Sum rules for baryon form factors of second-class currents

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ABSTRACT

By means of high energy constraints we relate the second-class current
vertices $\langle \Delta' | J_\mu^{ll} | \Delta \rangle$ and $\langle \Delta' | J_\mu^{ll} | N \rangle$ with the $\langle N' | J_\mu^{ll} | N \rangle$ vertex. (N denote
the nucleon and $\Delta$ the $\Delta(1232)$-resonance.) For this purpose superconvergence
sum rules are derived for a second-class vector ($J_\mu^{ll} = V_\mu^{ll}$) and axial vector
current ($J_\mu^{ll} = A_\mu^{ll}$). This report extends the previous work (ref. 1) to
second-class form factors.
Summenregeln für Baryon-Formfaktoren der Ströme zweiter Art

ZUSAMMENFASSUNG

Mit Hilfe von Hochenergiebedingungen lassen sich die Matrixelemente
\[ <\Delta'|J^{\mu}_{\text{II}}|\Delta> \] und \[ <\Delta'|J^{\mu}_{\text{II}}|N> \] der Ströme zweiter Art mit dem Matrixelement
\[ <N'|J^{\mu}_{\text{II}}|N> \] verknüpfen. (N bezeichnet das Nukleon und \( \Delta \) die \( \Delta(1232) \)-Resonanz.) Zu diesem Zweck werden Superkonvergenzsummenregeln für Vektor- (\( J^{\mu}_{\text{II}} = V^{\mu}_{\text{II}} \)) und Axialvektorströme (\( J^{\mu}_{\mu} = A^{\mu}_{\mu} \)) zweiter Art aufgestellt. In diesem Bericht wird das Verfahren der vorangegangenen Arbeit (Ref. \( |1| \)) auf Formfaktoren zweiter Art erweitert.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II.</td>
<td>NOTATIONS</td>
<td>4</td>
</tr>
<tr>
<td>III.</td>
<td>SUM RULES FOR THE VECTOR CURRENT $v_{\mu}^{II}$</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>III.1 $\Delta v_{\mu}^{II} \to \Delta \pi$</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>III.2 $N v_{\mu}^{II} \to \Delta \pi$</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>III.3 $N v_{\mu}^{II} \to N \pi$</td>
<td>15</td>
</tr>
<tr>
<td>IV.</td>
<td>SUM RULES FOR THE AXIAL VECTOR CURRENT $a_{\mu}^{II}$</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>IV.1 $\Delta a_{\mu}^{II} \to \Delta \pi$</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>IV.2 $N a_{\mu}^{II} \to \Delta \pi$</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>IV.3 $N a_{\mu}^{II} \to N \pi$</td>
<td>22</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

This is a technical report in which we quote explicitly sum rules for
the form factors of the second-class current vertices \( <N'|J^{II}_\mu|N> \), \( <\Delta'|J^{II}_\mu|\Delta> \) and
\( <\Delta'|J^{II}_\mu|\Delta> \) with \( N \) the nucleon and \( \Delta \) the \( \Delta(1232) \) resonance. It completes
the appendix C of |1| – furthermore denoted by I – by the corresponding sum
rules of second-class currents.

Second-class currents are distinguished from first-class currents in
the following way: Assume \( J_\mu^o \) represents the neutral member \( (I_3=0) \) of an
isospin current multiplet \( (I=0,1,...) \), then

\[
J_\mu^o = \begin{cases} 
+ & (J_\mu^o)^+ \\
(-) & (J_\mu^o)^- 
\end{cases}
\]  

(I.1)

defines a first- (second)-class current. There is an alternate definition by
means of the \( G \)-parity operator originally due to S. Weinberg |2| (see also
P. Langacker |3|).

\[
G J_\mu^o G^{-1} = (\pm)(-1)^I \eta_p J_\mu^o
\]  

(I.1')

which is equivalent to (I.1) as long as the TCP theorem holds \( \eta_p = +1(-1) \),
the parity of the vector (axial vector) current. For example in the usual de­
composition of the nucleon \( V-A \) current

\[
\bar{u}(p') \left[ \gamma_\mu \frac{p^\mu}{2m} + \frac{i}{2m} \sigma_{\mu\nu} (p' - p)^\nu \frac{p^\nu}{2} + \frac{1}{m} (p' - p) \gamma_5 \frac{p^\mu}{2} \right] \\
+ \gamma_\mu \gamma_5 \frac{F_1^A}{m} + \frac{1}{m} (p' - p) \gamma_\mu \gamma_5 \frac{F_1^A}{m} + \frac{1}{m} (p' + p) \gamma_5 \frac{F_1^A}{m} \right] u(p) ,
\]  

(I.2)

\[
F_1^V(0) = 1 , \quad F_1^A(0) = -1.26 ,
\]
the underlined terms represent the second-class contributions.

Second-class currents cannot be excluded a priori in weak interactions. Nevertheless they are mostly disregarded in theoretical quark and gauge models. On the other side the experimental verification of second-class currents in nuclear β-decay is an open question and quite controversial as the recent development has shown [4].

In [5] and [6] (see also I) we have used successfully pole term sum rules to determine the first-class matrix elements \( \langle \Delta' | J_{\mu}^I | N \rangle \) and \( \langle \Delta' | J_{\mu}^I | \Delta \rangle \) in terms of the experimentally known form factors of \( \langle N' | J_{\mu}^I | N \rangle \). In this report we summarize the corresponding sum rules for second-class currents, which are evaluated in [7]. There it is shown that in the framework of our approximation second-class contributions do not exist for small momentum transfers.

