KfK 2738
März 1979

TP1
A Computer Program for the Calculation of Reactivity and Kinetic Parameters by One-Dimensional Neutron Transport Perturbation Theory

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This work was partially supported by
the Alexander von Humboldt Stiftung.

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Kernforschungszentrum Karlsruhe GmbH
ISSN 0303-4003

Abstract

TP1, a FORTRAN-IV program based on transport theory, has been developed to determine reactivity effects and kinetic parameters such as effective delayed neutron fractions and mean generation time by applying the usual perturbation formalism for one-dimensional geometry. Direct and adjoint angular dependent neutron fluxes are read from an interface file prepared by using the one-dimensional S_n-code DTK which provides options for slab, cylindrical and spherical geometry. Multigroup cross sections which are equivalent to those of the DTK-calculations are supplied in the SIGMN-block which is also read from an interface file. This block which is usually produced by the code GRUCAL should contain the necessary delayed neutron data, which can be added to the original SIGMN-block by using the code SIGMUT.

Two perturbation options are included in TP1: a) the usual first order perturbation theory can be applied to determine probe reactivities, b) assuming that there are available direct fluxes for the unperturbed reactor system and adjoint fluxes for the perturbed system, the exact reactivity effect induced by the perturbation can be determined by an exact perturbation calculation. According to the input specifications, the output lists the reactivity contributions for each neutron reaction process in the desired detailed spatial and energy group resolution.

Since the DTK the somewhat approximate assumption of fission spectrum is used to calculate the criticality factor, TP1 derives an appropriate correction taking into account the individual fission spectra of the various isotopes. By this means it is possible to determine an improved criticality factor which is close enough to the exact value for almost all practical applications.

Numerical examples are presented to demonstrate the sensitivity of the calculated reactivity results with respect to the values used in DTK for the accuracy criterion, the number of spatial mesh points and the order n of the S_n calculations. The reactivity effects obtained from TP1 are compared with corresponding results from one-dimensional results given in the literature for the reactor model used for the present calculations.

TP1, ein Rechenprogramm zur Bestimmung von Reaktivitätswerten und kinetischen Parametern durch eindimensionale Neutronen-transport-Störungstheorie

Zusammenfassung

TP1 ist ein FORTRAN-IV Programm, das unter Benutzung transport-theoretischer Methoden nach den üblichen Verfahren der Störungs-theorie Reaktivitätseffekte und kinetische Parameter, wie den effektiven Anteil der verzögerten Neutronen und die mittlere Generationszeit, bestimmt. Die benötigten, direkten und adjungierten winkelabhängigen Neutronenflüsse werden auf einer Zwischendatei als Ergebnis einer eindimensionalen S_n-Rechnung mit dem Programm DTK erwartet, das die Behandlung von Problemen in Platten-, Zylinder- und Kugelgeometrie ermöglicht. Ebenso wie in den DTK-Rechnungen werden die Multigruppen-Wirkungsquerschnitte aus dem SIGMN-Block von einer weiteren Zwischendatei eingelesen. Für TP1-Rechnungen muß dieser Block, der im allgemeinen von dem Programm GRUCAL erstellt wird, die erforderlichen Daten für die verzögerten Neutronen enthalten. Diese Daten können mit Hilfe des Programms SIGMUT dem ursprünglichen SIGMN-Block angefügt werden.

TP1 enthält die beiden unterschiedlichen Störungstheorie-Optionen: a) Störungstheorie erster Ordnung zur Bestimmung des Reaktivitäts-effektes von Materialproben, b) exakte Störungstheorie zur exakten Ermittlung der durch eine Störung verursachten Reaktivitätsänderung, wobei die direkten Flüsse für das ungestörte Reaktorsystem und die adjungierten Flüsse für das gestörte System bereitgestellt werden müssen. Abhängig von den spezifizierten Eingabegrößen werden in der Programmausgabe die Einzelbeiträge zum Reaktivitätseffekt nach Neutronenreaktionen, Energiegruppen und Ortsabhängigkeit aufgegliedert.

Da DTK den Kritikalitätswert unter Benutzung der Näherungsannahme eines isotopenunabhängigen Spaltspektrums bestimmt, berechnet TP1 eine entsprechende Korrektur, die den Einfluß der für die einzelnen Isotope unterschiedlichen Spaltspektren berücksichtigt. Dadurch kann ein verbesserter Kritikalitätswert ermittelt werden, der für fast alle praktischen Anwendungen den exakten Wert genügend genau approximiert.

Die numerischen Beispiele zeigen die Abhängigkeit der TP1-Reaktivitätswerte von den in DTK benutzten Werten für die Genauigkeitsabfrage, die Anzahl der Orts-Maschenpunkte und die Ordnung n der S_n-Rechnungen. Die mit TP1 berechneten Reaktivitätswerte werden mit entsprechenden Resultaten einer Diffusions-Störungsrechnung sowie mit den in der Literatur verfügbaren experimentellen Resultaten für das in den vorliegenden Rechnungen verwendete Reaktormodell verglichen.

Acknowledgement

The author wishes to express his thanks to Prof. K. Wirtz, Dr. R. Fröhlich and Alexander von Humboldt Stiftung for their support of his stay in INR of Kernforschungszentrum as a Humboldt Stipendiat.

He thanks to Dr. R. Fröhlich and Dr. Kiefhaber for their valuable discussions to complete this work.

In particular, appreciation is expressed to Mr. K. Küfner for his kind cooperation to transfer all data from GRUCAL and DTK codes to TP1 code, to Mr. D. Woll for his sincere help to make use of the results of the GRUCAL code, and to Mr. A. Polch for his calculation of the reactivity in Inhour unit by LAMBDA code.

The author wishes also to thank all members of INR for their help in making FORTRAN program and for their carrying the output papers from computer room to INR.

CONTENTS

I. INTRODUCTION

This report describes transport perturbation theory for reactivity, the details and the use of the one dimensional perturbation code TP1 and some numerical results.The transport perturbation theory is already established[1], and in chapt II is given a full detail of the equations for the calculation of the reactivity, effective neutron generation time and delayed neutron fraction in TP1 code.

The code TP1 uses the direct and adjoint angular fluxes from the one dimensional S_n transport code DTK. In the present version of this code, use is made of a fission neutron spectrum which is independent of the fissionable isotope. It is found that the correction factor to the criticality factor to take into account the effect of the isotope dependency of fission neutron spectrum and the delayed neutron spectrum can be calculated by using perturbation theory. This theory is also described in chapt II and is incorporated in the TP1 code in which the corrected criticality factor is printed by the input option. In chapt III are given the details of the program and input description, and in chapt IV sample calculations are given to demonstrate the accuracy of the reactivity with respect to the error criterion used in the DTK code, the effect of the number of mesh points and the order of S_n method. Comparison with the diffusion method and with the experimental results are also given there.

II. THEORY

1. Transport Equation

We write the transport equation for the perturbed system as /1/, /2/

$$\frac{1}{v_g}\frac{\partial f_g(\vec{r},\vec{\Omega}_m,t)}{\partial t} = (G + \frac{1}{k_o}F_p)f_g(\vec{r},\vec{\Omega}_m,t) + \sum_i \chi_{ig}\lambda_i C_i(\vec{r},t) \quad (1)$$

$$\frac{\partial C_i(\vec{r},t)}{\partial t} = \frac{1}{k_o}F_i f_g(\vec{r},\vec{\Omega}_m,t) - \lambda_i C_i(\vec{r},t) \quad (2)$$

where in operator notation

$$G = -(\vec{\Omega}_m\vec{\nabla}) - \hat{\Sigma}_{tg} + \sum_{m'}\Delta\Omega_{m'}\sum_{g'}\Sigma_s(\vec{\Omega}_m, g\leftarrow\vec{\Omega}_{m'}, g') \quad (3)$$

$$F_p = \frac{1}{4\pi}\sum_j \chi_g^j \sum_{g'}(\nu_p\Sigma_f)_{g'}^j \sum_{m'}\Delta\vec{\Omega}_{m'} \quad (4)$$

$$F_i = \frac{1}{4\pi}\sum_{g'}(\nu_d^i\Sigma_t)_{g'}\sum_{m'}\Delta\vec{\Omega}_{m'}, \quad (\nu_d^i\Sigma_t)_{g'} = \sum_j(\nu_d^i\Sigma_f)_{g'}^j \quad (5)$$

$$\hat{\Sigma}_{tg} = \Sigma_{tg} + D_g B^2, \quad \Sigma_{tg} = \Sigma_{cg} + \Sigma_{fg} + \Sigma_{sg} \quad (6)$$

and

k_o: effective multiplication factor which is determined to make unperturbed equation critical,

$\vec{\Omega}_m$: direction vector,

v_g: neutron speed of the g-th group,

fg: angular flux of the g-th group

C_i: delayed neutron precursor population of i-th delayed group

$\hat{\Sigma}_{tg}$: total cross section including leakage term approximated by using Buckling,

Σ_{tg}: total cross section,

Σ_{cg}: capture cross section,

Σ_{fg}: fission cross section,

Σ_{sg}: scattering cross section,

D_g: diffusion coefficient,

B^2: buckling,

$(\vec{\Omega}_m\vec{\nabla})$: leakage operator, but the leakage approximated by $D_g B^2$ should be treated separately,

$\Sigma_s(\vec{\Omega}_m, g \leftarrow \vec{\Omega}_{m'}, g')$: the scattering cross section from $\vec{\Omega}_{m'}$ and g' to $\vec{\Omega}_m$ and g,

$(\nu_p \Sigma_f)_g^j$: the fission cross section times the number of prompt neutrons produced by the isotope j,

$(\nu_d^i \Sigma_f)_g^j$: the fission cross section times the number of delayed neutrons of i-th delayed neutron group for the isotope j. If ν_d^i is independent of the fission energy, it can be written as $\beta_i^j \nu \Sigma_{fg}^j$, where β_i^j is a i-th group delayed neutron fraction for the isotope j.

χ_g^j: the fission spectrum of prompt neutrons of isotope j,

χ_{ig}: the delayed neutron spectrum produced by i-th delayed group precursor,

λ_i: the decay constant of i-th delayed group precursor.

The indices g, i, j and m always indicate the energy group, the delayed neutron group, fissionable isotope and the angular direction respectively. Summation over i always indicates sum over all delayed neutron group, j over all fissionable isotope and m over all angular directions. As boundary conditions for the angular flux, vacuum, reflective or periodic conditions can be used at the outer most boundary. The initial condition for Eqs. (1) and (2) is that the angular flux and precursor populations should be given at t=0.

We assume that for the unperturbed system, the following equations hold:

$$(G^o + \frac{1}{k_o} F_p^o) f_{kog}(\vec{r},\vec{\Omega}) + \sum_i \chi_{ig} \lambda_i C_i^o(\vec{r}) = 0 \qquad (7)$$

$$\frac{1}{k_o} F_i^o f_{kog}(\vec{r},\vec{\Omega}) - \lambda_i C_i^o(\vec{r}) = 0 \qquad (8)$$

where the suffix "o" is used to characterize the quantities of the unperturbed system. Eliminating $\lambda_i C_i^o(r)$ in Eq.(7) and using Eq. (8), we get

$$(G^o + \frac{1}{k_o} F^o) f_{kog}(\vec{r},\vec{\Omega}) = 0 \qquad (9a)$$

where

$$F = \frac{1}{4\pi} \sum_j \chi_g^j \sum_{g'} (\nu_p \Sigma_f)_{g'}^j + \sum_i \chi_{ig} \sum_{g'} (\nu_d^i \Sigma_f)_{g'} \sum_{m'} \Delta\vec{\Omega}_{m'} \qquad (10)$$

The static equation for the perturbed system is

$$(G + \frac{1}{k} F) f_{kg}(\vec{r},\vec{\Omega}) = 0 \qquad (9b)$$

Adjoint equations for Egs.(9) are

$$(G^{o+} + \frac{1}{k_o} F^{o+}) f_{kog}^+(\vec{r},\vec{\Omega}) = 0 \qquad (11a)$$

$$(G^+ + \frac{1}{k} F^+) f_{kg}^+(\vec{r},\vec{\Omega}) = 0 \qquad (11b)$$

where G^{o+}, F^{o+}, G^+ and F^+ are the adjoint operators of G^o, F^o, G and F respectively, and

$$G = G^o + \delta G \quad , \qquad (12)$$

$$F = F^o + \delta F \quad . \qquad (13)$$

Eqs. (9) and (11) are solved by DTK code using the boundary conditions for the angular flux: vacuum , reflective or periodic.

2. Perturbation Theory

Multiplying Eq.(9a) by f_{kg}^+ and Eq.(11b) by f_{kog}, subtracting the resulting equations and integrating over space and solid angle, we obtain the static reactivity exactly without making approximation,

$$\rho \equiv \frac{1}{k_o} - \frac{1}{k} = \frac{\langle f_k^+ (\delta G + \frac{1}{k_o} \delta F) f_{k_o} \rangle}{\langle f_k^+ F f_{k_o} \rangle} \tag{14}$$

where the notation < > indicates

$$\langle f_k^+ \rangle = \int dV \sum_m \Delta\Omega_m \sum_g f_{kg}^+ (\vec{r}, \vec{\Omega}_m) \tag{15}$$

There is also another definition of the reactivity, the asymptotic period reactivity defined by Henry /2/, which is discussed in Appendix I.

We can assume that the asymptotic flux can be expressed by

$$f_g(\vec{r}, \vec{\Omega}, t) = f_{\omega g}(\vec{r}, \vec{\Omega}) e^{\omega t} \quad , \tag{16}$$

and

$$C_i(\vec{r}, t) = C_{\omega i}(\vec{r}) e^{\omega t} \quad . \tag{17}$$

Substituting Eqs. (16) and (17) into Eqs. (1) and (2), we obtain

$$(G + \frac{1}{k_o} F) f_{\omega g}(\vec{r}, \vec{\Omega}) = (\frac{\omega}{v_g} + \frac{\omega}{k_o} \sum_i \frac{\chi_{ig} F_i}{\lambda_i + \omega}) f_{\omega g}(\vec{r}, \vec{\Omega}) \tag{18}$$

Multiplying Eq. (8) by $f_{kg}^+(\vec{r}, \vec{\Omega})$ and Eq. (11b) by $f_{\omega g}(\vec{r}, \vec{\Omega})$, taking the difference of the resulting two equations and integrating over space and solid angle, we obtain

$$\rho = \omega \bar{\Lambda} + \omega \sum_i \frac{\bar{\beta}_i}{\lambda_i + \omega} \quad , \tag{19}$$

where

$$\rho = \frac{1}{k_o} - \frac{1}{k} \quad , \tag{20}$$

$$\bar{\Lambda} = \frac{\langle f_k^+ \frac{1}{v} f_\omega \rangle}{\langle f_k^+ F f_\omega \rangle} , \qquad (21)$$

and

$$\bar{\beta}_i = \frac{\langle f_k^+ \chi_{ig} F_i f_\omega \rangle}{\langle f_k^+ F f_\omega \rangle} . \qquad (22)$$

Eq. (19) is the reactivity which gives the asymptotic period, if the static reactivity ρ is given. $\bar{\Lambda}$ and $\bar{\beta}_i$ may be called as the effective generation time and the effective fraction of the delayed neutron of i-th delayed group, respectively.

For the analysis of pulsed neutron experiments, other definitions for the generation time and the delayed neutron fractions are necessary. These are given e.g. in Ref./3/. Further, we can define other kinds of reactivities: the Reactivity Factors, the Inherent Reactivity Factor and the Reactivity Integral. They can be obtained by making small modifications to the subroutines of the TP1 code in which cross section differences and reactivities are calculated. These modified quantities might be helpful or specifically suited for the interpretation of certain kinds of experiments as mentioned e.g. in Ref./4/.

3. Explicit Expression

1) Definition of reactivities for each reaction process

From Eqs. (3)-(5), the numerator of Eq.(14) becomes

$$\delta G + \frac{1}{k_o} \delta F = -\delta \hat{\Sigma}_{tg} + \sum_{m'} \Delta\vec{\Omega}_{m'} \sum_{g'} \delta\Sigma_s(\vec{\Omega}_m, g \leftarrow \vec{\Omega}_{m'}, g')$$

$$+ \frac{1}{4\pi k_o} \sum_j \chi_g^j \sum_{g'} \delta(\nu_p \Sigma_f)_{g'} \sum_{m'} \Delta\vec{\Omega}_{m'}$$

$$+ \frac{1}{4\pi k_o} \sum_i \chi_{ig} \sum_{g'} \delta(\nu_d^i \Sigma_f)_{g'} \sum_{m'} \Delta\vec{\Omega}_{m'} \qquad (23)$$

We define the group and space dependent static reactivity for each reaction process as follows:

$$\rho_{cg}(\vec{r}) = -\delta\hat{\Sigma}_{cg} \sum_m \Delta\vec{\Omega}_m f^+_{kg}(\vec{r},\vec{\Omega}_m) f_{kog}(\vec{r},\vec{\Omega}_m)/[F] \quad \text{(capture)} \tag{24}$$

$$\rho_{fg}(\vec{r}) = -\delta\Sigma_f \sum_m \Delta\vec{\Omega}_m f^+_{kg}(\vec{r},\vec{\Omega}_m) f_{kog}(\vec{r},\vec{\Omega}_m)/[F] \quad \text{(fission)} \tag{25}$$

$$\rho_{rg}(\vec{r}) = -\delta\hat{\Sigma}_{tg} \sum_m \Delta\vec{\Omega}_m f^+_{kg}(\vec{r},\vec{\Omega}_m) f_{kog}(\vec{r},\vec{\Omega}_m)/[F] \quad \text{(removal)} \tag{26}$$

$$\rho_{sog}(\vec{r}) = -\delta\Sigma_s \sum_m \Delta\vec{\Omega}_m f^+_{kg}(\vec{r},\vec{\Omega}_m) f_{kog}(\vec{r},\vec{\Omega}_m)/[F] \quad \text{(scattering out)} \tag{27}$$

$$\rho_{sig}(\vec{r}) = \sum_m \Delta\vec{\Omega}_m f^+_{kg}(\vec{r},\vec{\Omega}_m) \sum_{m'} \Delta\vec{\Omega}_{m'} \sum_{g'=1}^{g} \Sigma_s(\vec{\Omega}_m g \leftarrow \vec{\Omega}_{m'}, g') f_{kog'}(\vec{r},\vec{\Omega}_{m'})/[F]$$

(scattering in) (28)

$$\rho_{oig}(\vec{r}) = \rho_{sog}(\vec{r}) + \rho_{sig}(\vec{r}) \quad \text{(scattering out + in)} \tag{29}$$

$$\rho_{sig\leftarrow g-1}(\vec{r}) = \sum_m \Delta\vec{\Omega}_m f^+_{kg}(\vec{r},\vec{\Omega}_m) \sum_{m'} \Delta\vec{\Omega}_{m'} \Sigma_s(\vec{\Omega}_{m'}, g \leftarrow \vec{\Omega}_{m'}, g-1) f_{kog-1}(\vec{r},\vec{\Omega}_{m'})/[F]$$

(scattering in from g-1) (30)

$$\rho_{fsg}(\vec{r}) = \sum_m \Delta\vec{\Omega}_m f^+_{kg}(\vec{r},\vec{\Omega}_m) \Big(\sum_j \chi^j_g \sum_{g'} \delta(\nu_p\Sigma_f)^j_{g'} + \sum_i \chi_{ig} \sum_{g'} \delta(\nu^i_d\Sigma_f)_{g'}\Big) \sum_{m'} \Delta\vec{\Omega}_{m'} f_{kog'}(\vec{r},\vec{\Omega}_{m'})/(4\pi k_o[F])$$

(fission source) (31)

$$\rho_{afg}(\vec{r}) = \sum_{g'} \sum_m \Delta\vec{\Omega}_m f^+_{kg'}(\vec{r},\vec{\Omega}_m) \Big(\sum_j \chi^j_{g'} \delta(\nu_p\Sigma_f)^j_g + \sum_i \chi_{ig'} \delta(\nu^i_d\Sigma_f)_g\Big) \sum_{m'} \Delta\vec{\Omega}_{m'} f_{kog'}(\vec{r},\vec{\Omega}_{m'})/(4\pi k_o[F])$$

(adjoint fission source) (32)

$$\rho_g(\vec{r}) = \rho_{rg}(\vec{r}) + \rho_{sig}(\vec{r}) + \rho_{fsg}(\vec{r}) \quad \text{(total)} \tag{33}$$

$$[F] = \frac{1}{4\pi} \sum_p \Delta V_p \Big(\sum_g \sum_m \Delta\vec{\Omega}_m f^+_{kg}(\vec{r}_p,\vec{\Omega}_m) \sum_j \chi^j_g\Big)\Big(\sum_{g'} (\nu_p\Sigma_f)^j_{g'} \sum_{m'} \Delta\vec{\Omega}_{m'} f_{kog'}(\vec{r}_p,\vec{\Omega}_{m'})\Big)$$
$$+ \frac{1}{4\pi} \sum_p \Delta V_p \Big(\sum_g \sum_m \Delta\vec{\Omega}_m f^+_{kg}(\vec{r}_p,\vec{\Omega}_m) \sum_i \chi_{ig}\Big)\Big(\sum_{g'} (\nu^i_d\Sigma_f)_{g'} \sum_{m'} \Delta\vec{\Omega}_{m'} f_{kog'}(\vec{r}_p,\vec{\Omega}_{m'})\Big) \tag{34}$$

The scattering out and scattering in means the scattering in and out with respect to the group and angle variables. Therefore, these terms include within group scattering term, that is different from the perturbation expression by the diffusion equation. In the TP1 code, the scattering cross section is calculated as the difference of the total cross section and absorption cross section. (Scattering out and in) in Eq.(29) means the net scattering.

