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A Simple Analysis Method for Measuring in Real-Time Power Spectral Densities and Coherence Functions in a Large Frequency Range

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Abstract

This paper describes a real-time method which allows the measurement of auto and cross power spectral densities in a large frequency range with almost constant relative frequency resolution. Based on a normal digital frequency analysis the resolution at low frequencies can be increased to any extend without additional electronic equipment. The long time signals needed for the low frequencies are won from the high frequency data by a digital low pass filter. Due to this decimation of the time series only moderate storage region is needed allowing the use of a small digital computer for on-line application. The method is suitable to monitor the spectra in a wide frequency range without time delay.

Eine einfache Methode zur Echtzeitmessung von Frequenzspektren und Kohärenzfunktion in einem großen Frequenzbereich

Zusammenfassung

In diesem Bericht wird ein einfaches Echtzeitverfahren vorgestellt, das die Messung spektraler Leistungsdichten in einem großen Frequenzbereich erlaubt, wobei eine fast konstante relative Frequenzauflösung erreicht wird. Ausgehend von einer normalen digitalen Frequenzanalyse kann die Auflösung für kleine Frequenzen praktisch beliebig verbessert werden. Die für die kleinen Frequenzen benötigten langen Zeitsignale werden durch digitales Filtern erzeugt, das eine Reduktion der Daten und damit des Speicherplatzes erlaubt. Dadurch ist die Verwendung eines Kleinrechners im on-line Einsatz möglich. Die Methode ist geeignet, Spektren in einem großen Frequenzbereich ohne Verzögerung zu überwachen.

Contents

1. Introduction

In nuclear power plants it becomes recently more and more customary to observe and monitore not only the mean values of diverse signals (e.g. neutron flux, temperature, pressure, flow rate) but also the fluctuations of these signals. The additional information got from detailed analysis of these fluctuations yields a better picture of the state of the system. This information can help to recognize disturbances at an early state and thus larger damage can be avoided by proper acting.

The most used method for analysing signal fluctuations is the frequency analysis. Due to the availability of small digital computers the digital frequency analysis using Fourier transform of time series has become very popular. The spectra obtained by this technique and especially the changes of the spectra during the lifetime of the reactor system describe the system and its changes more precisely than the mere observation of the mean values. Looking for special effects in a distinctive frequency range monitoring the spectra in a rather narrow frequency band may be sufficient. But for overall surveillance one needs the spectra in a very large frequency range. Moreover in order to get the spectra as soon as possible they should be calculated in real-time by an on-line method.

This paper describes a simple real-time method for measurement of power spectral densities in a large frequency range. Based on a normal digital frequency analysis the frequency resolution at low frequencies is increased without additional electronic equipment. By this method spectra can be measured in a frequency range of several decades with an almost constant frequency resolution in the entire range.

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2. Digital Frequency Analysis and its Disadvantages

The time series $x(n \cdot \Delta t)$ needed for digital frequency analysis is won by sampling and digitizing the analog time signal(s) in an ADC at a constant rate. The sampling frequency $f_s = 1/\Delta t$ must be at least twice the highest frequency f_h one is interested in. To avoid aliasing of the spectra by higher frequency components a low pass filter suppressing all frequencies higher than $f_s/2$ has to be used. By Fourier transformation of a time interval with N samples the complex valued Fourier coefficients of this interval were calculated for frequencies $k \cdot \Delta f$

$$c(k \cdot \Delta f) = \sum_{n=o}^{N-1} x(n\Delta t) \cdot e^{-i\frac{2\pi}{N} \cdot n \cdot k} \qquad (1)$$

with

$$k = 0,1,2 \ldots N/2$$

and the bandwidth

$$\Delta f = \frac{1}{N \cdot \Delta t} = \frac{1}{T} .$$

T is the length of the time interval. Conjugate complex multiplication of the Fourier coefficients (by themselves or by the coefficients got from a second signal), averaging these values for several time intervals and dividing by the bandwidth yields the auto and cross power spectral density resp. of the signals /1,2,3/.

By this technique for the measurement of spectra only a few frequency decades can be covered depending on the number of samples N in a time interval. Since the bandwidth Δf is constant one gets only poor relative frequency resolution for low frequencies. Increasing of the resolution at low frequencies can be achieved by two ways:

a) decreasing the sampling frequency
b) increasing the number of samples N.

Both methods have severe disadvantages. In the first case the high frequency part of the spectra is lost. In the second case the highest analysed frequency remains the old one but one has to calculate the Fourier coefficients for a time series with a large number of samples. Due to the constant bandwidth Δf this results in an unnecessarily high resolution at high frequencies. Another disadvantage of a large N may be the fact that one has to wait for the high frequency part of the spectra the same long time which is needed for the low frequency spectrum namely the time of at least one time interval. In addition the computation time increases proportional to N^2 and $N \cdot lg_2 N$ for a discrete Fourier transform and for the Fast Fourier Transform (FFT) respectively.

Assuming still reasonable numbers of N = 1000 delivers the spectrum for about 2 1/2 frequency decades. Requiring a resolution $\Delta f/f \leq 10\%$ only 1 1/2 decades of the spectrum can be used. Certainly this frequency range is too small for surveillance. There is one way to extend the frequency range without these disadvantages: another ADC sampling the data at another frequency and also another low pass filter to avoid aliasing. Thus, for a large frequency range several analysing channels are required. If one uses a multiplexing device to connect the diverse filters to one common ADC at least several low pass filters must be used.

