KfK 3005
Oktober 1980

Analysis of Process Signals at a Reprocessing Plant

R. Avenhaus, G. Spannagel
Institut für Datenverarbeitung in der Technik
Projekt Wiederaufarbeitung und Abfallbehandlung

Kernforschungszentrum Karlsruhe

KERNFORSCHUNGSZENTRUM KARLSRUHE

Institut für Datenverarbeitung in der Technik
Projekt Wiederaufarbeitung und Abfallbehandlung

KfK 3005
PWA 52 / 80

ANALYSIS OF PROCESS SIGNALS AT A REPROCESSING PLANT

by

R. Avenhaus and G. Spannagel

Kernforschungszentrum Karlsruhe GmbH, Karlsruhe

Kernforschungszentrum Karlsruhe GmbH
ISSN 0303-4003

Abstract

In the framework of the 'Project of Reprocessing and Waste Treatment' of the Karlsruhe Nuclear Research Center, some time ago an activity has been initiated, the aim of which is to detect in time disturbances of production processes of nuclear installations with the help of process-signal analysis.

In this paper, the present state of this activity is described. The basic approach will be discussed: using two special cases, by means of appropriate process models it is demonstrated which behaviour of the process will lead to which signals; vice versa, the investigations of these signals by decision theoretical methods will provide information on possible disturbances of the process. Special attention will be paid to the influence of process variations on the decision procedure.

Implementation of this procedure in the GWK reprocessing plant, Karlsruhe, is underway.

Analyse von Prozeßsignalen einer Wiederaufarbeitungsanlage

Zusammenfassung

Im Rahmen des Projektes "Wiederaufarbeitung und Abfallbehandlung" des Kernforschungszentrums Karlsruhe wurde vor einiger Zeit eine Untersuchung begonnen, deren Ziel es ist, Betriebsstörungen kerntechnischer Anlagen unter Einsatz der Prozeßsignalanalyse rechtzeitig zu entdecken.

Diese Arbeit beschreibt den gegenwärtigen Stand dieser Untersuchung. Unter Verwendung zweier Spezialfälle wird die grundlegende Vorgehensweise diskutiert: mittels geeigneter Prozeßmodelle wird gezeigt, welches Prozeßverhalten zu welchen Signalen führt; umgekehrt liefert die Prüfung dieser Signale mit Methoden der Entscheidungstheorie Hinweise auf mögliche Betriebsstörungen. Insbesondere wird der Einfluß von Prozeßschwankungen auf die Entscheidungsprozedur behandelt.

Mit der Implementierung bei der GWK Wiederaufarbeitungsanlage, Karlsruhe, wurde begonnen.

Contents

1. Introduction

It is the objective of any control of complex industrial production processes to run the production lines in such a way that a high availability, i.e., a high productivity, is maintained. Such a control demands careful observation of relevant process signals as well as comprehensive plant experience. These requirements will be a prerequisite above all of the large German facility for reprocessing spent nuclear fuels. Therefore, it seemed to be meaningful to study methods and to propose tools which hopefully will support these objectives. The WAK reprocessing plant is used for demonstration purposes in this study.

As regards similar demands on nuclear power plants, considerable efforts have been spent for a long time. For example, the Halden Programme /1/, /2/ represents a well known investigation to this effect: It deals with the potentials of computer-based on-line analysis of process and component behavior within the frame of nuclear power plant surveillance systems; emphasis is laid on a combination of technological, methodological and procedural factors. This system of analysis has been installed in the Grafenrheinfeld nuclear power plant /3/. However, compared with the conditions encountered at reactor stations, the process behavior at a reprocessing facility is quite different. Just for illustration it might be mentioned that reprocessing is essentially related to chemical procedures which, in case of anomalous behavior, leave more time for counteraction compared to a similar situation at a nuclear reactor.

The following idea underlies the approach presented here: In the central control room of the plant under consideration all kinds of signals are available which describe the state of the production lines. Instead of continuously checking all these signals independently, it is tried to organize observation of these signals by process models such that, on the one hand, process disturbances are quickly recognized, and on the other hand, the number of false alarms is minimized.

The investigation carried out so far was confined to the anomalous process behavior induced by hydraulic disturbances in the so-called '2nd uranium cycle'; it should be pointed out that this analysis can be applied in general.

The methodological tools used for this investigation are taken from mathematical statistics, more specifically from the theory of hypothesis testing. As this presentation has been written for practitioners who might not always be very familiar with these tools, the analysis has been explained in great detail, although, hopefully, in such a manner that the findings can be generally understood. Especially ANNEX serves this purpose: Its contents do not provide new information for statisticians; however, it can hardly be found in the usual statistical textbooks.

Implementation of the procedures on a process computer is a prerequisite of successful use of the methods described: in a special report /4/ the interested reader will find the relevant details.

2. Anomalous hydraulic states in the 2nd uranium cycle of the WAK reprocessing plant

As demonstrated in figure 1, the 2nd uranium cycle represents an essential part of the process branch responsible for purification of the uranium product. A more detailed description of this cycle is given by figure 2: In the 2D mixer-settler the uranium is transferred from the 2DF-stream (aqueous solution) to the 2DX-stream (organic solution); the fission products remain in the stream while entering the container '42.01.1/2' as the 2DW-stream. In the 2E mixer-settler the uranium is retransferred into the aqueous solution (2EU-stream). As indicated, the fluids are usually transferred by airlifts, for example the 2DF-stream is transferred from the container '41.11' to the 2D mixer-settler through the airlifts A113 and A131.

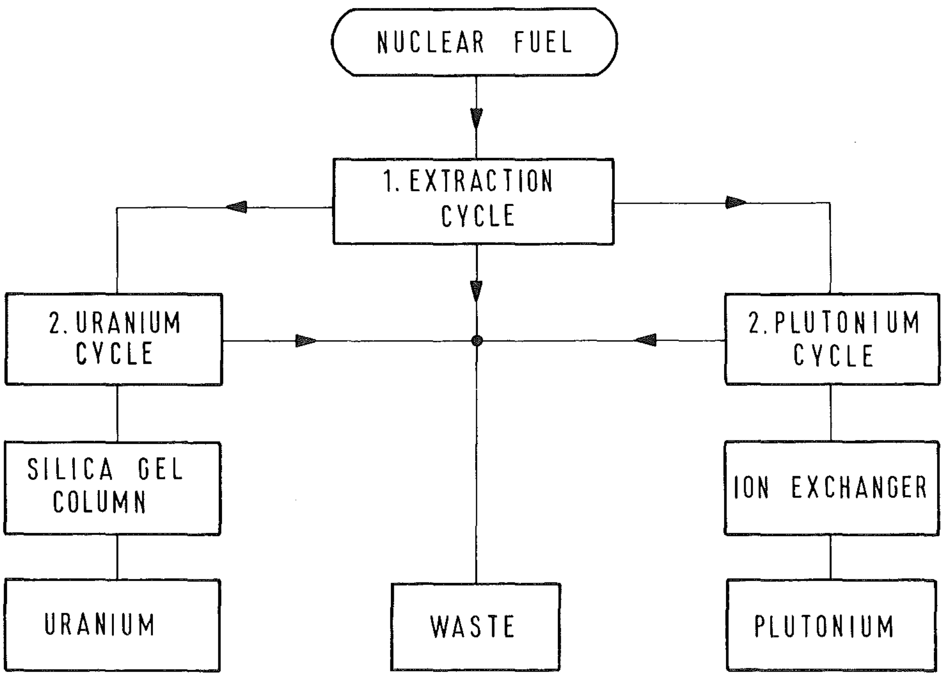


Fig. 1: Main process units of the WAK reprocessing plant.

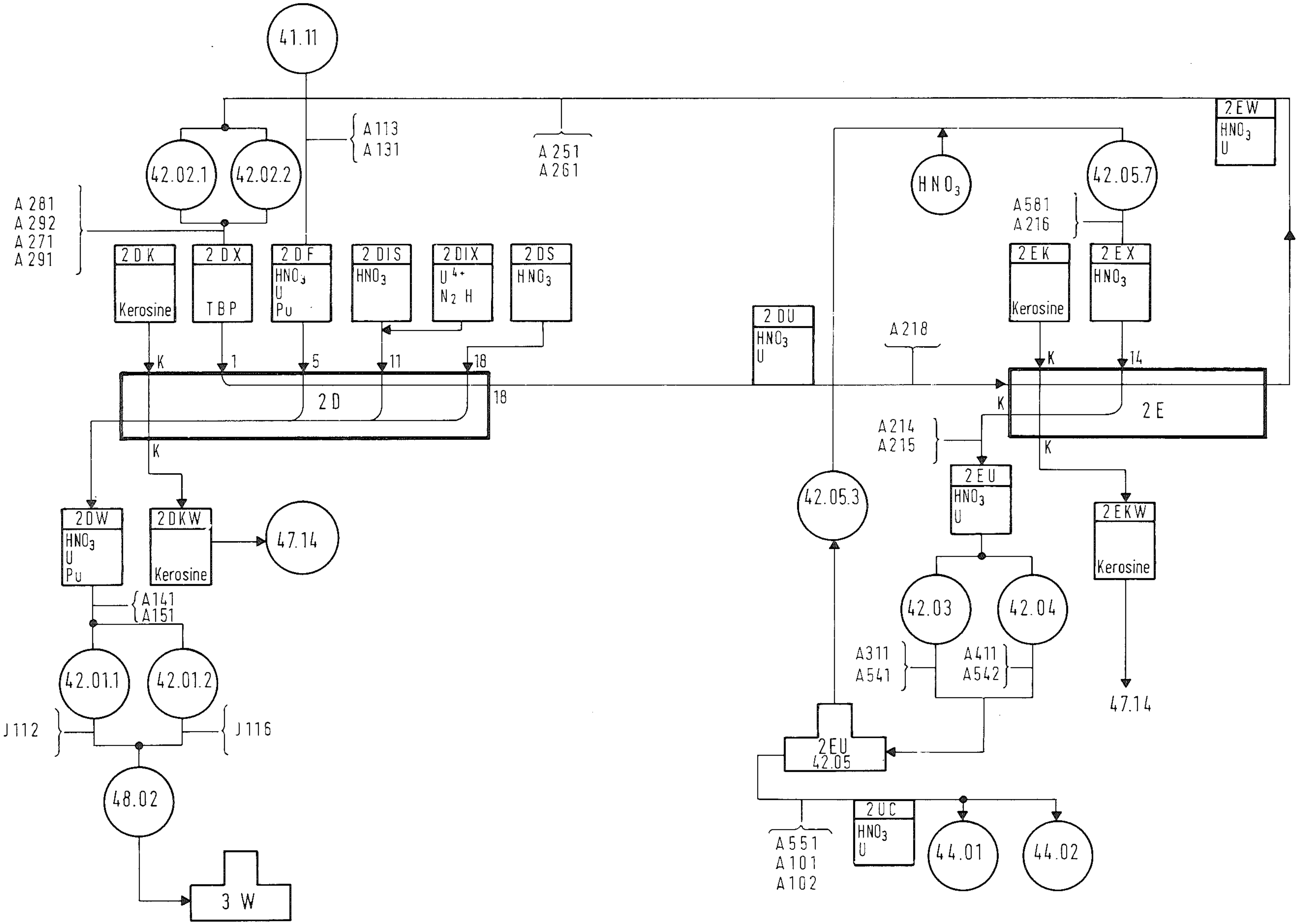


Fig. 2: The 2nd uranium cycle of the WAK reprocessing plant.

The most currently used method of liquid transfer control consists in recording the change in terms of time of the container filling levels by means of dip tubes. In certain pipes the liquid flow can be measured directly; furthermore, the air flow of the airlifts and the level between the aqueous and organic solutions in the mixer-settlers can be observed.

