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A PHENOMENOLOGICAL INTERPRETATION OF THE PRODUCTION CROSS SECTION FOR THE REACTION $p + p \rightarrow \pi + d$

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Abstract

Using new data on the unpolarized differential cross section for the reaction $pp\to\pi d$, obtained by the NESIKA collaboration at SIN, the energy dependence of the anisotropy parameters γ_0,γ_2, and γ_4 is interpreted phenomenologically. Especially the energy dependence of γ_4, which is stronger than predicted theoretically, could be understood as being due to the behaviour of a production amplitude with $\ell_\pi=3$ near threshold. Technical details are given in Appendix A,B, and C.

EINE PHENOMENOLOGISCHE INTERPRETATION DES PRODUKTIONSQUERSCHNITTS FÜR DIE REAKTION $p + p \to \pi + d$

Zusammenfassung

Unter Verwendung neuer Daten über den unpolarisierten differentiellen Wirkungsquerschnitt für die Reaktion $pp\to\pi d$, die von der NESIKA Kollaboration am SIN gewonnen wurden, wird die Energieabhängigkeit der Anisotropieparameter γ_0,γ_2 und γ_4 phenomenologisch interpretiert. Insbesondere könnte die Energieabhängigkeit von γ_4, die stärker ist als theoretisch vorhergesagt, durch das Schwellenverhalten einer Produktionsamplitude mit $\ell_\pi=3$ verstanden werden. Technische Details sind in den Anhängen A, B und C beschrieben.

At SIN the unpolarized differential cross section for the reaction $p+p\rightarrow\pi+d$ between 514 and 583 MeV in the laboratory system has been measured by the NESIKA collaboration. The experimental method is described elsewhere[1]. Preliminary data have been normalized using monitors measuring elastic pp scattering rates from the experimental target. The absolute normalization was obtained by using the calculated pp elastic cross sections of BUGG[2]. The data were analysed to obtain the usual anisotropy parameters γ_i (Tab.I) as defined by

$$\frac{d\sigma}{d\Omega} = \frac{1}{32\pi} (\gamma_o + \gamma_2 \cos^2\Theta_{CM} + \gamma_4 \cos^4\Theta_{CM}).$$

Tabelle I: Anisotropy parameters γ_i [mb/sr] for the reaction $p+p\rightarrow\pi+d$. Ep=proton lab. energy, Wp=kinetic energy of one proton in the C.M.system.

Ep/MeV	Wp/MeV	γ_o	γ_2	γ_4
514	120.7	9.47±.11	34.81± .67	- 3.67± .80
527	123.6	9.96±.14	38.40± .99	- 3.73±1.23
540	126.5	10.68±.12	40.91± .79	- 6.06± .94
554	129.6	11.11±.18	44.42±1.43	- 8.03±1.84
569	132.8	11.57±.14	46.56± .91	- 9.39±1.08
576	134.4	11.57±.13	47.87± .79	-11.88± .89
583	135.9	11.64±.14	49.43±1.00	-12.99±1.19

Fig.1 shows the results together with a selection of data from other experiments[3 to 9]. The γ parameters, and especially γ_4 show a much stronger energy dependence than predicted theoretically[10,11] as can be seen from Fig.2.

In order to clarity whether such a strong energy dependence calls for a possible dibaryonic resonance in addition to the well known $N\Delta$