Summation over group gives the space dependent, but group independent reactivity;

$$\rho_\alpha(\vec{r}) = \sum_g \rho_{\alpha g}(\vec{r}) \quad , (\alpha \text{ is a suffix for each process}) \tag{35}$$

Integration over space gives the group dependent, but space independent reactivity;

$$\rho_{\alpha g} = \sum_p \rho_{\alpha g}(\vec{r}_p)\Delta V_p \tag{36}$$

where ΔV_p is a volume element.

Summation over energy group and integration over space gives the net reactivity induced by the perturbation in the whole reactor;

$$\rho_\alpha = \sum_g \sum_p \rho_{\alpha g}(\vec{r}_p)\Delta V_p \quad \text{for each reaction.} \tag{37}$$

In the DTK code, the transport cross section is used for the total cross section. Therefore, the scattering cross section Σ_{sg} is replaced by $(1-\bar{\mu})\Sigma_{sg}$, where $\bar{\mu}$ is the average cosine of the scattering angle.

The scattering cross section should be expanded by the spherical harmonics functions in the same form as in DTK /5/ and SNOW /6/ code:

$$\Sigma_s(\vec{\Omega},g\leftarrow\vec{\Omega}',g') = \sum_{l=0}^{I_s} \frac{2l+1}{4\pi}\Sigma_{sl}(g\leftarrow g')P_l(\vec{\Omega}\cdot\vec{\Omega}')$$

$$= \frac{1}{4\pi}\sum_{l=0}^{I_s}(2l+1)\,\Sigma_{sl}\sum_{m=-l}^{l} Y_{lm}(\vec{\Omega})\,Y^*_{lm}(\vec{\Omega}') \qquad (38)$$

where $P_l(\mu)$ is Legendre polynomial of l-th order and

$$Y_{lm}(\vec{\Omega}) = \left[\frac{(l-m)!}{(l+m)!}\right]^{\frac{1}{2}} P_{lm}(\cos\theta)\; e^{im\phi} \qquad (39)$$

$$\int Y_{lm}(\vec{\Omega})\; Y_{l'm'}(\vec{\Omega})\; d\vec{\Omega} = \frac{4\pi}{2l+1}\,\delta_{ll'}\delta_{mm'} \qquad (40)$$

$$Y^*_{lm}(\vec{\Omega}) = (-)^m\, Y_{l,-m}(\vec{\Omega}) \qquad (41)$$

$P_{lm}(\mu)$ is an associated Legendre polynomial and the notation * indicates a complex conjugate. Similarly, we expand the angular flux in the form

$$f(\vec{r},\vec{\Omega}) = \frac{1}{4\pi}\sum_{l=0}^{\infty}(2l+1)\sum_{m=-l}^{l} f^{lm}(\vec{r})\; Y_{lm}(\vec{\Omega}) \qquad (42)$$

The spherical harmonics moment $f^{lm}(\vec{r})$ is given by

$$f^{lm}(\vec{r}) = \int d\vec{\Omega}\; Y_{lm}(\vec{\Omega})\; f(\vec{r},\vec{\Omega})$$

$$= \int d\vec{\Omega}\; [\,]^{\frac{1}{2}}\, P_{lm}(\cos\theta)(\cos m\phi - i \sin m\phi)\; f(\vec{r},\vec{\Omega})$$

$$\equiv f^{clm}(\vec{r}) - i f^{slm}(\vec{r}) \qquad (43)$$

In the one dimensional cylindrical geometry, the following symmetry excists for the angular flux as seen in Fig.1:

$$f_g(\vec{r},\theta,\phi) = f_g(\vec{r},\theta,-\phi) \qquad (44)$$

Using this relation in Eq.(42),

$$f(\vec{r},\theta,-\phi) = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1) \sum_{m=-l}^{l} f^{lm}(\vec{r})\ Y^{*}_{lm}(\vec{\Omega})$$

$$= \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1) \sum_{m=-l}^{l} f^{l,-m}(\vec{r})\ (-)^{m}\ Y_{lm}(\vec{\Omega}) \qquad (45)$$

From this equation, we obtain

$$f^{lm}(\vec{r}) = (-)^{m}\ f^{l,-m}(\vec{r}) \qquad \text{for cylindrical geometry of one dimension,} \qquad (46)$$

Since angular flux $f(\vec{r},\vec{\Omega})$ is real,

$$f(\vec{r},\vec{\Omega}) = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1) \sum_{m=-l}^{l} f^{*lm}(\vec{r})\ Y^{*}_{lm}(\vec{\Omega})$$

$$= \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1) \sum_{m=-l}^{l} f^{*l,-m}(\vec{r})\ (-)^{m}\ Y_{lm}(\vec{\Omega}) \qquad (47)$$

Then, the following relation holds for all geometries:

$$f^{l,m*}(\vec{r}) = (-)^{m}\ f^{l,-m}(\vec{r}) \qquad (48)$$

Therefore, only the real term is kept for the cylindrical geometry of one dimension;

$$f^{lm}(\vec{r}) = f^{lm*}(\vec{r}) \qquad (49)$$

The same result holds for $f^{+}_{kg}(\vec{r},\vec{\Omega})$.

2) Spherical harmonics functions

In the slab and spherical geometry of one dimension, only Legendre polynomials are used. In the cylindrical geometry, according to the reference /7/, we set (Fig.1),

$$\mu = \sin\theta \cos\phi \quad , \quad \eta = \sin\theta \sin\phi \quad , \quad \xi = \cos\theta \tag{50}$$

When $\phi=0$, $\mu=\sin\theta$, $\xi=\cos\theta$, then $\mu^2+\xi^2=1$ (51)

$$\text{In general, } \cos\phi = \frac{\mu}{\sin\theta} = \frac{\mu}{\sqrt{1-\xi^2}} \quad , \quad \mu^2+\eta^2+\xi^2 = 1 \tag{52}$$

$$\tan\phi = \frac{\eta}{\mu} = \frac{\sqrt{1-\mu^2-\xi^2}}{\mu} \tag{53}$$

These relations are identical with those given in Ref. /5/. The spherical harmonics functions defined by the following equation

$$\hat{Y}_{ln}(\theta,\phi) = \left[\frac{2(l-n)!}{(l+n)!}\right]^{\frac{1}{2}} P_{ln}(\xi_m) \cos n\phi_m \qquad \text{for } l \geq 1 \tag{54}$$

are transfered from the DTK code from the array CL(MM,NM). $\hat{Y}_{ln}(\theta,\phi)$ is assumed to be stored in the order shown in the Table 1.

3) Explicit expression for reactivity, effective neutron generation time and effective delayed neutron fraction

Using the expansion of scattering cross sections and angular flux of Eqs. (38) and (42), Eqs. (28), (30), (31), (32) and (34) become

$$\rho_{sig}(\vec{r}) = \sum_{g'=1}^{g-1} \sum_{l'=0}^{I_s} \frac{2l+1}{4\pi} \Sigma_{sl}(g\leftarrow g') \sum_{m=0}^{l} \epsilon_m \left(f_{kg}^{+clm}(\vec{r}) \cdot f_{kog'}^{clm}(\vec{r}) + f_{kg}^{+slm}(\vec{r}) \cdot f_{kog'}^{slm}(\vec{r})\right) / [F] \tag{55}$$

$$\rho_{sig\leftarrow g-1}(\vec{r}) = \sum_{l'=0}^{I_s} \frac{2l+1}{4\pi} \Sigma_{sl}(g\leftarrow g-1) \sum_{m=0}^{l} \epsilon_m (f_{kg}^{+clm}(\vec{r}) \cdot f_{kog-1}^{clm}(\vec{r}) + f_{kg}^{+slm}(\vec{r}) \cdot f_{kog-1}^{slm}(\vec{r}))/[F] \quad (56)$$

$$\rho_{fg}(\vec{r}) = f_{kg}^{+oo}(\vec{r}) \cdot (\sum_j \chi_g^j \sum_{g'} \delta(\nu_p \Sigma_f)_{g'}^j + \sum_i \chi_{ig} \sum_{g'} \delta(\nu_d^i \Sigma_f)_{g'}) f_{kog'}^{oo}(\vec{r})/(4\pi k_o[F]) \quad (57)$$

$$\rho_{afg}(\vec{r}) = \sum_{g'} f_{kg'}^{+oo}(\vec{r}) \cdot (\sum_j \chi_{g'}^j \delta(\nu_p \Sigma_f)_g^j + \sum_i \chi_{ig'} \delta(\nu_d^i \Sigma_f)_g) f_{kog}^{oo}(\vec{r})/(4\pi k_o[F]) \quad (58)$$

$$[F] = \frac{1}{4\pi} \sum_p \Delta V_p \sum_g f_{kg}^{+oo}(\vec{r}_p) \cdot (\sum_j \chi_g^j \sum_{g'} (\nu_p \Sigma_f)_{g'}^j + \sum_i \chi_{ig} \sum_{g'} (\nu_d^i \Sigma_f)_{g'}) f_{kog'}^{oo}(\vec{r}_p) \quad (59)$$

Approximating $f_{\omega g}(\vec{r},\vec{\Omega})$ by $f_{kog}(\vec{r},\vec{\Omega})$ in Eqs.(21) and (22), and using Eq.(42), Eqs.(21) and (22) become

$$\bar{\Lambda} = \sum_p \Delta V_p \sum_g \frac{1}{v_g} (\sum_m \Delta\vec{\Omega}_m f_{kg}^{+}(\vec{r}_p,\vec{\Omega}_m) f_{kog}(\vec{r}_p,\vec{\Omega}_m))/[F] \quad (60)$$

$$\bar{\beta}_i^j \equiv \sum_p \Delta V_p \sum_g (f_{kg}^{+oo}(\vec{r}_p) \chi_{ig}) \sum_{g'} ((\nu_d^i \Sigma_f)_g^j f_{kog'}^{oo}(\vec{r}_p))/[F] \quad (61)$$

$$\bar{\beta}_i = \sum_j \bar{\beta}_i^j \quad (62)$$

4) Dependence of the equation on geometry

The volume element ΔV_p, the weight of the solid angle ω_m and the number of unvanished spherical harmonics moment are dependent on geometry. $f_g^{sln}(\vec{r})=0$ for all one dimensional geometries.

a) Slab geometry

$$\Delta V_p = \Delta x_p, \quad \Delta\vec{\Omega}_m = \omega_m = 2\pi\Delta\mu_m \tag{63}$$

$$f_g^{cln}(\vec{r}) \neq 0 \quad \text{for } n=0, \qquad f_g^{cln}(\vec{r}) = 0 \quad \text{for } n\neq 0 \tag{64}$$

b) Cylindrical geometry

$$\Delta V_p = \pi(r_{p+1}^2 - r_p^2), \qquad \Delta\vec{\Omega}_m = \omega_m = \Delta\mu_m\Delta\phi_m \tag{65}$$

$$f_g^{cln}(\vec{r}) \neq 0 \quad \text{for } 1+n = \text{even}, \qquad f_g^{cln}(\vec{r}) = 0 \quad \text{for } 1+n = \text{odd}. \tag{66}$$

c) Spherical geometry

$$\Delta V_p = \frac{4\pi}{3}(r_{p+1}^3 - r_p^3), \qquad \Delta\vec{\Omega}_m = \omega_m = 2\pi\Delta\mu_m \tag{67}$$

$$f_g^{cln}(\vec{r}) \neq 0 \quad \text{for } n=0, \qquad f_g^{cln}(\vec{r}) = 0 \quad \text{for } n\neq 0 \tag{68}$$

5) A comment on the change of the buckling and size of reactor

If the height of the cylindrical reactor H_o, for example, is changed to H, the change of the leakage term approximated by using the buckling is

$$\delta(DB^2) = DB^2 - D^oB_o^2 = \frac{1}{3\Sigma_{tr}} \left(\frac{\pi}{H+2d}\right)^2 - \frac{1}{3\Sigma^o_{tr}} \left(\frac{\pi}{H_o+2d_o}\right)^2 , \qquad (69)$$

where the extrapolation length $d=0,71\ \lambda_{tr}$, $\lambda_{tr}= \frac{1}{\Sigma_{tr}}$.

Here we have assumed for simplicity reasons that the upper and lower extrapolation lengths are identical.

If the size of the reactor changes, for example, by the change of temperature, we can treat it in the perturbation calculation by considering the expanded system from the beginning and regard the outermost region as void or the region filled with some appropriate substance. Then the expansion of the system may be treated as the change of material filled, namely, the change of cross sections. Additional remarks on the treatment of movements of region boundaries can be found in the work of Stumbur /3/, /4/.

4. Correction to the Criticality Factor for Isotope Dependency of Fission Neutron Spectrum

The one dimensional S_n transport code DTK solves the equation

$$(G^o + \frac{1}{k_o} F^{o\prime}) f'_{kog}(\vec{r},\vec{\Omega}) = 0 \qquad (70)$$

for the unperturbed system, where an isotope independent fission spectrum χ_g is used in the fission source term of the present version of the code,

$$F'_o = \frac{1}{4\pi} \chi_g \sum_{g'} \nu \Sigma_{tg'} \sum_m \Delta\vec{\Omega}_m , \qquad (71)$$

instead of isotope dependent fission spectrum and delayed neutron spectrum in Eq.(10).

In order to calculate a correction to the criticality factor to take into account the isotope dependency of the fission neutron spectrum and the delayed neutron spectrum by the perturbation method, we consider the adjoint equation to Eq.(1),

$$(G^{o+} + \frac{1}{k_o} F^{o+})\ f^{+}_{kog}(\vec{r},\vec{\Omega}) = 0 \tag{72}$$

Multiplying Eq.(70) by f^{+}_{kog} and Eq.(72) by f'_{kog}, subtracting the resulting equations and integrating over space and solid angle, we obtain the correction factor ρ' as

$$\rho' \equiv \frac{1}{k_o'} - \frac{1}{k_o} = \frac{1}{k_o'} \frac{< f^{+}_{k_o} \delta F^{o} f'_{k_o} >}{< f^{+}_{k_o} F^{o} f'_{k_o} >} \tag{73}$$

where

$$\delta F^{o} = F^{o} - F^{o\prime} \tag{74}$$

$$F^{o} = \frac{1}{4\pi} \left[\sum_j \chi^{j}_{g} \sum_{g'} (\nu_p \Sigma_f)^{j}_{g'} + \sum_i \chi_{ig} \sum_{g'} (\nu^{i}_{d} \Sigma_f)_{g'} \right]^{o} \sum_{m'} \Delta\vec{\Omega}_{m'} \tag{75}$$

Therefore, if we calculate ρ' by Eq.(73), we can obtain the exact criticality factor k_o for unperturbed system by

$$k_o = \frac{k_o'}{1-\rho' k_o'} \tag{76}$$

In practice, we must use $f^{+\prime}_{k_o}$ in Eq.(73) which is obtained from the DTK code solving the equation

$$(G^{o+} + \frac{1}{k_o'} F^{o\prime+})\ f^{\prime+}_{kog}(\vec{r},\vec{\Omega}) = 0 \tag{77}$$

instead of $f^{+}_{k_o}(\vec{r},\vec{\Omega})$.

Usually, the difference δF^{o} is small and then the difference between $f^{+}_{k_o}$ and $f^{\prime+}_{k_o}$ is also small. Therefore the equation

$$\rho' = \frac{1}{k_o'} \frac{\langle f_{k_o}'^{+} \delta F^o f_{k_o}' \rangle}{\langle f_{k_o}'^{+} F^o f_{k_o}' \rangle} \tag{78}$$

may give a value for ρ' which is sufficiently accurate for practical purpose. This fact is numerically confirmed by the sample calculation as shown in chap. IV.

In the TP1 code, ρ' is calculated by the equation

$$\rho' = \frac{1}{k_o'} \frac{\langle f_k'^{+} \delta F \, f_{k_o}' \rangle}{\langle f_k'^{+} \, F \, f_{k_o}' \rangle} \quad , \quad \delta F = F - F^{o\,\prime} \tag{79}$$

where F is the fission operator including isotope dependent fission spectrum and $f_k'^{+}$ is the adjoint angular flux for perturbed system with the fission operator F'. Therefore, the perturbation should be small to obtain ρ' accurately ; namely, if the perturbation is small enough, Eq.(79) becomes a close approximation to Eq.(78).

III. COMPUTER PROGRAM

1. Problem Solved by TP1 Code

One dimensional S_n transport perturbation code TP1 calculates two cases: Exact perturbation and first order perturbation. TP1 reads direct and adjoint angular flux and cross section tabe from the disk (IDTK) written by the DTK S_n code. Further, fission spectrum, fission cross sections of isotope dependent and delayed neutron data are read from the disk (IGRUC) written by the group constant generation code GRUCAL /8/ and the SIGMUT code /9/ by which the delayed neutron data is arranged. One cross section table used in DTK code should contain cross sections for all mixtures and probe cross sections which are used in the perturbation calculation by TP1.