2.1. First example

The process state illustrated in figure 3 will serve as an example of mathematical analysis: If the transfer of the 2DW-stream is disturbed, the liquid level in the container 42.01.1 will vary less than it should. Furthermore, the separating layer in the 2D mixer-settler will be increased until

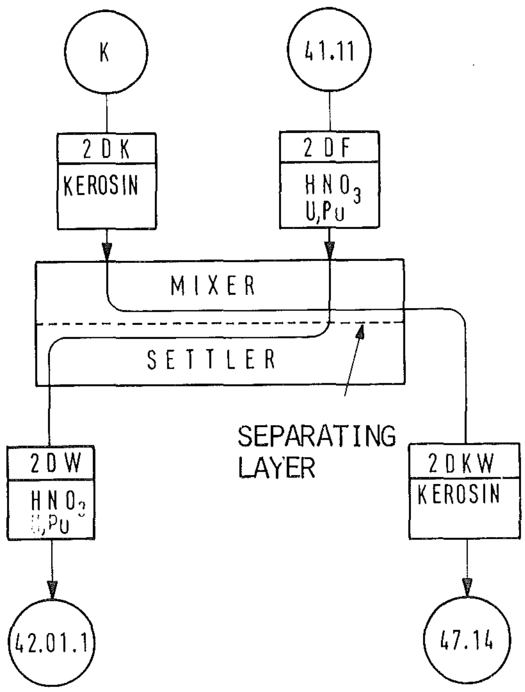


Fig. 3: Process section of the 2nd uranium cycle used for mathematical analysis.

the level is reached of the point where the 2DK-stream leaves the mixer-settler. Then, together with the 2DKW-stream, that part of the DW-stream will also leave the mixer-settler, which cannot enter the container 42.01.1 because of the disturbance. The consequence is that the variation of the liquid level in the container 47.14 is greater than it should be.

On account of the restricted measurement devices available at the time being, three types of information can be used to diagnose disturbances: the variations of the liquid levels in the containers 42.01.1 and 47.14, and the position of the separating layer - the latter will be an indication of the beginning of a disturbance, expressed by a steady increase from its regular position. Due to technical circumstances, this disturbance may have three causes: either a wrong adjustment of the air pressure, which influences the flow rate of the 2DW-stream, or a blocking of either the outlet of the 2D mixer-settler or of the airlift 141/151 (see also figure 4).

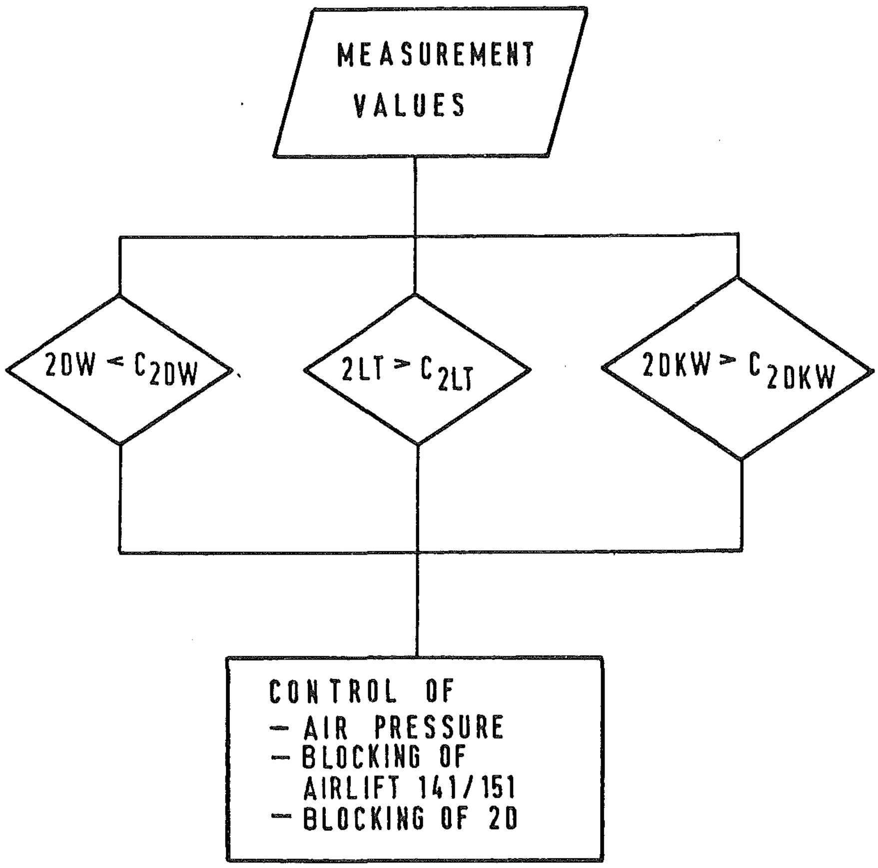


Fig. 4: Scheme of anomalous process state related to the process section of figure 3.

2.2. Second example

For demonstration figure 5 presents another process state which is related to the operation of the 2EU vaporizer belonging to the 2nd uranium cycle. This vaporizer concentrates the purified uranium product, whose density d must be kept at a constant value c_d. For instance, if this density is too high, the following reasons might contribute to this anomalous process behaviour:

- The 3EU feed flow is too small.
- The removal of the 2UC concentrate is too slow.
- The temperature 'T_{2UC}' of the pipes carrying the 2UC-stream is too low.
- The temperature 'T_{2EU}' within the vaporizer has a moderate value only.

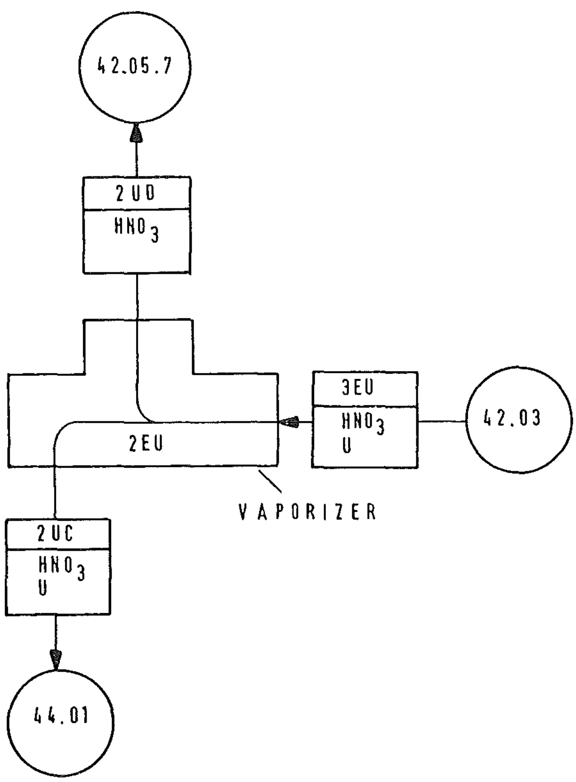


Fig. 5: Process section of the 2nd uranium cycle used for demonstration of a process state.

These disturbances can be induced by blocking of the corresponding air-lifts and/or tubing as well as by wrong adjustments of the heating systems (electric and steam heating). The relations are presented in figure 6.

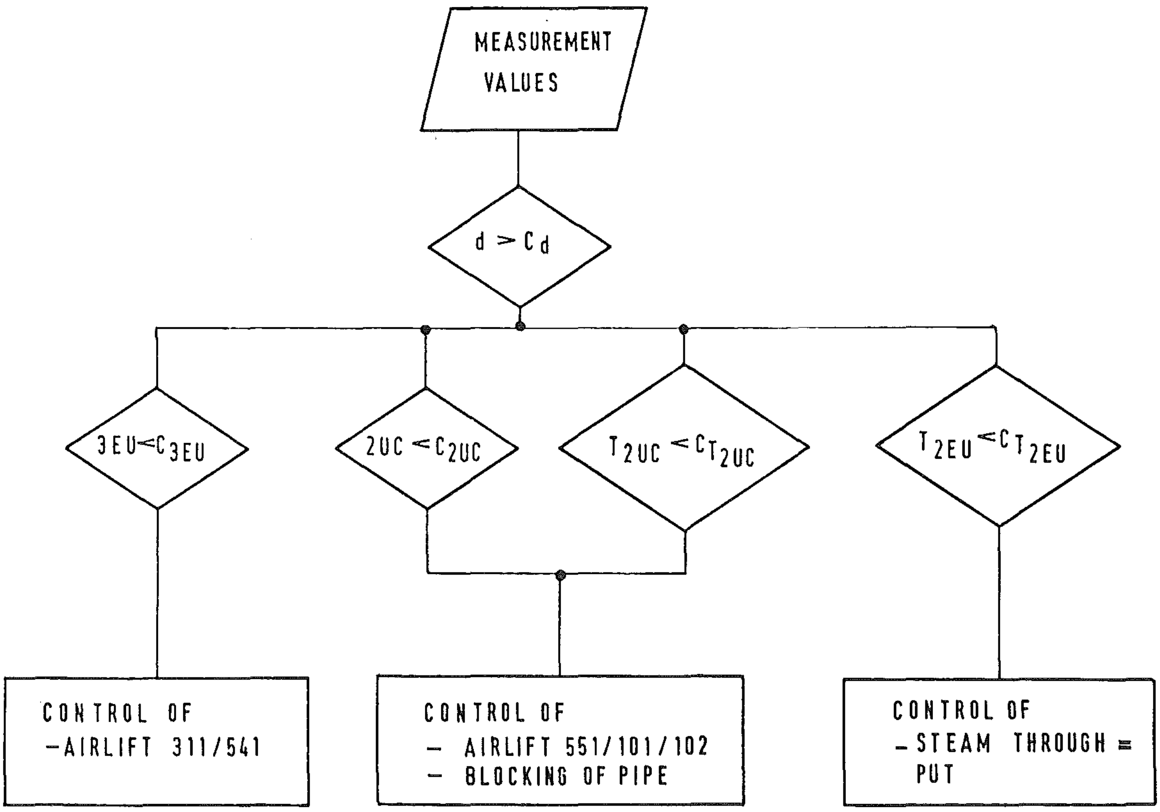


Fig. 6: Scheme of anomalous process state related to the process section of figure 5.

2.3. Procedure for the early detection of anomalous process states

If the operator in the central control room recognizes that the signals deviate from their nominal values, and he, consequently, has to decide whether or not he shall take an action, he must consider two aspects:

- The signals could deviate from their nominal values just because of measurement errors or process fluctuations not involving difficulties; thus, a shutdown would be false ('error of the first kind') and reduce the availability of the plant.

- Because of this possibility of a false alarm the operator could hesitate and take no action even if there were a disturbance ('error of the second kind'); too late an action, however, would cause major technical difficulties and again reduce the availability of the plant.

Therefore, the problem arises to choose appropriate *significance thresholds* for signal deviations from their nominal values, above which the operator must take an action. These thresholds are determined by the choice of appropriate *false alarm probabilities* (probability of error of the first kind) which, ultimately, must be determined with the help of availability considerations. A schematical representation of these relations is given in figure 7.

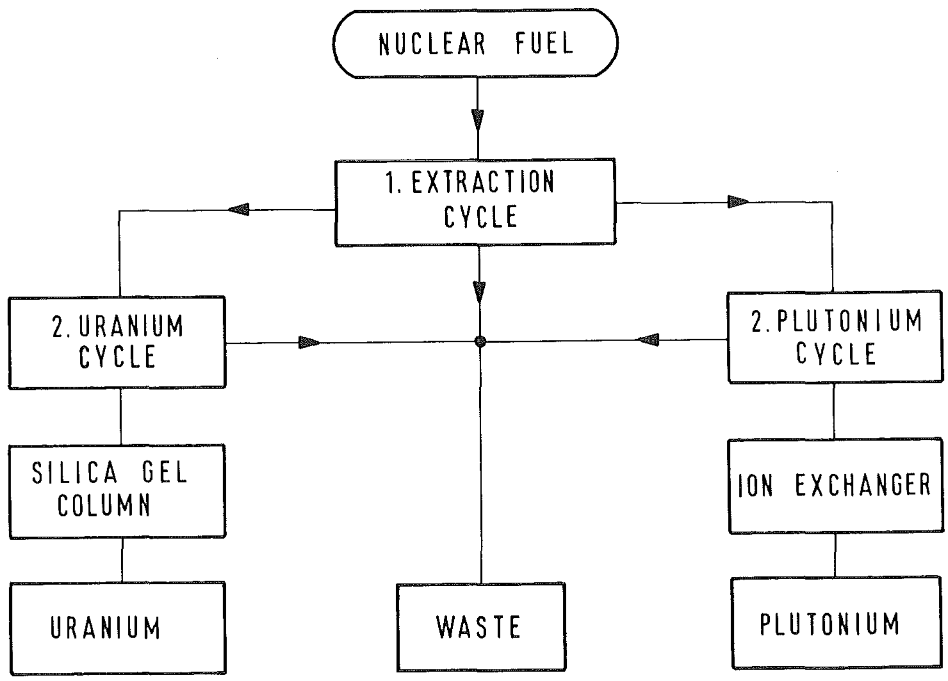


Fig. 7: Procedural model for the early detection of anomalous process states.

