735E-01	8.1	44.51	2.455E-01	2.000E-02
43.15	2.111E-01	9.9	46.09	1.900E-01	1.884E-02
46.15	1.843E-01	8.7	49.25	1.668E-01	1.448E-02
47.65	2.331E-01	14.8	50.83	2.115E-01	3.122E-02
49.15	1.885E-01	17.2	52.40	1.716E-01	2.950E-02
50.65	1.444E-01	22.7	53.97	1.318E-01	2.986E-02
52.15	1.480E-01	23.2	55.54	1.355E-01	3.141E-02