KfK 2723 B
Januar 1979

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FLUID-STRUCTURE INTERACTIONS IN ONE-DIMENSIONAL LINEAR CASES

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ISSN 0303-4003

FLUID - STRUCTURE INTERACTIONS IN ONE - DIMENSIONAL LINEAR CASES

Abstract

The interaction of pressure waves in a pipe with an elastic endwall (piston) is analyzed using a linear ("acoustic") model. Two transient and two periodic cases are investigated. In the transient cases the motions are initiated by either a sudden pressure drop at the open end (breaking membrane) or by a sudden release of the piston from a non-equilibrium position ("snapback"); in the latter case the other end of the pipe is closed. In the periodic cases harmonic oscillations of the piston and the fluid are investigated with the other end of the pipe being either closed or open (kept at constant pressure).

The problem is characterized by three non-dimensional numbers (e.g.: Mach-, Strouhal-, and an interaction-number). The solution of the wave equation for the pressure accounting for the coupling to the structure can be reduced analytically to the problem of integrating one ordinary differential equation of second order in time. This differential equation in turn can be integrated analytically at least for a certain time period. At later times this ordinary differential equation is integrated numerically. For the periodic cases eigenvalue-problems arise with an infinite number of solutions. The first few eigensolutions are given.

By comparison to the exact solutions the accuracy of fully numerical schemes and of approximate formulae are discussed. General insight in the physics of the coupled system is obtained. Finally cases are investigated which resemble to some extent the HDR-blowdown- or snapback-experiments.

FLUID - STRUKTUR WECHSELWIRKUNGEN IN EINDIMENSIONALEN LINEAREN FÄLLEN

Zusammenfassung

Die Wechselwirkung von Druckwellen in einem Rohr mit einer elastischen Endwand ("Kolben") wird anhand eines linearisierten ("akustischen") Modells analysiert. Zwei transiente und zwei periodische Fälle werden untersucht. In den transienten Fällen werden die Bewegungen entweder durch einen plötzlichen Druckabfall am offenen Ende (brechende Membran) oder durch ein plötzliches Loslassen des Kolbens aus einer Nichtgleichgewichtslage ("Zupfen") angeregt; im letzteren Fall ist das andere Ende des Rohres geschlossen. In den periodischen Fällen werden harmonische Schwingungen des Kolbens und des Fluids untersucht, wobei das andere Rohrende entweder geschlossen oder offen (konstanter Randdruck) sei.

Das Problem ist durch drei unabhängige Kennzahlen charakterisiert (z.B.: Mach-, Strouhal- und eine Kopplungs-Zahl). Die Lösung der Wellengleichung des Fluids kann unter Beachtung der Koppelung an die Struktur analytisch auf das Problem der Integration einer gewöhnlichen Differentialgleichung zweiter Ordnung in der Zeit reduziert werden. Für diese Differentialgleichung wiederum gibt es analytische Lösungen zumindest für eine gewisse Anfangszeit. Für spätere Zeiten erfolgt die Integration dieser gewöhnlichen Differentialgleichung numerisch. In den periodischen Fällen stößt man auf Eigenwert-Probleme mit unendlich vielen Lösungen. Die ersten paar Eigenlösungen werden ermittelt.

Anhand der Lösungen wird die Genauigkeit voll numerischer Verfahren sowie die von Näherungslösungen diskutiert. Man erhält generelle Einsichten in die Physik des gekoppelten Systems. Schließlich werden Fälle diskutiert, die eine gewisse Ähnlichkeit mit den HDR-Blowdown- bzw. Schwingungs-Experimenten aufweisen.

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NOMENCLATURE

1. General Notation

a	any non-dimensional parameter
$\hat{a}$	dimensionful parameter
$\dot{a}$	time derivative
a'	space derivative
a_o	initial value
a_1	value outside the pipe
a_a	value referring to the acceleration
a_{break}	value referring to the membran break
a_{max}	maximum value
a_s	static value

2. Frequently used symbols

$\hat{a}$	speed of sound	
c	piston deflection	$\hat{c}/\hat{L}$
E	stiftness ratio	$(\hat{s}\ \hat{L})/(\hat{\varrho}\ \hat{a}^2)$
K	interaction number	$\hat{L}\ \Delta\hat{p}/(\hat{m}\ \hat{a}^2)$
$\hat{L}$	length of pipe	
$\hat{m}$	mass of piston per unit area	
M	Mach-number	$(\Delta\hat{p}/(\hat{\varrho}\ \hat{a}^2))^{1/2}$
N	number of grid points per unity	
p	pressure	$(\hat{p}-\hat{p}_1)/\Delta\hat{p}$
R	energy-transfer number	$2(\hat{s}\ \hat{m})^{1/2}/(\hat{a}\ \hat{\varrho})$
s	stiffness of the piston per unit area	
S	Strouhal number	$(\hat{s}/\hat{m})^{1/2}\ \hat{L}/\hat{a}$
t	time	$\hat{t}\ \hat{a}\ /\ \hat{L}$
$\hat{u}$	fluid velocity	
U	mass ratio	$\hat{L}\ \hat{\varrho}\ /\ \hat{m}$
x	space coordinate	$\hat{x}\ /\ \hat{L}$

$\alpha = 1$	in case 1: broken membran, = 0 in case 2: snapback	
$\Delta\hat{p}$	initial pressure difference	
ε	relative error	
$\hat{\varrho}$	fluid density	
ω	angular eigenfrequency	$\hat{\omega}\,\hat{L}\,/\,\hat{a}$
ν	frequency	$\nu^2 = S^2 - U^2/4$

1. INTRODUCTION

Within the HDR-Blowdown-project [1] the coupled motion of fluid and structure represents a central subject of investigation. Several numerical methods are available (see e.g. [2,3]) for analysis of such fluid-structure interaction problems. For the understanding of the physical processes and for analysis of the accuracy of the numerical results it is very important to have analytical solutions at hand at least for simplified cases. The purpose of this report is to present such analytical solutions for a simple one-dimensional linear problem. The present work is an extension of an earlier study by Iwanicki [7].

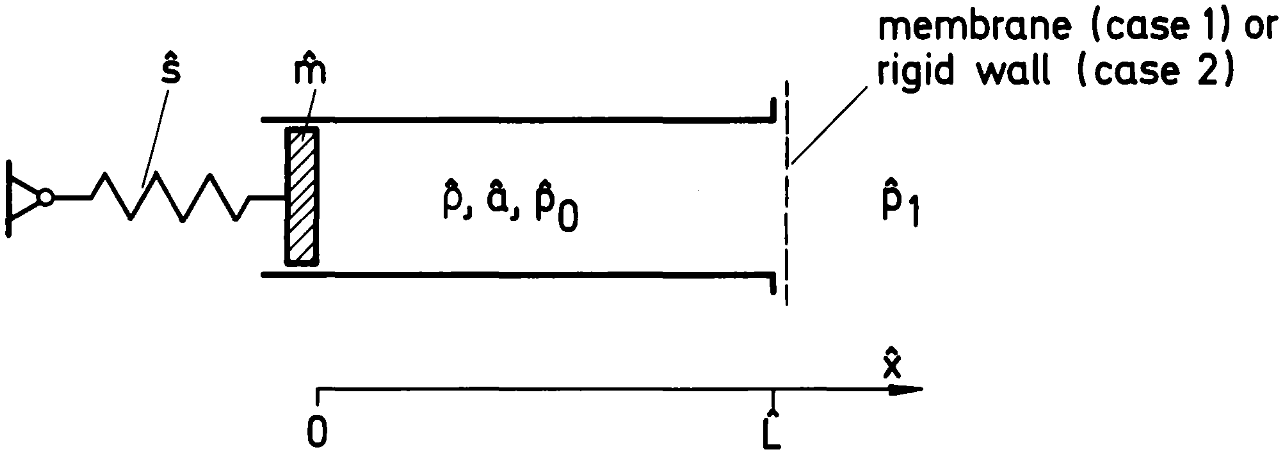


Fig. 1: The one-dimensional problem

We consider the geometry sketched in Fig. 1. A pipe of length $\hat{L}$ is filled with fluid of density $\hat{\rho}$, constant speed of sound $\hat{a}$ and at initial pressure $\hat{p}_0$. The right end of the pipe is either closed at all times by a rigid wall or closed by a membrane which is suddenly removed at time $\hat{t} = 0$. (A gradual change of the boundary pressure - as it would be appropriate for finite break time - can be treated with minor changes in the theory). The left side is closed by a piston of mass $\hat{m}$ per unit area which is fixed by a spring of stiffness $\hat{s}$ per unit area. The piston is at rest initially. We distinguish two transient cases:

Transient case 1:

The membrane breaks at $\hat{t} = 0$, the outer pressure $\hat{p}_1$ is lower than $\hat{p}_0$. The initial position $\hat{c}_0$ of the piston is a result of the pressure difference $\hat{p}_0 - \hat{p}_1$ and the stiffness.

$$\hat{c}_0 = (\hat{p}_1 - \hat{p}_0)/\hat{s}$$

Transient case 2:

The outer pressure $\hat{p}_1$ is the same as $\hat{p}_0$. The membrane does not break (rigid wall at right side). However the piston is not in its equilibrium position initially but rather at some value

$$\hat{c}_0 < 0$$

Both cases can also be investigated under the assumption that the system is in a quasi-steady state with harmonic oscillations. These "Periodic cases 1 and 2" will be discussed too.

In order to allow for an analytical solution the following assumptions are used:

A1) the fluid pressure satisfies the linear wave equation

$$\frac{1}{\hat{a}^2}\frac{\partial^2 \hat{p}}{\partial \hat{t}^2} - \frac{\partial^2 \hat{p}}{\partial \hat{x}^2} = 0, \quad \hat{a} = const \tag{1-1}$$

A2) the piston-movement follows

$$\hat{m}\,\ddot{\hat{c}} + \hat{s}\,\hat{c} = \hat{p}_1 - \hat{p}/_W \tag{1-2}$$

where $\hat{p}/_W$ = pressure at the inner piston surface.

A3) the maximum deflection of the piston $\hat{c}_{max}$ be small in comparison to the length $\hat{L}$ of the pipe, so that

$$\hat{p}/_W = \hat{p}\,(\hat{x}=0) \tag{1-3}$$

A4) At the same position the boundary condition for the pressure follows from the momentum equation and is

$$\frac{\partial \hat{p}}{\partial \hat{x}}(\hat{x}=0) = -\hat{\varrho}\,\ddot{\hat{c}} \qquad (1\text{-}4)$$

($\hat{\rho}$ = const.)

A5) At the right end we have the boundary condition

$$\alpha\left[\hat{p}(\hat{x}=\hat{L}) - \hat{p}_1\right] + (1-\alpha)\frac{\partial \hat{p}}{\partial \hat{x}}(\hat{x}=\hat{L}) = 0 \qquad (1\text{-}5)$$

where in case of the broken membrane

$$\alpha = 1 \qquad (1\text{-}6a)$$

and in case of a rigid wall

$$\alpha = 0 \qquad (1\text{-}6b)$$

(Values of α between 0 and 1 are not considered here).

2. DIMENSIONAL ANALYSIS

The problem is characterized by the following independent parameters

$$\hat{L}\,,\ \hat{\rho}\,,\ \hat{a}\,,\ \hat{m}\,,\ \hat{s} \tag{2-1a}$$

and a pressure difference $\Delta\hat{p}$ which in case 1 is

$$\Delta\hat{p} \equiv \hat{p}_0 - \hat{p}_1 > 0 \tag{2-1b}$$

and in case 2 is defined according to the initial deflection c_o:

$$\Delta\hat{p} \equiv -\hat{s}\,\hat{c}_o \tag{2-1c}$$

From $\Delta\hat{p}$ we can define a characteristic flow velocity

$$\hat{u}_0 \equiv (\Delta\hat{p}/\hat{\rho})^{1/2} \tag{2-1d}$$

With these parameters three independent dimensionless numbers can be defined:

"Mach-number" $$M = \hat{u}_0/\hat{a} \tag{2-2}$$

"Strouhal-number" $$S = \left(\frac{\hat{s}}{\hat{m}}\right)^{1/2}\frac{\hat{L}}{\hat{a}} \tag{2-3}$$

"mass-ratio" $$U = \hat{L}\,\hat{\rho}/\hat{m} \tag{2-4}$$

Also appearing is the combination

"interaction number" $$K \equiv M^2 U = \frac{\hat{L}\,\Delta\hat{p}}{\hat{m}\,\hat{a}^2} \tag{2-5}$$

We use the following characteristic quantities to make all equations dimensionless:

length: $\hat{L}$
time : $\hat{L}/\hat{a}$
pressure: $\Delta\hat{p}$

It should be noted that there are several alternatives (e.g. $\hat{c}_o$ instead of $\hat{L}$, $\hat{L}/\hat{u}_o$ instead of $\hat{L}/\hat{a}$), however the present choice has the advantage to give a very simple set of equations, in particular for the pressure. Accordingly we define:

$$\begin{aligned} p &\equiv (\hat{p} - \hat{p}_1) / \hat{\Delta p} \\ t &\equiv \hat{t}\, \hat{a}/\hat{L} \\ x &\equiv \hat{x}/\hat{L} \\ c &\equiv \hat{c}/\hat{L} \\ u &\equiv \hat{u}/\hat{a} \end{aligned} \tag{2-6}$$

The resultant non-dimensional equations are

$$\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0 , \quad p(x,t) , \quad \begin{matrix} 0 \leq x \leq 1, \\ 0 \leq t \end{matrix} \tag{2-7}$$

(2-8)

Initial conditions:

$$p(x,0) = \alpha = \begin{cases} 1 & \text{in case 1} \\ 0 & \text{in case 2} \end{cases} \tag{2-9}$$

$$c(0) = c_o \equiv -k/s^2 \tag{2-10}$$

$$\dot{c}(0) = 0 \tag{2-11}$$

Boundary conditions:

left fluid boundary:

$$\frac{\partial p}{\partial x}(0,t) = -\frac{1}{M^2}\, \ddot{c}(t) \tag{2-12}$$

right fluid boundary, t > 0:

$$\alpha\, p(1,t) + (1-\alpha) \frac{\partial p}{\partial x}(1,t) = 0 \tag{2-13}$$

3. TRANSIENT CASES

3.1 Principle Solution Scheme

The general solution of the wave equation is d'Alembert's ansatz, see e.g. [4],

$$p(x,t) = F_1(x+t) + F_2(x-t) \tag{3-1}$$

From the initial values (2-9) we get

$$F_1(x) + F_2(x) = \alpha \quad , \quad 0 \leq x \leq 1$$

There is no other condition (which would reflect the past), therefore we arbitrarily set

$$F_1(x) = F_2(x) = \frac{\alpha}{2} \quad , \quad 0 \leq x \leq 1. \tag{3-2}$$

From the boundary condition (2-12) at the piston we get

$$\frac{\partial p}{\partial t}(0,t) = \dot{F}_1(t) + \dot{F}_2(-t) = -\frac{1}{M^2}\ddot{c}(t)$$

Integration over the time 0 to t gives

$$F_1(t) - F_2(-t) = -\frac{1}{M^2}\dot{c}(t) \tag{3-3}$$

where we made use of the initial values (3-2) and (2-11). From the second boundary condition (2-13) at x=1 we get

$$\alpha\left(F_1(1+t) + F_2(1-t)\right) + (1-\alpha)\left(\dot{F}_1(1+t) + F_1(1-t)\right) = 0$$

In case 1 with $\alpha = 1$ this is

$$F_1(1+t) + F_2(1-t) = 0$$

In case 2 with $\alpha = 0$ we can integrate and have

$$F_1(1+t) - F_2(1-t) = 0$$

or in general

$$F_1(1+t) + (2\alpha-1)\, F_2(1-t) = 0. \qquad (3\text{-}4)$$

With (3-1) the structural equation (2-8) results

$$\ddot{c} + S^2 c = -K\,(F_1(t) + F_2(-t))$$

By means of (3-3) we may eliminate $F_2(-t)$

$$\ddot{c}(t) + S^2 c(t) = -K\left(2F_1(t) - \frac{1}{M^2}\,\dot{c}(t)\right)$$

with the result (using 2-5)

$$\ddot{c}(t) + U\dot{c}(t) + S^2 c(t) = -2K\,F_1(t). \qquad (3\text{-}5)$$

These equations allow for construction of the following solution method:

1) Set initial values

$$F_1(x) = F_2(x) = \alpha/2 \quad , \quad 0 \le x < 1, \qquad (3\text{-}6)$$

$$\dot{c}(0) = 0 \quad , \quad c(0) = -K/S^2, \qquad (3\text{-}7)$$

set initial time $t_o = 0$.

2) Integrate from time t_o to the arbitrary end time t_{max} by

a) from (3-4) set

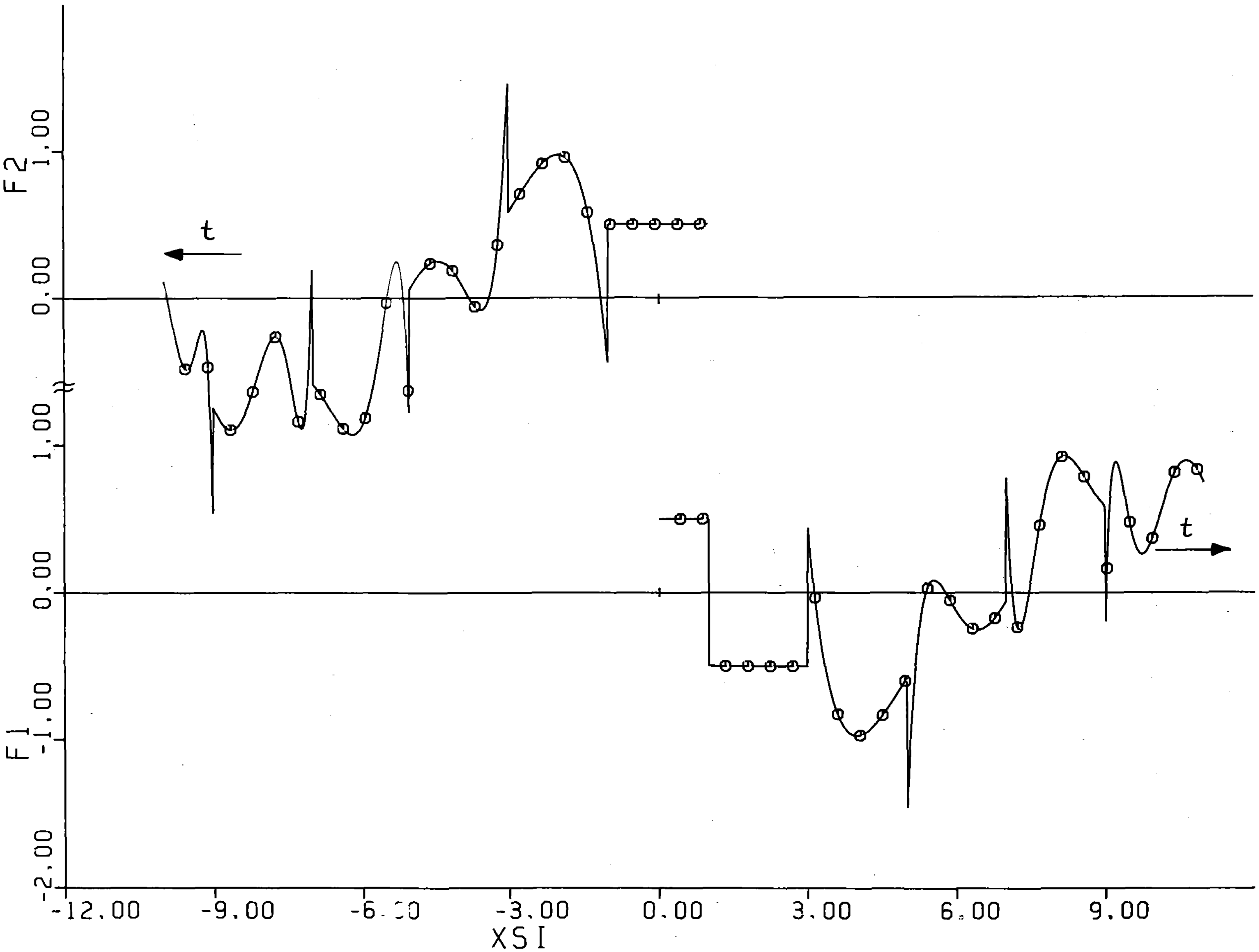
$$F_1(1+t) = -(2\alpha-1)\, F_2(1-t) \qquad (3\text{-}8)$$

b) integrate according to (3-5)

$$\ddot{c} + U\dot{c} + S^2 c = -2K\,F_1(t) \qquad (3\text{-}9)$$

c) from (3-3) set

$$F_2(-t) = F_1(t) + \frac{1}{M^2}\,\dot{c}(t) \qquad (3\text{-}10)$$



<u>Fig. 2</u>: Function F_1 and F_2 versus ζ for S=1, U=2; transient case, breaking membrane.

3) From the resulting functions

$$F_1(\xi), \quad 0 \le \xi \le t_{max}+1$$
$$F_2(\xi), \quad -t_{max} \le \xi \le 1$$

we may compute the pressure according to (3-1):

$$p(x,t) = F_1(x+t) + F_2(x-t), \qquad 0 \le x \le 1, \; 0 \le t \le t_{max}$$

Note that the function F_1 is integrated from zero to positive arguments, the function F_2 from one to negative values, see Fig. 2.

3.2 Analytical solution for the first two time units of structural oscillations

3.2.1 Case 1 (broken membrane)

The solution scheme gives (see Fig. 2):

t = 0:

$$F_1(x) = F_2(x) = \frac{1}{2}, \quad 0 \le x \le 1$$
$$\dot{c}(0) = 0, \quad c(0) = -K/S^2$$

$0 < t \le 1$

$$F_1(1+t) = -\frac{1}{2}$$
$$\dot{c}(t) = 0, \quad c(t) = -K/S^2$$
$$F_2(-t) = \frac{1}{2}$$

At time t = 1 the pressure wave arrives at the piston so that the pressure load at the piston changes:

$1 < t \le 2$

$$F_1(1+t) = -\frac{1}{2}$$

$$\ddot{c} + U\dot{c} + S^2 c = K$$

Here an anlytical integration is possible. The result depends on the parameter

$$\nu^2 = S^2 - U^2/4. \qquad (t_0 \equiv 1)$$

In case of $\nu^2 > 0$:

$$c(t) = \frac{K}{S^2}\left\{1 - 2\,e^{-U(t-t_0)/2}\left[\cos(\nu(t-t_0)) + \frac{U}{2\nu}\sin(\nu(t-t_0))\right]\right\} \tag{3-11}$$

In case of $\nu^2 = 0$:

$$c(t) = \frac{K}{S^2}\left\{1 - e^{-U(t-t_0)/2}\left[U(t-t_0)+2\right]\right\} \tag{3-12}$$

In case of $\nu^2 < 0$ we use $\nu'^2 = -\nu^2$ and have:

$$c(t) = \frac{K}{S^2}\left\{1 - 2\,e^{-U(t-t_0)/2}\left[\cosh(\nu'(t-t_0)) + \frac{U}{2\nu'}\sinh(\nu'(t-t_0))\right]\right\} \tag{3-13}$$

The function $F_2(-t)$ has to be determined from these results according to (3-10).

$2 < t \leq 3$

At time t=2 the pressure wave comes back to the open end. In the subsequent time interval the only change appears with respect to

$$F_1(1+t) = -F_2(1-t)$$

which is no longer a constant. However c(t) and $F_2(-t)$ are computable as before.

$3 < t$

For later times, the function $F_1(t)$ in (3-9) is no longer a constant. No analytical solution has been found by the author. We have to use numerical methods. However, the only errors possible are those due to approximations of the time differentials; space differentials are still described exactly.

3.2.2 Case 2 (closed pipe)

The solution scheme gives

t = 0: $F_1(x) = F_2(x) = 0 \quad , \quad 0 \leq x \leq 1$
$\dot{c}(0) = 0, \quad c(0) = -K/S^2$

0 < t < 1: $F_1(1+t) = 0$
$c(t)$ is the integral of (3-9) with $F_1(t) = 0$.

The result can again be found analytically; it depends on

$$\nu^2 = S^2 - U^2/4.$$

In case of $\nu^2 > 0$:

$$c(t) = -\frac{K}{S^2} e^{-Ut/2} \left[\cos(\nu t) + \frac{U}{2\nu} \sin(\nu t)\right] \tag{3-14}$$

In case of $\nu^2 = 0$:

$$c(t) = -\frac{K}{S^2} e^{-Ut/2} \left[\frac{U}{2} t + 1\right] \tag{3-15}$$

In case of $\nu^2 < 0$ with $\nu'^2 = -\nu^2$:

$$c(t) = -\frac{K}{S^2} e^{-Ut/2} \left[\cosh(\nu' t) + \frac{U}{2\nu'} \sinh(\nu' t)\right] \tag{3-16}$$

The function $F_2(-t)$ has to be determined from these results according to (3-10).

$1 < t < 2$

In the subsequent time interval the only change appears with respect to

$$F_1(1+t) = + F_2(1-t)$$

which is no longer a constant. However c(t) and $F_2(-t)$ are computable as before.

$2 < t$

At t = 2 the pressure wave has returned to the piston. For later times, numerical integration is required as discussed in 3.2.1 for 3 < t.

3.3 Discussion

3.3.1 Transient Case with Breaking Membrane (Case 1)

Figs. 3-6 show the resultant non-dimensional deformation of the structure (piston) c(t) normalized by the initial amplitude $-c_o = M^2\, U/S^2$, the acceleration $\ddot{c}(t)$ divided by its maximum value $K = M^2 U$, the pressure $p_{/w}(t)$ at the wall as a function of time, and the pressure profiles p(x) as a function of the space coordinate x at early times.

The computations are for S = 1, M = 0.1, and U = 0.5, 2, 4. From the analytical solutions we have learned that a principally different behaviour is to be expected for negative, zero, and positive values of $\nu^2 = S^2 - U^2/4$, respectively. One can use either this difference or a new non-dimensional number

$$R = \frac{2S}{U} = \frac{2\,(\hat{\jmath}\,\hat{m})^{1/2}}{\hat{\alpha}\;\hat{\varrho}} \tag{3-17}$$

to characterize the solution type. This ratio can be interpreted as

$$R = \pi \frac{\omega_S}{\omega_F} \frac{m_S}{m_F} \tag{3-18}$$

where ω_S, ω_F are the eigenfrequencies, and m_S, m_F the masses of the structure and the fluid, respectively. In the three cases under discussion R has the values 4, 1, and 1/2, respectively, representing either a

stiff and heavy (R > 1),
medium (R ≈ 1),
or weak and light (R < 1) structure.

As we see from the figures, the value of R determines the speed by which energy is exchanged between the fluid and the structure. This energy transfer rate is largest for large values of R.