If the direct angular flux is calculated using unperturbed cross section by Eq.(70), and adjoint flux using perturbed cross section by Eq.(77), in DTK code, and if the option of exact perturbation is chosen in TP1, the exact reactivities defined by Eqs.(24)-(33) are calculated. In this case, the net reactivity calculated from both criticality factors by $\rho = \frac{1}{k_o} - \frac{1}{k}$ should, in principle, become identical to the one by the perturbation calculation of Eq.(39). However, there might be a small difference between them. In TP1 code the fission source term F is calculated using isotope dependent prompt fission neutron spectrum and delayed neutron spectrum as seen in Eq.(10). On the other hand, in the source term of Eq.(70) and (77) of the DTK code, only one isotope independent prompt neutron fission spectrum is used for all regions and mixtures of the reactor. This may give rise to a small difference, perhaps less than 0,1%, between both reactivities obtained by the criticality factors and perturbation calculations.

If the direct and adjoint angular fluxes are calculated using the same cross sections, and if the option of probe perturbation is chosen, the reactivity due to the change of the original cross sections by the mixture of the probe is calculated in the specified region by the first order approximation. In the code, the difference of the cross sections between that used for the calculation of direct angular flux and the probe mixture cross section

is used to calculate the probe reactivity. If the cross sections used for adjoint angular flux is different from the one used for direct angular flux, however, if the difference is small, the reactivity of the probe perturbation may be obtained in good accuracy.

All reactivity obtained by Eqs.(24)-(33) are printed in group and space dependent form according to the option choosen in the TP1 input. The effective delayed neutron fraction $\bar{\beta}_i$ and isotope dependent fraction $\bar{\beta}_i^j$ given by Eqs.(61) and (62), and generation time given by Eq.(60) are always computed and printed.

A corrected criticality factor which takes into account the isotope dependency of fission neutron spectrum and the delayed neutron spectrum is calculated and printed by the input option.

2. Program Chain and Data Flow for TP1 Calculation

The computation is performed by using the codes GRUCAL, SIGMUT, DTK, which are modules of the KAPROS system, and the transport perturbation code TP1 in the following order.

1) GRUCAL — computes all specified cross sections in the form of SIGMN file for the unperturbed and perturbed systems, and writes the SIGMN file on a disk, IGRUC.

2) SIGMUT — reads the delayed neutron data from card input and computes delayed fission cross section and adds this to SIGMN file on the disk by using KAPROS utility.

3) DTK — computes first the direct angular flux and criticality factor and then the adjoint angular flux and criticality factor by using the cross sections from the disk written by GRUCAL, and writes them on the disk IDTK. This interface file is created only by the latest version of DTK within KAPROS system /10/.

4) TP1 — reads the direct and adjoint angular flux and cross sections from the disk written by DTK. Adjoint angular flux is calculated and written in DTK code in inverse order with respect to the group and angle indexes. Therefore,

at the beginning of the computation, the order of the adjoint angular flux read from the disk is inverted with respect to group and angle indexes. TP1 reads also isotope dependent fission cross sections and delayed neutron data from the disk written by the GRUCAL and the SIGMUT codes. Then exact perturbation or probe perturbation calculations are performed.

3. Explanation of the Program TP1

1) Subroutines used in TP1

The following subroutines are used in TP1

a) DUMMY: Dimensions of the working arrays A and IA are declared.

b) RDINC: All input data to TP1 are read from cards.

c) REDTK1: Following data are read from the disk IDTK written by DTK. After reading, these are printed. All notations given here and in the following follow closely those of the DTK code /3/.

ISCT: Order of anisotropic scattering

ISN: Quadrature order, 2,4,6,..., etc.

IGE: Geometry (1/2/3 = plane/cylinder/sphere)

IZM: Number of zones

IM: Number of intervals

IGM: Number of groups

MT: Total number of cross section table

MTP: Number of mixtures

NM: Number of spherical harmonics moments. NM=1 for ISCT=0, and NM=ISCT for ISCT$\geq$1 and IGE=1,3. NM=ISCT(ISCT+4)/4 for ISCT$\geq$1 and IGE=2.

MM: =ISN+1, if IGE=1,3. MM=ISN(ISN+4)/4, if IGE=2.

IHS: Location of selfscattering cross section, (IHS=6)

IHT: Location of total cross section, (IHT=5)

IHM: Length of cross section table, (IHM=IHS+IGM-1)

MBK: Index for buckling

BF: Buckling factor

XT: $= 2Z_o = 2 \times 0{,}7104$

d) REDTK2: If MBK$\neq$0, the following data are read from the disk IDTK written by DTK, first for unperturbed and next for perturbed system.
If MBK=0, this record does not exist on the disk.

DY: Buckling height (in cm) for slab and cylindrical geometry if MBK=1

DZ: Buckling height (in cm) for slab geometry, if MBK=1

BK(IGM): If MBK=-1 or -11

BK(IZM,IGM): If MBK=-2 or -12

e) REDTK3: Following data are read from the disk IDTK written by the DTK.

MA(IM): Integers defining which zone a space cell is in.

IAL(MTP): Mixture index

W(MM): W_m, Quadrature weight for the integration over solid angle.

CL(MM,NM): $=\sqrt{2}\left[\frac{(1-n)!}{(1+n)!}\right]^{\frac{1}{2}} P_{1n}(\mu_m)\cos n\phi$, spherical harmonics functions.

V(IM): ΔV_i, volume element

R(IMP): r_i, Meshpoint, IMP=IM+1. After reading r_i, R(I) is replaced by $R(I)=(r_i+r_{i+1})/2$.

IPLATZ(MTP): Location of o-th order scattering cross section in cross section table C.

C(IHM,IGM,MT): Cross section table.

XKI(IGM): Fission spectrum used in DTK code.

f) REDTK4: MZ(IZM), location of isotropic scattering cross section of cross section table, are read from the disk IDTK written by DTK first for unperturbed ans next for perturbed system, and IZ(IZM) are computed for unperturbed and perturbed system.

g) REDTK5: First criticality factor k_0 and angular flux FKO(IMP,MM,IGM) for unperturbed system and then those for perturbed system are read from the disk IDTK written by the DTK.

h) LIES2: This subroutine is taken from DTK code, and is rewritten such that the program for the preparation of cross section table C is deleted and only the SIGMN file prepared by GRUCAL and SIGMUT is read from the disk file IGRUC to the core memory to prepare delayed neutron data for READD.

i) WQORG: This subroutine is the same as that used in the DTK code. This code is used in READD to read cross sections from SIGMN file.

j) READD: Following data are read from the core memory prepared by LIES2.

SNFTJ(IGM,IFM,MTP) = $(\nu\Sigma_f)_g^j$: Number of total fission neutrons times fission cross section of j-isotope,

SNFDJ(IGM,IDM,IFM,MTP) = $(\nu_d^i\Sigma_f)_g^j$: Number of i-th group delayed neutrons times fission cross section of j-isotope,

XKIJ(IGM,IFM,MTP) = χ_g^j : Prompt fission spectrum of j-isotope,

DKI(IGM,IDM) = χ_{ig} : Delayed neutron spectrum of i-th delayed neutron group,

where MTP, IFM and IDM are Number of mixtures, Number of fissile isotope and Number of delayed neutron groups, respectively.

After reading these data, the following cross sections are computed.

$$SNFP(IGM,MTP) = (\nu_p\Sigma_f)_g = \sum_j (\nu_p\Sigma_f)_g^j$$

$$SNFPJ(IGM,IFM,MTP) = (\nu_p\Sigma_f)_g^j$$

$$SNFD(IGM,IDM,MTP) = (\nu_d^i\Sigma_f)_g = \sum_j (\nu_d^i\Sigma_f)_g^j$$

k) CLEAR: This subroutine is used to make array set zero.

l) PRINT1: Print one dimensional array,

m) PRINT2: Print two dimensional array

n) PRINT3: Print three dimensional array.

o) PERT1: Denominator [F], effective generation time and delayed neutron fractions are computed.

p) PRRT: Reactivities for each process are computed and printed.

q) BUCK: Buckling leakage term is computed.

r) INVAF: Adjoint angular flux is inversely reordered with respect to the indices for group and angular direction.

2) Buckling correction

The loss of neutrons by leakage can be treated approximately by using a Buckling. This is done in the following way:

$$DB^2 = D(B_y^2 + B_z^2) = \frac{1}{3\Sigma_{tr}}\left[\left(\frac{\pi}{L_y+2Z_o\lambda_{tr}}\right)^2 + \left(\frac{\pi}{L_z+2Z_o\lambda_{tr}}\right)^2\right]$$

$$= \frac{\pi^2}{3}\Sigma_{tr}\left[\frac{1}{(L_y\Sigma_{tr} + 2Z_o)^2} + \frac{1}{(L_z\Sigma_{tr} + 2Z_o)^2}\right]$$

for x-y-z geometry, where Z_o=0,7104 and $\lambda_{tr} = \frac{1}{\Sigma_{tr}}$, and L_y and L_z are

the thicknesses of the reactor to x and y directions in units of cm. The leakage in the r-direction in r-z geometry is

$$DB^2 = \frac{1}{3\Sigma_{tr}} \left(\frac{2,405}{r_o + Z_o\lambda_{tr}}\right)^2 = \frac{2,405^2}{3} \Sigma_{tr} \frac{1}{(r_o\Sigma_{tr} + Z_o)^2}$$

In the code, the following notations are used:

$$SRI(IZM,IGM) = BF^2 \cdot AZ \left[(DY \cdot AZ + XT)^{-2} + (DZ\Lambda Z + XT)^{-2}\right]$$

where $AZ = \Sigma_{tr}$,

$BF = \frac{\pi}{\sqrt{3}}$, $XT = 2Z_o$, $DY = L_y$, $DZ = L_z$ for x-y-z geometry,

and

$BF = \frac{2,405}{\sqrt{3}}$, $XT = Z_o$, $DY = r_o$ for cylindrical geometry.

3) Arrays used in TP1

NAIST(2*IFM):	Name of fissile isotope.
MA(IM):	Integers defining which zone a space cell is in.
MZ(IZM):	Location of the o-th moment of scattering cross section in the cross section table C for each zone (unperturbed system)
MZP(IZM):	MZ(IZM) for perturbed system
IAL(MTP):	Mixture index
IPLATZ(MTP):	Location of the o-th moment of scattering cross section in the cross section table C for each mixture.
IZ(IZM):	Mixture index for each zone (unperturbed system)
IZP(IZM):	IZ(IZM) for perturbed system
IMTN(IM):	Indices of mesh intervals where the probe is inserted.

VE(IGM) = $1/v_g$,

XKI(IGM) = χ_g, Isotope independent fission spectrum used in DTK.

C(IHM,IGM,MT): cross section table,

$C(1,G,N)=\Sigma^m_{trg}$, $C(2,G,N)=\Sigma^m_{fg}$

$C(3,G,N)=\Sigma^m_{ag}$, $C(4,G,N)=\nu\Sigma^m_{fg}$

$C(5,G,N)=\Sigma^m_{tg}$, $C(IHS+G-G',G,N)=\Sigma^m_{so}(g\leftarrow g')$

$C(IHS+G-G',G,N+L)=\Sigma^m_{sl}(g\leftarrow g')$ for $1\leq L\leq ISCT\leq 6$,

where L=MA(I), M=ABS(IZ(L)) and N=IPLATZ(M), and I is an index for space mesh point. M=m is mixture index. In the present code, IHS=6.

SNFDJ(IGM,IDM,IFM,MTP) = $(\nu^i_d\Sigma_f)^j_g$

SNFD(IGM,IDM,MTP) = $(\nu^i_d\Sigma_f)_g = \sum_j(\nu^i_d\Sigma_f)^j_g$

SNFTJ(IGM,IFM,MTP) = $(\nu_t\Sigma_f)^j_g$

SNFPJ(IGM,IFM,MTP) = $(\nu_p\Sigma_f)^j_g$

SNFP(IGM,MTP) = $(\nu_p\Sigma_f)_g = \sum_j(\nu_p\Sigma_f)^j_g$

XKIJ(IGM,IFM,MTP) = χ^j_g

SDK(IDM): Temporally

SMFD(IDM): Temporally

SMFDJ(IFM,IDM,MTP): Temporally

SMDFD(IDM): Temporally

BETAJ(IFMP,IDMP,MTP1): $\bar{\beta}^j_i$, IFMP=IFM+1, IDMP=IDM+1, MTP1=MTP+1

DKI(IGM,IDM) = χ_{ig}

R(IMP) = $\frac{1}{2}(r_i + r_{i+1})$

V(IM) = ΔV_i

W(MM) = ω_m

CL(MM,NM) = $\left[\frac{2(l-n)!}{(l+n)!}\right]^{\frac{1}{2}} P_{ln}(\xi_m)\cos n\phi_m$ for $l\geq 1$.

FKD(IMP,MM,IGM) = $f_{kog}(r_i,\vec{\Omega}_m)$. After the angular fluxes at the mesh points are read from

the disk IDTK, the average values for mesh intervales are calculated;

$$FKO(I,M,G) = \frac{1}{2}(f_{kog}(r_i,\vec{\Omega}_m) + f_{kog}(r_{i+1},\vec{\Omega}_m)).$$

FKD(IMP,MM,IGM) = $f^{+}_{kg}(r_i,\vec{\Omega}_m)$

FTKO(IM,IGM) = $f^{oo}_{kog}(r_i)$

FTKD(IM,IGM) = $f^{+oo}_{kg}(r_i)$

BK(ISP) = B_g^2 , ISP=IGM , group dependent buckling, if MBK=-1 or -11, (unperturbed system)

BK(ISP) = $B_g^2(r)$, ISP=IZM·IGM, group and zone dependent buckling, if MBK=-2 or -12. (unperturbed system)

BKP(ISP): BK(ISP) for perturbed system

LSRIP(ISPP): = $D_g B_g^2$, ISPP=IZM·IGM, if MBK≠0. (unperturbed system)

LSRI(ISPP): LSRI(ISPP) for perturbed system.

RG(10,IGM): Reactivity for each process.

SMXD(IDM): Temporally,

SMXP(IFM): Temporally,

U(IFM): Temporally,

UP(IFM): Temporally.

4) Total size of a core memory

The integer array IA(LEN) and the real array A(LAENGE) are declared in the subroutine DUMMY.

The number of the core storage for them can be given by
LEN = 36+2IFM+2IM+4IZM+2MTP for IA,
and

LAENGE = LMAX+IRIS for A,

where LMAX $\doteq$ 2IMP·IGM·(MM+1)+IHM·IGM·MT+IGM(IDM+1)(IFM+1)MTP .

IFM: Number of fissile isotopes,

MTP: Number of mixtures,

MT: Total number of cross section tables,

IHM =IGM+5,

IGM: Number of groups,

IM: Number of space mesh points,

IMP =IM+1,

IDM: Number of the delayed neutron groups,

IZM: Number of zones.

The last IRIS words of the array A are used by the WQORG as working array to prepare the cross section table C(IHM,IGM,MT). Therefore, IRIS calculated as a difference between LAENGE and actually needed core storage should not be too small. In the case of sample problems shown in chapt. IV, the amount of core storage needed by TP1 is nearly twice of that used in the DTK calculation.

4. Input Description

1) Input data to GRUCAL

Type, CHI, NUSF, SCAPT, SFISS, SBE, SREM, STR, STRTR, 1/V, STOT and SMTOT are necessary. Further option of 'AUSWERT', three 'ZUSATZ' CHI(micro), NUSF(macro) and SFISS(macro) are necessary to produce isotope dependent quantities for fissile isotope. All fissile isotopes should be assumed to be contained in all mixtures, because isotope dependent fission spectrum and cross sections are stored in the arrayes XKIJ(IGM,IFM,MTP), SNFDJ(IGM,IDM,IFM,MTP) and SNFTJ(IGM,IFM,MTP). The option of microscopic fission spectrum is necessary for the CHI. The input example is shown in the Appendix IV.

2) Input data to SIGMUT

Isotope independent delayed neutron spectrum for each delayed group and the delayed neutron fraction for each isotope should be read from cards.

3) Input data to DTK

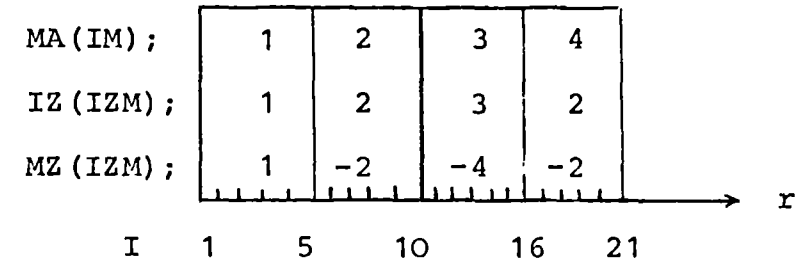
The option of k_{eff} computation and ITH=-1 should be chosen to create the interface file for TP1. An error criteriom 10^{-5} is desirable for the criticality factor and for the flux. For the adjoint caculation, only an input specification card is necessary for the adjoint equation. All other specifications, for example, the number of mesh points, the order of S_N and error criterions, are automatically reproduced to the adjoint calculation in the KAPROS system from the input data of the direct equation.

An example of the input data to DTK code is shown for the exact perturbation calculation.

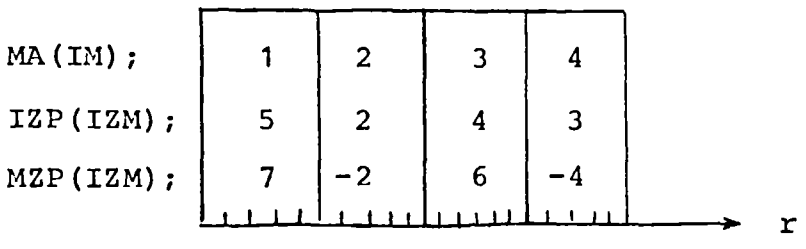
```
ISCT = 1  ;  Order of anisotropic scattering,
IM   = 20 ;  Total number of intervals,
IZM  = 4  ;  Number of zone,
MT   = 7  ;  Total number of cross section table,
MTP  = 5  ;  Number of mixtures.

IAL (MTP)      ;  1, -2, -3, 4, 5
IPLATZ(MTP)    ;  1, 2, 4, 6, 7
```

unperturbed system



perturbed system



Mixtures 1, 3 and 2 for the first, third and forth zone of the unperturbed system are replaced in the perturbed system by mixtures 5, 4 and 3 respectively.

4) Input data for TP1

Card	Variable		Meaning
K1	TITLE(18) (18A4)		Information
K2	IFM(24I3),		Number of fissile isotope
	IDM,		Number of delayed neutron group
	IFEX	1	Exact perturbation is calculated.
		0	This is not done.
	IFGS	1	Group and space dependent reactivity is printed.
		0	This is not done.
	IFS	1	Space dependent reactivity is printed.
		0	This is not done.
	IFG	1	Group dependent reactivity is printed.
		0	This is not done.
	IFPP	1	The first order perturbation by the probe is calculated.
		0	This is not done.
	IFPG	1	Global input of perturbed region (K4)
		0	Pointwise input of perturbed region (K5)
	IFK	1	Corrected criticality factor for isotope dependent fission spectrum is calculated.
		0	This is not done.
	IPAF	1	Direct and adjoint angular flux are printed
		0	This is not done.
	IPFX	1	Direct and adjoint total flux are printed.
		0	This is not done.
	IPCS	1	Cross sections are printed.
		0	This is not done.
K3	NAIST(IFM) (9A8)		Name of fissile isotope

SI If IFFP=1 and IFPG=1, K3, if IFFP=1 and IFPG=0, K4 is read repeatedly for all cases as needed.