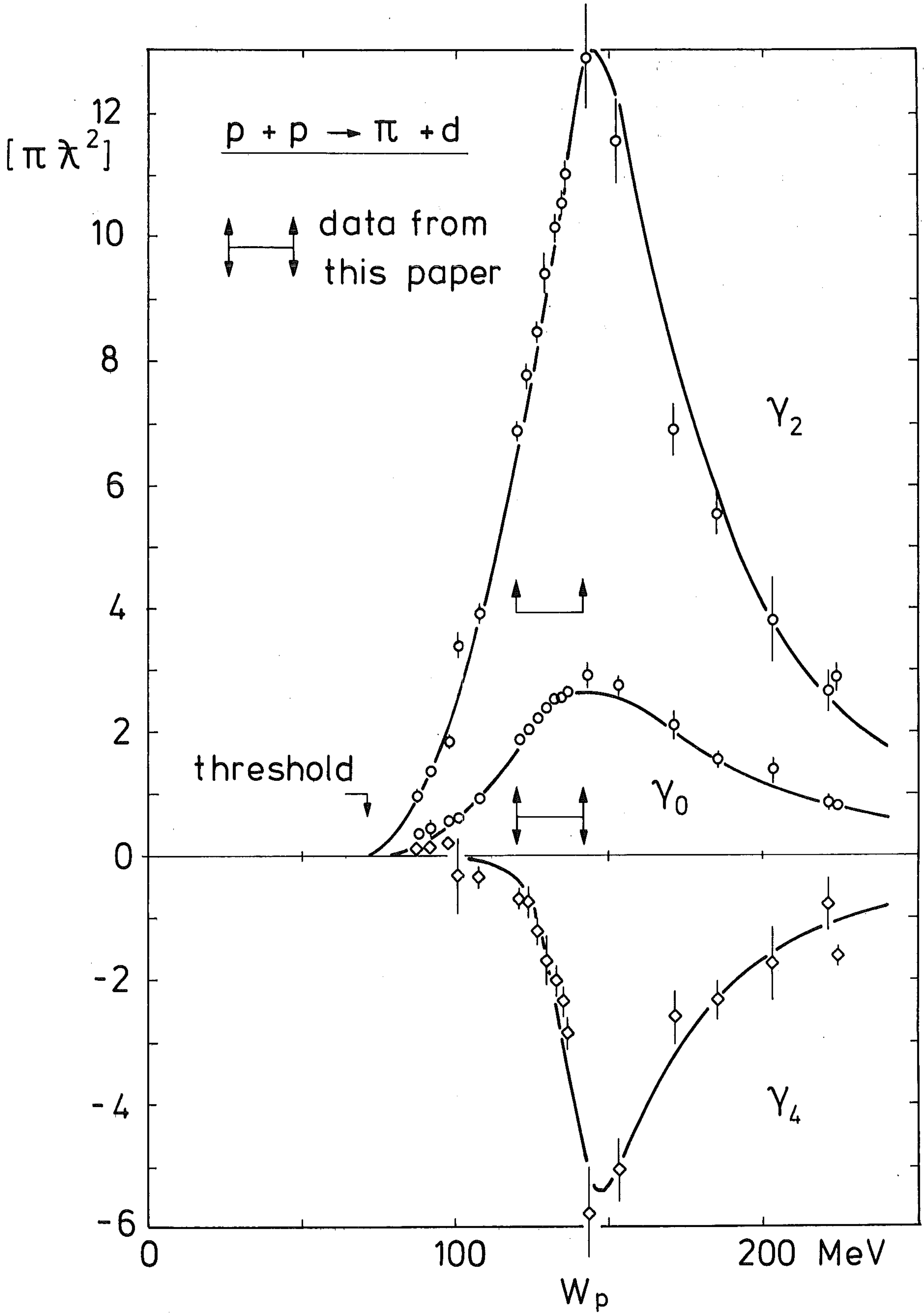

Fig.1: Anisotropy parameters γ_i are in unities of $\pi ƛ^2$ where λ in the wave length of one proton at the kinetic energy W_p in the C.M. system (see Appendix A). The curves are calculated using a Breit-Wigner formalism.

resonant production mechanism, the data was interpreted qualitatively by means of a Breit-Wigner energy-dependent width approximation[12].

Only initial singlet states were taken into account: A $^1D_2$ resonant state decaying via ℓ_π=1 and 3, and a $^1S_0$ non-resonant background. The corresponding Breit-Wigner amplitudes[13] $a_2(\ell_\pi=1)$ and $a_7(\ell_\pi=3)$ from the $^1D_2$ state had to have the same parameters for the resonant energy and total width but could have different phase and relative energy dependence due to different threshold behaviour with respect to the centrifugal barrier in the final state. The non resonant $^1S_0$ contribution is described by an additional amplitude a_0.