On the other hand, since S is kept constant, one can conclude that for large values of U, which means a large fluid mass compared to the structural mass, the frequency of the motion is reduced.

As is shown in appendix 1, the maximum amplitude of the structure, c_{max}, is bounded due to the finite amount of initial energy by

$$\frac{c_{max}}{c_0} = 1 + E \quad , \quad E = S^2/U. \tag{3-19}$$

This ratio is 3, 3/2, 9/8 in the three cases. In fact, we find the largest amplitudes in the first case (S = 1, U = 0.5), see Fig. 2. The computed ratio is however only slightly larger than unity and thus by far below the theoretical limit.

One might ask under which conditions the theoretical limit is reached. It has been found that this is about the case if the eigenfrequency of the structural motion including the reducing effect of the fluid inertia is equal to the eigenfrequency of the pressure wave oscillation. The latter is determined by the running time of the pressure wave twice back and forth which is 4 in the non-dimensional system. Thus the angular eigenfrequency should be

$$\omega_{critical} = \pi/2 . \tag{3-20}$$

In this case the pressure wave arrives at the structure and forces it every time the structure has reached a maximum or a minimum value. Since the eigenfrequency is not yet known exactly but close to S if the fluid inertia is small, we demonstrate this effect for the case $S = \pi/2$, $U = 0.05$, see Figs. 7, 8.

The magnitude of the Mach number M has not been varied simply because it effects the magnitude of the initial structural amplitude only ($c_0 = - M^2 U/S^2$).

- Accuracy of the time integration

All the results shown up to now were obtained by numerically integrating eq. (3-9) using an explicit second order scheme, see Appendix 4. According to an input parameter N the time step was set to 1/N for this purpose. One can show [3] that the explicit integration scheme is stable as long as $N > S/2$, which can easily be insured for moderate values of S. By comparison with the analytical solutions, at least during the first two time intervals of the structural motion, one can evaluate the error ε defined as the maximum difference over the maximum solution value in $0 < t < 3$ (Case 1) or $0 < t < 2$ (Case 2). The values observed in the above mentioned cases are given in Table I. The resultant values are so small that the plotted results can be claimed to be exact within the accuracy of the drawings.

- Comparison to a fully numerical solution scheme (FLUX1D)

The given problem has been solved also with a finite difference scheme within the code FLUX1D, see Appendix 5. This code is a special one-dimensional variant of the code FLUX2 (three-dimensional) described in [3]. Second order finite differences are used in space. In time a second order scheme (Newmark) or a first order scheme (fully implicit ICE analogy) can be used. The resultant equations are solved by means of direct elliptic solvers (i.e. Gaussian elimination of tridiagonal systems in the one-dimensional case). The numerical results for the cases S = 1, U = 0.5, 2, 4 as discussed above are computed using a space and time interval 1/N, N = 64. The resultant curves are shown in Figs. 9 to 12. By comparison of Figs. 3 and 9, we see that the numerical result is quite accurate for the structural motion amplitude c(t). However,for the acceleration and the pressure relatively "wild" oscillations appear which are solely due to space discretization errors.

If we repeat the computations with the fully implicit version (ICE analogy) these oscillations do not appear but the structural motion amplitude and in particular the pressure shock wave are damped due to numerical damping, see Figs. 13 to 16. Also the acceleration is grossly underestimated by this first order scheme.

Much reduced spurious oscillations appear, see Figs. 17a and 17b, if a steady pressure change at the membrane is prescribed instead of a sudden one:

$$p(x=1,t) = \begin{cases} 0 & \text{for } t \geq t_{break} \\ \cos^2\left(\pi t/(2t_{break})\right) & \text{for } t < t_{break}. \end{cases} \tag{3-21}$$

Thus, the spurious oscillations are a consequence of non-continuous boundary conditions.

3.3.2 Transient case 2 with closed end (snap-back)

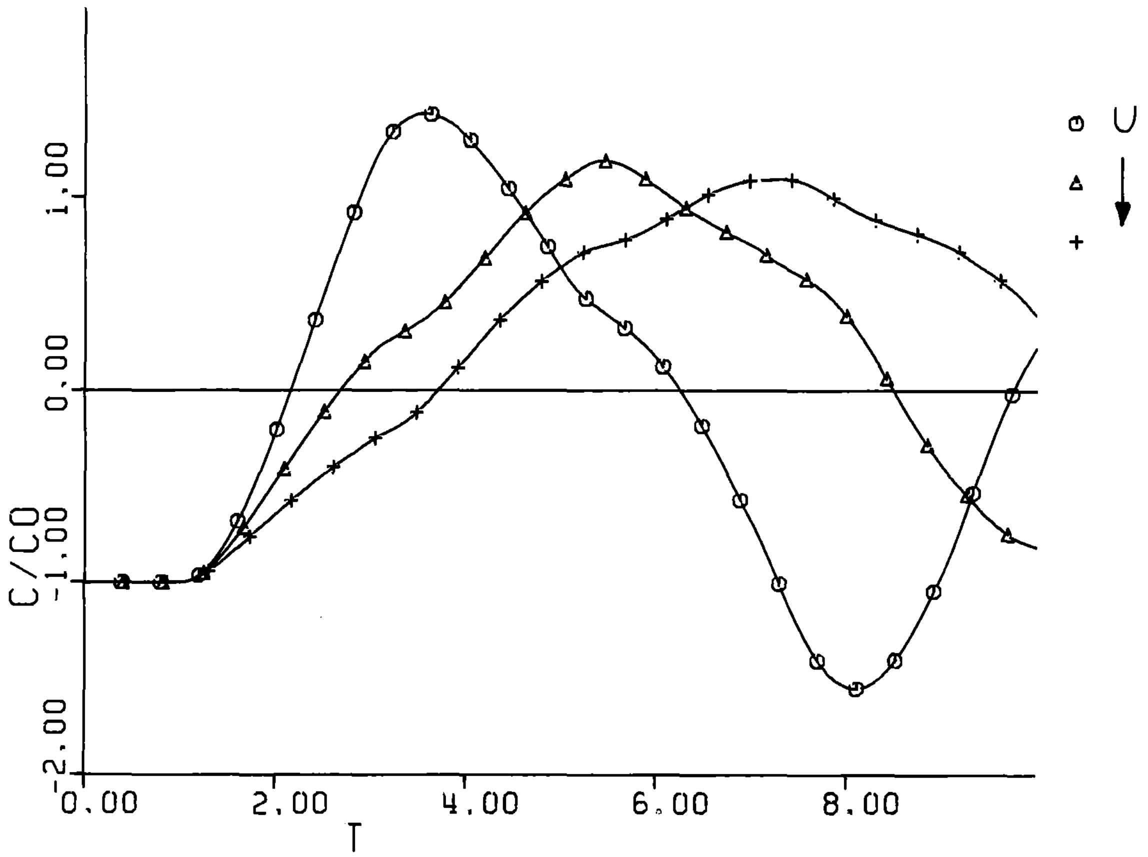
Much of what has been said for case 1 (open end) can be said again for case 2 where the structure (piston) snaps back from an initially deflected position into the fluid region with closed end. See Figs. 18 to 21 for the "exact" results.

In case of large values of U, the value c(t) remains negative all the time. In fact, the static value is given by (see Appendix 2)

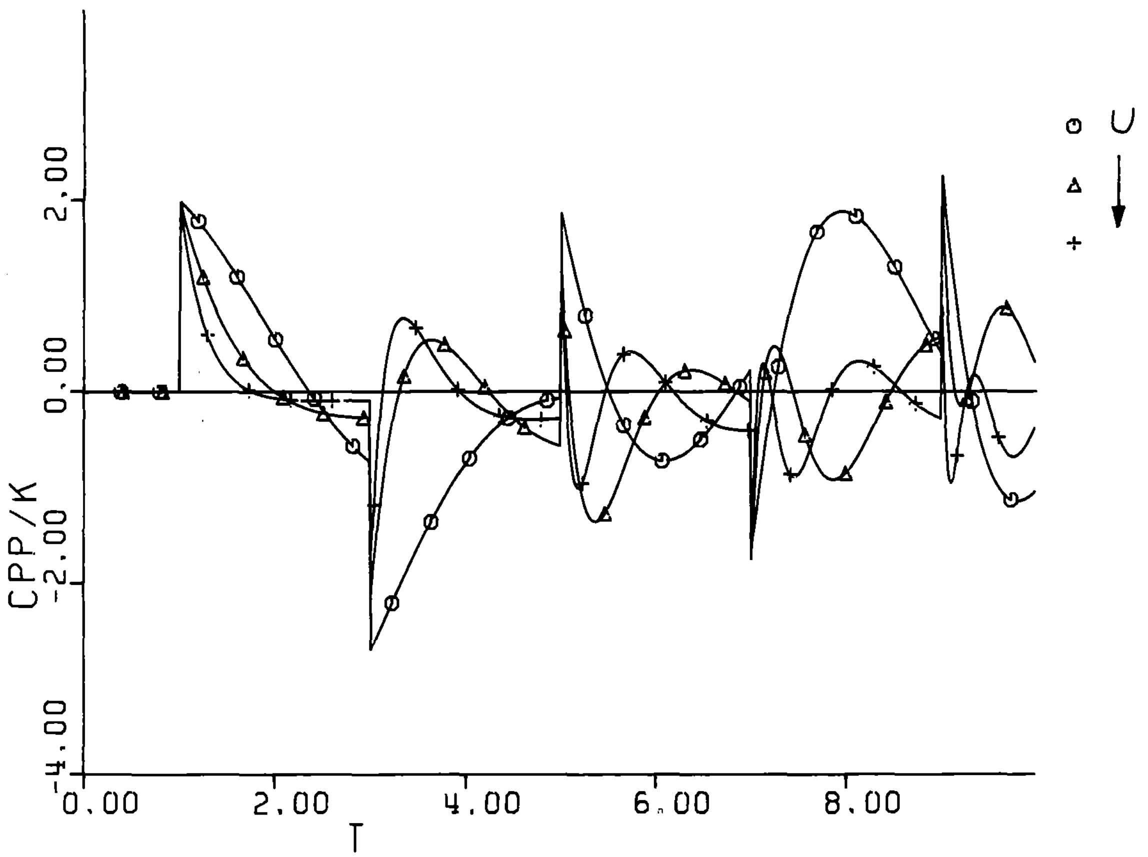
$$\frac{c_s}{c_0} = \frac{U}{U + S^2} = \frac{1}{1 + E} \tag{3-22}$$

which is smallest for $E = S^2/U \rightarrow \infty$.

The fully numerical scheme (here FLUX1D) gives significantly better accuracy in this case than in case 1. As can be seen from Figs. 22 to 25, not only the structural motion amplitude but also its acceleration and the pressure are computed with good accuracy if compared to the "exact" solutions shown in Figs. 18 - 21. This can be explained by the fact that in case 1 a step appears in the boundary value for the pressure, whereas in case 2 such a step appears only with respect to the boundary pressure gradient. A small "noise" signal is still shown however by the acceleration and pressure results (Figs. 23 to 25) in particular for large values of U. One should, therefore, use a steady change in the structural acceleration in a manner similar to Eq. (3-21).



<u>Fig. 3</u>: Deformation versus time, "exact" solution; S=1, U=0.5, 2, 4.



<u>Fig. 4</u>: Acceleration versus time, "exact" solution; S=1, U=0.5, 2, 4.

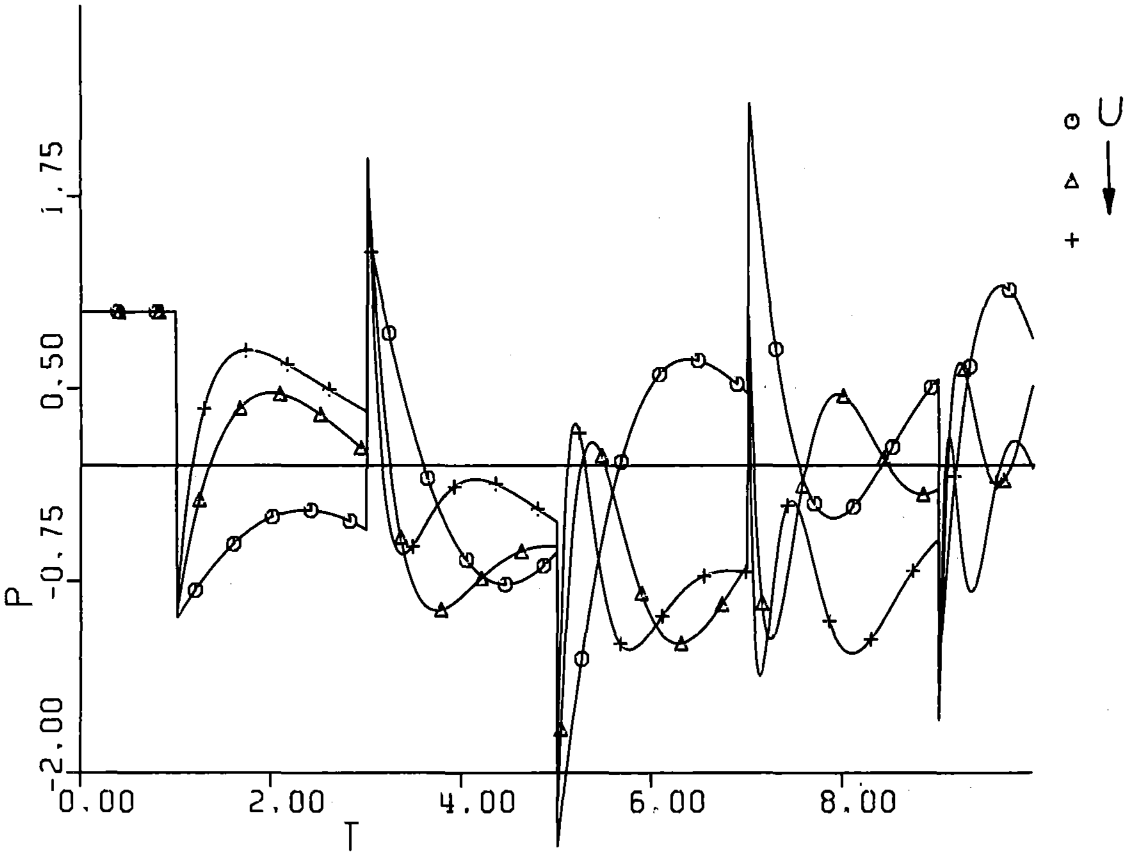


Fig. 5: Pressure at the piston versus time, "exact" solution; S=1, U=0.5, 2, 4.

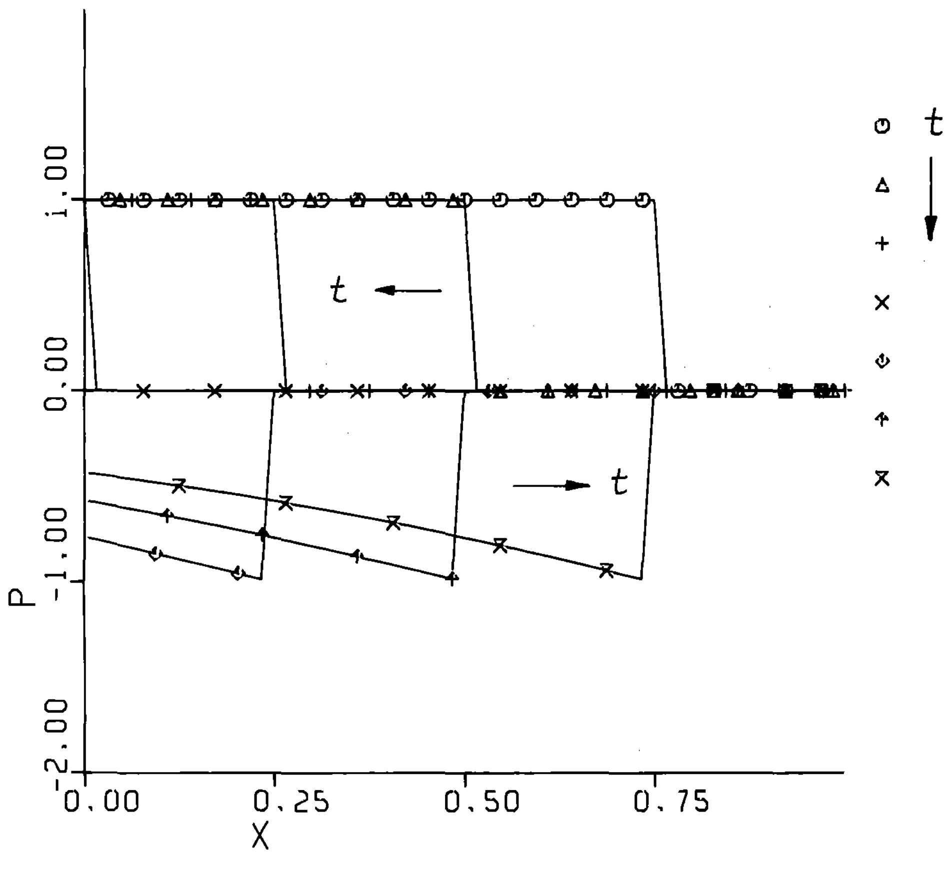


Fig. 6a: Pressure profiles at times 0.25, 0.5,... 1.75 for S=1, U=0.5.

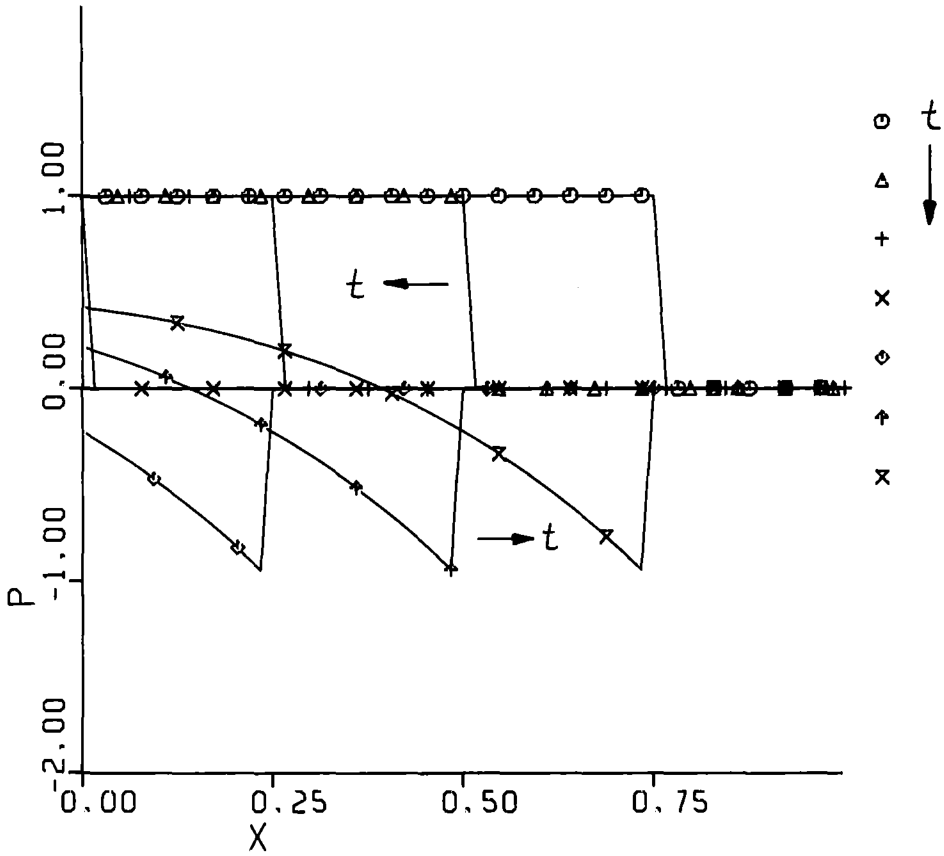


Fig. 6b: The same for U=2.

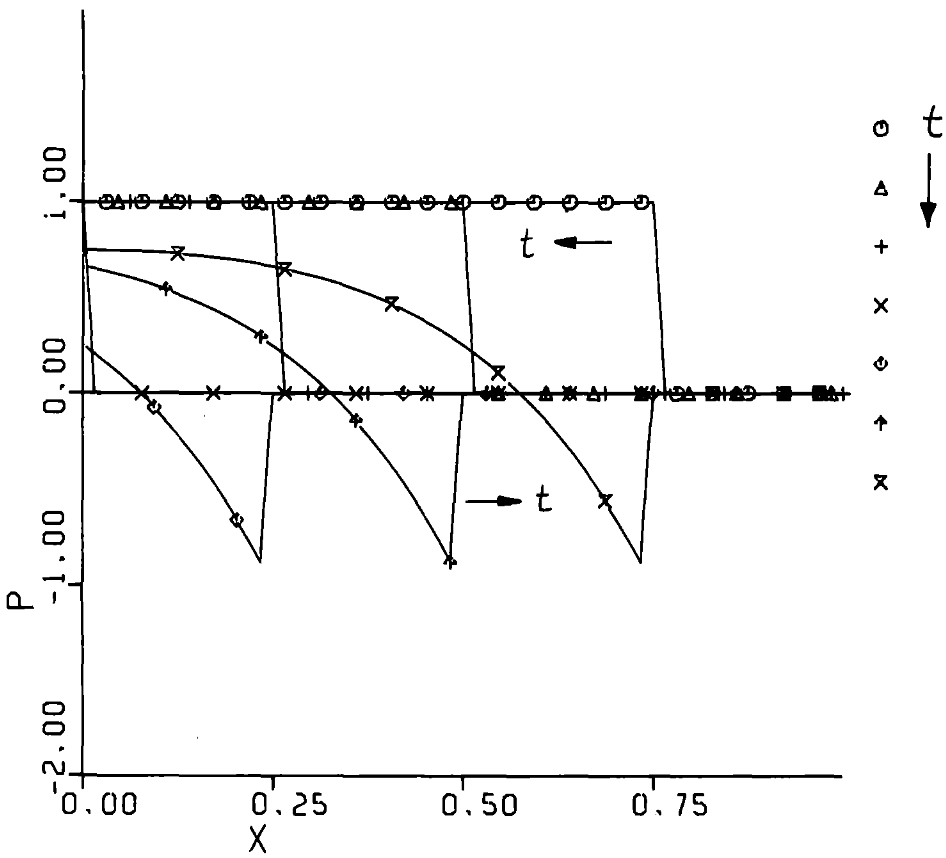
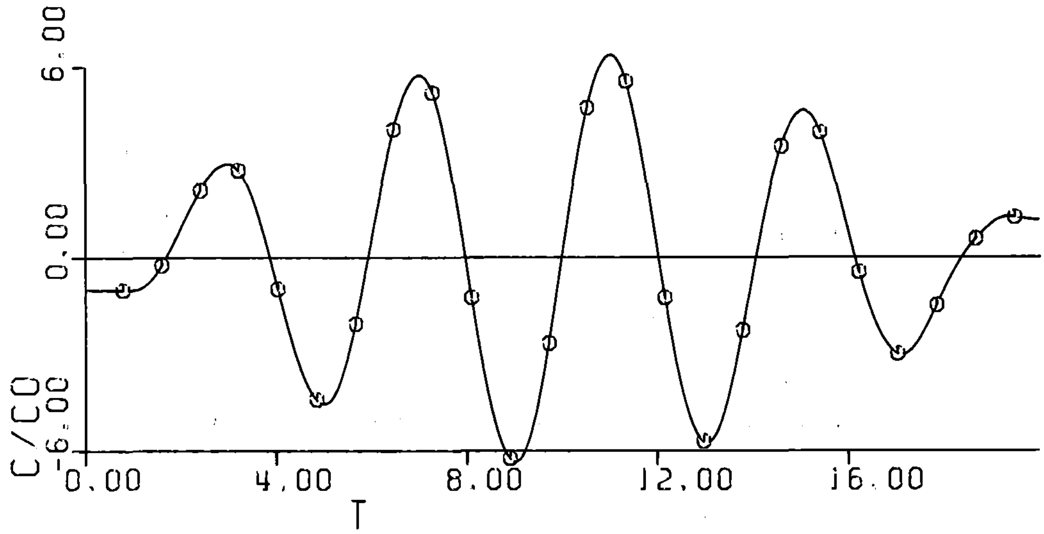
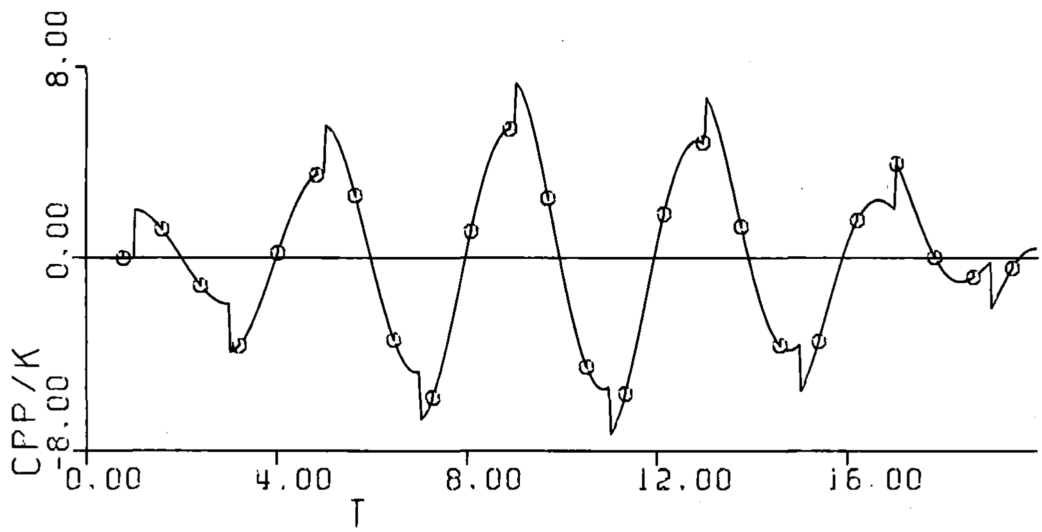


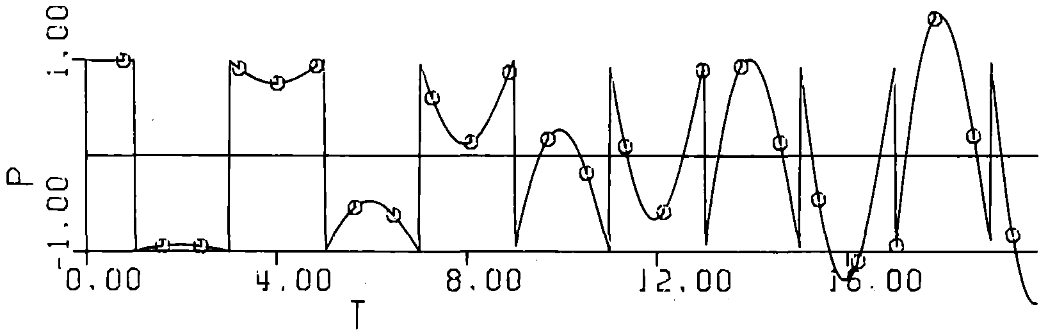
Fig. 6c: The same for U=4.