3. Mathematical analysis

In the following paragraphs a detailed mathematical analysis will be given using the first example in chapter 2; the analysis of the second example in chapter 2 will just be outlined at the end of this chapter.

3.1. Analysis of the first example

Let us make the following assumptions:

i) The 2DF-stream has a fixed value and must not be checked at all.

ii) The 2DW-stream may be disturbed; it can, however, not be checked directly.

iii) The 2DK-stream, now called X_1, is subject to process variations which are, however, not considered to be disturbances; we write

$$X_1 := 2DK = \mu_{DK}+e_{DK}+d$$

where μ_{DK} is the nominal value, e_{DK} the measurement error, and d the process variation; furthermore

$$\mathrm{var}(e_{DK}) =: \sigma^2_{DK} , \quad \mathrm{var}(d) =: \sigma^2_P , \quad \mathrm{cov}(e_{DK},d) = 0 .$$

iv) A process variation of 2DK influences directly 2DKW, now called X_2. We write for the normal state (*null hypothesis* H_0),

$$X_2 := 2DKW = \mu_{DKW}+e_{DKW}+d \quad \text{under} \quad H_0 .$$

A decrease of the true value of 2DW by the value Δ leads to an increase of the true value of 2DKW by the same value Δ (*alternative hypothesis* H_1)

$$X_2 := 2DKW = \mu_{DKW}+\Delta+e_{DKW}+d \quad \text{under} \quad H_1 .$$

In both cases we write

$$\mathrm{var}(e_{DKW}) =: \sigma^2_{DKW} , \quad \mathrm{cov}(e_{DKW},d) = 0 .$$

v) The position of the separating layer LT, now called X_3, is proportionally influenced by process variations. We write for the normal state

$$X_3 = \mu_{LT}+e_{LT}+a\cdot d \quad \text{under} \quad H_O \ .$$

Furthermore, we assume that in case 2DKW is increased by Δ, X_3 increases proportionally:

$$X_3 = \mu_{LT}+b\cdot\Delta+e+a\cdot d \quad \text{under} \quad H_1 \ .$$

In both cases we write

$$\mathrm{var}(e_{LT}) =: \sigma_{LT}^2 \ , \qquad \mathrm{cov}(e_{LT},d) = 0 \ .$$

vi) The random variables e_{DK}, e_{DKW}, e_{LT} and d are normally distributed with zero expectation values and known variances.

Because of these assumptions the random variables X_1, X_2, X_3 are normally distributed, i.e.,

$$\mathrm{prob}\{X_1\le x_1\} = \int_{-\infty}^{x_1} f_{X_1}(t)\,dt = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_{DK}^2+\sigma_P^2}} \cdot \int_{-\infty}^{x_1} \exp\left[-\frac{1}{2}\cdot\frac{(t-\mu_{DK})^2}{\sigma_{DK}^2+\sigma_P^2}\right] dt$$

$$\mathrm{prob}\{X_2\le x_2|H_i\} = \int_{-\infty}^{x_2} f_{X_2}(t)\,dt = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_{DKW}^2+\sigma_P^2}} \cdot \int_{-\infty}^{x_2} \exp\left[-\frac{1}{2}\cdot\frac{(t-E_i(X_2))^2}{\sigma_{DKW}^2+\sigma_P^2}\right] dt$$

$$\mathrm{prob}\{X_3\le x_3|H_i\} = \int_{-\infty}^{x_3} f_{X_3}(t)\,dt = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_{DKW}^2+\sigma_P^2}} \cdot \int_{-\infty}^{x_3} \exp\left[-\frac{1}{2}\cdot\frac{(t-E_i(X_3))^2}{\sigma_{LT}^2+\sigma_P^2}\right] dt \ ,$$

$$i=0,1,$$

where the expectation values of X_2 and X_3 under H_O are

$$(E_O(X_2),\ E_O(X_3)) = (\mu_{DKW},\ \mu_{LT}) \ ,$$

and, furthermore, under H_1,

$$(E_1(X_2),\ E_1(X_3)) = (\mu_{DKW}+\Delta,\ \mu_{LT}+b\cdot\Delta) \ .$$

Using the well known symbol $\phi(.)$ for the normal distribution function,

$$\phi(x) := \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{x} \exp(-\frac{t^2}{2})dt$$

we write this as follows

$$\text{prob}\{X_1 \le x_1\} = \phi\left(\frac{x_1-\mu_{DK}}{\sqrt{\sigma_{DK}^2+\sigma_P^2}}\right)$$

$$\text{prob}\{X_2 \le x_2\} = \phi\left(\frac{x_2-E_i(X_2)}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}\right)$$

$$\text{prob}\{X_3 \le x_3\} = \phi\left(\frac{x_3-E_i(X_3)}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}\right), \quad i=0,1 \quad .$$

Because of the process variation the random variables X_1, X_2 and X_3 are not independent. The joint density $f_i(\underline{x})$ of the random vector $\underline{X}'=(X_1,X_2,X_3)$ under the hypotheses H_i, $i=0,1$, defined by

$$\text{prob}\{X_1 \le x_1,\ X_2 \le x_2,\ X_3 \le x_3 | H_i\} = \int_{-\infty}^{x_1} dt_1 \int_{-\infty}^{x_2} dt_2 \int_{-\infty}^{x_3} dt_3\ f_i(\underline{t}),\ i=0,1 \ ,$$

is given by the expression[1])

$$f_i(\underline{x}) = (2\pi)^{-\frac{3}{2}} \cdot |\underline{\underline{\Sigma}}|^{-\frac{3}{2}} \cdot \exp\left[-\frac{1}{2}(\underline{x}-\underline{\mu}_i)' \cdot \underline{\underline{\Sigma}}^{-1} \cdot (\underline{x}-\underline{\mu}_i)\right] \quad , \quad i=0,1 \ ,$$

where the *covariance matrix* $\underline{\underline{\Sigma}}$ is given by

---

[1]) In ANNEX it has been shown that the common distribution of *two* normally distributed random variables with non-vanishing correlation is a general bivariate normal distribution. For *three* normally distributed random variables with non-vanishing correlations similar considerations can be made which leads to the expression given here.

$$\underline{\underline{\Sigma}} = \begin{pmatrix} \sigma_1^2 & \rho_{12}\cdot\sigma_1\cdot\sigma_2 & \rho_{13}\cdot\sigma_1\cdot\sigma_3 \\ \rho_{12}\cdot\sigma_1\cdot\sigma_2 & \sigma_2^2 & \rho_{23}\cdot\sigma_2\cdot\sigma_3 \\ \rho_{13}\cdot\sigma_1\cdot\sigma_3 & \rho_{23}\cdot\sigma_2\cdot\sigma_3 & \sigma_3^2 \end{pmatrix}$$

where the matrix elements are defined as follows

$$\sigma_1^2 := \mathrm{var}(X_1) = \sigma_{DK}^2+\sigma_P^2 , \qquad \rho_{12}\cdot\sigma_1\cdot\sigma_2 := \mathrm{cov}(X_1,X_2) = \sigma_P^2 ,$$

$$\sigma_2^2 := \mathrm{var}(X_2) = \sigma_{DKW}^2+\sigma_P^2 , \qquad \rho_{13}\cdot\sigma_1\cdot\sigma_3 := \mathrm{cov}(X_1,X_3) = a\cdot\sigma_P^2 ,$$

$$\sigma_3^2 := \mathrm{var}(X_3) = \sigma_{LT}^2+a^2\cdot\sigma_P^2 , \qquad \rho_{23}\cdot\sigma_2\cdot\sigma_3 := \mathrm{cov}(X_2,X_3) = a\cdot\sigma_P^2 ,$$

and where the expectation vectors $\underline{\mu}_i$ under the hypotheses H_i, i=0,1, are given by

$$\underline{\mu}_0' := (\mu_{DK},\mu_{DKW},\mu_{LT})$$

$$\underline{\mu}_1' := \underline{\mu}_0'+\underline{\Delta}, \text{ where } \underline{\Delta} := (0,\Delta,b\cdot\Delta) .$$

Elementary Test Procedure

Let us assume that we use only the measured value of the 2DKW-stream in order to decide whether or not there is a disturbance of the 2DKW-stream. This is an intuitive approach and it can be proven formally that the best decision procedure in decision making is:

2DKW≤s: no disturbance
2DKW>s: disturbance

We call s the *significance threshold* of the test.

We call the statement 'no disturbance' the *null hypothesis* H_0 and describe it according to our assumptions by

$$H_0: E(DKW) = \mu_{DKW} .$$

Furthermore, we call the statement 'disturbance' the *alternative hypothesis* H_1 and describe it by

$$H_1: E(DKW) = \mu_{DKW}+\Delta ,$$

where Δ is a parameter of the problem.

If we call the region 2DKW≤s the *acceptance region* A and the region 2DKW>s the *rejection region* K, we can represent the test procedure as follows

$$2DKW \leq s: \quad H_0 \text{ correct}$$

$$2DKW > s: \quad H_1 \text{ correct} .$$

In this way two errors may be committed: We call the error that we decide 'H_1 to be correct' if in fact H_0 is correct, the *error of the first kind,* and the error that we decide 'H_0 to be correct' if in fact H_1 is correct, the error of the second kind. The corresponding probabilities

$$\alpha := \text{prob}\left\{2DKW \leq s \,\middle|\, H_0\right\} = \phi\left(\frac{s-\mu_{DKW}}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}\right)$$

$$\beta := \text{prob}\left\{2DKW > s \,\middle|\, H_0\right\} = \phi\left(\frac{\mu_{DKW}-\Delta-s}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}\right)$$

are generally called probabilities of the first and second kind. In our case, we call for intuitive reasons α the *false alarm probability* and $1-\beta$ the *probability of detection.* In total, four outcomes are possible the probabilities of which are listed in the following table

decision \ reality	H_0 correct	H_1 correct
H_0 correct	$1-\alpha$	β
H_1 correct	α	$1-\beta$

For example, if in reality H_0 is correct, then the decision 'H_0 correct' will be made with probability $1-\alpha$; however, if in reality H_1 is correct, then the decision 'H_0 correct' will be made with probability β.

It is common practice to fix the value of the significance threshold s by postulating a value of the false alarm probability α; we get

$$s = \sqrt{\sigma_{DKW}^2+\sigma_P^2} \cdot U_{1-\alpha} + \mu_{DKW} ,$$

where U. is the inverse of the normal distribution function.

In the simple case discussed here the test is completely determined by postulating the value of α; in fact, we get the following expression for the probability of detection if we eliminate s with the help of α:

$$1-\beta = \phi\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}} - U_{1-\alpha}\right) .$$

The probability of detection as a function of Δ, $\sigma_{DKW}^2+\sigma_P^2$ and α describes the *efficiency* of the test procedure. As can be seen immediately, $1-\beta$ increases with increasing Δ and α, and it decreases with increasing $\sigma_{DKW}^2+\sigma_P^2$.

