KERNFORSCHUNGSZENTRUM

KARLSRUHE

Januar 1976 KFK 2241

Institut für Reaktorentwicklung
Projekt Schneller Brüter

FLEXA
A Convenient FORTRAN Program for Flexibility and Stress Analyses of Plain Piping Systems

H. Schnauder

GESELLSCHAFT
FÜR
KERNFORSCHUNG M.B.H.

KARLSRUHE

GESELLSCHAFT FÜR KERNFORSCHUNG M.B.H.

KARLSRUHE

KERNFORSCHUNGSZENTRUM KARLSRUHE

KFK 2241

Institut für Reaktorentwicklung
Projekt Schneller Brüter

F L E X A

A Convenient FORTRAN Program for Flexibility and Stress Analyses of Plain Piping Systems

by

H. Schnauder

Gesellschaft für Kernforschung m.b.H., Karlsruhe

Abstract

The report describes a method and a computer program for flexibility and stress analyses of plain piping systems located between two fixed anchor points. The program can be used in time sharing operation through TSO (time sharing option of OS/360) as well as in batch operation. Only few data are necessary for the input, e.g. only the length and inclination for a straight piping element and the radius of curvature and the two angles including the bend for a bent piping element. The temperature dependent material data for the types of steel are taken from a program internal library. Besides the usual list output, it is possible to plot the shape of the piping system in a simple true-to-scale drawing. In time sharing operation the representation is achieved via the display, in batch operation via the plotter. Besides the fixed anchor point forces the program also calculates the equivalent stresses at the point of the maximum bending moment as well as the safety with respect to the creep limit ($\sigma_{1/100\ 000}$).

Kurzbeschreibung

FLEXA - Ein handliches FORTRAN-Programm zur Flexibilitäts- und Spannungsberechnung von ebenen Rohrsystemen.

Es wird eine Methode und ein Rechenprogramm zur Flexibilitäts- und Spannungsberechnung von ebenen Rohrsystemen zwischen zwei Einspannfestpunkten beschrieben. Das Programm kann sowohl im Time-Sharing-Betrieb über TSO (Time sharing option des OS/360) als auch im Batch-Betrieb verwendet werden. Für die Eingabe sind nur sehr wenig Daten erforderlich, so z.B. für die Definition eines geraden Rohrelements Länge und Steigungswinkel und für einen Rohrbogen Krümmungsradius und der einschließende Winkel. Die temperaturabhängigen Stoffdaten entstammen der programmeigenen Bibliothek. Neben der üblichen Ausgabe in Listenform kann der Verlauf des betreffenden Rohrsystems graphisch dargestellt werden; im Time-Sharing-Betrieb auf Bildschirm und im Batch-Betrieb als Zeichnung auf Papier.
Das Programm errechnet neben den Festpunktskräften die Spannungen an der am stärksten belasteten Stelle und ermittelt an Hand der vorhandenen Werkstoffdaten die Sicherheiten gegenüber der Zeitdehngrenze ($\sigma_{1/100\,000}$).

Index

List of Figures

1. Capabilities of the Program

The program is qualified for flexibility and stress analyses of plain piping systems which are located between two fixed anchor points. The calculation takes into consideration the change of temperature and the internal pressure. Cold springs and fixed point movements are input data. The program can be used for time sharing operation by TSO as well as for normal batch operation. A high value was set for a simple and clear input, e.g. knowledge of the coordinates for the individual piping elements is not necessary for the program input. The data needed are calculated by the program. Some restrictions result from the simplifications.

- The piping system consists only of straight parts and bends.
- Nozzles or branches are not allowed.
- The piping system begins and ends with a straight part.
- A straight part must be followed by a bend and vice versa.
- Pipe bends and straight parts must meet each other tangentially, which means that knees must not be provided in the piping system.
- The pipe cross section and the temperature are constant for the piping system.
- A piping system can be made up of a maximum of 21 part components. (This limitation is imposed by the computer program used and can be easily extended, if so desired.)

The input data for a straight part are the pipe length and the inclination only. For a bend the bending radius related to the neutral axis and the two angles comprising the bend must be indicated.

The calculation of the forces acting at the fixed anchor points is based on a procedure recommended by Hampel [1]. The stress analysis is based on recommendations of Jürgensonn [2].

The material data needed for calculation, e.g. stress intensity factors, Young's modulus, and the coefficient of thermal expansion are temperature dependent and are taken from a program internal library. (Types of steals see chapt.4)

Besides the moments and forces of the fixed anchor points the output of the program also contains the strains caused by change of temperature and internal pressure and the maximum equivalent stresses at the point of the maximum bending moment. The coordinates of the beginning and end of each individual part (straight pipe element or bend), the coordinates of the centroid, and the coordinates of the center of curvature for the bends are also output data. The shape of the piping system can be plotted in a simple true-to-scale drawing. The recognition of errors in the input data, is e.g., facilitated by this means.

2. Mathematical Basis of Flexibility Analysis of Piping Systems

2.1 Generalities

Since in statically determinate systems compulsive forces are neither exerted by expansions nor by movements of anchor points, piping systems are always statically indeterminate systems. Therefore, the first task consists in making a system statically determinate. For piping systems this is e.g. achieved by assuming one fixed extremity while the other can move freely.

The load at the end points is obtained if forces are allowed to act on the freely moving extremities, which are sufficiently great to neutralize the displacement. With respect to the end points, a distinction must be made between fixed anchor and hinged anchor points. Fixed anchor points can accomodate torsion moments in addition to forces and bending moments, while hinged anchor points only accomodate forces.

Investigations have revealed that fixed anchor points are very close to practice. The most frequent case is a piping system contained between 2 fixed anchor points. Therefore, it is reasonable to restrict oneself to this case. Since, in practice, fixed anchor points are always a bit flexible, the computation yields higher stresses, which means that the assumption of fixed anchor points is on the safe side.

The single-plane system is a special case of a three dimensional system. Transverse forces and torsion moments do not occur. The supporting loads are restricted to one normal force and one bending moment each.

It should be noted that the relations applicable to the single-plane system are also valid for a three dimensional system. The principal difference consists in the occurrence of transverse forces and torsion moments.

2.1.1 Coordinate System

Since the case of the single-plane system treated here is a special case of the multi-plane system, the axes are designated a,b instead of x,y. This facilitates the transition to the three-dimensional system.

A distinction must be made between the outer and the inner coordinate systems. The outer system applies to the entire piping system while each part component has its own inner coordinate system. The corresponding axes are parallel,only the origins differ from each other.

2.1.2 Determination for each Individual Part Component of the Initial and End Coordinates, the Centroid Coordinates and the Centers of the Bending Radii

For straight part components the inner coordinate system refers to the initial point marked by the index 1 in Fig.1. For bends the inner coordinate system is placed into the center of curvature. The angles φ_1 and φ_2 describe the included angle α. φ_1 is always the smaller and φ_2 is always the greater of the two angles. It is of no significance in this respect which angle is related to the initial point of the bend and which is related to the end point of the bend, as represented in Fig.2. All angle data are related to the positive a-axis and are given in the positive sense mathematically. For pipe bends extending from the negative to the positive quadrant, a value > 360 degrees must be given for the angle φ_2 in the positive quadrant. For example, instead of the angle φ_2 = 50 degrees the value φ_2 = 410 degrees must be given, if φ_1 is smaller than 360 degrees.

The connection of a pipe bend with a straight part component or vice versa is determined via the tangent at the point of connection, which is possible by assuming a continuous transition.