<u>Fig. 7</u>: Deformation versus time, "exact" solution, "critical" case; S=π/2, U=0.05



<u>Fig. 8a</u>: Same as Fig. 7 for acceleration.



<u>Fig. 8b</u>: Same as Fig. 7 for the pressure at the piston.

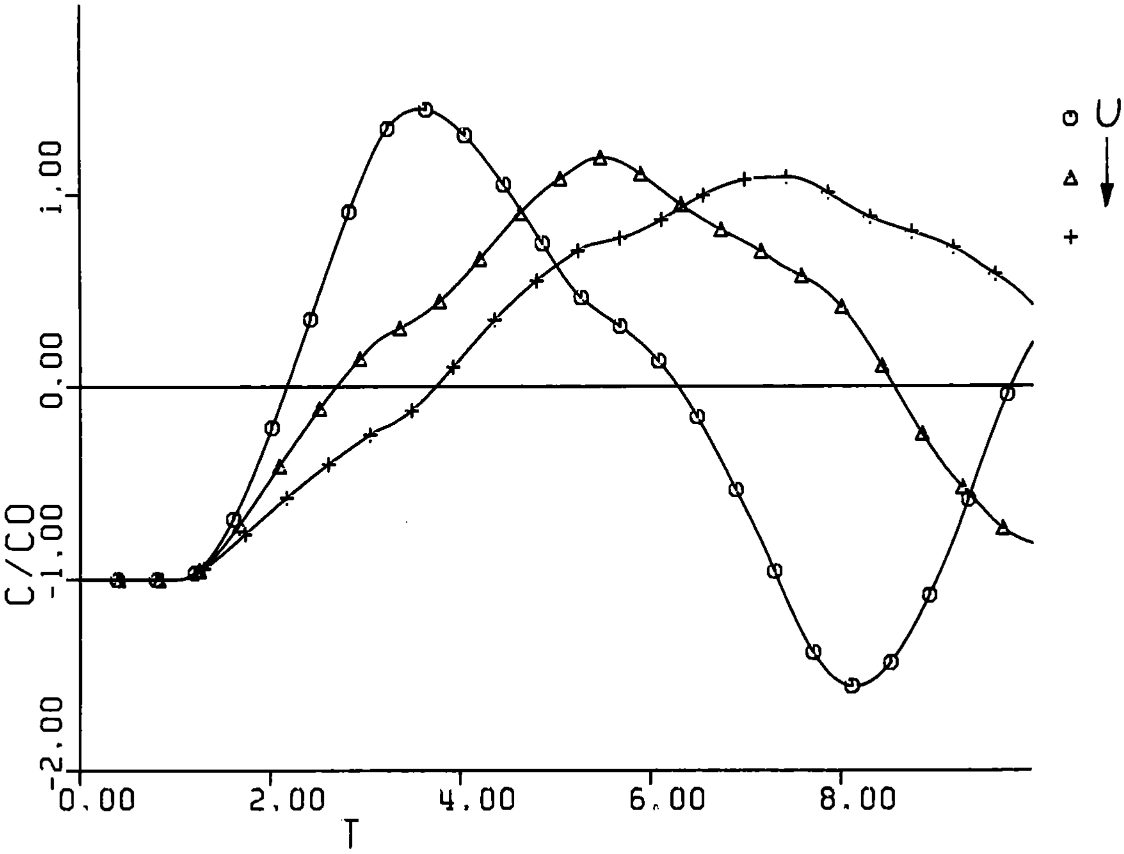


Fig. 9: Deformation versus time, finite difference solution; S=1, U=0.5, 2, 4, N=64.

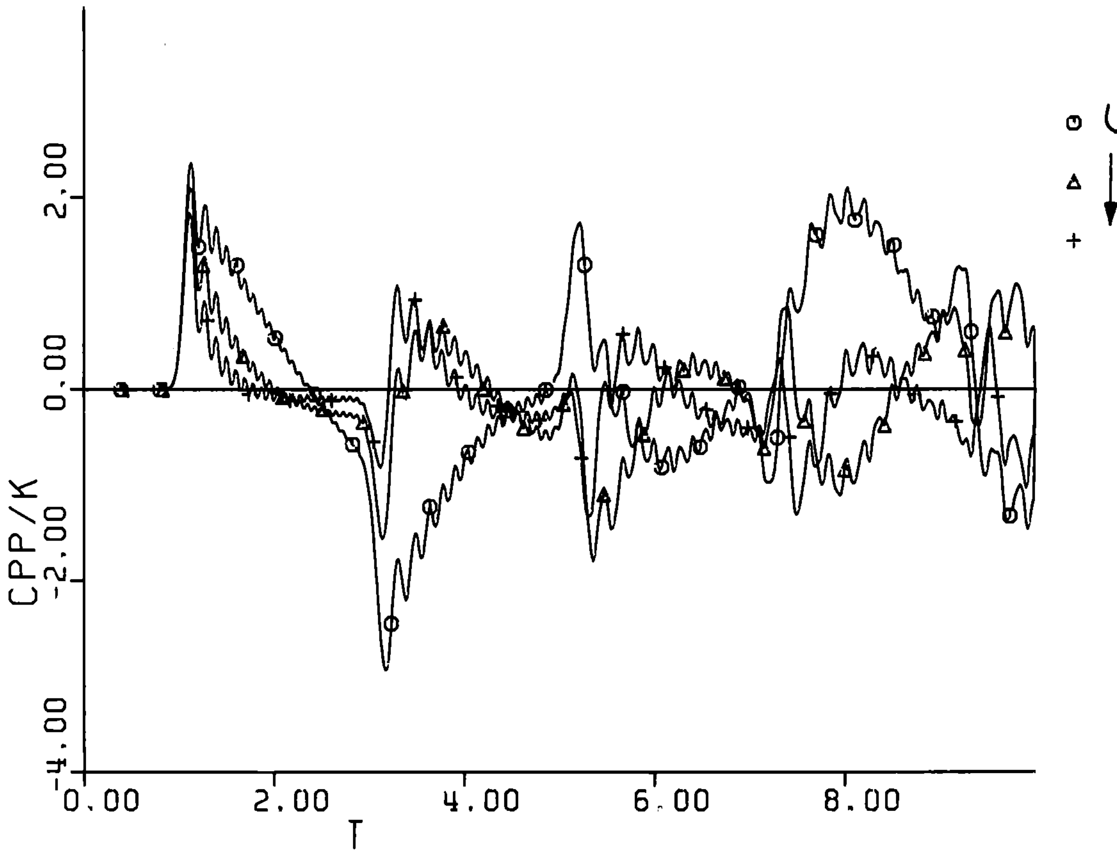


Fig. 10: Acceleration versus time, finite difference solution; S=1, U=0.5, 2, 4; N=64.

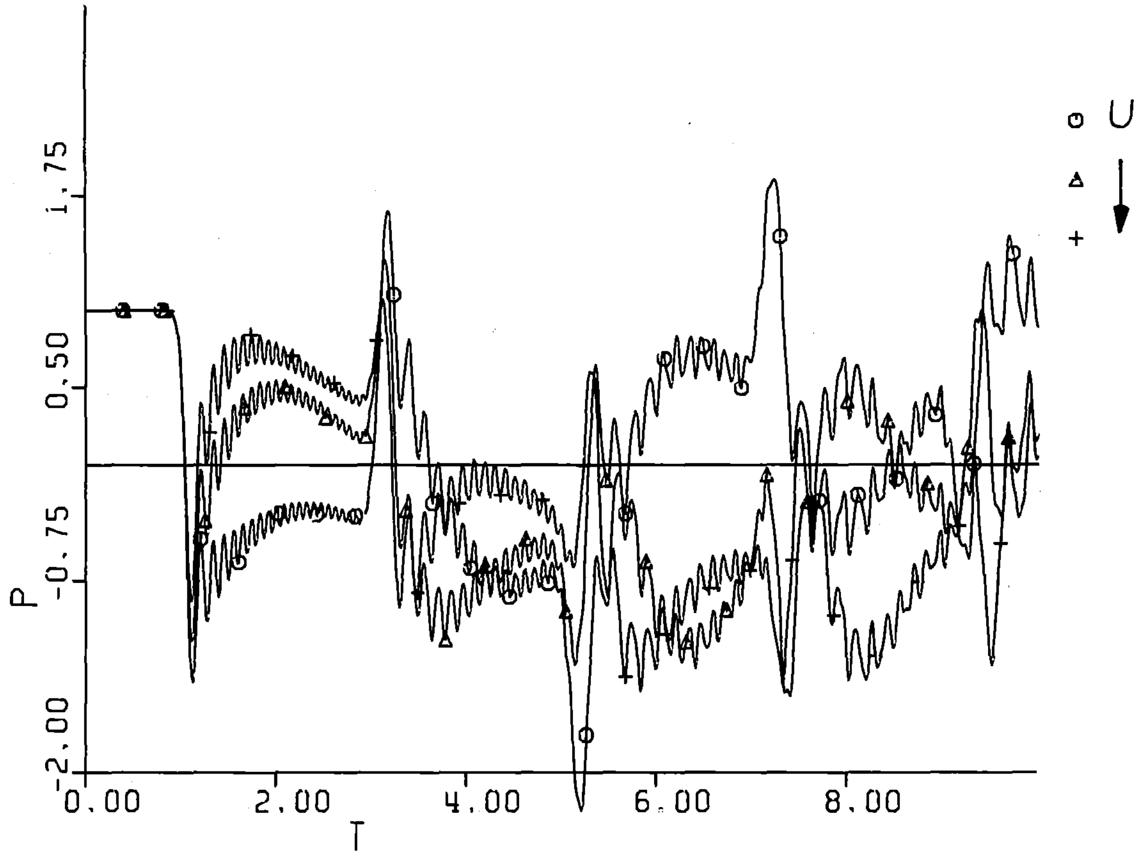


Fig. 11: Pressure at the piston versus time, finite difference solution; S=1, U=0.5, 2, 4; N=64.

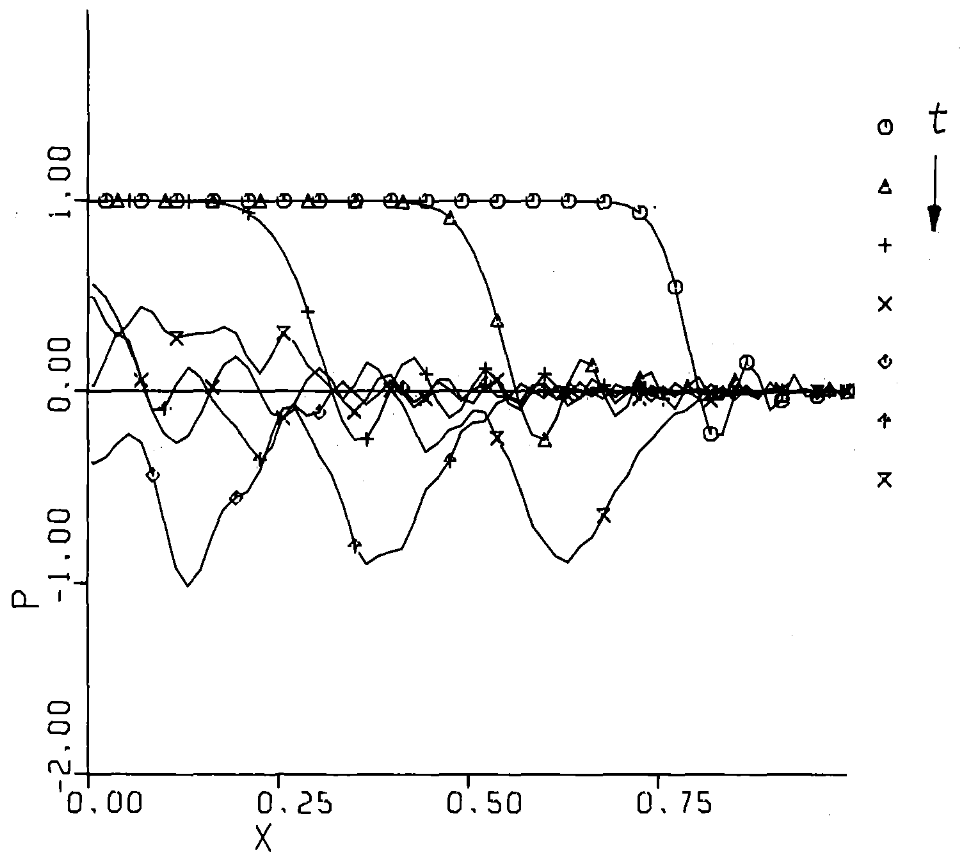


Fig. 12: Pressure profiles at times 0.25, 0.5 ..., 1.75, finite difference solution; S=1, U=2.

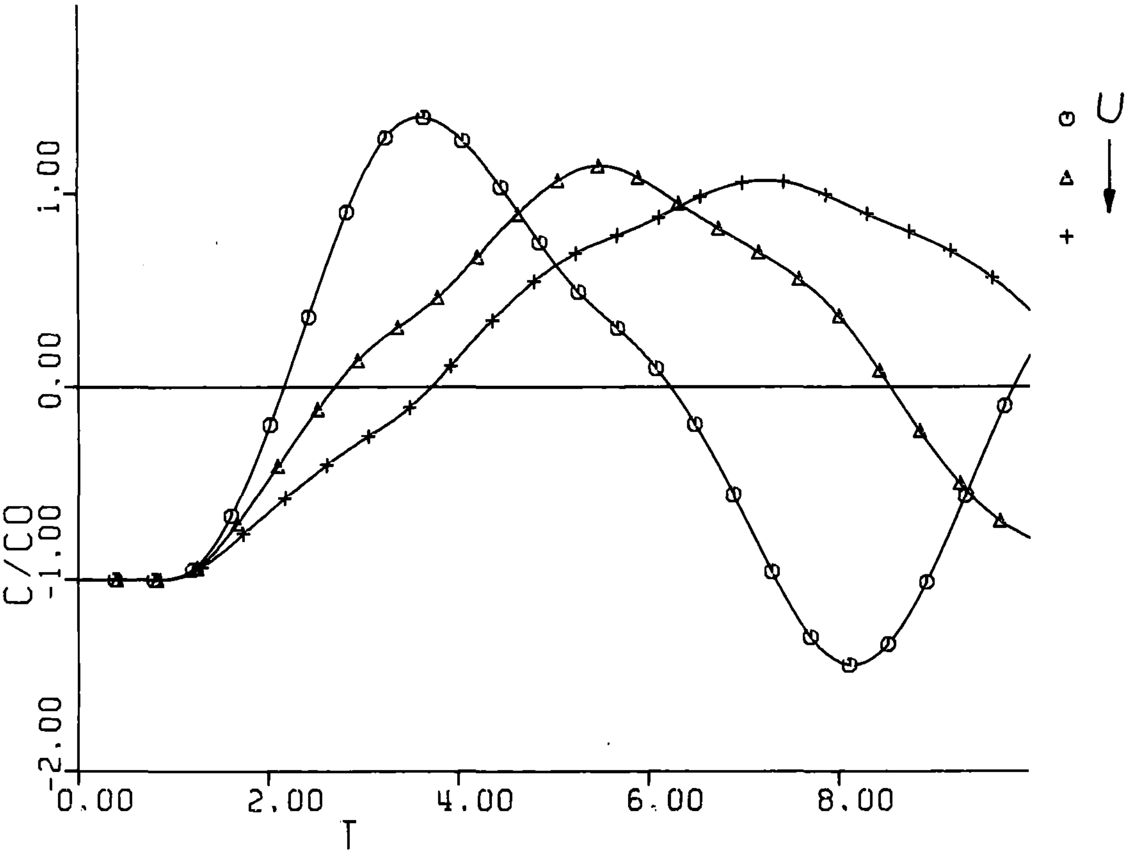


Fig. 13: Deformation versus time, same as Fig. 9 but fully implict first order time integration.

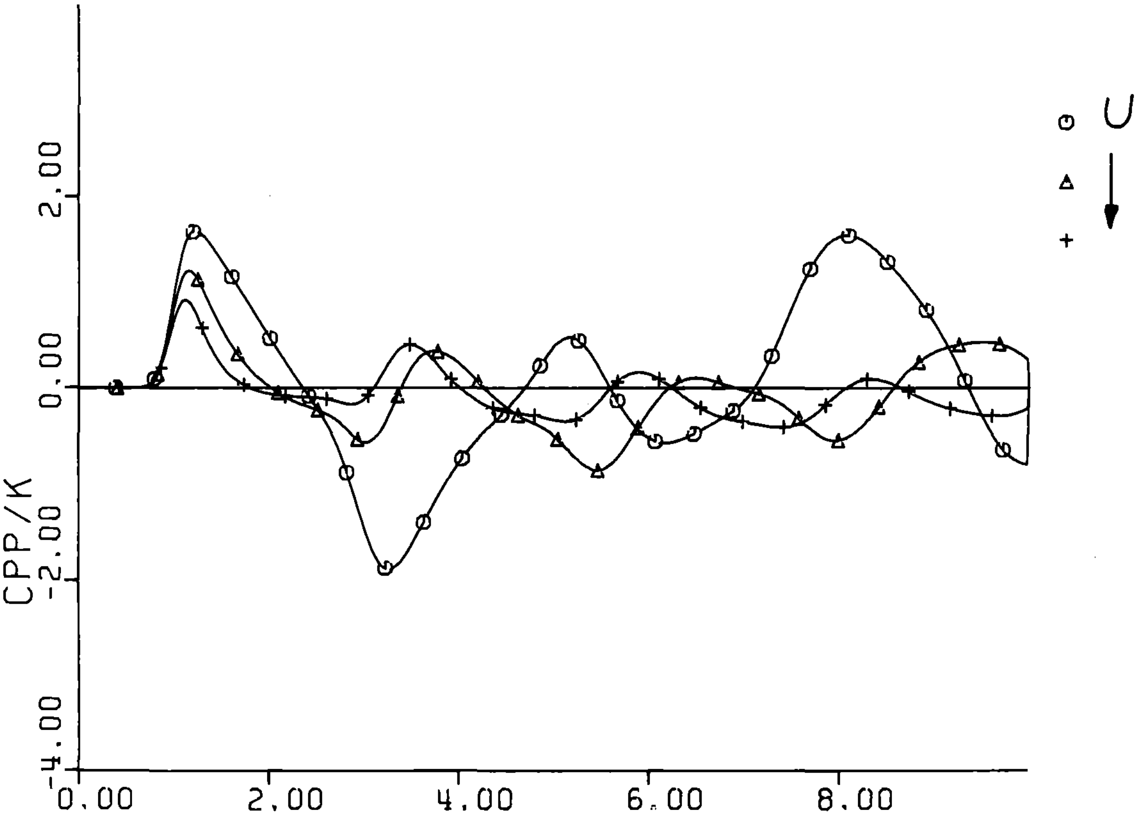


Fig. 14: Acceleration versus time, like Fig. 10, fully implicit scheme.

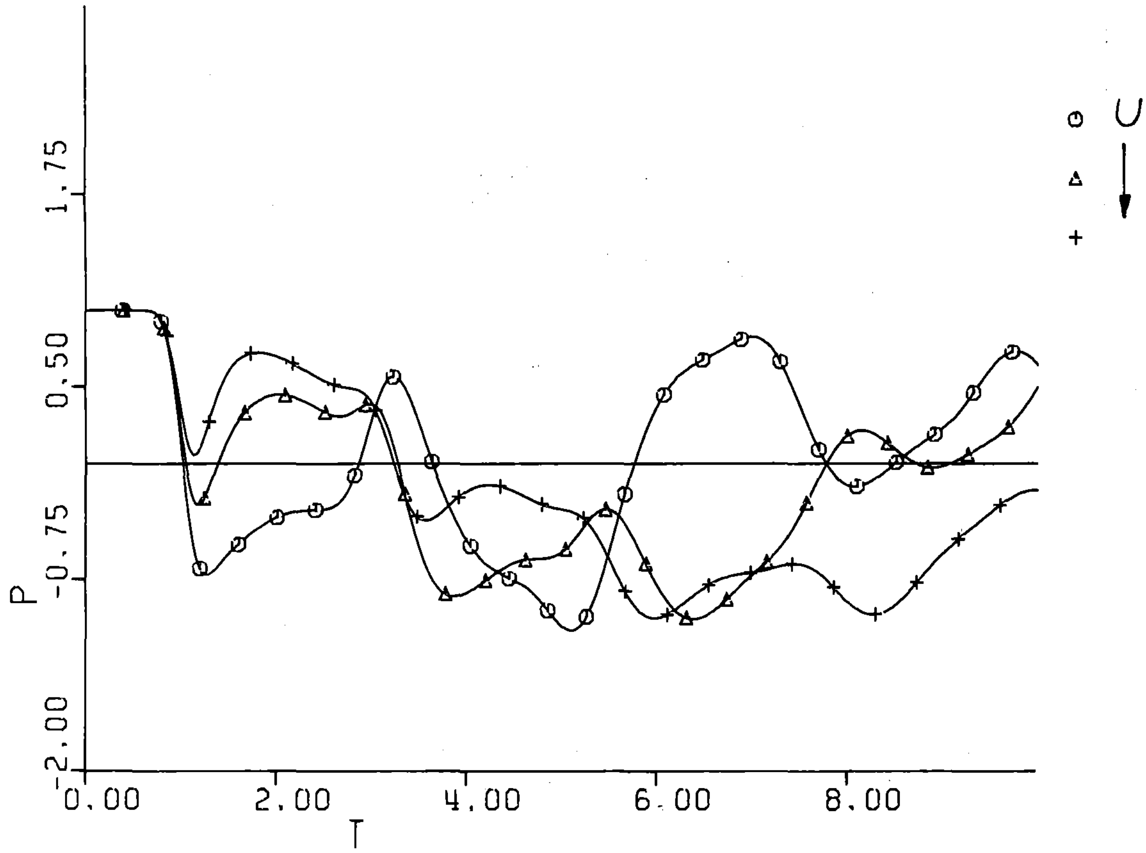


Fig. 15: Pressure at the piston versus time, like Fig. 11, fully implicit scheme.

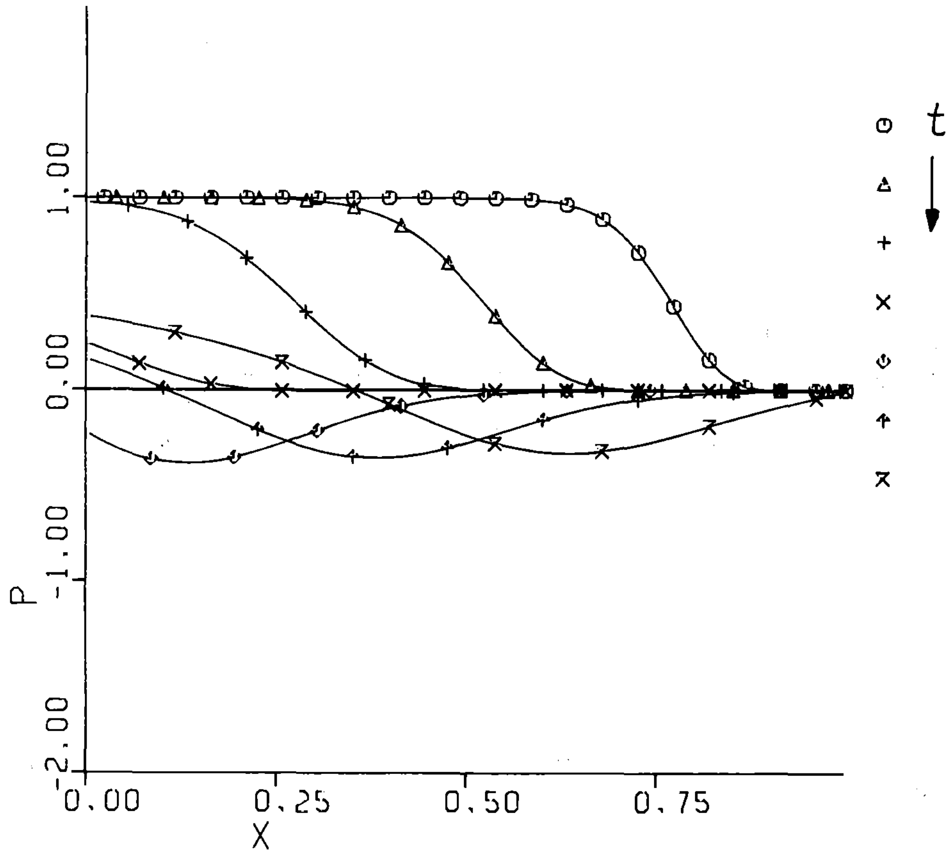


Fig. 16: Pressure profiles as in Fig. 12, fully implicit scheme; S=1, U=2.

