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RADIAL SENSITIVITY OF HADRONIC PROBES

AND

HOW ACCURATELY ARE NUCLEAR RADII DETERMINED

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Abstract

The radial sensitivity of hadronic probes is studied by applying a local perturbation to the neutron density in optical model calculations of elastic scattering of 104 MeV α particles, 1 GeV protons and 130 MeV pions by ^{48}Ca. Also calculated are level shift and width for the 2p state in pionic atoms of ^{48}Ca. All calculations refer to recent experimental results. From comparisons with the error analysis available in the Fourier Bessel description of optical potentials it is concluded that in many analyses gross underestimates of the uncertainties of the results are made.

DIE EMPFINDLICHKEIT HADRONISCHER SONDEN AUF DIE NEUTRONENVERTEILUNG IN KERNEN

Die Empfindlichkeit hadronischer Sonden auf die Dichteverteilung der Neutronen in Kernen wird untersucht, indem eine lokale Störung der Neutronendichte in Berechnungen des optischen Potentials für die Streuung von 104 MeV α-Teilchen, 1 GeV Protonen und 130 MeV Pionen an ^{48}Ca eingeführt wird. Ebenfalls werden die Verschiebung und die Breite des 2p Zustandes in pionischen ^{48}Ca-Atomen berechnet. Die Studien beziehen sich auf neuere experimentelle Untersuchungen der Neutronenverteilung. Aus dem Vergleich mit der Fehleranalyse, welche die Fourier-Bessel-Beschreibung des optischen Potentials liefert, kann man schließen, daß bisher in vielen Analysen die Unsicherheiten stark unterschätzt wurden.

I. INTRODUCTION

The radial distribution of nucleons in nuclei is a topic of current interest as it provides a sensitive test of theories of nuclear structure. While the distribution of protons can be studied most precisely via the electromagnetic interaction, studies of the total matter or neutron density distribution inevitably rely on a strongly interacting probe which implies more difficulties in interpreting experimental observations in terms of nuclear properties. Nevertheless, hadronic probes have been shown to be quite useful in providing at least partial answers to the question of nuclear densities, in particular by comparative studies when the "apparatus function" (effective probe-nucleus interaction) could be "calibrated" on a nucleus with a presumably known neutron distribution. The results of various types of experiments such as elastic scattering of α particles, of protons or pions etc. (see for examples ref 1), although showing internal consistency seem to be sometimes in conflict with each other. Apart from the residual uncertainty of the effective interaction (which in most cases has not been taken into account in evaluating the errors of the final results) and in addition to deficiencies and constraints in the analysis itself, the above-mentioned discrepancies could originate from differences in the radial sensitivity of different types of experiments which are currently used to probe the nuclear density distribution. It is therefore interesting to investigate which parts of the nucleus are well probed, and how uncertainties of the radial moments are affected by the radial sensitivity. Also important is a critical comparison of the methods of evaluating the uncertainties in the different analyses.

In the present paper we consider four different experiments which are typical of their kind in the quality of the data they provide: (1) The elastic scattering of α particles[2] around 100 MeV where only data extending to large angles are considered thus probing the interior of the nucleus beyond the surface region. (2) The elastic scattering of protons in the GeV region[3] where the analysis in terms of the fundamental proton-nucleon interaction is a characteristic feature. (3) The elastic scattering of 130 MeV pions[4] where the availability of both π^+ and π^- beams together with the strong isospin dependence of the interaction are of particular interest. (4) Strong interaction level shifts and widths in pionic atoms[5] where the very good experimental accuracy together with the isospin dependence of the interaction are interesting features for probing neutron distributions in nuclei.

Experiments on the elastic scattering of α particles and of protons have been analyzed in recent years using methods which are efficient and instructive in studying the radial distributions of the interaction potential or of the nuclear density. These methods, guided by the "model-independent" procedures used in electron scattering, overcome the constraints of using simple analytical forms and also provide a realistic analysis of the uncertainties in the studied distribution as a function of r, the distance from the center of the nucleus. Several variants of these "model-independent" methods have been successfully used: (i) the Fourier-Bessel (FB) method[2,6,7]. (ii) The sum-of-Gaussians (SOG) method[8]. (iii) The spline function method[9].

(iv) A method based on a set of orthogonal polynomials[10]. These methods could also be introduced into microscopic models[7,11] relating the interaction potential to the density distribution of protons and neutrons in the nucleus. In the case of a zero-range probe-bound nucleon interaction the potential is proportional to the nuclear density unless there are strong isospin effects which warrant separate handling of neutrons and protons (the latter distribution is generally assumed to be known from the accurately measured charge distribution). If a finite range is assumed for the interaction the "microscopic" description implies some type of a folding model[11,12]. Due to the smearing effects of the folding integral the densities are then more remote from the experimental data than are the potentials themselves with the consequence that the relative uncertainties in the densities are larger than those for the potential, particularly at the interior of the nucleus. All these features are clearly revealed when applying the "model independent" procedures in the analyses of scattering data.