We now make a few comments on the derivation of the sum rules to give some background information which will facilitate the orientation in the following chapters. More details are found in [5], [6] or I. Only isoscalar and isovector currents are considered. The sum rules are derived for the superconvergent parity conserving and regularized t-channel helicity amplitudes \( G_{\mu}^{1,2}(s,t,u) \) of [8] for the peripheral processes

\[
\begin{align*}
\Delta + J_{\mu}^{II} & \rightarrow \Delta + \pi, \\
N + J_{\mu}^{II} & \rightarrow \Delta + \pi, \\
N + J_{\mu}^{II} & \rightarrow N + \pi.
\end{align*}
\]

The reactions (I.3 - I.5) are initiated by a vector or axial vector field \( J_{\mu}^{II} = V_{\mu}^{II}, A_{\mu}^{II} \).

It is assumed that in forward direction, \( t=0 \), the high energy behaviour of \( G_{\mu}^{1,2} \) is determined by the leading t-channel Regge-trajectory
In (I.6) \( m \) denotes the maximal helicity flip; \( m \) is a function of the helicities \( \Lambda \) of all particles. Superconvergence means \( \alpha_{\text{eff}}^{-m} < -1 \). If \( \alpha_{\text{eff}}^{-m+1} < -n \), \( n \) integer \( \geq 0 \), the following sum rules are valid:

\[
\int_{s_0}^{\infty} (s')^n \Im G_{\Lambda}^{1,2}(s', t, u') - \int_{u_0}^{\infty} (\Sigma-t-u')^n \Im G_{\Lambda}^{1,2}(s', t, u') = 0 \quad (I.7)
\]

with \( s+t+u = \Sigma \). In practice usually \( n=0 \). If \( m=3 \) also \( n=1 \) is possible, in which case we speak of a moment sum rule. The effective Regge-trajectories are specified in table 1.

**TABLE 1: Leading trajectories of the t-channel**

<table>
<thead>
<tr>
<th>particles ( \mu )</th>
<th>( I_t = 0 )</th>
<th>( I_t = 1 )</th>
<th>( I_t = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi )</td>
<td>( \alpha_{\text{eff}}(0) &lt; 1 )</td>
<td>( \eta )</td>
<td>( \rho )</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-</td>
<td>-</td>
<td>( A_2 )</td>
</tr>
<tr>
<td>( \omega )</td>
<td>( \alpha_{\text{eff}}(0) &lt; 0 )</td>
<td>( A_2 )</td>
<td>( A_1 )</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-</td>
<td>-</td>
<td>( \rho )</td>
</tr>
<tr>
<td>( \mu )</td>
<td>( \alpha_{\text{eff}}(0) &lt; 1 ), ( \alpha_{\rho}(0) \approx \alpha_{A_2}(0) \approx \alpha_{\omega}(0) \approx 0.5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_{\pi}(0) \approx \alpha_{A_1}(0) \approx 0.0 ), ( \alpha_{\pi}(0) \approx \alpha_{B}(0) &lt; 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The sum rules (1.7) are saturated by the N- and Δ-intermediate contributions in the zero width approximation \( \Gamma_\Delta = 0 \), so that we obtain the pole term sum rules

\[
\sum_{i=N,\Delta} \{ \text{Res} \left[ s^2 G^{1,2}_\Lambda (s', t, u) \right] - \text{Res} \left[ (\Sigma-t-u)_m G^{1,2}_\Lambda (s', t, u') \right] \} = 0
\]

These sum rules and some of their first derivatives with respect to t at t=0 are listed in the sections III and IV (we call them \( t^0 \)- and \( t^1 \)-sum rules, respectively). Sum rules for higher derivatives yield no information since with increasing number of derivatives more and more of the low partial amplitudes get truncated. Among the \( t^1 \)-sum rules we consider only those as reliable which converge very fast, i.e.

\[
|s^n G^{1,2}_\Lambda (s, t, u)| < |s|^{-2}, \quad n = 0, 1, \quad s \to \infty
\]

Furthermore we restrict ourselves only to sum rules with helicity flip \( m \geq 2 \) for the reactions (1.3) and (1.4). For \( m < 2 \) the sum rules are not useful on account of their complicated kinematical structure.

II. NOTATIONS

The sum rules in the following sections III and IV are compiled for \( G^{1,2}_\Lambda \) amplitudes with definite isospin in the t-channel \( I_t = 0, 1, 2 \). The definitions of the vertices, phase conventions and other technicalities are taken from [5]. Each sum rule contains the complete N- and Δ-contribution. The abbreviations \( L^{N(\Delta)}_N(\text{NO}) \) have been introduced to denote the N(Δ)-contribution of the lefthand side of equation (NO); analogously \( L(\text{NO}) \) denotes the complete lefthand side of (NO).
For the reaction (I.3) we distinguish three classes of sum rules:

1. sum rules of the helicity flip $m=3$ amplitudes,
2. moment sum rules ($n=1$) of the $m=3$ amplitudes,
3. sum rules of the $m=2$ amplitudes.

Only the crossing antisymmetrical amplitudes lead to non-trivial sum rules in the cases (1) and (3) and the crossing symmetrical amplitudes in the case (2). The amplitudes of the reaction (I.4), $N + J^{\Pi}_\mu \rightarrow \Delta + \pi$, have no definite crossing properties. To separate first-class sum rules from second-class ones, one has to combine the sum rules for (I.4) with the corresponding equations for the reaction $\Delta + J^{\Pi}_\mu \rightarrow N + \pi$. For the reaction (I.5) again only the crossing antisymmetrical amplitudes lead to non-trivial sum rules.