A method /4,5/ avoiding these disadvantages computes the spectrum only for distinct frequencies with a desired frequency resolution. Any frequency and any resolution may be chosen. However, the computation time of this method is proportional to the number of frequency points one is interested in.

3. The Extended Frequency Analysis

By the technique to be described now the frequency range is extended to lower frequencies without receiving the disadvantages mentioned above.

3.1 The Principle

(see also fig.1)

First, the analog time signal is sampled with a frequency according to the highest frequency of interest. Only a moderate number of samples (e.g. $N = 256$) is needed for the well known procedure, described above. This delivers the PSD of the original time signal. So far normal digital frequency analysis was performed.

Now the Fourier coefficients (Eq. 1) are used for further calculations. By an inverse Fourier transform of the coefficients c one gets a time signal $x_1(t)$

$$x_1\,(n\Delta t) = \sum_{k=0}^{N/2K} c(k\Delta f) \cdot e^{+i\,\frac{2\pi}{N}\,n\cdot k} \tag{2}$$

$$n = 0,1,2,\ \dots\ N-1$$

For $K = 1$ the original time signal x is obtained. If $K > 1$ is chosen the calculated time signal x_1 does not contain high frequencies. This signal $x_1(t)$ is a least squares fit to the original time signal $x(t)$ /6,9/. Since the maximum frequency is $N_{/2K} < N_{/2}$ it is sufficient to take only $N_{/K} < N$ samples of the signal. Indeed by a discrete inverse Fourier transform only $N_{/K}$ equidistant points are computed

$$x_1\ (m\Delta t_1) = \sum_{k=o}^{N/2K} c(k\cdot\Delta f)\ e^{+\ i\frac{2\pi K}{N} m\cdot k} \tag{3}$$

$$m = 0,1,2,\ \ldots\ \frac{N}{K} - 1$$

The sample frequency $1/\Delta t_1$ of the time signal $x_1(t)$ is reduced according to the maximum frequencies:

$$\frac{1}{\Delta t_1} = \frac{1}{K} \cdot \frac{1}{\Delta t} \tag{4}$$

That means: The original time interval with N samples has been low pass filtered and is now represented by only $N_{/K}$ samples.

Putting together those $N_{/K}$ samples calculated from K original time intervals following each other without gap a new time series $x_1(t)$ with again N samples is composed. This time interval T_1 is longer than the original time interval T by the factor K. The time series $x_1(t)$ is now used for a normal digital frequency analysis. In comparison with the original spectrum the frequency range of this 1st order spectrum is shifted to lower frequencies by a factor of K (see eqs. 1 and 4). As a consequence the frequency resolution for low frequencies has been increased.

Obviously the 1st order time signal $x_1(t)$ can be processed in the same way as the original signal in order to produce a 2nd order signal $x_2(t)$ and the related 2nd order spectrum the frequency range of which will be shifted again to smaller values. This procedure can be repeated as often as one likes. Taking a constant factor K for all shifts the spectra of the diverse orders are placed equidistantly on a log. frequency scale. Although the bandwidth remains constant within each spectrum the composed spectrum approximates a constant relative frequency resolution. Thus a spectrum can be measured in a large frequency range (several decades) without the disadvantages described earlier.

Fig.2 shows an example: a white noise signal passes simultaneously two electronic filters set as a low pass (f = 2 Hz) and as a band pass (10 Hz to 200 Hz) resp. The sum of the filter outputs was digitized at a sample frequency of 512 Hz. N = 256 samples have been taken for one time interval. The original spectrum and the 1st, 2nd and 3rd order spectra each shifted by a factor of K = 4 are shown in a linear plot in fig. 2a. The entire real-time measured spectrum composed of the four individual spectra is shown in fig. 2b now using logarithmic scales.

3.2 Refinements

Though this results are quite encouraging the method must be refined to minimize the errors due to fact that signals of finite length are transformed.

3.2.1 Window Function

Before transforming time series x(t) of finite length T it is recommended /2,3,7,8,9/ to apply special spectral windows in order to reduce the side-lobes as far as possible. This produces an increase of the effective bandwidth resulting in a smoother spectrum. The windows w(t) are usually normalized to

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \; W(\omega) = 1 \qquad (5)$$

with W(ω) being the Fourier transform of the window function w(t). The normalization (eq. 5) fits well for spectra computed as a Fourier transform of a correlation function. Since in the direct method used here the spectra are calculated as square modules of the Fourier coefficients the power spectral density must be normalized to

$$\frac{1}{2\pi T} \int_{-\infty}^{\infty} d\omega \; W^2(\omega) \leq 1 \qquad (6)$$

A window function damps the signal amplitudes near the ends of the time interval. Applying the procedure described above to a time series $x(t)$ modified by a window $w(t)$ one gets a low pass filtered time signal $x_1(t)$ which must be divided by the window function $w(t)$ to have the correct signal. Obviously there appear large errors at both ends of the time interval, because of the small values of the window function. As a consequence of this fact only the middle part (e.g. the second and third quarter) of the interval can be used to compose the new time signal $x_1(t)$. If a taper window with $w(t) = 1$ for the second and third quarter is used computation time can be saved because in this case there is no need for deviding the filtered signal by the window. But, if only time intervals are used which follow each other without gaps, the composed signal $x_1(t)$ will have large gaps. This can be avoided by using not only the intervals alone but also overlapped intervals which are shifted by half the interval length (see fig. 1). This results not only in a correctly composed signal $x_1(t)$ but it also improves the accuracy of the spectrum of the original time signal $x(t)$ (see below).

