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# **CRISS Power Spectral Density — A Method for Correlating Signal Components from Different Frequency Ranges**

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## Abstract

The correlation of signal components at different frequencies like higher harmonics cannot be detected by a normal power spectral density measurement, since this technique correlates only components at the same frequency.

This paper describes a special method for measuring the correlation of two signal components at different frequencies: the CRISS power spectral density. From this new function in frequency analysis, the correlation of two components can be determined quantitatively either they stem from one signal or from two diverse signals. The principle of the method, suitable for the higher harmonics of a signal as well as for any other frequency combination is shown for the digital frequency analysis technique. Two examples of CRISS power spectral densities demonstrate the operation of the new method.

Spektrale QUER Leistungsdichte -  
eine Methode zur Korrelation von Signalkomponenten  
aus verschiedenen Frequenzbereichen

## Zusammenfassung

Die Messung einer (normalen) Spektraldichte korreliert nur Signalanteile bei derselben Frequenz, so daß damit die Korrelation von Oberwellen nicht gemessen werden kann.

Hier wird eine Methode beschrieben, die die Korrelation von Signalkomponenten bei verschiedenen Frequenzen mißt: die spektrale QUER Leistungsdichte (CRISS power spectral density). Mit dieser Funktion kann die Korrelation verschiedener Frequenzanteile von einem oder zwei Signalen quantitativ erfaßt werden. Das Prinzip erlaubt die Detektion sowohl von Oberwellen als auch der Korrelation von Signalanteilen bei beliebiger Frequenzkombination. Das Meßverfahren ist für die digitale Frequenzanalyse erläutert. Zwei Beispiele zeigen seine Funktionsfähigkeit.

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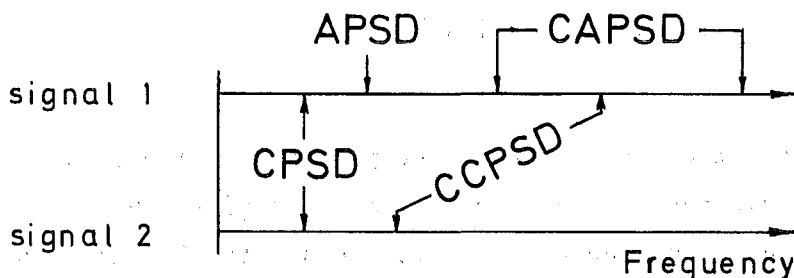
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1. Introduction

For the surveillance of a nuclear power plant and investigation of the system's dynamics power spectral density functions of diverse signals are measured, e.g. neutron flux, temperature, barrel movement, etc. These power spectral densities often include resonances at frequencies which are harmonic multiples of a fundamental frequency. These characteristics suggest that there may be harmonic components in the signal. However, the normal frequency analysis technique can correlate only signal components at the same frequency. Thus it cannot be decided whether two resonances in a power spectral density are correlated or not, i.e. whether they are caused by the same source. If the resonances are really sharp and exact at harmonic frequencies (integer multiples of a fundamental mode) their correlation is obvious but cannot be guaranteed. An attempt to correlate harmonics is made by the cepstrum technique /e.g. 1/. However, this also gives only evidence of the presence of harmonic components.

The method described in this note correlates signal components coming from different frequency ranges. It is suitable for higher harmonics as well as for spectral components at any other frequency combination. In addition, the signal components, either deterministic or stochastic, may be superposed by some uncorrelated noise. The CRISS auto power spectral density (CAPSD) is measured from two components of the same signal, e.g. two harmonics of a square wave. The CRISS cross power spectral density (CCPSD) correlates components from both, diverse signals and different frequencies. See the sketch.



## 2. The Normal Frequency Analysis

Since nowadays power spectral densities are measured mostly by the digital frequency analysis using small computers, the principle of the new method is described for this analyzing technique. The digital frequency analysis for measuring auto and cross power spectral densities (APSD, CPSD) is well known from the literature /2-4/. To get a PSD five main steps have to be done:

- 1 Select a part of the signal within the time interval  $k$  of length  $T$ ;
- 2 Sampling at a constant frequency and digitizing this signal interval gives a time series;
- 3 A discrete Fourier transform (e.g. FFT) of this time series gives the complex Fourier coefficients

$$c(n) = r_n \cdot e^{j\omega_n \tau_n} \quad (2.1)$$

at equidistant frequency points  $\omega_n = n \cdot \omega_0 = n \cdot \frac{2\pi}{T}$  ;