The following abbreviations are used:

\[ IS = -\frac{8}{15} \left( \frac{2}{3} \right) \quad (\text{II.1}) \]

for the isospin $I_t = 0(2)$.

\[ IS_1 = 2(-2), \quad IS_2 = -\frac{5}{9} \left( \frac{1}{3} \right), \quad IS_3 = -\frac{5}{12} \left( -\frac{1}{12} \right) \quad (\text{II.2}) \]

for $I_t = 1(2)$. The masses are denoted by

pion: $m_\pi = 0.1396$ GeV, nucleon: $m = 0.9383$ GeV,

$\Lambda(1232)$-resonance: $M = 1.232$ GeV \quad (\text{II.3})

Finally the following kinematical symbols will be used

\[ M_+ = M + m, \quad M_- = M - m, \quad MDN2 = M^2 + m^2 - M_m^2 - m^2 \quad (\text{II.4}) \]
\begin{equation}
\gamma_N = - [2m_\pi m_\pi + m_\pi^2 + k^2], \quad \gamma_\Delta = - [m_\Delta^2 + k^2], \tag{II.5}
\end{equation}

\begin{equation}
\frac{g_{12}}{g_1} = \frac{2m^4 g_1 + m_\pi^2 g_2}{g_2^2} \quad \frac{g_{2}}{g_2} = \frac{g_{12}}{g_1^2} - 3g_2, \tag{II.6}
\end{equation}

\begin{equation}
f(G_i, G_j) = 4 g_2 G_i - \left( \frac{g_{12}}{g_1^2} - \frac{k^2}{g_2^2} \right) G_j. \tag{II.7}
\end{equation}

In the following we quote explicitly only the sum rules of the isovector current \( J^\mu \). The corresponding sum rules of the isoscalar current are given by the following substitutions: For all three reactions (I.3-1.5) only \( I_\ell = 1 \) sum rules exist. They are obtained for the reaction (I.3) from the \( I_\ell = 0, 2 \) isovector sum rules by setting \( I_S = 0 \), since \( \langle N'|J^\mu_\mu|N \rangle \) - vertex does not contribute. The form factors of the \( \langle N'|J^\mu_\mu|\Delta \rangle \)-vertex \( G_\mu \) are to be replaced by the isoscalar form factors \( G^S \). For the second reaction (I.4) one has to put \( I_S_1 = I_S_2 = 0 \), \( I_S_3 = -1/12 \) and to replace the isovector form factors of the \( \langle N'|J^\mu_\mu|N \rangle \)- and \( \langle \Delta'|J^\mu_\mu|\Delta \rangle \)-vertex by the corresponding isoscalar ones. There are no non-trivial sum rules for the last reaction (I.5).

For typographical reasons we denote in the sections III and IV the isospin in the t-channel by \( I \) instead of \( I_\ell \).

The parity conserving, regularized helicity amplitudes \( c_{1,2}^{\lambda_1 \lambda_2 \lambda_b, \lambda_2 \lambda_a} \) corresponds to the t-channel reaction \( \tilde{\gamma}(q, S) + \gamma^\mu_\mu(K, \lambda_a) + B_2(p_2, \lambda_d) + \overline{B_1}(p_1, \lambda_c) \). For further details in particular the definition of the hadronic coupling constants \( g, g_1, g_2 \) of the \( \pi N N, \pi N \Delta, \pi \Delta \Delta \) vertices see the appendix of I or [5]. There one finds also the definition of the nucleon, \( N-\Delta \) excitation and \( \Delta \) form factors: \( F_i, C_i, G_i \).
III. SUM RULES FOR THE VECTOR CURRENT $V^{II}_{\mu}$

III.1 $\Delta V^{II}_{\mu} \rightarrow \Delta \pi$

III.1.1 Sum rules of the $m=3, I=0,2$ amplitudes

$G_{2V}^{I=0,2}, \quad G_{2V}^{I=0,2}, \quad G_{2V}^{I=0,2}$ are crossing symmetrical.

\begin{align*}
G_{3/2,3/2,S1}^{2V} & = 0 \quad (\text{III.1})
\end{align*}

a) $t^0$-sum rules

\begin{align*}
G_{3/2,3/2,S1}^{1V} I=0,2 : \quad IS\cdot Mg^{\ast} \left( \frac{m}{2M} C_3^V + C_4^V \right) + \frac{1}{9} \frac{2}{M^2} \frac{g_{12}}{M} G_7^V & = 0 \quad (\text{III.1})
\end{align*}

b) $t^1$-sum rules

\begin{align*}
G_{3/2,3/2,S1}^{1V} I=0,2 : \quad IS\cdot Mg^{\ast} C_4^V + \frac{2}{9} \frac{2}{M^2} \frac{g_{12}}{M} C_7^V & = 0 \quad (\text{III.2})
\end{align*}

III.1.1.2 Sum rules of the $m=3, I=1$ amplitudes

$G_{3/2,3/2,S1}^{IV I=1}$ is crossing symmetrical.
a) $t^0$-sum rules

\[ G_{2V}^{I=1} \text{ has no } t^0\text{-contribution.} \]

\[ G_{3}^{2V} \frac{I=1}{\frac{3}{2}, \frac{3}{2}, S1} \]

\[ G_{3}^{2V} \frac{I=1}{\frac{3}{2}, \frac{3}{2}, S1} = \left\{ -\frac{4}{3} M_g \left[ K^2 (C_4^{V'} - C_5^{V'}) - m^2 C_6^{V'} \right] + \frac{4}{9} \frac{m^2}{M^2} g_{12} C_7^{V'} (m^2 - K^2) = 0 \right. \quad (III.3) \]

\[ G_{3}^{2V} \frac{I=1}{\frac{3}{2}, \frac{3}{2}, S1} = -\frac{4}{3} M_g \frac{C_4^{V'}}{C_6^{V'}} + \frac{1}{9} \frac{K^2}{M^2} f \left( G_5^{V'}, G_6^{V'} \right) - \frac{4}{9} \frac{g_{12}}{M^2} - \frac{K^2}{M} g_2^{2V} = 0 \quad (III.4) \]