In this simplified model, γ_4 is given by the expression

$$\gamma_4 = \frac{15}{2}\,|a_7|^2 - 15\sqrt{6}\ \mathrm{Re}\ a_2 a_7^{*}.$$

(The amplitudes a_i are defined as by MANDL and REGGE[16].)

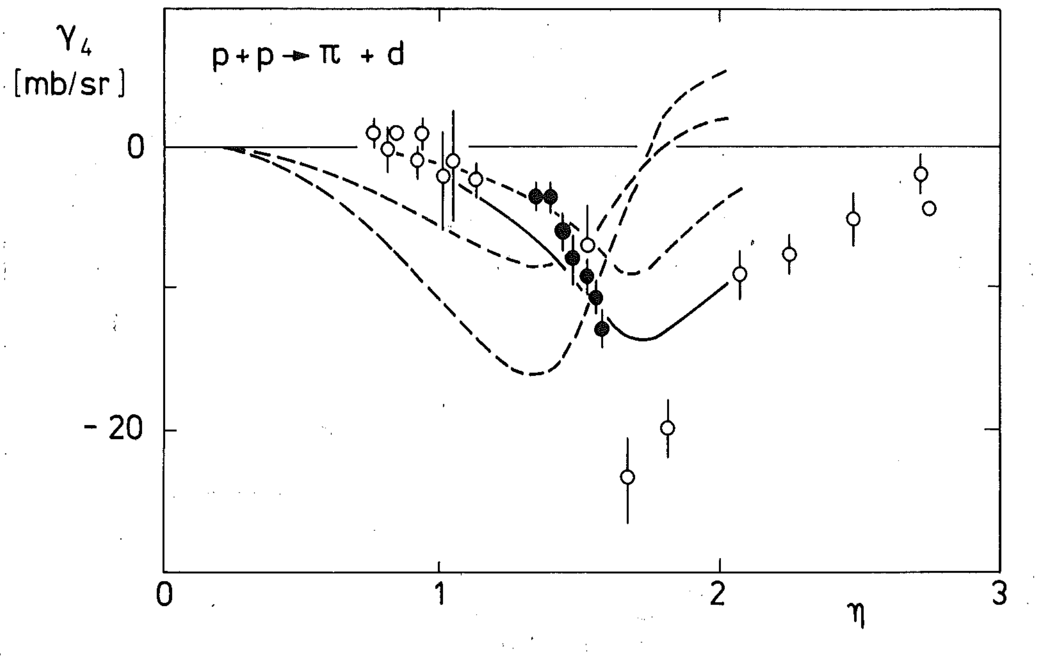

Fig.2: Comparison of the anisotropy parameter γ_4 with theoretical predictions. $\eta = p\pi/(m\pi c)$ is the reduced momentum of the pion in the C.M. system. The full points are preliminary results of the NESIKA collaboration, the other points from ref. 3 to 9. Full line: NISKANEN[10], broken lines: MAXWELL, WEISE, and BRACK[11].

As shown in Fig.1, the sudden increase of γ_4 as a function of energy can indeed be explained by this model. (Details for calculating the curves shown in Fig.1 and 2 using the MINUIT code[14]) are given in Appendix B and C.) It turns out that γ_4 can be understood essentially as an interference between a predominant amplitude a_2 and a small contribution of a_7. (The importance of a_7 was first pointed out by J.A. NISKANEN[10]).) Their relative contributions can be seen in Fig.3, where two linear combinations of the γ's have been used to better illustrate the amplitude behaviour.