Neyman Pearson Test

The optimal test in the sense of guaranteeing the highest probability of disturbance detection for a fixed false alarm probability is given by the Neyman Pearson test /5/. The *critical region* K of this test, i.e., that region of $\underline{x}$-values whose realization leads to an acceptance of the alternative hypothesis H_1, is defined by the set

$$K := \left\{\underline{x} = (x_1,x_2,x_3): \frac{f_1(\underline{x})}{f_O(\underline{x})} > k\right\} ,$$

where k has to be determined in such a way that the false alarm probability defined by

$$\alpha := \text{prob}\left\{\underline{x}\in K \middle| H_O\right\}$$

does not exceed a given value.

Explicitly, the critical region of the Neyman Pearson test is determined as follows: As the inequality

$$\frac{f_1(\underline{x})}{f_O(\underline{x})} > k$$

is equivalent to

$$-\frac{1}{2}\cdot(\underline{x}-\underline{\mu}_1)'\cdot\underline{\underline{\Sigma}}^{-1}\cdot(\underline{x}-\underline{\mu}_1)+\frac{1}{2}\cdot(\underline{x}-\underline{\mu}_0)\cdot\underline{\underline{\Sigma}}^{-1}\cdot(\underline{x}-\underline{\mu}_0) > \ln k\ ,$$

which is, furthermore, equivalent to

$$\frac{1}{2}\cdot\underline{x}\cdot\underline{\underline{\Sigma}}^{-1}\cdot\underline{\mu}_1+\frac{1}{2}\cdot\underline{\mu}_1'\cdot\underline{\underline{\Sigma}}^{-1}\cdot\underline{x}-\frac{1}{2}\cdot\underline{x}'\cdot\underline{\underline{\Sigma}}^{-1}\cdot\underline{\mu}_0-\frac{1}{2}\cdot\underline{\mu}_0\cdot\underline{\underline{\Sigma}}^{-1}\cdot\underline{x}$$

$$> \ln k+\frac{1}{2}\cdot\underline{\mu}_1'\cdot\underline{\underline{\Sigma}}^{-1}\cdot\underline{\mu}_1-\frac{1}{2}\cdot\underline{\mu}_0'\cdot\underline{\underline{\Sigma}}^{-1}\cdot\underline{\mu}_0\ ,$$

which is, finally, equivalent to

$$\underline{x}'\cdot\underline{\underline{\Sigma}}^{-1}\cdot(\underline{\mu}_1-\underline{\mu}_0) > k'\ ,$$

where k' is given by

$$\ln k+\frac{1}{2}\cdot\underline{\mu}_1'\cdot\underline{\underline{\Sigma}}^{-1}\cdot\underline{\mu}_1-\frac{1}{2}\cdot\underline{\mu}'\cdot\underline{\underline{\Sigma}}^{-1}\cdot\underline{\mu}_0\ ,$$

the critical region of the Neyman Pearson test with

$$\underline{\mu}_1-\underline{\mu}_0=\underline{\Delta}=(0,\Delta,b\cdot\Delta)$$

is given by the region

$$K=\left\{\underline{x}=(x_1,x_2,x_3):\ \underline{x}'\cdot\underline{\underline{\Sigma}}^{-1}\cdot\underline{\Delta} > k'\right\}\ .$$

Using the explicit form of $\underline{\underline{\Sigma}}^{-1}$, we get the following critical region:

$$\begin{aligned}K=\{\underline{x}:\ &\sigma_2\cdot\sigma_3\cdot[\sigma_3\cdot(-\rho_{12}+\rho_{13}\cdot\rho_{23})+b\cdot\sigma_2\cdot(\rho_{12}\cdot\rho_{13}-\rho_{23})]\cdot x_1+\\&+\sigma_1\cdot\sigma_3\cdot[\sigma_3\cdot(1-\rho_{13}^2)+b\cdot\sigma_2\cdot(-\rho_{23}+\rho_{12}\cdot\rho_{13})]\cdot x_2+\\&+\sigma_1\cdot\sigma_2\cdot[\sigma_3\cdot(-\rho_{23}+\rho_{12}\cdot\rho_{13})+b\cdot\sigma_2\cdot(1-\rho_{12}^2)]\cdot x_3\geq k''\}\ ,\end{aligned}$$

where k" differs from k' by a constant factor only. As this set does not contain the value of Δ, this test is a *uniformly most powerful test*.

On the basis of our assumptions the test statistics defining the critical region is a normally distributed random variable; therefore, it would not be difficult, in principle, to determine the relation between the false alarm probability α and the significance threshold k". As the determination of the variance, however, leads to long algebraic formulas, we will discuss here only the special case a=b=1 which in practice might be achieved by appropriate scale transformations.

Returning to the process related nomenclature, the test statistics in this case is reduced to

$$Z := -\sigma_P^2\cdot(\sigma_{LT}^2+\sigma_{DKW}^2)\cdot 2DK+\sigma_{LT}^2\cdot(\sigma_{DK}^2+\sigma_P^2)\cdot 2DKW+\sigma_{DKW}^2\cdot(\sigma_{DK}^2+\sigma_P^2)\cdot LT \ .$$

According to our assumptions, the expectation value of this statistics is

$$E_O(Z) = -\sigma_P^2\cdot(\sigma_{LT}^2+\sigma_{DKW}^2)\cdot\mu_{DK}+\sigma_{LT}^2\cdot(\sigma_{DK}^2+\sigma_P^2)\cdot\mu_{DKW}+\sigma_{DKW}^2\cdot(\sigma_{DK}^2+\sigma_P^2)\cdot\mu_{LT}$$

under H_O ,

and furthermore

$$E_1(Z) = -\sigma_P^2\cdot(\sigma_{LT}^2+\sigma_{DKW}^2)\cdot\mu_{DK}+\sigma_{LT}^2\cdot(\sigma_{DK}^2+\sigma_P^2)\cdot(\mu_{DKW}+\Delta)+\sigma_{DKW}^2\cdot(\sigma_{DK}^2+\sigma_P^2)\cdot(\mu_{LT}+\Delta)$$

under H_1 .

The variance of Z can be written conveniently as

$$\begin{aligned}\mathrm{var}(Z) = \mathrm{var}[&\sigma_P^2\cdot(\sigma_{LT}^2+\sigma_{DKW}^2)\cdot e_{DK}+\sigma_{LT}^2\cdot(\sigma_{DK}^2+\sigma_P^2)\cdot e_{DKW}+\sigma_{DKW}^2\cdot(\sigma_{DK}^2+\sigma_P^2)\cdot e_{LT}\\ &+(\sigma_P^2\cdot(\sigma_{LT}^2+\sigma_{DKW}^2)+\sigma_{LT}^2\cdot(\sigma_{DK}^2+\sigma_P^2)+\sigma_{DKW}^2\cdot(\sigma_{DK}^2+\sigma_P^2))\cdot d]\end{aligned}$$

which leads to the following expression:

$$\mathrm{var}(Z) = (\sigma_P^2+\sigma_{DK}^2)\cdot(\sigma_{LT}^2+\sigma_{DKW}^2)[(\sigma_{LT}^2+\sigma_{DKW}^2)\cdot\sigma_P^2\cdot\sigma_{DK}^2+(\sigma_{DK}^2+\sigma_P^2)\cdot\sigma_{LT}^2\cdot\sigma_{DKW}^2] \ .$$

According to the definition of the critical region we have

$$1-\alpha = \mathrm{prob}\left\{\frac{Z-E_O(Z)}{\sqrt{\mathrm{var}(Z)}} \le \frac{k'-E_O(Z)}{\sqrt{\mathrm{var}(Z)}}\right\} = \phi\left(\frac{k'-E_O(Z)}{\sqrt{\mathrm{var}(Z)}}\right) \ ,$$

and furthermore,

$$\beta = \text{prob}\left\{\frac{Z-E_1(Z)}{\sqrt{\text{var}(Z)}} \le \frac{k'-E_1(Z)}{\sqrt{\text{var}(Z)}}\right\} = \phi\left(U_{1-\alpha} - \frac{E_1(Z)-E_O(Z)}{\sqrt{\text{var}(Z)}}\right) .$$

Using the relation

$$E_1(Z) - E_O(Z) = (\sigma_{DK}^2+\sigma_P^2)\cdot(\sigma_{LT}^2+\sigma_{DKW}^2)\cdot\Delta ,$$

we therefore finally get the following expression for the probability $1-\beta$ of detecting a disturbance Δ with a given false alarm probability α

$$1-\beta = \phi\left(\frac{\Delta}{\sqrt{\dfrac{\sigma_{LT}^2}{\sigma_{LT}^2+\sigma_{DKW}^2}\cdot\sigma_{DKW}^2+\dfrac{\sigma_P^2}{\sigma_P^2+\sigma_{DK}^2}\cdot\sigma_{DK}^2}} - U_{1-\alpha}\right) .$$

Let us consider some special cases.

If the uncertainty of the position of the separating layer is much greater than the uncertainty of the measurement of 2DKW, i.e., if $\sigma_{LT}^2 \gg \sigma_{DKW}^2$, we get the following expression for the detection probability

$$1-\beta = \phi\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\dfrac{\sigma_P^2}{\sigma_P^2+\sigma_{DK}^2}\cdot\sigma_{DK}^2}} - U_{1-\alpha}\right) .$$

Now, as we have

$$\text{var}\left(2DKW - \frac{\sigma_P^2}{\sigma_P^2+\sigma_{DK}^2}\cdot 2DK\right) = \sigma_{DKW}^2 + \frac{\sigma_P^2}{\sigma_{DK}^2+\sigma_P^2}\cdot\sigma_{DK}^2 ,$$

this means that the Neyman Pearson test statistics is given by

$$2DKW - \frac{\sigma_P^2}{\sigma_P^2+\sigma_{DK}^2}\cdot 2DK .$$

If we compare this probability of detection with that for the test which uses only the 2DKW signal, we see that the latter is smaller than the first because of

$$\sigma^2_{DKW} + \frac{\sigma^2_P}{\sigma^2_P+\sigma^2_{DK}} \cdot \sigma^2_{DK} < \sigma^2_{DKW}+\sigma^2_P \quad ,$$

although the disturbance of the 2DKW-stream does not have any influence on the 2DK-stream. The reason for this is the process variation of the 2DK-stream which influences also the 2DKW-stream: The use of the 2DK-signal allows a sharper discrimination between process variation and disturbances of the 2DKW-stream.

Furthermore, because of

$$\sigma^2_{DKW}+ \frac{\sigma^2_P}{\sigma^2_P+\sigma^2_{DK}} \cdot \sigma^2_{DK} = \begin{cases} \sigma^2_{DKW}+\sigma^2_{DK} & \text{for } \sigma^2_{DK}<<\sigma^2_P \\ \sigma^2_{DKW}+\sigma^2_P & \phantom{\text{for }} \sigma^2_{DK}>>\sigma^2_P \end{cases}$$

we get the following intuitive result:

i) In case the measurement error of 2DK is small compared to the process variation, we use the *difference* 2DKW-2DK as test statistics; in this way the process variation is completely eliminated.

ii) In the opposite case *simply* 2DKW is taken as test statistics.

If the process variation is large compared to the measurement uncertainty of 2DK, i.e., if $\sigma^2_P>>\sigma^2_{DK}$, we get for the detection probability

$$1-\beta = \phi\left(\frac{\Delta}{\sqrt{\frac{\sigma^2_{LT}}{\sigma^2_{LT}+\sigma^2_{DKW}} \cdot \sigma^2_{DKW}+\sigma^2_{DK}}} - U_{1-\alpha} \right) ,$$

which means that in this case the Neyman Pearson test statistics is given by

$$\frac{\sigma^2_{LT}}{\sigma^2_{LT}+\sigma^2_{DKW}} \cdot 2DKW+ \frac{\sigma^2_{DKW}}{\sigma^2_{LT}+\sigma^2_{DKW}} \cdot LT - 2DK \quad .$$

Multivariate Test

The Neyman Pearson test does not give any information about significant deviations of the *single* process signals from their nominal values. There may be reasons, however, to get this information which is obtained - at the expense of the overall probability of detection - by performing tests on the single signals. We will analyse this multivariate test procedure for the case of the 2DK- and the 2DKW-stream, thus neglecting the separating layer signal LT in this section.