K4	NPR, (24I3)	Mixture index of the probe which replaces the mixture of unperturbed system.
	ML,	Left mesh index where the probe is inserted.
	MR,	Right mesh index where the probe is inserted.
K5	NPR, (24I3)	Mixture index of the probe which replaces the mixture of the unperturbed system at the mesh points IMTN(ITNMI).
	ITNMI,	Total number of mesh intervals.
	IMTN(ITNMI),	Indices of mesh intervals where the probe is inserted.

5. Output Description

The mean neutron generation time and the delayed neutron fraction for each fissionable isotope and for each mixture are always printed. The criticality factor corrected for the isotope dependency of fission neutron spectrum is printed according to the input option. The space and energy group dependent reactivities can be printed by the input option in the following order; mesh index, average radius, group index, reactivities due to capture $\rho_{cg}(\vec{r}_i)$, fission $\rho_{fg}(\vec{r}_i)$, removal $\rho_{rg}(\vec{r}_i)$, scattering out $\rho_{sog}(\vec{r}_i)$, scattering in $\rho_{sig}(\vec{r}_i)$, scattering out + in $\rho_{sog}(\vec{r}_i) + \rho_{sig}(\vec{r}_i)$, scattering in from g-1 $\rho_{sig\leftarrow g-1}(\vec{r}_i)$, fission source $\rho_{fsg}(\vec{r}_i)$, adjoint fission source $\rho_{afg}(\vec{r}_i)$ and total $\rho_g(\vec{r}_i)$. The mixture dependent delayed neutron fraction is written on the disk IDSK4 for the calculation of the effective decay constant of the delayed neutron by the LAMBDA code made by Polch /11/.

IV NUMERICAL EXAMPLES

1. Sample Calculations for ZPR-3, Assembly 48

Sample calculations are performed for ZPR-3 assembly 48, a plutonium fueled fast critical assembly, of fast reactor benchmark

No.3 /11/. Number density for each isotope is taken from Table I of Ref. /11/ to the spherical model and is shown in Table 2. Since the atom Mn is not available in the group constants library GRUBA used by the GRUCAL program, the number density of the atom Cr is simply increased by adding that of Mn. Core radius of 45.245 cm and blanket thickness of 30.0 cm given in Table 2 are used. A reflected boundary condition is used at the origin and a vacuum condition at the outermost boundary. The sample calculations are also performed for the slab reactor which is derived simply by replacing the index for the spherical geometry by the slab geometry in the DTK code. Therefore, in the slab reactor, the criticality factor exceeds unity. An average fission spectrum for the DTK code is calculated from the isotope dependent fission spectrum χ_g^j using weight ω^j by

$$\chi_g = \frac{\sum_j \omega_j \chi_g^j}{\sum_j \omega_j} \tag{80}$$

in the GRUCAL code. Use is made of the weights which are derived using the flux determined by the diffusion calculation for the core /9/. The weights for the blanket ω^j(Blanket) are calculated by the crude approximation

$$\omega^j(\text{Blanket}) = \omega^j(\text{core}) \times \frac{n^j(\text{Blanket})}{n^j(\text{core})} \tag{81}$$

where n^j is the number density of j-th fissionable isotope. The weights used in the calculations are shown in Table 3. Results of sample calculations are shown in Table 4~9. In the Tables, k_o and k^+ are the criticality factors for direct and adjoint equations respectively. Mixture indices 1 and 3 indicate the mixtures prepared by using the number density of the core and by the number density of the blanket given in the Table 2 respectively. Mixture 2 is obtained by multiplying the number density of Pu^{239} in mixture 1 by 1.01. The number densities of other isotopes in the mixture 2 are the same as those of the mixture 1. The net reactivity ρ_k is calculated from the criticality factors k_o and k^+ by Eq.(20) and the net reactivity ρ_p by the perturbation equation of Eq.(37). $\bar{k}$ is the corrected criticality factor of k_o by taking into account the isotope dependent fission spectrum using ρ' of

Eqs.(73) and (76). Computations are performed by using IBM370-168 in Kernforschungszentrum Karlsruhe.

In the DTK code, there is a negative fix up routine in which the angular flux is set equal to zero, if the angular flux becomes negative. Since it might be considered that this method introduces some complication for the understanding of the numerical results, this negative fix up routine is skipped for the perturbation calculation. In all sample calculations, use is made of the options of Chebyshew extrapolation for the outer iterations and the initial guess is provided by the code.

In Table 4, use is made of the error criterions for the flux, 0, 10^{-3} and 10^{-4} to check the effect of the error criterion on the flux. (In the case of the error criterion 0, the flux convergence test is skipped). As the error criterion to the criticality factor, 10^{-5} is used in all cases. From Table 4, it is seen that the criticality factor for the direct equation converged within the error of 10^{-5} independent of the error criterion for the flux. The criticality factor for direct and adjoint equations for the slab geometry should be identical for the same system. This is numerically confirmed in case 5 for mixtures 2 and 3. However, the criticality factor for the adjoint equation has errors of 2~4 $\times$ 10^{-4} in the case of no error criterion for the flux, 1 $\times$ 10^{-4} in the case of the error criterion of 10^{-3} for the flux. In case 5, the criticality factor of adjoint equation converged to the exact value within the error of 10^{-5} as stated above, however, in case 6 of Table 4 there is yet an error of 2 $\times$ $1o^{-4}$. It may seem curious that there are some cases in which the number of outer iterations and CPU time for the error criterion of 10^{-4} are less than for no error criterion for the flux. From this result, it is known that the error criterion of 10^{-4} for the flux is not sufficient to obtain the criticality factor with the accuracy of 10^{-5}. Therefore, in the following calculations, an error criterion of 10^{-5} is used for the flux as well as for the criticality factor.

In Tables 5 and 6 are shown the results for the slab and spherical geometry to check the mesh effect. It is seen that if the mesh intervals of 28 and 16 are used for the core and blanket, an accuracy of 1 $\times$ 10^{-4} is obtained for the criticality factor. In the benchmark book of Ref. /11/, a mesh of 2 cm which gives the mesh intervals of

23 and 15 for the core and blanket is recommended for the diffusion calculation. In the case of slab reactor, criticality factor of the direct equation agrees to that of the adjoint equation within the accuracy of 10^{-5} for the cases 1-4. As the number of mesh intervals increases, this difference becomes larger, 3×10^{-5} for 14+8 mesh intervals, and 5×10^{-5} for 28+16 mesh intervals.

In the case of the spherical reactor, there is a much larger difference, about 10^{-3} between the criticality factors of the direct and adjoint equations. This introduces a large error to the reactivity calculated from the criticality factors. As seen in the Tables 5 and 6, the reactivity from the perturbation method has higher accuracy, 1×10^{-6} for the slab reactor and 3×10^{-6} for the spherical reactor, than that from the criticality factors, 5×10^{-5} for the slab reactor and 8×10^{-4} for the spherical reactor in absolute value.

The difference of criticality factors between the direct and adjoint equations for curved geometry may be due to the non adjointness of the finite difference form of the adjoint equation for curved geometry, because this difference becomes less as the order of S_n increases as seen in Table 7.In the case of S_{16}, there is no difference between them, and then the reactivity calculated from the criticality factors agrees with that from the perturbation calculations within the error of 10^{-5}.

The reactivity from the S_4 method of the perturbation method agree to that from the S_{16} method with the accuracy of 10^{-5}, wheras the reactivity from the S_2 method has an error of 8×10^{-4} compared with the S_{16} method. In all cases of the S_2, S_4 and S_8 methods, the perturbation method gives more accurate results than that from the criticality factors.

In the DTK code, the approximate fission source term of Eq.(71) is used instead of exact one of Eq.(10). In Table 8 are shown the corrected criticality factors by Eqs.(76) and (79) for the several weights to the average fission spectrum in the GRUCAL code. The computations in DTK given in Table 8 are performed using S_8 method with 28+16 mesh intervals. In the case 1 of Table 8, use is made of the fission spectrum in DTK from the GRUCAL in which only the

weight of the fission spectrum for the isotope Pu^{239} is unity and all other weights are zero. The case 4 is the another extreme case, in which only the the weights for the isotope U^{238} are unity and all other weights are zero. This case gives largest correction factor to the criticality factor about 0.4%. Since all corrected criticality factors agree within the accuracy of 10^{-5}, this correction seems reasonably done to take into account the effect of isotope dependent fission spectra and delayed neutron spectra. More accurately, following Eq.(78), the adjoint flux for the unperturbed system should be used in Eq.(79) for the correction of this effect. This effect of the approximation is seen in the column 6 of the Table 9, in which the results are shown to demonstrate how small reactivity can be calculated by the perturbation code.

From the cases 1 to 4 of Table 9, the number density of Pu^{239} in the perturbed core is increased by 1.01, 1.oo1 and 1.ooo1 from the unperturbed core, where as all other number densities are the same as those in the unperturbed core. The adjoint flux of case 3 of Table 9 may be almost the same as that of the unperturbed equation. Then the criticality factor of $\bar{k}$ of case 3, which is less than the previous value by 2×10^{-5}, may be most accurately corrected for the isotope dependent fission spectra.

It is seen that the reactivities calculated by the perturbation method change almost linearly as the change in number density of Pu^{239}. On the other hand, the reactivity from the criticality factors calculations does not change linearly and has an error of about 2×10^{-4} as pointed out before. From the case 4 to 7, the number density of Na in the perturbed core is changed by 0.99, 0.9, 0.5 and 0 from the unperturbed core to see the Na void effect, keeping all other isotopes unchanged. In this case, also, the reactivity calculated by the perturbation method changes almost linearly as the change of the number density of Na.

The numerical error of the reactivity calculated by the perturbation method, $\Delta\rho$ can be expressed as

$$\left|\frac{\Delta\rho}{\rho}\right| \leq \left|\frac{\Delta k}{k_o}\right| + \left|\frac{\Delta\langle f_k^+ F f_{k_o}\rangle}{\langle f_k^+ F f_{k_o}\rangle}\right| + \left|\frac{\Delta\langle f_k^+ \delta G f_{k_o}\rangle}{\langle f_k^+ \delta G f_{k_o}\rangle}\right| + \left|\frac{\Delta\langle f_k^+ \delta F f_{k_o}\rangle}{\langle f_k^+ \delta F f_{k_o}\rangle}\right| \qquad (82)$$

from Eq.(14), where ΔX signifies the numerical error of X. Because of mesh effect, $|\frac{\Delta k}{k_o}| \simeq 10^{-4}$ in the present case. The neutron balance in the DTK calculations is of the order of 4×10^{-4} for the present cases. Then, the error of the second term may be

$$|\frac{\Delta<f_k^+ F f_{k_o}>}{<f_k^+ F f_{k_o}>}| = 10^{-3} \tag{83}$$

Roughly, we may write

$$|\frac{\Delta<f_k^+ \delta G f_{k_o}>}{<f_k^+ \delta G f_{k_o}>}| + |\frac{\Delta<f_k^+ \delta F f_{k_o}>}{<f_k^+ \delta F f_{k_o}>}| \doteq |\frac{\Delta<f_k^+ f_{k_o}>}{<f_k^+ f_{k_o}>}| + |\frac{\Delta\delta\Sigma}{\delta\Sigma}|$$

$$\doteq 10^{-3} + |\frac{\Delta\delta\Sigma/\Sigma}{\delta\Sigma/\Sigma}| \tag{84}$$

where Σ is the typical perturbed cross section. As a result,

$$|\frac{\Delta\rho}{\rho}| \leq 10^{-3} + |\frac{\Delta\delta\Sigma/\Sigma}{\delta\Sigma/\Sigma}| \tag{85}$$

If the perturbation of the cross section is of the order of $\frac{\delta\Sigma}{\Sigma} = 10^{-4}$, $\frac{\Delta\delta\Sigma}{\delta\Sigma} = 10^{-2}$ and then $|\frac{\Delta\rho}{\rho}| \leq 10^{-2}$, because the single precision of IBM 370 has about 6.7 significant digit. The results given in Table 9 do not contradict to this argument.

Summary of the sample calculations

1. An error criterion of 10^{-5} should be used for criticality factor and flux in DTK code to obtain the accuracy of 5×10^{-5} for the criticality factor of the adjoint equation.

2. There is a difference of about 8×10^{-4} between the criticality factors of direct and adjoint equations for the spherical geometry and the S_2 method. This difference is reduced to 10^{-5}, if S16 method is used.

3. The reactivity calculated from the criticality factors has errors of 5×10^{-5} for slab geometry and 8×10^{-4} for spherical geometry using the S_2 method.

4. The accuracy of the reactivity calculated by the perturbation method may be restricted by the numerical error of the flux as a whole, about 10^{-3} and by the round-off error of the perturbed cross sections in the form:

$$\left|\frac{\Delta\rho}{\rho}\right| \leq 10^{-3} + \left|\frac{\Delta\delta\Sigma/\Sigma}{\delta\Sigma/\Sigma}\right|$$

where $\delta\Sigma$ is a typical perturbed cross section, and $\Delta\delta\Sigma$ is its numerical error due to round-off. The accuracy of this reactivity is almost independent of the number of mesh points, contrary to those from the criticality factors.

5. The relative difference of the reactivities $\delta\rho/\rho$ determined by the perturbation calculations is 1.4×10^{-2} between the S2 method and the S4, S8 or S16 method for the present problem.

6. Correction to the criticality factor to take into account the isotope dependency of fission neutron spectrum and delayed neutron spectrum is successful with an accuracy of 2×10^{-5} independent of the weights for the fissionable isotope applied to the GRUCAL program.

2. Comparison with the Results of the Diffusion Theory

A diffusion perturbation calculation is also performed using the PERT1 code /14/ for the case shown in Tables 6 and 7. The diffusion calculation is made using the same cross section as used for Tables 6 and 7 except the transport cross section, and using 56 and 32 mesh intervals, i.e. mesh sizes of about 0.8 cm and 0.94 cm in the core and blanket respectively and the error criterion of 10^{-5} for the criticality factor and flux in the diffusion code D1F1D /15/. The result is shown in Table 10. The diffusion theory should give nearly the same accuracy as the S_2 method, because the S_2 equation becomes equivalent to the diffusion equation, if the two discrete ordinates over angle are chosen as those of the gaußian quadrature formula /16/, and the difference between the gaußian quadrature formula and the trapezoidal rule used in S_n method may not be so large. The criticality factors of the diffusion and S_2 method have about the same error of 1 % from that of S_{16} method, but opposite sign.

The GRUCAL code provides two kinds of transport cross sections, designated by STR and STRTR. In the diffusion calculation, the transport cross section STR is used to calculate the diffusion coefficient. On the other hand, in the DTK transport calculation, the transport cross section STRTR is used to represent the total cross section. (Normally STR is used in DTK code only to determine the buckling leakage in a way equivalent to diffusion theory.) If the transport cross section STR is used for DTK calculation, the criticality factor by the S_{16} method is found to be 0.99389 and the difference from that of the diffusion calculation is 0.72 %, which is nearly the same as the transport correction reported in Ref. /12/ and /17/. The problem of using STR or STRTR in DTK is beyond the scope of the present report. The reactivities from the diffusion and S_2 method have 1 % and -1.5 % errors from S_{16} method, however this error may be small compared with experimental error. This might be considered due to the fact that the perturbation of 1 % change of ^{239}Pu density is distributed in a whole core. If the perturbation is limited to a small area, the difference between the diffusion method and higher order S_n method might become larger. This is not the case for ^{239}Pu. The central reactivity worth

for ^{239}Pu by S_2 method has only 0.07 % difference from the S_8 method, whereas other isotopes have 3 ~ 9 % difference from S_8 method as described in the next section.

3. Central Reactivity Worth

The central reactivity worths are calculated by the first order perturbation method to compare with the experimental values given in Table VI in Ref. /12/ for ZPR-3 Assembly 48.

In the case of 28 mesh intervals in the core, the volume of the first mesh is

$$V = \frac{4\pi r^3}{3} = \frac{4\pi}{3}(1.6159\text{cm})^3 = 1.76739\text{x}10\text{cm}^3 .$$

Then, the number density of 1 mol atom in this value is

$$n = \frac{6.02486\text{x}10^{23}}{V} = 0.034089\text{x}10^{24}/\text{cm}^3 .$$

If this number density is added to the number density for core material as a perturbed cross section, the TP1 code calculates the central reactivity worth for the added isotope of one mole by taking the difference between the perturbed cross section and the unperturbed core cross section. The addition of probe isotope increases the cross section and further causes a change in selfshielding factor for resonant isotope, and this introduces the additional change in cross section. To avoid nonlinear changes of the selfshielding effect and the corresponding cross section differences, the number density which is added is reduced by 10^{-3} in case A and 10^{-2} in case B of Table 11.

Therefore, to obtain a reactivity per one mole, the output reactivitiy from TP1 code is multiplied by 10^3 and 10^2 for cases A and B respectively. The difference between case A and B is due to the change of selfshielding effect or a rounding off error in taking the cross section difference in perturbation calculation. The results are shown in Table 11 for the S_2 and S_8 method together with

experimental value and calculation value given in Ref. /11/. In the Table are also shown the ratios of those to S_8 method in case B. It is seen that the difference between case A and B is less than 0.7 % which is negligibly small. In the Table are shown also the results by the perturbation diffusion method using PERT1 code, in which the same cross section as the case B for S_m method is used.

The difference of the diffusion and S_2 method from S_8 method are nearly same, but in opposite direction except for ^{239}Pu and ^{235}U.

The largest difference of the S_2 value from S_8 value is in carbon, about 9 %. For ^{238}U, Na and Al, there are about 3 to 5 % difference between S_2 and S_8 methods.

There is a large difference between experimental and calculated values, from 7 to 85 %. Assuming that a part of this difference is due to the error in the delayed neutron data, the experimental material worths are recalculated using the delayed neutron data obtained from TP1 code. If the reactivity is small, the reactivity in inhour unit is given by the following equation /18/,

$$\rho_{ih} = 3600 \frac{\lambda_{eff}}{\beta_{eff}} \cdot \rho$$

where ρ is the reactivity defined by Eq. (14) and

$$\beta_{eff} = \sum_i \bar{\beta}_i$$

$$\lambda_{eff} = \frac{\beta_{eff}}{\sum_i \bar{\beta}/\lambda i} .$$

The experimental reactivity given in Table 11 was converted from the measured period using the conversion factor 1 % = 981 Ih in Ref. /12/.