3.2.2 Filter Function

For the same reason, spectral windows are applied to time series one has to apply a filter function to the frequency data i.e. the Fourier coefficients should be filtered properly before an inverse Fourier transformation is made. The filtering as indicated in eq. (2,3) means using a rectangular filter with the transfer function the real part of which is given by

$$F(k \cdot \Delta f) = \begin{cases} 1 & \text{for } k = 0,1,2 \ldots {}^{N}/{}_{2K} \\ 0 & \text{otherwise} \end{cases}$$

The imaginary part equals zero for all frequencies. It is well known that a filter of this type causes considerably high side-lobes in the transformed signal. To reduce this side-lobes it is recommended to use a filter function with a smooth cut off. For the measurements shown in figs. 3, 4, 5 a filter with a cosine taper was applied the transfer function of which was

$$F(k\Delta f) = \begin{cases} 1 & \text{for } k = 0 \ldots \frac{N}{4K} \\ 1/2 \cdot \left[1-\cos\left(\frac{4\pi K}{N} \cdot k\right)\right] & \text{for } k = \frac{N}{4K} \ldots \frac{N}{2K} \\ 0 & \text{otherwise} \end{cases}$$

Of course this filtering influences the spectrum of the next order which is shifted to lower frequencies by a factor K. The power spectral density will be modified by the square of this filter function, which results from eqs.(1) and (4) in

$$F^2(k \cdot \Delta f_1) = \begin{cases} 1 & \text{for } k = 0 \ldots \frac{N}{4} \\ 1/4 \cdot \left[1-\cos\left(\frac{4\pi}{N} \cdot k\right)\right]^2 & \text{for } k = \frac{N}{4} \ldots \frac{N}{2} \end{cases}$$

with $\Delta f_1 = \Delta f/_K$. For simplicity reasons only the first half of this spectrum is plotted, where the filter function equals 1.

With these two refinements the dynamic range of the extended digital frequency analysis is improved. It should be noted that the extended frequency analysis works best for white spectra, a fact which is well known from the normal frequency analysis. If the dynamic range of the spectrum to be measured is too large prewhitening of the signal is recommended.

4. The Measuring Equipment

The electronic equipment for the measurement of the spectrum of an analog signal consists only of a low pass filter (for anti-aliasing) and an ADC. The sampled data are stored alternately in two buffers each representing a time interval. During the time one buffer is filled up the content of the other buffer must be worked up and the computation of the higher ordered signals and spectra must be completed too. As described above and to be seen in the scheme in fig. 1 all work is done by the computer. Only the original time signal x(t) must be digitized by the ADC. The filtered signals $x_1(t)$, $x_2(t)$ ... are produced and stored in the computer.

4.1 Storage Region

Though there have to be stored high frequency data as well as low frequency data only moderate storage region is needed because only N samples for each signal x(t), $x_1(t)$, $x_2(t)$ ... $x_n(t)$ are kept. Assuming N = 256 and n = 5 a storage region of (n+1)N = 1536 words is required. With a shifting factor K = 4 the nth order interval is K^n = 1024 times longer than the original intervals. For comparison,in a normal digital frequency analysis one has to transform a record with $K^n \cdot N \simeq$ 250,000 samples in order to cover the same frequency range. When performing a cross correlation measurement between two signals there is a need for

$(n+1)N \cdot 2$	samples of the two time signals
$(n+1)\frac{N}{2} \cdot 2$	values of the auto power spectral densities
$(n+1)N$	values of the cross power spectral density

that is a total of 4(n+1)N words. Since the higher ordered signals and spectra have to be computed very seldom they may be stored outside the core memory e.g. on a disc storage.

4.2 Speed

Although there is a lot of computation to be done the method works still pretty fast. If one uses numbers $N = 2^p$ (p an integer) the FFT (Fast Fourier Transform) algorithm can be used. The example shown in fig. 3 was measured in real-time using a small digital computer (hp 2100, 16 k core memory) with a microprogrammed FFT and a very flexible operating system made for performing digital frequency analysis. In this example two APSD's and the CPSD of two signals have been measured at once in a frequency range of about 5 decades with an upper frequency limit of 12 Hz. In fact with the parameters chosen for this measurements the program can be run two times faster. Since a great deal of the measuring program was written in FORTRAN without paying attention to speed there can be won some speed up by using assembler language and proper programming. It should be noted that increasing of the number of shifts does not decrease the programs' speed and hence the highest analysed frequency. This is due to the fact that the higher ordered signals and spectra must be computed very seldom. This can be done when the computer is not busy for the original and the 1st order signals and spectra respectively.

5. Accuracy, Dynamics

As mentioned above the frequency analysis works best for a white spectrum. The accuracy test is shown in fig.4. White noise was band pass filtered with a lower and upper cut off frequency of .01 Hz and 50 Hz resp. (Near the cut off frequencies the electronic filter which was used has a gain of 1 dB). It is clearly to be seen that white noise is measured with very high accuracy. Apart from the statistical error there is no recognizable difference between the spectra of the diverse orders.