Unfortunately, it is not always possible to use these model-independent methods. For example, measurements of total or reaction cross section yield one or two experimental numbers, and this is also the case when strong interaction level shifts and widths are measured in pionic atoms. Furthermore, model-independent analyses of the elastic scattering of pions at about 100 MeV do not seem feasible at present due to the insufficient knowledge of important details of the pion-nucleus potential.

In order to study the radial sensitivity of different types of experiments on equal footing we applied in the present work the notch test method[13,14], which is in several aspects less satisfactory, somewhat unphysical and rather crude in comparison to the above mentioned methods. In order to make it more acceptable we introduced some modifications compared to previous uses[14]. The notch was applied only to the neutron density distribution with fixed proton density, and was limited to only 30 % of the neutron density, which seems a reasonable value when comparing with the estimates of the uncertainties obtained from the other methods. In addition, the finite range of the effective interaction makes the perturbation in the potential even smoother and it damps unphysical effects induced by the notch. With these precautions we use the notch test for the comparison of radial sensitivities, but demonstrate also its deficiencies in the well-studied case of α particle scattering. We emphasize that all the present results, although referring to typical experimental data are only for demonstration purposes, and they should not be regarded as final results of analyses for any of the experiments discussed.

II METHODS

The sensitivity of the various experiments to the neutron distribution was studied by introducing a local perturbation[13,14] ("notch") into that distribution. The neutron density distribution $\rho_n(r)$ was therefore multiplied by the factor

$$f(r,R_{No}) = 1 - d \exp[-((r-R_{No})/a)^2] \qquad (1)$$

where d measures the amount of density removed by the notch, a measures the radial extent of the perturbation and R_{No} is its location. The factor f was applied to $\rho_n(r)$ in a calculation which, without f, produced a best fit to the data. By varying the value of R_{No}, the dependence of χ^2 (the sum of squares) on R_{No} was scanned throughout the nucleus, thus demonstrating the radial sensitivity to the neutron density of any particular experiment. Several values of a and d were used, in order to check the numerical stability of the results. The results shown in the present work are for d = 0.3 and a = 0.5 fm, values which are very reasonable when the sensitivity to the neutron distribution is studied. These values are well within the range of parameters which produced smooth perturbation-like behavior of the results. This smooth behavior of the results was one of the indications for the numerical stability of the calculations, which is due to the mild nature of the perturbation in the potential. Additionally, checks were made by varying the radial integration step in all calculations and confirming the stability of the results.

In the following we describe the potentials and types of calculations used for each of the four kinds of experiments. In each case we used the type of analysis which is currently being used in interpreting experimental data, thus we avoided gross simplifications which could be introduced by adopting a common method for all experiments. Some simplifications were, however, made which do not affect the radial sensitivity, but may affect the precise value of nuclear radii. Examples for such simplifications are the neglect of spin-orbit interaction in proton scattering and the neglect of finite range effects in the pion experiments.

A. Elastic scattering of α particles

The fit to the data was made using the density-dependent folding model, which had been shown[11,16] to be successful in fitting elastic scattering data extending to large angles. The real part of the potential was written as

$$\mathrm{Re}U(r) = \int -V_G \exp[-|\vec{r}\,'-\vec{r}|^2/a_G^2]\ [1-\gamma\rho_m^{2/3}(r')]\rho_m(r')d^3r' \qquad (2)$$

where ρ_m is the nuclear density and the term $(1-\gamma\rho_m^{2/3})$ represents the density dependence of the α particle-bound nucleon interaction. Writing $\rho_m = \rho_n+\rho_p$, the sum of neutron and proton densities, respectively, the notch test was introduced into ρ_n, as described above. One of our standard optical model programs, MODINA[17], was used for these tests.

B. Elastic scattering of 1 GeV protons

It is commonly accepted that at energies of the order of 1 GeV one can reliably use the impulse approximation to obtain the proton-nucleus optical potential. The optical potential was therefore written as[18]

$$U(r) = \frac{-\hbar^2c^2}{(2\pi)^2E_L}\frac{k_L}{k_o}\int e^{-i\vec{q}\cdot\vec{r}}\ [f_n(q)F_n(q)+f_p(q)F_p(q)]d^3q \qquad (3)$$

where $f_{n(p)}$ are the proton-neutron (proton) scattering amplitudes and F are the nuclear form-factors,

$$F_{n(p)}(q) = \int e^{i\vec{q}\cdot\vec{R}} \rho_{n(p)}(R)\, d^3R \quad . \tag{4}$$

E_L is the total energy in the laboratory system, k_L and k_o are the wave numbers in the laboratory and in the nucleon-nucleon systems respectively.