- 4 Multiplication of each coefficient with its complex conjugate gives the PSD values

$$\text{APSD}_k(n) = |c(n)|^2 = r_n^2 \cdot e^{j\omega_n(\tau_n - \tau_n)} = r_n^2 \quad (2.2)$$

$$\text{CPSD}_k(n) = c_1^+(n) \cdot c_2(n) = r_1 \cdot r_2 \cdot e^{j\omega_n(\tau_{2n} - \tau_{1n})} ;$$

- 5 Averaging these PSD values from many time intervals of the same length gives the desired power spectral density

$$\text{PSD}(n) = \langle \text{PSD}_k(n) \rangle$$

The phase of each coefficient  $c_n$  calculated in step 3 is determined by  $\tau_n$  and the frequency  $\omega_n$  of this coefficient. The  $\tau_n$  value is given by the time position of the signal's component at  $\omega_n$  with regard to the analyzing



frequency  $\omega_n$ . Since usually the analyzer runs independently of the signal the  $\tau_n$  from each time interval  $k$  will have another value and can be considered as a stochastic quantity. Two coefficients  $c1_n$  and  $c2_n$  of two signals will have two stochastic values  $\tau1_n$  and  $\tau2_n$ . However, if the two components are correlated their time relation with regard to each other is fixed. Therefore, the differences  $\tau1_n - \tau2_n$  will be constant values independent of the actual time interval  $k$ . By step 4, in which the phase values are subtracted, the stochastic part of the phase is eliminated. The resulting APSD from one signal is real valued. A measured CPSD from two signals is a complex function but with a phase independent of the actual time interval  $k$  and determined only by the correlation of the two signals. Thus step 5 can be applied successfully.

Note: The stochastic part of the phase is eliminated due to the fact that step 4 is done with coefficients at the same frequency  $\omega_n$ .

The correlation of signal components at different frequencies by mere multiplication of a coefficient with the complex conjugate of a second coefficient at another frequency, according to step 4, has to fail.

In the product 
$$r_n \cdot r_1 \cdot e^{j(\omega_n \cdot \tau_n - \omega_1 \cdot \tau_1)} \quad (2.3)$$

the stochastic part of the phase is not eliminated since the  $\tau$  values of the two components are multiplied by different frequencies  $\omega_n$  and  $\omega_1$ . As a consequence the average of these products from many time intervals will be always zero, even if there is a correlation between the two chosen components.

As a conclusion from this chapter it is seen that correlating of components at different frequencies will be successful if it can be attained that the  $\tau$  values of equation 2.3 are multiplied by a common frequency value. This can be achieved since the frequency values  $\omega_n$  of the two coefficients are multiples of a common fundamental frequency  $\omega_0$  (see equation 2.1). There are three mathematical operations to get the same frequency values in equation 2.3:

- a) the phase of each coefficient is multiplied by a factor proportional to the frequency of the other coefficient;
- b) each (entire) coefficient is rised to a power proportional to the frequency of the other coefficient;
- c) the time signals of the components to be correlated are rised to a power corresponding to b).

### 3. How to Measure The CRISS PSD?

Like in a normal PSD measurement the steps 1 to 3 are performed. Now the coefficients of the two components to be correlated are fetched:  $c(n)$  and  $c(1)$ . The phase of each coefficient is multiplied by a factor proportional to the frequency of the other coefficient:

$$c^{(1)}(n) = r_n \cdot e^{j\omega_n \tau_n \cdot l} = r_n \cdot e^{j \cdot n \cdot l \cdot \omega_o \cdot \tau_n} \quad (3.1)$$

$$c^{(n)}(1) = r_1 \cdot e^{j\omega_1 \cdot \tau_1 \cdot n} = r_1 \cdot e^{j \cdot l \cdot n \cdot \omega_o \cdot \tau_1}$$

Using these new coefficients in step 4 a quantity is calculated, the average of which gives the CRISS auto power spectral density

$$CAPSD(n,1) = \left\langle r_n \cdot r_1 \cdot e^{j \cdot l \cdot n \cdot \omega_o (\tau_1 - \tau_n)} \right\rangle \quad (3.2)$$

As to be seen from the formula only the difference  $(\tau_1 - \tau_n)$  appears in the exponent. Similar to equation 2.2 the phase of the  $CAPSD_k$  is independent of the actual time interval  $k$ . Averaging according to step 5 gives the CAPSD, the phase of which is determined by the signal's characteristics only. By the same procedure, but fetching the two coefficients from different signals, the CRISS cross power spectral density results

$$CCPSD(n,1) = \left\langle r1_n \cdot r2_1 \cdot e^{j \cdot l \cdot n \cdot \omega_o (\tau2_1 - \tau1_n)} \right\rangle \quad (3.3)$$

As mentioned in the previous chapter there is a second way to criss-correlate. The multiplication of the phase of the Fourier coefficients may be replaced by rising each entire coefficient to a power proportional to the frequency of the other coefficient.