From this conversion factor, we obtain

$$C_o \equiv \frac{\lambda_{eff}}{\beta_{eff}} = \frac{\rho_{ih}}{3600\ \rho} = \frac{981}{3600 \times 10^{-2}} sec^{-1} = 27.25\ sec^{-1}.$$

From Eq. (19), if $\omega\lambda_i << 1$ and $\omega\Lambda << 1$, we obtain

$$\rho = \frac{1}{T} \sum_i \frac{\bar{\beta}_i}{\lambda_i} = \frac{1}{TC}\ ,\quad T = \frac{1}{\omega}\ ,\quad C = \frac{\lambda_{eff}}{\beta_{eff}}$$

Then we obtain the corrected reactivity ρ_c from the original reactivity ρ,

$$\rho_c = \frac{C_o}{C}\ \rho\ .$$

C is calculated by the LAMBDA code /11/ using the delayed neutron fraction calculated by TP1 code and the data from Ref. /19/

$$C \equiv \frac{\lambda_{eff}}{\beta_{eff}} = \frac{9.3050 \times 10^{-2} sec^{-1}}{3.7479 \times 10^{-3}} = 2.4827\ sec^{-1}\ .$$

On the other hand, the LAMBDA code calculates the reactivity which corresponds to the period of 1 hour,

$$\rho(1\ hour) = 1.1271 \times 10^{-5}$$

This means that the number of Inhours per % ρ is 887.2 Ih. From this value, we obtain

$$C = \frac{887.2\ Ih}{3600sec\ x\ 10^{-2}} = 24.65 sec^{-1}\ .$$

Using this value, we get

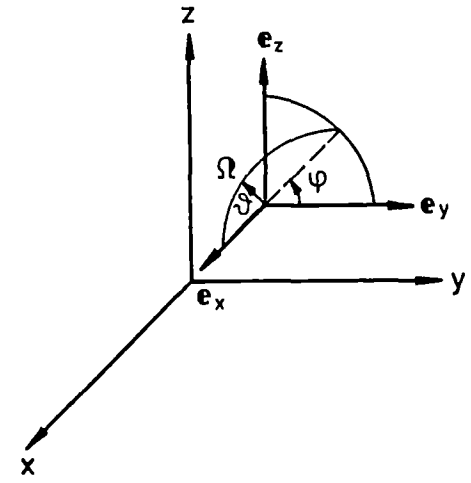
$$\rho_c = \frac{C_o}{C}\ \rho = \frac{27.25 sec^{-1}}{24.65 sec^{-1}}\ \rho = 1.105\rho\ .$$

The corrected reactivity is given in Tables 11 and 13. In Table 13 are also shown the results calculated by using ENDF/B-III and IV

reported by Hardie et al. /17/. The experimental value used to compute the ratio calculated to experimental in ENDF/B-III and IV is corrected using the number of Inhours per % ρ shown in Table 12, which is calculated also using the ENDF/B data.

As seen in Tables, the corrected values approach to the experimental values by making the delayed neutron correction. However, there still remain large difference between theory and experiment of about 3 % ~ 50 % except ^{23}Na and C, part of which might be due to the inadequacy of the spherical model compared to the cylindrically shaped experimental configuration. The fairly good agreement for ^{238}U and ^{10}B is probably fortuitous.

For ^{23}Na and C, the errors are very large, amounting to factors of 3 and 6, respectively, and are far beyond the delayed neutron correction.

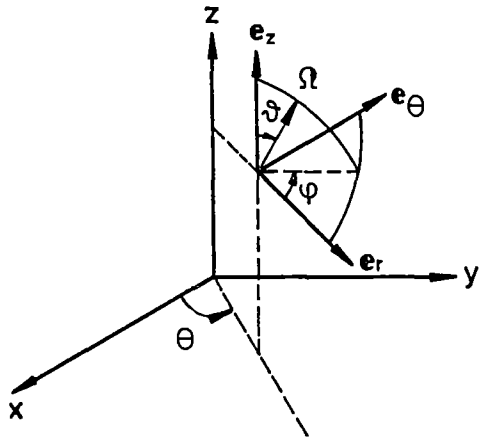


$\mu = \Omega \cdot e_x = \cos\vartheta$

$\eta = \Omega \cdot e_y = \sin\vartheta \cos\varphi$

$\xi = \Omega \cdot e_z = \sin\vartheta \sin\varphi$

slab geometry (x-coordinate)

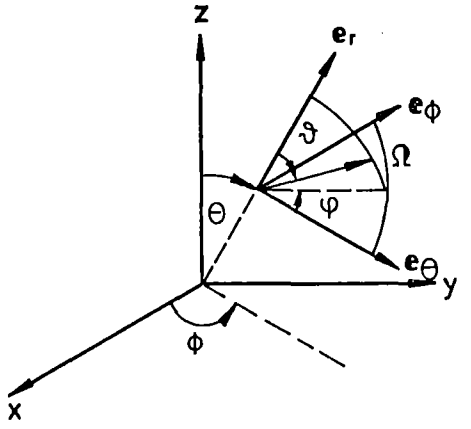


$\mu = \Omega \cdot e_r = \sin\vartheta \cos\varphi$

$\eta = \Omega \cdot e_\theta = \sin\vartheta \sin\varphi$

$\xi = \Omega \cdot e_z = \cos\vartheta$

cylindrical geometry (r -coordinate)



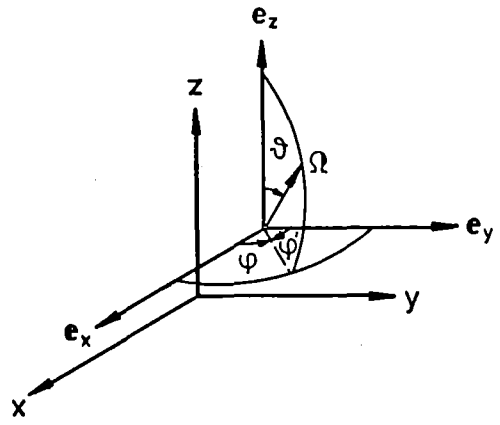
$\mu = \Omega \cdot e_r = \cos\vartheta$

$\eta = \Omega \cdot e_\theta = \sin\vartheta \cos\varphi$

$\xi = \Omega \cdot e_\phi = \sin\vartheta \sin\varphi$

spherical geometry (r- coordinate)

Fig.1 Coordinate system for one dimensional geometry

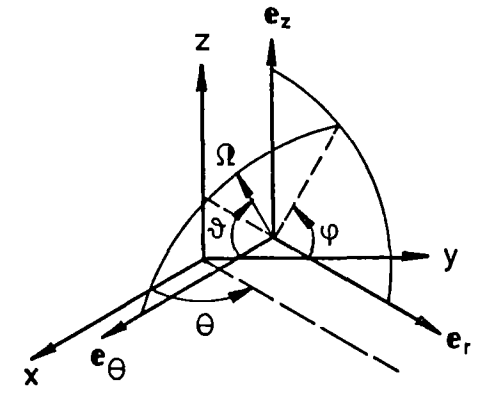


$\mu = \Omega \cdot e_x = \sin\vartheta \cos\varphi$

$\eta = \Omega \cdot e_y = \sin\vartheta \sin\varphi$

$\xi = \Omega \cdot e_z = \cos\vartheta$

x-y and triangular geometries on x-y plane

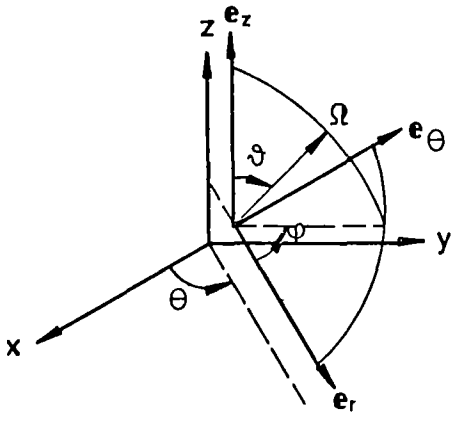


$\mu = \Omega \cdot e_r = \sin\vartheta \cos\varphi$

$\eta = \Omega \cdot e_z = \sin\vartheta \sin\varphi$

$\xi = \Omega \cdot e_\theta = \cos\vartheta$

r-z geometry



$\mu = \Omega \cdot e_r = \sin\vartheta \cos\varphi$

$\eta = \Omega \cdot e_\theta = \sin\vartheta \sin\varphi$

$\xi = \Omega \cdot e_z = \cos\vartheta$

r-θ geometry

Fig. 2 Coordinate system for two dimensional geometry

Table 1: Spherical Harmonics Functions in One-dimensional Cylindrical Geometry.

N	Spherical Harmonics Functions	NM	IRS	IRT
1	Y_{11}	1	1	1
2	Y_{20}, Y_{22}	3	2	3
3	Y_{31}, Y_{33}	5	4	5
4	Y_{40}, Y_{42}, Y_{44}	8	6	8
5	Y_{51}, Y_{53}, Y_{55}	11	9	11

N: The order of spherical harmonics functions

NM = ISCT (ISCT+4)/4: Number of spherical harmonics functions for the ISCT-th order anisotropic scattering

IRS = $[(N+1)^2/4]$: Initial address of N-th order spherical harmonics functions

IRT = IRS + $[(2N+1)/4]$: Final address of N-th order spherical harmonics functions.

Table 2: ZPR-3 Assembly 48 Spherical Model Atom Densities, atom/barn-cm

Isotope	Core Radius = 45.245 cm	Blanket Thickness = 30.0 cm
^{239}Pu	0.001645	-
^{240}Pu	0.000106	-
^{241}Pu	0.000011	-
^{242}Pu	0.0000004	-
^{235}U	0.000016	0.000083
^{238}U	0.007405	0.03969
C	0.02077	-
Na	0.006231	-
Fe	0.01018	0.004925
Cr	0.002531	0.001225
Ni	0.001119	0.000536
Mo	0.000206	-
Al	0.000109	-
Mn	0.000106	0.000051
Si	0.000124	0.000060

Table 3: Input Weights ω^j to GRUCAL Code for Averaging Isotope Dependent Fission Spectrum.
(GRUCAL renormalizes the weights to unity)

	Core	Blanket
Pu239	0.844059	
Pu240	0.151694×10^{-1}	
Pu241	0.769374×10^{-2}	
Pu242	0.438405×10^{-4}	
U235	0.709431×10^{-2}	0.877913×10^{-2}
U238	0.125940	0.155849

Table 4: The variation of criticality factor and reactivity due to the variation of the error criterion for the flux for the slab reactor by S_2 method with 28+16 mesh intervals.

	DTK							TP1		
Case	Error criterion	k_0[1] (upper) k^+ (lower)	Mixture index core, blanket	CPU time[2] (min)	Number of outer iteration	Number of inner iteration	ρ_k ($\times 10^{-3}$)	ρ_p ($\times 10^{-3}$)	$\bar{k}$[3]	ρ' ($\times 10^{-3}$)
1	-	1.34066	1, 3	0.39	16	3820	3.19	3.322	1.34003	-0.350
		1.34641	2, 3	0.52	34	3402				
2	-	1.34664	2, 3	0.41	16	3812	-3.54	-3.322	1.34599	-0.357
		1.34025	1, 3	0.47	43	3663				
3	10^{-3}	1.34066	1, 3	0.42	16	4257	3.25	3.321	1.34003	-0.350
		1.34653	2, 3	0.48	31	3317				
4	10^{-3}	1.34664	2, 3	0.42	16	4268	-3.38	-3.322	1.34599	-0.356
		1.34053	1, 3	0.45	28	3228				
5	10^{-4}	1.34066	1, 3	0.47	16	4838	3.31	3.321	1.34003	-0.350
		1.34664	2, 3	0.37	13	4075				
6	10^{-4}	1.34664	2, 3	0.46	16	4860	-3.45	-3.322	1.34599	-0.357
		1.34042	1, 3	0.33	13	4012				

1) The error criterion of 10^{-5} is used for the criticality factor.

2) IBM 370-168 is used.

3) $\bar{k}$ is the corrected criticality factor of k_0 for the isotope dependency of prompt fission neutron and for the delayed neutron spectrum.

Table 5: Criticality factor and reactivity for the slab reactor by the S_2 method.

Case	Number of mesh intervals core + blanket	DTK k_0(upper)[1] k^+(lower)	Mixture index core, blanket	DTK CPU time[2] (min)	Number of outer iteration	Number of inner iteration	Number of core memory	DTK ρ_k ($\times 10^{-3}$)	TP1 ρ_p ($\times 10^{-3}$)	TP1 $\bar{k}$	TP1 ρ' ($\times 10^{-3}$)	TP1 CPU time[2] (sec)	TP1 Number of core memory
1	2 + 2	1.33386	1, 3	0.18	25	9720	3.6 K	3.35	3.348	1.33321	-0.364	3	8.8 K
		1.33984	2, 3	0.06	12	4487							
2	2 + 2	1.33984	2, 3	0.19	25	9712		-3.34	-3.348	1.33917	-0.372	3	
		1.33387	1, 3	0.06	12	4463							
3	7 + 4	1.34017	1, 3	0.18	16	5906	4.4 K	3.32	3.323	1.33954	-0.351	3	10.2 K
		1.34615	2, 3	0.17	15	6062							
4	7 + 4	1.34616	2, 3	0.19	16	5913		-3.33	-3.323	1.34551	-0.358	3	
		1.34016	1, 3	0.17	15	6048							
5	14 + 8	1.34055	1, 3	0.31	16	5700	5.7 K	3.30	3.322	1.33993	-0.350	4	12.5 K
		1.34651	2, 3	0.30	15	6061							
6	14 + 8	1.34654	2, 3	0.31	16	5684		-3.33	-3.322	1.34589	-0.357	4	
		1.34053	1, 3	0.30	15	6055							
7	28 + 16	1.34065	1, 3	0.55	16	5682	8.4 K	3.29	3.322	1.34002	-0.350	5	17.2 K
		1.34659	2, 3	0.55	16	6126							
8	28 + 16	1.34663	2, 3	0.55	16	5684		3.34	-3.322	1.34599	-0.357	5	
		1.34060	1, 3	0.56	16	6114							

1) Error criterion 10^{-5} is used for the criticality factor and flux.

2) IBM 370-168 is used.

Table 6: Criticality factor and reactivity for the spherical reactor by the S_2 method.

Case	Number of mesh intervals core + blanket	DTK							TP1				
		k_0(upper)[1] k^+(lower)	Mixture index core, blanket	CPU time (min)	Number of outer iteration	Number of inner iteration	Number of core memory	ρ_k ($\times 10^{-3}$)	ρ_p ($\times 10^{-3}$)	$\bar{k}$	ρ' ($\times 10^{-3}$)	CPU time (sec)	Number of core memory
1	2 + 2	1.00276	1, 3	0.23	32	10845	3.6 K	3.25	5.060	1.00160	-1.163	3	9 K
		1.00604	2, 3	0.06	13	4029							
2	2 + 2	1.00793	2, 3	0.23	33	11113		-6.97	-5.060	1.00673	-1.180	3	
		1.00089	1, 3	0.06	12	3977							
3	7 + 4	1.00751	1, 3	0.17	16	4949	4.4 K	4.32	5.074	1.00652	-0.981	3	10 K
		1.01192	2, 3	0.19	16	6196							
4	7 + 4	1.01269	2, 3	0.17	16	4943		-5.84	-5.074	1.01167	-0.996	3	
		1.00674	1, 3	0.18	16	6190							
5	14 + 8	1.00705	1, 3	0.27	16	4630	5.7 K	4.33	5.084	1.00608	-0.963	4	13 K
		1.01146	2, 3	0.28	15	5796							
6	14 + 8	1.01223	2, 3	0.27	16	4637		-5.84	-5.084	1.01123	-0.978	4	
		1.00629	1, 3	0.28	15	5795							
7	28 + 16	1.00692	1, 3	0.46	16	4650	8.4 K	4.33	5.087	1.00594	-0.959	5	17 K
		1.01132	2, 3	0.51	15	5801							
8	28 + 16	1.01209	2, 3	0.44	16	4661		-5.84	-5.087	1.01110	-0.974	5	
		1.00615	1, 3	0.49	15	5809							

1) Error criterion of 10^{-5} is used for the criticality factor and flux.

Table 7: Criticality factor and reactivity for the spherical reactor by S_4, S_8 and S_{16} method with 28 + 16 mesh intervals.

Case	Order of S_n	DTK							TP1				
		k_o(upper)[1] k^+(lower)	Mixture index core, blanket	CPU time (min)	Number of outer iteration	Number of inner iteration	Number of core memory	ρ_k ($\times 10^{-3}$)	ρ_p ($\times 10^{-3}$)	$\bar{k}$	ρ' ($\times 10^{-3}$)	CPU time (sec)	Number of core memory
1	4	0.99720	1, 3	0.62	16	4570	9.1 K	4.98	5.157	0.99623	-0.977	5	21.9 K
		1.00218	2, 3	0.70	15	5738							
2	4	1.00235	2, 3	0.62	16	4567		-5.31	-5.157	1.00135	-0.992	5	
		0.99704	1, 3	0.71	15	5735							
3	8	0.99656	1, 3	0.89	16	4563	10.6 K	5.12	5.161	0.99558	-0.979	6	31.2 K
		1.00167	2, 3	1.06	15	5744							
4	8	1.00169	2, 3	0.90	16	4559		-5.18	-5.161	1.00070	-0.994	5	
		0.99653	1, 3	1.04	15	5741							
5	16	0.99627	1, 3	1.59	16	4560	13.5 K	5.16	5.163	0.99530	-0.979	6	50.0 K
		1.00141	2, 3	1.92	15	5736							
6	16	1.00141	2, 3	1.55	16	4555		-5.15	-5.163	1.00041	-0.994	6	
		0.99627	1, 3	1.88	15	5735							

1) Error criterion of 10^{-5} is used for the criticality factor and flux.

Table 8: The variation of correction factor to the criticality factor due to the variation of the fission spectrum weights. The criticality factor is calculated for the spherical reactor by S_8 method with 28 + 16 mesh intervals.

Case	DTK			TP1			
	k_0 (upper)[1)] k^+ (lower)	Mixture index core, blanket	ρ_k ($\times 10^{-3}$)	ρ_p ($\times 10^{-3}$)	$\bar{k}$	ρ' ($\times 10^{-3}$)	
1	0.99744 1.00255	1, 3 2, 3	5.11	5.149	0.99558	-1.873	ω(Pu239) = 1, ω^j = 0 for other fissile isotope.
2	0.99297 0.99809	1, 3 2, 3	5.17	5.212	0.99559	2.653	ω^j = 1 for all fissile isotope.
3	0.99273 0.99785	1, 3 2, 3	5.17	5.216	0.99559	2.895	ω^j is proportional to the number density.
4	0.99166 0.99679	1, 3 2, 3	5.18	5.231	0.99559	3.982	ω(U238) = 1, ω^j = 0 for other fissile isotope.

1) Error criterion of 10^{-5} is used for the criticality factor and flux.

Table 9: The variation of reactivity due to the variation of the number density of Pu^{239} or Na in the core. The results are for the spherical reactor by S_4 method with 28 + 16 mesh intervals.

Case	DTK			TP1			Perturbation
	k_0 (upper) [1] k^+ (lower)	Mixture index core, blanket	ρ_k ($\times 10^{-3}$)	ρ_p ($\times 10^{-3}$)	$\bar{k}$	ρ' ($\times 10^{-3}$)	
1	0.99720	1, 3	4.98	5.157	0.99623	-0.977	
	1.00218	2, 3					n^{Pu239} x 1.01
2	0.99720	1, 3	0.36	0.519	0.99622	-0.994	
	0.99755	4, 3					n^{Pu239} x 1.001
3	0.99720	1, 3	-0.11	0.052	0.99621	-0.995	
	0.99709	5, 3					n^{Pu239} x 1.0001
4	0.99720	1, 3	-0.25	-0.090	0.99621	-0.996	
	0.99695	6, 3					n^{Na} x 0.99
5	0.99720	1, 3	-1.07	-0.912	0.99621	-1.001	
	0.99613	7, 3					n^{Na} x 0.9
6	0.99720	1, 3	-4.88	-4.724	0.99618	-1.027	
	0.99237	8, 3					n^{Na} x 0.5
7	0.99720	1, 3	-10.87	-10.712	0.99614	-1.066	
	0.98651	9, 3					n^{Na} x 0

1) Error criterion of 10^{-5} is used for the criticality factor and flux.