The dependence on the momentum-transfer q was written as

$$f_{n(p)}(q) = f_{n(p)}(0)\, e^{-\frac{1}{2}\beta^2_{n(p)} q^2} \tag{5}$$

and finally the optical theorem was used, namely,

$$f_{n(p)}(0) = \frac{k_o}{4\pi}\, \sigma_{T_{n(p)}} (i+\alpha_{n(p)}) \quad . \tag{6}$$

An additional kinematical factor and the (A-1)/A factor[18] were included in U(r) but not written in eq. (3) for simplicity. In any case, these factors are not essential in the present application because the amplitudes were slightly adjusted in order to improve the fit to the data.

In the calculations the Fourier transformation in eq. (3) was not performed explicitly. Instead, a Gaussian folding was performed in coordinate space[19], where eq. (3) becomes

$$U(r) = \frac{-\hbar^2 c^2 k_L}{2E_L} (2\pi)^{-3/2} \int \{ \rho_n(r')(i+\alpha_n)\sigma_{T_n} \exp[-|\vec{r}\,'-\vec{r}|^2/2\beta_n^2]$$

$$+ \rho_p(r')(i+\alpha_p)\sigma_{T_p} \exp[-|\vec{r}\,'-\vec{r}|^2/2\beta_p^2] \} d^3r' \quad . \tag{7}$$

Not included in the present calculation was a spin-orbit term, but that should not affect our results regarding radial sensitivity because of the small adjustments to the parameters mentioned above made before introducing the notch test. The same program, MODINA[17], was used for these tests as for the α particle scattering calculations.

C. Elastic scattering of pions

The elastic scattering of pions was calculated using the simplest potential possible, namely, the standard Kisslinger form[20]

$$U(r) = -k_{cm}^2[\beta_{0n}\rho_n(r)+\beta_{0p}\rho_p(r)] + \beta_{1n}\vec{\nabla}\cdot\rho_n\vec{\nabla} + \beta_{1p}\vec{\nabla}\cdot\rho_p\vec{\nabla} \qquad (8)$$

where the coefficients β_0 and β_1 are complex numbers. This potential does not contain several of the ingredients of the more modern versions[21] (such as ρ^2 terms, Lorentz-Lorenz effect and angle transformation terms), which were developed for analyses of pionic atoms and also used to analyze elastic scattering. Nevertheless, reasonable fits to the data are possible using this simple potential and it was regarded as an adequate approximation for the purposes of the present work. The above potential (eq. (8)) assumes a zero range of the pion-nucleon interaction. In view of the many ambiguities in the potential it was not considered worthwhile to introduce here a finite range (see also comment on pionic atoms). A modified form of the program PIRK[22] (new version) was used for the calculations.

D. Pionic atoms

Strong interaction level shifts and widths in pionic atoms have been calculated recently using many different versions of the Ericson-Ericson potential. It was shown[23,24] that equivalent fits to the data were obtained throughout the periodic table with any of the versions tested. Therefore, only version

I of ref. 24 was used in the present work, namely

$$U(r) = \frac{1}{2\mu}\left[q(r)+\vec{\nabla}\cdot\alpha(r)\vec{\nabla}\right] \qquad (9)$$

where μ is the reduced mass, q is the s-wave part and α the p-wave part of the potential, written in terms of the nuclear densities as follows:

$$q(r) = -4\pi\{(1+\frac{\mu}{m})b_0(\rho_n+\rho_p)+b_1(\rho_n-\rho_p)+(1+\frac{\mu}{2m})4B_0\rho_n\rho_p\} \qquad (10)$$

$$\alpha(r) = \frac{\alpha_0(r)}{1+\frac{1}{3}\alpha_0(r)} \qquad (11)$$

$$\alpha_0(r) = 4\pi\{(1+\frac{\mu}{m})^{-1}\left[c_0(\rho_n+\rho_p)+c_1(\rho_n-\rho_p)\right]+(1+\frac{\mu}{2m})^{-1}4C_0\rho_n\rho_p\} \qquad (12)$$

m is the nucleon mass and the coefficients b_0, b_1, B_0, c_0, c_1 and C_0 were taken from fits to data[24]. The above potential assumes a zero range for the pion-nucleon interaction. Introducing a finite range to the p-wave part of eq. (9) may have, in principle, far reaching consequences[25]. However, it was shown recently by Alexander et al.[26] that for pionic atoms the only effect of introducing finite range is to change the values of parameters, but otherwise maintaining the same overall picture. We therefore used in this work the zero-range version of the potential. A modified version of the program ANATBND[27] was used.