The test procedure is defined by the following requirement. If

$$2DKW \le s_{DKW} \wedge 2DK \ge s_{DK}$$

$$\text{or} \qquad 2DKW \le s_{DKW} \wedge 2DK < s_{DK}$$

$$\text{or} \qquad 2DKW > s_{DKW} \wedge 2DK \ge s_{DK}$$

then it is decided that there is no disturbance, otherwise, i.e., if

$$2DKW > s_{DKW} \wedge 2DK < s_{DK} ,$$

it is decided that there is a disturbance.

The two significance thresholds s_{DKW} and s_{DK} are related to the single false alarm probabilities α_1 and α_2 according to

$$1-\alpha_1 := \text{prob}\{2DKW \le s_{DKW} | H_O\} = \phi\left(\frac{s_{DKW}-\mu_{DKW}}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}\right)$$

$$1-\alpha_2 := \text{prob}\{2DK > s_{DK} | H_O\} = 1-\phi\left(\frac{s_{DK}-\mu_{DK}}{\sqrt{\sigma_{DK}^2+\sigma_P^2}}\right) .$$

The total false alarm probability α, which is defined by

$$\alpha := \text{prob}\{DKW>s_{DKW} \wedge DK<s_{DK} | H_O\} ,$$

is related to the single false alarm probabilities by

$$\alpha = \frac{1}{2\pi\cdot\sqrt{1-\rho^2}} \cdot \int_{-\infty}^{U\alpha_1} dt_1 \int_{-\infty}^{U\alpha_2} dt_2 \, \exp\left[-\frac{t_1^2-2\rho t_1 t_2+t_2^2}{2(1-\rho^2)}\right] ,$$

where ρ stands for the negative correlation of the 2DKW- and the 2DK-stream:

$$\rho := -\frac{\mathrm{cov}(2DKW,2DK)}{\sqrt{\mathrm{var}(2DKW)\cdot\mathrm{var}(2DK)}} = -\frac{\sigma_P^2}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}\cdot\sqrt{\sigma_{DK}^2+\sigma_P^2}} .$$

Before going on, let us consider some special cases to which this relation applies. For a vanishing process fluctuation, i.e., $\sigma_P^2=0$ we have

$$\alpha = \alpha_1\cdot\alpha_2 \quad \text{for} \quad \rho=0 .$$

In addition the properties of the bivariate distribution (s. also /6/) yield the limits:

$$\left.\begin{array}{ll} \min(\alpha_1,\alpha_2) \geq \alpha \geq \alpha_1\cdot\alpha_2 & \rho\geq 0 \\ & \text{for} \\ \alpha_1\cdot\alpha_2 \geq \alpha \geq \alpha_1+\alpha_2-1 & \rho<0 \end{array}\right.$$

(our purposes are only served by the second pair of inequalities).

Finally, we have

$$\alpha = \begin{cases} \alpha_1 & \alpha_2=1 \\ & \text{for} \\ \alpha_2 & \alpha_1=1 \end{cases} .$$

Figure 8 gives a graphical representation of the relation between α, α_1, α_2 and ρ for a total false alarm probability $\alpha=0.1$ and selected values of the parameter ρ.

For a disturbance Δ the probability of detection $1-\beta(\Delta)$ is defined by

$$1-\beta(\Delta) := \mathrm{prob}\{2DKW>s_{DKW} \wedge 2DK<s_{DK} | H_1\} ,$$

which leads to the following expression

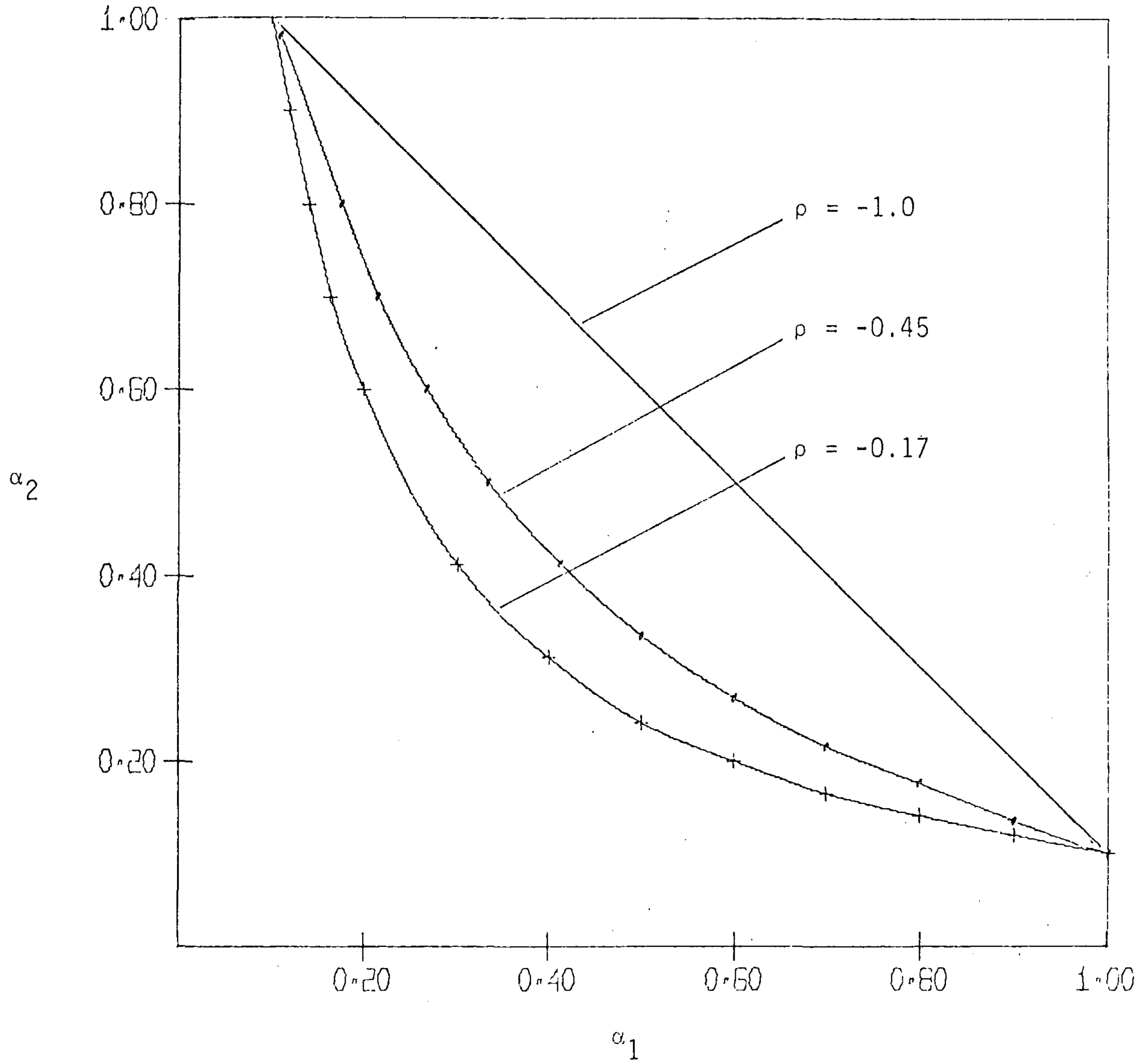


Fig. 8: Presentation of the relation between α, α_1, α_2, and ρ false alarm probability α=0.1 and selected values of the parameter ρ.

$$1-\beta(\Delta) = \frac{1}{2\pi\cdot\sqrt{1-\rho^2}}\cdot\int_{-\infty}^{\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}-U_{1-\alpha_1}} dt_1 \int_{-\infty}^{U\alpha_2} dt_2 \exp\left[-\frac{t_1^2-2\rho t_1 t_2+t_2^2}{2\cdot(1-\rho^2)}\right] .$$

For $\sigma_P^2=0$ we obtain

$$1-\beta(\Delta) = \phi\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}-U_{1-\alpha_1}\right)\cdot\alpha_2 .$$

In addition the following limits are valid:

$$\min\left(\phi\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}-U_{1-\alpha_1}\right),\ \alpha_2\right) \geq 1-\beta(\Delta) \geq \phi\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}-U_{1-\alpha_1}\right)\cdot\alpha_2 \qquad \rho\geq 0$$

for

$$\phi\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}-U_{1-\alpha_1}\right)\cdot\alpha_2 \geq 1-\beta(\Delta) \geq \phi\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}-U_{1-\alpha_1}\right)+\alpha_2-1 \qquad \rho\leq 0 .$$

Finally, we have

$$\lim_{\Delta\to\infty} 1-\beta(\Delta) = \frac{1}{2\pi\cdot\sqrt{1-\rho^2}}\cdot\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{U\alpha_2} dt_2 \exp\left[-\frac{1}{2}\cdot\frac{t_1^2-2\rho t_1 t_2+t_1^2}{1-\rho^2}\right] = \alpha_2 .$$

In the attempt to get an optimal availability of the process for a given value of the overall false alarm probability α, the values of the single false alarm probabilities α_1 and α_2 have to be chosen in such a way that the overall probability of detection is maximized; i.e. the values of α_1 and α_2 must be obtained as solutions of the problem of maximization

$$\max_{\alpha_1,\alpha_2}(1-\beta(\alpha_1,\alpha_2))$$

where α_1 and α_2 are subject to the boundary condition of a fixed total false alarm probability.

We shall eliminate now α_2 by the total false alarm probability relation and look for the maximum of the overall probability of detection with respect to α_1.

Using for the derivation of a function of the type

$$F(x) = \int_{-\infty}^{g(x)} dt\ f(t,x)$$

the well-known formula

$$\frac{d}{dx} F(x) = f(g(x),x) \cdot \frac{d}{dx} g(x) + \int_{-\infty}^{g(x)} dt\ \frac{d}{dx} f(t,x) \quad ,$$

we get with the definition

$$\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}} =: a$$

the following expression for the derivation of $1-\beta$ with respect to α_1:

$$\frac{d}{d\alpha_1} (1-\beta) =$$

$$= \frac{d}{d\alpha_1}\left[\frac{1}{2\pi\cdot\sqrt{1-\rho^2}} \cdot \int_{-\infty}^{a+U\alpha_1} dt_1 \int_{-\infty}^{U\alpha_2} dt_2\ \exp\left[-\frac{1}{2}\cdot\frac{t_1^2-2\rho t_1 t_2+t_2^2}{1-\rho^2}\right]\right] =$$

$$= \frac{1}{2\pi\cdot\sqrt{1-\rho^2}} \cdot \int_{-\infty}^{U\alpha_2} dt_2\ \exp\left[-\frac{1}{2}\cdot\frac{(a+U\alpha_1)^2-2\rho(a+U\alpha_1)t_2+t_2^2}{1-\rho^2}\right] \cdot \frac{dU\alpha_1}{d\alpha_1} +$$

$$+ \frac{1}{2\pi \cdot \sqrt{1-\rho^2}} \cdot \int_{-\infty}^{a+U\alpha_1} dt_1 \; \exp\left[-\frac{1}{2} \cdot \frac{t_1^2 - 2\rho t_1 U\alpha_2 + U^2\alpha_2}{1-\rho^2}\right] \cdot \frac{dU\alpha_2}{d\alpha_1} =$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \exp\left[-\frac{(a+U\alpha_1)^2}{2}\right] \cdot \phi\left(\frac{U\alpha_2 - \rho \cdot (a+U\alpha_1)}{\sqrt{1-\rho^2}}\right) \cdot \frac{dU\alpha_1}{d\alpha_1} +$$

$$+ \frac{1}{\sqrt{2\pi}} \cdot \exp\left[-\frac{U^2\alpha_2}{2}\right] \cdot \phi\left(\frac{a+U\alpha_1 - \rho \cdot U\alpha_2}{\sqrt{1-\rho^2}}\right) \cdot \frac{dU\alpha_2}{d\alpha_2} \cdot \frac{d\alpha_2}{d\alpha_1} \quad .$$