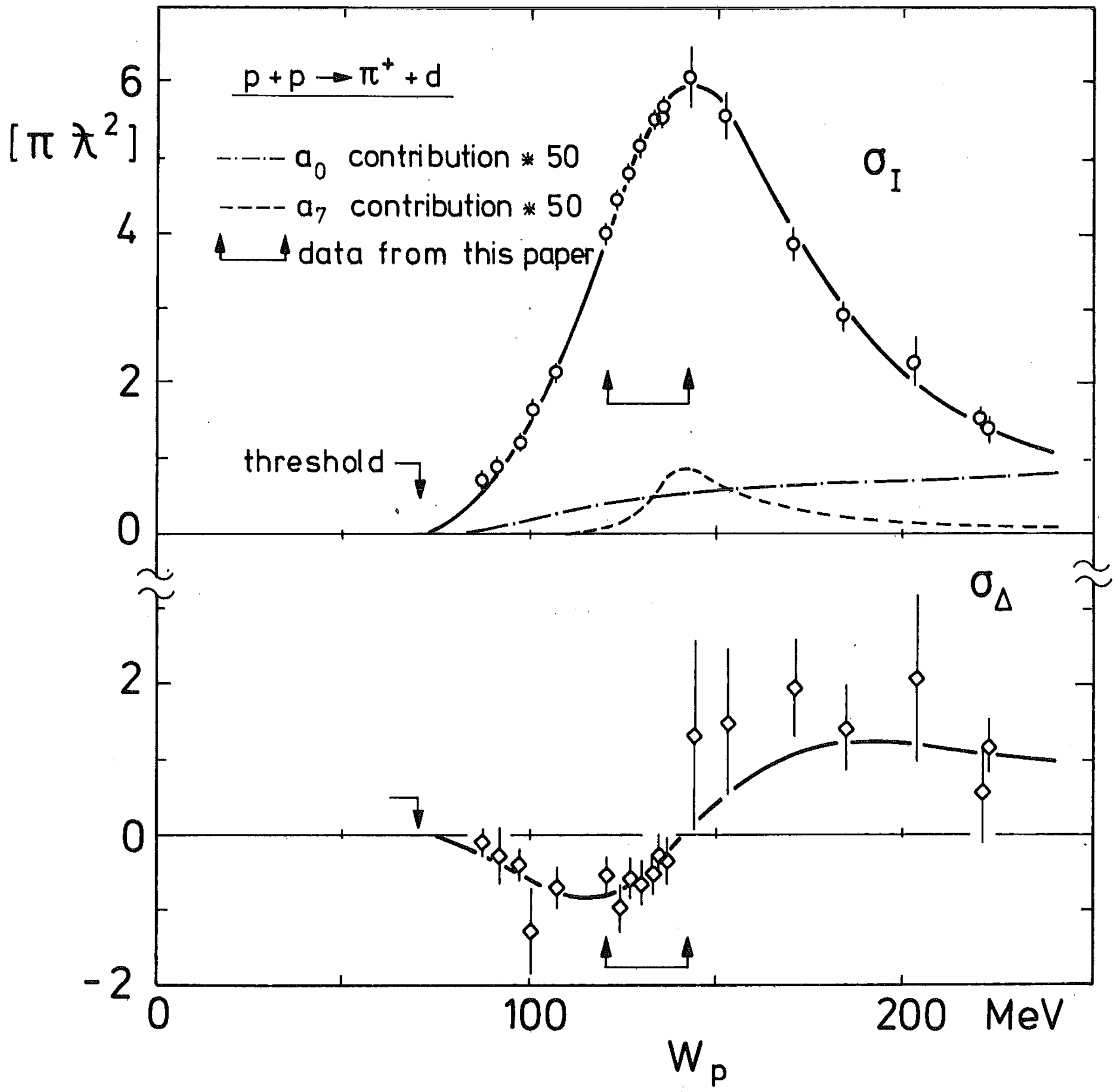

Fig.3: σ_I and σ_Δ as discussed in the text. The curves correspond to those in Fig.1. γ_0 and γ_2 are obtained from the relations

$$\gamma_0 = \frac{1}{2}\sigma_I + \frac{1}{6}\sigma_\Delta + \frac{1}{15}\gamma_4, \quad \gamma_2 = \frac{3}{2}\sigma_I - \frac{1}{2}\sigma_\Delta - \frac{4}{5}\gamma_4.$$

The quantity σ_I, which is

$$\sigma_I = 2\,(|a_0|^2 + |a_2|^2 + |a_7|^2)$$

$$= \gamma_0 + \gamma_2/3 + \gamma_4/5,$$

is proportional to the integral production cross section. Aratio of $|a_7|^2/|a_2|^2 = 3\cdot10^{-3}$ at the resonance energy is sufficient to explain γ_4.