III RESULTS

In order to enable a comparison between the four types of experiments, the same nucleus - ^{48}Ca - was chosen for all cases. This nucleus has been extensively studied by many different groups using a variety of methods and good quality data are available to base on it the present studies. This nucleus with its relatively large neutron excess is typical of medium-weight to heavy nuclei, which form the prime object of investigations of neutron density distributions.

A. Elastic scattering of alpha particles

The data on which the present sensitivity tests are based are those from Karlsruhe[2] which were extensively analyzed using the FB method[2,11,16]. For the purpose of the present work new fits were made using the density dependent folding model with a 3 parameter Fermi function description of ρ_n, the neutron density distribution. Figure 1 shows the results of the notch test applied to ρ_n of the best-fit density. If the doubling of χ^2/F signifies the radial region which is sensitive to ρ_n, then the present results show it to be from 6 fm down to 2.5 fm, or from 2 % of the central density to the beginning of the central plateau of ρ_n. Adopting other criteria for the shift of χ^2/F from its minimum value will hardly affect this conclusion.

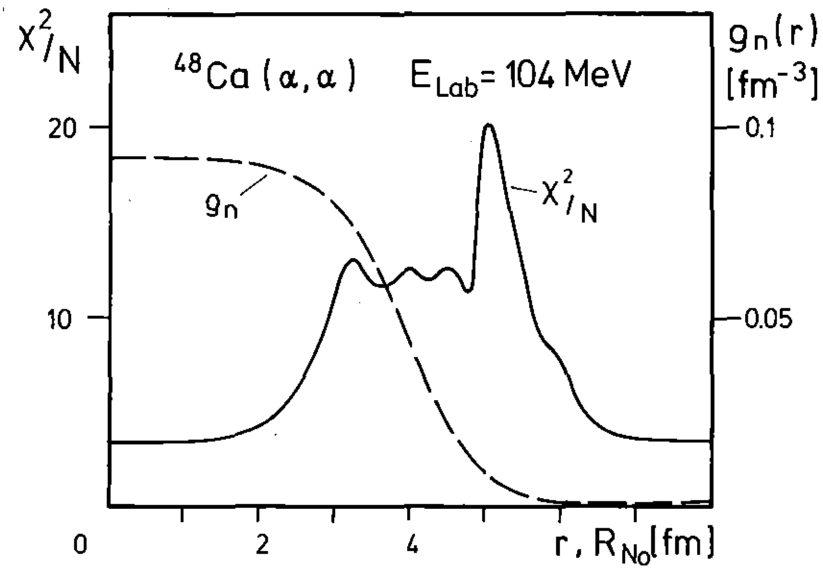

Fig. 1: χ^2 per point for the elastic scattering of 104 MeV alpha particles by ^{48}Ca, calculated as a function of the position of a 30 % notch in the neutron density. Also displayed is the neutron density distribution as a reference for the position of the notch.

B. Elastic scattering of 1 GeV protons

The experimental cross sections on which the present work is based are those from the Saclay-Gatchina group[3]. We have used the potential of eq. (7) in analyzing the results for ^{40}Ca assuming a known[11] neutron density ρ_n. Adjusting very slightly the interaction parameters in (7), a very good fit to the data was obtained. Using then the same parameters for ^{48}Ca and adjusting its ρ_n, a very good fit to the data was also obtained, with reasonable ρ_n. That fit served as the basis for the present notch tests. We reiterate that the present fits were used only as basis for the notch tests and that they should not be regarded as providing final results or as competing with more refined analyses[32].

Figure 2 shows the results of the present notch tests, where the range of sensitivity to the present 30 % notch in ρ_n is between 5.5 fm down to 2.5 fm, or from about 4 % of the central neutron density to the beginning of the central plateau.

