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THE DEPENDENCE ON ENERGY AND MASS-NUMBER OF THE α-PARTICLE OPTICAL POTENTIAL: A JUSTIFICATION FOR THE FOLDING MODEL APPROACH

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Abstract

Data for elastic scattering of α-particles by $^{40,42,44,48}Ca$, ^{50}Ti, ^{52}Cr and ^{90}Zr at 104 MeV, by ^{40}Ca, $^{46,48,50}Ti$, ^{58}Ni, ^{90}Zr and ^{208}Pb at 140 MeV and by $^{58,60,62,64}Ni$ at 173 MeV are analysed using a Fourier-Bessel description of the optical potential. All data extend to large angles thus allowing unique determination of volume integrals and rms radii of the potentials. The variations with mass number and energy of these quantities are investigated and conclusions are drawn about studies of nuclear radii with the help of optical potentials.

Zusammenfassung

DIE ENERGIE- UND MASSENABHÄNGIGKEIT DES OPTISCHEN POTENTIALS DER α-TEILCHEN STREUUNG: EINE RECHTFERTIGUNG DES FALTUNGSMODELLS

Die differentiellen Wirkungsquerschnitte für die elastische Streuung von α-Teilchen an $^{40,42,44,48}Ca$, ^{50}Ti, ^{52}Cr und ^{90}Zr bei 104 MeV, an ^{40}Ca, $^{46,48,50}Ti$, ^{58}Ni, ^{90}Zr and ^{208}Pb bei 140 MeV und an $^{58,60,62,64}Ni$ bei 173 MeV wurden auf der Basis der Fourier-Bessel Beschreibung des optischen Potentials analysiert. Alle experimentellen Daten erstrecken sich zu großen Streuwinkeln und erlauben daher eine eindeutige Bestimmung der Volumenintegrale und mittleren quadratischen Radien der Potentiale. Die Variation mit der Massenzahl und der Energie dieser Größen wird untersucht und die Information, die über Kernradien aus den optischen Potentialen gewonnen werden kann, beleuchtet.

1. Introduction

The optical model is a convenient and useful approximation to the interaction of strongly interacting particles with nuclei and it has been most successful in describing elastic scattering of particles by nuclei over a wide energy range and over the whole range of the periodic table. Being a simplified approximation to the true interaction and thus lacking its fine details the optical potential is expected to display rather smooth variations with energy and with mass number provided too light nuclei and too low bombarding energies are not considered (where effects due to particular reaction channels or compound nucleus effects may be significant). Among the interesting and important characteristics of the optical potential are its variations with the target mass number (A) and with the energy (E) of the projectile. Numerous studies of the A- and E-dependence of the optical potential for protons, neutrons and light complex particles have been made in the past [1-3]. A standard procedure was usually to describe the shape of the potential by preselected functional forms such as the widely used Woods-Saxon (WS) function, and then to study the A- and E-dependence of the parameters of the potentials obtained from best fits to experimental data. Sometimes further assumptions (and constraints) were introduced into the analysis such as an $A^{1/3}$ dependence of the radii which could complicate the studies and smooth out important details. When analysing the experimental data two serious problems emerged: (i) the quality of the best fits as measured by the χ^2 per degree of freedom (χ^2/F) was not always as required for a meaningful comparison between a hypothesis and experimental results (ii) the experimental data considered proved sometimes to be insufficient to determine the optical potentials in a unique way. In particular, the potentials found for elastic scattering of strongly absorbed particles exhibit discrete ambiguities so that different combinations of parameters yielded equivalent descriptions of the experimental cross sections. In addition, one should mention the continuous ambiguities where small changes of one parameter value are compensated by small changes of another thus making the study of the A- and E-dependence of the optical potential parameters difficult.