Using the relation

$$\frac{dU\alpha}{d\alpha} = \sqrt{2\pi} \cdot \exp\left[\frac{U^2\alpha}{2}\right]$$

we get

$$\frac{d}{d\alpha_1}(1-\beta) = \exp\left[-\frac{\alpha^2}{2} - a \cdot U\alpha_1\right] \cdot \phi\left(\frac{U\alpha_2 - \rho \cdot (a+U\alpha_1)}{\sqrt{1-\rho^2}}\right) + \phi\left(\frac{a+U\alpha_1 - \rho \cdot U\alpha_2}{\sqrt{1-\rho^2}}\right) \cdot \frac{d\alpha_2}{d\alpha_1} \quad .$$

By implicit differentiation of the total false alarm relation we get

$$\phi\left(\frac{U\alpha_2 - \rho \cdot U\alpha_1}{\sqrt{1-\rho^2}}\right) + \phi\left(\frac{U\alpha_1 - \rho \cdot U\alpha_2}{\sqrt{1-\rho^2}}\right) \cdot \frac{d\alpha_2}{d\alpha_1} = 0 \quad ;$$

therefore, we finally get

$$\frac{d}{d\alpha_1}(1-\beta) = \exp\left[-\frac{a^2}{2} - a \cdot U\alpha_1\right] \cdot \phi\left(\frac{U\alpha_2 - \rho \cdot (a+U\alpha_1)}{\sqrt{1-\rho^2}}\right) - \phi\left(\frac{a+U\alpha_1 - \rho \cdot U\alpha_2}{\sqrt{1-\rho^2}}\right) \cdot \frac{\phi\left(\frac{U\alpha_2 - \rho \cdot U\alpha_1}{\sqrt{1-\rho^2}}\right)}{\phi\left(\frac{U\alpha_1 - \rho \cdot U\alpha_2}{\sqrt{1-\rho^2}}\right)} \quad .$$

For $\rho=0$ we get with $\alpha=\alpha_1\cdot\alpha_2$

$$\frac{d}{d\alpha_1}(1-\beta) = \exp\left(-\frac{a^2}{2} - a\cdot U\alpha_1\right)\cdot\alpha_2 - \phi(a+U\alpha_1)\cdot\frac{\alpha_2}{\alpha_1} =$$

$$= \frac{\alpha}{\alpha_1^2}\cdot\left[\exp - \frac{a^2}{2} - a\cdot U\alpha_1 \cdot\alpha_1 - \phi(a+U\alpha_1)\right] =$$

$$=: \frac{\alpha}{\alpha_1^2}\cdot f(\alpha_1)\ .$$

Now it can be shown that

$$\lim_{\alpha_1\to 0} f(\alpha_1) = 0 \quad \text{and} \quad \frac{d}{d\alpha_1} f(\alpha_1) < 0 \quad ,$$

which means that the optimum value of α_1 is given by the smallest possible value of α_1, which is

$$\alpha_1 = \alpha \quad \text{and therefore} \quad \alpha_2 = 1 \quad .$$

This can be understood immediately: $\rho=0$ means no process variation implying that the 2DK-stream has nothing to do with the 2DKW-stream; therefore, a disturbance of the 2DKW-stream is detected best by concentrating the testing efforts on the 2DKW-stream alone.

In the case $\rho<0$ numerical calculations indicate that the optimum value of α_1 again is given by the smallest possible value of α_1, i.e., $\alpha_1=\alpha$, $\alpha_2=1$. This result cannot be understood so easily; in the case of the Neyman Pearson test we found that the use of the 2DK signal improved the total probability of detection. Nevertheless, as already mentioned, there may be reasons not to use those α_1 and α_2 values which maximize the total probability of detection.

Discussion

Table 1 summarizes the testing procedures discussed in some detail above. With respect to the efficiency, i.e. the probability of detecting a disturb-

Table 1 : Summary of testing procedures discussed

Procedure	Probability to detect an anomalous process state $1-\beta(\Delta)$ [1]	False alarm probability	Advantages	Disadvantages
Checking of 2DKW	$\phi\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}-U_{1-\alpha}\right)$	α	Simple procedure; efficient if $\sigma_P^2 \ll \sigma_{DK}^2$	Provides no information on 2DK
Checking of 2DKW-2DK	$\phi\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_{DK}^2}}-U_{1-\alpha}\right)$	α	Relatively simple procedure; efficient, if $\sigma_{DK}^2 \ll \sigma_P^2$	Provides no separate information on 2DK and 2DKW
Parallel checking of 2DKW and 2DK	$B\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_P^2}}-U_{1-\alpha_1},U_{\alpha_2},\rho\right)$ [2]	$B(U_{\alpha_1},U_{\alpha_2},\rho)$	Provides separate information on 2DK and 2DKW	Data intense as well as arithmetically labourious procedure
Checking of $2DKW-\frac{\sigma_P^2}{\sigma_{DK}^2+\sigma_P^2}\cdot 2DK$ (Neyman Pearson)	$\phi\left(\frac{\Delta}{\sqrt{\sigma_{DKW}^2+\sigma_{DK}^2\cdot\frac{\sigma_P^2}{\sigma_{DK}^2+\sigma_P^2}}}-U_{1-\alpha}\right)$	α	With respect to the probability of detection the best 'parallel' procedure	Provides no separate information on 2DK and 2DKW

[1]) e.g. blocking of 2DW results in an increase by Δ of the 2DKW flow.

[2]) $$B(x,y,\rho) := \frac{1}{2\pi\cdot\sqrt{1-\rho^2}}\cdot\int_{-\infty}^{x} dt_1 \int_{-\infty}^{y} dt_2 \exp\left[-\frac{1}{2\cdot(1-\rho^2)}\cdot(t_1^2-2\rho t_1 t_2+t_2^2)\right].$$

ance (for a given probability of error), the Neyman Pearson procedure is much better than the other testing procedures mentioned. For demonstration, the next chapter will present some results obtained with numerical exercises.

The qualitative advantages and disadvantages are discussed in the last two columns of table 1. It should be pointed out that generally, the procedures of checking separately, the 2DKW- and the 2DK-stream provide specific information on the two streams. Finally, it should be mentioned that all procedures might be generalized. Of course, the first two as well as the Neyman-Pearson test benefit by a generalization to be performed quite easily. A generalization of the fourth procedure could induce arithmetic difficulties because multivariate distributions must then be used.

3.2. Outline of the analysis of the second example

Without giving the full analysis of the second example presented in the second chapter (2.2.) we will outline it below in order to point to the new aspects arising in a situation where more than one cause of a deviation from nominal values may exist.

Let us make the following assumptions

i) Under H_0 we have

$$X_1 := 3EU = \mu_{3EU} + e_{3EU} + d$$

$$X_2 := 2UC = \mu_{2UC} + e_{2UC} + d$$

$$X_3 := \rho = \mu_\rho + e_\rho + a \cdot d$$

$$X_4 := T_{2UC} = \mu_{T2UC} + e_{T2UC} + d$$

where 3EU, 2UC, ρ, and T_{2UC} are the measurement values and where μ, e, d, and a represent the nominal value, the measurement error, the process variation, and a constant factor, respectively.

ii) As already described for the second example (section 2.2.), an anomalous process behaviour might be induced through blocking of the airlifts 311/541 related to the transfer of the 3EU-stream. This blocking may result in a decrease of 3EU by Δ_1: therefore, under $H_1^{(1)}$ we have:

$$X_1 = 3EU = \mu_{3EU} - \Delta_1 + e_{3EU} + d ,$$

$$X_2 = 2UC = \mu_{2UC} - \Delta_1 + e_{2UC} + d ,$$

$$X_3 = \rho = \mu_\rho + b\cdot\Delta_1 + e_\rho + a\cdot d ,$$

$$X_4 = T_{2UC} = \mu_{T2UC} - c\cdot\Delta_1 + e_{T2UC} + d .$$

iii) Anomalous process behaviour might also be induced through blocking of the airlifts 551/101/102 as well as of the piping carrying the 2UC-stream. This may result in a decrease by Δ_2 of 2UC: therefore, under $H_1^{(2)}$ we have:

$$X_1 = 3EU = \mu_{3EU} + e_{3EU} + d \quad \text{(same as under } H_O) ,$$

$$X_2 = 2UC = \mu_{2UC} - \Delta_2 + e_{2UC} + d ,$$

$$X_3 = \rho = \mu_\rho + f\cdot\Delta_2 + e_\rho + a\cdot d ,$$

$$X_4 = T_{2UC} = \mu_{T2UC} - \rho\cdot\Delta_2 + e_{T2UC} + d .$$

Among others, the following test procedures seem to be reasonable: Firstly, we perform an overall check (Neyman Pearson test) using all data ($X_1 \ldots X_4$) available. In this case we have to make a decision about the alternative hypothesis H_1: Either we take the combination of $H_1^{(1)}$ and $H_1^{(2)}$ or we take only $H_1^{(1)}$ or $H_1^{(2)}$. In the first case the test is not optimal, if, in fact, only one disturbance is given, in the second case vice versa. Furthermore, it should be mentioned that in case the test result is significant, we cannot make out which cause led to the significance.

To avoid these two difficulties, separate tests (X_1 and X_3, on the one hand, and X_2, X_3 and X_4, on the other hand) can be performed. However, such a procedure has also two disadvantages: First, it is not optimal for any alternative hypothesis, i.e., the overall probability of detecting any disturbance is lower than for the procedure described before. Second, in order to guarantee a given overall false alarm probability, we have to use the rather complicated formalism described before.

The quantitative evaluation of these two procedures will be the subject of a forthcoming paper.

4. Availability considerations

In the study presented so far the false alarm rate has been introduced as a parameter of the problem. In the following paragraphs it will be described in which way the value of the false alarm rate can be determined by means of availability considerations.

Evidently, the availability will decrease if too many errors of the operator result in unnecessary shutoffs of the process; vice versa, the availability will possibly decrease even more seriously if one hesitates to interrupt the process in case there is a disturbance. Therefore, an optimal value of this false alarm rate must exist at least from the viewpoint of an optimal availability of the process.

Let us assume, e.g., that the Neyman Pearson test is performed at equidistant times, and furthermore, that at such a time the process is disturbed in the way described above with the probability p. The losses in production time in case of a shut-down are

- a   if there is no disturbance,
- b   if there is a disturbance and no action is taken,
- c   if there is a disturbance and no action is taken.

Here, we assume $0<a<b<c$. Then the expected loss in time is

$a\cdot\alpha+0\cdot(1-\alpha)$ if there is no disturbance,

$b\cdot(1-\beta)+c\cdot\beta$ if there is a disturbance.

Therefore, the (unconditional) expected loss in time $E(\alpha)$ is

$$E(\alpha) = a\cdot\alpha\cdot(1-p)+((b+(c-b)\cdot\beta))\cdot p \quad .$$

The optimal value of α is the value which minimizes the expected loss in time. It is determined by the relation

$$a\cdot(1-p)+(c-b)\cdot p\cdot\beta'(\alpha) = 0 \ .$$

As the derivative $\beta'(\alpha)$ of β with respect to α is a negative, monotonously increasing function of α with

$$\beta'(0) = -\infty \ , \quad \beta'(1) = 0 \ ,$$

there exists exactly one optimum value α_{opt} with the following properties

i) α_{opt} decreases with increasing a for fixed values of b, c and p: If the time loss in case of a false alarm is high, one should be careful with shut-downs.

ii) α_{opt} increases with increasing c-b for fixed values of a and p: If the time loss in case of a not detected disturbance is relatively high, one should not hesitate to shut down the cycle.

iii) α_{opt} increases with increasing p for fixed values of a, b and c: If the frequency of disturbances is great, one should not hesitate to shut down the cycle.