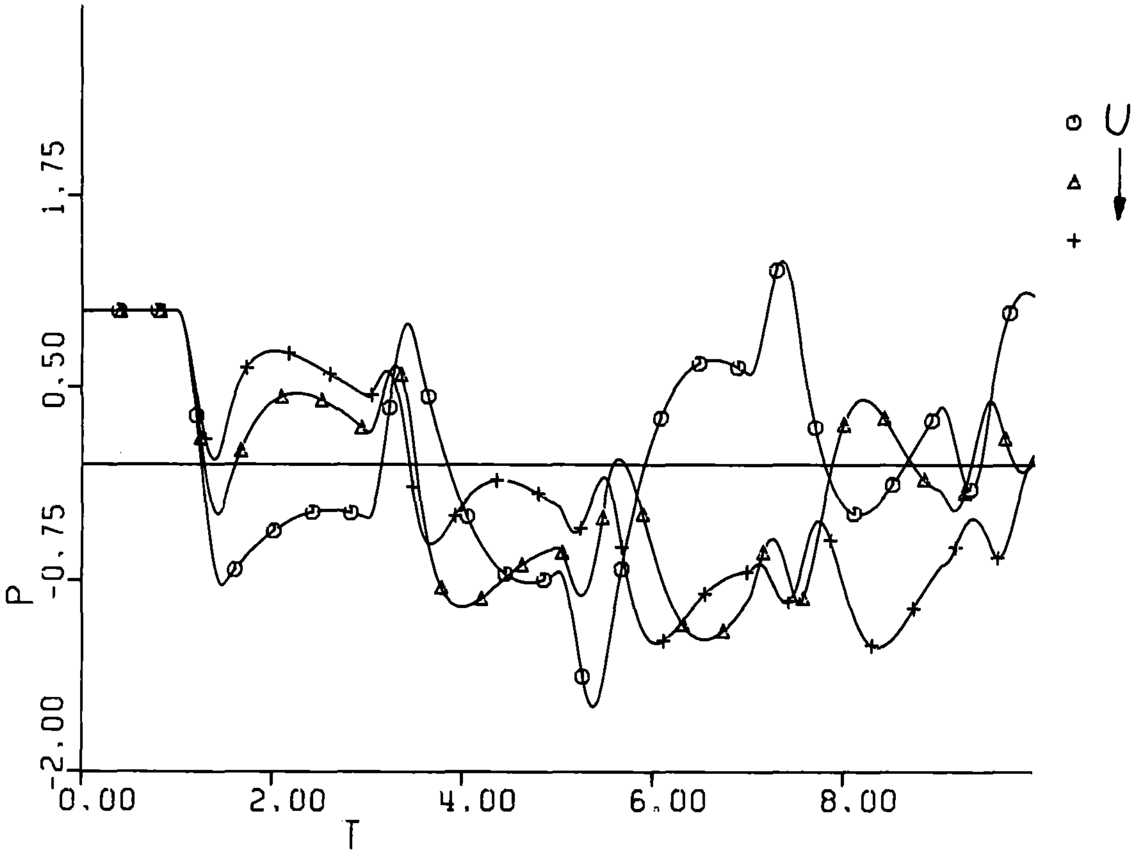


Fig. 17a: Pressure at the piston versus time for finite break time t_{break}=0.5, S=1, U= 0.5,2,4, "exact" solution

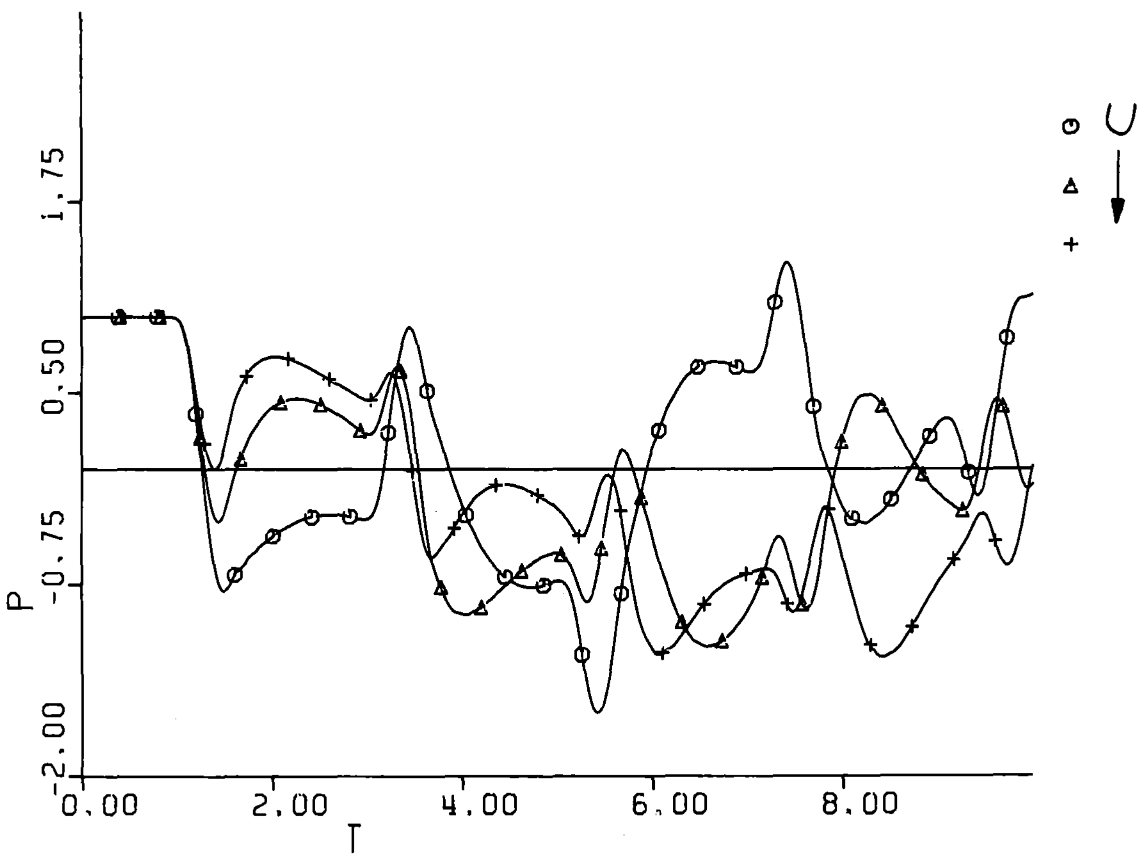


Fig. 17b: Same as Fig. 17a for finite difference solution.

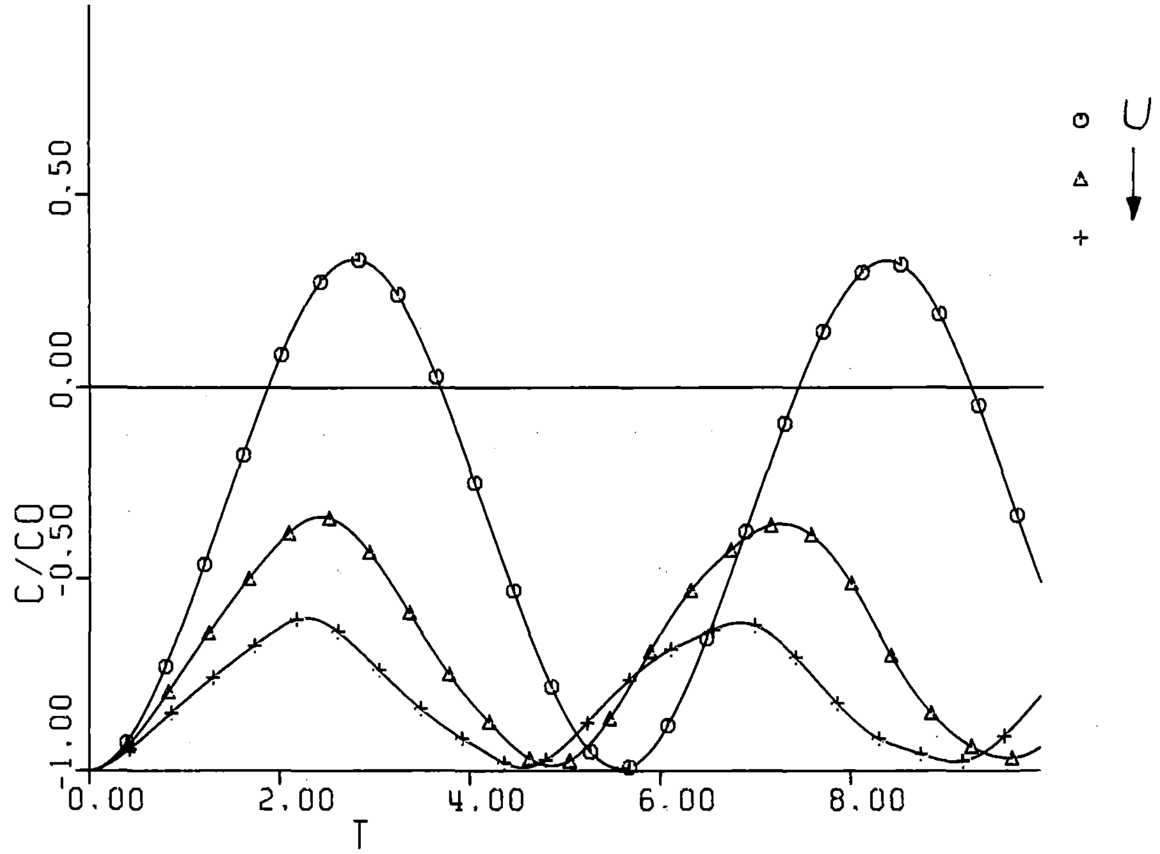


Fig. 18: Snapback - Deformation versus time, "exact" solution; S=1, U=0.5, 2, 4.

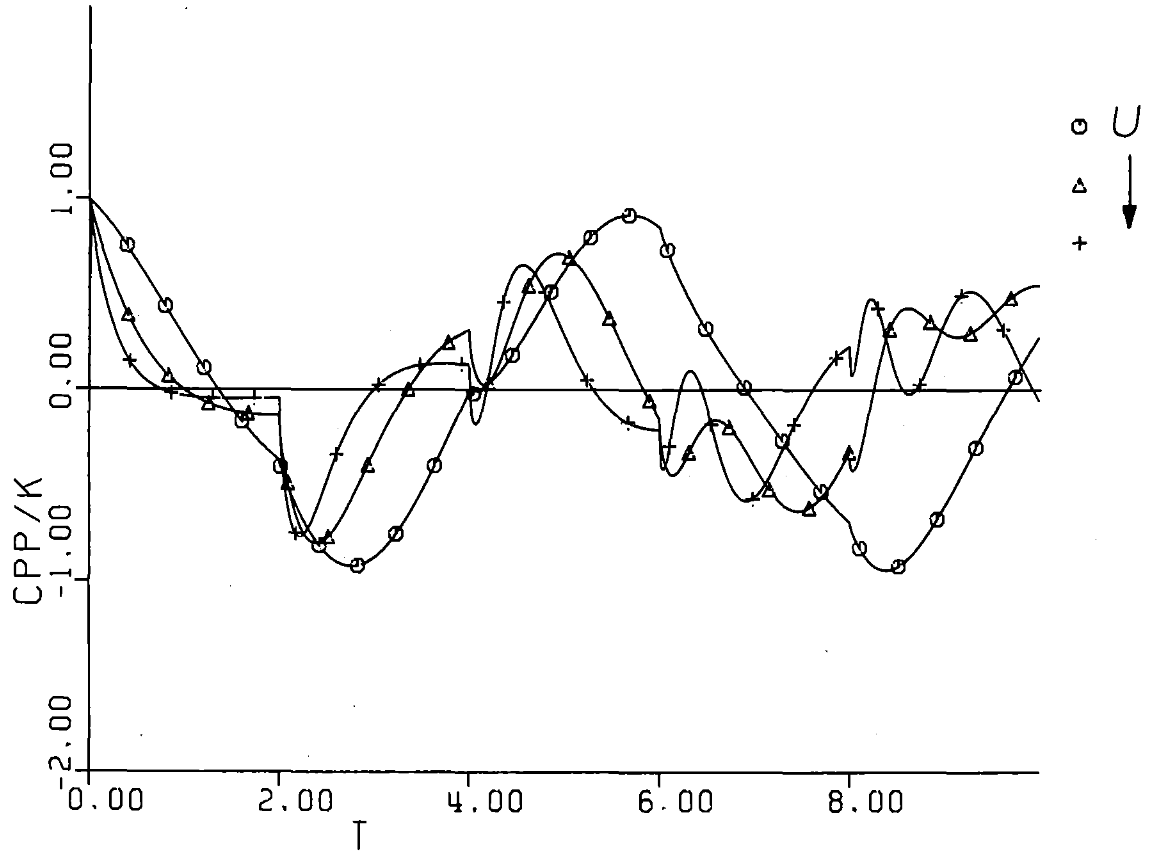
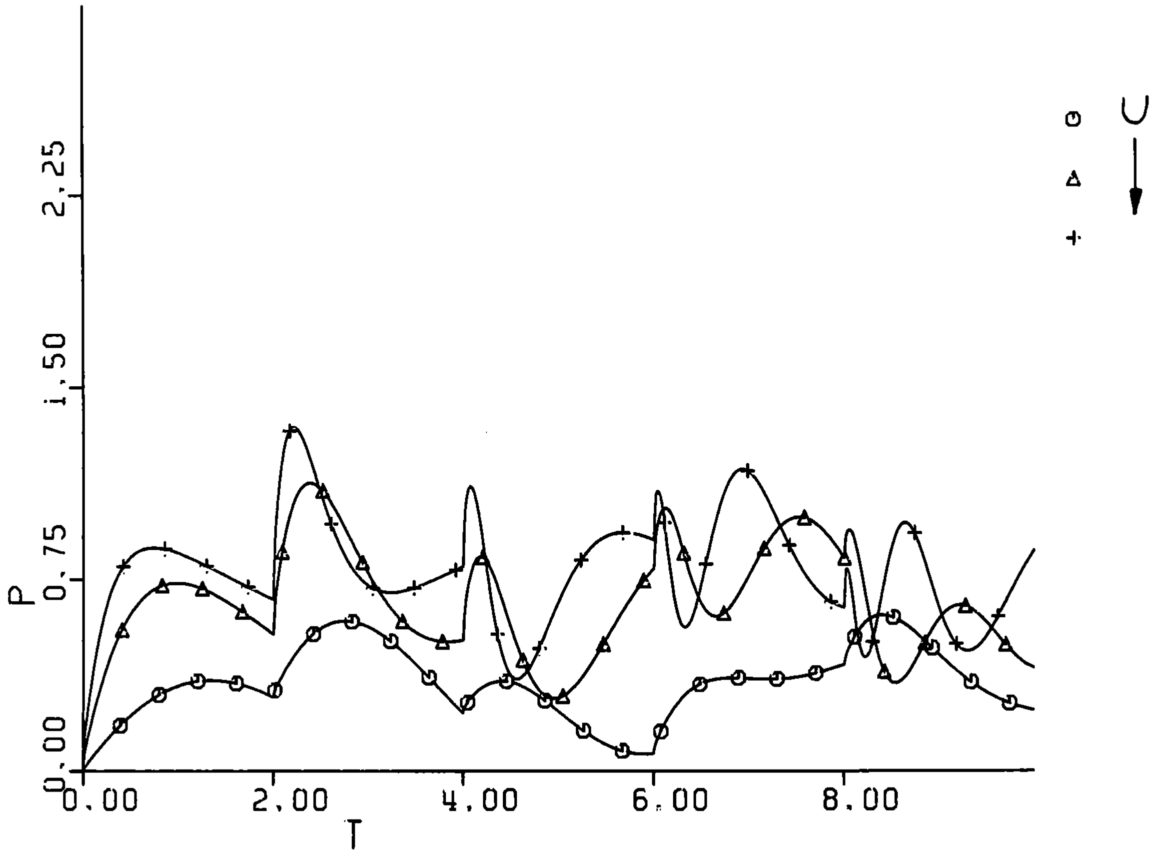
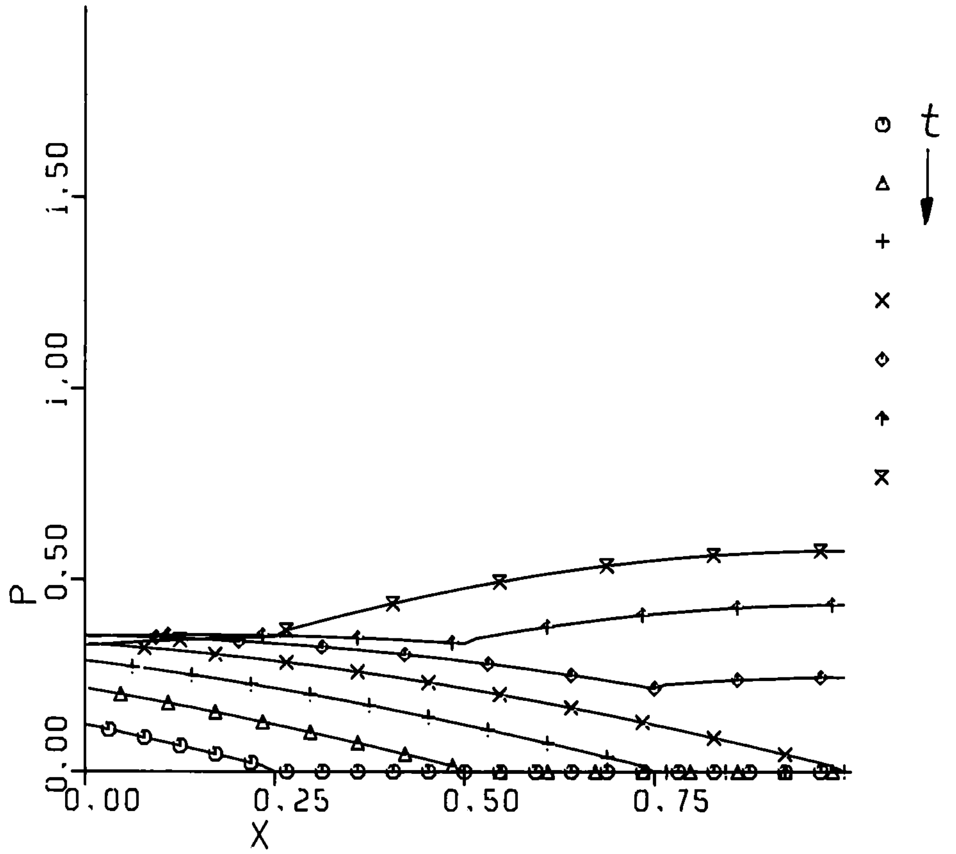


Fig. 19: Snapback - Acceleration versus time, "exact" solution; S=1, U=0.5, 2, 4.



<u>Fig. 20</u>: Snapback - Pressure at the piston versus time, "exact" solution; S=1, U=0.5, 2, 4.



<u>Fig. 21a</u>: Snapback - Pressure profiles at times 0.25, 0.5,...., 1.75, "exact" solution; S=1, U=0.5.

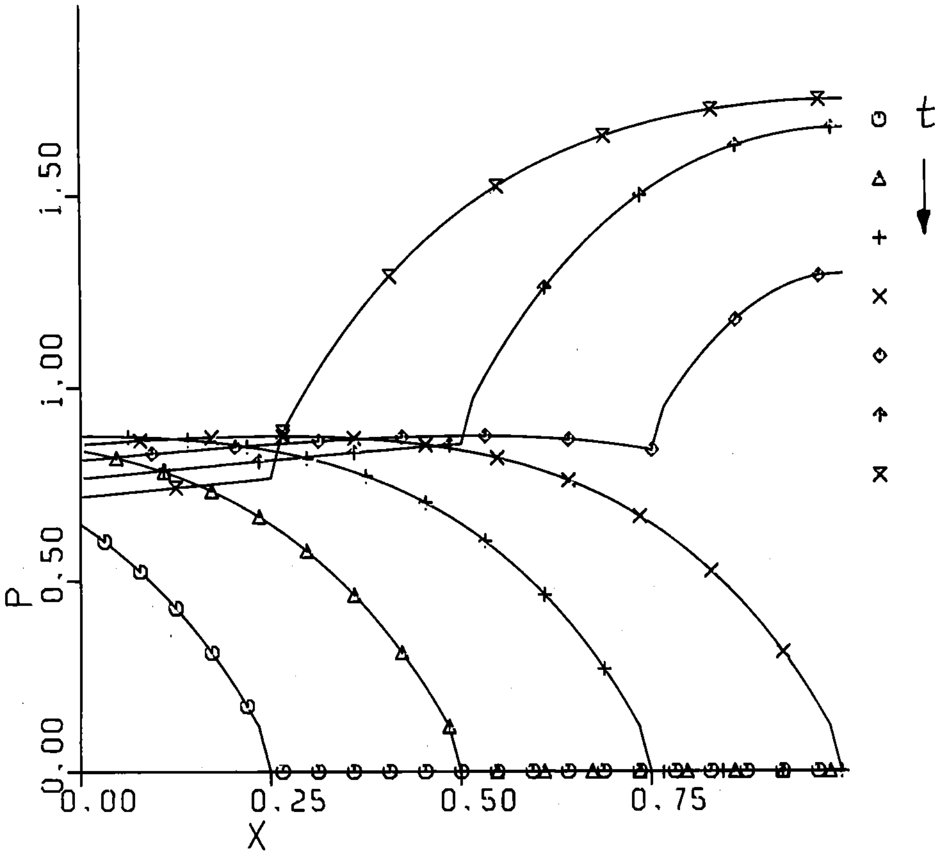


Fig. 21b: The same for U=4.

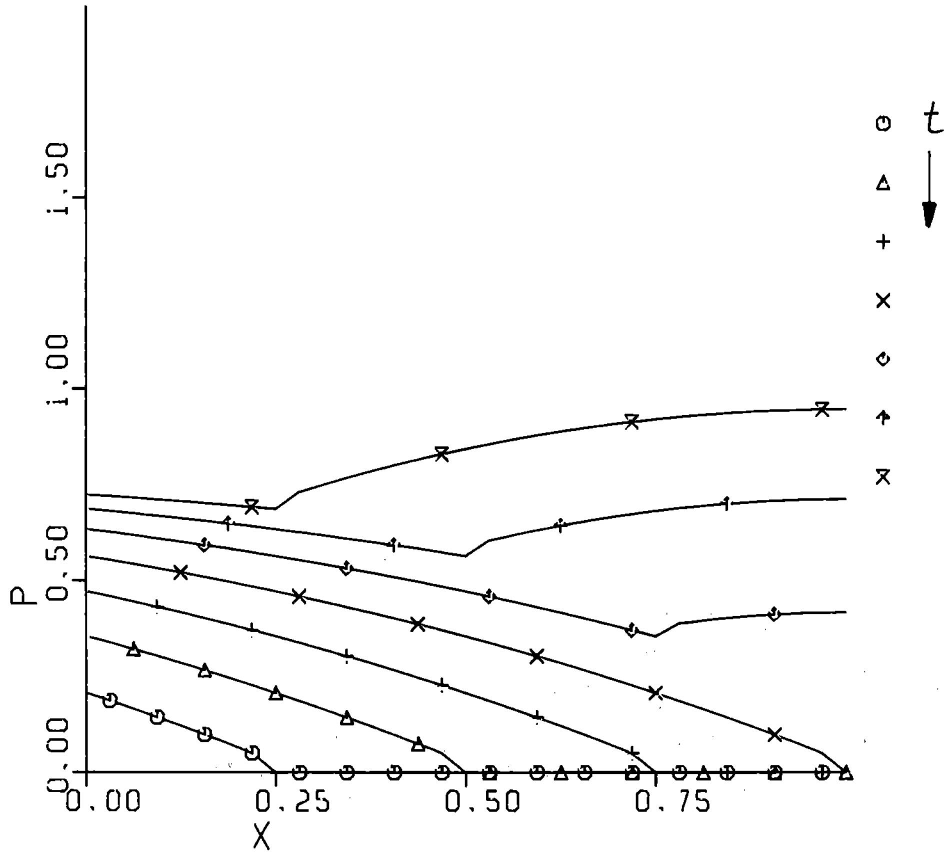
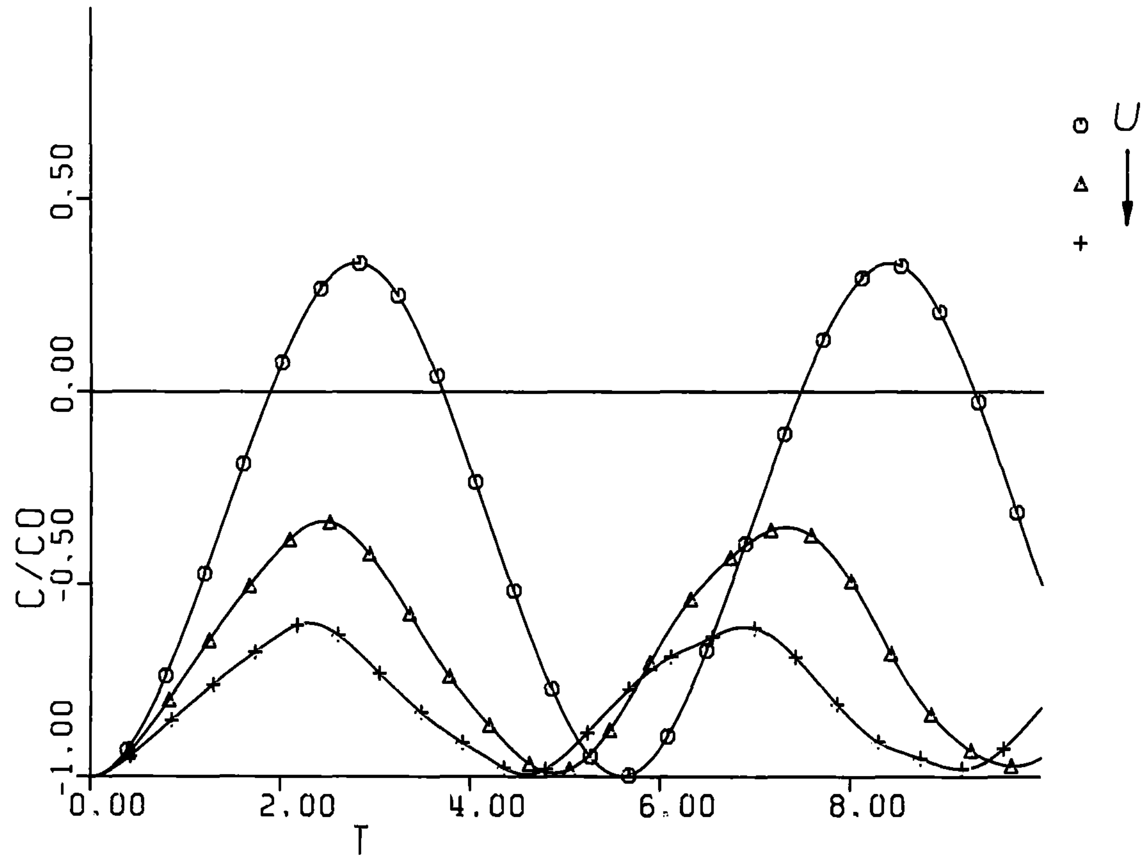
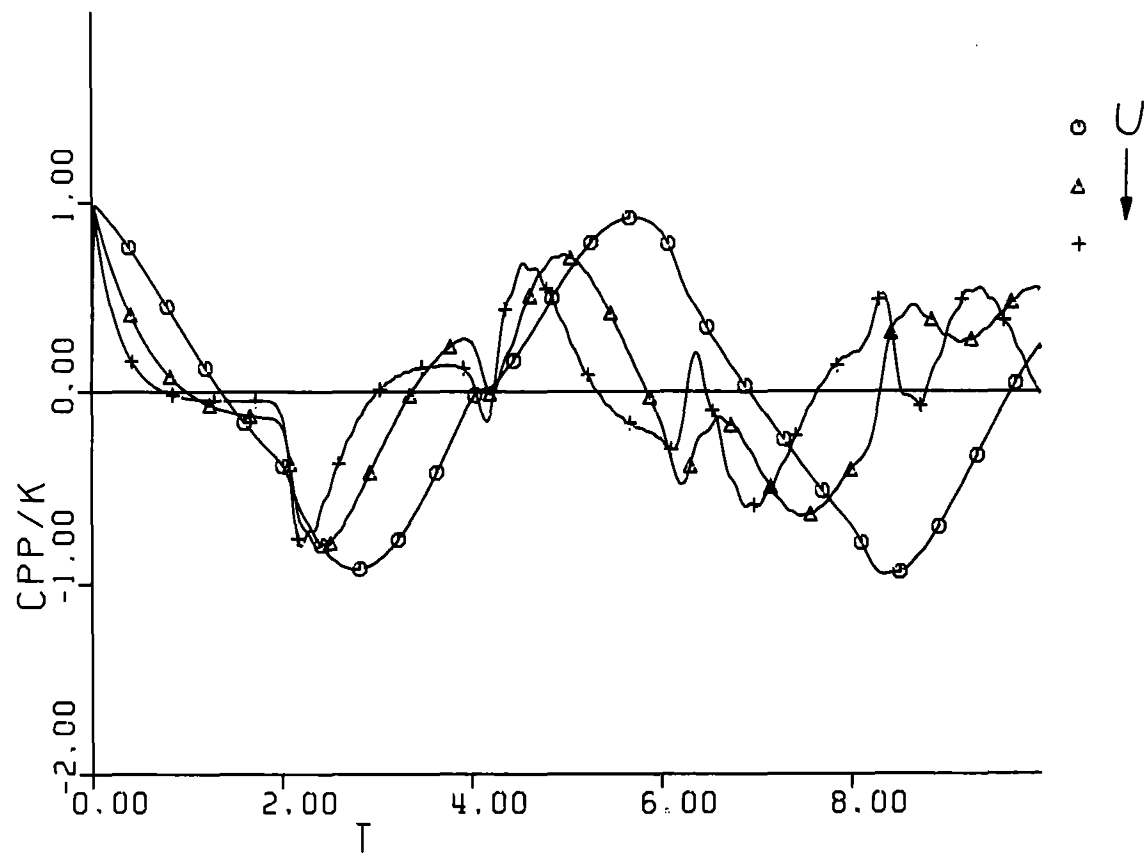


Fig. 21c: The same for U=0.835, S=0.359 (HDR).