In recent years a significant progress has been made in understanding the optical potential which describes the elastic scattering of α-particles by nuclei. For incident energies of about 100 MeV and higher one observes a qualitative change in the nature of the interaction from diffractive to refractive scattering [4] as evidenced by the "nuclear rainbow" which is understood in terms of a semiclassical description as the largest possible deflection angle. Differential scattering cross sections measured beyond the "nuclear rainbow" peak show an exponential fall off. The most striking conclusion resulting from the fact that the absorption is apparently not strong enough to obliterate probing the nuclear interior is the elimination [5] of the discrete ambiguities in the optical potential. An additional consequence is an increased sensitivity to the detailed shape of the potential, and it has been found [6] that a squared Woods-Saxon (WS2) form for the real part is more adequate to get the correct shape than the often used WS form. This increased sensitivity can be fully exploited by introducing so called model-independent analyses where the optical potential (usually its real part) is described not by a pre-chosen function but by a Fourier-Bessel series [7], by a sum of Gaussian functions [8], by spline functions [9,10] or by a complete set of orthogonal polynomials [11]. The flexibility provided by these functions removes to a very large degree the dependence of the results on the choice of analytical functions and also allows a more realistic analysis of the uncertainties of the results. In these methods the conventional parameters of the optical potential (depth, radius parameter etc.) become irrelevant and other quantities emerge as being significant, in particular the volume integral and various radial moments of the potential.

The present work is concerned with the A- and E-dependence of these well-defined quantities of the optical potential for α-particle scattering between 100 and 180 MeV. The motivation for our studies is the recent investigations of nuclear sizes by elastic scattering of α-particles [12] where the interpretation of the experimental results invokes explicitely or implicitely a folding model description. It is, therefore, important to know to which extent the phenomenologically extracted optical potentials exhibit features which are consistent with folded potentials. There are definite theoretical

predictions [13-16] for the energy-dependence derived within the folding model approach. Until recently, however, the comparison with the phenomenological optical potentials [16,17] was hampered by the above indicated uncertainties about the true shape of the potentials and by the restriction to a few particular nuclei [9,18] and to lower α-particle energies (where the folding model approach appears to be more doubtful due to exchange and antisymmetrization effects).

2. Method and Procedure

The differential cross sections which the present work is based on come from three sources: (i) the 104 MeV data measured by the Karlsruhe group [12,19,20] for $^{40,42,44,48}Ca$, ^{50}Ti, ^{52}Cr and ^{90}Zr. (ii) the 140 MeV data from Maryland [4,21] for ^{40}Ca, $^{46,48,50}Ti$, ^{58}Ni, ^{90}Zr and ^{208}Pb. (iii) the 173 MeV data from the Jülich-Cracow collaboration [22] for $^{58,60,62,64}Ni$. The overlap between the data at different energies is quite reasonable, particularly that between the 104 and 140 MeV data.

The analysis is based on the Fourier-Bessel method [7] describing the real potential by adding to a conventional (say Woods-Saxon or squared Woods-Saxon) form an extra-potential given by a Fourier-Bessel series.

$$V(r) = V_o(r) + \sum_{n=1}^{N} b_n \, j_o(q_n \, r) \qquad (1)$$

The quantities j_o are spherical Bessel functions, $q_n = n\pi/R_{cut}$ and R_{cut} is a suitably chosen cut-off radius beyond which the extra-potential vanishes. The coefficents b_n are determined by least-squares fit to the data. Within the framework of the FB-procedure the mean-square uncertainty of the potential value at the distance r is given by

$$[\delta V(r)]^2 = 2 \sum_{m,n=1} \langle \delta b_m \delta b_n \rangle_{av} \, j_o \, (q_m r) \; j_o(q_n r) \qquad (2)$$

with $\langle \delta b_m \; \delta b_n \rangle_{av}$ being the correlation matrix between the coefficients b_n. The FB method has been shown to lead to very good values of χ^2/F and to well defined integral quantities of the potential, and at the same time providing realistic estimates of errors. However,

before studying variations over different nuclei and different energies two additional effects which up to now were not explicitely taken into account in FB analyses have to be considered: (i) possible coupling effects between real and imaginary parts of the potential and the results of introducing more flexibility into the imaginary potential, (ii) possible effects due to coupling with strongly excited reaction channels. In principle, both types of effects are interrelated since the nonelastic channels are approximately represented by the absorptive part of the optical potential for elastic scattering.