The pipe system should be so arranged in the outer coordinate system that it develops from the right to the left side, which is shown in the example, Fig.3. The initial point of the system is A, the end point B. The first part component of the length l_1 starts at point A coinciding with point 1 and ends at point 2. In this case, the angle of the first straight part component is 180 degrees. According to the definition the second part component must be a pipe bend extending from point 2 to point 3. The radius is r_2, the angle φ_1 = 180 degrees, the angle φ_2 = 270 degrees.

2.2 Flexibility Analysis

The flexibility of a piping system can be illustrated by the following relation [2]:

$$\text{flexibility} = C \frac{f(L)}{EJ} \qquad (1)$$

where C is a constant given by the shape of the piping system. f(L) is an initially unknown function of the length L, E is the Young's modulus, and J the second moment of inertia.

According to the assumption mentioned before the fixed point A of the system is rigid, while B can move dependent on the deformations of the system. A force is supposed acting upon B, which cancels the displacement. It is reasonable for this study to place the coordinate system in point B, as shown in Fig.4.

Now the elastic values of the pipe length and of the first moments of the line related to the a- and b-coordinate axes must be calculated for the system, which yields the coordinates of the elastic centroid. Likewise, the elastic values of the second moments of the line and the product of inertia of the line related to the a- and b-axes are calculated which form the basis of calculation of the end reactions.

The elastic values differ from the purely geometrical values by the fact that, in accordance with reality, a higher flexibility is obtained than would be admitted by the purely geometrical values. A discussion lateron will extensively deal with the reasons underlying this effect.

2.2.1 Determination of the Statical Moments, the Moments of Inertia and the Product of the Inertia of Lines

Fig.5 lists the designations of a straight part component in the a,b-coordinate system.
The statical moment related to the a-axis, i.e. the b-plane, is derived from the relation

$$Sa = \int b dl = L b_g \qquad (2)$$

and, accordingly, the statical moment related to the b-axis, i.e. the a-plane, is

$$Sb = \int a dl = L a_g \qquad (3)$$

The moment of inertia related to the a'-axis, i.e. to the centroid G, is calculated after the relation:

$$Ta' = \int b'^2 dl = L^3/12 \sin^2\psi \qquad (4)$$

The conversion to the a-axis is carried out using the Steiner's theorem

$$T_{new} = T_{old} + La^2 \qquad (5)$$

which appears in the following relation.

$$Ta = \int b^2 dl = Ta' + Lb_g^2 \qquad (6)$$

The moment of inertia related to the b'-axis reads accordingly

$$Tb' = \int a'^2 \, dl = L^3/12 \cos^2\psi \qquad (7)$$

and related to the b-axis

$$Tb = \int a^2 dl = Tb' + La_g^2 \qquad (8)$$

The product of inertia related to the axes of gravity a' and b' reads

$$Dab' = \int a'b'dl = L^3/12 \sin\psi \cos\psi \qquad (9)$$

and related to the a,b-axes

$$Dab = \int a \, b \, dl = Dab' + La_g \, b_g \qquad (10)$$

The length L is obtained to be

$$\int dl = L$$

and the distances a' and b' of the line element dl from the axes through the centroid

$$a' = (\ell - L/2) \cos \psi \qquad (11)$$

$$b' = (\ell - L/2) \sin \psi \qquad (12)$$

2.2.2 Determination of the Statical Moment, the Moments of Inertia and the Product of the Inertia of Circular Bends

Fig.6 contains the designations of a circular bend in the a,b-coordinate system.

The moments are first related to the center of curvature of the bend, which lies in the origin of the coordinate axes a" and b". Then, applying the Steiner's theorem as in the preceding section the conversion is carried out to the a' b' coordinate system originating in the centroid of G of the bend.

Introducing now the constants G,A,B and C (the constant G is different from the centroid of the bend G) which depend only on the aperture angle of the bend, very simple relations are obtained for the moments related to the axes through the centroid of the arc. These relations incorporate only the inclination ψ of the bend related to the positive a-axis. Fig.7 exhibits the development of the constants G,A,B and C for aperture angles α in the range between 0 and 180 degrees.

The dependence is described by the following relations:

$$G = \frac{2}{\alpha} \cdot \sin \frac{\alpha}{2} \qquad (13)$$

$$A = \frac{\alpha + \sin\alpha}{2} - \frac{4}{\alpha} \cdot \sin^2 \frac{\alpha}{2} \qquad (14)$$

$$B = \frac{\alpha - \sin\alpha}{2} \qquad (15)$$

$$C = \sin \alpha - \frac{4}{\alpha} \cdot \sin^2 \frac{\alpha}{2} \qquad (16)$$

The distances of the centroid a_g'' and b_g'' related to the a",b"-coordinate system are obtained according to

$$a_g'' = R \cdot G \cos \psi \qquad (17)$$

$$b_g'' = R \cdot G \sin \psi \qquad (18)$$

Related to the centroid, i.e. the a',b'-coordinate system, the moment of inertia of the bend line is for the a'-axis, i.e. the b'-plane

$$Ta' = R^3 (A \sin^2\psi + B \cos^2\psi) \tag{19}$$

and related to the b'-axis, i.e. the a'-plane

$$Tb' = R^3 (A \cos^2\psi + B \sin^2\psi) \tag{20}$$

The product of inertia of the bend line related to the centroid is obtained to be

$$Dab = R^3 C \sin\psi \cos\psi \tag{21}$$

The statical moments Sa' and Sb' related to the centroid are zero, since the distance from the centroid is zero.

The next step now consists in converting the moments to the a,b-coordinate system.

For the statical moments the following value is obtained related to the a-axis

$$Sa = b_g L \tag{22}$$

and for the b-axis, respectively,

$$Sb = a_g L \tag{23}$$

where L is the length of the bend in accordance with the relation

$$L = \int dl = R\,\alpha \tag{24}$$

The moment of inertia related to the a-axis is obtained to be

$$Ta = Ta' + L\, b_g^2 \tag{25}$$

and for the b-axis

$$Tb = Tb' + L\, a_g^2 \tag{26}$$

The product of inertia of the bend line related to the a,b-coordinate system is

$$Dab = Dab' + L\, a_g\, b_g$$

2.2.3 Cross Section Flattening of a Bend

If a bend is exposed to a bending moment, the pipe cross section becomes flattened as exhibited in Fig.8. Flattening produces the effect that the moments required to deform the bend are lower than expected from the theory of bending. This means that a bend is less resistant to bending and behaves like a bend with a greater bending radius, i.e. a bend of greater length than obtained in accordance with the relation

$$L = R\,\alpha \qquad (24)$$

Von Kàrmàn was the first to indicate this phenomenon and he proposed a suitable method of correction. The method relies on the introduction of a correcting factor yielding a lower bending moment as compared to the theory of bending. The correcting factor is generally termed the Kàrmàn flexibility factor K. In practice, the reciprocal value k_k, also termed bend flexibility factor, has proved to be useful in the calculation.

$$k_k = \frac{1}{K} \qquad (27)$$

There are different approximations for the flexibility factor. For smooth bends the factor proposed by Clark and Reissner is recommended, which is the flexibility factor used in the American Piping Code [3]

$$k_k = \frac{1.65}{h}$$

where

$$h = \frac{4\,Rs}{dm^2} \qquad (28)$$

The value h = λ is also termed flexibility characteristic. R is the radius to the centerline of the bend related to the neutral axis, s is the wall thickness, and dm is the average pipe diameter, as represented in Fig.9.