b) $t^1$-sum rules

\[ G_{3}^{2V} \frac{I=1}{\frac{3}{2}, \frac{3}{2}, S1} = \left\{ -\frac{4}{3} M_g \frac{C_4^{V'}}{C_6^{V'}} + \frac{2}{9} \frac{m^2}{M^2} (g_{2}-3g_{2}) C_7^{V'} (m^2 - K^2) + \frac{2}{m_{n}^2 - K^2} L(III.3) = 0 \right. \quad (III.5) \]

\[ G_{3}^{2V} \frac{I=1}{\frac{3}{2}, \frac{3}{2}, S1} = \left\{ -\frac{4}{3} M_g \frac{C_4^{V'} + \frac{2}{9} \frac{m^2}{M^2} (g_{2}-3g_{2}) C_7^{V'} K^2 \frac{2}{3} \frac{m^2}{M^2} (m^2 - K^2) g_2^{2V} - \frac{1}{2} \frac{1}{m_{n}^2 - K^2} L(III.3) = 0 \right. \quad (III.6) \]

\[ G_{3}^{2V} \frac{I=1}{\frac{3}{2}, \frac{3}{2}, S1} = g_2 \left( C_7^{V'} + \frac{K^2}{4M^2} C_6^{V'} \right) = 0 \quad (III.7) \]

III.1.2.1 Moment sum rules of the $m=3$, $I=0,2$ crossing symmetrical amplitudes

a) $t^0$-sum rules

\[ G_{3}^{2V} \frac{I=0,2}{\frac{3}{2}, \frac{3}{2}, S1} \text{ has no } t^0\text{-contribution.} \]
\[ G^2_{3} \frac{I=0,2}{2} - \frac{3}{4} IS \cdot y_{N} L_{N}(III.3) + y_{\Delta} L_{\Delta}(III.3) = 0 \]  
(III.8)

\[ G^2_{3} \frac{I=0,2}{2} - \frac{3}{4} IS \cdot y_{N} L_{N}(III.4) + y_{\Delta} L_{\Delta}(III.4) = 0 \]  
(III.9)

b) \( t^1 \)-sum rules

\[ G^2_{3} \frac{I=0,2}{2} - \frac{3}{4} IS \cdot y_{N} L_{N}(III.5) + y_{\Delta} L_{\Delta}(III.5) = 0 \]  
(III.10)

\[ G^2_{3} \frac{I=0,2}{2} - \frac{3}{4} IS \cdot \frac{y_{N}}{M^2} L_{N}(III.6) + \frac{y_{\Delta}}{M^2} L_{\Delta}(III.6) - \frac{3}{4} IS \cdot L_{N}(III.3) + L_{\Delta}(III.3) = 0 \]  
(III.11)

\[ G^2_{3} \frac{I=0,2}{2} - \frac{3}{4} \frac{y_{\Delta}}{M^2} \frac{4}{3} L(III.7) - \frac{3}{4} IS \cdot L_{N}(III.4) + L_{\Delta}(III.4) = 0 \]  
(III.12)

III.1.2.2 Moment sum rules of the \( m=3, I=1 \) crossing symmetrical amplitudes

a) \( t^0 \)-sum rules

\[ G^4_{3} \frac{I=1}{2} - \frac{4}{3} y_{N} Mg_{\ast} \left( \frac{m}{2M} C_3 + C_4^{V'} \right) + y_{\Delta} \frac{m^2}{M^2} g_{12} \frac{2}{M^2} C_7^{V} = 0 \]  
(III.13)

\( t^1 \)-sum rules are not considered on account of the convergence criterion (I.9).
III.1.3 Sum rules of the m=2 amplitudes

We use the linear combinations

\[ A^1 V(±), A^2 V(±), A^IV(±) \]

of I and

\[ A^IV(±) := \frac{1}{2} G^IV \pm \frac{1}{2} G^1 V \]

III.1.3.1 Sum rules of the m=2, I=0,2 amplitudes

\[ A^1 V(+)I=0,2, A^2 V(-)I=0,2, A^IV(+)I=0,2, A^IV(+)I=0,2 \]

are crossing symmetrical.

a) \( t^0 \)-sum rules

\[ A^1 V(-)I=0,2, A^2 V(-)I=0,2, A^IV(-)I=0,2, A^IV(-)I=0,2 \]

have no \( t^0 \)-contributions.

\[ A^2 V(+)I=0,2: \] The sum rules are identical with (III.1).

b) \( t^1 \)-sum rules

\[ A^1 V(-)I=0,2: \{ IS \cdot M_g \} \left[ \left( M_+ - \frac{m^2 - k^2}{4M} \right) m^3 V' + \frac{1}{9} \left( \frac{m^2}{M^2} - \frac{m^2 - k^2}{M^2} \right) g_{12}^7 \right] \]

\[ + \frac{1}{6} \frac{m^2}{M^2} (m^2 + K^2) (g_{-2} - g_2) C_7^V + \frac{1}{9} \frac{m^2}{M^2} \frac{4M^2 - m^2}{M^2} g_{12}^7 \frac{m^2 - k^2}{M^2} \left( m^2 - k^2 \right) = 0 \]

(III.14)
\[ A_{IV(-)}^{I=0,2} : \{ IS \cdot Mg \left[ \frac{m}{2M} \frac{K^2 C_3 V'}{4M^2} \right] + \frac{1}{9} \frac{m}{M^2} \frac{2m^2 - m^2}{m} g_{12} G_7 + \frac{4}{3} \frac{m^2}{M^2} g_2 G_7 (m^2 - K^2) + \frac{2K^2}{m^2 - K^2} L(III.14) = 0 \] (III.15)

\[ A_{IV(-)}^{I=0,2} : \{ IS \cdot Mg \left[ \frac{m}{2M} \frac{K^2 C_3 V'}{4M^2} \right] + \frac{1}{9} \frac{m}{M^2} \frac{2m^2 - m^2}{m} g_{12} G_7 + \frac{4}{3} \frac{m^2}{M^2} g_2 G_7 (m^2 - K^2) + \frac{2K^2}{m^2 - K^2} L(III.14) = 0 \] (III.16)

III.1.3.2 Sum rules of the \( m=2, I=1 \) amplitudes

\( A_1^{IV(-)}I=1, A_1^{2V(+)}I=1, A_0^{IV(-)}I=1, A_S^{IV(-)}I=1 \) are crossing symmetrical.

a) \( t^o \)-sum rules

\( A_1^{IV(+)}I=1, A_1^{2V(-)}I=1 \) have no \( t^o \)-contributions.