In order to ascertain that there is no significant coupling between real and imaginary parts we introduced various forms of the imaginary potential [3], such as the usual Woods-Saxon form (WS), its square (WS2), a combination of WS and its derivative and finally even a FB form (analoguous to eq. (1)). From fits to the data using these forms (see tab. 1a) one concludes that once a FB series is used for the real potential the added flexibility in the shape of the imaginary potential proves to be unnecessary. This observation is supported by systematic studies of elastic α-particle scattering form ^{90}Zr at several energies where the absorptive form factor was described by spline functions [17]. The improvement in the goodness of the fits obtained by the use of more flexible forms is marginal and the integral quantities of interest (the volume integral J_V and the rms radius of the real potential) change very little. The extra flexibility in the imaginary potential could cause, however, difficulties in the convergence in the parameter search.

An interesting by-product of these calculations is the result that the WS2-form of the imaginary optical potential is not a proper parametrization. Both forms of the imaginary potential (WS and WS2) do produce equivalent fits to the data but the volume integral of the real potential shows clear signs of instability when the WS2 form is used for the imaginary potential. These signs of instability and of coupling with the real part lead us to reject the WS2 form of imaginary part. In fact, microscopic treatments [2 3] of the α-particle optical potential suggest the imaginary part to be close to the WS form.

Effects which may result from a coupling of the elastic channel to strongly excited reaction channels were studied by exploratory coupled channels calculations explicitely coupling the first two

$-J_V/4A$ (MeV·fm^3)	$\langle r_V^2 \rangle^{1/2}$ (fm)	χ^2/F	Form of the Imaginary Part
327 ± 2	4.37 ± 0.06	2.0	WS
318 ± 3	4.37 ± 0.06	2.0	WS2
322 ± 3	4.34 ± 0.07	1.6	WS + FB

Tab. 1a: Specific volume integral and rms radius of the real potential (FB-form) from elastic α-particle scattering on ^{40}Ca at E_{Lab} = 104 MeV

$-J_V/4A$ (MeV·fm^3)	$\langle r_V^2 \rangle^{1/2}$ (fm)	χ^2/F	Procedure of the Analysis	
307.5	4.47	2.0	Elastic Sc. Data: WS-Imaginary Part	Real Part: WS2
307.4	4.48	4.2	Simultaneous Fit to Elastic and Inelastic Sc. Data 0^+-2^+-4^+ Coupling WS-Imaginary Part	Real Part: WS2
320.0	4.61	4.0	Elastic Sc. Data: WS-Imaginary Part	Real Part: WS
328.6	4.66	28.2	Simultaneous Fit to Elastic and Inelastic Sc. Data: 0^+-2^+-4^+ Coupling WS-Imaginary Part	Real Part: WS

Tab. 1b: Specific volume integral and rms radius of the real potential (SW2 or SW form) from α-particle scattering on ^{50}Ti at E_{Lab} = 104 MeV

excited states to the ground state and simultaneously fitting elastic and inelastic scattering data [24].

In these calculations which used the coupled channels code ECIS [25] the radial shape of the (deformed) optical potential was not of the FB type, rather a conventional WS form or a WS2 form was taken for the real part. In particular, the WS2 shape was shown [12] to be a very good approximation of the diagonal real optical potential, so that such a simplification is justified in the coupled channels calculations. The real coupling potentials are specified to be derivatives (including second order) of the WS and WS2 form, respectively and complex coupling was included. Details of the analysis of inelastic scattering will be given elsewhere [24]. In the context of the present paper the results show (see Tab. 1b), that the values of the volume integral of the real potential are very little affected, by coupled channel effects, and this is true in particular when using the WS2 shape, which is a very good approximation to the FB potential. The rms radius proves to be quite stable against all variations in the procedures. As expected, the explicit coupling reduces somewhat the strength of the imaginary potential.

Finally we remark that the FB-procedure which starts from a best-fit WS or WS2 form (see eq. (1)) implies some residual model-dependence of the final results on the initial conditions since beyond R_{cut} the potential is given by the initial analytic form [26]. This brings a systematic uncertainty of 2 - 3 % for the volume integral and 1 % for the values of rms radius, concerning their absolute values, but it is completely negligible when considering variations with respect to one reference nucleus, e.g. ^{40}Ca.