Table 10: Comparison of reactivity by the diffusion method and S_n method

	k_o	k^+	ρ_k 1) ($\times 10^{-3}$)	ρ_p ($\times 10^{-3}$)	difference of k_o from S_{16} (%)	difference of ρ_p from S_{16} (%)
Diffusion	0.98671	0.99181	5.216	5.213	-1.0	1.0
S_2	1.00692	1.01132	4.33	5.087	1.1	-1.5
S_4	0.99720	1.00218	4.98	5.157	0.1	-0.1
S_{16}	0.99627	1.00141	5.16	5.163	0.0	0.0

1) $\rho_k = \frac{1}{k_o} - \frac{1}{k^+}$

Table 11: Material worth at the center of spherical reactor of ZPR-3 Assembly 48

	Material Worth								Ratios to the values of S_8 case B						
Isotope	Experiment ($\times 10^{-5}$/mol)	Experiment[2] (corrected)	Calculation[1]	S_8 case A	S_2 case A	S_8 case B	S_2 case B	PERT1	Experiment (uncorrected)	Experiment[2] (corrected)	Calculation	S_8 case A	S_2 case A	S_2 case B	PERT1
^{239}Pu	108.4±1.0	119.8	138.06	136.7	136.8	136.7	136.8	132.9	0.7928	0.8760	1.0	1.0000	1.0007	1.0007	0.9717
^{235}U	80.0±1.2	88.4	103.10	99.73	99.77	99.63	99.67	98.17	0.8030	0.8873	1.035	1.0010	1.0014	1.0004	0.9853
^{238}U	-5.72±0.17	-6.32	-6.894	-6.118	-6.319	-6.113	-6.315	-5.950	0.9357	1.0339	1.128	1.0008	1.0337	1.0330	0.9733
^{23}Na	-0.148±0.007	-0.164	-0.2543	-0.4706	-0.4920	-0.4736	-0.4951	-0.4483	0.3125	0.3453	0.5370	0.9937	1.0389	1.0454	0.9466
^{10}B	90.96±0.61	100.51	-93.57	-97.28	-98.73	-97.27	-98.72	-95.74	0.9351	1.0333	0.9620	1.0001	1.0150	1.0149	0.9843
Fe	-0.700±0.023	-0.774		-1.075	-1.094	-1.079	-1.098	-1.0537	0.6487	0.7168		0.9963	1.0139	1.0176	0.9766
Cr	-0.652±0.079	-0.720				-0.9596	-0.9780	-0.9368	0.6794	0.7507				1.0192	0.9762
Ni	-1.09±0.01	-1.20		-1.838	-1.866	-1.833	-1.861	-1.7575	0.5947	0.6571		1.0027	1.0180	1.0153	0.9752
Mn	-1.28±0.06	-1.41													
Al	-0.432±0.022	-0.477		-0.6949	-0.7166	-0.6991	-0.7208	-0.6803	0.6179	0.6828		0.9940	1.0250	1.0310	0.9731
Ta	-30.25±0.92	-33.43													
Mo	-4.24±0.04	-4.69		-6.359	-6.456	-6.312	-6.408	-6.1908	0.6717	0.7422		1.0074	1.0228	1.0152	0.9808
C	-0.055±0.015	-0.061		-0.3636	-0.3963	-0.3631	-0.3958	-0.3280	0.1517	0.1676		1.0014	1.0914	1.0901	0.9033

1) This value is taken from Ref. /11/.

2) Corrected value is obtained by multiplying the uncorrected value by 1.105.

Table 12: β_{eff}, Inhours per % ρ and generation time

	Benchmark [1]	ENDF/B [2]	TP1 and LAMBDA	
			S_2	S_8
β_{eff}		0.003588	0.003698	0.003748
Inhours per % ρ	981	932.5	898.9	887.2
generation time		0.2529 µsec	0.2631 µsec	0.2640 µsec

1) From Ref. /11/ where the delayed neutron data of Keepin was used.

2) From Ref. /18/ where the delayed neutron data from ENDF/B-III and IV was used.

Table 13: Comparison of central reactivities in terms of calculated to experimental values by TP1 and those obtained by using ENDF/B-III and IV

	ENDF/B-III	ENDF/B-IV	TP1 (uncorrected)	TP1 (corrected) [1)]
^{239}Pu	1.216	1.185	1.261	1.141
^{235}U	1.232	1.196	1.245	1.127
^{238}U	1.164	1.038	1.069	0.967
^{23}Na	1.595	2.078	3.200	2.896
^{10}B	0.988	0.959	1.069	0.967
Fe	1.147	1.260	1.542	1.395
Cr	1.529	1.546	1.472	1.332
Ni	1.382	1.403	1.682	1.522
Al			1.618	1.464
Ta	1.035	1.024		
Mo			1.489	1.348
C			6.601	5.974

1) Corrected value is obtained by dividing the uncorrected value by 1.105.

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Appendix I Asymptotic Period Reactivity

Henry defined another form of reactivity, asymptotic period reactivity. He writes the angular flux in the form /2/,

$$f_g(\vec{r},\vec{\Omega},t) = \Phi_g(\vec{r},\vec{\Omega},t)\ T(t) \tag{A-1}$$

Substituting Eq.(A-1) into Eq.(1), multiplying it by $f^+_{k_o}(\vec{r},\vec{\Omega})$ and integrating over space and solid angle, we obtain

$$\frac{\partial}{\partial t}\langle f^+_{kog}(\vec{r},\vec{\Omega})\frac{1}{v_g}\Phi_g(\vec{r},\vec{\Omega},t)\rangle T(t) = \langle f^+_{kog}(G^o+\delta G+\frac{1}{k_o}(F^o_p+\delta F_p)\Phi_g(\vec{r},\vec{\Omega},t)\rangle T(t)$$

$$+ \sum_i \lambda_i \langle f^+_{kog}\ \chi_{ig}\ C_i(\vec{r},t)\rangle \tag{A-2}$$

If we define $\Phi_g(\vec{r},\vec{\Omega},t)$ such that

$$\frac{\partial}{\partial t}\langle f^+_{kog}(\vec{r},\vec{\Omega})\ \frac{1}{v_g}\ \Phi_g(\vec{r},\vec{\Omega},t)\rangle = 0 \tag{A-3}$$

Eq.(A-2) becomes

$$\langle f^+_{kog}(\vec{r},\vec{\Omega})\frac{1}{v_g}\Phi_g(\vec{r},\vec{\Omega},t)\rangle \frac{dT(t)}{dt} = \langle f^+_{kog}(G^o+\delta G+\frac{1}{k_o}(F^o_p+\delta F_p))\Phi_g(\vec{r},\vec{\Omega},t)\rangle\ T(t)$$

$$+ \sum_i \lambda_i \langle f^+_{kog}\ \chi_{ig}\ C_i(\vec{r},t)\rangle \tag{A-4}$$

Using the following relation

$$\langle f^+_{kog}(G^o+\frac{1}{k_o}F^o_p)\Phi_g(\vec{r},\vec{\Omega},t)\rangle = \langle f^+_{kog}(G^o+\frac{1}{k_o}F^o-\frac{1}{k_o}\sum_i \chi_{ig}\ F^o_i)\Phi_g(\vec{r},\vec{\Omega},t)\rangle$$

$$= -\frac{1}{k_o}\langle f^+_{kog}\sum_i \chi_{ig}\ F^o_i\ \Phi_g(\vec{r},\vec{\Omega},t)\rangle \tag{A-5}$$

and the definitions

$$\rho(t) = \frac{1}{N} \langle f^+_{kog} (\delta G + \frac{1}{k_o} \delta F_p) \Phi_g(\vec{r},\vec{\Omega},t) \rangle \quad (A\text{-}6)$$

$$\beta(t) = \frac{1}{N} \langle f^+_{kog} \frac{1}{k_o} \sum_i \chi_{ig} F^o_i \Phi_g(\vec{r},\vec{\Omega},t) \rangle \quad (A\text{-}7)$$

$$Ci(t) = \frac{1}{\Lambda N} \langle f^+_{kog} \chi_{ig} Ci(\vec{r},t) \rangle \quad (A\text{-}8)$$

$$\Lambda = \frac{1}{N} \langle f^+_{kog} \frac{1}{v_g} \Phi_g(\vec{r},\vec{\Omega},t) \rangle \quad (A\text{-}9)$$

$$N = \langle f^+_{kog} F \Phi_g(\vec{r},\vec{\Omega},t) \rangle \quad (A\text{-}10)$$

Eq.(A-4) can be written as

$$\frac{dT(t)}{dt} = \frac{\rho(t)-\beta(t)}{\Lambda} T(t) + \sum_i \lambda_i C_i(t) \quad (A\text{-}11)$$

Multiplying Eq.(2) by $f^+_{kog}(\vec{r},\vec{\Omega})\chi_{ig}$, we obtain

$$\frac{\partial}{\partial t} \langle f^+_{kog}(\vec{r},\vec{\Omega}) \chi_{ig} C_i(\vec{r},t) \rangle = \frac{1}{k_o} \langle f^+_{kog} \chi_{ig} F_i \Phi_g(\vec{r},\vec{\Omega},t) \rangle T(t)$$

$$- \lambda_i \langle f^+_{kog} \chi_{ig} C_i(\vec{r},t) \rangle \quad (A\text{-}12)$$

Defining

$$\bar{\beta}_i = \frac{1}{Nk_o} \langle f^+_{kog} \chi_{ig} F_i \Phi_g(\vec{r},\vec{\Omega},t) \rangle \quad (A\text{-}13)$$

Eq.(A-12) becomes

$$\frac{dC_i(t)}{dt} = \frac{\bar{\beta}_i}{\Lambda} T(t) - \lambda_i C_i(t) \quad (A\text{-}14)$$

It should be noticed that in the definitions (A-7) and (A-13),

$$\bar{\beta} \neq \sum_i \bar{\beta}_i \quad (A\text{-}15)$$

In general, all parameters defined by Eqs.(A-6), (A-7), (A-9) and (A-13) are time dependent. However, in the case of asymptotic beha-

viour, Eq.(A-1) can be written in the form of Eq.(16), namely, $\Phi_g(\vec{r},\vec{\Omega},t) = f_{\omega g}(\vec{r},\vec{\Omega})$ and $\Phi_g(\vec{r},\vec{\Omega},t)$ is independent of time. In this asymptotic case, all parameters are independent of time and given by

$$\rho_\omega = \frac{1}{N} \langle f^+_{kog} (\delta G + \frac{1}{k_o} \delta F_p) f_{\omega g} \rangle \qquad \text{(A-16)}$$

$$\bar{\beta} = \frac{1}{N} \langle f^+_{kog} \frac{1}{k_o} \chi_{ig} F^o_i f_{\omega g} \rangle \qquad \text{(A-17)}$$

$$\Lambda = \frac{1}{N} \langle f^+_{kog} \frac{1}{\nu_g} f_{\omega g} \rangle \qquad \text{(A-18)}$$

$$\bar{\beta}_i = \frac{1}{N} \langle f^+_{kog} \frac{1}{k_o} \chi_{ig} F_i f_{\omega g} \rangle \qquad \text{(A-19)}$$

$$N = \langle f^+_{kog} F f_{\omega g} \rangle \qquad \text{(A-20)}$$

Using Eqs.(16) and (17) in Eqs.(A-11) and (A-14), we obtain

$$\rho_\omega = \Lambda\omega + \omega \sum_i \frac{\bar{\beta}_i}{\omega+\lambda_i} + \bar{\beta} - \sum_i \bar{\beta}_i \qquad \text{(A-21)}$$

Eq.(A-21) is a little different from Eq.(19) and also the definitions of all parameters, (A-16)∿(A-20) are a little different from those for the static reactivity. Practically, the asymptotic period reactivity ρ_ω and all other parameters of Eqs.(A-17)∿(A-20) must be calculated by approximating $f_{\omega g}(\vec{r},\vec{\Omega})$ by $f_{kg}(\vec{r},\vec{\Omega})$.

As shown in Appendix II, the approximation of $f_{\omega g}(\vec{r},\vec{\Omega})$ by $f_{kg}(\vec{r},\vec{\Omega})$ has higher accuracy than the approximation of $f_{\omega g}(\vec{r},\vec{\Omega})$ by $f_{kog}(\vec{r},\vec{\Omega})$ which is used in the method by the static reactivity of TP1 code. Therefore, the asymptotic period may be obtained by the method of the asymptotic period reactivity more accurately than the method by the static reactivity for the problems in which the difference between $f_{kg}(\vec{r},\vec{\Omega})$ and $f_{kog}(\vec{r},\vec{\Omega})$ is large.

Nevertheless, use is made of the method by the static reactivity in TP1 code, because the static reactivity itself can be obtained accurately without approximation by the perturbation calculation and also from the two criticality factors by Eq.(20) which does

not hold for the asymptotic period reactivity.

If Eq.(96) is multiplied by $f^{+}_{kog}(\vec{r},\vec{\Omega})$ and Eq.(11a) by $f_{kg}(\vec{r},\vec{\Omega})$, we can also obtain the expression for the static reactivity as

$$\rho \equiv \frac{1}{k_o} - \frac{1}{k} = \frac{\langle f^{+}_{k_o} (\delta G + \frac{1}{k_o} \delta F) f_k \rangle}{\langle f^{+}_{k} F f_{k_o} \rangle} \qquad \text{(A-22)}$$

Comparing Eqs.(A-16) and (A-22), we can see clearly the difference between the static and asymptotic period reactivities.

Appendix II On Approximation of $f_{\omega g}(\vec{r},\vec{\Omega})$ by $f_{kog}(\vec{r},\vec{\Omega})$

Now, we consider about the approximation of replacing $f_{\omega g}(\vec{r},\vec{\Omega})$ by $f_{kog}(\vec{r},\vec{\Omega})$ in Eqs.(21) and (22). From Eq.(18), we can derive the following equation

$$(G + \frac{1}{k} F) f_{\omega g}(\vec{r},\vec{\Omega}) = \left[-\rho_k F + \frac{\omega}{v_g} + \frac{\omega}{k_o} \sum_i \frac{\chi_{ig} F_i}{\lambda_i + \omega} \right] f_{\omega g}(\vec{r},\vec{\Omega}) \qquad \text{(A-23)}$$

where use is made of Eq.(20).

If we assume that all the emitted spectra of the delayed neutrons are equal to the prompt neutron fission spectrum and this fission spectrum is independent of the isotope, ie; $\chi_{ig} = \chi^{j}_{g} = \chi_g$, and the delayed neutron fraction β_i is independent of the energy, i.e. $\nu^{i}_{d}\Sigma_{fg} = \beta_i \nu\Sigma_{fg}$, the fission operators Eqs.(5) and (10) can be written in the form

$$F_i = \beta_i \sum_{g'} \nu \Sigma_{fg'} \frac{1}{4\pi} \sum_{m'} \Delta\vec{\Omega}_{m'} \qquad \text{(A-24)}$$

$$F = \chi_g \sum_{g'} \nu \Sigma_{fg'} \frac{1}{4\pi} \sum_{m'} \Delta\vec{\Omega}_{m'} \qquad \text{(A-25)}$$

The assumption of the isotope independency of fission spectrum

is used in the DTK code and the error to the criticality factor in the sample calculations shown in Chap. IV is less than 0.4%.

Substituting Eqs.(A-24) and (A-25) into Eq.(22), we obtain

$$\bar{\beta}_i = \frac{\langle f_k^+ \chi_{ig} F_i f_\omega \rangle}{\langle f_k^+ F f_\omega \rangle} = \frac{\langle f_k^+ \chi_{ig} \sum_g \nu_d^i \Sigma_{fg}, \frac{1}{4\pi} \sum_m \Delta\Omega_m, f_\omega \rangle}{\langle f_k^+ \chi_g \sum_g \nu \Sigma_{fg}, \frac{1}{4\pi} \sum_m \Delta\Omega_m, f_\omega \rangle} = \beta_i \qquad \text{(A-26)}$$

If we use this assumption in Eq.(A-23), we obtain /12/

$$(G + \frac{1}{k} F) f_{\omega g}(\vec{r},\vec{\Omega}) = \left[(-\rho_k + \frac{\omega}{k_o} \sum_i \frac{\beta_i}{\lambda_i + \omega}) F + \frac{\omega}{v_g}\right] f_{\omega g}(\vec{r},\vec{\Omega}) \qquad \text{(A-27)}$$

Usually, the ratio of the asymptotic period to the neutron velocity is much smaller than the macroscopic cross sections of the any real reactors. Then the right hand side of Eq.(A-27) is small, because of Eq.(19), and Eq.(A-27) becomes a close approximation to Eq.(9b). Namely, $f_{\omega g}(\vec{r},\vec{\Omega}) = f_{kg}(\vec{r},\vec{\Omega})$. Unfortunately, we can not use $f_{kg}(\vec{r},\vec{\Omega})$ if we apply the TP1 code in the usual manner. Instead of $f_{\omega g}(\vec{r},\vec{\Omega})$, we must use $f_{kog}(\vec{r},\vec{\Omega})$.

From Eq.(96), we obtain

$$(G^\circ + \frac{1}{k_o} F^\circ) f_{kg}(\vec{r},\vec{\Omega}) = \left[\rho_k F^\circ - (\delta G + \frac{1}{k} \delta F)\right] f_{kg}(\vec{r},\vec{\Omega}) \qquad \text{(A-28)}$$

For the approximation, $f_{kg}(\vec{r},\vec{\Omega}) = f_{kog}(\vec{r},\vec{\Omega})$ to be valid, the right hand side of Eq.(A-28) must be small enough, namely, the perturbation must be small. This requirement is equivalent to the applicability of first order perturbation theory.

Appendix III Structure of the Interfacefile (IDTK) for Perturbation Calculation from the DTK Code

1st record: ISCT, ISN, IGE, IZM, IM, IGM

MT, MTP, NM, MM, IHS, IHT, IHM, MBK, BF, XT, RLOG

2^{nd} record:
- i(i) DY, DZ — if MBK>0
- (ii) BK(IGM) — if MBK=-1 or -11
- (iii) BK(IZM, IGM) — if MBK=-2 or -12
- (iv) nothing — if MBK=0

3^{rd} record: (If MBK=0, this is the second record)
MA(IM), IAL(MTP), W(MM), CL(MM,NM), V(IM), R(IMP), IPLATZ(MTP), C(IHM, IGM, MT), XKI(IGM)

4^{th} record: MZ(IZM)

5^{th} record: k_o

6^{th} record to $(5+IGM)^{th}$ records: (FKO(IM, MM, IG), IG=1, IGM)

$(6+IGM)^{th}$ record: The buckling values for the adjoint equation as in 2^{nd} record, if MBK≠0.

$(7+IGM)^{th}$ record: k

$(8+IGM)^{th}$ record to $(7+2IGM)^{th}$ record: (FKD(IM, MM, IG), IG=1, IGM)

Appendix IV Sample Problem Input and Output

In the following, sample problem input and output for the exact perturbation is shown for the case 1 of Table 6, the spherical model of ZPR-3 Assembly 48 by S_2 method with 2 + 2 mesh intervals.