\( A_0^{IV(+)}I=1, A_S^{IV(+)}I=1 \): The sum rules are identical with (III.3, III.4).
b) $\pm^1$-sum rules

\[ \text{A}^1_{V(+)} I=1: \quad [\text{L}(A13) - 2\text{L}(A5)](m^2_\pi - K^2) = 0 \quad (\text{III.17}) \]

\[ \text{A}^1_{V(+)} I=1: \quad \left[ -\frac{4}{3} M g^* K^2 \left( \frac{m}{2 M} c^V_6 + c^V_4 \right) - \frac{1}{9} \frac{M^2}{m^2 - K^2} \frac{2 M - K^2}{m^2} g_{12} c^V_7 \right] (m^2_\pi - K^2) + (\frac{m^2_\pi - K^2}{m^2}) \text{L}(\text{III.6}) - K^2 \text{L}(\text{III.3}) - \left[ \frac{m^2_\pi + K^2}{m^2 - K^2} + \left( \frac{m^2_\pi - K^2}{4 M^2} \right) \right] \text{L}(\text{III.3}) = 0 \quad (\text{III.18}) \]

\[ \text{A}^1_{V(+)} I=1: \quad -\frac{4}{3} M g^* C^V_6 + \frac{1}{9} \frac{M^2}{m^2 - K^2} \left\{ (4(3g_2 - 2g_2) c^V_5 - \frac{1}{4 M^2} \frac{K^2}{M^2} (3g_2 - 2g_2) \right\} c^V_7 \left[ -\frac{8}{9} \frac{K^2}{M^2} (3g_2 - 2g_2) \right] \]

\[ -\frac{4}{3} M g^* (3g_2 - 2g_2) c^V_7 - \frac{4}{3} M \frac{m^2_\pi - K^2}{m^2} \text{L}(\text{III.7}) + \frac{1}{2} \frac{m^2_\pi - K^2}{m^2} \text{L}(\text{III.4}) = 0 \quad (\text{III.19}) \]

III.2 $N_V^{\pi II} \rightarrow \Delta \pi$

III.2.1 Sum rules of the $I=1,2$ amplitudes

a) $\mp^0$-sum rules

\[ G^1_{V I=1(2)}: \quad NV^d_1 + IS_1 NV^u_1 + IS_2 DV^d_1 + IS_3 DV^u_1 = 0 \quad (\text{III.20}) \]

\[ G^2_{V I=1(2)}: \quad \text{The sum rules are identical with (III.20).} \]
\[ G_{1V}^{IV} I=1(2) : \left( NV_v^2 + IS_1 NV_u^1 + IS_2 DV_v^1 + IS_3 DV_u^1 \right) \frac{m^2 - k^2}{2m^4} + K^2 L(III, 20) = 0 \] (III.21)

\[ G_{3}^{IV} I=1(2) : NV_v^3 + IS_1 NV_u^3 + IS_2 DV_v^3 + IS_3 DV_u^3 = 0 \] (III.22)

with

\[ NV_v^S = 0 \]

\[ NV_u^1 = g(C_3^V + \frac{M}{m} C_4^V) \]

\[ DV_v^S = \left( m^2 - k^2 \frac{g_1^2}{g_2^2} \right) C_3^V - m \left( \frac{g_{12}}{2} - \frac{k^2}{g_2^2} \right) C_4^V + \left( \frac{k^2}{2} \frac{g_{12}}{g_2^2} \right) C_5^V + \left( m^2 \frac{g_{12}}{2} \right) C_6^V \]

\[ DV_u^1 = mg \left( 3 + \frac{m}{M} \frac{MDN2}{M^2} \right) C_7^V \]

\[ NV_v^S = \frac{g}{2} \left[ g \left( C_4^V - C_5^V \right) - m^2 C_6^V \right] \]

\[ DV_v^2 = 2 \left( \frac{m^2}{2} + \frac{k^2}{g_2^2} C_3^V + \left( \frac{M^2 - Mm}{2} + \frac{2M_4 - Mm}{2} \right) C_4^V + \left( \frac{2M_4 - Mm}{2} + \frac{k^2}{g_2^2} \right) C_5^V + \left( \frac{M^2 - Mm}{2} + \frac{k^2}{g_2^2} \right) C_6^V \right) \]

\[ DV_u^2 = mg \left( \frac{m}{M} \frac{4(2M^2 + m^2)}{2} \right) C_7^V \]

\[ NV_v^S = \frac{g}{2} \left[ \frac{K^2}{m} \right] \]

\[ NV_u^3 = g \left( C_6^V \right) \]

\[ DV_v^3 = - \left( \frac{M^2 - Mm}{2} \right) \left( \frac{g_{12}}{2} - \frac{k^2}{g_2^2} \right) C_6^V \]

\[ DV_u^3 = -mg \left( \frac{MDN2}{M^2} \frac{K^2}{4m^2} (C_5^V + C_5^V) + \frac{2M^2 + m^2}{2} \right) \left( \frac{m^2}{4} \frac{C_7^V}{C_6^V} \right) \]
b) \( t^1 \)-sum rules