3. Results

3.1 RADIAL SENSITIVITY AND THE DEPENDENCE ON QUALITY OF DATA

Before presenting the results on the A- and E-dependence of the optical potentials we discuss the question of how sensitive are the experiments analysed in the present work to these potentials and in particular, what is the dependence of the results on the quality of the data.

The first indications for the sensitivity of large angle data to the nuclear interior were presented by Goldberg et al. [4] who showed that the discrete ambiguity in the real part of the α-particle optical potential was eliminated when large angle data was analyzed at 140 MeV. More quantitative results are given by e.q. (2) and it was already shown [12,26] based on FB analyses, that data of the kind used in the present work probes the optical potential down to about r = 2 - 2.5 fm. The crude and somewhat unphysical method of the "notch test", where part of the potential is artificially removed, was also used [3,27,28] to study the radial sensitivity of the elastic scattering of α particles. In the following we show that the notch test is unreliable and could be misleading in studying the radial sensitivity and quality of data.
Figure 1 shows results for 140 MeV α-particles [4] scattered by ^{90}Zr. In addition to analysing (a) the full data, we also analysed two partial sets of the data: (b) retaining only every third data point throughout the angular range. (c) retaining only the data points for angles smaller than 40^{o}. The lower part of the figure shows the relative error of the real part of the potential calculated using e.q. (2), where the information on the quality of the data is carried by the correlation matrix. It is clearly seen that the partial data lead to less accurate potentials: case (b) leads to increased errors compared to (a) while case (c) shows that information on the interior of the nucleus is lost when large angle data is excluded. Most interesting are the errors of the volume integral and rms radius, which almost double when only every third data point is retained.