The phase of these new coefficients  $c^1(n)$  and  $c^n(1)$  will be the same as in equation 3.1, their magnitudes, however, will be  $r_n^1$  and  $r_1^n$  respectively. Therefore, the resulting CRISS PDSs will be

$$\begin{aligned} \text{CAPSD}_p(n,1) &= \left\langle r_n^1 \cdot r_1^n \cdot e^{j \cdot 1 \cdot n \cdot \omega_o (\tau_1 - \tau_n)} \right\rangle \\ \text{CCPSD}_p(n,1) &= \left\langle r_n^1 \cdot r_1^n \cdot e^{j \cdot 1 \cdot n \cdot \omega_o (\tau_2 - \tau_1)} \right\rangle \end{aligned} \quad (3.4)$$

The index p denotes that the powers of the Fourier coefficients have been used in step 4 and 5.

Instead of rising the Fourier coefficients to the appropriate power, this may be done with the time signal itself, too. This third way seems to be advantageous if a signal component to be correlated contains more than one Fourier coefficient, i.e. the component is a narrow band signal. However, in order to achieve a better signal-to-noise ratio it is recommended to take not the entire signal but only the band which contains the desired component. Therefore, some band-pass filtering must be done. To avoid analog filters a band-pass filtered signal can be calculated by the inverse Fourier transform applied to the coefficients got in step 3 which lie within the desired pass-band at center frequencies  $\omega_n$  and  $\omega_1$ . From the pass-band filtered time signals  $x(n)$  and  $x(1)$  the new time signals  $x^1(n)$  and  $x^n(1)$  are computed. The normal cross power spectral density (steps 3 to 5) of these two signals gives the CRISS PSD (time version) of the two original components

$$\text{Criss PSD}_t(n,1) = \left\langle \left[ \text{FT}(x^1(n)) \right]^+ \cdot \left[ \text{FT}(x^n(1)) \right] \right\rangle \quad (3.5)$$

$\text{FT}(x^1(n))$  is the Fourier transform of the time signal  $x^1(n)$ .

Of course, if there is a common factor p in the frequency values  $\omega_n = n \cdot \omega_o$  and  $\omega_1 = 1 \cdot \omega_o$  the multiplication and rising to a power may be done with

$1/p$  and  $n/p$ , respectively. From the point of view of signal theory, multiplying the phase of a coefficient by a factor of  $n$  means shifting this component to a frequency  $n$  times higher. Thus by the procedures described above the two components are shifted to the same frequency.

The principle of measuring CRISS PSDs was shown for signals which are free from uncorrelated noise.

It can be shown that the method works, too, if there is a contribution to the signals and hence to the Fourier coefficients by some uncorrelated noise. However, in the formulae of the CRISS PSDs only the quantities  $r$  and  $(\tau_1 - \tau_n)$  of the correlated components will appear. Of course, the signal-to-noise ratio will be reduced. Well known coherence criteria will help to decide about correlation if the CRISS PSD is normalized to the APSDs of the modified components. Using the first way, namely phase multiplying, the normalization is done with the normal APSD values.

Some characteristics of the CRISS PSD should be mentioned. Because of the fact that each frequency point may be correlated with all the other ones the CRISS PSD is in general a two-dimensional function. The CRISS APSD - in contrary to a normal APSD - may have a phase. The phase of a measured CRISS PSD will reproduce the phase lag of the components; thus the wave form of the time signal whose harmonic components have been criss-correlated can be reproduced. Usually a signal's wave form is measured by signal recovery using a trigger signal. But if the signal is a stochastic signal or a deterministic one hidden by uncorrelated noise no trigger signal is available and therefore signal recovery cannot be applied.