The dynamics of the method has been tested using signals the spectra of which are proportional to f^2 and $(1/f)^2$ resp. The result of this test is shown in fig. 5 and shows that at least three decades in the amplitude of the spectra are measured with an error less than 10 %. Since the dynamic range of the spectra in fig. 3 is larger than 6 decades the analogue signals have been prewhitened by amplifying the high frequency part by 100. By this prewhitening the spectra measured actually have a dynamic range of about 2 decades.

6. Statistical Error

Of course, the statistical errors of the higher ordered spectra must be larger than those of the original and low ordered spectra because of the very different numbers of intervals which can be used for averaging the spectra. In the example in fig.3 measured by true averaging the total signal length was only 8 hours. Therefore only 3 intervals of the 5th order signal have been computed whereas the original spectrum is averaged from about 3000 intervals. Assuming validity of the error estimation in /10/ for small numbers too, this results in a statistical error of the low frequency part of the spectrum (5th order) to be 32 times larger than that one of the high frequency part (original).

Since the application of window functions to time signals results in an increase of the effective bandwidth the spectrum becomes smoother and the statistical error of the spectral values would decrease. But by window functions part of the signal is omitted and therefore the statistical error would increase. Indeed, as can be shown from results in /9/ the relative error ε of an auto power spectral density value is not influenced by the window function and is given by

$$\varepsilon^2 = \frac{1}{M} \tag{7}$$

when averaging M independent (non-overlapping) intervals.

The spectrum is computed as square modulus of the Fourier coefficients. Therefore, when applying a window function w(t), only a part

$$\frac{1}{T}\int_0^T dt\ w^2(t) \leq 1 \quad (\max(w(t)) = 1) \tag{8}$$

of the available information is used. With a taper window as indicated in chapter 3.2.1

$$w(t) = \begin{cases} 1/2(1-\cos \frac{4\pi}{T}t) & \text{for } 0 \leq t \leq \frac{T}{4} \\ 1 & \text{for } \frac{T}{4} \leq t \leq \frac{3T}{4} \\ 1/2(1-\cos \frac{4\pi}{T}t) & \text{for } \frac{3T}{4} \leq t \leq T \end{cases} \qquad (9)$$

only 11/16 of the signal is used effectively. Hence, the variance of the power spectral density values is expected to be 16/11 times larger than the variance which will be found by using all information. Indeed by calculating the spectrum from overlapping intervals and using the window function defined above no information is thrown away. The ratio of the variances with and without overlapping intervals for a fixed averaging time was measured and found to be 0.67 ± 5%, which agrees quite well with the theoretical value 11/16 $\simeq$ 0.69.

Averaging the power spectral density values from overlapping intervals and using a window function w(t) the relative error ε_o results from eq. 7 and 8 in

$$\varepsilon_o^2 = \frac{1}{M \cdot T} \int_o^T dt\, w^2(t). \qquad (10)$$

Since M is the number of non-overlapping intervals M•T is the total measuring time. This formula is only valid for windows with $w(t) = 1$ for $T/_4 \leq t \leq 3T/_4$ and $M >> 1$.

7. Conclusions

It was the purpose of this paper to show that the digital frequency analysis can be used for real-time measurements in a large frequency range avoiding the well known disadvantages. The main advantages of the method describes above are

1. Large frequency range with almost constant relative frequency resolution in the entire range.
2. Real-time analysis without additional electronic equipment. Changes in the high frequency part of the spectrum can be seen earlier than those in the low frequency region because of the different time constants which can be taken when the RC-averaging mode is used.

Therefore this method is suitable for surveillance of a system by monitoring the frequency spectra of relevant signals.