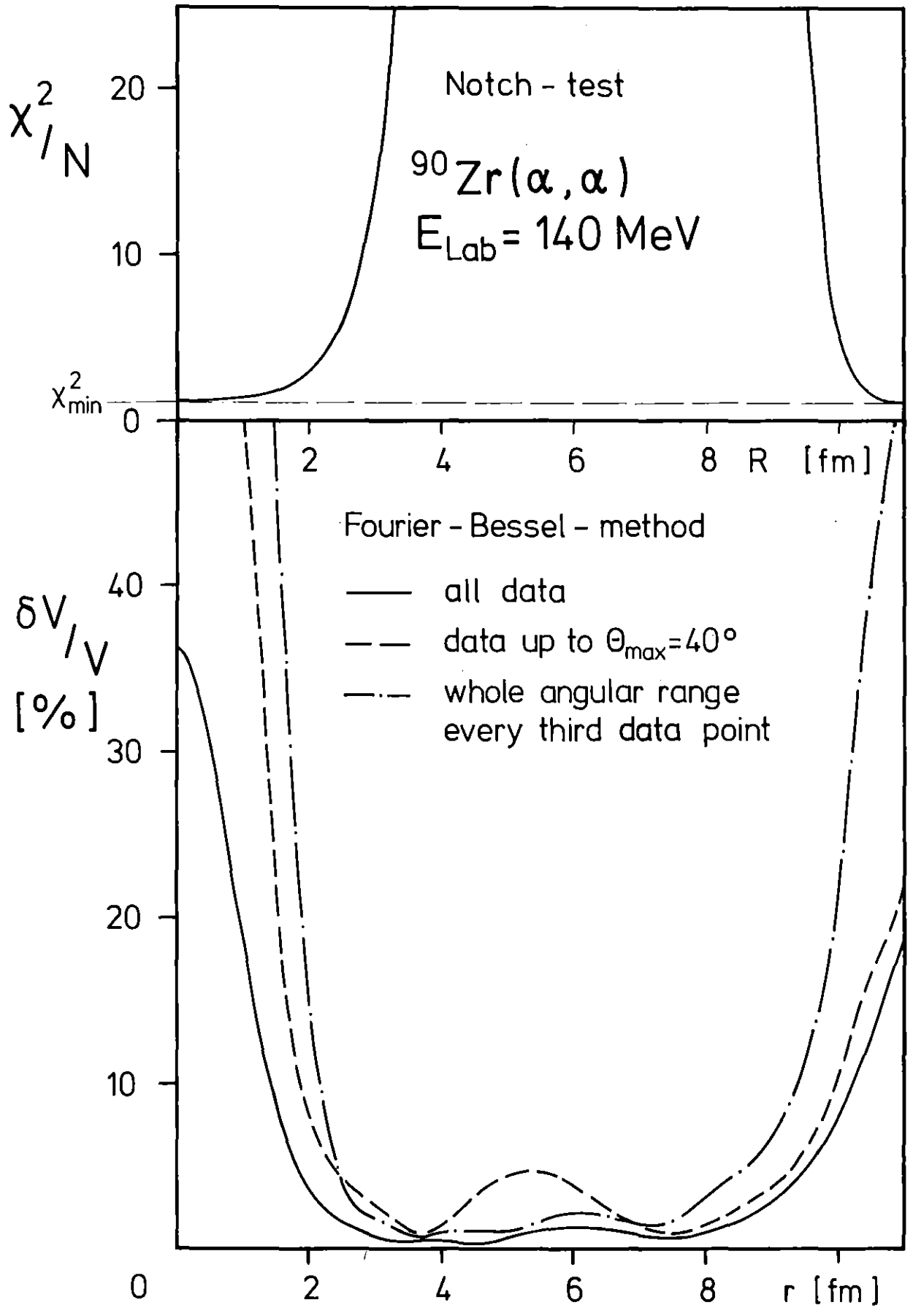

Fig. 1: Lower part: Relative errors of the real potential for 140 MeV α-particle scattering from ^{90}Zr obtained by FB fits using: (a) the full data set (b) every third data point and (c) only data for $\theta_{cm} < 40^{o}$. Upper part: χ^2/F vs position of notch which is unable to distinguish the three different cases considered in the lower part.

The upper part of fig. 1 shows results of a notch test, where the real part of the potential was multiplied by a factor

$$f(r,R) = 1 - d \exp[-\{(r-R)/a\}^2]. \qquad (3)$$

In the present calculations we used d = 0.3 and a = 0.5 fm, thus removing up to 30 % of the potential around r = R. The goodness of the fit, χ^2 was calculated for different values of R, and an increase in χ^2 indicates sensitivity (at the 30 % level in this case) to the potential. While there is generally qualitative agreement between the FB results and those of the notch test, it is clear that the latter is incapable of providing quantitative information on the quality of the data and of the fit achieved. Obviously,when χ^2 exceeds the minimum χ^2 by a factor of 5 and more, very little information is carried then by the value of χ^2.

We conclude: (i) the experimental data used in the present work indeed probe the interior of the nucleus. (ii) FB-analysis is reliable in evaluating properties of the real potential and is quite more instructive than the notch technique which in fact could lead to wrong conclusions.

3.2 A- AND E-DEPENDENCE OF OPTICAL POTENTIALS

Figure 2 shows examples of the differential cross sections of several cases studied in this paper. Tables 2-4 summarize the resulting χ^2/F, the specific volume integral (volume integral per nucleon pair -J/4A) and the rms radius for the real and imaginary potentials at 104, 140 and 173 MeV. The real potential was described by the WS2 form plus a FB series whereas the imaginary potential was of the WS form. Uncertainties are not quoted for the imaginary potential because the error analysis may be unreliable when a simple analytical form is used. However, additional calculations with the FB option included also in the imaginary potential (see sect. 2) showed that the errors quoted for the real potential are typical also for those in the imaginary part.