If a bend is exposed to internal pressure, its flexibility decreases again, since the cross section flattening is partly reversed. In this case k_k can be corrected by the relation /¯1_7

$$k_{kp} = \frac{k_k}{1 + 6 \cdot \frac{p}{E} \cdot \left(\frac{r}{s}\right)^2 \cdot \sqrt[3]{\frac{R}{s}}} \geqq 1 \qquad (29)$$

implying that k_{kp} substitutes k_k.

2.2.4 Further Corrections Influencing the Flexibility of a Piping System

Further corrections are necessary both on a bend or a straight pipe section in case that different pipe dimensions are encountered in a pipe system and/or the sections of a piping system are at different temperature levels. (These cases are presently not implemented in the program.)

The flexibility factor k_I resulting from different pipe dimensions would then be obtained from the relation

$$k_I = \frac{I_o}{I} = \frac{r_{ao}{}^4 - r_{io}{}^4}{r_a{}^4 - r_i{}^4} \qquad (30)$$

and the flexibility factor k_E resulting from different temperature levels is given by

$$k_E = \frac{E_o}{E} \qquad (31)$$

I_o and E_o are the most frequently encountered or - with a nearly identical frequency - the smaller values of the moment of inertia of the cross section and the Young's modulus, respectively. r_a and r_i are the outer and inner radii, respectively, of the pipe cross section. In the subsequent computation the flexibility factors should reasonably be combined according to

$$k = k_k \cdot k_I \cdot k_E \tag{32}$$

with the value $k_k = 1$ inserted for straight pipe sections.

2.2.5 Calculation of the Elastic Centroid and the Elastic Moments of the Line of the Piping System

As already mentioned, the flexibility analysis does not rely on the geometrical but rather on the elastic values of the system centroid and related to it on the elastic values of the moments of the line.

The relationship between the geometrical and elastic values is described by the relation

$$C_{el} = k\, C_{geo} = C_{geo} + (k - 1)\, C_{geo} \tag{33}$$

C being put for the length, the statical moment, the moment of inertia, and the product of inertia of the line.

The second form of the equation is meaningful if different operating conditions or pipe dimensions are to be calculated. In this case, the geometrical value must be assessed but once which enables to determine the elastic value dependent on the different flexibility factors k.

It is essential that the equation cannot be applied to the pipe system as a whole but separately to each component part, which means separately for each bend and each straight pipe section because of the variation from element to element of the value k.

Starting from the fact that a piping system is made up of n elements - the first element being i = 1 and the last accordingly i = n - the pipe length L_i, the statical moments Sa_i and Sb_i, the moments of inertia of the line Ta_i and Tb_i, the product of inertia of the line Dab_i, and likewise the flexibility factor k_i are determined for each element. The elastic values for the whole system result from the relations

$$L_{el} = \sum_{i=1}^{n} L_i + \sum_{i=1}^{n} L_i \, (k_i - 1) \tag{34}$$

$$Sa_{el} = \sum_{i=1}^{n} Sa_i + \sum_{i=1}^{n} Sa_i \, (k_i - 1) \tag{35}$$

$$Sb_{el} = \sum_{i=1}^{n} Sb_i + \sum_{i=1}^{n} Sb_i \, (k_i - 1) \tag{36}$$

$$Ta_{el} = \sum_{i=1}^{n} Ta_i + \sum_{i=1}^{n} Ta_i \, (k_i - 1) \tag{37}$$

$$Tb_{el} = \sum_{i=1}^{n} Tb_i + \sum_{i=1}^{n} Tb_i \, (k_i - 1) \tag{38}$$

$$Dab_{el} = \sum_{i=1}^{n} Dab_i + \sum_{i=1}^{n} Dab_i \, (k_i - 1) \tag{39}$$

related to the a,b-coordinate system of the whole system. The next step consists in the conversion to the coordinates of the elastic centroid. The coordinates of the elastic centroid η_a and η_b are found to be

$$\eta_a = \frac{Sa_{el}}{L_{el}} \tag{40}$$

and

$$\eta_b = \frac{Sb_{el}}{L_{el}} \tag{41}$$

respectively.

The elastic moments of inertia Tag and Tbg as well as the elastic product of inertia of the line are obtained from the relations

$$\mathrm{Tag} = \mathrm{Ta}_{el} - \mathrm{Sa}_{el}^{2}/L_{el} = \mathrm{Ta}_{el} - L_{el} \cdot \eta_b^{2} \tag{42}$$

$$\mathrm{Tbg} = \mathrm{Tb}_{el} - \mathrm{Sb}_{el}^{2}/L_{el} = \mathrm{Tb}_{el} - L_{el} \cdot \eta_a^{2} \tag{43}$$

$$\mathrm{Dagbg} = \mathrm{Dab}_{el} - (\mathrm{Sa}_{el} \cdot \mathrm{Sb}_{el})/L_{el} = \mathrm{Dab}_{el} - L_{el} \cdot \eta_a \eta_b \tag{44}$$

The computation scheme is so conceived that in a calculation by hand tables can be used.

2.2.6 Systems between two Fixed Anchor Points in the Absence of Torsion

In systems placed between two fixed anchor points the resulting force passes through the centroid with the thrust line of the system normally not passing through the fixed anchor points. To determine the two components Pa and Pb of the resulting force P, the expansions of the free end in a and b direction due to temperature changes and internal pressure must first be determined.

The thermal expansion due to temperature change is obtained from the relation

$$\varepsilon_{\ell\vartheta} = \alpha_{\vartheta} \; \Delta \cdot \vartheta \tag{45}$$

α_{ϑ} [K^{-1}] ϑ [^{o}C]

where α_{ϑ} is the coefficient of thermal expansion and $\Delta\vartheta$ is the temperature difference.

The <u>expansion due to internal pressure</u> is calculated by the following relation:

$$\varepsilon_{\ell p} = \frac{\sigma_\ell - \nu\,(\sigma_u + \sigma_r')}{E} = \frac{p}{E} \cdot \frac{1 \div 2\nu}{u^2 - 1} \qquad (46)$$

yielding $\nu = 0.3$

$$\varepsilon_{\ell p} = \frac{p}{E} \cdot \frac{0.4}{u^2 - 1} \qquad (47)$$

where p is the internal pressure, E is the Young's modulus, and u is the ratio of external pipe diameter to internal pipe diameter.

This gives a total strain

$$\varepsilon_\ell = \varepsilon_{\ell\vartheta} + \varepsilon_{\ell p} \qquad (48)$$

yielding the total expansions in the a and b directions

$$\delta_{\ell a} = \ell_a \cdot \varepsilon_\ell \qquad (49)$$

$$\delta_{\ell b} = \ell_b \cdot \varepsilon_\ell \qquad (50)$$

If an extraneous movement also called PUC (pull-up-cold) of the end point A and B, respectively is possible, the following total expansion of the system in the a-direction is found

$$\Delta_a' = (\delta_{\ell a} + \delta_{Aa} \div \delta_{Ba}) \qquad (51)$$

and in the b-direction

$$\Delta_b' = (\delta_{\ell b} + \delta_{Ab} - \delta_{Bb}) \qquad (52)$$

where δ_A is the movement of the initial point A and δ_B is the movement of the end point B.
If, furthermore, the system is prestressed, the total expansion Δ_a and Δ_b, respectively,

$$\Delta_a = \frac{v_a}{100\ \Delta_a'} \tag{53}$$

$$\Delta_b = \frac{v_b}{100\ \Delta_b'} \tag{54}$$

is obtained, where v_a and v_b indicate the percentage of prestressing.