Figures 9 and 10 display in a qualitative way the dependence on α of $E(\alpha)$ and $E'(\alpha)$, respectively.

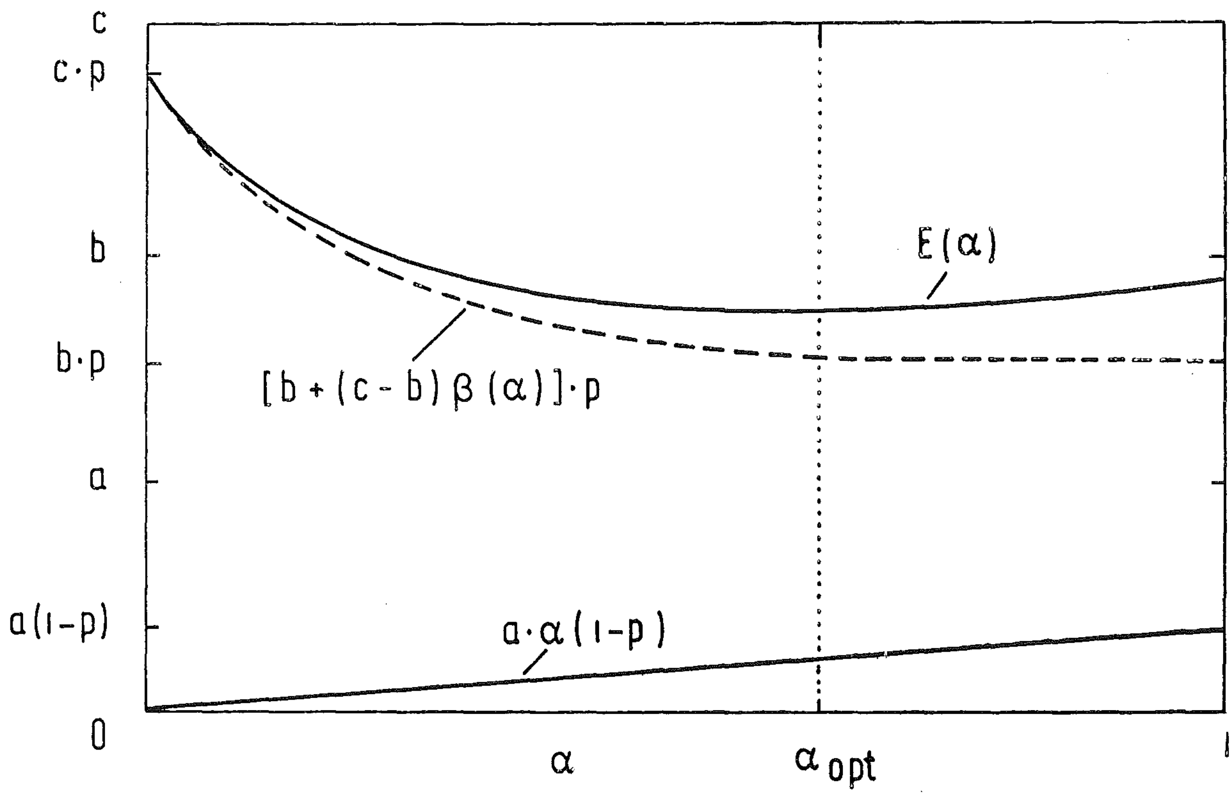


Fig. 9: E(α) as a function of α.

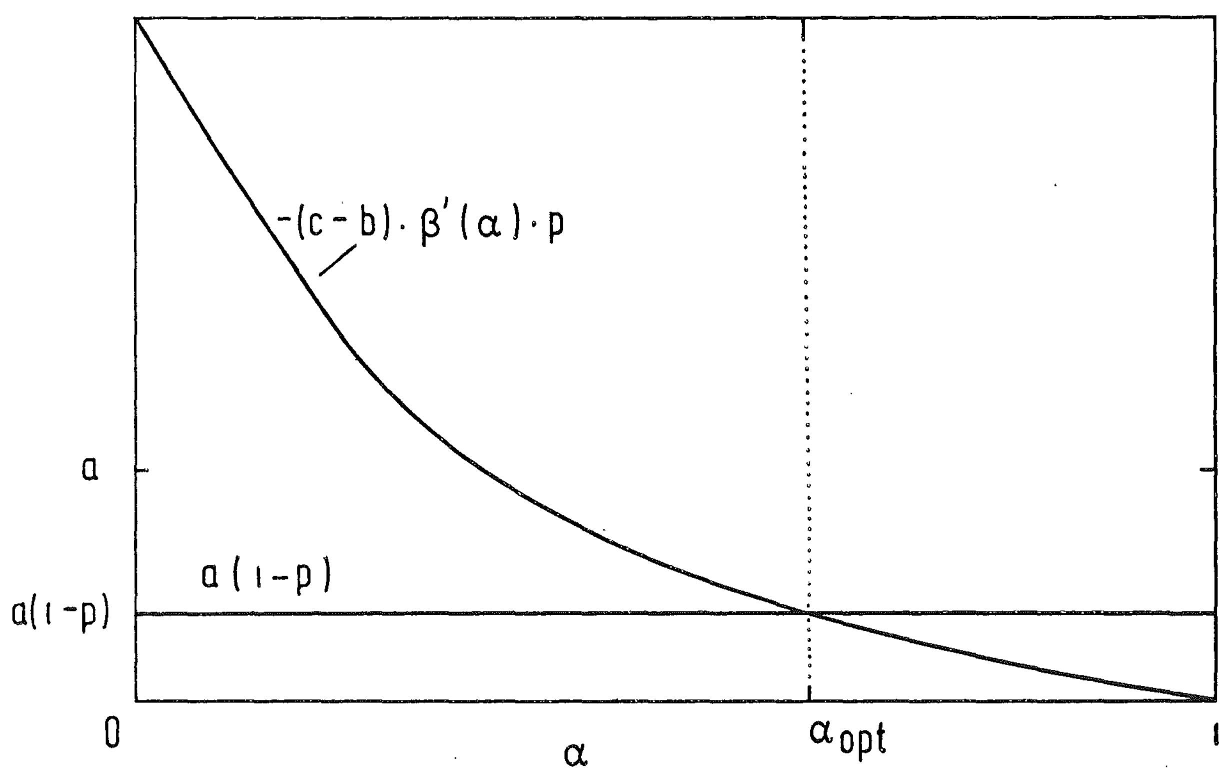


Fig. 10: E'(α) as a function of α.

5. Numerical illustration

For simplicity we only consider the case that the uncertainty of the position of the separating layer is much greater than the measurement uncertainties and the process variation. We assume

$$\mu_{2DK} = 1[\ell/h] \qquad \sigma^2_{DK} = .10^2[(\ell/h)^2]$$

$$\mu_{2DKW} = 4[\ell/h]^{*)} \quad \sigma^2_{DKW} = .12^2[(\ell/h)^2] \quad .$$

Experience shows that the process variation is of the order of 10 %; we shall consider now the two cases σ^2_P=0.10^2 and 0.15^2[$(\ell/h)^2$].

Figures 11 and 12 show the results of the determination of the power of the test procedure as a function of the disturbance. One can see in which way the process variation influences the probability of detection. Furthermore, it is shown that isolated checking of the 2DKW-stream results in a detection probability, which is considerably higher than that obtained by parallel checking (α_1=α_2) of the 2DK- and 2DKW-streams. Application of the Neyman Pearson test gives the highest detection probability.

A difficulty with respect to the determination of the optimal values of the false alarm probability lies in the fact that it depends upon the value Δ of the disturbance. For illustrative purposes we assume

$$\Delta^2 = 1.5^2(\sigma^2_{DKW}+\sigma^2_P\cdot\sigma^2_{DK}/(\sigma^2_P+\sigma^2_{DK})) \ ,$$

and furthermore a=1 resp. 2[h], b=6[h] and c=12[h]. The result of the calculation is shown in figure 13 which proves the qualitative discussion above.

---

*) It should be noted that the 2DKW-stream alone has a lower value; this stream, however, is measured together with three further kerosine streams, which leads to the value indicated.