<u>Fig. 22</u>: Snapback - Deformation versus time, finite difference solution; S=1, U=0.5, 2, 4; N=64.



<u>Fig. 23</u>: Snapback - Acceleration versus time, finite difference solution; S=1, U=0.5, 2, 4; N=64.

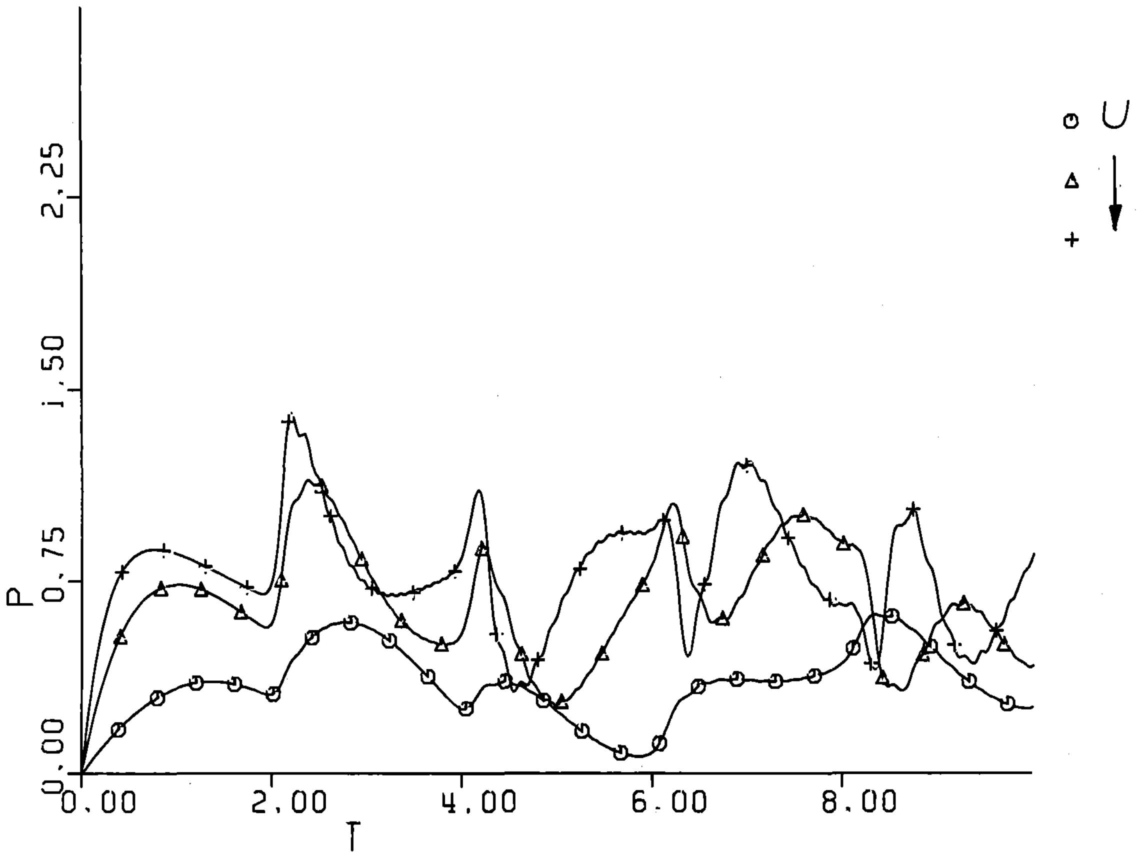


Fig. 24: Snapback - Pressure at the piston versus time, finite difference solution; S=1, U=0.5, 2, 4; N=64.

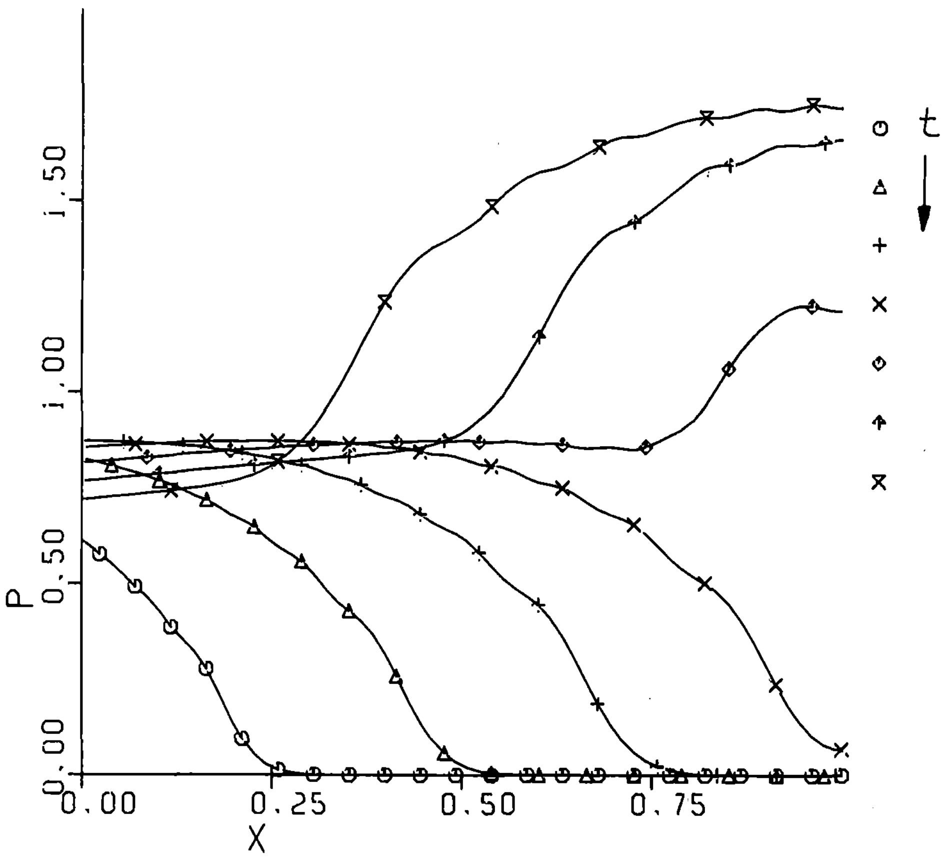


Fig. 25: Snapback - Pressure profiles at times 0.25, 0.5,...,1.75, finite difference solution; S=1, U=4; N=64.

4. PERIODIC CASES

4.1 General Solution

We use the harmonic ansatz $(i = \sqrt{-1})$

$$c(t) = c\, e^{i\omega t}, \quad p(x,t) = p(x)\, e^{i\omega t} \tag{4-1}$$

The equations (1-7, 2-8, 2-12, 2-13) give then

$$-\omega^2 c + S^2 c = -K\, p/w \tag{4-2}$$

$$-\omega^2 p - p'' = 0 \qquad \left(p' \equiv \frac{\partial p}{\partial x}\right) \tag{4-3}$$

$$p'(0) = \frac{\omega^2}{M^2}\, c \tag{4-4}$$

$$\alpha\, p(1) + (1-\alpha)\, p'(1) = 0 \tag{4-5}$$

From (4-2) we eliminate c:

$$c = \frac{K\, p(0)}{\omega^2 - S^2}$$

For p(x) we use the ansatz

$$p(x) = \frac{p_0}{(1+B^2)^{1/2}} \left(\cos(\omega x) + B \sin(\omega x)\right) \tag{4-6}$$

where p_0 is an arbitrary amplitude and B and integration constant. From the boundary condition at x=1 we obtain

$$B = \frac{(1-\alpha)\omega \sin\omega - \alpha\cos\omega}{(1-\alpha)\omega\cos\omega + \alpha\sin\omega}. \tag{4-7}$$

From the boundary condition at x=0 we get the characteristic equation

$$G(\omega) = \left[(1-\alpha)\omega\sin\omega - \alpha\cos\omega\right](\omega^2 - S^2) - U\omega\left[(1-\alpha)\omega\cos\omega + \alpha\sin\omega\right] = 0, \tag{4-8}$$

the roots ω_i of which are the eigenfrequencies of the system. Obviously an infinite number of eigenfrequencies exists.

In the special case 1 (open end), $\alpha = 1$, we have

$$G_1(\omega) = \cos\omega\,(\omega^2 - S^2) + U\omega \sin\omega = 0. \qquad (4\text{-}9)$$

In the special case 2 (closed end), $\alpha = 0$, we have to find the roots of

$$G_2(\omega) = \sin\omega\,(\omega^2 - S^2) - U\omega\cos\omega = 0 \qquad (4\text{-}10)$$

In both cases we get as expected $\omega^2 = S^2$ if $U = 0$ (no fluid mass).

These relations are programmed in the procedure PERI, see Appendix 6.

4.2 Discussion

The first five eigenfrequencies $\omega^{(i)}$, $i=1,2,..,5$ and eigensolutions $p^{(i)}(x)$, $c^{(i)}$ have been computed for the cases $M=0.1$, $S=1$, $U=0.5,2,4$ and $S=\pi/2$, $U=0.05$, and are given in Table II and Figs. 26 and 27.

For the lowest eigenvalue, the pressure profile is virtually linear in case 1 and quadratic in case 2. At the left boundary the pressure gradient corresponds to the structural acceleration, at the right boundary the pressure obviously satisfies the boundary conditions, zero value in case 1, zero gradient in case 2.

The second highest eigenfrequency in case 1 is similar to the first one except for the opposite sign of the gradient (and acceleration) at the piston. Thus, the first eigenfrequency corresponds to the case where the structure and the fluid oscillate in the same direction whereas the second one belongs to the case of fluid and structure oscillating in opposite directions. In the first mode the eigenfrequency is lower than in vacuo (S) due to the added fluid inertia, in the second it is larger due to the added fluid stiffness.

In case 2 only one mode appears which results from the fluid-structure coupling.

The other eigenfrequencies belong to the standing pressure waves of which there exist infinitely many. The fact that the eigenfrequencies for these standing waves are higher than those for the main structural oscillations is incidentally a consequence of the rather small value S = 1.

In the case of the lowest eigenfrequency being equal to the lowest eigenfrequency of the standing pressure waves, a double eigenvalue appears and two eigensolutions coincide, this can be seen from Fig. 26d, where the lowest eigenvalue is close to $\pi/2$.

The resultant eigenfrequencies can be compared with those predictable by approximating formulae. By replacing space derivates by division by lengths one obtains, see Appendix 3:

<u>Case 1:</u>

$$\omega_{1/2}^2 = \frac{1}{2}\,\frac{1}{1+\alpha U}\left\{\left(1+S^2+(1+\alpha)U\right) \pm \sqrt{\left(1+S^2+(1+\alpha)U\right)^2 - 4S^2(1+\alpha U)}\right\} \qquad (4\text{-}11)$$

<u>Case 2:</u>

$$\omega = \sqrt{\frac{S^2+U}{1+\alpha U}} \qquad (4\text{-}12)$$

The coefficient α accounts for that fraction of the fluid mass which effectively adds to the structural mass. Different proposals are in use for this fraction. Dienes et al. [5] use α=0 in the SOLA-FLX code; Schlechtendahl [6] uses $\alpha=1/3$ in STRUYA-computations. If the code FLUX2 is applied with only one radial mesh cell in the downcomer model then this corresponds to $\alpha=1/2$.

The resultant eigenfrequencies obtained from these approximative relations are tabulated in Table II with $\alpha=0$. Also listed are those values of α for which Eqs. (4-11,12) give the same result as the exact theory. Obviously, the correct value lies between 1/3 and 1/2.

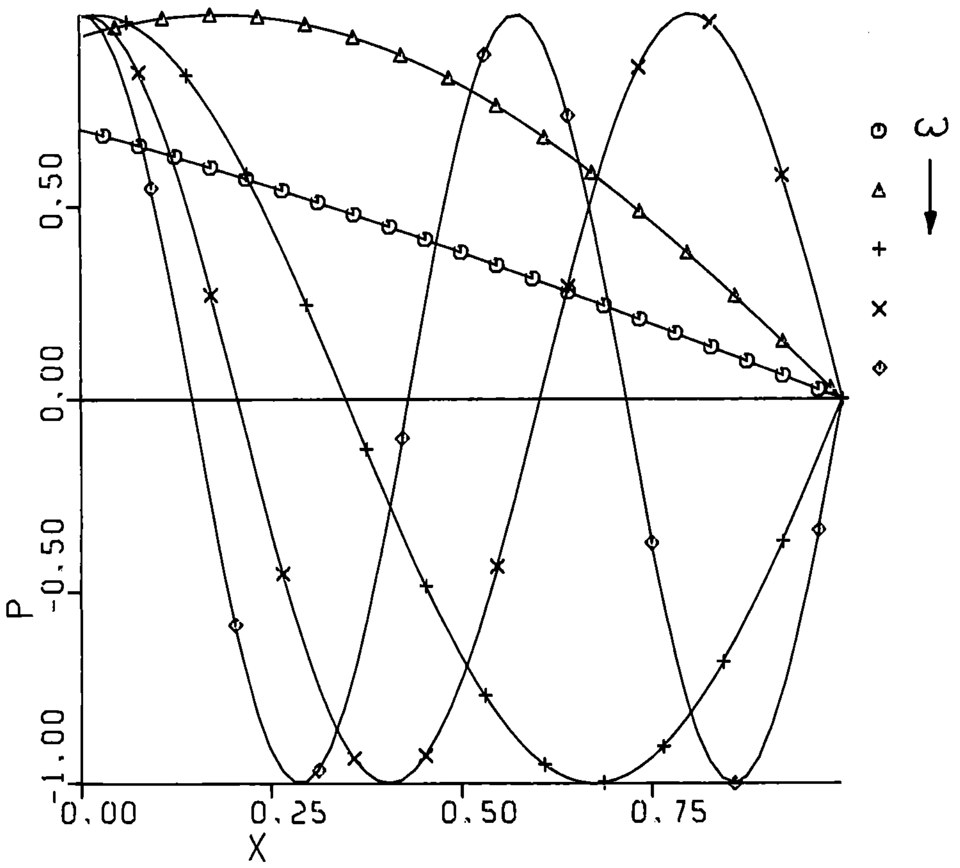


Fig. 26a: Open ended pipe - eigensolutions for the pressure profile for the first five eigenfrequencies; S=1, U=0.5.

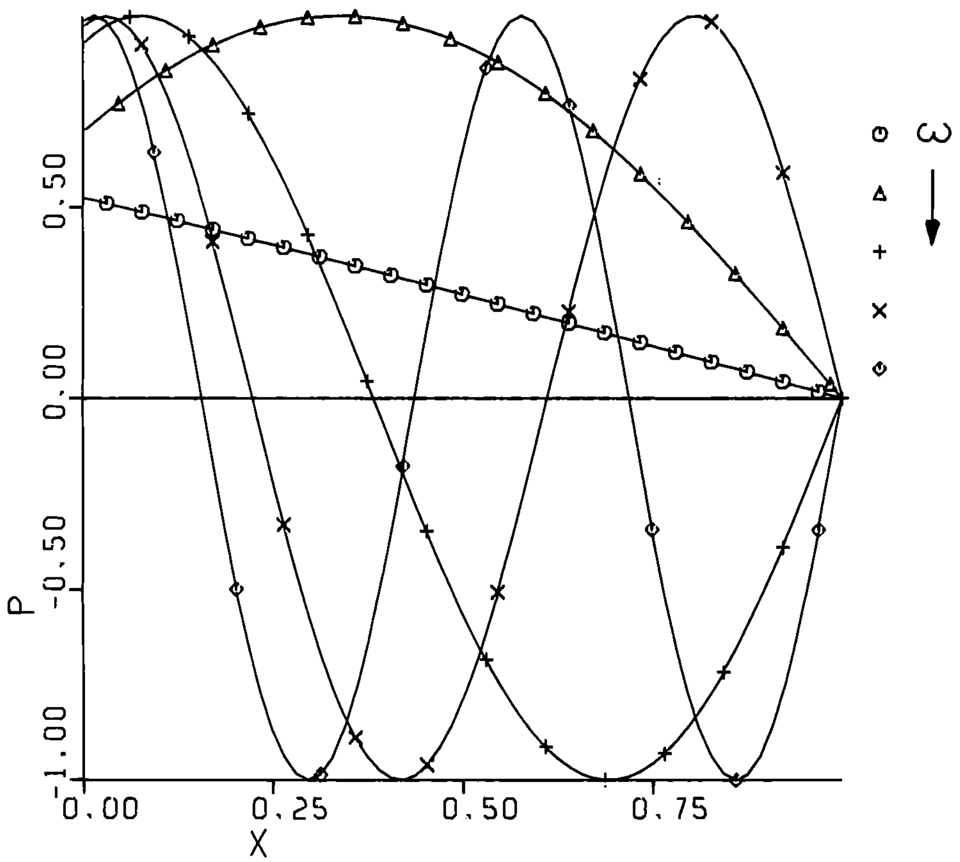


Fig. 26b: Same for S=1, U=2.

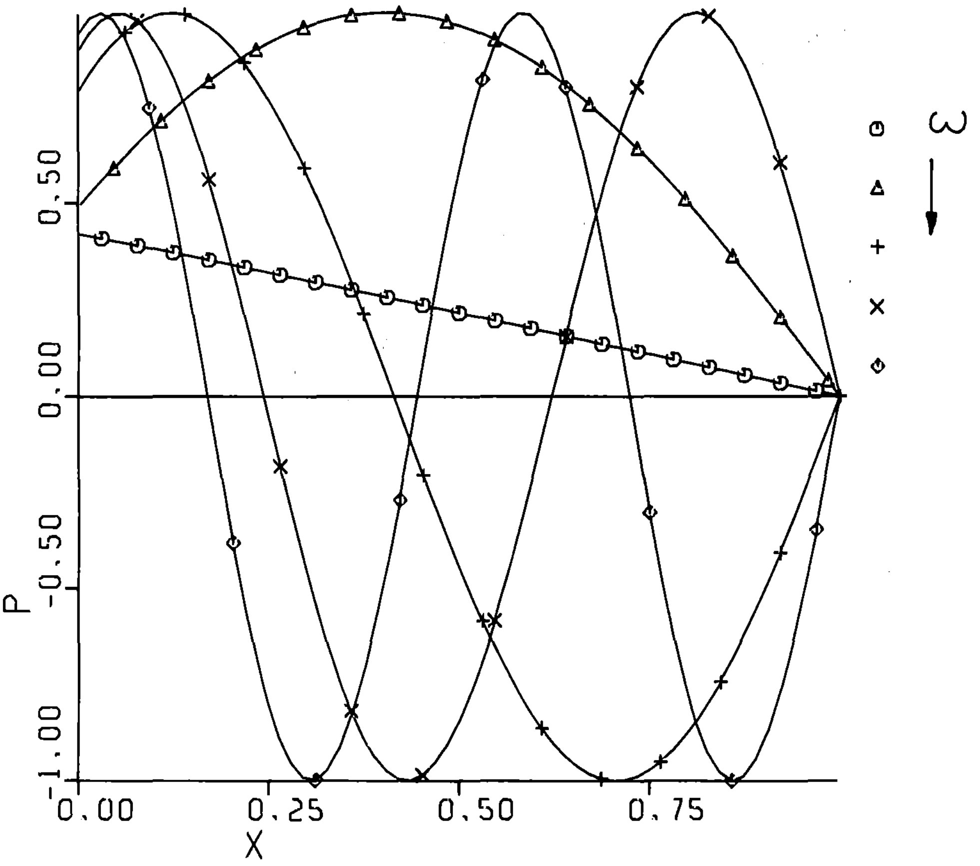


Fig. 26c: Same for S=1, U=4.

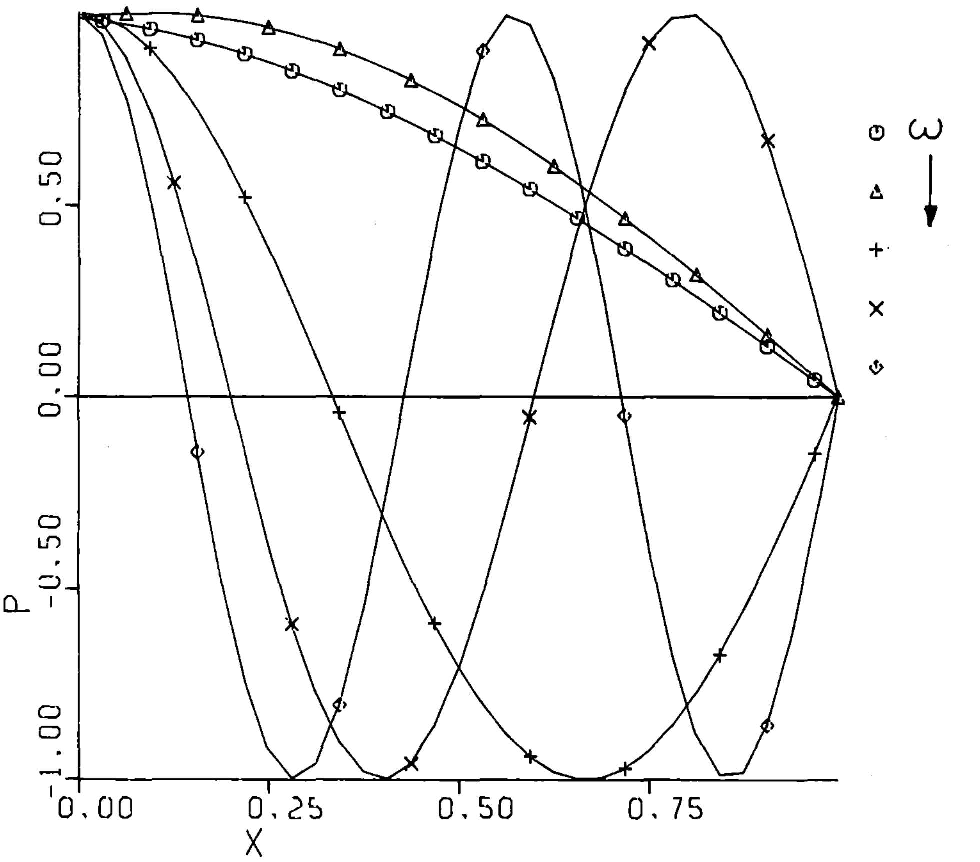


Fig. 26d: Same for S=π/2, U=0.5, note the closeness between the first and the second eigensolution.

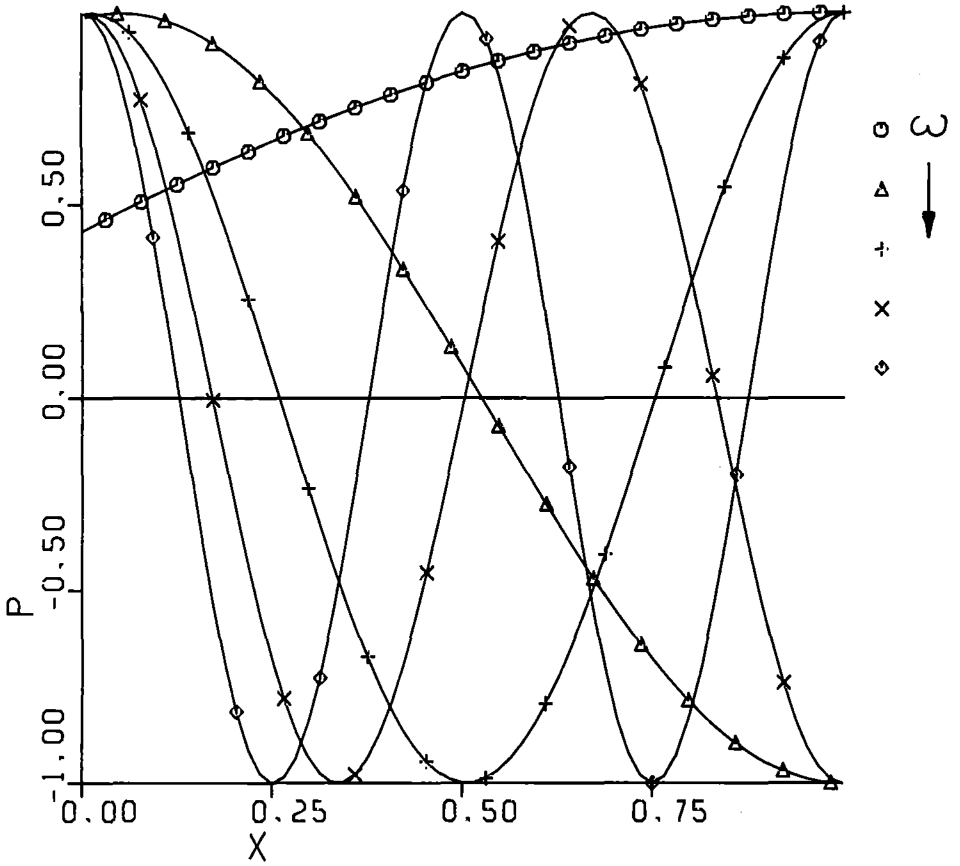


Fig. 27a: Closed pipe - eigensolutions for the pressure profiles for the first five eigenfrequencies, S=1, U=0.5.