In Fig.3 the other quantity

$$\sigma_\Delta = 6|a_0|^2 + 12\,\mathrm{Re}\,(\sqrt{2}\,a_2 - \sqrt{3}\,a_7)\,a_0^*$$

$$= 3\gamma_0 - \gamma_2 - \gamma_4$$

is also shown. It has a typical interference pattern of a resonance amplitude near threshold with a non resonant background. This might be regarded as some kind of justification of the phenomenological approach.

Finally it should be mentioned, that the contribution of a small γ_6 term can affect the results for γ_4 dramatically. Indeed it has been shown recently[15] that γ_6 is definitively not zero at higher energies. However, even if γ_6 were present, it has been shown from this model that pronounced energy dependencies which are contradictory to theoretical predictions could be explained as a threshold effect from a purely phenomenological point of view.

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Appendix A

Some Useful Relations

In the Fig.1 and 3 the anisotropy parameters γ_i are given in units of $\pi\lambda\!\!\!^{-}{}^2$. This reduction compensates for the dependence of Breit-Wigner (W.-B.) amplitudes from the kinetic energy Wp in the C.M. system. One obtains γ_i in unities of $\pi\lambda\!\!\!^{-}{}^2$ by

$$(\gamma_i)_{red} = \frac{\gamma_i}{\pi\lambda\!\!\!^{-}{}^2} = \frac{\gamma_i}{\pi}\left(\frac{pc}{\hbar}\right)^2, \tag{1A}$$

where pc is derived from the proton lab. kinetic energy Ep. In detail this is

$$Wp = Mp(\sqrt{1\text{-}Ep/2Mp}-1), \text{ and } pc = \sqrt{Wp(Wp+2Mp)}$$

with $\hbar = 6.58\cdot 10^{-22}$ MeVs, $c = 3\cdot 10^{10}$ cm/s, and Mp = .93826 GeV. With all these relations substituted into (1A), one gets the numerical relation

$$(\gamma_i)_{red} = (\gamma_i/\text{mb/sr}) \times (\text{pc/GeV})^2 \times .81687.$$

Another quantity needed for B.-W. calculations is the reduced momentum $\eta = p_\pi/(m_\pi c)$ of the pion in the C.M. system. By replacing the deuteron mass with 2 Mp, the mass of two protons, and with the abbreviations $w = Wp/m_\pi$ and $m = Mp/m_\pi$ = 938.26 MeV/139.57 MeV one obtains

$$\eta = \frac{1}{w+m}\sqrt{(w(w+2m) - \tfrac{1}{4})^2 - m^2}.$$

Appendix B

The Energy-Dependent Width Approximation and Parametrization

The amplitudes a_o, a_2 and a_7 described in the text were parametrized in the following way:

$$a_o = e^{i(\delta_1+\alpha_o)}\,\Gamma_o\,P_1 = |a_o|\,e^{i\phi_o},$$

$$a_2 = e^{i\delta_1}\,\frac{\Gamma_2 P_1}{(Wp-W_{res})+i\frac{\Gamma}{2}P_1} = |a_2|\,e^{i\phi_2},$$

$$a_7 = e^{i(\delta_3+\alpha_7)}\,\frac{\Gamma_7\cdot P_3}{(Wp-W_{res})+i\frac{\Gamma}{2}P_3} = |a_7|\,e^{i\phi_7}.$$

The phase shifts δ_ℓ and the penetration coefficient P_ℓ should be proportional to $\eta^{2\ell+1}$ near threshold and become constant at large η. (ℓ = pion angular momentum with respect to the deuteron.) δ_ℓ and P_ℓ are model dependent. For δ_ℓ we used the phase shift of a hard sphere[12] with a radius of one pionic unit (1.3 fm):

$$\delta_1 = -\eta + \operatorname{artan}\eta,$$

$$\delta_3 = -\eta + \operatorname{artan}\left(\eta\,\frac{1-\frac{\eta^2}{15}}{1-\frac{2}{3}\eta^2}\right).$$