#### 4. Two examples

In order to demonstrate the new method an artificial signal was generated and analyzed. The analysis was done with an on-line system for the digital frequency analysis based on a small digital computer /7/. The system has been extended by special programs performing the mathematical operations described here. Fig. 1 shows the result. A square wave at 20 cps, but not synchronized with the analyzer, was mixed with a band limited white noise signal. The auto power spectral density APSD was measured by the well known

digital frequency analysis technique. In addition from each interval the Fourier coefficient at 20 cps was picked and its phase was multiplied by 3 in order to look for the harmonic component at three times the fundamental frequency. This new coefficient was put to all frequency points of the Fourier transformed interval. Of course, its APSD is a constant equal to the signal's APSD value at 20 cps. Multiplying this modified Fourier-coefficient by the conjugated complex of the signal's Fourier-coefficients - i.e. normal Cross PSD - results in the CRISS PSD. As expected only at  $60 = 20 \cdot 3$  cps a correlation is to be found. The coherence

$$q = \frac{| \text{CRISS PSD} |}{\sqrt{\text{APSD}(\text{signal}) \cdot \text{APSD}(20)}}$$

shown in the middle part of Fig. 1 confirms the correlation at 60 cps. Due to the finite number of 5000 averaged intervals the coherence function shows a bias error which is to be seen besides 60 cps, where no correlation is expected. For a number of 5000 averaged intervals and no correlation the magnitude coherence  $q$  will have a bias error of  $.012 \pm 53\%$  /5,6/. This agrees well with the experimentally found coherence values in Fig. 1. In the lower part of the figure the real and imaginary part of the CRISS (auto) power spectral density is plotted. The CAPSD at 60 cps shows a negative real part and an imaginary part near zero. The negative sign can be explained by considering that a square wave is composed of sine waves  $\sin(\omega t) + 1/3 \sin(3\omega t) + \dots$  and that the third power of  $\sin(\omega t)$  equals  $-1/4 \sin(3\omega t) + \dots$ . In normal power spectral density measurements the Cross PSD equals the geometric mean of the two auto PSDs. If there exists uncorrelated noise the Cross PSD equals the geometric mean of the correlated parts in the auto PSDs. Unfortunately this is not true for the CRISS PSD if uncorrelated noise is present: the CRISS PSD is reduced due to the modification of the coefficients. This reduction is the greater the higher the number is which is used as factor in the phase or for rising the coefficients to a power. Thus for practical use small factors may be used only if uncorrelated noise is present.

For the second example a true reactor signal was used: the neutron flux fluctuation from the KNK II /8/ a sodium cooled fast Reactor at a power

level of about  $29 \text{ MW}_{\text{th}}$  ( $\approx 50\%$  full power). The APSD of the signal up to 4 cps is shown in Fig. 2. The signal was high-pass filtered at .25 cps. Therefore, the first 5 frequency points are not used for the criss correlation. The signal's APSD shows two big resonances, the second of which at  $\approx 3.4$  cps is at two times the frequency value of the first one at 1.7 cps. Therefore, the CRISS PSD for the second harmonic was measured: each Fourier coefficient of the first half of the frequency range was squared and put to the place at double the frequency. In addition two adjacent coefficients are multiplied and put to the frequency point at the sum of their frequencies. Performing step 4, these new coefficients are multiplied by the complex conjugates of the original coefficients. The coherence from the resulting CRISS PSD and the APSDs of the modified coefficients and the original coefficients, respectively, is shown in Fig. 2, too. It confirms the second harmonic correlation for two peaks. Due to 1791 averaged intervals ( $\approx 16$  hours) a bias error of the coherence of  $.02 \pm 53\%$  is expected.

In addition, an attempt was made to find third harmonic correlation in the signal. For this purpose new coefficients were calculated

$$c_{\text{new}}(3n + k) = c^{3-k}(n) \cdot c^k(n + 1) \quad k = 0, 1, 2$$

The coherence shown in the lower part of Fig. 2 indicates no third harmonic correlation.

## 5. Conclusion

The CRISS power spectral density introduced here is a useful tool for measuring the correlation of higher harmonics. With this function their correlation can be determined quantitatively. In general the CRISS PSD correlates signal components coming from different frequency ranges. The normal PSD may be considered as a special case of the CRISS PSD: namely the correlation of components at the same frequency.

The performance of the CRISS PSD measurement is described for the digital frequency analysis technique. The quintessence of the signal processing

for a CRISS PSD is the elimination of the stochastic relation between the signals and the analyzer without disturbing the phase relation between the signal components themselves. This can be achieved by different operations applied to the Fourier coefficients of the two components. As a common basis all effective operations for this purpose must multiply the phase of one coefficient by a factor proportional to the frequency of the other coefficient.