The force components Pa and Pb are now derived in accordance with the following relations:

in the a-direction

$$Pa = - E_o I_o \frac{\Delta_a\ Tag + \Delta_b\ Dagbg}{Tag\ Tbg - Dagbg^2} \tag{55}$$

and in the b-direction

$$Pb = - E_o I_o \frac{\Delta_b\ Tbg + \Delta_a\ Dagbg}{Tag\ Tbg - Dagbg^2} \tag{56}$$

with the resulting force P

$$P = \sqrt{Pa^2 + Pb^2} \tag{57}$$

With

$$\sin\gamma = \frac{Pa}{P} \tag{58}$$

and

$$\cos\gamma = \frac{Pb}{P} \tag{59}$$

the distance can be determined of the thrust line of the system of the force P from the origin of the coordinates of the a,b-system

$$h_g = \eta_a \sin\gamma + \eta_b \cos\gamma \tag{60}$$

Consequently, the slope of the thrust line is

$$tg\ \gamma = \frac{\sin\gamma}{\cos\gamma} = \frac{Pa}{Pb} \tag{61}$$

Substituting in equation (60) the coordinates η_a and η_b by the coordinates of the single part components, their distance h_g' from the origin of coordinates parallel to the perpendicular distance h_g of the thrust line of the system is obtained.

$$h_{gi}' = a_i \sin \gamma + b_i \cos \gamma \tag{62}$$

The distance of the points a_i, b_i from the line of action of the resulting force P becomes

$$h_{gi} = h_{gi}' - h_g \tag{63}$$

and thus, the bending moment can be calculated for point i, which is the basis of the stress analysis

$$M_i = P \cdot h_{gi} \tag{64}$$

At the point of maximum distance $h_{g\ max}$, the maximum bending moment occurs in accordance with this equation, which must be used to dimension the piping system.

2.3 Stress Analysis

The stress will be determined according to a proposal by Jürgensonn [2]. The underlying stress hypothesis is the Mises criterion (distortion energy theory).

The maximum bending stress occurring in the external fibre of a pipe cross section is

$$\sigma_{ba} = \frac{M}{W} = \frac{M \cdot D}{200 \cdot J} \tag{65}$$

and, accordingly, for the inner fibre

$$\sigma_{bi} = \frac{M \cdot d}{200 \cdot J} \tag{66}$$

M is the bending moment [kp/cm^2] derived from the previous computation; D is the external pipe diameter [cm];

d is the internal pipe diameter /‾cm_7; J is the second moment of inertia of the cross section /‾cm^4_7.

The shear stress is calculated to be

$$\tau_a = \frac{Mt}{2\ W} = \frac{Mt\ D}{4\ J} \qquad (67)$$

Mt is the torsion moment which is zero in our case and, hence, also the shear stress becomes zero.

The stresses generated by the internal pressure are axial stresses

$$\sigma_{aa} = \sigma_{ai} = \frac{p\ d^2}{400\ (d+s)s} \qquad (68)$$

and tangential stresses

$$\sigma_{ta} = \frac{p\ d^2}{200\ (d+s)s} \qquad (69)$$

$$\sigma_{ti} = \sigma_{ta} + p/100 \qquad (70)$$

and radial stresses

$$\sigma_{ra} = 0 \qquad (71)$$

$$\sigma_{ri} = -p/100 \qquad (72)$$

The subscripts a ind i refer to the external and to the internal axes, respectively; p is the internal pressure /‾kp/cm^2_7, and s is the wall thickness of the pipe /‾cm_7. Bending stresses and internal pressure stresses are added and combined to become the equivalent stress.

Using

$$\sigma_{xa} = \sigma_{ba} + \sigma_{aa} \qquad (73)$$

$$\sigma_{ya} = \sigma_{ta} \qquad (74)$$

$$\sigma_{za} = 0 \qquad (75)$$

the following equivalent stress is obtained for the external axis

$$\sigma_{va} = \sqrt{\sigma_{xa}^{2} + \sigma_{ya}^{2} - \sigma_{xa}\,\sigma_{ya} + (3\,\tau^{2})} \qquad (76)$$

In our case the term $3\tau^2$ is eliminated, since no torsion moments occur in accordance with the assumption.

Using

$$\sigma_{xi} = \sigma_{bi} + \sigma_{aa} = \frac{\sigma_{ba} \cdot d}{D} + \sigma_{aa}\ ;\ \sigma_{aa} = \sigma_{ai} \qquad (77)$$

$$\sigma_{yi} = \sigma_{ti} \qquad (78)$$

$$\sigma_{zi} = \sigma_{ri} \qquad (79)$$

the following equivalent stress is obtained for the internal axis

$$\sigma_{vi} = \sqrt{0.5\,[(\sigma_{xi}-\sigma_{yi})^{2} + (\sigma_{yi}-\sigma_{zi})^{2} + (\sigma_{zi}-\sigma_{xi})^{2}] + 3\tau^{2}} \qquad (80)$$

where the last term of the equation is also zero in accordance with the statement above.

3. Working Cycle of the Program

Starting with the input data, e.g. the length of the piping elements and the radii of curvature, respectively, and the angle, the program calculates the coordinates of the starting and end points of each individual piping element. Because of the assumption of tangential connections between the individual piping elements, the inclination of the tangent is a criterion of the connection. Parallel to these calculations the centroids for all piping elements and the center points of curvature for the bends are calculated. All coordinates will be related to the coordinate system which is shifted to the end point of the piping system. Now the statical moments, the moments of inertia and the product of inertia are calculated, first for each individual part, then for the whole piping system.

The elastic pipe length and moments and, based on them, the coordinates of the elastic centroid of the piping system can be found together with the flexibility constants.

The next step is the determination of the strains caused by change of temperature and internal pressure. Using the elastic moments it is possible to find the vertical and horizontal components of the acting fixed point force allowing also to determine the inclination of the trust line of the system. The flexibility analysis is completed and subsequently the stress analysis can be made. Based on the resulting force and the distances to the piping elements, the bending moments can be calculated. At the point of maximum bending moment, the equivalent stresses according to the Mises criterion will be found.

Concerning the input and output, the program is relatively variable. The possibilities are shown schematically in Fig.10. Besides the usual card input, an input via TSO-terminal is possible. For time sharing operation it is helpful to set the control constant "TSO" (in the program) equal to 1 (one). Then the input becomes format free and the output list will be reduced.

The output for time sharing operation is made via terminal and display, and for batch operation via printer and plotter.

4. Program Input for Batch Operation

All input data have to be written in FØRMAT 6 G 12.6 and 4 G 12.6, respectively, An example for input data is given in the appendix.

1.Card:	P	[bar]	Internal design pressure
	T	[°C]	Design temperature of the piping system
	DA	[m]	Outer diameter of the pipe
	S	[m]	Wall thickness
	VSH	[ ]	Horizontal cold spring. 100 % cold spring means that under hot conditions 100 % of the cold spring is acting. (no prestressing under cold conditions) The value of VSH must be used as absolute value, e.g. 100 % ≙ 1.0 50 % ≙ 0.5
	VSV	[ ]	Vertical cold spring, as described before
2.Card:	CVKH	[m]	Horizontal fixpoint movement
	CVKV	[m]	Vertical fixpoint movement
	MAT	[ ]	Material constant MAT=1 ≙ Material No.7380=10CrMo 9 10 =2 ≙ " " 4961=X8CrNiNb 16 13 =3 ≙ " " 4981=X8CrNiMoNb 16 16 =4 ≙ " " 4988=X8CrNiMoVNb 16 13 =5 ≙ " " 4922=X20CrMoV 12 1 =6 ≙ " " 4301=X5CrNi 18 9 ≙Typ 304 =7 ≙ Typ 316 =8 ≙ " " 4948=X6CrNi 18 1
	NFALL	[ ]	Identification number for the run (number of case)

3.Card to n.Card: This type of card specifies the shape of the piping system. One card is necessary for each piping element, at the maximum 20 plus one to point out that the data are complete.