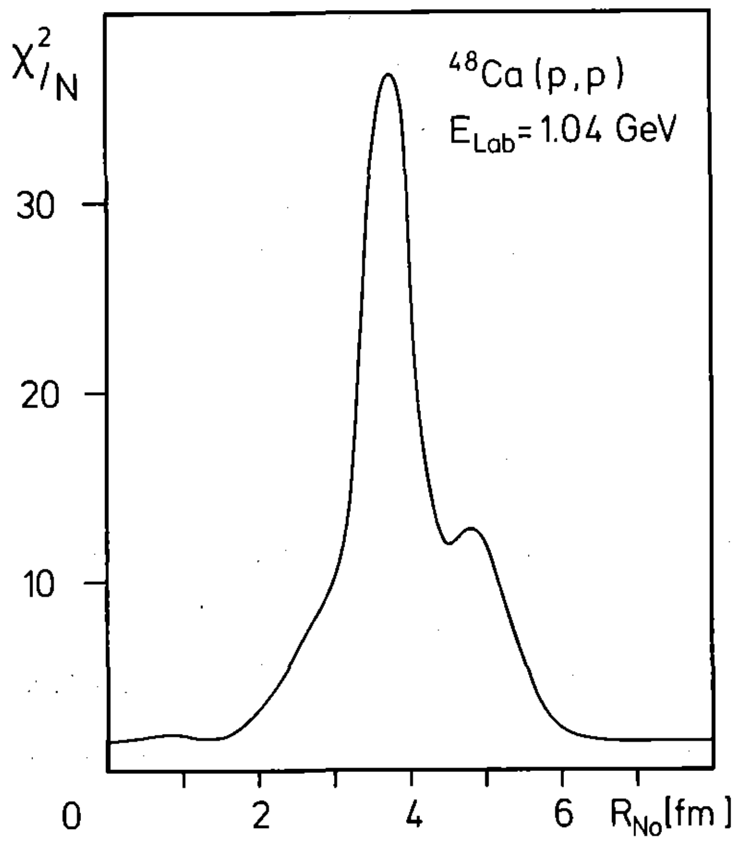

Fig. 2:
χ^2 per point for the elastic scattering of 1 GeV protons by ^{48}Ca, calculated as a function of the position of a 30 % notch in the neutron density.

C. Pion scattering

The sensitivity tests for pion scattering are based on the results[4] of the elastic scattering of 130 MeV $\pi^{\pm}$ by ^{48}Ca. As the potential used in the present work did not include some of the refinements of more recent versions, no effort was made to achieve very good fit to the data. Calculating the coefficients β_0 and β_1 (eq. 8) from pion-nucleon scattering amplitudes around 100 MeV and making minor adjustments, a χ^2 per point of about 10 was obtained when comparing calculations with data[4], using a neutron density distribution taken from a fit to pionic atoms (see below). No obvious systematic deficiencies are observed in the fits. The optical model was then used to generated "pseudo-data" for the purpose of the notch tests, by using the angles included in the experiment and randomly shifting calculated cross-sections within the quoted experimental errors. Using this procedure a χ^2 per point of $\approx$ 1 was obtained.

Figure 3 shows the results of the notch test applied both to π^+ and π^- scattering. As in the other cases, a 30 % notch with a = 0.5 fm was introduced into ρ_n. It is well-known that at these energies π^- interact mainly with neutrons and π^+ with protons and this is clearly observed in Fig. 3. If the increase of χ^2/N beyond 2 is a measure of sensitivity to the neutron distribution, then the π^- scattering probes the neutron density between 5 and 2.5 fm which is the region between 10 % of the central density and almost the beginning of the central plateau of the neutron density. π^+ scattering, on the other hand, is almost not sensitive to the neutron density (within the present 30 % notch test), a result which is expected. The elastic scattering of π^+ is, however, very useful as a further check on the consistency of the analysis.

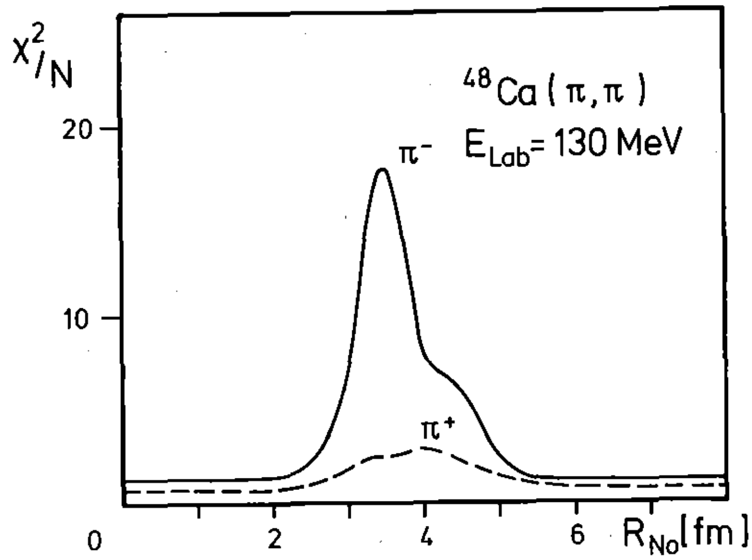

Fig. 3: χ^2 per point for the elastic scattering of 130 MeV π^+ and π^- pions by ^{48}Ca calculated as a function of the position of a 30 % notch in the neutron density.