1) Input Data

Input data for GRUCAL, SIGMUT, DTK and TP1 are shown in the following.

```
//INR05927 JOB (0659,101,P6M2B),KOBAYASHI,
//  REGION=900K,TIME=2
//*MAIN SYSTEM=M168
//*MAIN LINES=6
//*FORMAT PR,DDNAME=SYSPRINL                                              00000020
//*FORMAT PR,DDNAME=FT42F001                                              00000030
//  EXEC KSCLG                                                            00000040
//K.FT44F001 DD SPACE=(3064,200)                                          00000050
//K.FT01F001 DD UNIT=3330,VOL=SER=KAPROS,DISP=SHR,DSN=KSDA.GRUBA.KFKINR  00000060
//K.FT09F001 DD UNIT=3330,VOL=SER=KAPROS,DISP=SHR,DSN=KSDA.JBGRUC        00000070
//K.FT10F001 DD UNIT=3330,VOL=SER=KAPROS,DISP=SHR,DSN=GRSTAB             00000080
//K.FT11F001 DD UNIT=3330,VOL=SER=KAPROS,DISP=SHR,DSN=F26                00000090
//K.FT12F001 DD DSN=&&BAND,DISP=(NEW,PASS),UNIT=SYSDA,                    00000100
//       SPACE=(TRK,(150))                                                00000110
//K.FT04F001 DD DSN=GROUCO,VOL=SER=KAPROS,UNIT=3330,DISP=SHR              00000120
//K.FT20F001 DD UNIT=SYSDA,SPACE=(TRK,(10))                               00000130
//K.FT21F001 DD UNIT=SYSDA,SPACE=(TRK,(10))                               00000140
//K.FT30F001 DD UNIT=SYSDA,SPACE=(TRK,(10)),DCB=BLKSIZE=400               00000150
//K.FT31F001 DD UNIT=SYSDA,SPACE=(TRK,(10)),DCB=BLKSIZE=400               00000160
//K.FT32F001 DD UNIT=SYSDA,DSN=&&FLUX,DISP=(NEW,PASS),SPACE=(TRK,(10))    00000170
//K.FT39F001 DD UNIT=SYSDA,SPACE=(TRK,10),DSN=&&DTKINPUT,
// DCB=(LRECL=80,RECFM=FB,BLKSIZE=800),DISP=(NEW,PASS)
//K.FT02F001 DD  UNIT=SYSDA,DISP=(NEW,PASS),SPACE=(TRK,50),
//  DCB=(RECFM=VBS,BLKSIZE=3200),DSN=&&INTFAC
//K.SYSIN DD *                                                            00000200
*COMPILE G                                                               00000210
C                                                                        00000220
C     ***** KAPROS DRIVER ROUTINE FOR CALCULATION OF                     00000230
C     ***** SIMGN BLOCKS (MODULE GRUCAL),                                00000240
C     ***** INCLUSION OF DELAYED NEUTRON CS (MODULE SIGMUT)              00000250
C     ***** AND 1D TRANSPORT CALCULATION (MODULE KADTK)                  00000260
C     ***** SIGMN BLOCK IS WRITTEN ON UNIT 12 BY MODULE UTKS             00000270
C                                                                        00000280
      SUBROUTINE STEUER                                                  00000290
C                                                                        00000300
      REAL GRUCAL(4),SIG26(4),FLUX(4),SIGALT(4),DTK(4),SIGMUT(4),UTKS(4)00000310
      REAL PERTIN(4)
C                                                                        00000320
C     ***** INITIALISATION OF DATABLOCK NAMES                            00000330
C                                                                        00000340
      DATA                                                               00000350
     1  GRUCAL /'GRUC','AL  ','    ','    '/,                           00000360
     2  SIG26  /'SIGM','N   ','    ','    '/,                           00000370
     3  FLUX   /'FLUX','1   ','    ','    '/,                           00000380
     4  SIGALT /'SIGM','ALT ','    ','    '/,                           00000390
     5  DTK    /'DTKI','NPUT','    ','    '/,                           00000400
     6  SIGMUT /'INPU','T SI','GMUT','    '/,                           00000410
     7  UTKS   /'INPU','T UT','KS  ','    '/,                           00000420
     8  PERTIN /'PERT','UBAT','IONI','NPUT'/
C                                                                        00000430
C     ***** INITIALISATION OF KAPROS SYSTEM                              00000440
C                                                                        00000450
      CALL KSINIT(N1,N2,N3,N4,N5)                                        00000460
C                                                                        00000470
C     ***** CALL FOR MODULE GRUCAL (CREATES SIGMN BLOCK                  00000480
C     ***** WITH NAME SIGMALT NOT INCLUDING DELAYED PART                 00000490
C                                                                        00000500
      CALL KSEXEC ('GRUCAL  ',2,0,GRUCAL,GRUCAL,SIG26,SIGALT,IQ)        00000510
      IF(IQ.NE.0) GOTO 1001                                              00000520
C                                                                        00000530
C     ***** CALL FOR MODULE SIGMUT (ADDS THE DELAYED CS PART TO          00000540
C     ***** THE SIGMN BLOCK SIGMALT AND STORES THE RESULT                00000550
C     ***** IN KAPROS DATABLOCK SIGMN                                    00000560
C                                                                        00000570
      CALL KSEXEC ('SIGMUT  ',3,0,SIGMUT,SIGMUT,SIG26,SIG26,            00000580
     1             SIGALT,SIGALT,IQ)                                     00000590
```

```
      IF(IQ.NE.0) GOTO 1002                                              00000600
C                                                                        00000610
C     ***** CALL FOR MODULE UTKS WHICH WRITES OUT THE KAPROS             00000620
C     ***** DATABLOCK SIGMN ON EXTERNAL UNIT 12 FOR LATER                00000630
C     ***** USE IN KADTK AND PERTUBATION PROGRAM                         00000640
C                                                                        00000650
      CALL KSEXEC('UTKS    ',2,0,UTKS,UTKS,SIG26,SIG26,IQ)               00000660
      IF(IQ.NE.0) GO TO 1003                                             00000670
C                                                                        00000680
C     ***** CALL FOR MODULE KADTK WHICH PERFORMS 1D TRANSPORT            00000690
C     ***** CALCULATIONS REQUIRED BY THE INPUT DATABLOCK DTKINPUT        00000700
C                                                                        00000710
      CALL KSEXEC('DTK     ',4,0,DTK,DTK,SIG26,SIG26,FLUX,FLUX           00000720
     1,PERTIN,PERTIN,IQ)
      IF(IQ.NE.0) GOTO 1004                                              00000730
C                                                                        00000740
C     ***** REGULAR END OF KAPROS JOB (IQ.EQ.0)                          00000750
C                                                                        00000760
      RETURN                                                             00000770
C                                                                        00000780
C     ***** ERROR - MESSAGES (IQ.NE.0)                                   00000790
C                                                                        00000800
 1001 WRITE(N5,101) IQ                                                   00000810
      GOTO 1009                                                          00000820
 1002 WRITE(N5,102) IQ                                                   00000830
      GOTO 1009                                                          00000840
 1003 WRITE(N5,103) IQ                                                   00000850
      GOTO 1009                                                          00000860
 1004 WRITE(N5,104) IQ                                                   00000870
      GOTO 1009                                                          00000880
C                                                                        00000890
C     ***** FORMAT STATEMENTS                                            00000900
C                                                                        00000910
  104 FORMAT(1H ,' ERROR IN MODULE CALL 4,  IQ = ',I6)                   00000920
  103 FORMAT(1H ,' ERROR IN MODULE CALL 3,  IQ = ',I6)                   00000930
  102 FORMAT(1H ,' ERROR IN MODULE CALL 2,  IQ = ',I6)                   00000940
  101 FORMAT(1H ,' ERROR IN MODULE CALL 1,  IQ = ',I6)                   00000950
C                                                                        00000960
C     ***** RETURN CONTROL TO KAPROS                                     00000970
C                                                                        00000980
 1009 RETURN                                                             00000990
      END                                                                00001000
*$*$                                                                     00001010
*LINK MAP,LIST                                                          00001020
 ENTRY STEUER                                                            00001030
 NAME STEUER                                                             00001040
*$*$                                                                     00001050
*KSIOX DBN=INPUT UTKS,TYP=CARD,PMN=PRUTKS                               00001060
'BWRITE  ' 'SIGMN           ' 12 'STRUC1NU' 'END UTKS'                  00001070
*$*$                                                                     00001080
*KSIOX DBN=GRUCAL,TYP=CARD,PMN=PRGRUC                                   00001090
'GRUCAL  '
'KFKINR  ' 'SN-RECH.' '
 10 11
'MISCH   '
3
```

```
14            'KFKINR  '    '          '    '          '                    00001150
'C       '       300.        0.02077                                            1
'NA      '       300.        0.006231                                           2
'FE      '       300.        0.01018                                            3
'CR      '       300.        0.002637                                           4
'NI      '       300.        0.001119                                           5
'MO      '       300.        0.000206                                           6
'AL      '       300.        0.000109                                           7
'SI      '       300.        0.000124                                           8
'PU239   '       300.        0.001645                                           9
'PU240   '       300.        0.000106                                          10
'PU241   '       300.        0.000011                                          11
'PU242   '       300.        0.0000004                                         12
'U 235   '       300.        0.000016                                          13
'U 238   '       300.        0.007405                                          14
14            'KFKINR  '    '          '    '          '                    00001150
'C       '       300.        0.02077                                            1
'NA      '       300.        0.006231                                           2
'FE      '       300.        0.01018                                            3
'CR      '       300.        0.002637                                           4
'NI      '       300.        0.001119                                           5
'MO      '       300.        0.000206                                           6
'AL      '       300.        0.000109                                           7
'SI      '       300.        0.000124                                           8
'PU239   '       300.        0.00166145                                         9
'PU240   '       300.        0.000106                                          10
'PU241   '       300.        0.000011                                          11
'PU242   '       300.        0.0000004                                         12
'U 235   '       300.        0.000016                                          13
'U 238   '       300.        0.007405                                          14
6             'KFKINR  '    '          '    '          '                    00001150
'FE      '       300.        0.004925                                           1
'CR      '       300.        0.001276                                           2
'NI      '       300.        0.000536                                           3
'SI      '       300.        0.000060                                           4
'U 235   '       300.        0.000083                                           5
'U 238   '       300.        0.03969                                            6
'GEW.CHI '
3
6
'PU239   '  0.844059
'PU240   '  0.151694E-1
'PU241   '  0.769374E-2
'PU242   '  0.438405E-4
'U 235   '  0.709431E-2
'U 238   '  0.125940
6
'PU239   '  0.844059
'PU240   '  0.151694E-1
'PU241   '  0.769374E-2
'PU242   '  0.438405E-4
'U 235   '  0.709431E-2
'U 238   '  0.125940
2
'U 235   '  0.877913E-2
'U 238   '  0.155849
'TYP     '
'ALLEIN  '  11
'CHI     '
'NUSF    '
'SCAPT   '
'SFISS   '
'SBE     '
'SREM    '
'STR     '
'STRTR   '
'1/V     '
'STOT    '
'SMTOT   '
'AUSWERT '
'ZUSATZ  '  3
'SFISS   '  '          '  0 0
'CHI     '  '          '  1 0
'NUSF    '  '          '  0 0
'GRUCEND '
*$*$
*KSIOX DBN=PERTUBATIONINPUT,PMN=PRDUM,TYP=CARD
  5*0
```

```
    2     3                                                                  11
*$*$
*KSIOX DBN=INPUT SIGMUT,TYP=CARD,PMN=PRSIGM                                00001420
'BETA'                                                                     00001430
'SIGMALT          ' 'SIGMN            '  26  3  6  6                       00001440
'FISS' '    '                                                              00001450
4*0.0 0.06 0.17 0.23 0.27 0.16 0.08 0.02 0.01                              00001460
14*0.0                                                                     00001470
4*0.0 0.14 0.37 0.28 0.11 0.06 0.03 0.01 0.0                               00001480
14*0.0                                                                     00001490
4*0.0 0.09 0.36 0.25 0.16 0.09 0.04 0.01 0.0                               00001500
14*0.0                                                                     00001510
0.0 0.0 0.0 0.07 0.16 0.30 0.22 0.15 0.06 0.03 0.01 0.0                    00001520
14*0.0                                                                     00001530
0.0 0.0 0.0 0.04 0.14 0.36 0.26 0.14 0.05 0.01 0.0 0.0                     00001540
14*0.0                                                                     00001550
0.0 0.0 0.0 0.04 0.14 0.36 0.26 0.14 0.05 0.01 0.0 0.0                     00001560
14*0.0                                                                     00001570
'PU239    ' 0.2523E-3 1.8592E-3 1.4343E-3 2.1779E-3 0.6839E-3 0.2324E-3    00001580
'PU240    ' 0.2688E-3 2.6208E-3 1.8432E-3 3.3600E-3 1.2288E-3 0.2784E-3    00001590
'PU241    ' 0.160E-3 3.664E-3 2.768E-3 6.240E-3 2.912E-3 0.256E-3          00001600
'PU242    ' 0.0912E-3 4.4460E-3 3.6708E-3 9.3936E-3 4.9704E-3 0.2280E-3    00001610
'U 235    ' 0.6449-3 3.6146-3 3.1904-3 6.9068-3 2.1721-3 0.4412-3
'U 238    ' 0.0586-2 0.6176-2 0.7303-2 1.7491-2 1.0143-2 0.3381-2
'END'                                                                      00001630
*KSIOX DBN=DTKINPUT,PMN=PRUEF,TYP=CARD,LMN=LESE                            00001640
 ZPR-3 ASSEMBLY 48 SPHERICAL MODEL DTK INPUT (6.3.1978)                      1
 -6001    -1     0     2     3     1     0     2     1    26     0     0     2
     3     3     0     0     4     0   100    20    20     0     0     1     3
   -12     1     0    30     1    31     0                                  4
         1.0       1.E-5          0.0                                      5
      1.0E-5         0.0                                                   6
      45.245      75.245                                                   7
     2     2                                                               8
     1     3                                                               9
99999                                                                       10
*GO SM=STEUER                                                              00001750
/*                                                                         00001760
//DEL EXEC PGM=IEFBR14
//A1  DD UNIT=3330,VOL=SER=TSTLIB,DISP=(OLD,DELETE),
//    DSN=KO.INR659.LAMBDA
/*
//  EXEC FGCLG
//C.SYSPRINT DD DUMMY
//C.SYSIN DD  *
C     REACTIVITY AND KINETIC PARAMETERS BY THE PERTURBATION METHOD USING    MAIN
C     ONE DIMENSIONAL MULTIGROUP SN CODE DTK MADE BY KEISUKE KOBAYASHI      MAIN
C     AT 26 AUG 1977.                                                       MAIN
C     MAIN PROGRAM                                                          MAIN
/*
/*
//L.MYOB3 DD UNIT=3330,VOL=SER=TSTLIB,DISP=SHR,DSN=OB3.INR659.LIES2
//L.SYSIN DD *
 INCLUDE MYOB3
 ENTRY MAIN
//G.FT10F001 DD  UNIT=SYSDA,DISP=(OLD,PASS),DSN=&&INTFAC
//G.FT11F001 DD  UNIT=SYSDA,DISP=(OLD,PASS),DSN=&&BAND
//G.FT13F001 DD UNIT=3330,VOL=SER=TSTLIB,
// DISP=(NEW,KEEP),SPACE=(TRK,2),DSN=KO.INR659.LAMBDA
//G.SYSIN DD *
 TEST DATA FOR TP-1  PERTURBATION FOR ZPR-3 ASSEMBLY 48 SPHERICAL MODEL.
  6  6  1  1  1  1  0  0  1  0  0  0
PU239   PU240   PU241   PU242   U 235   U 238
/*
//
```

REACTIVITY AND KINETIC PARAMETERS BY THE PERTURBATION METHOD USING ONE DIMENSIONAL MULTIGROUP SN CODE DTK.

TEST DATA FOR TP-1 PERTURBATION FOR ZPR-3 ASSEMBLY 48 SPHERICAL MODEL.
NUMBER OF FISSILE ISOTOPE IFM= 6
NUMBER OF DELAYED NEUTRON GROUPS 6

NAME OF ISOTOPE PU239 PU240 PU241 PU242 U 235 U 238

INPUT DATA FROM DTK PROGRAM

ISCT= 0 ISN= 2 IGE= 3 IZM= 2 IM= 4 IGM= 26 MT= 3 MTP= 3

NM= 1 MM= 3 IHS= 6 IHT= 5 IHM= 31 MBK= 0

BF= 0.0 XT= 1.420892E+00

MA(IM)
1 1 2 2

****

FLUXES AND ADJOINTS FOR ALL FOLLOWING PERTURBATION CALUCULATIONS ARE OBTAINED BY USING THE FOLLOWING COMPOSITIONS
IN SUCCEEDING REGIONS

****

FOR DIRECT EQUATION

MZ(IZM)

1 3

FOR ADJOINT EQUATION

MZ(IZM)

2 3

LAENGE= 200000 LASTR= 8992 IRIS= 191008

LEN= 500 LASTI= 71 IDIF= 429

CRITICALITY FACTOR FOR DIRECT EQUATION 1.002763E+00
CRITICALITY FACTOR FOR ADJOINT EQUATION 1.006042E+00
EXACT REACTIVITY FROM THE CRITICALITY FACTOR 3.250062E-03

CROSS SECTIONS CORRECTLY PREPARED

CORRECTION TO THE CRITICALITY FACTOR OF DTK WHERE AN ISOTOPE INDEPENDENT FISSION SPECTRUM IS USED FOR PROMPT AND DELAYED
FISSION NEUTRONS
CORRECTED CRITICALITY FACTOR FOR NORMAL EQUATION 1.001595E+00 (RO=-1.163E-03)

MEAN GENERATION TIME= 2.595200E-07SEC

EFFECTIVE DELAYED NEUTRON FRACTION FOR EACH ISOTOPE

MIXTURE INDEX= 1

GROUP	PU239	PU240	PU241	PU242	U 235	U 238	ROW SUM
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SUM	0.0	0.0	0.0	0.0	0.0	0.0	0.0

MIXTURE INDEX= 2

GROUP	PU239	PU240	PU241	PU242	U 235	U 238	ROW SUM
1	5.80305E-05	9.83343E-07	3.34634E-07	9.40804E-10	1.49094E-06	1.92195E-05	8.00598E-05
2	4.37983E-04	9.81850E-06	7.84861E-06	4.69668E-08	8.55880E-06	2.07408E-04	6.71664E-04
3	3.35774E-04	6.86235E-06	5.89225E-06	3.85368E-08	7.50714E-06	2.43739E-04	5.99812E-04
4	5.19558E-04	1.27469E-05	1.35359E-05	1.00485E-07	1.65612E-05	5.94806E-04	1.15731E-03
5	1.62790E-04	4.65141E-06	6.30279E-06	5.30517E-08	5.19681E-06	3.44161E-04	5.23155E-04
6	5.53188E-05	1.05384E-06	5.54092E-07	2.43356E-09	1.05558E-06	1.14720E-04	1.72705E-04
SUM	1.56945E-03	3.61163E-05	3.44682E-05	2.42415E-07	4.03705E-05	1.52405E-03	3.20470E-03

MIXTURE INDEX= 3

GROUP	PU239	PU240	PU241	PU242	U 235	U 238	ROW SUM
1	0.0	0.0	0.0	0.0	8.98963E-07	6.34870E-06	7.24767E-06
2	0.0	0.0	0.0	0.0	5.75262E-06	7.42776E-05	8.00302E-05
3	0.0	0.0	0.0	0.0	4.92246E-06	8.56790E-05	9.06014E-05
4	0.0	0.0	0.0	0.0	1.12850E-05	2.15576E-04	2.26861E-04
5	0.0	0.0	0.0	0.0	3.56746E-06	1.25402E-04	1.28969E-04
6	0.0	0.0	0.0	0.0	7.24631E-07	4.18006E-05	4.25252E-05
SUM	0.0	0.0	0.0	0.0	2.71511E-05	5.49083E-04	5.76234E-04