IG [ ] Identification constant
IG=1 for a straight piping element
IG=2 for a bend
According to the assumptions for calculating the piping system the first and the last piping element have to be straight elements.

RL [m] Length of a straight piping element or radius of the curvature of a bend, respectively.

PHI1 [grd] Inclination of a straight piping element or first angle for a bend, respetively.

PHI2 [grd] For a straight piping element is this constant equal 0.(zero).
For a bend is it the second angle.

If the piping system is complete, the first value IG must be equal to zero and the data for the next example must follow, starting with card No.1.

If the values of the first and second cards of the following example are equal to the example calculated before, the value IG must be equal to -1 (minus one). Then the following example starts with the third card.

If the calculation is finished, the input card for the program stop must follow. (see Chapt. 6)

5. Program Input for Time Sharing Operation

The input for time sharing operation is format free. The sequence of the data is equal to the batch operation described before. The program prints out via terminal which are the input data needed. A complete example of a time sharing job is given in the appendix.

6. Stop of the Program

The program stop is possible in two ways. In the first case, when the stop is to follow a card of the type 3 where the first value IG was zero, the third value on the next card (type 1) had to be DA, also equal to zero. In the second case, when the first value on the last card (type 3) was equal to -1, a card of the same type had to follow with the first value IG equal to or greater than 999.

7. Program Output for Batch Operation

A complete output list is given in Appendix 1.

MAX.STRESS OUTS.	$[N/mm^2]$	Maximum equivalent stress in the outer fiber of the piping system
MAX.STRESS INS.	$[N/mm^2]$	Maximum equivalent stress in the inner fiber of the piping system
HORIZONTAL FORCE	$[N]$	Horizontal force component of the fixed anchor point reaction
VERTICAL FORCE	$[N]$	Vertical force component of the fixed anchor point reaction
RESULTING FORCE	$[N]$	Resulting force of the fixed anchor point reaction
MOMENT TXS	$[m^4]$	Elastic value of the moment of inertia related to the a(x) coordinate axis
MOMENT TYS	$[m^4]$	Elastic value of the moment of inertia related to the b(y) coordinate axis

MOMENT ZS	[m^4]	Elastic value of the product of inertia
MOMENT SLIXI	[m^2]	Elastic value of the statical moment related to the a(x) axis
YOUNG'S MODULUS	[kN/mm^2]	Young's Modulus at design temperature
SIGMA X OUTSIDE	[N/mm^2]	Axial stress component for the outside of the pipe at the point of the maximum bending moment
SIGMA Y OUTSIDE	[N/mm^2]	Tangential stress component for the outside of the pipe at the point of the maximum bending moment
MAX.BEND.MOMENT	[Nm]	Maximum bending moment
SAFETY OUTS.		Safety factor for the creep limit ($\sigma_{1/100\ 000}$) for the maximum stress in the outer fiber
SAFETY INS.		Safety factor for the creep limit ($\sigma_{1/100\ 000}$) for the maximum stress in the inner fiber
HORIZ.ELONG.	[m]	Horizontal elongation caused by internal pressure and design temperature taking into consideration possible fixed anchor point movements
VERTICAL ELONG.	[m]	Elongation in the vertical direction according to HORIZ.ELONG.
DISTANCE TO O,O	[m]	Perpendicular distance of the thrust line of the system from the origin of the coordinate system used. (The thrust line of the system is the action line of the resulting force)
LENGTH OF SYSTEM	[m]	Elastic value of the length of the piping system

YETA	[m]	b-(y) coordinate of the elastic centroid of the system
XKSI	[m]	a-(x) coordinate of the elastic centroid of the system
MOMENT SLIYI	[m^2]	Elastic value of the statical moment related to the b(y) axis
STRESS INT.VALUE	[N/mm^2]	Stress intensity value $\sigma_{1/100\,000}$ (creep limit) at design temperature as a basis to calculate the safety factor
SIGMA X INSIDE	[N/mm^2]	Axial stress component for the inside of the pipe at the point of the maximum bending moment
SIGMA Y INSIDE	[N/mm^2]	Tangential stress component
SIGMA Z INSIDE	[N/mm^2]	Radial stress component

The following table contains the geometrical data for each piping element starting with a straight tube.

A1, B1	[m]	a- and b-coordinates (x,y) of the beginning of an individual piping element under consideration
A2, B2	[m]	a- and b-coordinates (x,y) of the end of an individual piping element under consideration
SA, SB	[m]	a- and b-coordinates of the centroid of the individual piping element
BEND MOMENT	[kp m]	Bending moment at the starting point of the piping element under consideration
LENGTH	[m]	Distance between the first point of the piping element and the thrust line of the system

RA, RB a- and b-coordinates of the center point of curvature for bend pipe element

8. Program Output for Time Sharing Operation

The output list is very short during time sharing operation. It contains only the maximum stresses (at the point of the maximum bending moment) on the inner and outer sides of the pipe and the safeguards with respect to the creep limit ($\sigma_{100\ 000}$). An example is given in Appendix 2.

9. References

/¯1_7 Hampel, H.: Rohrleitungsstatik, Springer-Verlag, Berlin Heidelberg New York (1972)

/¯2_7 Jürgensonn, H.V.: Elastizität und Festigkeit im Rohrleitungsbau, Springer-Verlag, Berlin Göttingen Heidelberg (1953) 2.Edition