A first example shows the measurement of the third harmonic correlation of a noise contaminated square wave. As another example the second harmonic correlation of the neutron flux fluctuation of a nuclear power reactor was measured. No third harmonic correlation was found in this signal. Statements about the correlation between two or more frequency ranges in a power spectral density function will reduce the number of possible noise sources. On the other hand a statement for no correlation between two resonances of similar shape will be useful, too. Thus the CRISS PSD will help to understand normal power spectral densities, especially those from signals with higher harmonics.

From the coherence of the CRISS PSD and the auto PSDs of the components the degree of correlation can be determined quantitatively. The CRISS PSD itself is a complex quantity and reproduces the phase lag between the two components. Thus the wave form of a deterministic signal hidden in uncorrelated noise can be determined by the CRISS correlation of the signal's harmonic components.

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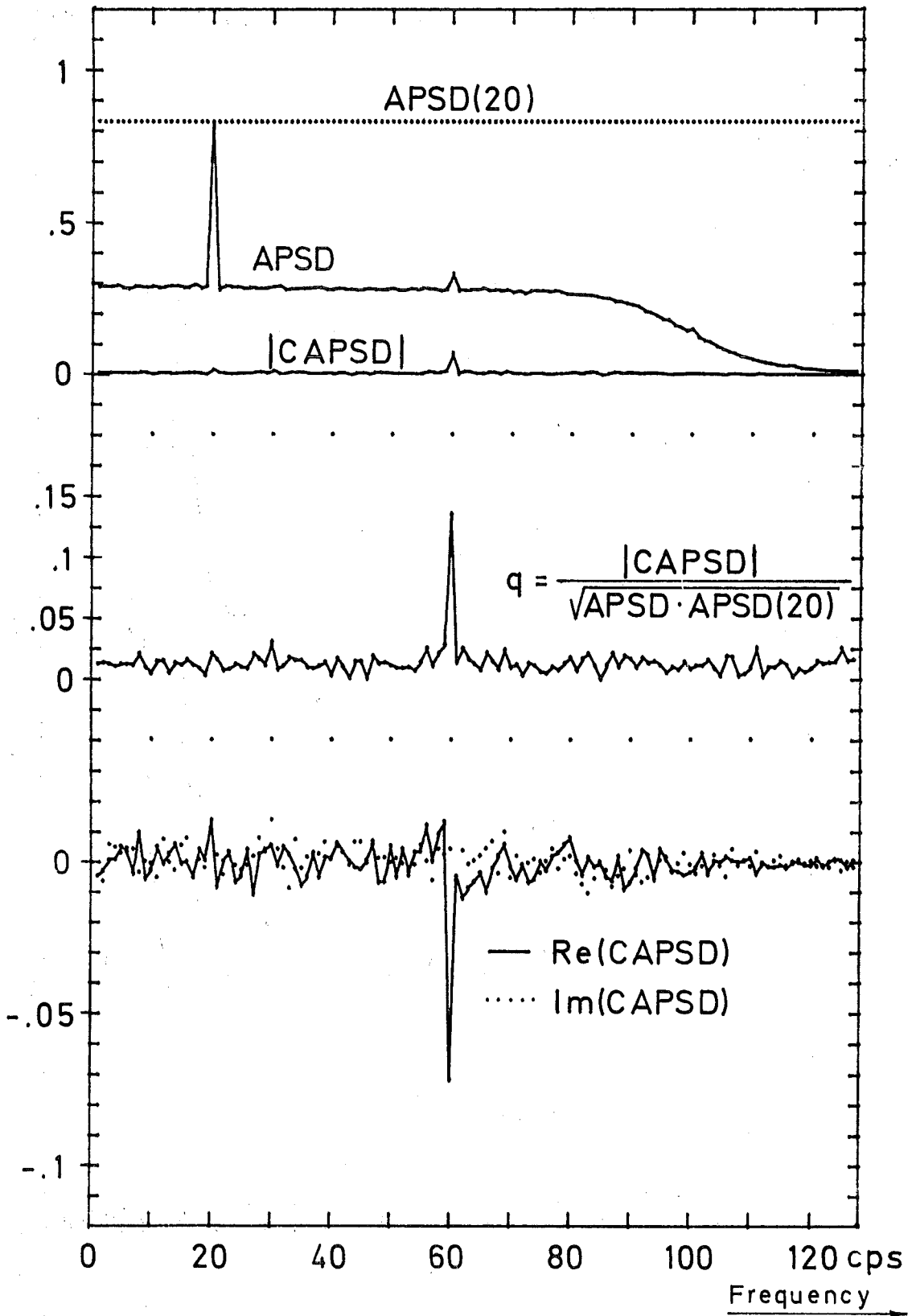