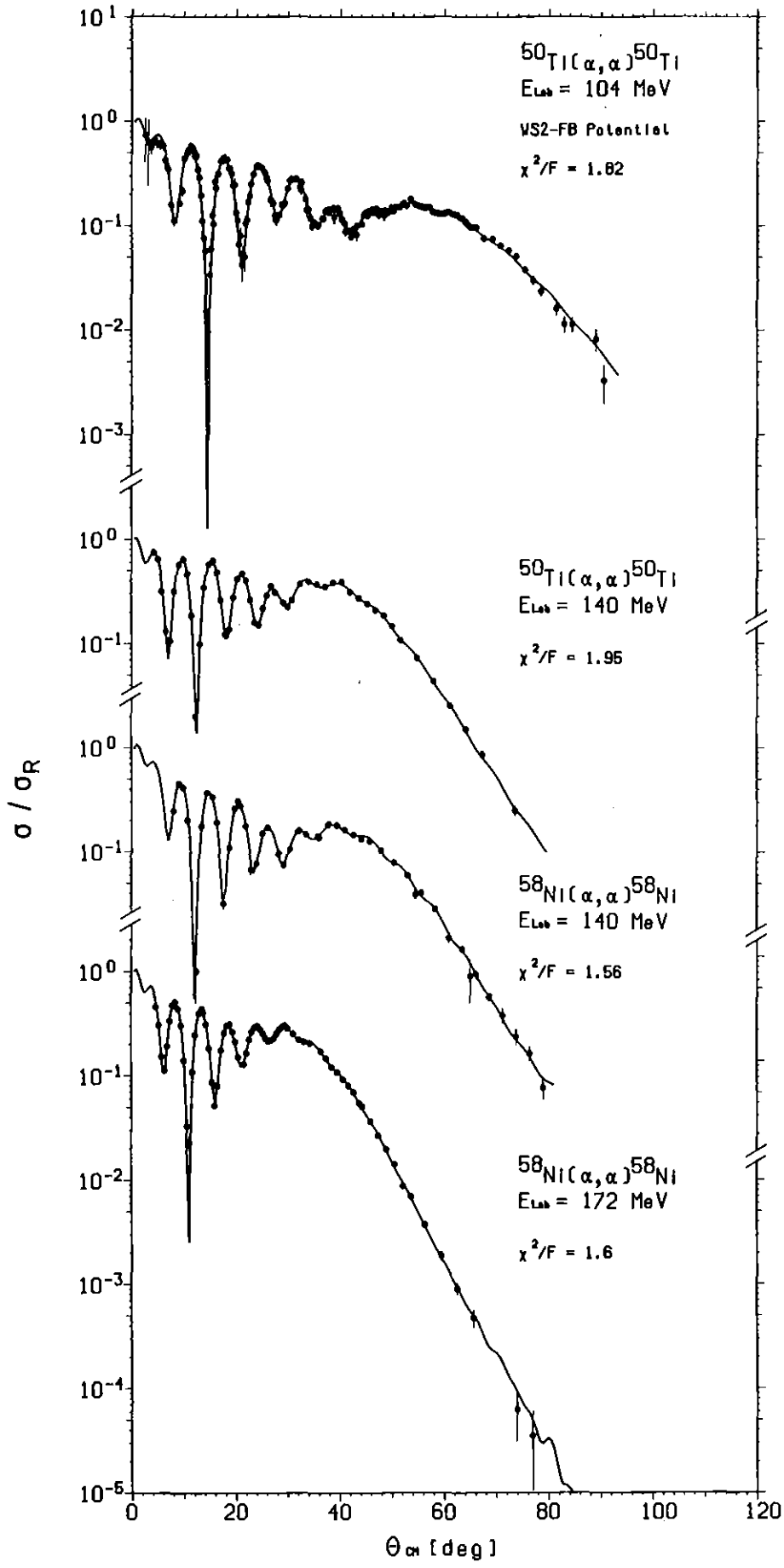

Fig. 2: Experimental and best fit calculated differential cross sections.

Tables 2 and 3 demonstrate that the specific volume integral of the potential is rather constant over wide regions of the periodic table with the exception of ^{40}Ca whose value of $-J_V/4A$ is a little larger. In the context of a folding model interpretation (see below) the present results suggest that the effective interaction does not vary strongly from one nucleus to another. Also no significant dependence on the energy is observed in the rms radii thus indicating a rather constant shape of the optical potential for any given nucleus over the energy range of 100 - 170 MeV. Another feature observed in the tables is that there is very little or no correlation at all between the volume integrals of the real and imaginary parts of the potential. The small isotopic variation in $-J_W/4A$ can possibly be related to the residual dependence on the target structure and the influence of inelastic channels. Beyond the uncertainties a slow increase of $-J_W/4A$ with the energy and a decrease with increasing mass number are observed.