/¯3_7 American Power Piping Code USA S B 31.1.0

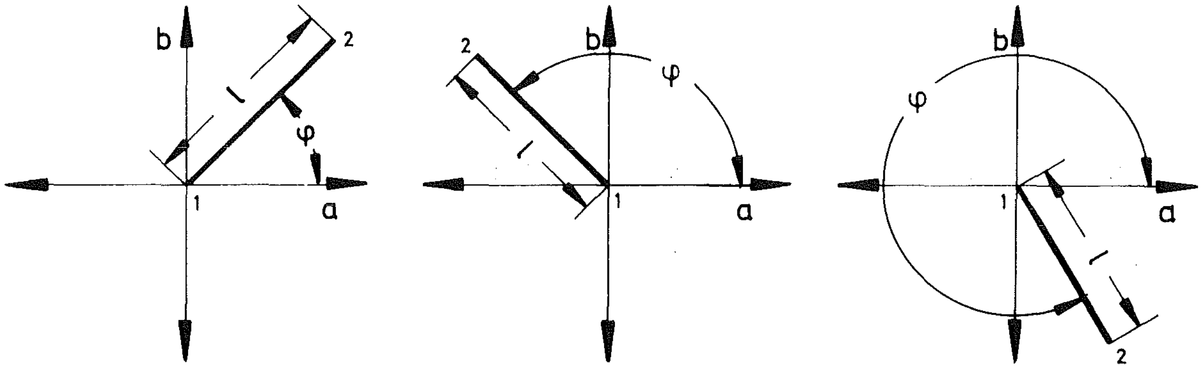


FIG. 1 INNER COORDINATE SYSTEM FOR A STRAIGHT PIPING ELEMENT

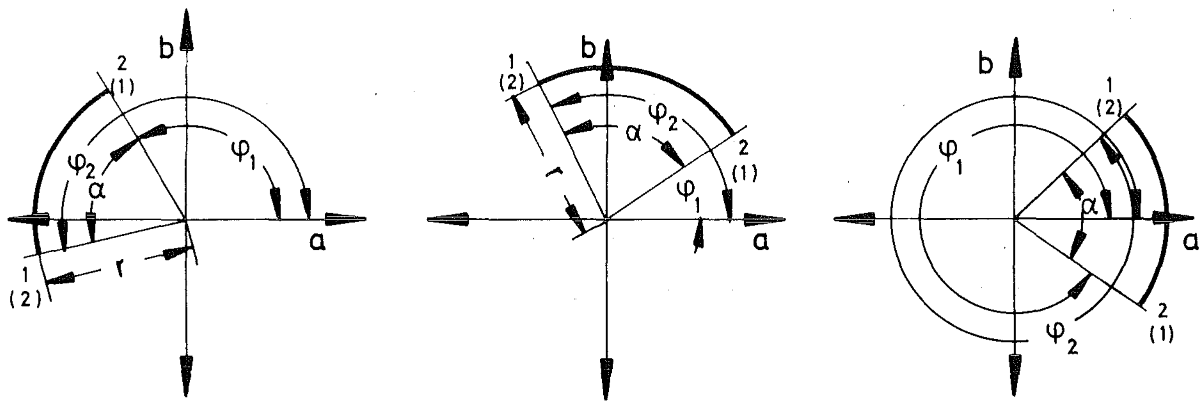


FIG. 2 INNER COORDINATE SYSTEM FOR A BEND

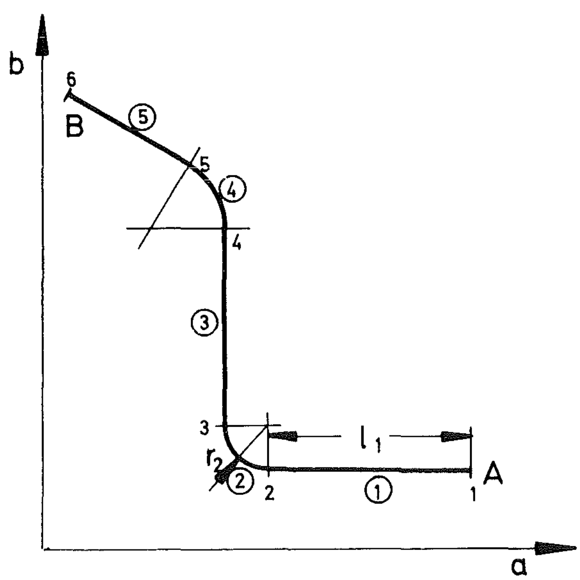


FIG. 3 EXAMPLE OF A PIPING SYSTEM

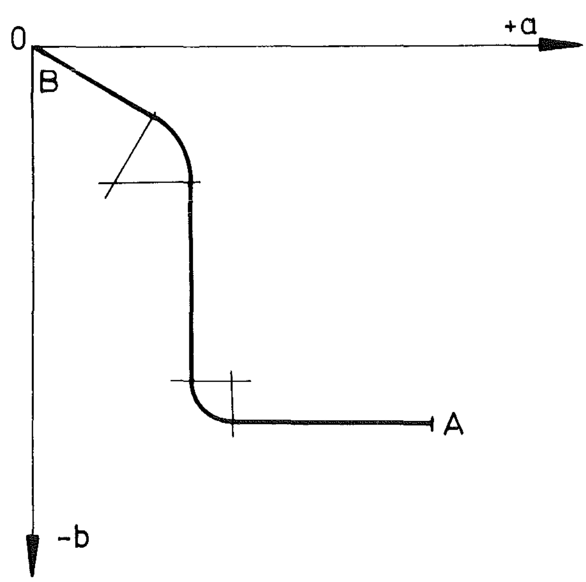


FIG. 4 ARRANGEMENT OF THE PIPING SYSTEM IN THE OUTER COORDINATE SYSTEM

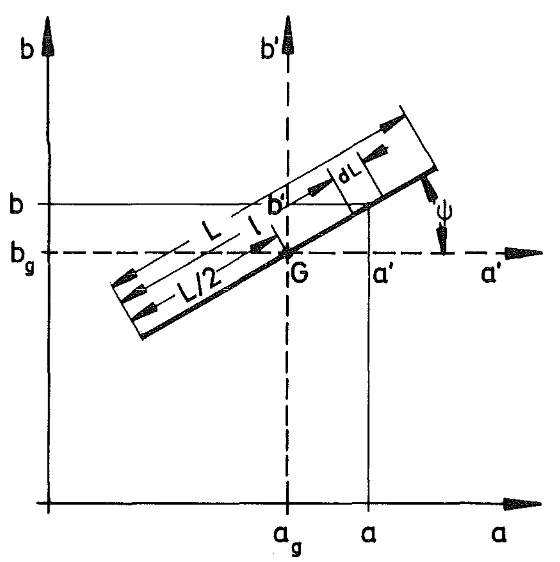


FIG. 5 DENOTATION FOR A STRAIGHT PIPING ELEMENT IN THE a, b - COORDINATE SYSTEM [1]

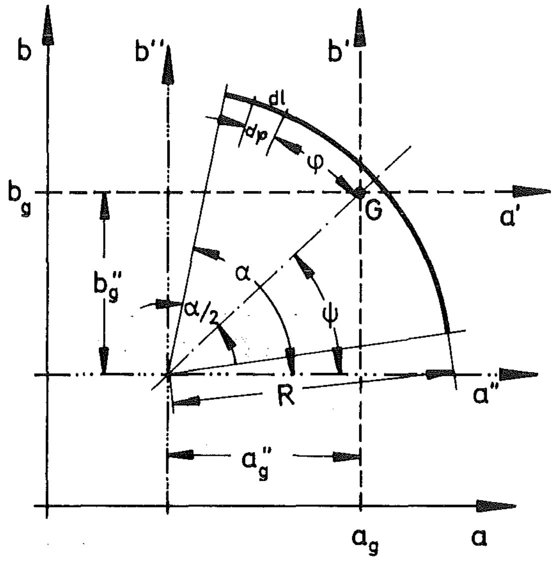


FIG.6 DENOTATION FOR A BEND IN THE a,b-COORDINATE SYSTEM [1]

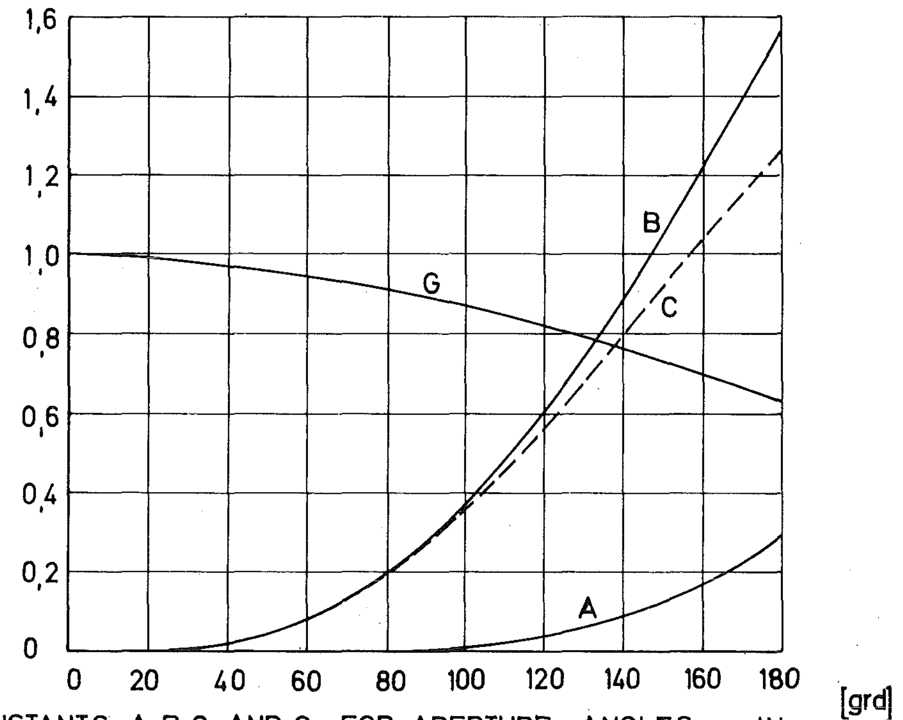


FIG. 7 CONSTANTS A,B,C AND G FOR APERTURE ANGLES α IN THE RANGE BETWEEN 0° AND 180° [1]