\[
G^{IV}_{\frac{3}{2}, \frac{1}{2}, S1} \equiv \left( NV^S_4 + 2NV^u_4 - \frac{5}{9} DV^S_5 - \frac{5}{12} DV^u_5 \right) \left( m^2 - K^2 \right) \\
+ \left( NV^S_2 + 2NV^u_2 - \frac{5}{9} DV^S_5 - \frac{5}{12} DV^u_5 \right) \frac{m^2 + K^2}{m^2} - \frac{M^+}{m} K^2 L(III.20, I=1) = 0 \quad (III.23)
\]

\[
G^{IV}_{\frac{3}{2}, \frac{1}{2}, S0} \equiv NV^S_5 + 2NV^u_5 - \frac{5}{9} DV^S_5 - \frac{5}{12} DV^u_5 = 0 \quad (III.24)
\]

\[
G^{IV}_{\frac{3}{2}, \frac{1}{2}, SS} \equiv NV^S_6 + 2NV^u_6 - \frac{5}{9} DV^S_6 - \frac{5}{12} DV^u_6 = 0 \quad (III.25)
\]

with

\[
NV^S_4 = 0
\]

\[
NV^u_4 = g \left( \frac{K_2^2}{m^2} C^V_5 + C^V_6 \right)
\]

\[
DV^S_4 = - \frac{K_2^2}{m^2} \frac{g_{12}}{M} \left( m^2 + M \right) C^V_3 - \frac{K_2}{m^2} \frac{M^+ - M \varphi}{m} C^V_4 - \frac{g_{12}}{M} \left( m^2 + M \varphi \right) C^V_5 + \frac{K_2^2}{m^2} \frac{K^2}{M^2} \left( \frac{K_2}{M} C^V_5 + C^V_6 \right)
\]

\[
DV^u_4 = - M g^* \left( 2 \frac{2M^2 + M^2 \varphi}{m^2} + \frac{K^2}{M^2} \left( 3 + \frac{MDN^2}{M^2} \right) \right) C^V_7
\]

\[
NV^S_5 = 0
\]

\[
NV^u_5 = g \left[ \frac{K_2^2}{m^2} \left( C^V_4 + C^V_5 \right) + C^V_6 \right]
\]

\[
DV^S_5 = - \frac{2K_2^2 m}{m^2} \frac{g_{12}}{M^2} C^V_3 - \frac{K_2^2 m}{M^2} \frac{g_{2} C^V_4}{m^2} + \frac{M^+ - M \varphi}{M^2} \frac{g_{12}}{g_2} C_4 - \frac{K_2^2}{M^2} \left( m^2 + M \varphi \right) \frac{g_{12}}{2} \left( M^2 + M \varphi \right) \left( \frac{K^2}{2} (C^V_4 - C^V_5) + C^V_6 \right)
\]

\[
DV^u_5 = - 4 M g^* \left( 2 \frac{2M^2 + M^2 \varphi}{m^2} + \frac{K^2}{M^2} \frac{MDN^2}{M^2} \right) C^V_7
\]
\[ NV_s^6 = NV_{12}^6 = 0 \]

\[ DV_s^6 = -s_2 \, C_s^6 \]

\[ DV_{12}^6 = -4 \, N \, g^* \left( G_7^V + \frac{K}{4M^2} \, G_s^V \right) \]

III.3. \[ N \stackrel{\text{III}}{\mu} \rightarrow N \pi \]

### III.3.1 \( t^0 \)-sum rules of the \( m=1, I=0 \) amplitudes

\[ G_{2V}^{I=0} = G_{2V}^{I=0} = G_{2V}^{I=0} \text{ are crossing symmetrical.} \]

\[ G_{2V}^{I=0} = g^* \left( \left[ K^2 \, MDN_2 - m \, \frac{M^2 + M}{m} - \frac{2}{m} \, m(2M+m) \right] \right) C_s^V \]

\[ + \left[ MDN_2(M^2 - 2K^2) - M^2(M^2 + 2m^2) \right] C_s^V \]

\[ + \left[ MDN_2 (2 + \frac{M}{m}) - 3M^2 \right] (K^2 C_s^V + m^2 C_s^V) \left( \frac{m^2 - K^2}{m^2} \right) = 0 \]  

### III.3.1.2 \( t^0 \)-sum rules of the \( m=1, I=1 \) amplitudes

\[ G_{2V}^{I=1} \text{ is crossing symmetrical.} \]

\[ G_{2V}^{I=1} \text{ has no } t^0 \text{-contribution.} \]
IV. SUM RULES FOR THE AXIAL VECTOR CURRENT $A^\mu_{\pi}$

IV.1 $\Delta A^\mu_{\pi} \rightarrow \Delta \pi$

IV.1.1.1 Sum rules of the $m=3, I=0,2$ amplitudes

$G^I_{1A, I=0,2}$, $G^I_{2A, I=0,2}$, $C^I_{1A, I=0,2}$ are crossing symmetrical.