D. Pionic atoms

The experimental results on which the present sensitivity tests of pionic atoms are based are those of Powers et al.[5] for the 2p level in pionic ^{48}Ca. Using the parameters of potential I given by Friedman and Gal[24], the calculated strong interaction level shift and width agree with the experimental results using a neutron density distribution with a rms radius 0.16 fm larger than that of the proton density distribution. The present notch tests were applied to such a neutron distribution.

Figure 4 shows the results for a 30 % notch introduced into ρ_n, with a = 0.5 fm. The small increase in the value of χ^2/N near 2 fm is a consequence of the gradient term in the potential (eq. (9)); the notch introduces a gradient in an otherwise flat region of the nucleus. If the increase of χ^2/N beyond 2 is a measure of the sensitivity, then the present test shows a sensitivity to neutron distributions between 5.5 and 3.2 fm, where ρ_n changes from 5 % to 85 % of its central density.

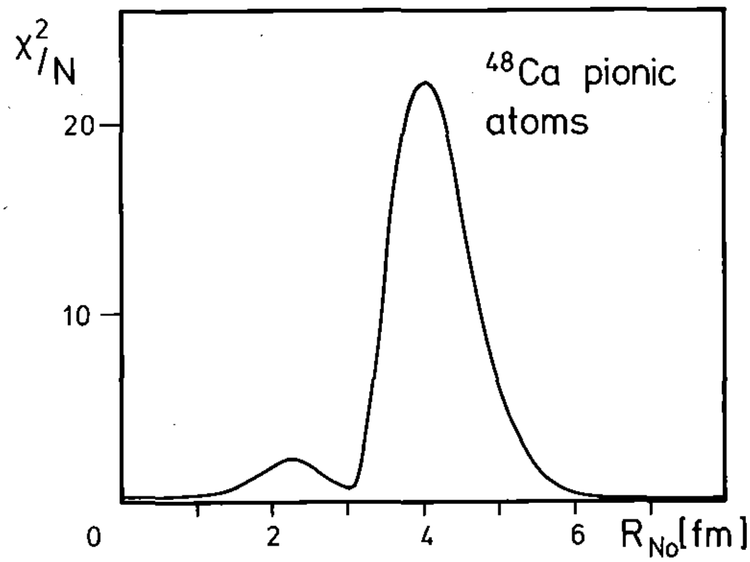

Fig. 4: χ^2/N for the shift and width of the 2p level in pionic atoms of ^{48}Ca, calculated as a function of a 30 % notch in the neutron density.

IV DISCUSSION

The purpose of the present study was to explore which regions of the neutron density distribution are determined by several hadronic probes. We considered four kinds of typical examples of current interest. In order to get a realistic picture it was important to analyse each kind of experiment within its own specific theoretical description as over-simplifications might lead to conclusions which reflect rather the limits of the approximation and not those of the experiments. This could be possibly the case in the feasibility study of Meyer[28] whose considerations are confined to high energy hadron-nucleus interaction (with emphasis on total cross section measurements) using the optical limit of the Glauber multiple scattering theory[29]. In order to enable a consistent comparison of the radial sensitivities a perturbation of ρ_n has been introduced ("notch technique") in the present work scanning ρ_n and exploring the effects of variations of ρ_n on the observable quantities, within the framework of a realistic reaction model. Although the notch test is accompanied by some inherent difficulties and it overlooks some details, it provides semi-quantitative information about the radial sensitivity.

The results of the calculations indicate slightly different radial sensitivity for the different probes. 100 MeV α particles appear to be not only capable of probing the nuclear surface but they are also able to compete with 1 GeV protons in providing information on the interior of the nucleus. This fact is due to the refractive behavior of α particles scattered at large angles and has been already demonstrated in "model-independent" analyses[11,15,16]. It has also been revealed[7] that it is not true that the higher the bombarding energy the greater will be the penetration of the projectile.

Comparing nuclear radii determined by different experiments[1] it seems unlikely that the small differences in radial sensitivity are the origin of the discrepancies in the quotes values of rms radii. A more likely explanation can be found in the inadequate analysis of uncertainties (including those of the effective interaction) combined with the constraints in some analyses, in particular those introduced by the use of simple parametrisations[28] of ρ_n. For example, the 1 GeV proton scattering result[30] of the Saclay group based on a three-parameter Fermi shape of ρ_n which yields $\langle r^2\rangle_n^{1/2} - \langle r^2\rangle_p^{1/2} = 0.10 \pm 0.03$ fm for ^{48}Ca seems to contradict the α particle scattering result[16] of $\langle r^2\rangle_n^{1/2} - \langle r^2\rangle_p^{1/2} = 0.25 \pm 0.12$ fm derived by a FB-description of ρ_n. However, when comparing the densities in the well-determined region of ρ_n obtained in the two experiments, one finds that there is no significant difference. This is a strong hint that the quoted error in the proton scattering result does not reflect the uncertainties in the less well-determined part of ρ_n. In fact, the consequent use of "model-independent" techniques in proton scattering analyses[12] has lead recently to more consistent results and more realistic errors.