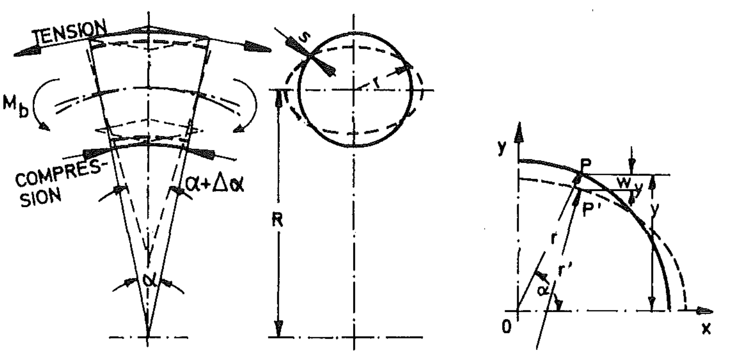


FIG. 8 CROSS SECTION DEFORMATION OF A BEND BY A BENDING MOMENT [2]

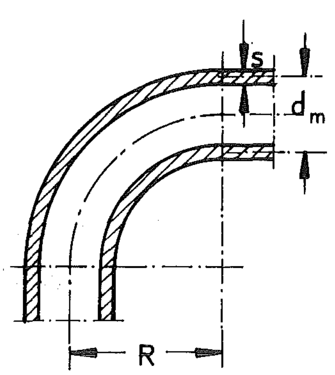


FIG.9 DENOTATION FOR A BEND

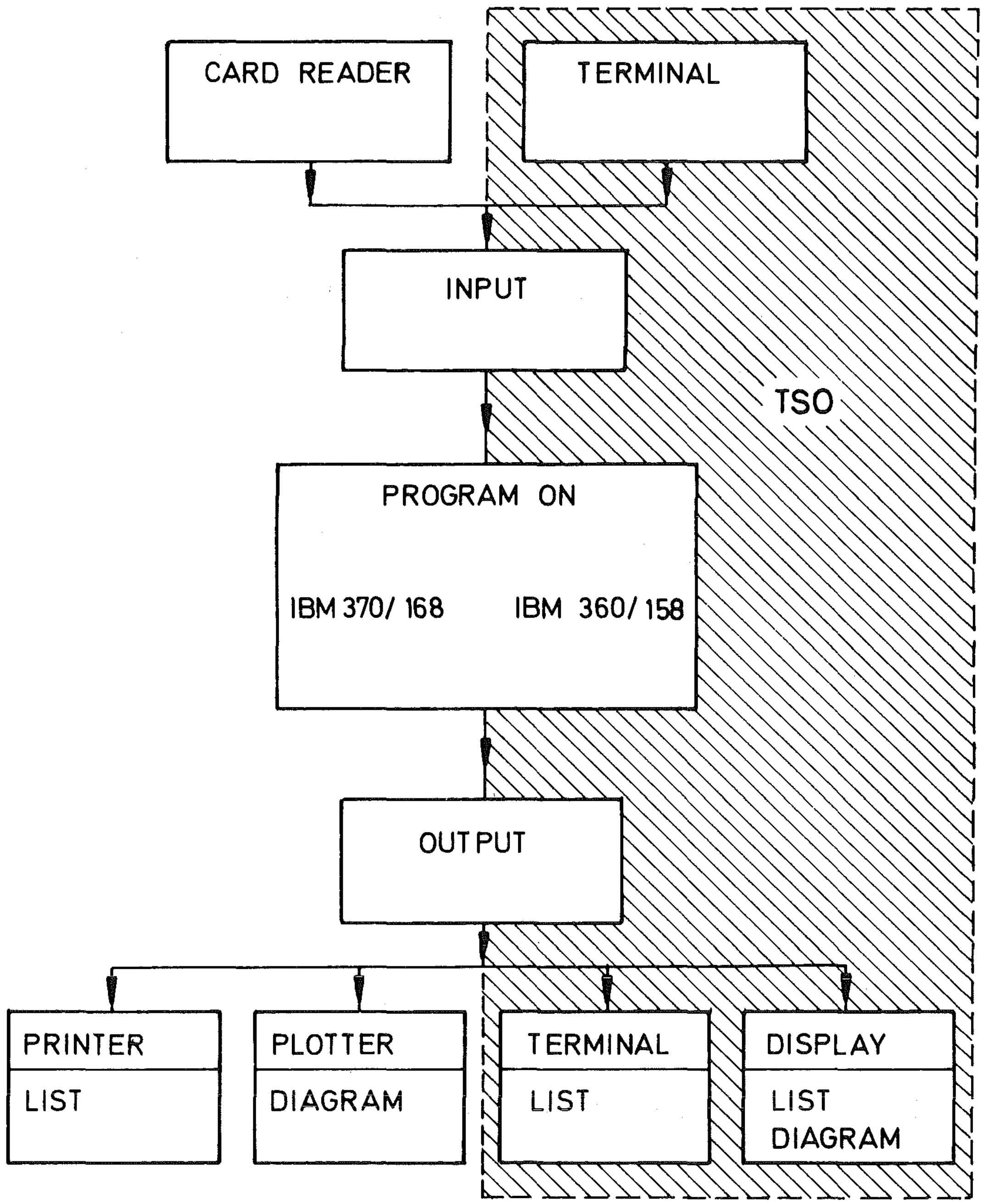


FIG. 10 POSSIBILITIES OF INPUT AND OUTPUT FOR THE PROGRAM

Appendix 1

DATENKARTEN

Programm FLEXA Datum Name Blatt-Nr.

50.		220.	.324		.0071		1.	1.
0.		0.		2		0		
	1	4.8	180.		0.			
	2	1.2	180.		270.			
	1	4.236	90.		0.			
	2	1.0	0.		60.			
	1	5.196	150.		0.			
	0	0.	0.		0.			
0.		0.	0.		0.		0.	0.