For the penetration coefficients P_ℓ we used the following expression[12]:

$$P_\ell = \frac{1}{j_\ell^2(\eta R_\ell) + y_\ell^2(\eta R_\ell)}\;\frac{2\eta R_\ell}{\pi}\;.$$

where j_ℓ and y_ℓ are the regular and irregular spherical Bessel functions. P_ℓ approaches unity for large η, but however for small η it is proportional to $\eta^{2\ell}$ only. The resulting expressions are

$$P_1 = \frac{1}{1+\frac{1}{(\eta R_1)^2}},$$

and

$$P_3 = \frac{1}{1 + \frac{6}{(\eta R_3)^2} + \frac{45}{(\eta R_3)^4} + \frac{225}{(\eta R_3)^6}},$$

where R_1 and R_3 were used as adjustable parameters.

The angles α_0 and α_7 are constant adjustable parameters in order to account for phase differences in the production mechanism. The energy dependence of the phases in the initial channel was neglected.

In total there were nine real parameters allowed to vary:

- Γ_0 reduced amplitude of a_0
- Γ_2, Γ_7 reduced partial widths for a_2 and a_7
- Γ, W_{res} reduced total width and resonance energy of the B.-W. amplitudes
- α_0, α_7 constant phase differences as explained above
- R_1, R_3 effective 'ranges' in the penetration coefficients for $\ell_\pi=1$ and 3

Appendix C

Fitting Procedure and Numerical Results

The curves shown in Fig.1 and 3 were obtained by a stepwise fitting procedure using the MINUIT code[14].

1st step: σ_I was fitted by the amplitude a_2 alone, varying $\Gamma_2, \Gamma, W_{res}$, and R_1

2nd step: γ_4 was fitted with the additional amplitude a_7, varying Γ_7, α_7, and R_3 in addition

3rd step: σ_Δ was fitted with the additional amplitude a_0, varying Γ_0 and α_0

4th step: γ_0, γ_2, and γ_4 were fitted directly allowing all nine real parameters to vary.

In order to reproduce the sudden increase of γ_4 it was necessary to increase the errors of the positive values of γ_4 near Wp = 90 MeV by a factor of three and to reduce the errors of the two low lying γ_4 points near Wp = 150 MeV