Acknowledgement

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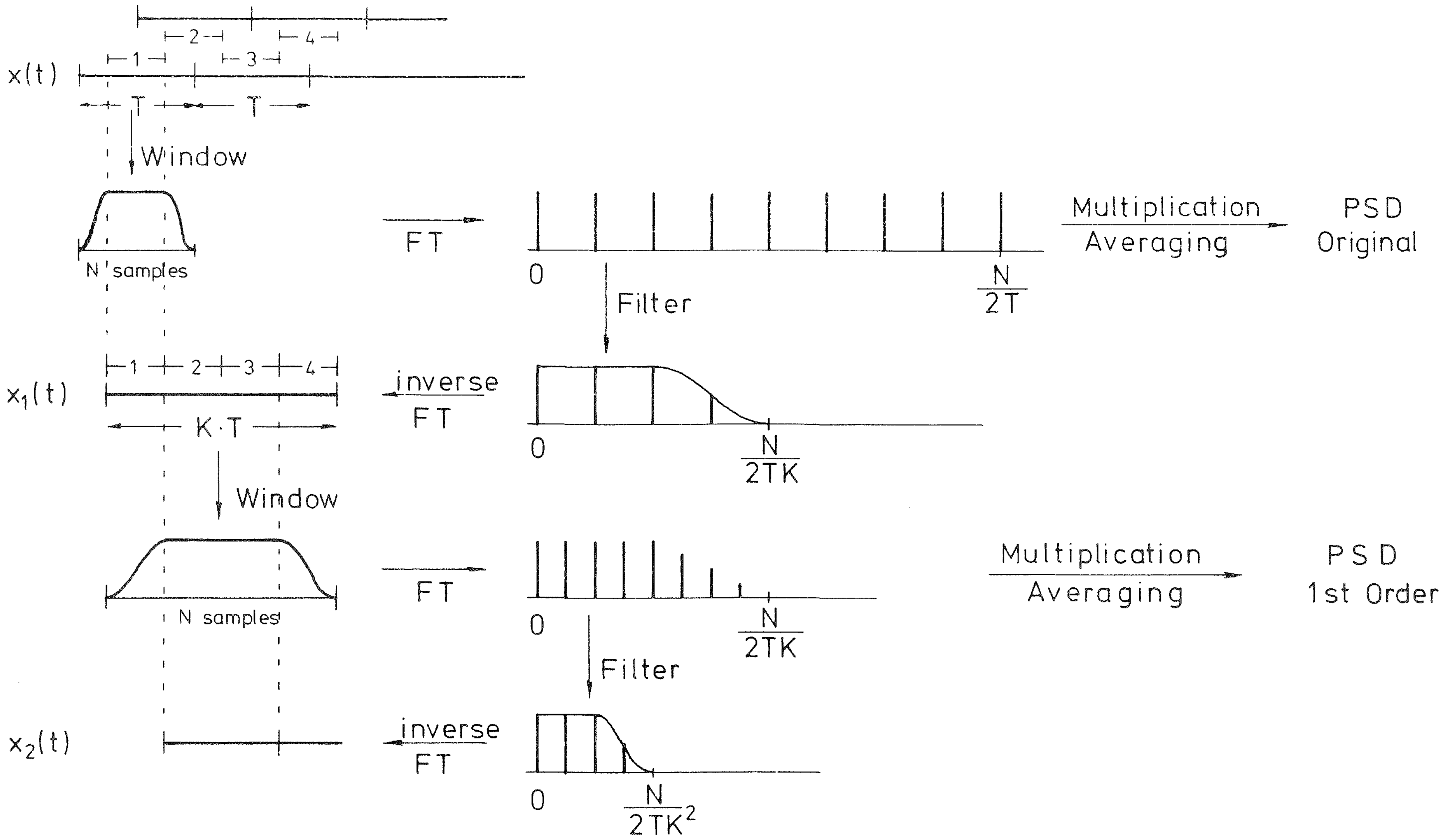

Fig. 1 Scheme of the Extended Digital Frequency Analysis

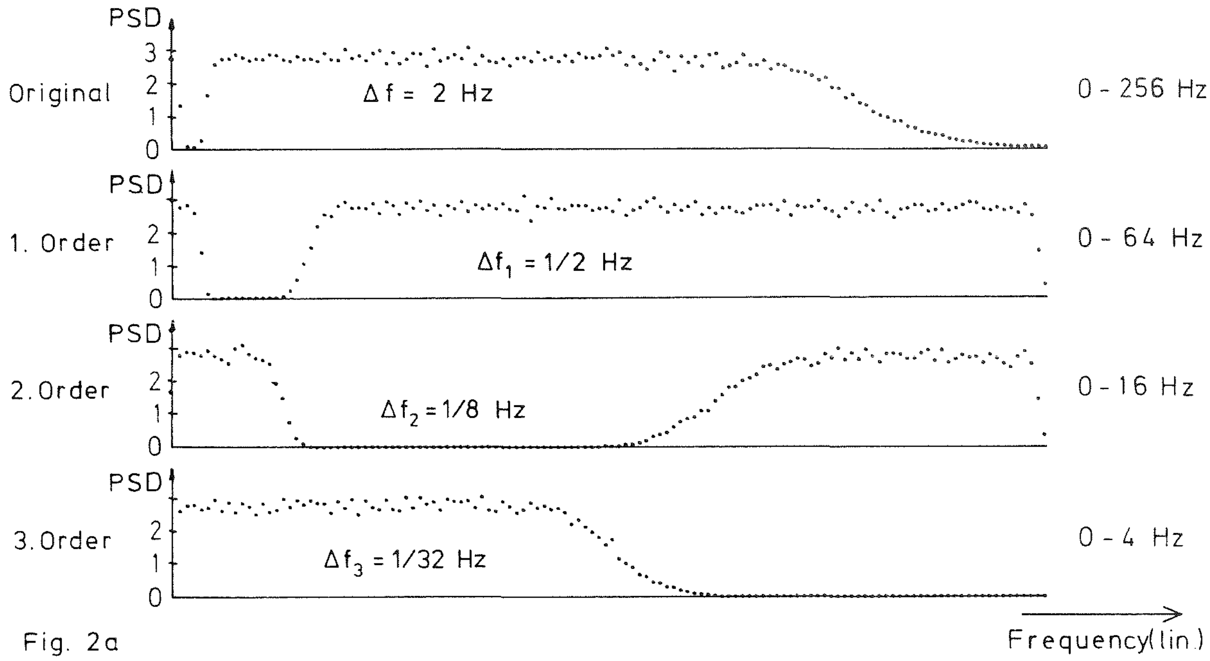
PSD
Original
Δf = 2 Hz
0 - 256 Hz
1. Order
Δf1 = 1/2 Hz
0 - 64 Hz
2. Order
Δf2 = 1/8 Hz
0 - 16 Hz
3. Order
Δf3 = 1/32 Hz
0 - 4 Hz
Frequency(lin.)