PIPING SYSTEM NO. 0

INPUT DATA

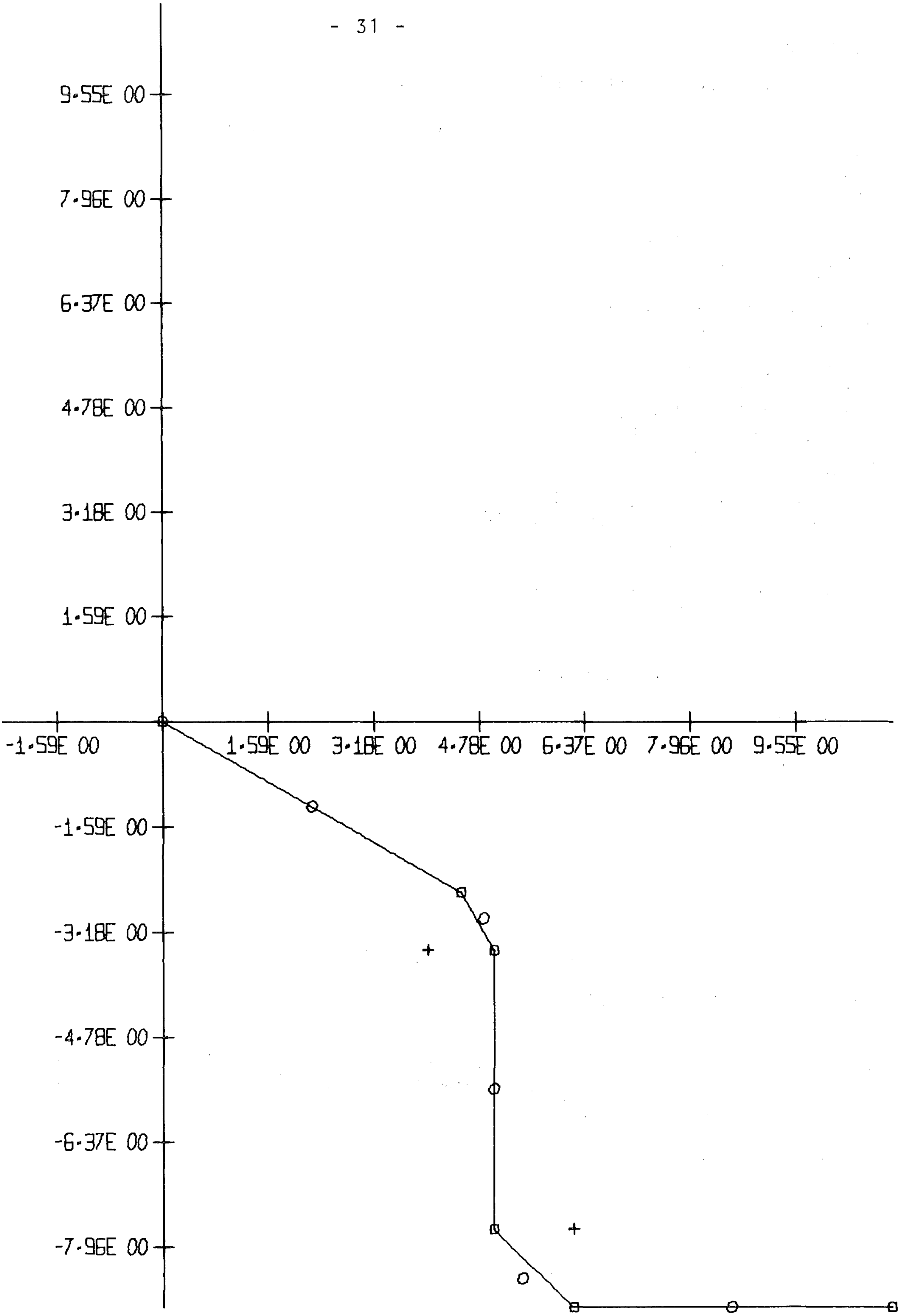
INTERNAL PRESSURE =	50.000 BAR	HORIZ.FIXP.MOV. =	0.0
PIPE TEMPERATURE =	220.000 DEG.C	VERT.FIXP.MOV. =	0.0
OUTER DIAMETER =	0.324 M	MATERIAL =	2
WALL THICKNESS =	0.007 M		
HORIZ.COLD SPRING =	1.000 0/0		
VERT. COLD SPRING =	1.000 0/0		

NO.OF ELEMENT	TYPE	LENGTH/RADIUS	ANGLE 1	ANGLE 2
1	1	4.800	180.00	0.0
2	2	1.200	180.00	270.00
3	1	4.236	90.00	0.0
4	2	1.000	0.0	60.00
5	1	5.196	150.00	0.0

RESULTS OF CALCULATION

MAX. STRESS OUTS. =	147.55 N/MM**2	SAFETY OUTSIDE =	1.037
MAX. STRESS INS. =	149.37 N/MM**2	SAFETY INSIDE =	1.025
HORIZONTAL FORCE =	-10868.6 N	HORIZ. ELONG. =	0.041 M
VERTICAL FORCE =	16818.0 N	VERTICAL ELONG. =	-0.033 M
RESULTING FORCE =	20024.3 N	DISTANCE TO 0,0 =	1.261 M
MOMENT TXS =	275.7 M**4	LENGTH OF SYSTEM=	29.497 M
MOMENT TYS =	121.6 M**4	COORD. YETA =	-5.725 M
MOMENT ZS =	-138.4 M**4	COORD. XKSI =	5.201 M
MOMENT SLIXI =	-168.9 M**2	MOMENT SLIYI =	153.4 M**2
YOUNGS MODULUS =	184. KN/MM**2	STRESS INT.VALUE=	153.04 N/MM**2
SIGMA X OUTSIDE =	168.39 N/MM**2	SIGMA X INSIDE =	163.35 N/MM**2
SIGMA Y OUTSIDE =	106.64 N/MM**2	SIGMA Y INSIDE =	111.64 N/MM**2
MAX.BEND.MOMENT	63021. NM	SIGMA Z INSIDE =	-5.00 N/MM**2

A1	B1	A2	B2	SA	SB	BEND MOMENT	LENGTH	RA	RB
11.00	-8.90	6.20	-8.90	8.60	-8.90	0.63021E+05	3.15		
6.20	-8.90	5.00	-7.70	5.44	-8.46	0.17706E+05	-0.88	6.20	-7.70
5.00	-7.70	5.00	-3.46	5.00	-5.58	0.24845E+05	-1.24		
5.00	-3.46	4.50	-2.60	4.83	-2.99	0.21194E+05	1.06	4.00	-3.46
4.50	-2.60	0.0	0.0	2.25	-1.30	0.22197E+05	1.11		



EXAMPLE FOR A PLOT OUTPUT

Appendix 2

```
attrib a lrecl(80) blksize(1680) recfm(f  b)
READY
alloc da(plot) f(ft07f001) space(462) block(1680) using(a)
READY
edit piptso fortgi
EDIT
run
G1 COMPILER ENTERED
SOURCE ANALYZED
PROGRAM NAME = MAIN
*  NO DIAGNOSTICS GENERATED
SOURCE ANALYZED
PROGRAM NAME = SIGMA1
*  NO DIAGNOSTICS GENERATED
SOURCE ANALYZED
PROGRAM NAME = EMODUL
*  NO DIAGNOSTICS GENERATED
SOURCE ANALYZED
PROGRAM NAME = ALFA
*  NO DIAGNOSTICS GENERATED
    *STATISTICS*  NO DIAGNOSTICS THIS STEP :
 TYPE IN P,T,DA,S,VSH,VSV
?
50. 220. 0.3239 0.0071 1. 1.
 TYPE IN CVKH,CVKV,MAT,NFALL
?
0. 0. 2 0
 TYPE IN IG(N),RL(N),PHI2(N),PHI1(N)
?
1 4.8 180. 0.
?
2 1.2 180. 270.
?
1 4.236 90. 0.
?
2 1. 0. 60.
?
1 5.196 150. 0.
?
0 0. 0. 0.
                     PIPING SYSTEM NO.   0

 RESULTS OF CALCULATION
 ----------------------

 MAX. STRESS OUTS. =   147.55 N/MM**2    SAFETY OUTSIDE   =    1.037
 MAX. STRESS INS.  =   149.37 N/MM**2    SAFETY INSIDE    =    1.025
 TYPE IN P,T,DA,S,VSH,VSV
?
0. 0. 0. 0. 0. 0.
EDIT
end
READY
```