Because of the great importance of the error analysis we discuss now in some detail two of the recently used procedures of evaluating the uncertainties. In the FB method the nuclear density distribution is described as a first approximation function $\rho_o(r)$ (which has the correct volume integral of A nucleons) plus a Fourier-Bessel series

$$\rho(r) = \rho_o(r) + \sum_{n=1}^{N} \beta_n j_o(q_n r) \qquad (13)$$

where $q_n = n\pi/R$ and the series is included in $\rho(r)$ only for $r \leqq R_c$. The coefficients $\beta_1 \ldots \beta_N$ are obtained by requiring a best-fit to the data while constraining the above series to have a zero volume integral. The following expression is obtained for the uncertainties[6]

$$\langle \Delta \beta_i \Delta \beta_j \rangle = 2(M^{-1})_{ij} \qquad (14)$$

where (M^{-1}) is the covariance matrix obtained numerically in the course of performing the χ^2 fit. This expression represents the statistical 60 % confidence limit and is valid only in the case of purely statistical deviations between calculation and experiment, which implies a χ^2 per degree of freedom (χ^2/F) close to 1. When χ^2/F is larger than 1, it is a common practice to increase the quoted errors by multiplying eq. (14) by χ^2/F, which means that the error in e.g. the rms radius is proportional to $(\chi^2/F)^{1/2}$. Whereas there is no rigourous justification to this prescription, particularly when the deviation of χ^2/F from 1 is due to some non-statistical deficiency, it appears to be a plausible one at least when $\chi^2/F \leq 3$. In any case, when χ^2/F is considerably greater than 1 it is indicative of some fundamental problems in the analysis and a straight-forward error analysis is inadequate.

Another method which was recently used to estimate the uncertainties[12,31] is based on introducing long-range perturbations into the density and finding the limits of these such that any calculated point will not deviate beyond its estimated experimental error. In principle this method corresponds to the above mentioned statistical approach, based on an increase of χ^2 by 1. However, when χ^2/F is significantly larger than 1 this method leads to much smaller estimated errors as compared to the covariance matrix method.

As a general rule, the best value of χ^2/F achieved in analyzing experimental data should be used as a guide to possible systematic errors. Obviously when χ^2/F approaches 1 the different methods of evaluating uncertainties should be aquivalent. With these comments in mind we note that while values of χ^2/F achieved[31] in some of the analyses are of the order of 10, the errors quoted are as though χ^2/F was 1, which may lead to unrealistically small errors.

Returning now to the notch test which was used in the present work for rough comparisons, we emphasize that it gives only semi-quantitative results and that it should not be used to calculate the uncertainties. This point is demonstrated by fig. 5 showing the results of an error analysis for the elastic scattering of 104 MeV α particles from ^{48}Ca. The errors are obtained using the FB method* and are compared to the sensitivity function obtained by the notch test. One of the striking features observed is that the errors from the FB-method clearly reflect the quality of data. When only every second data point is retained, a marked increase is evident in the deduced errors whereas the notch test is unable to really distinguish between the different data sets. The same is true when only forward angles are included. In this case (both in scattering of α particles and of protons) simple analytical forms for the densities lead to excellent fits with

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*Meyer[28] has pointed out that the use of "model-independent" methods could produce some "fake sensitivity" due to couplings of $\rho_n(r)$ at different radii. This may be true in cases where actually a too naive and simplified procedure is applied, but it can be avoided by random choices of the initial conditions.