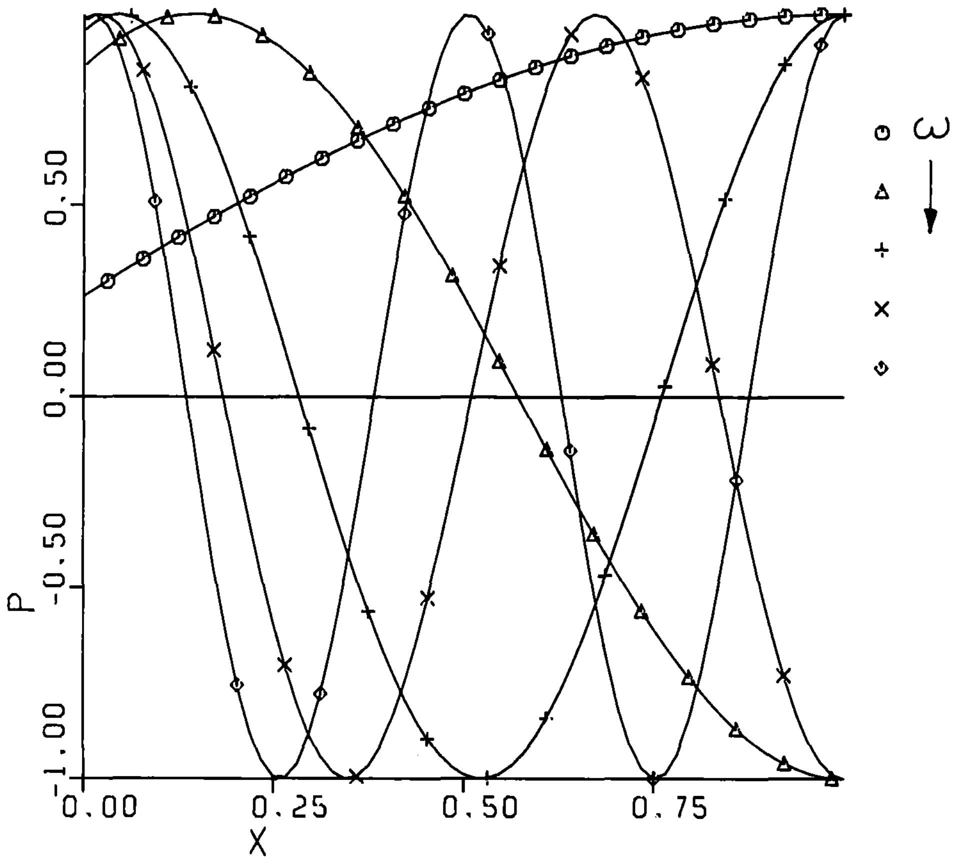


Fig. 27b: Same for S=1, U=2.

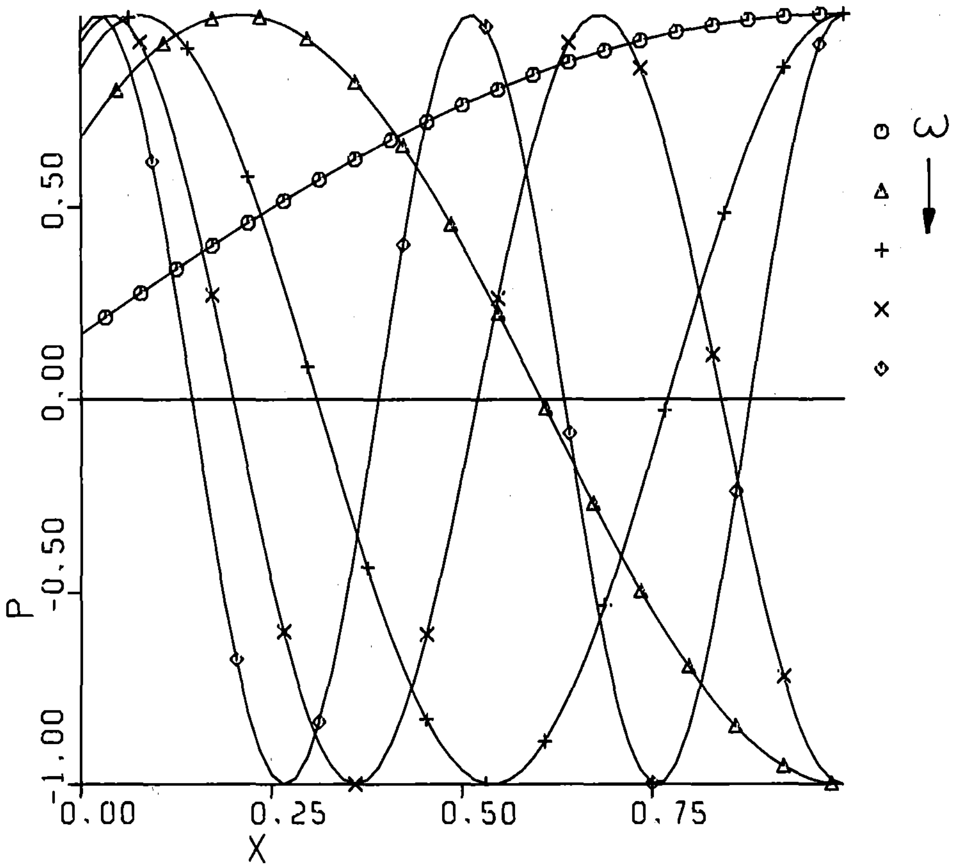


Fig. 27c: Same for S=1, U=4.

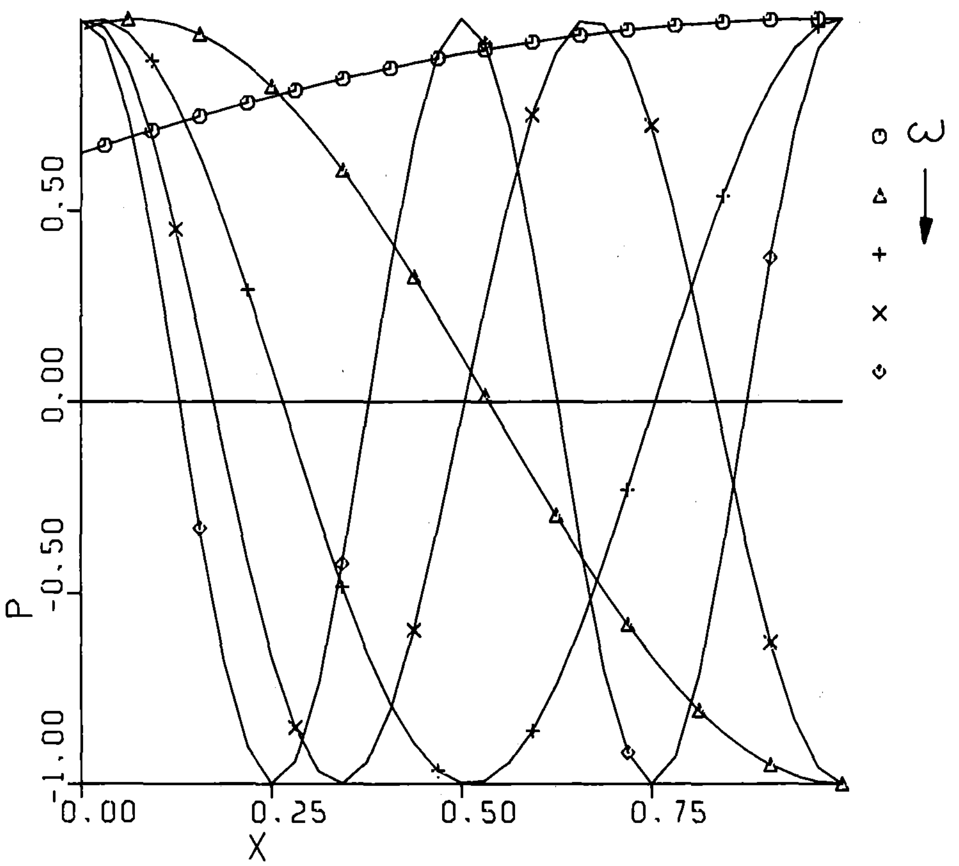


Fig. 27d: Same for S=0.359, U=0.835 (HDR).

5. DISCUSSION FOR CASE OF THE HDR-BLOWDOWN - AND SNAPBACK-EXPERIMENTS

5.1 Case 1: Broken Membrane

As a very crude approximation one can use the present model as representative for the blowdown-pipe of the HDR blowdown-experiments; the piston would model the core barrel. If we assume that the core barrel flexibility is controlled solely by the membrane stresses according to a simple pipe model, than one can assume that the following parameter values are valid:

$\hat{L} = 1.1$ m, $\hat{\rho} = 780$ kg/m^3, $\hat{a} = 1088$ m/s,
$\hat{m} = 179.4$ kg/m^2, $\hat{s} = 2.25 \times 10^9$ N/m^3
$\Delta\hat{p} = 5.665 \times 10^6$ N/m^2

This corresponds to

$M = 0.0783, \quad S = 3.58, \quad U = 4.78$

$$K = M^2 U = 0.0293$$
$$R = 2\, S/U = 1.498$$
$$E = S^2/U = 2.68$$

Figs. 28 to 30 show the resultant motions in non-dimensional form. The results are obtained from the quasi exact solution method with 32 time steps per unit. This might be not enough. Some errors become obvious from the change in the shock wave size.

In absolute values we have

$$\hat{c}_o = \hat{L}\; K/S^2 = 2.5 \text{ mm}$$

$$\hat{\ddot{c}}_{max} = K \hat{a}^2/\hat{L} = 31\,531 \text{ m/s}^2$$

The characteristic time during which this extreme acceleration persists is of the order 2/U or in dimensionful units

$$\hat{t}_a = (2/U)\; \hat{L}/\hat{a} \tag{5-1}$$
$$= 0.42 \text{ ms}$$

If a finite break time $\hat{t}_{break}$ = 1 ms corresponding to

$$t_{break} = 0.989$$

is applied, the maximum acceleration value is reduced to

$$\ddot{c}_{max} = 11971 \ m/s^2$$

In reality the acceleration will be even smaller by approximately a factor of ten [8] due to the geometric expansion between the pipe and the core barrel near the nozzle.

5.2 Case 2: Snapback-Experiment

With respect to the snapback-experiment [2], the present model can be assumed to be representative for the gap between core barrel and pressure vessel in the downcomer. The following parameter values are characteristic for this case: $\hat{L}$ = 0.15 m, $\hat{\rho}$ = 999.2 kg/m^3, $\hat{a}$ = 1480 m/s, $\hat{m}$ = 179.4 kg/m^2, $\hat{s}$ = 2.25x10^9 N/m^3, $\Delta\hat{p} = \hat{s} \cdot \hat{c}_o$ = 2.25 x 10^6 N/m^2, $\hat{c}_o$ = 1 mm

or M = 0.0321, S = 0.359, U = 0.835
K = 0.00086, R = 0.860, E = 0.154

Figs. 31 to 34 show the resultant motions in non-dimensional form. In absolute values we have

$$\ddot{\hat{c}}_{max} = K\hat{a}^2/L = 12560 \ m/s^2$$

$$\hat{t}_a = (2/U)\,\hat{L}/\hat{a} = 0.24 \ ms$$

The maximum pressure change is found to be 3.50 MPa. If the initial deformation would be of opposite sign, this pressure change can cause cavitation.

The first five eigenfrequencies are given in Table II. In vacuo the angular eigenfrequency would be equal to S = 0.359. We see that the coupled system has a larger eigenfrequency. So with respect to this type of coupling the fluid stiffness is more important than the fluid mass. The fraction of water which is added to the structural inertia amounts to a=0.351. If one uses the approximative equation (4-12) with a = 0 an error of 14 % is made with respect to the first eigenfrequency.

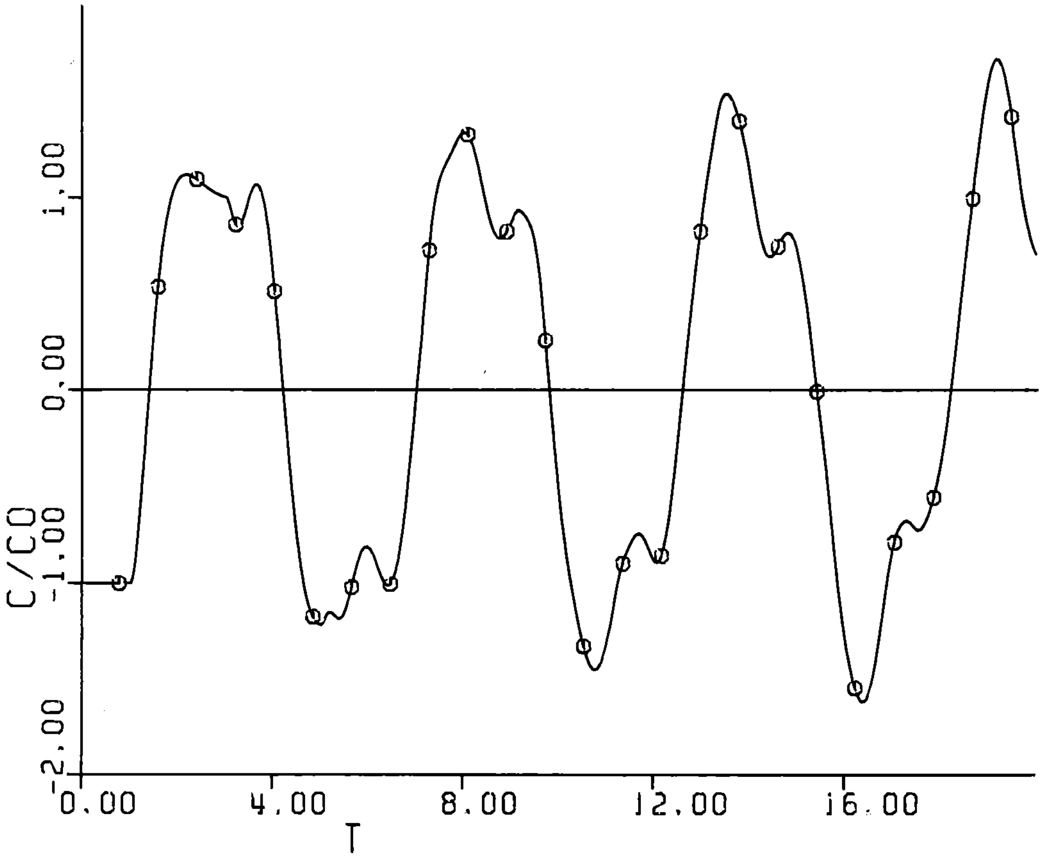


Fig. 28: HDR-Blowdown-Pipe - Deformation versus time, "exact" solution; S=3.58, U=4.78.

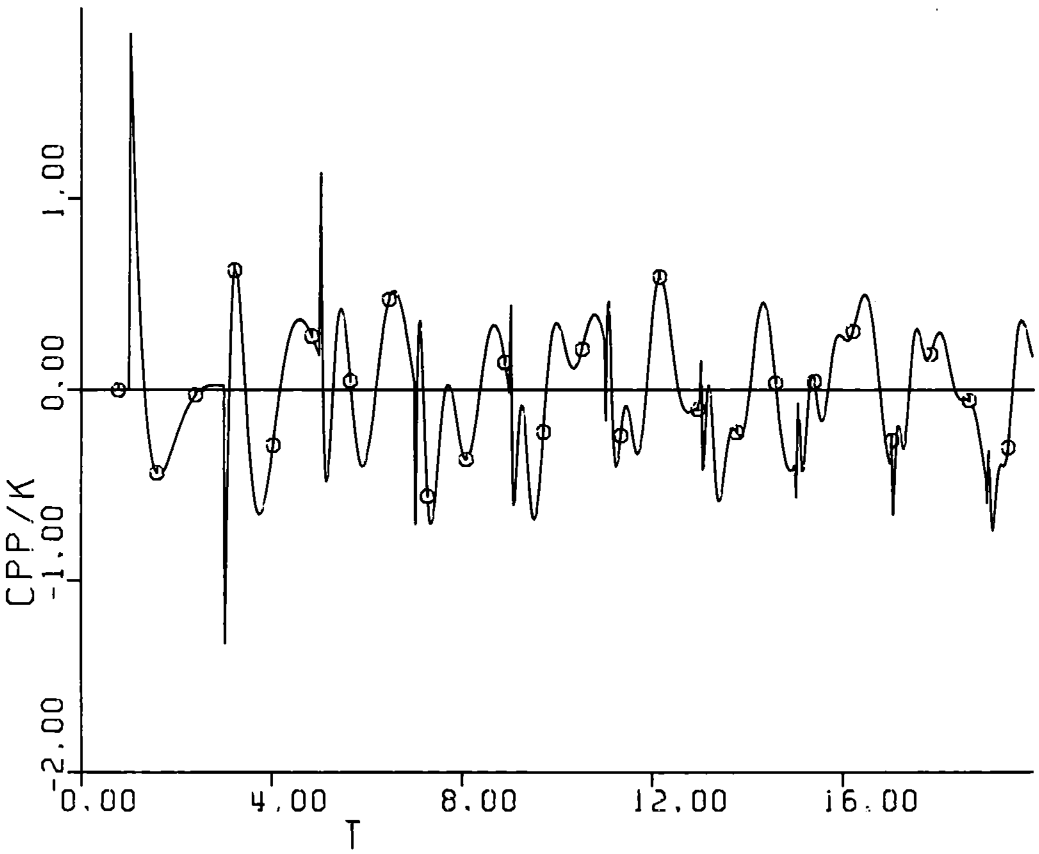


Fig. 29: Same as Fig. 28 for acceleration.

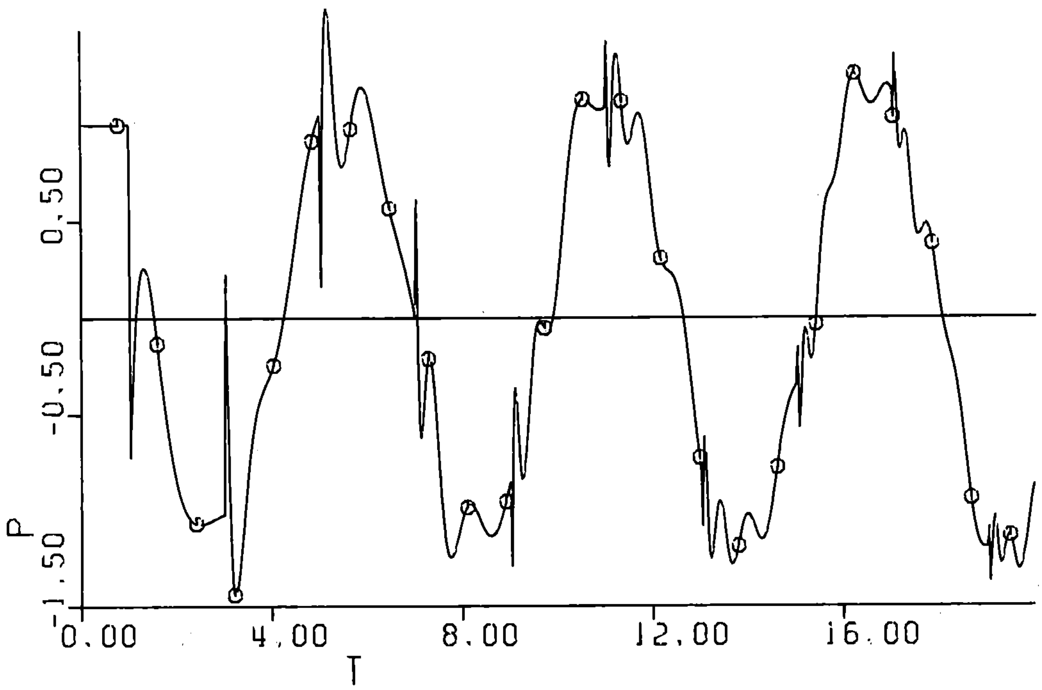


Fig. 30: Same as Fig. 28 for pressure at piston.

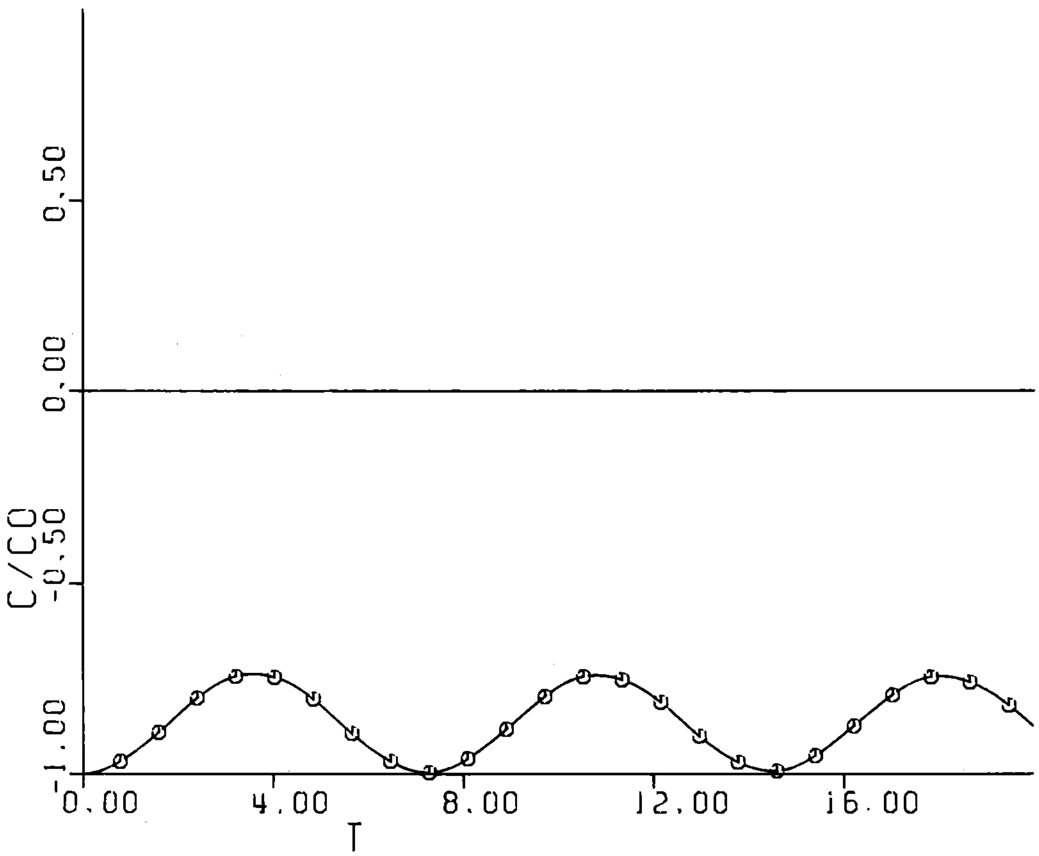


Fig. 31: HDR-Snapback - Deformation versus time, "exact" solution; S=0.359, U=0.835.

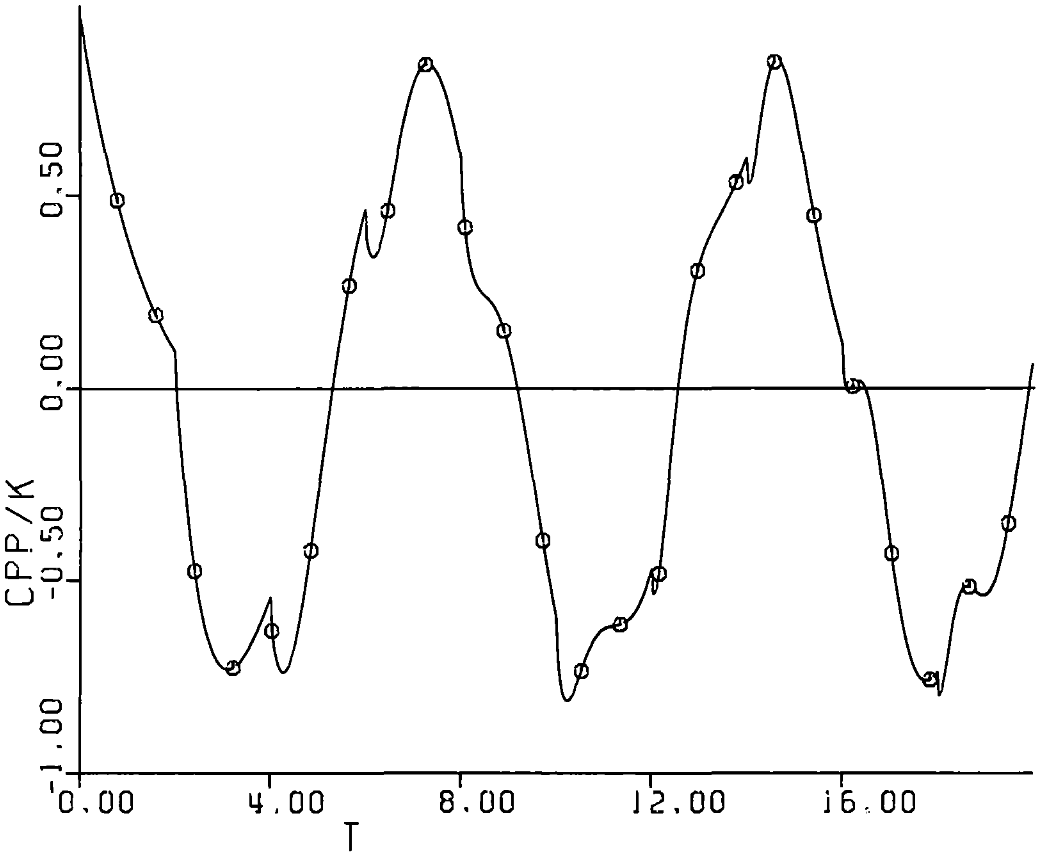


Fig. 32: Same as Fig. 31 for acceleration.

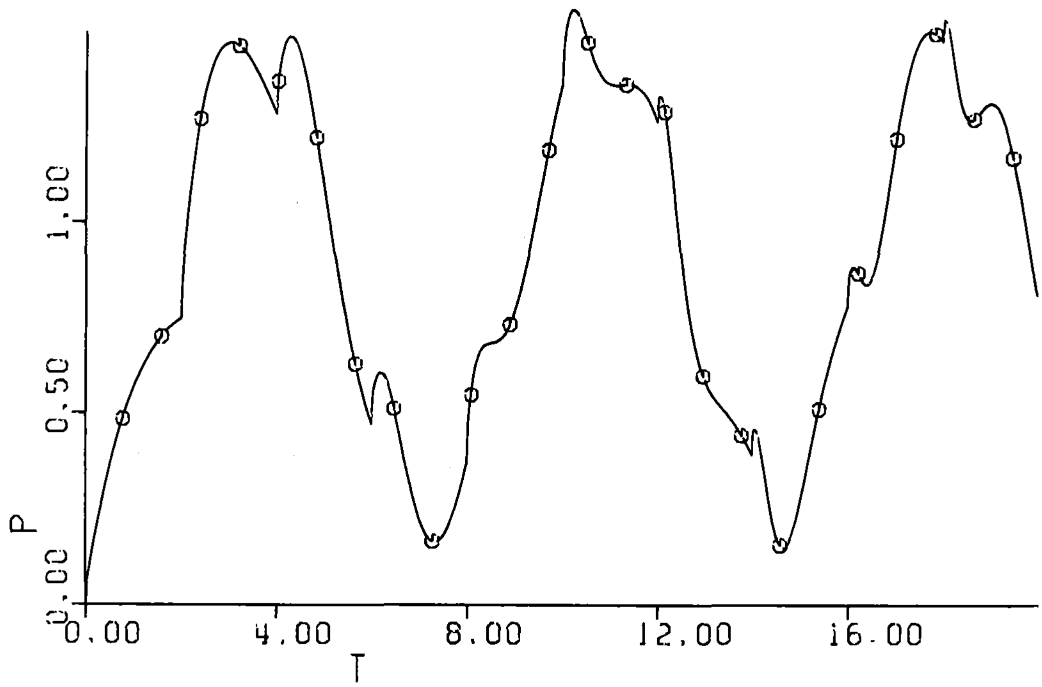


Fig. 33: Same as Fig. 31 for pressure at piston.