Table 2: Integral parameters of the α-particle potential at 104 MeV: specific volume integral and rms radius of real and imaginary potentials.

Target	χ^2/F	$-J_V/4A$ (MeV·fm^3)	$\langle r_V^2\rangle^{1/2}$ (fm)	$-J_W/4A$ (MeV·fm^3)	$\langle r_W^2\rangle^{1/2}$ (fm)
^{40}Ca	2.0	327 ± 2	4.37 ± 0.06	103	4.94
^{42}Ca	2.5	317 ± 3	4.38 ± 0.06	110	4.93
^{44}Ca	2.7	314 ± 3	4.41 ± 0.07	112	4.96
^{48}Ca	2.3	319 ± 5	4.49 ± 0.09	96	5.09
^{50}Ti	1.8	306 ± 6	4.47 ± 0.12	94	5.05
^{52}Cr	1.8	303 ± 4	4.48 ± 0.09	96	5.09
^{90}Zr	1.9	313 ± 4	5.14 ± 0.04	87	5.87

Table 3: Integral parameters of the α-particle potential at 140 MeV: specific volume integral and rms radius of real and imaginary potentials.

Target	χ^2/F	$-J_v/4A$ (MeV·fm^3)	$\langle r_v^2\rangle^{1/2}$ (fm)	$-J_w/4A$ (MeV·fm^3)	$\langle r_w^2\rangle^{1/2}$ (fm)
^{40}Ca	0.8	322 ± 3	4.41 ± 0.06	107	4.88
^{46}Ti	1.7	302 ± 3	4.51 ± 0.06	108	5.04
^{48}Ti	1.9	298 ± 3	4.51 ± 0.06	101	5.08
^{50}Ti	2.0	306 ± 3	4.57 ± 0.06	96	5.10
^{58}Ni	1.3	287 ± 5	4.66 ± 0.12	93	5.27
^{90}Zr	0.7	289 ± 2	5.17 ± 0.04	84	5.88
^{208}Pb	3.1	295 ± 5	6.34 ± 0.12	74	7.37

Table 4: Integral parameters of the α-particle potential at 173 MeV: specific volume integral and rms radius of real and imaginary potentials.

Target	χ^2/F	$-J_v/4A$ (MeV·fm^3)	$\langle r_v^2\rangle^{1/2}$ (fm)	$-J_w/4A$ (MeV·fm^3)	$\langle r_w^2\rangle^{1/2}$ (fm)
^{58}Ni	1.5	269 ± 3	4.61 ± 0.08	98	5.23
^{60}Ni	1.9	276 ± 3	4.68 ± 0.06	100	5.31
^{62}Ni	2.4	264 ± 3	4.64 ± 0.08	100	5.37
^{64}Ni	1.9	277 ± 3	4.77 ± 0.06	100	5.42

The variation with energy of the properties of the phenomenological real potential has received considerable attention in the past [29,30]. From the tables we deduce a variation of $-J_V/4A$ of - 0.50 MeV·fm^3/MeV over the region from 104 to 140 MeV, and - 0.47 MeV·fm^3/MeV from 140 to 173 MeV. Normalizing to 290 MeV·fm^3 the energy variation can be described in a linear approximation by the factor $1-\alpha E$ with $\alpha = (0.00165 \pm 0.0004)$ MeV^{-1}. This result is in excellent agreement with the prediction of Jackson and Johnson [13] who assumed a folding model approach (see below).

4. Discussion

In the present work we analyzed elastic scattering of α-particles from several nuclei at 104, 140 and 173 MeV. In all cases the data extended beyond the rainbow angle and thus the real part of the optical potentials could be unambiguously determined. The quality of the fits to the data is good in all cases and the volume integrals and rms radii are determined to a good accuracy so that meaningful comparisons can be made between the various results.