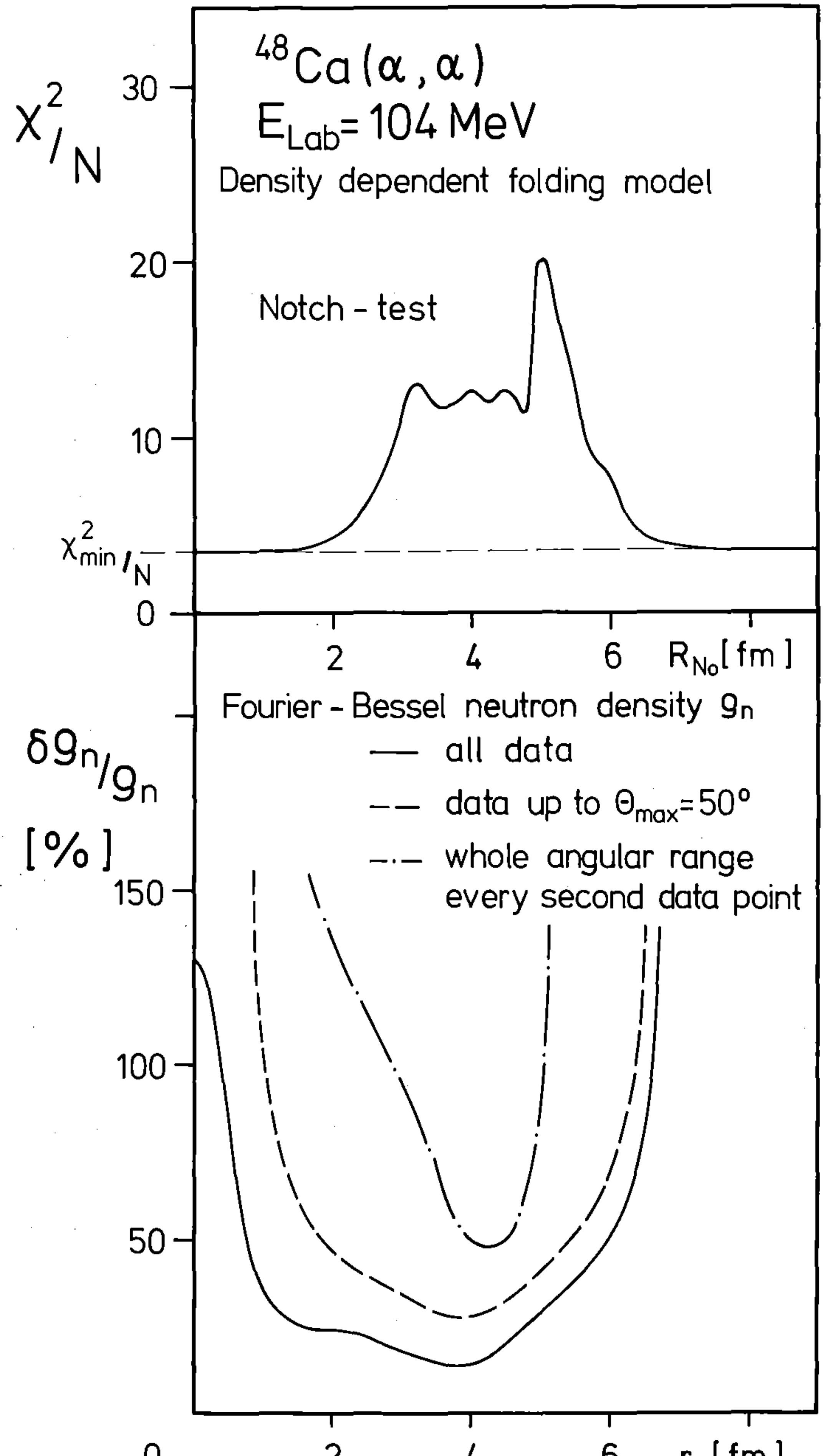

Fig. 5: Lower part: Relative errors of the neutron density from 104 MeV α particle scattering from ^{48}Ca obtained by FB fits using: (a) the full data set (b) every second data point (c) only data for θ_{cm}<40°. Upper part: χ^2/N vs. position of notch in the neutron density, which is unable to distinguish between the three cases shown in the lower part.

unrealistically small errors for e.g. the rms radii while a FB-analysis reveals the poorer accuracy of the derived quantities.

The case of elastic scattering of α particles in the 100 MeV region has been most extensively analyzed concerning possible sources of uncertainties[2,15,16]. In addition to a model-independent description uncertainties in the effective α particle-bound-nucleon interaction have been taken into account. Typical realistic errors in the rms radii of the neutron distribution are ± 0.15 fm. It is unlikely that the true uncertainties in 1 GeV proton scattering are smaller since only forward angles are measured and as the uncertainties in the nucleon-nucleon amplitudes above 500 MeV are at least as large as for the α particle effective interaction[32].

The scattering of negative pions and level shift and width in pionic atoms are most useful probes of the neutron densities in nuclei because π^- interact predominantly with neutrons. The π^- scattering has also been analyzed in terms of phenomenological diffraction models[33] the extracted size parameter of which are not directly comparable with the rms radii of ρ_n.

At present the uncertainties in the many parameters of the π^- nucleus optical potential prevent to fully exploit the pion as a probe of neutron densities.

A progress in this field may be expected from a simultaneous analysis of π^- data with those from another hadron scattering experiment.

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