Table I: Erorrs of the quasi exact solution

S	Strouhal no.		1	1	1	3.58	0.359
U	mass ratio		0.5	2	4	4.78	0.835
N	number of intervals		64	64	64	32	32
$\varepsilon \cdot 10^2$	relative error in deformation	open pipe	0.97	0.57	0.34		
$\varepsilon \cdot 10^2$	in velocity	open pipe	0.004	0.007	0.016		
$\varepsilon \cdot 10^2$	in acceleration	open pipe	0.63	1.55	3.04		
$\varepsilon \cdot 10^2$	in deformation	closed pipe	0.56	0.29	0.17		
$\varepsilon \cdot 10^2$	in velocity	closed pipe	2.17	4.05	6.50		
$\varepsilon \cdot 10^2$	in acceleration	closed pipe	0.63	1.50	2.86		

Table II: Eigenfrequencies and relative amplitudes*)

Table IIa: open ended pipe

S Strouhal	1	1	1	$\pi/2$	3.58
U mass ratio	0.5	2	4	0.05	4.78
$\omega^{(1)}$ eigenfrequency	0.782	0.556	0.435	1.417	1.132
$c^{(1)} \cdot 10^3$ structural amplitude	-15.3	-9.07	-20.8	-1.08	-2.3
$\omega^{(2)}$	1.915	2.370	2.629	1.732	3.275
$c^{(2)} \cdot 10^3$	3.02	1.76	3.31	0.93	-1.86
$\omega^{(3)}$	4.820	5.100	5.370	4.724	5.665
$c^{(3)} \cdot 10^3$	0.74	0.22	1.14	0.025	0.882
$\omega^{(4)}$	7.918	8.100	8.308	7.861	8.457
$c^{(4)} \cdot 10^3$	0.30	0.08	0.53	0.008	0.411
$\omega^{(5)}$	11.041	11.174	11.337	11.000	11.43
$c^{(5)} \cdot 10^3$	0.16	0.04	0.30	0.004	0.226
ω_1 approximative**) eq. (4-11)	0.707	0.518	0.414	0.984	0.847
ω_2 approximative**) eq. (4-11)	1.414	1.932	2.414	1.597	4.228

*) for unit pressure amplitude **) computed with a = 0

Table IIb: closed pipe

S Strouhal no.	1	1	1	0.359
U mass ratio	0.5	2	4	0.835
$\omega^{(1)}$ eigenfrequency	1.126	1.307	1.401	0.863
$c^{(1)} \cdot 10^3$ structural amplitude	8.02	7.39	7.04	0.907
$\omega^{(2)}$	3.307	3.673	3.964	3.386
$c^{(2)} \cdot 10^3$	0.50	1.38	1.85	0.0736
$\omega^{(3)}$	6.364	6.585	6.823	6.413
$c^{(3)} \cdot 10^3$	0.13	0.45	0.75	0.0208
$\omega^{(4)}$	9.478	9.632	9.815	9.512
$c^{(4)} \cdot 10^3$	0.056	0.21	0.39	0.0095
$\omega^{(5)}$	12.606	12.723	12.869	12.632
$c^{(5)} \cdot 10^3$	0.032	0.12	0.23	0.0054
a effective mass fraction in first mode	0.365	0.378	0.387	0.351
ω approximative, eq. (4-12) computed with a = 0	1.225	1.732	2.236	0.982

APPENDIX 1: ENERGETIC ANALYSIS OF THE BREAKING MEMBRANE CASE

Of general interest is the maximum amplitude the structural oscillations can assume. An upper limit c_{max} can be obtained from an energy balance.

Initially, the potential energy in the system is apportioned on the structure and the fluid and equal to

$$\hat{E} = \frac{1}{2}\,\hat{s}\,\hat{c}_o^{\,2} + \frac{1}{2}\,\frac{\hat{L}}{\hat{\rho}\,\hat{a}^2}\,(\hat{p}_o - \hat{p}_1)^2 \qquad \text{(A1-1)}$$

This is equal to the work required to bring the system from a state where $\hat{c}_o = 0$ and $\hat{p}_o - \hat{p}_1 = 0$ to the given initial state under the assumption of linearity and small pressure changes (see also Appendix 2).

The largest oscillations appear if all this energy is transferred into the structure (neglecting any energy losses):

$$\hat{E} = \frac{1}{2}\,\hat{s}\,\hat{c}_{max}^{\,2} \qquad \text{(A1-2)}$$

With $c_o = -(p_o - p_1)/s$ we obtain:

$$\frac{\hat{c}_{max}}{\hat{c}_o} = \frac{c_{max}}{c_o} = 1 + \frac{\hat{s}\,\hat{L}}{\hat{\rho}\,\hat{a}^2} \qquad \text{(A1-3)}$$

The ratio

$$E \equiv \frac{\hat{s}\,\hat{L}}{\hat{\rho}\,\hat{a}^2} = S^2/U \qquad \text{(A1-4)}$$

is a new characteristic number characterizing the ratio of the stiffnesses of the structure and the fluid.

Thus, large amplitudes c_{max} are possible if the structure is stiff if compared to the fluid.

APPENDIX 2: STATIC PRESSURE IN THE CLOSED PIPE

We consider the closed pipe with the membrane at zero position. The density is given by the fluid mass m_F per unit area and length of the pipe.

$$\hat{\rho}_o = \frac{\hat{m}_F}{\hat{L}} \quad , \quad \hat{\rho} = \frac{\hat{m}_F}{\hat{L}-\hat{c}} = \hat{\rho}_o \frac{\hat{L}}{\hat{L}-\hat{c}}$$

From the equation of state we have

$$\hat{p} - \hat{p}_o = \hat{a}^2 (\hat{\rho} - \hat{\rho}_o)$$
$$= \hat{\rho}_o \hat{a}^2 \left(\frac{\hat{L}}{\hat{L}-\hat{c}} - 1 \right)$$

For small values of $\hat{c}$, respectively $\hat{p} - \hat{p}_o$, we have

$$\hat{p} - \hat{p}_o = \hat{\rho}_o \hat{a}^2 \hat{c} / \hat{L} \tag{A2-1}$$

or in non-dimensional quantities

$$p = M^2 c \tag{A2-2}$$

If the piston was not at zero position rather than at c_o with $p_o = 0$, then

$$p = M^2 (c - c_o) . \tag{A2-3}$$

In the static case we have from eq. (2-8)

$$S^2 c = -K p \quad , \quad K = M^2 U$$

With the above result we obtain

$$S^2 c_S = - U (c_S - c_o)$$

or

$$c_S = \frac{U}{U + S^2} c_o . \tag{A2-4}$$

APPENDIX 3: APPROXIMATE FORMUAE FOR EIGENFREQUENCIES

We start from eqs. (4-2 to 4-5) and approximate

$$p'' \approx \left(p'(1) - p'(0) \right)/1. \tag{A3-1}$$

In case 1 we set

$$p'(1) = - p/1, \tag{A3-2}$$

in case 2 we have

$$p'(1) = 0. \tag{A3-3}$$

Also, we set

$$p|_W = p - a\, p'(0), \tag{A3-4}$$

where a can be chosen between zero and one half. With these approximations the characteristic equations are

$$-\omega^2 c + S^2 c = -K \left(p - a\, \omega^2 c / M^2 \right) \tag{A3-5}$$

and in case 1

$$-\omega^2 p + p + \frac{\omega^2}{M^2} c = 0 \tag{A3-6}$$

or in case 2

$$-\omega^2 p \qquad + \frac{\omega^2}{M^2} c = 0 . \tag{A3-7}$$

One can eliminate p and c and obtains

in case 1:

$$\omega^2_{1/2} = \frac{1}{2}\,\frac{1}{1+aU}\left\{\left(1+S^2+(1+a)U\right) \pm \sqrt{\left(1+S^2+(1+a)U\right)^2-4\,S^2(1+aU)}\;\right\} \tag{A3-8}$$

or in case 2:

$$\omega = \sqrt{\frac{S^2+U}{1+\alpha U}} \tag{A3-9}$$

APPENDIX 4: THE PROGRAM TRANS FOR ANALYSIS OF THE TRANSIENT CASES

```
TRANS:PROC (S,U,EM,ICASE,N,M,F1,F2,PO,PX,X,C,CP,CPP,T,NPM,NP,PRAND
          ,BTEST
          )  REORDER;
   DCL  (S,U,EM,KA    /* NON-DIMENSIONAL NUMBERS */
        )  BIN FLOAT(53);
   DCL  (ICASE      /*  1  BROKEN MEMBRAN ,  2  SNAP-BACK */
        ,N          /*  NUMBER OF TIME INTERVAL PER UNIT  */
        ,M          /*  NUMBER OF TIME UNITS  */
        ,NPM        /*  NUMBER OF TIME STEPS OF PRESSURE EVALUATION */
        ,NM         /*  N*M */
        , I,J
        )  BIN FIXED(15);
   DCL  NP(*)  /* 1:NPM */   BIN  FIXED(15) CONN;
   DCL  ( F1(*)    /*  (0: N*(M+1)) */
        , F2(*)    /*  (-N*M : N)   */
        , (PO,C,CP,CPP,T ) (*)   /* (0: N*M) */
        ,  PX (*,*) /*  1:NPM, 0:N */
        ,  X(*)      /*  0 : N */
        ) BIN FLOAT(53) CONNECTED;
   DCL PRAND ENTRY(BIN FLOAT(53)) RETURNS(BIN FLOAT(53))
    /* PRAND(T) RETURNS BOUNDARY VALUE */;
     DCL BTEST BIT(1) /* DECIDES ON PRINT-OUT */ ;
     DCL (X1,X2) BIN FLOAT(53);
   KA = EM*EM*U;
   NM = N*M;
   DCL (ALPHA,BETA) BIN FLOAT(53);
     IF  ICASE=1  THEN DO; ALPHA=1.; BETA= 0.; END;
                  ELSE  DO; ALPHA=0.; BETA= 1.; END;
   DCL (DT,FAK1,FAK2,FAK3
          ) BIN FLOAT(53);
   DT= 1.E0/N;
   FAK1= N*(N+U*1E-18);
   FAK2= (2*N*N-S*S)/FAK1;
   FAK3= N*(U*1E-18-N)/FAK1;
   FAK1= 1.E0/FAK1;
   /* INITIAL  VALUES */
     DO I= 0 TO N;
      F1(I),F2(I) = ALPHA*1E-18;
     END;
     CP(0)=0;
   C(0)=-KA/(S*S);
     T(0)=0;
   /* INTEGRATE */
      C(1) = -FAK1*(2.E0*KA*F1(0)) + FAK2* C(0)+FAK3*(C(0)-CP(0)
                                                          /N);
     DO   I= 1 TO NM-1;
          T(I)=I*DT;
        F1(N+I)= F2(N-I)*(BETA-ALPHA)/(BETA+ALPHA)
```

```
                +ALPHA*PRAND(T(I));
    X1=F1(I)*2.;
        C (I+1)=-FAK1*X1*KA+FAK2*C(I)+FAK3*C(I-1);
        CP(I)  =  (C(I+1)-C(I))*N;
        CPP(I) =  (C(I+1)-2.E0*C(I)+C(I-1))*(N*N);
        F2(-I) =  F1(I) + CP(I)/(EM*EM);
     END;
 /*  COMPUTE PRESSURE  */
     /*  P AT WALL */
     DO   I=0  TO  NM-1;
        PO(I) = F1(I) + F2(-I);
     END;
   T(NM)=T(NM-1); C(NM)=C(NM-1); CP(NM)=CP(NM-1); CPP(NM)=CPP(NM-1);
   PO(NM)=PO(NM-1);
 /*  PRESSURE  PROFILE  AT  T=   NP(J)*DT  */
    DO J= 1 TO NPM;
     DO   I= 0  TO N;
        PX(J,I)= F1(I+NP(J))+F2(I-NP(J));
     END;
    END;
    DO I = 0 TO N;
     X(I)=I*(1.E0/N);
    END;
   PUT SKIP LIST ('PRESSURE PROFILE AT TIMES');
   PUT SKIP;
   DO J= 1 TO MIN(NPM,17);
       X1= NP(J)*DT;
       PUT EDIT (X1) (F(7,3));
   END;
   PUT   SKIP(2);
   DO I= 0 TO N;
     PUT SKIP EDIT ( (PX(J,I) DO J=1 TO MIN(NPM,17)))
                   ( 17 F(7,3));
   END;
 /*  CHECK  ACCURACY  */
DCL (EMAX,EPMAX,EPPMAX,CMAX,CPMAX,CPPMAX) INIT(1.E-40) BIN FLOAT(53);
   PUT SKIP LIST('COMPARISON WITH EXACT SOLUTION');
    IF BTEST THEN
   PUT SKIP LIST('T,C,CP,CPP,P,ERR,ERRP,ERRPP');
   DCL (CA,CPA,CPPA) (0:3*N) BIN FLOAT(53);
   DCL (ERR,ERRP,ERRPP)(0:3*N) BIN FLOAT(53);
   CA,CPA,CPPA =0;
   ERR,ERRP,ERRPP =0;
   IF ICASE=1 THEN DO; NM=3*N; END;
             ELSE      NM=2*N;
   DO   I= 0  TO  NM;
      CALL  CANAL(T(I),S,KA,U,ICASE,CA(I),CPA(I),CPPA(I));
      ERR(I)= CA(I)-C(I);
      ERRP(I)= CPA(I)-CP(I);
```

```
      ERRPP(I)= CPPA(I)-CPP(I);
     IF (I=0 & ICASE=2) | (I=N & ICASE=1) THEN ERRP(I),ERRPP(I)=0;
    EMAX=MAX(EMAX,ABS(ERR(I)));
    EPMAX=MAX(EPMAX,ABS(ERRP(I)));
    EPPMAX=MAX(EPPMAX,ABS(ERRPP(I)));
    CMAX=MAX(CMAX,ABS(CA(I)));
    CPMAX=MAX(CPMAX,ABS(CPA(I)));
    CPPMAX=MAX(CPPMAX,ABS(CPPA(I)));
    IF BTEST THEN
   PUT SKIP EDIT (T(I),C(I),CP(I),CPP(I),PO(I),ERR(I),ERRP(I),ERRPP
                (I) )
          ( 8 E(14,5));
   END;
    EMAX=EMAX/CMAX; EPMAX=EPMAX/CPMAX; EPPMAX=EPPMAX/CPPMAX;
    PUT SKIP DATA(EMAX,EPMAX,EPPMAX);
   NM =  N*M-1;
    IF BTEST THEN DO;
   PUT SKIP LIST ('X1,F1,X2,F2');
   DO  I= 0  TO  NM+N;   X1= I*DT;  X2= 1.-X1;
      PUT  SKIP EDIT (X1,F1(I),X2, F2(N-I))
                     ( 4  F(15,3));
   END;
   END;
    IF BTEST THEN DO;
   PUT SKIP LIST ('SOLUTION  T,C,CP,CPP,PO');
   DO I= 0  TO NM;
     PUT SKIP EDIT  (T(I),C(I),CP(I),CPP(I),PO(I))
                    (5 E(15,3));
   END;
    END;
CANAL: PROC (T,S,KA,U,ICASE,C,CP,CPP);
     DCL      ICASE   BIN FIXED(15)
       ,(T,S,KA,U,C,CP,CPP,NUE2,NUE,U2,CO,SI,EX,CO,TS
       ) BIN FLOAT(53);
       U2=U*1.E-18;
   IF  ICASE = 1  &  T <  1.E0  THEN  DO;
         C= -KA/(S*S);
         CP=0;
         CPP=0;
         RETURN;
   END;
       NUE2= S*S-U2*U2;
       NUE=  SQRT(ABS(NUE2));
       CO =  KA/(S*S);  IF   ICASE=1 THEN CO=CO*1.E18;
       TS =  T;         IF   ICASE=1 THEN TS= T-1.E0;
       EX =  U2*TS;
       IF  EX> 160.E0  THEN  EX=0.;ELSE EX = EXP(-EX);
       IF NUE2 >  1.E-15  THEN DO;
           CO= COS(NUE*TS);
```

```
            SI= SIN(NUE*TS);
            C = -CO*EX*(CO+SI*U2/NUE);
            CP=  CO*EX*(U2*U2/NUE+NUE)*SI;
            CPP= CO*EX*(S*S*CO - U2*(U2*U2/NUE+NUE)*SI);
         END; ELSE DO;
         IF NUE2 <  -1.E-15  THEN DO;
            CO= COSH(NUE*TS);
            SI= SINH(NUE*TS);
            C = -CO*EX*(CO+SI*U2/NUE);
            CP=  CO*EX*(U2*U2/NUE-NUE)*SI;
            CPP= CO*EX*(S*S*CO + U2*(NUE-U2*U2/NUE)*SI);
         END;
         ELSE DO;
            C =   -CO*EX*(U2*TS+1.E0);
            CP =   CO*EX*U2*U2*TS;
            CPP=   CO*EX*(1.E0-U2*TS)*S*S;
         END;
         END;
       IF ICASE = 1  THEN
          C = C + KA/(S*S);
   END  CANAL;
  END TRANS;
```