The most important result is that at a given energy the value of $-J_V/4A$, the specific volume integral of the real potential, is almost constant. This result is expected for a simple folding model, where the real part of the optical potential is written in terms of the nuclear matter density ρ_m and an effective interaction $V_{\alpha N}$ as follows

$$V(r) = \int V_{\alpha N}(|\vec{r}-\vec{r}'|)\rho_m(r')d^3r' \qquad (4)$$

However, it should be emphasized that a simple folding model, if used explicitly in fitting the data, is totally inadequate and one must introduce saturation [31] into the interaction before a good fit to the data is obtained (this refers only to data extending to large angles). Such a saturation is achieved [12], e.g. by introducing into e.q. (4) an additional factor

$$f(r') = 1-\gamma\rho_m^{2/3}(r') \qquad (5)$$

The present results show, nevertheless, that an *implicit* folding interpretation of the real potential is consistent with the data. The energy dependence of the volume integral of the real potential is found to agree with the folding model approach worked out by Jackson and Johnson [13]. The present results, therefore, provide an *a posteriori* justification to their use of the folding model.

The motivation for the present analysis was the application of elastic α-particle scattering in the study of nuclear matter radii. The rather constant value of the specific volume integral which is a basic property of a folding approach makes it reasonable to assume that a further basic relationship, the additivity of the ms radii is also valid. Denoting by $\langle r^2_{eff} \rangle$ the ms radius of α-particle-bound nucleon effective interaction, one writes

$$\langle r^2_v \rangle = \langle r^2_m \rangle + \langle r^2_{eff} \rangle \qquad (6)$$

where $\langle r^2_m \rangle$ is the ms radius of the nuclear matter distribution. Without any explicit specification of the α-particle-bound-nucleon interaction we may relate the difference between ms radii of the nuclear matter distribution in two nuclei to the corresponding difference between ms radii of the real optical potentials:

$$\Delta\langle r_m{}^2 \rangle = \Delta\langle r^2_v \rangle \qquad (7)$$

The latter can be regarded as experimentally determined quantities and the uncertainty of this difference is obtained from the errors in $\langle r^2_v \rangle^{1/2}$ (listed in the tables for the cases considered). Combining the errors in quadrature, the errors in values of $\Delta\langle r^2_m \rangle^{1/2}$ are typically $\pm$ 0.1 fm. Such disappointingly large uncertainties are inevitable if one considers the accuracies of the experimental quantities. In the past, much better accuracies have been claimed for values of $\langle r^2_m \rangle^{1/2}$, but these relied on additional assumptions in the analysis such as introduced by using an explicit folding model and ignoring the uncertainties in the α-N potential.

If it is assumed that $\langle r_m^2 \rangle^{1/2}$ is precisely known for one particular nucleus[+)] one may then use the optical potential for that nucleus to "calibrate" $\langle r_{eff}^2 \rangle$ by

$$\langle r_{eff}^2 \rangle = \langle r_v^2 \rangle - \langle r_m^2 \rangle \qquad (8)$$

with an error determined by that of the potential. Using this calibrated value of $\langle r_{eff}^2 \rangle$ for any other nucleus is, of course, equivalent to the case of eq. (7) with the "calibrating" nucleus as a reference. However, it is obvious that when comparing values of $\delta\langle r_m^2 \rangle$ (with reference to the calibrating nucleus) the uncertainty in $\langle r_{eff}^2 \rangle$ should not be included once more.

5. Concluding Remarks

In this paper we considered experimentally well defined quantities characterizing α-particle scattering in the energy region from 100 - 170 MeV and we have avoided any specific assumption about the microcopic structure of the α-particle optical potential. Nevertheless we conclude that differences in the ms radii of the real potential are related to differences in the nuclear matter distributions in a transparent way. In fact, it has been already shown [32)] by theoretically more specific calculations that indeed $\delta\langle r_v^2 \rangle$ is equal to $\delta\langle r_m^2 \rangle$ for the ^{40}Ca - ^{48}Ca case. Therefore, it seems to be a matter of experimental accuracy in α-particle scattering investigations to further exploit this source of information.

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+) *Usually one takes the ^{40}Ca case as a reference because the rms radius of the nuclear matter distribution is believed to be rather well known. The small deviation of the specific volume integral of ^{40}Ca from that of the other nuclei (see tabs. 2 and 3) need not affect eq. (6) since it is observed [26)] that generally the dependence of $\langle r_v^2 \rangle$ on the details of the calculation is considerably smaller than that of $-J_v/4A$.*

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