Fig. 2a

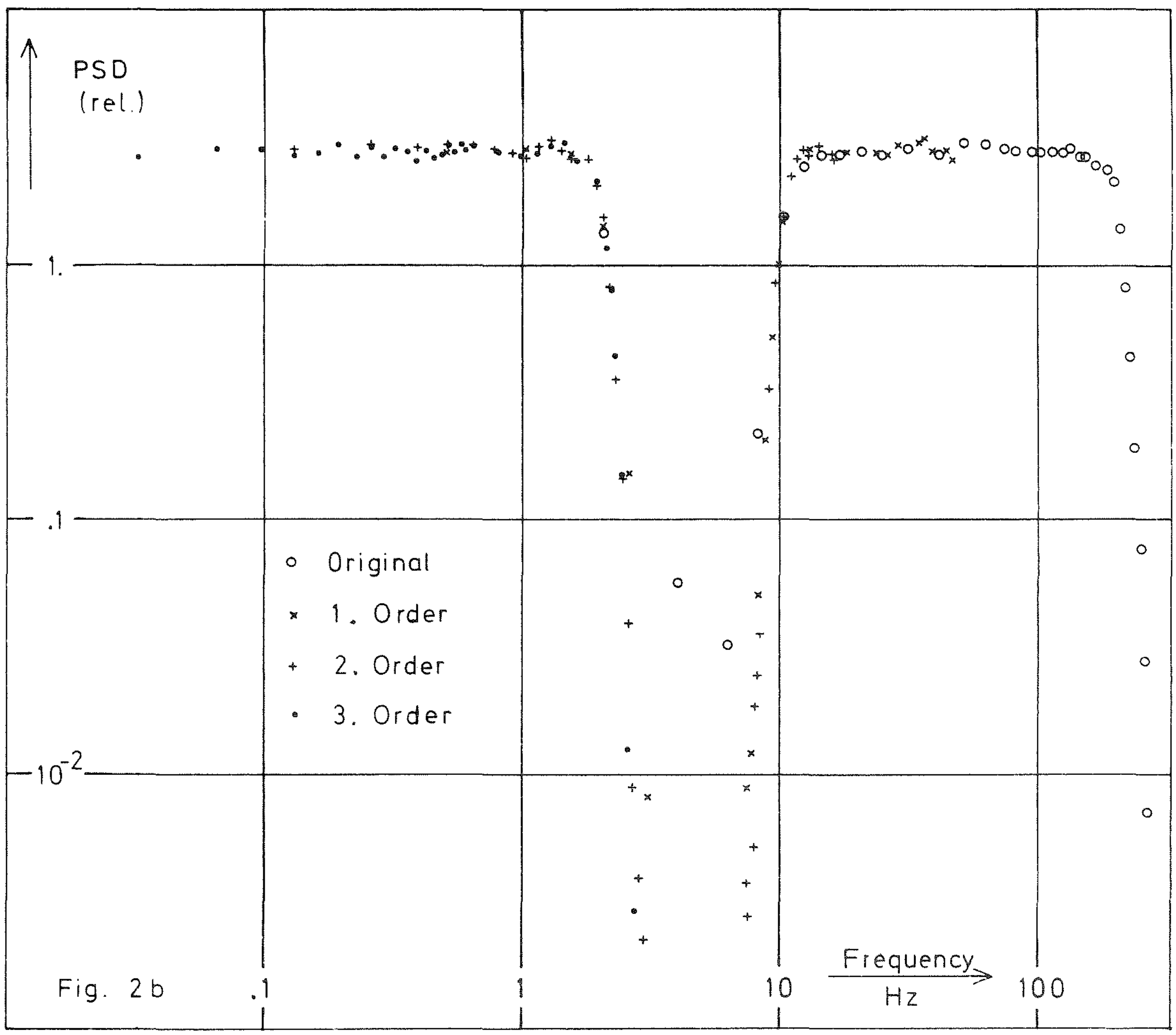
PSD
(rel.)
1.
.1
10-2
Original
1. Order
2. Order
3. Order
.1
1
10
100
Frequency
Hz