SUM OF BETA

GROUP	PU239	PU240	PU241	PU242	U 235	U 238	ROW SUM
1	5.80305E-05	9.83343E-07	3.34634E-07	9.40804E-10	2.38990E-06	2.55682E-05	8.73074E-05
2	4.37983E-04	9.81850E-06	7.84861E-06	4.69668E-08	1.43114E-05	2.81686E-04	7.51694E-04
3	3.35774E-04	6.86235E-06	5.89225E-06	3.85368E-08	1.24296E-05	3.29418E-04	6.90414E-04
4	5.19558E-04	1.27469E-05	1.35359E-05	1.00485E-07	2.78463E-05	8.10381E-04	1.38417E-03
5	1.62790E-04	4.65141E-06	6.30279E-06	5.30517E-08	8.76426E-06	4.69562E-04	6.52124E-04
6	5.53188E-05	1.05384E-06	5.54092E-07	2.43356E-09	1.78021E-06	1.56521E-04	2.15230E-04
SUM	1.56945E-03	3.61163E-05	3.44682E-05	2.42415E-07	6.75216E-05	2.07314E-03	3.78094E-03

EXACT PERTURBATION

I	R(I)	GROUP	CAPTURE	FISSION	REMOVAL	SCATT.OUT	SCATT.IN	OUT+IN	(G-1) TO G	FISS.SOURCE	AD.FISS.SOURCE	TOTAL
1	11.31	1	-3.0249E-14	-6.9921E-11	-1.3382E-10	-6.3780E-11	1.9555E-11	-4.4225E-11	0.0	1.3462E-09	2.1761E-10	1.2319E-09
1	11.31	2	-2.6518E-13	-2.0194E-10	-5.2018E-10	-3.1766E-10	1.1757E-10	-2.0009E-10	1.4125E-12	4.3178E-09	6.6720E-10	3.9152E-09
1	11.31	3	-1.2611E-12	-4.7223E-10	-1.2217E-09	-7.4818E-10	3.5520E-10	-3.9298E-10	6.7204E-12	7.8038E-09	1.4410E-09	6.9373E-09
1	11.31	4	-5.5131E-12	-9.7086E-10	-2.4320E-09	-1.4544E-09	9.5660E-10	-4.9784E-10	2.7064E-11	1.0355E-08	2.9568E-09	8.8792E-09
1	11.31	5	-1.2048E-11	-9.5710E-10	-2.7402E-09	-1.7698E-09	1.4731E-09	-2.9664E-10	7.0538E-11	6.7902E-09	3.1833E-09	5.5231E-09
1	11.31	6	-6.4040E-11	-1.5825E-09	-6.1626E-09	-4.5158E-09	4.4961E-09	-1.9764E-11	1.5241E-10	4.7195E-09	5.2002E-09	3.0530E-09
1	11.31	7	-1.4669E-10	-1.4026E-09	-7.5426E-09	-5.9911E-09	6.5383E-09	5.4717E-10	2.9257E-10	2.0531E-09	4.5950E-09	1.0488E-09
1	11.31	8	-1.9005E-10	-1.3401E-09	-8.8050E-09	-7.2741E-09	7.6429E-09	3.6879E-10	1.6315E-10	8.0155E-10	4.4162E-09	-3.6058E-10
1	11.31	9	-1.9092E-10	-1.1855E-09	-9.0821E-09	-7.7051E-09	7.8755E-09	1.7044E-10	1.5632E-10	3.1199E-10	3.9593E-09	-8.9463E-10
1	11.31	10	-3.1411E-10	-1.0389E-09	-8.8747E-09	-7.5195E-09	7.5930E-09	7.3449E-11	1.3361E-10	1.0147E-10	3.5108E-09	-1.1803E-09
1	11.31	11	-3.3525E-10	-7.3256E-10	-5.9230E-09	-4.8547E-09	4.8900E-09	3.5371E-11	1.1153E-10	4.7418E-11	2.4440E-09	-9.8560E-10
1	11.31	12	-4.0935E-10	-5.8152E-10	-4.1493E-09	-3.1578E-09	3.2100E-09	5.2192E-11	2.9812E-14	8.0550E-11	1.8473E-09	-9.3929E-10
1	11.31	13	-2.8836E-10	-3.1811E-10	-2.3005E-09	-1.6939E-09	1.6724E-09	-2.1543E-11	2.9464E-11	0.0	9.5309E-10	-6.2812E-10
1	11.31	14	-4.7325E-10	-5.3903E-10	-2.3696E-09	-1.3571E-09	1.4194E-09	6.2267E-11	7.0252E-11	0.0	1.5435E-09	-9.5024E-10
1	11.31	15	-3.3555E-10	-4.1676E-10	-1.5554E-09	-8.0288E-10	8.1293E-10	1.0052E-11	1.4222E-11	0.0	1.0596E-09	-7.4248E-10
1	11.31	16	-1.4067E-10	-2.0398E-10	-6.2091E-10	-2.7621E-10	2.8186E-10	5.6575E-12	6.4744E-12	0.0	4.6522E-10	-3.3905E-10
1	11.31	17	-3.9924E-11	-6.4231E-11	-1.8504E-10	-8.0877E-11	8.2033E-11	1.1557E-12	1.3264E-12	0.0	1.4143E-10	-1.0300E-10
1	11.31	18	-8.8912E-12	-2.4866E-11	-4.5044E-11	-1.1287E-11	1.1590E-11	3.0319E-13	3.0757E-13	0.0	4.4736E-11	-3.3454E-11
1	11.31	19	-4.7477E-13	-2.4917E-13	-1.3508E-12	-6.2690E-13	6.3694E-13	1.0035E-14	9.3138E-15	0.0	1.0907E-12	-7.1391E-13
1	11.31	20	-1.2159E-13	-1.4047E-13	-3.3264E-13	-7.0574E-14	7.3333E-14	2.7588E-15	2.1302E-15	0.0	3.5318E-13	-2.5931E-13
1	11.31	21	-2.6560E-15	-2.1006E-15	-6.8424E-15	-2.0854E-15	2.2360E-15	1.5068E-16	6.6999E-17	0.0	8.7319E-15	-4.6064E-15
1	11.31	22	-1.5986E-16	-1.2603E-15	-2.6886E-15	-1.2682E-15	1.3246E-15	5.6401E-17	1.9817E-17	0.0	2.5883E-15	-1.3640E-15
1	11.31	23	-4.7167E-17	-1.2130E-16	-2.3390E-16	-6.5426E-17	7.2856E-17	7.4301E-18	2.7355E-18	0.0	4.4866E-16	-1.6104E-16
1	11.31	24	-1.0254E-17	-1.9250E-17	-3.3233E-17	-3.7293E-18	5.6366E-18	1.9072E-18	2.6735E-19	0.0	4.5525E-17	-2.7597E-17
1	11.31	25	0.0	0.0	0.0	0.0	5.2105E-19	5.2105E-19	1.0735E-20	0.0	0.0	5.2105E-19
1	11.31	26	0.0	0.0	0.0	0.0	1.5909E-19	1.5909E-19	0.0	0.0	0.0	1.5909E-19
1	11.31	SUM	-2.9568E-09	-1.2103E-08	-6.4665E-08	-4.9595E-08	4.9449E-08	-1.4621E-10	1.3179E-09	3.8648E-08	3.8648E-08	2.3431E-08

2	33.93	1	-1.3738E-14	-3.1756E-11	-6.0778E-11	-2.8967E-11	9.2164E-12	-1.9750E-11	0.0	6.7177E-10	9.5327E-11	6.2021E-10
2	33.93	2	-1.1535E-13	-9.7836E-11	-2.2626E-10	-1.3817E-10	5.2944E-11	-8.5229E-11	6.4138E-13	2.0758E-09	2.8984E-10	1.9025E-09
2	33.93	3	-5.3228E-13	-1.9931E-10	-5.1563E-10	-3.1579E-10	1.5416E-10	-1.6163E-10	2.9690E-12	3.6811E-09	6.1465E-10	3.3196E-09
2	33.93	4	-2.2911E-12	-4.0347E-10	-1.0107E-09	-6.0443E-10	4.0851E-10	-1.9592E-10	1.1665E-11	4.8534E-09	1.2482E-09	4.2512E-09
2	33.93	5	-5.0602E-12	-4.0200E-10	-1.1509E-09	-7.4334E-10	6.2933E-10	-1.1401E-10	2.9062E-11	3.0738E-09	1.3980E-09	2.5522E-09
2	33.93	6	-2.9017E-11	-7.1703E-10	-2.7923E-09	-2.0461E-09	2.0377E-09	-8.4657E-12	6.4806E-11	2.1194E-09	2.4587E-09	1.3648E-09
2	33.93	7	-6.8749E-11	-6.5735E-10	-3.5350E-09	-2.8078E-09	3.0515E-09	2.4369E-10	1.3205E-10	9.0905E-10	2.2701E-09	4.2558E-10
2	33.93	8	-8.7460E-11	-6.1669E-10	-4.0520E-09	-3.3475E-09	3.5189E-09	1.7138E-10	7.5324E-11	3.4743E-10	2.1858E-09	-1.8571E-10
2	33.93	9	-8.6379E-11	-5.3634E-10	-4.1091E-09	-3.4861E-09	3.5707E-09	8.4620E-11	7.0594E-11	1.3203E-10	1.9711E-09	-4.0638E-10
2	33.93	10	-1.3628E-10	-4.5073E-10	-3.8503E-09	-3.2624E-09	3.3071E-09	4.4791E-11	5.9569E-11	4.2147E-11	1.7074E-09	-5.0100E-10
2	33.93	11	-1.3701E-10	-2.9938E-10	-2.4206E-09	-1.9840E-09	2.0106E-09	2.6620E-11	4.8014E-11	1.9468E-11	1.1340E-09	-3.9054E-10
2	33.93	12	-1.5714E-10	-2.2323E-10	-1.5928E-09	-1.2122E-09	1.2386E-09	2.6346E-11	3.2753E-11	1.2133E-14	8.1213E-10	-3.5426E-10
2	33.93	13	-1.0870E-10	-1.1991E-10	-9.6719E-10	-6.3853E-10	6.3266E-10	-5.8773E-12	1.1507E-11	0.0	4.0527E-10	-2.3453E-10
2	33.93	14	-1.7582E-10	-2.0025E-10	-8.8032E-10	-5.0417E-10	5.3109E-10	2.6917E-11	2.7066E-11	0.0	6.3627E-10	-3.4923E-10
2	33.93	15	-1.2589E-10	-1.5636E-10	-5.8356E-10	-3.0123E-10	3.0659E-10	5.3648E-12	5.3935E-12	0.0	4.3429E-10	-2.7697E-10
2	33.93	16	-5.3101E-11	-7.7000E-11	-2.3439E-10	-1.0427E-10	1.0690E-10	2.6326E-12	2.4703E-12	0.0	1.8948E-10	-1.2749E-10
2	33.93	17	-1.4647E-11	-2.3564E-11	-6.7884E-11	-2.9671E-11	3.0254E-11	5.8235E-13	4.9786E-13	0.0	5.6577E-11	-3.7631E-11
2	33.93	18	-3.3227E-12	-9.2926E-12	-1.6833E-11	-4.2179E-12	4.3525E-12	1.3464E-13	1.1693E-13	0.0	1.7668E-11	-1.2481E-11
2	33.93	19	-1.6595E-13	-8.7096E-14	-4.7218E-13	-2.1913E-13	2.2411E-13	4.9800E-15	3.3793E-15	0.0	4.1737E-13	-2.4807E-13
2	33.93	20	-4.2823E-14	-4.9471E-14	-1.1715E-13	-2.4855E-14	2.6039E-14	1.1843E-15	7.6982E-16	0.0	1.3248E-13	-9.1109E-14
2	33.93	21	-8.7906E-16	-6.9524E-16	-2.2647E-15	-6.9020E-16	7.5295E-16	6.2750E-17	2.3007E-17	0.0	3.1815E-15	-1.5117E-15
2	33.93	22	-5.2769E-17	-4.1601E-16	-8.8748E-16	-4.1861E-16	4.4340E-16	2.4793E-17	6.5713E-18	0.0	9.4608E-16	-4.4408E-16
2	33.93	23	-1.6817E-17	-4.3247E-17	-8.3394E-17	-2.3327E-17	2.6401E-17	3.0742E-18	9.7014E-19	0.0	1.6549E-16	-5.6993E-17
2	33.93	24	-3.7510E-18	-7.0422E-18	-1.2157E-17	-1.3643E-18	2.1737E-18	8.0947E-19	9.6874E-20	0.0	1.6993E-17	-9.9837E-18
2	33.93	25	0.0	0.0	0.0	0.0	2.2820E-19	2.2820E-19	4.0008E-21	0.0	0.0	2.2820E-19
2	33.93	26	0.0	0.0	0.0	0.0	6.9812E-20	6.9812E-20	0.0	0.0	0.0	6.9812E-20
2	33.93	SUM	-1.1917E-09	-5.2116E-09	-2.7967E-08	-2.1559E-08	2.1601E-08	4.2210E-11	5.7450E-10	1.7925E-08	1.7925E-08	1.1559E-08
INTEGRAL		1	-6.1307E-09	-1.4171E-05	-2.7122E-05	-1.2927E-05	4.0771E-06	-8.8495E-06	0.0	2.9333E-04	4.2915E-05	2.7029E-04
INTEGRAL		2	-5.2018E-08	-3.9611E-05	-1.0204E-04	-6.2312E-05	2.3675E-05	-3.8637E-05	2.8624E-07	9.1409E-04	1.3075E-04	8.3573E-04
INTEGRAL		3	-2.4185E-07	-9.0564E-05	-2.3429E-04	-1.4349E-04	6.9559E-05	-7.3927E-05	1.3338E-06	1.6281E-03	2.7854E-04	1.4634E-03
INTEGRAL		4	-1.0451E-06	-1.8405E-04	-4.6105E-04	-2.7572E-04	1.8507E-04	-9.0652E-05	5.2726E-06	2.1498E-03	5.6711E-04	1.8738E-03
INTEGRAL		5	-2.3021E-06	-1.8288E-04	-5.2360E-04	-3.3817E-04	2.8508E-04	-5.3089E-05	1.3287E-05	1.3728E-03	6.2898E-04	1.1343E-03
INTEGRAL		6	-1.2956E-05	-3.2016E-04	-1.2468E-03	-9.1362E-04	9.0979E-04	-3.8324E-06	2.9391E-05	9.4835E-04	1.0869E-03	6.1136E-04
INTEGRAL		7	-3.0453E-05	-2.9118E-04	-1.5658E-03	-1.2437E-03	1.3530E-03	1.0926E-04	5.9017E-05	4.0817E-04	9.9348E-04	1.9534E-04
INTEGRAL		8	-3.8907E-05	-2.7434E-04	-1.8026E-03	-1.4892E-03	1.5652E-03	7.6065E-05	3.3483E-05	1.5681E-04	9.5619E-04	-8.0532E-05
INTEGRAL		9	-3.8583E-05	-2.3957E-04	-1.8354E-03	-1.5571E-03	1.5941E-03	3.6992E-05	3.1546E-05	5.9952E-05	8.6116E-04	-1.8134E-04
INTEGRAL		10	-6.1496E-05	-2.0340E-04	-1.7375E-03	-1.4722E-03	1.4909E-03	1.8768E-05	2.6702E-05	1.9229E-05	7.4989E-04	-2.2732E-04
INTEGRAL		11	-6.2770E-05	-1.3716E-04	-1.1090E-03	-9.0895E-04	9.1971E-04	1.0752E-05	2.1708E-05	8.9086E-06	5.0348E-04	-1.8038E-04
INTEGRAL		12	-7.3198E-05	-1.0398E-04	-7.4196E-04	-5.6466E-04	5.7614E-04	1.1475E-05	1.5025E-05	5.5647E-09	3.6528E-04	-1.6582E-04
INTEGRAL		13	-5.0884E-05	-5.6134E-05	-4.0595E-04	-2.9892E-04	2.9589E-04	-3.0400E-06	5.3352E-06	0.0	1.8380E-04	-1.1008E-04
INTEGRAL		14	-8.2636E-05	-9.4121E-05	-4.1377E-04	-2.3697E-04	2.4913E-04	1.2157E-05	1.2595E-05	0.0	2.9085E-04	-1.6464E-04
INTEGRAL		15	-5.9010E-05	-7.3292E-05	-2.7354E-04	-1.4120E-04	1.4350E-04	2.3087E-06	2.5207E-06	0.0	1.9882E-04	-1.3003E-04
INTEGRAL		16	-2.4848E-05	-3.6032E-05	-1.0968E-04	-4.8791E-05	4.9959E-05	1.1681E-06	1.1526E-06	0.0	8.6886E-05	-5.9723E-05
INTEGRAL		17	-6.9084E-06	-1.1114E-05	-3.2019E-05	-1.3995E-05	1.4249E-05	2.5374E-07	2.3334E-07	0.0	2.6065E-05	-1.7770E-05
INTEGRAL		18	-1.5592E-06	-4.3605E-06	-7.8989E-06	-1.9792E-06	2.0396E-06	6.0411E-08	5.4610E-08	0.0	8.1673E-06	-5.8593E-06
INTEGRAL		19	-7.9361E-08	-4.1651E-08	-2.2580E-07	-1.0479E-07	1.0697E-07	2.1773E-09	1.5989E-09	0.0	1.9458E-07	-1.1883E-07
INTEGRAL		20	-2.0434E-08	-2.3607E-08	-5.5901E-08	-1.1860E-08	1.2396E-08	5.3584E-10	3.6464E-10	0.0	6.2102E-08	-4.3505E-08
INTEGRAL		21	-4.2722E-10	-3.3789E-10	-1.1006E-09	-3.3544E-10	3.6405E-10	2.8609E-11	1.1059E-11	0.0	1.5035E-09	-7.3658E-10
INTEGRAL		22	-2.5666E-11	-2.0234E-10	-4.3167E-10	-2.0361E-10	2.1476E-10	1.1152E-11	3.1918E-12	0.0	4.4669E-10	-2.1690E-10
INTEGRAL		23	-7.9963E-12	-2.0564E-11	-3.9654E-11	-1.1092E-11	1.2496E-11	1.4039E-12	4.6200E-13	0.0	7.7938E-11	-2.7158E-11
INTEGRAL		24	-1.7706E-12	-3.3242E-12	-5.7389E-12	-6.4400E-13	1.0113E-12	3.6729E-13	4.5852E-14	0.0	7.9764E-12	-4.7276E-12
INTEGRAL		25	0.0	0.0	0.0	0.0	1.0274E-13	1.0274E-13	1.8788E-15	0.0	0.0	1.0274E-13
INTEGRAL		26	0.0	0.0	0.0	0.0	3.1415E-14	3.1415E-14	0.0	0.0	0.0	3.1415E-14
INTEGRAL		SUM	-5.4795E-04	-2.3562E-03	-1.2630E-02	-9.7240E-03	9.7312E-03	7.2385E-06	2.5894E-04	7.9595E-03	7.9595E-03	5.0604E-03