$G^I_{3, I=0,2}$, $G^I_{3, I=0,2}$, $C^I_{2, I=0,2}$, $C^I_{2, I=0,2}$, $C^I_{2, I=0,2}$, $C^I_{2, I=0,2}$ are crossing symmetrical.

a) $t^0$-sum rules

$G^2_{I=0,2}$ has no $t^0$-contribution.
b) \( t^1 \) - sum rules

\[ G^{2A}_{3/2, 3/2, S1} \quad \text{is crossing symmetrical.} \]

\[ G^{1A}_{3/2, 3/2, S1} : \quad \left[- \frac{4}{3} \text{Mg} \cdot C^A_5 \right] + \frac{m^2}{2} \left[ \frac{1}{9} \left( \frac{g_{12}}{M^2} \right) + \frac{2}{m^2} \left( \frac{g_{12}}{M^2} - \frac{g_{12}}{M^2} \right) \right] \quad (m^2 \cdot K^2) = 0 \]  

\[ G^{1A}_{3/2, 3/2, SS} : \quad - \frac{4}{3} \text{Mg} \cdot \left( C^A_5 \right) + \frac{m^2}{2} \cdot \left( C^A_4 \right) = 0 \]  

b) \( t^1 \) - sum rules

\[ G^{1A}_{3/2, 3/2, S1} : \quad \left[- \frac{4}{3} \text{Mg} \cdot C^A_5 \right] + \frac{m^2}{2} \left[ \frac{1}{9} \left( \frac{g_{12}}{M^2} \right) + \frac{2}{m^2} \left( \frac{g_{12}}{M^2} - \frac{g_{12}}{M^2} \right) \right] \quad (m^2 \cdot K^2) + \frac{m^2}{2} \cdot \left( \frac{1}{3} \text{Mg} \cdot C^A_4 \right) + \frac{1}{18} \left[ \frac{1}{9} \left( \frac{g_{12}}{M^2} \right) + \frac{2}{m^2} \left( \frac{g_{12}}{M^2} - \frac{g_{12}}{M^2} \right) \right] = 0 \]  

IV. 1.1.2 Sum rules of the \( m=3, I=1 \) amplitudes
\[ G_{1A}^{I=1} \mid \frac{3}{2} - \frac{3}{2}, SS \mid \frac{3}{2} + \frac{3}{2}, SO \mid = -\frac{4}{3} M_K [C^A_5 + \frac{K^2}{m} C^A_4] - \frac{1}{9} \frac{K^2}{M^2} \left[ f(G^A_5, C^A_6) - \frac{8 \sigma_2}{M^2} C^A_6 - 4 \frac{\sigma_2}{M^2} G^A_7 \right] \]

\[ \frac{1}{9} \frac{K^2}{M^2} \left[ 4 \frac{\sigma_2}{M^2} C^A_7 + 3 \frac{\sigma_2}{M^2} \frac{m^2 - K^2}{M^2} G^A_6 \right] = 0 \quad (IV.6) \]

\[ G_{1A}^{I=1} \mid \frac{3}{2} - \frac{3}{2}, SS \mid \frac{3}{2} + \frac{3}{2}, SO \mid = \text{The } N\text{-exchange does not contribute anymore.} \]

**IV.1.2.1 Moment sum rules of the } m=3, I=0,2 \text{ crossing symmetrical amplitudes**

**a) } t^0\text{-sum rules**

\[ G_{1A}^{I=0,2} \mid \frac{3}{2} - \frac{3}{2}, SL \mid \frac{3}{2} + \frac{3}{2}, SO \mid = -\frac{3}{4} IS \cdot y_N L_N^{(IV.2)} + y_\Delta L_\Delta^{(IV.2)} = 0 \quad (IV.7) \]

\[ G_{1A}^{I=0,2} \mid \frac{3}{2} - \frac{3}{2}, SO \mid \frac{3}{2} + \frac{3}{2}, SO \mid = -\frac{3}{4} IS \cdot y_N L_N^{(IV.3)} + y_\Delta L_\Delta^{(IV.3)} = 0 \quad (IV.8) \]

\[ G_{1A}^{I=0,2} \mid \frac{3}{2} - \frac{3}{2}, SS \mid \frac{3}{2} + \frac{3}{2}, SS \mid = IS \cdot M_K \cdot y_N (C^A_5 + \frac{K^2}{m} C^A_6) = 0 \quad (IV.9) \]

\[ t^1\text{-sum rules are not considered.} \]
IV.1.2.2 Moment sum rules of the $m=3, I=1$ crossing symmetrical amplitudes

a) $t^0$-sum rules

\[ G_{2A}^{I=1} \] has no $t^0$-contribution.

b) $t^1$-sum rules

\[ G_{2A}^{I=1} : - \frac{4}{3} \frac{m^2}{M^2} \frac{g_{12}}{\Delta} \frac{4}{9} \frac{m^2}{M^2} \frac{g_{12}}{\Delta} = 0 \quad (IV.10) \]

IV.1.3 Sum rules of the $m=2$ amplitudes

Again we use the linear combinations

\[ B_1^{1A(\pm)} , B_1^{2A(\pm)} , B_0^{2A(\pm)} , B_S^{2A(\pm)} \]

of $I$.

IV.1.3.1 Sum rules of the $m=2, I=0, 2$ amplitudes

$B_1^{1A(-)}I=0, 2 , B_1^{2A(+)I=0, 2} , B_0^{2A(+)I=0, 2} , B_S^{2A(+)I=0, 2}$ are crossing symmetrical.

a) $t^0$-sum rules

$B_1^{1A(+)I=0, 2} \quad \text{and} \quad B_1^{2A(-)}I=0, 2 \quad \text{have no} \ t^0\text{-contributions.}$
\[ B_0^{2A(-)I=0,2} : \{ IS \cdot M^g \left[ K^2 c_A' \left\{ \frac{1}{4} M^2 \left( 2M_+^2 - m_\pi^2 - K^2 \right) c_A' \right\} \right] \]

\[ + \frac{1}{36} \frac{m}{M} \left( K^2 + m_\pi^2 \right) \frac{K^2}{M^2} \left[ f(G_A', G_5') - 2 \frac{g_{12}}{M^2} G_A' - 4 \frac{g_2}{G_7} c_A' \right] \] (IV.11)

\[ + \frac{4}{9} \frac{m}{M} K^2 \frac{g_{12}}{M^2} \left[ G_A' + \frac{K^2}{4M^2} c_A' - \frac{1}{2} G_7 c_A' \right] \left( m_\pi^2 - K^2 \right) = 0 \]

\[ B_S^{2A(-)I=0,2} : IS \cdot M^g \left( 2M_+^2 - m_\pi^2 - K^2 \right) \left( c_A' + \frac{K^2}{m_\pi^2 - K^2} A_A' \right) = 0 \] (IV.12)

b) \( t^1 \)-sum rules

\[ B_1^{1A(+)I=0,2} : L(IIV.1) \left( m_\pi^2 - K^2 \right) + \frac{1}{2} \frac{m}{M} \frac{L(IIV.7)}{m_\pi^2 - K^2} = 0 \] (IV.13)

Further \( t^1 \)-sum rules are not considered.