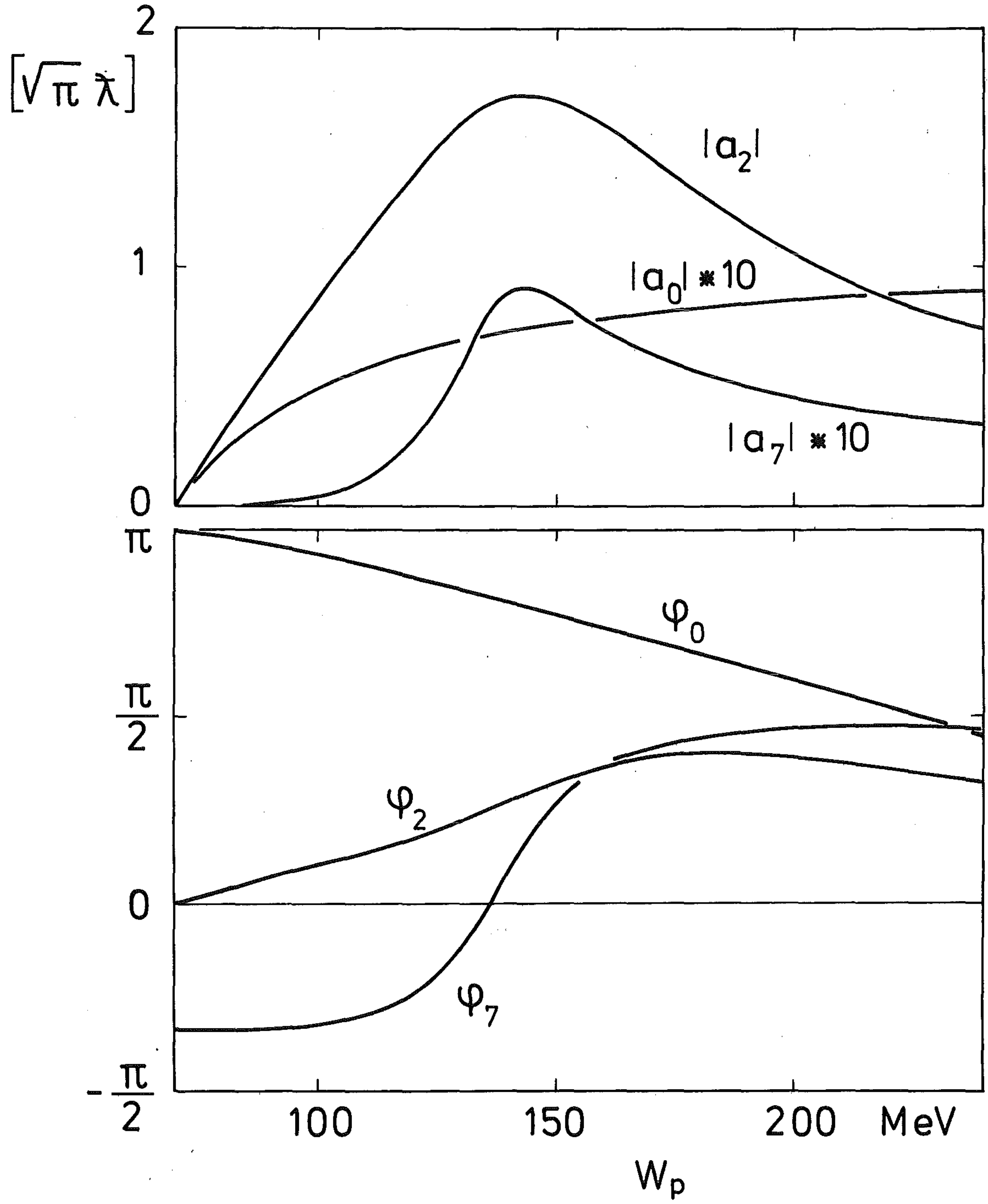

Fig.4: Absolute values and phase angles ϕ for the amplitudes a_0, a_2, and a_7 as a result of a Breit-Wigner interpretation of the differential cross section for $p+p \rightarrow \pi+d$.

by the same factor. (The positive values of γ_4 could easily be due to interferences between initial triplet state amplitudes with ℓ=2, which are not taken into account in our approach.) The resultant numerical values are:

$$
\begin{array}{lcll}
W_{res} & = & 143.4 \pm .5 & \text{MeV} \\
\Gamma/2 & = & 50.5 \pm 1.5 & \text{MeV} \\
\Gamma_2 & = & 87.1 \pm 2.4 & \text{MeV} \\
\Gamma_7 & = & 4.6 \pm .2 & \text{MeV} \\
\Gamma_o & = & .10 \pm .01 & \text{MeV} \\
\alpha_o & = & 3.12 \pm .07 & \text{rad} \\
\alpha_7 & = & -1.1 \pm 0.1 & \text{rad} \\
R_1 & = & 1.00 \pm .03 & \\
R_3 & = & 1.62 \pm .08 &
\end{array}
$$

The corresponding absolute values and phase angles ϕ for the amplitudes a_o,a_2 and a_7 are shown in Fig.4.

In this very crude phenomenological description, the sudden increase of γ_4 at Wp = 140 MeV is due to two effects:

- A sudden increase of $|a_7|$ and
- a change of $(\phi_2-\phi_7)$ from nearly $\frac{\pi}{2}$ to zero between Wp=130 and 150 MeV.

Both effects are essentially caused by the threshold behaviour of a_7.