APPENDIX 5: THE PROGRAM FLUX1D - A FINITE DIFFERENCE SCHEME FOR FLUID-STRUCTURE ANALYSIS

```
  /*   FLUX1D    FLUX-COMPRESSIBLE- 1D - MODELL-PROBLEM */
 FLUX1D:PROC(L,QC2,RHO,KAPPA,LAM,QQP,KAPPAS,CO,PO,BWAND,LINEAR,BITS
           ,BETA,GAMMA,BTEST,M,NSTEP,TMAX
           ,PWAND,CX,CPX,CPPX,TI,PVONX,XX,TX,NTX,POFT
           ) REORDER;
 DCL (L,QC2,RHO,KAPPA,LAM,QQP,KAPPAS,CO,BETA,GAMMA,TMAX,PO
     ,(PWAND,CX,CPX,CPPX,TI)(*) CONN /* 0:NSTEP */
     ,PVONX(*,*) CONN /* 1:NTX, 0: M+1 */
     ,XX(*) CONN /* 0: M+1  */
     ,TX(*) CONN /* NTX */
     ) BIN FLOAT(53);
DCL (BWAND,LINEAR,BITS,BTEST
    ) BIT(1);
DCL (M,NSTEP,NTX
    ) BIN FIXED(15);
DCL POFT ENTRY(BIN FLOAT(53)) RETURNS(BIN FLOAT(53)) VARIABLE;
  /*       1D   PIPE
           LEFT BOUNDARY  FLEXIBLE WALL
           RIGHT  BOUNDARY  OPEN  (CONSTANT PRESSURE  POFT(T))
           OR CLOSED WALL (DEPENDING ON BWAND)
           INITIAL STATE:
                   U=0,  P= PO ,  C=CO

   L     =  PIPE-LENGTH
   QC2   = (1./A**2)   , A=SPEED OF SOUND
   RHO   =  DENSITY
   KAPPA  = FLUID BULK VISCOSITY
   LAM   =  (S*S), LAM= STIFFNESS COEFFICIENT
   QQP   = LOAD COEFFICIENT:  C..+KAPPAS*C.+LAM*S=QQP*DRUCK
   KAPPAS = STRUCTURAL DAMPING COEFFICIENT
   CO    INITIAL DEFLECTION
   PO    INITIAL PRESSURE
   BWAND='1'B IF CLOSED PIPE, ='0'B IF OPEN ENDED PIPE (AT X=L)
   LINEAR='1'B IF KINETIC ENERGY TO BE NEGLECTED
   BITS = '0'B IF STIFF LEFT BOUNDARY WALL
   BETA,GAMMA = TIME INTEGRATION PARAMETERS
      0 < BETA <= 0.5, 0 <= GAMMA <= 0.5
     NEWMARK: STABLE IF BETA>=0.25, GAMMA=0
     ICE: STABLE IF BETA=GAMMA=0.5
   BTEST = '1'B IF MANY TEST-WRITES TO BE ISSUED
   M     = NUMBER OF FLUID MESH-CELLS
   NSTEP = MAXIMUM TIME-STEP NUMBER
   TMAX = MAXIMUM PROBLEM-TIME    (DT=TMAX/NSTEP)

    RESULTS
   PWAND(*) WALL PRESSURE
   CX(*)    C
   CPX(*)   C.
```

```
   CPPX(*)  C..
   TI(*)    T
   PVONX(*,*)    SPATIAL PRESSURE PROFILE AT TIMES TX(*)
   XX(*)    SPATIAL COORDINATES (CORRESPONDS TO PVONX(,*))
   TX(*)    TIMES FOR WHICH PRESSURE PROFILE IS TO BE SAVED
   NTX      NUMBER OF SUCH TIME STEPS
   POFT(T)  FUNCTION: PRESSURE AT OPENING VS TIME
*/
   DCL 1 STRU, 2(LAM, V,KAPPAS,OMEGA,QP) BIN FLOAT(53),PMAX BIN
        FLOAT(53);
     DCL (DT,DX,PI EXTERNAL,LAST,LASTO INIT(0)
        ,T, NUE,STAT_VERF
    ) BIN FLOAT(53);
    DCL 1 PARA, 2( FAC(17)) BIN FLOAT(53);
   STRU.LAM=LAM;
   STRU.QP=QQP;
   STRU.KAPPAS=KAPPAS;
    T=0.;
    DCL  (I,J)  BIN FIXED(15);
    PI=0.;PI=2*ACOS(PI);
    FAC=0.;
    STRU.V =  1.E0;
   DX    =  L/M;                    /*  MESH - SPACING  */
   DT=TMAX/NSTEP;
    STRU.OMEGA=SQRT(STRU.LAM);
   NUE   =  STRU.OMEGA / (2.*PI);  /* EIGEN-FREQUENZ IM VACUUM */
  PUT SKIP(3) DATA( LINEAR,BITS,BETA,DT,TMAX,NSTEP,QC2,
        PO,GAMMA,RHO,M,STRU,DX,NUE);
   FAC(1)=  (1.+KAPPA*(0.5+GAMMA)*DT)*QC2/((BETA+GAMMA)*DT*DT);
   FAC(2)=-(BFTA-GAMMA)/(BETA+GAMMA);
   FAC(3)=  (1.-KAPPA*(0.5-GAMMA)*DT)*RHO/((BETA+GAMMA)*DT*DT);
   FAC(4)= 1.E0/(BETA+GAMMA);
   FAC(5)= (1.-KAPPAS*STRU.OMEGA*(0.5-GAMMA)*DT)/(DT*DT);
   FAC(6)=  DX/2.;
   FAC(8)= DT/(1.+KAPPA*DT);
   FAC(9)= 1./(1.+KAPPA*DT);
   FAC(10)=0;
   FAC(11)=(1.+KAPPA*(0.5+GAMMA)*DT)/(DT*DT*RHO);
   FAC(12)= 1.E0/(DX*DX);
   FAC(13)= 1.0E0/ DX;
   FAC(14)=  -1.E0/(RHO*DX);
   FAC(15)=  FAC(4)*FAC(5)*QC2*(1.-KAPPA*(0.5-GAMMA)*DT);
   FAC(16)=(1.+KAPPAS*STRU.OMEGA
    *(0.5+GAMMA)*DT)/(DT*DT)+(BETA+GAMMA)*STRU.LAM;
   FAC(17)=(RHO/((BETA+GAMMA)*DT*DT))*(1.+KAPPA*(0.5+GAMMA)*DT);
   FAC(7) = (1.-KAPPA*(0.5-GAMMA)*DT)/(DT*DT*RHO);
    IF BTEST THEN PUT SKIP DATA(FAC);
   /*   UNCONDITIONALLY  STABLE FOR BETA >= 1/4
        ACCORDING TO  LINEAR  STABILITY  THEORY   */
```

```
   DCL DRUCK FILE PRINT;
 PUT FILE(DRUCK) PAGE DATA(BETA,M,STRU,LINEAR,BITS,QC2);
 DCL  1   P(3)
        ,2(PJ(M),GPW,PO,W,C
          ) BIN FLOAT(53);
 DCL  1   PH(3)
        ,2(PHJ(M),GPHW,GPHO) BIN FLOAT(53);
 DCL  1   E
        ,2 (EJ(M), GEW,EO)   BIN FLOAT(53);
 DCL  1   QPQ
        , 2 (PJ(M),GPW,PO,W,C)  BIN FLOAT(53);
 DCL  1   QPH
        , 2(PHJ(M),GPHW,GPHO) BIN FLOAT(53);
  DCL U(0:M) BIN FLOAT(53);
 DCL 1   LAPLP , 2 (AL,BL,CL) (M) BIN FLOAT(53)
     ,1   HELMH , 2(AH,BH,CH) (M)  BIN FLOAT(53)
     ,1  LAPLPH , 2 (AP,BP,CP) (M) BIN FLOAT(53);
 DCL CAP BIN FLOAT(53);
 DCL (IPM, IP,IPP ,ISTEP) BIN  FIXED(15);
 /*  SETUP  OPERATORS  */
 /*  HELMHOLTZ */
    AH= FAC(12);  CH=AH;
    BH= -AH-CH;
    BH= BH- FAC(1);
    BH(1)=BH(1)+AH(1);
 /*  LAPLACE (P+E) */
    AL= FAC(12); CL=AL;
    BL= -AL-CL;
    BL(1)= BL(1)+AL(1);
    AL(1)=-AL(1)*DX;      /*   AL(1) *  GW */
 /*  LAPLACE  PH   */
    AP =  FAC(12); CP= AP;
    BP = -AP-CP;
    /* NEUMANN  LINKS ERSETZT DURCH DIRICHLET */
    BP(M)= BP(M)+CP(M);
 IF BWAND THEN DO;
  BH(M)=BH(M)+CH(M); CH(M)=0;
  BL(M)=BL(M)+CL(M); CL(M)=0;
  BP(M)=BP(M)+CP(M); CP(M)=0;
 END;
  IF BTEST THEN DO;
CALL PRINT1(AH,'AH'); CALL PRINT1(BH,'BH'); CALL PRINT1(CH,'CH');
CALL PRINT1(AL,'AL'); CALL PRINT1(BL,'BL'); CALL PRINT1(CL,'CL');
CALL PRINT1(AP,'AP'); CALL PRINT1(BP,'BP'); CALL PRINT1(CP,'CP');
   END;
 IF BITS THEN CALL CSOLVE(CAP);
  /*   SET INITIAL VALUES */
 IPM=1;  IP=2;  IPP=3;
 P=0;
```

```
P(2).PJ(*) = PO;
   IF BITS THEN DO;
    P(2).C=CO;
   LASTO=(STRU.LAM*CO-STRU.QP*PO);
P(*).W = P(*).C*STRU.V;
   END;
   LAST=(1.-BETA-GAMMA)*LASTO;
P(1).PO=0; P(2).PO=  PO;
PH=0.;  E=0.; U=0.;
  IF BTEST THEN DO;
CALL PRP(P(1),'INIT DP(1)');
CALL PRP(P(2),'INIT P(2)');
   END;
PWAND(0)=PO;
CX(0)=CO;
CPX(0),CPPX(0)=0;
XX(0)=0;
DO I = 1 TO M;
 XX(I)=(I-0.5)*DX;
END;
XX(M+1)=L;
    ISTEP=0;
    TI(ISTEP)=T;
/*  LOOP  */
DO  WHILE ( ISTEP<NSTEP);
   ISTEP=ISTEP+1;
 T= T+DT;
  IF BTEST THEN
 PUT SKIP LIST ('************************ NEW TIME',T);
   IF BTEST THEN DO;
CALL PRINT1(E.EJ     ,'ENERGIE');
PUT SKIP DATA(E.GEW,E.EO);
    END;
/*  GET R.H.S. FOR PRESSURE AND DEFORMATION  */
CALL  SRP (P(IP),P(IPM),E,QPQ);
IF BTEST THEN
CALL PRP(QPQ,' QPQ');
/*  SOLVE FOR DELTA_P AND DELTA_C  */
CALL  PSOLVE( P(IPP),QPQ);
    IF BTEST THEN DO;
      CALL PRP(P(IPP),'DELTA P');
   CALL SEN(PH(IP),E,U);
 CALL PTEST(P(IPM),P(IP),P(IPP),E);
 END;
IF LINEAR  THEN DO;  PH(IPP)=0.; E=0.; U=0.; END; ELSE DO;
/*  GET R.H.S. FOR POTENTIAL TIME DERIVATIVE */
  CALL SRPH(P(IPM),P(IP),P(IPP),QPH);
IF BTEST THEN
CALL PRPH(QPH, 'QPH');
```

```
      CALL PHSOLV ( PH(IPP), QPH);
         IF BTEST THEN
    CALL PRPH(QPH,' PH_PUNKT');
      /*  INTEGRATE POTENTIAL IN TIME */
      CALL  SPHNEU ( PH(IPM),PH(IP),PH(IPP));
       IF BTEST THEN
     CALL PRPH(PH(IPP),'POTENTIAL');
      CALL SEN(PH(IPP),E,U);
    END /* ¬LINEAR */;
      /*  INTEGRATE PRESSURE AND DEFORMATION IN TIME */
    P(IPM)=P(IPP); P(IPP)=P(IP)+P(IPP); P(IP)=P(IPM);
       IF (MOD(ISTEP,10)=0) THEN DO;
       PUT SKIP FILE(DRUCK) (T,ISTEP);
       PUT FILE(DRUCK) EDIT((P(IPP).PJ(I) DO I = 1 TO M))
                   (SKIP,10  E(12,4));
      IF ¬ LINEAR THEN
     PUT FILE(DRUCK) EDIT((U(I) DO I = 1 TO M))
                   (SKIP,10 E (12,4));
    IF BITS THEN PUT FILE(DRUCK) EDIT(P(IPP).W) (SKIP, E(12,4));
    END;
    IF BTEST THEN
  CALL PRP(P(IPP),'PRESSURE');
    /* STEP - END */
     /*   EVALUATION */
    TI(ISTEP)=T;
    PWAND(ISTEP)=P(IPP).PJ(1)-FAC(6)*P(IPP).GPW;
    CX(ISTEP)=P(IPP).W;
    IF ISTEP>1 THEN DO;
     CPX(ISTEP-1)=(CX(ISTEP)-CX(ISTEP-2))/(2.*DT);
     CPPX(ISTEP-1)=(CX(ISTEP)-2.*CX(ISTEP-1)+CX(ISTEP-2))/(DT*DT);
    END;
    DO J = 1 TO NTX;
     IF ISTEP=FLOOR(TX(J)/DT+0.5) THEN DO;
       PVONX(J,0)=PWAND(ISTEP);
       DO I = 1 TO M;
        PVONX(J,I)=P(IPP).PJ(I);
       END;
       PVONX(J,M+1)=P(IPP).PO;
      END;
     END;
      IF ISTEP>1 THEN LAST=0;
                 ELSE LAST=(BETA-GAMMA)*LASTO;
     I= IPM;  IPM=IP; IP=IPP; IPP=I;
    END  /*  LOOP  WHILE (T< TMAX) */;
    IF BTEST THEN DO;
     /*  PRINT OUT  */
   PUT SKIP LIST('PRESSURE PROFILE AT TIMES');
PUT SKIP(2) EDIT((TX(I) DO I = 1 TO MIN(17,NTX) WHILE(TX(I)<=TMAX)))
          (17 F(7,3));
```

```
      PUT SKIP;
    DO I = 0 TO M+1;
 PUT SKIP EDIT((PVONX(J,I) DO J = 1 TO MIN(17,NTX) WHILE(TX(I)<=TMAX)))
   (17 F(7,3));
    END;
    PUT SKIP LIST('SOLUTION, T,PWAND,C,CP,CPP  VERSUS TIME');
    DO I = 0 TO NSTEP;
    PUT SKIP EDIT(TI(I),PWAND(I),CX(I),CPX(I),CPPX(I)) (5 E(15,3));
    END;
      END;
     PUT SKIP LIST (' END FLUX1D' );

     /*  SUBROUTINES */
     PRINT1:  PROC (F,TEXT) REORDER;
         DCL  F(*) BIN FLOAT(53), TEXT  CHAR(*);
         DCL(I,LB1,HB1)  BIN FIXED(15);
          LB1=LBOUND(F,1);
          HB1=HBOUND(F,1);
         PUT  SKIP  LIST(TEXT);  PUT SKIP DATA (LB1,HB1);
         PUT  EDIT  ((F(I)     DO I= LB1  TO HB1))
                     (SKIP, 10 E(12,4));
     END PRINT1;

   (NOUNDERFLOW):
TRIX: PROC(DB,DC,DIM,$A,$B,$C,$Y,$TCOS,$D,$W,$S) REORDER;
  /* SOLUTION OF TRIDIAGONAL LINEAR EQUATION SYSTEM */
   /* VERSION CODED BY A.STEIL */
   /* TOP OF PROCEDURE */
DCL (DB, DC) BIN FIXED(15);
DCL DIM BIN FIXED(15);
DCL $S PTR; /* UNUSED IN THIS VERSION */
DCL ($A, $B, $C, $Y, $TCOS, $D, $W) PTR;
DCL (PA, PB, PC, PY, PTCOS, PD, PW) PTR;
DCL (A BASED(PA), B BASED(PB), C BASED(PC), Y BASED(PY),
     TCOS BASED(PTCOS), D BASED(PD), W BASED(PW)) (0:10)
    BIN FLOAT(53);
PA = $A; PB = $B; PC = $C; PY = $Y;
PTCOS = $TCOS; PD = $D; PW = $W;
DCL (DIMM1, DIMM2) BIN FIXED(15);
DIMM1 = DIM - 1;
DIMM2 = DIMM1 - 1;
DCL (DBM1, DBM1P2, SDBM1P2, DCP1) BIN FIXED(15);
DBM1 = DB - 1; SDBM1P2,DBM1P2 = DBM1 + 2;
DCP1 = DC + 1;
     DCL (I,K,L,LINT) BIN FIXED(15) STATIC;
     DCL (X,Z) BIN FLOAT(53) STATIC;
L = SDBM1P2 / DCP1 - 1;
LINT = 1;
     DO K=0 TO DBM1;
```

```
IF (K=L) THEN DO I = 0 TO DIMM1; W(I) = Y(I); END;
X = TCOS(K);
Z = B(0) - X;
     D(0)=C(0)/Z;
     Y(0)=Y(0)/Z;
     DO I = 0 TO DIMM2;
Z = B(I+1) - X - A(I+1) * D(I);
     D(I+1)=C(I+1) / Z;
     Y(I+1)=(Y(I+1)-A(I+1)*Y(I)) / Z;
     END;
     DO I = DIMM2 TO 0 BY -1;
     Y(I)=Y(I)-D(I)*Y(I+1);
     END;
     IF(K=L) THEN DO;
  X = X - TCOS(DBM1+LINT);
  DO I = 0 TO DIMM1;
    Y(I) = Y(I) * X + W(I);
  END;
     LINT=LINT+1;
SDBM1P2 = SDBM1P2 + DBM1P2; /* EQUALS 'LINT * DBM1P2' */
L = SDBM1P2 / DCP1 - 1;
    END;
END;
     END TRIX;

    A1U:PROC ( P , F );
       DCL 1 P CONN
           , 2 (PJ(*),GPW,PO,W,C) BIN FLOAT(53)
           , F BIN FLOAT(53);
       F = FAC(16)*C - (BETA+GAMMA)*STRU.QP*(PJ(1)-FAC(6)*GPW);
    END A1U;

     BSOLVE: PROC( P, Q);
         DCL (TCOS INIT(0), W1(M),W2(M)) BIN FLOAT(53);
     DCL 1  P CONNECTED
          ,2 (PJ(*),GPW,PO,W,C) BIN FLOAT(53);
         DCL 1 Q  LIKE P;
         P.PO = Q.PO;
         IF BITS THEN DO;
           P.C=Q.C/FAC(16);
           P.GPW = Q.GPW-FAC(17)*STRU.V*P.C;
       END;
       ELSE DO; P.GPW=Q.GPW; P.C=0.; END;
            P.PJ=Q.PJ;
         P.PJ(1) =     P.PJ(1) +P.GPW*DX*AH(1);
         P.PJ(M) =     P.PJ(M) -  P.PO*CH(M);
        IF M>1 THEN
         CALL  TRIX(1,0,M,ADDR(AH),ADDR(BH),ADDR(CH),ADDR(P.PJ),
                    ADDR(TCOS), ADDR(W1),ADDR(W2),ADDR(TCOS));
```

```
   ELSE P.PJ(1)=P.PJ(1)/BH(1);
 P.W=STRU.V*P.C;
 IF BWAND THEN P.PO=P.PJ(M);
 END  BSOLVE;

CSOLVE: PROC(CAP);
  DCL (CAP,F) BIN FLOAT(53);
  DCL 1 P , 2(PJ(M),GPW,PO,W,C) BIN FLOAT(53)
      , 1 Q LIKE P;
  Q=0;
  Q.C=1.;
  CALL BSOLVE(P,Q);
  CALL A1U(P,F);
  CAP=1./F;
  PUT SKIP FILE(DRUCK) DATA(CAP);
END CSOLVE;

PSOLVE: PROC(P,Q);
  DCL (F) BIN FLOAT(53);
  DCL 1 P CONNECTED
      , 2 (PJ(*),GPW,PO,W,C) BIN FLOAT(53);
  DCL 1 Q LIKE P;
  IF BITS THEN DO;
    CALL BSOLVE(P,Q);
    CALL A1U(P,F);
    Q.C=Q.C+(Q.C-F)*CAP;
  END;
  CALL BSOLVE(P,Q);
END PSOLVE;

 PHSOLV:  PROC ( PH,Q);
 DCL 1  PH CONNECTED
      ,2 (PHJ(*),GPHW,GPHO) BIN FLOAT(53);
 DCL 1 Q LIKE PH, S BIN FLOAT(53);
       DCL((W1,W2)(M),TCOS INIT(0)) BIN FLOAT(53);
       PH.PHJ = Q.PHJ;
       PH.PHJ(1) =    PH.PHJ(1) + AP(1)*Q.GPHW*DX;
       PH.PHJ(M) =    PH.PHJ(M) - CP(M)*Q.GPHO*DX;
       IF BTEST THEN DO;
       S=0.;
       DO  I=1 TO M;  S=S+PH.PHJ(I); END;
       PUT SKIP LIST(' KONSISTENZ-TEST, 0= S=',S);
       END;
       CALL TRIX(1,0,M,ADDR(AP),ADDR(BP),ADDR(CP),ADDR(PH.PHJ)
                ,ADDR(TCOS),ADDR(W1),ADDR(W2),ADDR(TCOS));
       PH.GPHW= Q.GPHW;  PH.GPHO=Q.GPHO;
 END PHSOLV;

 SEN: PROC (PH,E,U);
```

```
DCL 1  PH CONNECTED
     ,2 (PHJ(*),GPHW,GPHO) BIN FLOAT(53);
DCL 1  E  CONNECTED
     ,2 (EJ(*) , GEW, EO ) BIN FLOAT(53);
    DCL  U(*)  BIN FLOAT(53);
    U(0) = GPHW;
    U(M) = GPHO;
    DO I= 1 TO  M-1;
      U(I)= (PHJ(I+1)-PHJ(I))* FAC(13);
    END;
    IF BTEST THEN CALL PRINT1( U, 'VELOCITY U' );
    DO  I= 1 TO  M;
      EJ(I)= (RHO/4.)*(U(I)*U(I)+U(I-1)*U(I-1));
    END;
   GEW=0;
    EO  =  EJ(M);
END SEN;

SRP: PROC ( P,PM ,E, Q);
DCL 1  P CONNECTED
     ,2 (PJ(*),GPW,PO,W,C) BIN FLOAT(53);
DCL 1  E  CONNECTED
     ,2 (EJ(*) , GEW, EO ) BIN FLOAT(53);
        DCL   ( 1 PM , 1 Q)  LIKE P;
    DO  I= 1 TO M;
      Q.PJ(I)= FAC(4)*(P.PJ(I)+E.EJ(I))+FAC(2)*PM.PJ(I);
    END;
      Q.PO   = FAC(4)*(P.PO+E.EO)+FAC(2)*PM.PO;
      Q.GPW  = FAC(4)*(P.GPW+E.GEW)+FAC(2)*PM.GPW;
      IF M > 1 THEN DO;
      E.EJ(1)=   BL(1)*Q.PJ(1)+CL(1)*Q.PJ(2)+ AL(1)*Q.GPW;
      E.EJ(M)=   CL(M)*Q.PO + BL(M)*Q.PJ(M)+ AL(M)* Q.PJ(M-1);
      END;
      ELSE
      E.EJ(1)=BL(1)*Q.PJ(1)+CL(1)*Q.PO+AL(1)*Q.GPW;
      DO I = 2  TO M-1;
      E.EJ(I) =   AL(I)*Q.PJ(I-1)+ BL(I)*Q.PJ(I)+CL(I)*Q.PJ(I+1);
      END;
    Q.PJ=-FAC(15)*PM.PJ- E.EJ;
    Q.PO=  POFT( T)-P.PO;
    IF BITS THEN DO;
    Q.GPW=  ( FAC(3)*PM.W
              -Q.GPW);
    Q.C =  ((P.PJ(1)-FAC(6)*P.GPW)
           -(BETA-GAMMA)*(PM.PJ(1)-FAC(6)*PM.GPW))*STRU.QP
           +LAST
           +FAC(5)*PM.C
           -STRU.LAM*(P.C-(BETA-GAMMA)*PM.C);
       END; ELSE Q.GPW,Q.C=0.;
```

```
        Q.W=0;
END SRP;

SRPH:PROC ( PM,P,PP,QPH);
DCL 1  P CONNECTED
      ,2 (PJ(*),GPW,PO,W,C) BIN FLOAT(53);
   DCL  (1 PM, 1 PP) LIKE P;
   DCL  1 QPH, 2 (PHJ(*),GPHW,GPHO)  BIN FLOAT(53);
 QPH.PHJ(*)=(PM .PJ(*)*FAC(7)-PP.PJ(*)*FAC(11))*QC2/RHO;
 IF BITS THEN QPH.GPHW=PP.W*FAC(11)-PM.W*FAC(7);
 ELSE          QPH.GPHW=0.;
 IF BWAND THEN QPH.GPHO=0;
 ELSE DO;
DCL (GRADPM,GRADP,GRADPP) BIN FLOAT(53);
 GRADPM=PM.PO-PM.PJ(M);
 GRADP =P .PO-P .PJ(M);
 GRADPP=PP.PO-PP.PJ(M);
   QPH.GPHO =  FAC(14)*(BETA*(GRADPP-GRADPM)
                      +GAMMA*(GRADPP+GRADPM)
                      +GRADP);
  END;
END  SRPH;

    SPHNEU: PROC ( PHM,PH,PHP);
DCL 1  PH CONNECTED
      ,2 (PHJ(*),GPHW,GPHO) BIN FLOAT(53);
DCL (1 PHM,1 PHP)LIKE PH;
    PHP  =   FAC(8)* PHP + FAC(9)*PH+ FAC(10)* PHM;
END SPHNEU;

PTEST:PROC(PM,P,PP,E);
  DCL 1 P CONNECTED,2(PJ(*),GPW,PO,W,C) BIN FLOAT(53)
        ,1 (PM,PP) LIKE P
       ,1 E CONNECTED,2 (EJ(*),GEW,EO) BIN FLOAT(53)
       ,(X(M),XM,XP,XX,BM,GE,CE)BIN FLOAT(53);
XM=BETA*(PP.GPW-PM.GPW)+GAMMA*(PP.GPW+PM.GPW)
  +P.GPW+E.GEW;
     DO I = 1 TO M;
     IF I < M THEN
   XP=BETA*(PP.PJ(I+1)-PM.PJ(I+1))+GAMMA*(PP.PJ(I+1)+PM.PJ(I+1))
       +P.PJ(I+1)+E.EJ(I+1);
        ELSE
       XP=BETA*(PP.PO-PM.PO)+GAMMA*(PP.PO+PM.PO)+P.PO+E.EO;
  XX=BETA*(PP.PJ(I)-PM.PJ(I))+GAMMA*(PP.PJ(I)+PM.PJ(I))
    +P.PJ(I)+E.EJ(I);
  X(I)=(PP.PJ(I)-PM.PJ(I))*QC2/(DT*DT)
 +(KAPPA*QC2/DT)*((0.5+GAMMA)*PP.PJ(I)+(0.5-GAMMA)*PM.PJ(I))
       -((XP-XX)/DX -XM)/DX;
  XM=(XP-XX)/DX;
```

```
    END;
    CALL PRINT1(X,'RESIDUAL');
   IF BITS THEN DO;
     DCL (QQ,QQP,QQM) BIN FLOAT(53);
     QQP=PP.PJ(1)-0.5*DX*PP.GPW;
     QQ =P. PJ(1)-0.5*DX*P. GPW;
     QQM=PM.PJ(1)-0.5*DX*PM.GPW;
   CE=(PP.C-PM.C)/(DT*DT)
  +(KAPPAS*STRU.OMEGA/DT)*((0.5+GAMMA)*PP.C+(0.5-GAMMA)*PM.C)
     +(BETA*(PP.C-PM.C)+GAMMA*(PP.C+PM.C)+P.C)*STRU.LAM
     -(BETA*(QQP-QQM)+GAMMA*(QQP+QQM)+QQ)*STRU.QP;
        END;
   ELSE CE=0;
   GE=RHO*(PP.W-PM.W)/(DT*DT)+(RHO*KAPPA/DT)
        *((0.5+GAMMA)*PP.W+(0.5-GAMMA)*PM.W)
     +BETA*(PP.GPW-PM.GPW)+GAMMA*(PP.GPW+PM.GPW)
     +P.GPW+E.GEW;
    PUT SKIP DATA(CE,GE);
  END PTEST;

  PRPH: PROC (PH,TEXT);
  DCL 1 PH CONNECTED,2(PHJ(*),GPHW,GPHO) BIN FLOAT(53);
  DCL TEXT CHAR(*);
  CALL PRINT1(PH.PHJ,TEXT||' PHJ');
  PUT SKIP DATA(TEXT,PH.GPHW,PH.GPHO);
  END PRPH;

   PRP: PROC(P,TEXT);
   DCL 1 P CONNECTED, 2(PJ(*),GPW,PO,W,C)BIN FLOAT(53);
   CALL PRINT1(P.PJ,TEXT||' PJ');
   DCL TEXT CHAR(*);
   PUT SKIP DATA (TEXT,P.GPW,P.PO,P.W,P.C);
   END PRP;
 END FLUX1D;
```

APPENDIX 6: THE PROGRAM PERI FOR ANALYSIS OF THE PERIODIC CASES

```
PERI: PROC (S,U,EM,ICASE,N,M,PX,C,OM,X,BTEST
          ) REORDER;
    DCL   (S,U,EM,KA     /* NON-DIMENSIONAL NUMBERS */,XMIN,XMAX,PI
           , (PX(*,*)     /* (1:M,0:N)  PRESSURE PROFILE
                            FOR DIFFERENT EIGENFREQUENCIES */
            ,C (*)       /*  STRUCTURAL AMPLITUDES   (1:M)   */
            ,OM(*)       /*  EIGEN-FREQUENCIES       (1:M)   */
            ,X (*)       /*  SPACE  COORDINATE       (1:N)   */
          )  CONN
          )  BIN FLOAT(53)
        , (ICASE     /* 1  OPEN  , 2  CLOSED FLUID REGION
                     */
          , N        /*  NUMBER  OF SPACE INTERVALS  */
          , M        /*  NUMBER  OF EIGEN-SOLUTIONS */
          ,IX)  BIN FIXED(15);
      DCL BTEST BIT(1)  /* DECIDES ON PRINT-OUT */ ;
  ZERO: PROC(XMIN,XMAX,NX,XO,IX,EPS,NDX,F)  RECURSIVE;
    /*   ZERO  DETERMINES ZEROS OF FUNCTION  F(X) IN THE
         INTERVAL  (XMIN,XMAX)
         NX  =  NUMBER  OF ZEROS REQUESTED
         IX  =  NUMBER   OF ZEROS FOUND   (RETURNED)
         EPS =  REQUESTED ACCURACY OF THE ZEROS  (ABSOLUTE)
                   FOR  CONTINOUS FUNCTIONS THE RESULTANT
                   ACCURACY IS USUALLY MUCH BETTER
         F    FUNCTION
         ***
          NO  DERIVATIVES COMPUTED
         ***
         NDX  = NUMBER OF INTERVALS TO BE USED FOR
                SEARCH OF SIGN-CHANGES;
                IF  F IS  CLOSE TO A LINEAR FUNCTION,
                    NX CAN BE AS SMALL AS 3;
                    OTHERWIZE  NX SHOULD BE TAKEN SUCH THAT
                    NO ZERO CAN  BE  OVERLOOKED
                       RECOMMENDED: NDX= 10*NX
    */
    DCL (NX,IX,IXX,NDX
        ) BIN FIXED(15)
      , (XMIN,XMAX,XO(*) CONN,EPS              ,DX,X1,X2,F1,F2
        )BIN FLOAT(53);
    DCL  F  ENTRY (BIN FLOAT(53)) RETURNS(BIN FLOAT(53)) VARIABLE;
        IX=0;
        DX= (XMAX-XMIN)/ NDX;
    START: X1=XMIN;  F1= F(X1);
        DO WHILE(X1<XMAX & IX<NX);
          X2=X1+DX;
          F2=F(X2);
          IF  F1*F2 <=0  THEN DO;
```

```
            IX=IX+1;
            IF  ABS(DX) > EPS THEN DO;
                DCL XOO(2) BIN FLOAT(53);
                CALL ZERO(X1,X2,2,XOO,IXX,EPS,3,F);
                IF IXX=1 THEN XO(IX)= XOO(1);
                         ELSE DO;
                            DX=DX/3.;
                            GOTO START;
                         END;
            END;
            ELSE
               XO(IX)= (X1*F2-X2*F1)/(F2-F1);
         END;
         X1=X2;
         F1=F2;
       END;
END ZERO;
   G2: PROC(X) RETURNS (BIN FLOAT(53));
      DCL X BIN FLOAT(53);
      RETURN(SIN(X)*(X*X-S*S)- U*X*COS(X));
   END G2;
   G1: PROC(X) RETURNS (BIN FLOAT(53));
     DCL X BIN FLOAT(53);
      RETURN(U*X*SIN(X)+COS(X)*(X*X-S*S));
   END G1;
   KA =  EM*EM*U;
   PUT SKIP DATA (S,U,EM,ICASE,N,M);
   XMAX =  MAX(10.*3.1415,S)* 1.2;
   XMAX=S+4.;
   DO I= 0  TO N;
       X(I) =  I * (1.E0/N);
   END;
   DCL XZ(M) BIN FLOAT(53);
   J=0;
     XMIN=1.E-3;
        NEXT:
  IF  ICASE=1  THEN CALL ZERO(XMIN,XMAX,M-J,XZ,IX,1.E-3,1000,G1);
            ELSE CALL ZERO(XMIN,XMAX,M-J,XZ,IX,1.E-3,1000,G2);
   PUT SKIP DATA (IX,(XZ(I) DO I = 1 TO IX));
    DO I = 1 TO IX;
       OM(I+J)=XZ(I);
    END;
    J=IX+J;
    XMIN=XMAX;
    XMAX=XMAX+31.41592E0;
    IF J<M THEN GOTO NEXT;
   PUT SKIP LIST('EIGENFREQUENCIES');
   PUT SKIP EDIT(OM)(10  F(12,4),SKIP);
   DCL (ALPHA,BETA,B
```

```
      ) BIN FLOAT(53)
      ,(I,J
       ) BIN FIXED(15);
    IF ICASE=1 THEN DO; ALPHA=1.  ; BETA=0.  ;   END;
              ELSE DO; ALPHA=0.  ; BETA=1.  ;   END;
  DO  J= 1 TO M;
    B=(BETA*OM(J)*SIN(OM(J))-ALPHA*COS(OM(J)))
     /(BETA*OM(J)*COS(OM(J))+ALPHA*SIN(OM(J)));
   DO I= 0  TO N;
     PX(J,I) = (COS(OM(J)*X(I))+ B*SIN(OM(J)*X(I)))
              / SQRT(1.+B*B);
   END;
   C(J) = KA*PX(J,0)/ (OM(J)*OM(J)-S*S);
  PUT SKIP  DATA ( J,OM(J),C(J));
   IF BTEST THEN
  PUT SKIP  EDIT ( PX(J,*)) ( 10 F(12,4), SKIP);
  END;
END   PERI;
```

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