Fig. 2b

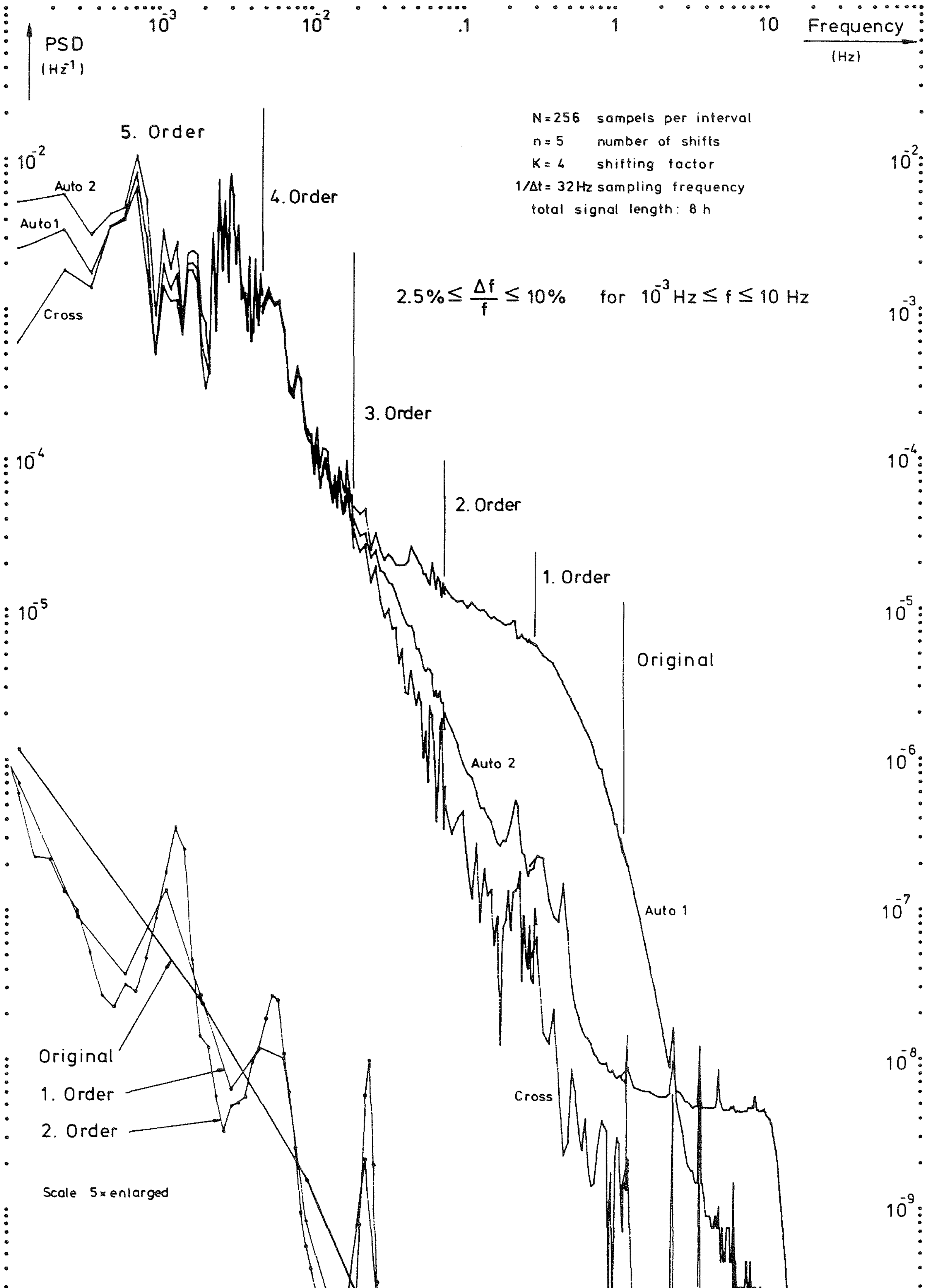

Fig. 3 Auto and Cross Power Spectral Densities measured in Real-Time