Fig.1 Third Harmonic Correlation of a square wave

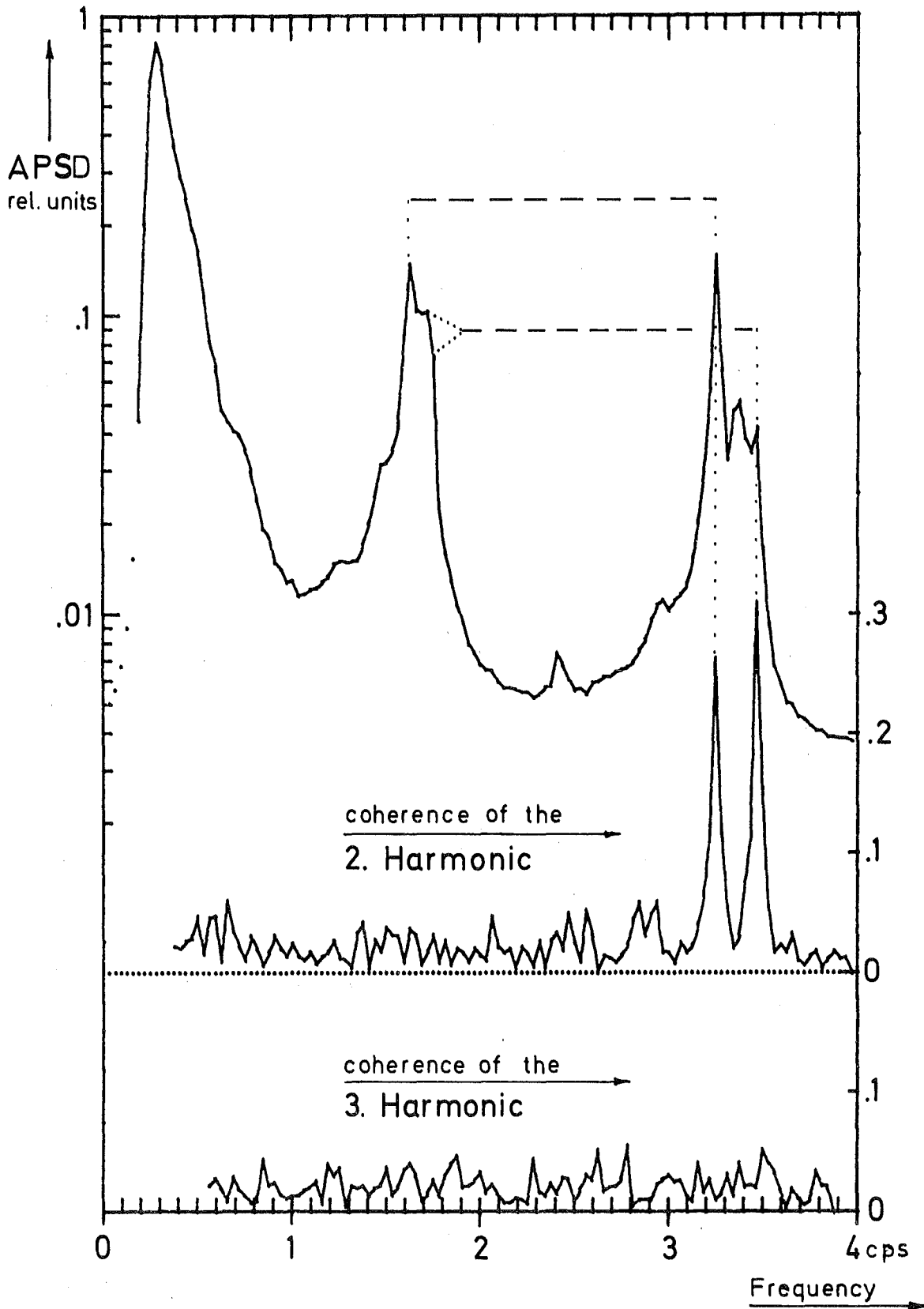


Fig.2 Second Harmonic Correlation in the neutron flux of a nuclear power reactor