IV.1.3.2 Sum rules of the \( m=2, I=1 \) amplitudes

\( B_1^{1A(+)I=1}, B_1^{2A(-)I=1}, B_0^{2A(-)I=1}, B_0^{2A(-)I=1} \) are crossing symmetrical.

a) \( t^0 \)-sum rules

\( B_1^{1A(-)I=1} \) has no \( t^0 \)-contribution

\( B_1^{2A(+)}I=1 \): The sum rule is identical with (IV.2).

\( B_0^{2A(+)I=1} \): The sum rule is identical with (IV.3)
The sum rule is identical with (IV.4)

t$^1$-sum rules are not considered.

According to (I.9) only t$^0$-sum rules are considered.

IV.2.1 t$^0$-sum rules of the I=1,2 amplitudes

\begin{equation}
G_3^{1A} I=1,2: \quad \frac{N A_{1}^{A} I=1,2}{2} \quad \text{is considered.}
\end{equation}

The sum rules are identical with (IV.14).

\begin{equation}
G_3^{1A} I=1,2: \quad \frac{N A_{1}^{A} I=1,2}{2} \quad \text{is considered.}
\end{equation}

\begin{equation}
G_3^{1A} I=1,2: \quad \frac{(N A_{2}^{A} I=1,2 + N A_{1}^{A} I=1,2 + N A_{2}^{A} I=1,2 + N A_{3}^{A} I=1,2)}{2} \quad \text{is considered.}
\end{equation}

with

\begin{equation}
N A_{1}^{A} = \frac{m A_{3}^{A}}{g s},
\end{equation}

\begin{equation}
N A_{1}^{A} = \frac{g s}{2} \left( \frac{m}{M_{+}} C_{3}^{A} + C_{4}^{A} \right),
\end{equation}

\begin{equation}
D A_{1}^{A} = \frac{1}{2} \left( \frac{g_{12}}{m} - \frac{K_{2}^{2}}{M_{+}} \right) \frac{m}{M_{+}} C_{3}^{A} + \frac{1}{2} \left( \frac{K_{2}^{2}}{M_{+}} \right) \frac{g_{12}}{m} - \frac{K_{2}^{2}}{M_{+}} \frac{g_{12}}{M_{+}} C_{4}^{A} + \frac{m}{M_{+}} \frac{m}{M_{+}} \frac{g_{12}}{M_{+}} C_{5}^{A}.
\end{equation}
\[
DA_1^u = m g \frac{m}{M} \left[ \frac{4}{M^2} \frac{\text{MDN}^2}{M^2} \left( \frac{G_5^A + K^2}{4M^2} G_6^A + \frac{3m^2}{M^2} G_6^A + 2 \left( \frac{M}{M^2} \right) + \frac{2(2N+m)^2}{M^2} G_7^A \right) \right],
\]

\[
NA_2^s = m g \frac{K^2}{2} F_3^A,
\]

\[
NA_2^u = g \frac{m^2}{M} C_5^A,
\]

\[
DA_2^s = 2 \frac{mM}{g_2} \frac{K^2}{2} C_3^A' + \left[ \frac{K^2}{2} \frac{g_12}{g_2} + \frac{M}{M^2} g_2 \right] C_4^A + m \left( \frac{M}{M^2} \right) g_2 \frac{K^2}{2} C_5^A',
\]

\[
DA_2^u = -m g \frac{m}{M} \frac{K^2}{2} \left[ \frac{4}{M^2} \frac{\text{MDN}^2}{M^2} \left( \frac{G_5^A + K^2}{4M^2} G_6^A - G_7^A \right) + \frac{3m^2}{M^2} G_6^A \right],
\]

\[
NA_3^s = 0
\]

\[
NA_3^u = g \left( C_5^A' + \frac{K^2}{m^2} C_6^A \right),
\]

\[
DA_3^s = \left( \frac{K^2}{M^2} - \frac{g_12}{g_2} \right) C_5^A' + \frac{K^2}{m^2} C_6^A',
\]

\[
DA_3^u = 0
\]

IV.3 \[ N^A_{\mu} \rightarrow N^\pi \]

Among the four \( m=1 \) amplitudes belonging to unnatural parity exchange in the \( t \)-channel \( G^A_{2A \mu} I=0 \) and \( G^A_{2A \mu} I=1 \) are crossing symmetrical and \( G^A_{2A \mu} I=0 \) \( \frac{1}{2}, \frac{1}{2}, S_1 \) and \( G^A_{2A \mu} I=1 \) \( \frac{1}{2}, \frac{1}{2}, S_1 \) has no \( t^0 \)-contributions. The remaining \( I=1 \) amplitude \( G^A_{2A \mu} I=1 \) although su-
perconvergent at \( t=0 \) will not be used because the \( A_1 \)-exchange with \( \alpha_{A_1}(0) \approx 0 \) contributes. It is not to be expected that an approximation keeping only the low energy contributions gives a good result because of slow convergence for \( t \approx 0 \).
REFERENCES