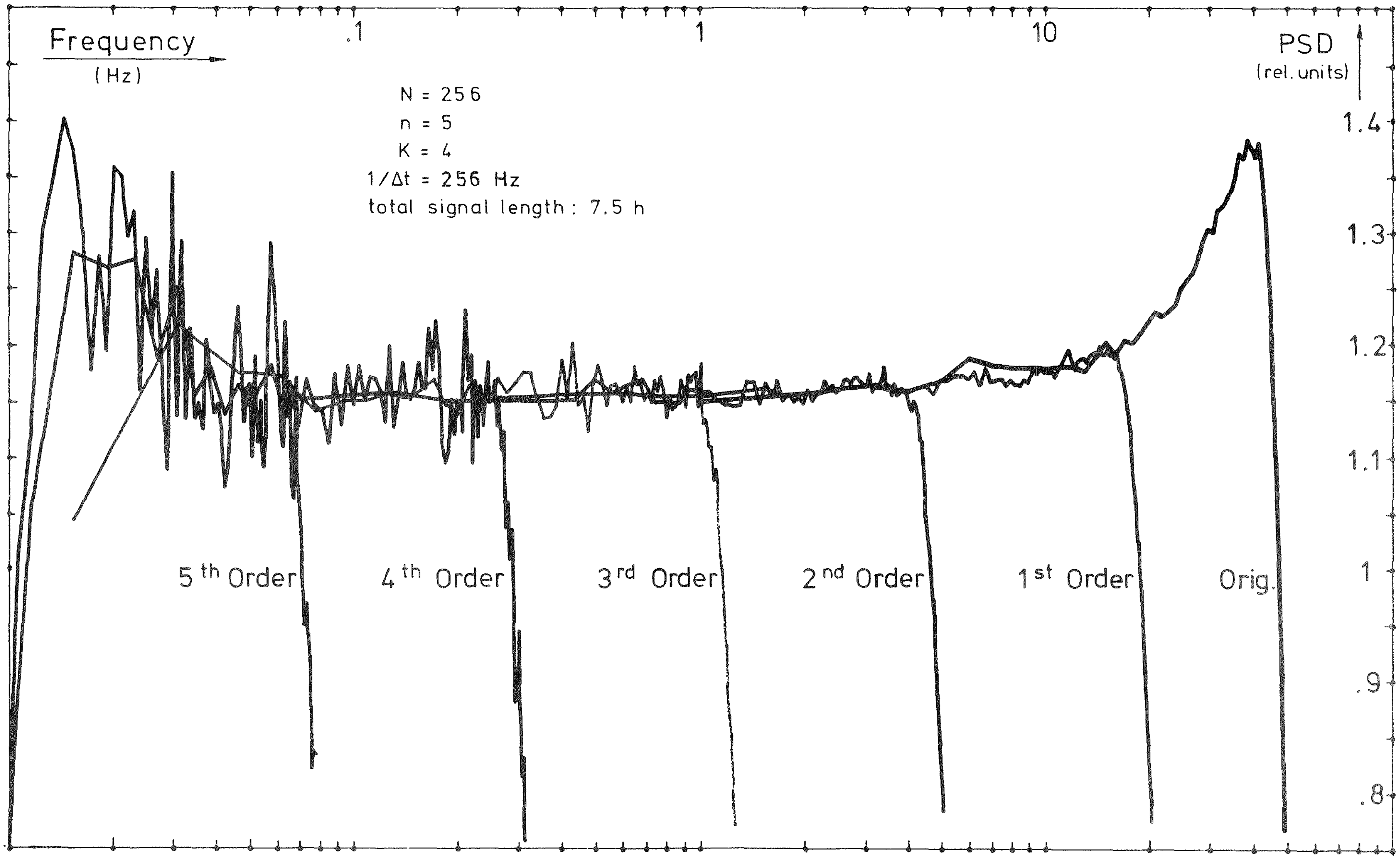

Fig. 4 Accuracy Test

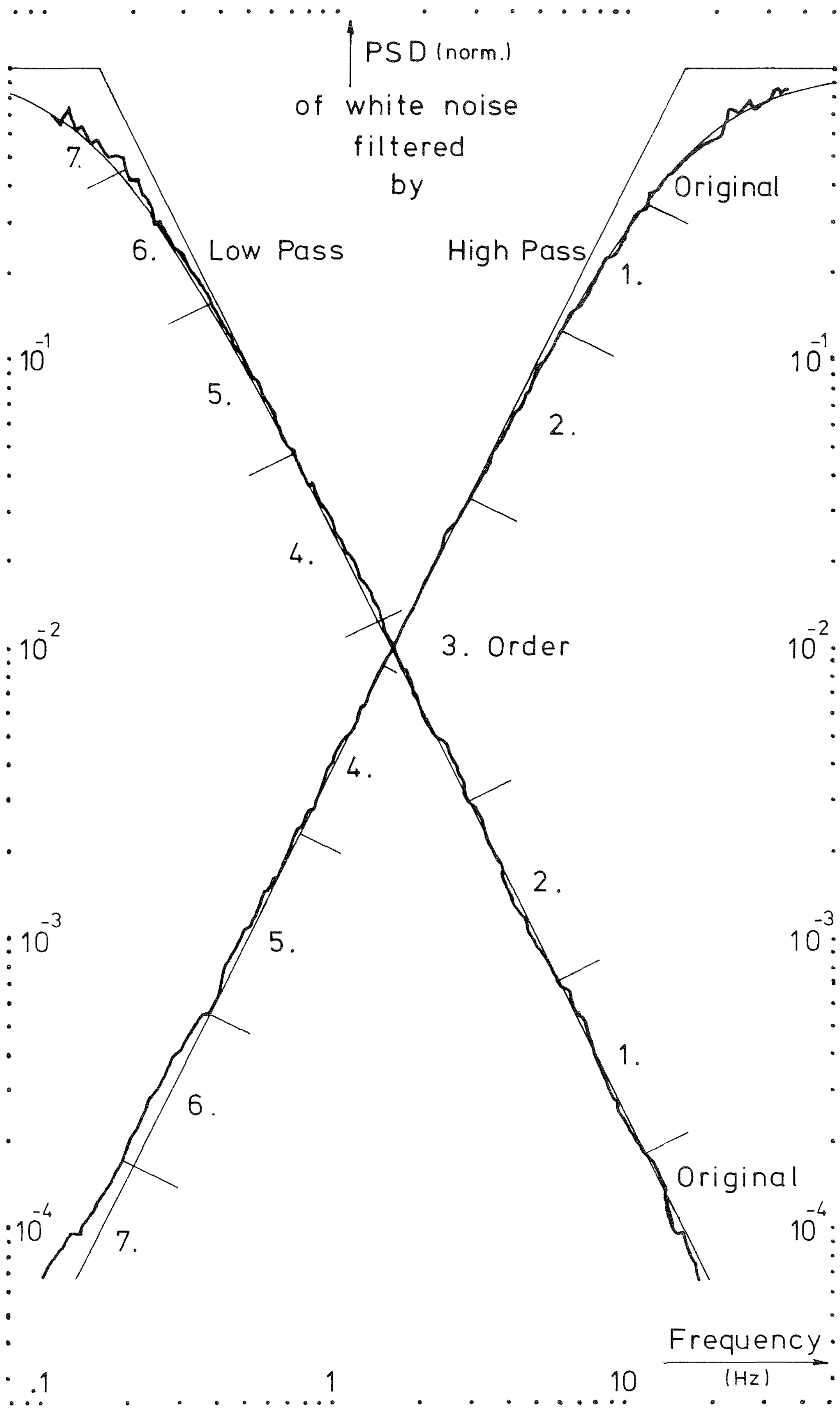

Fig. 5 Dynamics Test