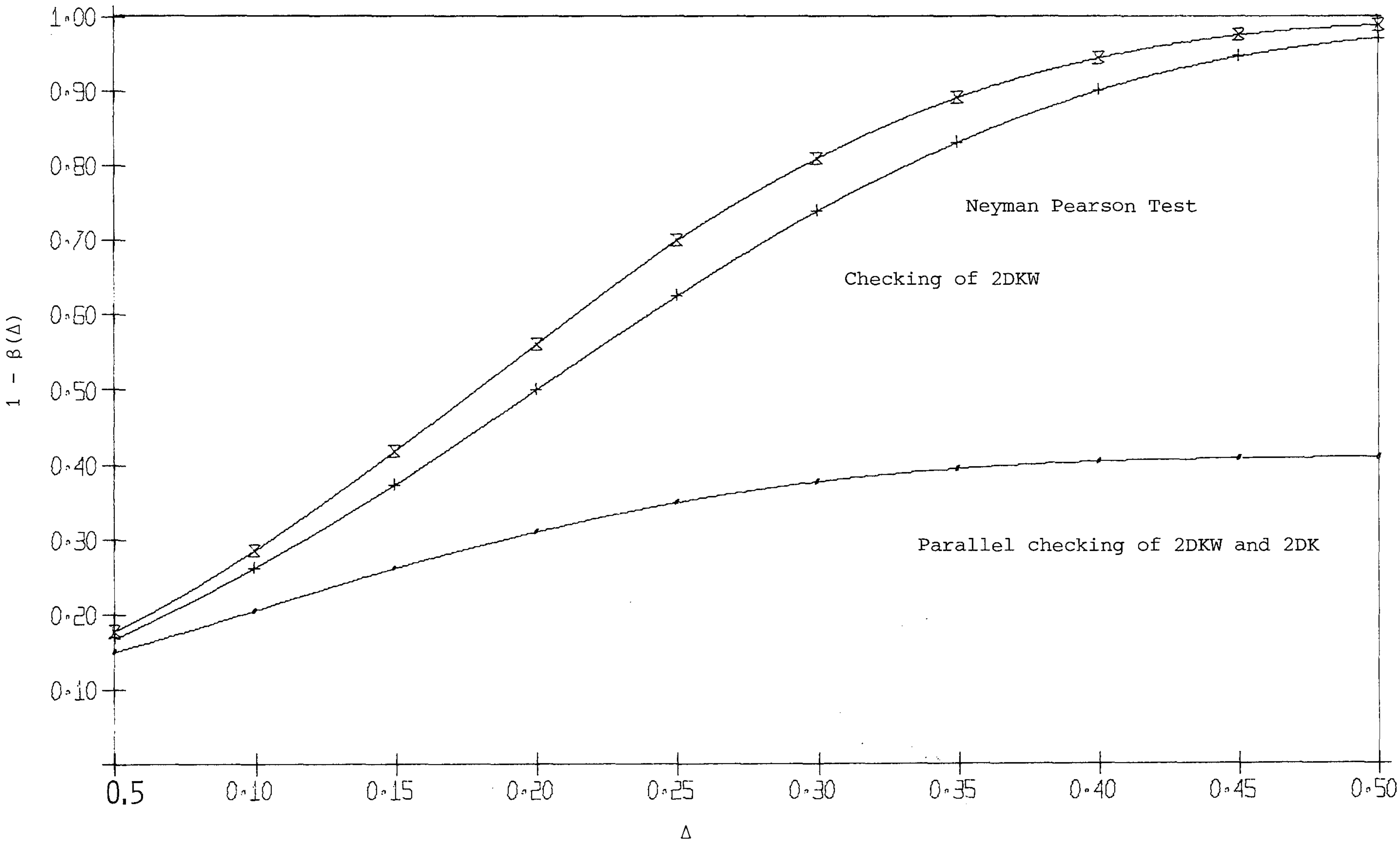
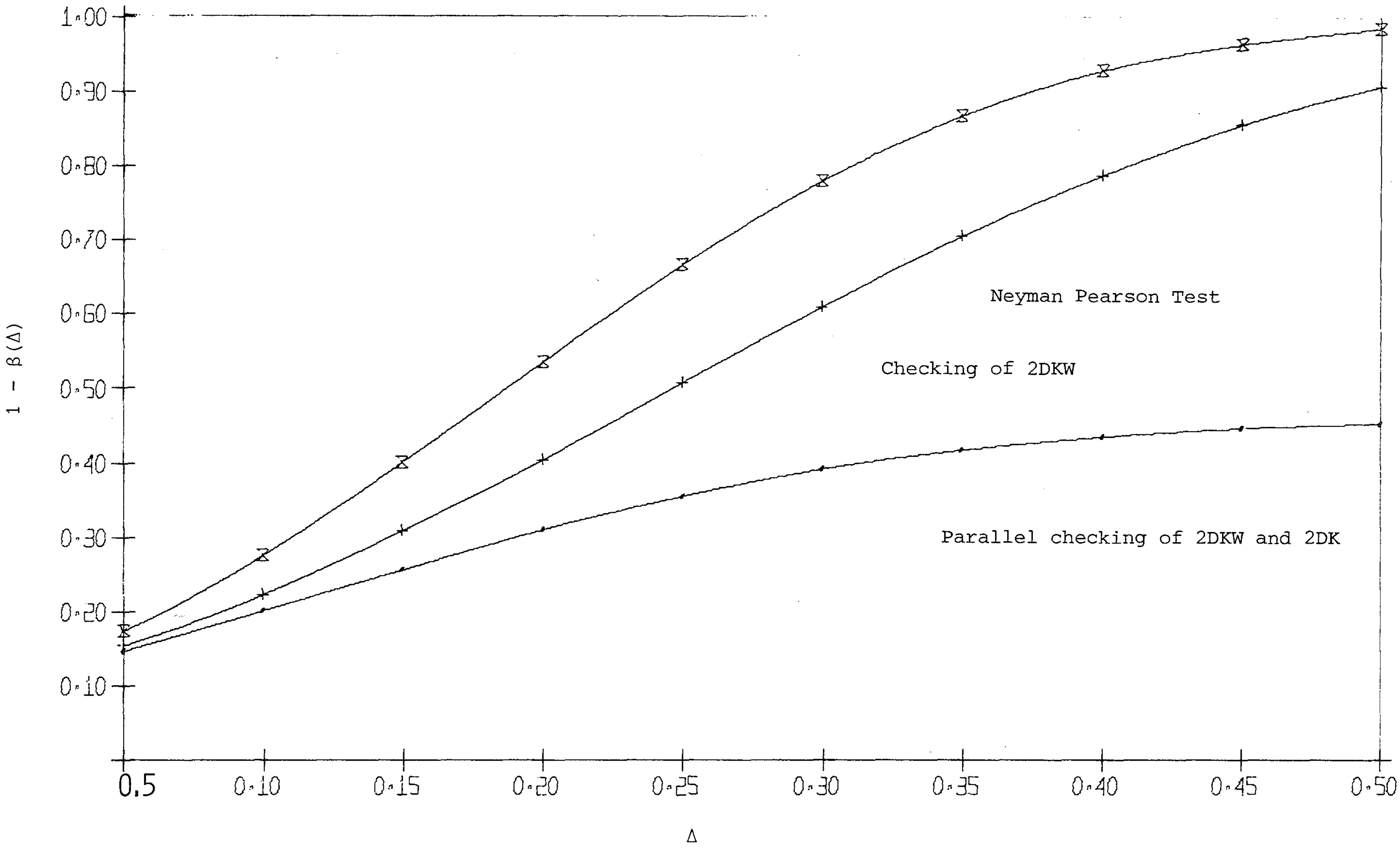


Fig. 11: Probability of detection as a function of the disturbance Δ with a false alarm rate α=0.1 for different test procedures and a process variation σ_P=0.10.



<u>**Fig. 12:**</u> Probability of detection as a function of the disturbance Δ with a false alarm rate α=0.1 for different test procedures and a process variation σ_p=0.15.

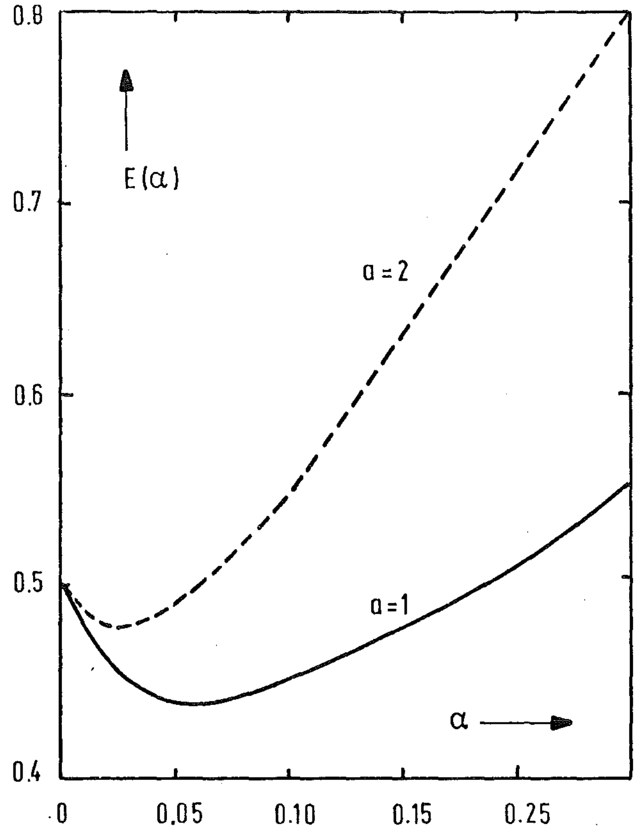


Fig. 13: Dependence of the expected loss in time E(α) on the false alarm probability with the parameter 'a' (loss in production time in case of shutdown if there is no disturbance).

References

/1/ R. Grumbach, unpublished results.

/2/ W.E. Büttner, L. Felkel, R. Grumbach, H.G. Herdtle, F. Öwre
'Data base and operational features of the disturbance analysis system for the Grafenrheinfeld Nuclear Power Plant',
Proceedings of the EHPG Meeting on water reactor fuel performance and application of process computers in reactor operation, Loen, 4-9 June 1978, HPR 221.14, Halden, Nov. 1978.

/3/ L. Felkel, R. Grumbach, A. Zapp (GRS),
F. Öwre, J.-K. Trengereid &Halden/
'Analytical Methods and Performance Evaluation of the STAR Application in the Grafenrheinfeld Nuclear Power Plant',
Proceedings of the Specialists' Meeting on 'Procedures and Systems for Assisting an Operator during Normal and Anomalous Nuclear Power Plant Operation Situations', Munich, 5-7 Dec. 1979.

/4/ R. Friehmelt, A. Jaeschke, K. Landmark
'Process Information Systems in Nuclear Fuel Reprocessing',
KfK-Nachrichten 11 (1979), p. 72.

/5/ K.A. Brownlee
'Statistical Theory and Methodology in Science and Technology',
John Wiley & Sons, New York (1967).

/6/ R. Avenhaus
'Significance Thresholds of One-sided Tests of Means of Bivariate, Normally Distributed Variables',
Communications in Statistics, Vol. A8, No. 3, P. 223-235, 1979.

/7/ H. Störmer
'Mathematische Theorie der Zuverlässigkeit'
R. Oldenbourg Verlag München 1970, p. 36.

ANNEX:

The common distribution of two measurements with independent, normally distributed measurement errors and a common, normally distributed calibration error

Let us assume that there are two measurements X_1 and X_2 which can be written as

$$X_1 = \mu_1+e_1+d$$

$$X_2 = \mu_2+e_2+d \ ,$$

where μ_1 and μ_2 are the true values, e_1 and e_2 the independent, normally distributed random measurement errors and d is the normally distributed random calibration error which is the same for both measurements. Let us furthermore assume that the first two moments of these errors are known:

$$E(e_1) = E(e_2) = E(d) = 0$$

$$\mathrm{var}(e_1) = \sigma_1^2$$

$$\mathrm{var}(e_2) = \sigma_2^2$$

$$\mathrm{var}(d) = \sigma_d^2$$

$$\mathrm{cov}(e_1,e_2) = \mathrm{cov}(e_1,d) = \mathrm{cov}(e_2,d) = 0 \ .$$

This means that the common density of e_1, e_2 and d is given by

$$f_{e_1,e_2,d}(U_1,U_2,U_3) = \frac{1}{(2\pi)^{\frac{3}{2}}} \cdot \frac{1}{\sigma_1} \cdot \frac{1}{\sigma_2} \cdot \frac{1}{\sigma_3} = \exp\left[-\frac{1}{2}\left[\frac{U_1^2}{\sigma_1^2} + \frac{U_2^2}{\sigma_2^2} + \frac{U_3^2}{\sigma_d^2}\right]\right] \ .$$

In order to determine the common density of X_1 and X_2, we define the following transformation

$$X_1 = \mu_1+e_1+d$$

$$X_2 = \mu_2+e_2+d$$

$$X_3 = d \ ,$$

the inverse transformation of which is given by

$$e_1 = X_1 - X_3 - \mu_1$$
$$e_2 = X_2 - X_3 - \mu_2$$
$$d = X_3 \;.$$

According to the transformation law of random variables /7/ the common density of (X_1, X_2, X_3) is given by

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3) = (2\pi)^{-\frac{3}{2}} \cdot \frac{1}{\sigma_1} \cdot \frac{1}{\sigma_2} \cdot \frac{1}{\sigma_3} \cdot \frac{1}{|J|} \cdot \exp\left[-\frac{1}{2} \cdot \left[\frac{(x_1-x_3-\mu_1)^2}{\sigma_1^2} + \right.\right.$$

$$\left.\left. + \frac{(x_2-x_3-\mu_2)^2}{\sigma_2^2} + \frac{x_3^2}{\sigma_3^2}\right]\right] \;,$$

where $|J|$ is the absolute value of the Jacobian determinant of the transformation, defined by

$$J = \begin{vmatrix} \frac{\partial X_1}{\partial e_1} & \frac{\partial X_1}{\partial e_2} & \frac{\partial X_1}{\partial d} \\ \frac{\partial X_2}{\partial e_1} & \frac{\partial X_2}{\partial e_2} & \frac{\partial X_2}{\partial d} \\ \frac{\partial X_3}{\partial e_1} & \frac{\partial X_3}{\partial e_2} & \frac{\partial X_3}{\partial d} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

The density of (X_1, X_2) therefore is given as the marginal density of (X_1, X_2, X_3), i.e.,

$$f_{X_1,X_2}(x_1,x_2) = \int_{-\infty}^{\infty} dx_3 \; f_{X_1,X_2,X_3}(x_1,x_2,x_3) =$$

$$= (2\pi)^{-\frac{3}{2}} \cdot \frac{1}{\sigma_1} \cdot \frac{1}{\sigma_2} \cdot \frac{1}{\sigma_3} \cdot \int_{-\infty}^{\infty} dx_3 \cdot \exp\left[-\frac{1}{2} \cdot \left[\frac{(x_1-x_3-\mu_1)^2}{\sigma_1^2} + \frac{(x_2-x_3-\mu_2)^2}{\sigma_2^2} + \frac{x_3^2}{\sigma_3^2}\right]\right] \;.$$

Now, because of

$$\frac{(x_1-x_3-\mu_1)^2}{\sigma_1^2}+\frac{(x_2-x_3-\mu_2)^2}{\sigma_2^2}+\frac{x_3^2}{\sigma_3^2}=$$

$$=\left[\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}+\frac{1}{\sigma_3^2}\right]\cdot\left(x_3-\frac{\frac{x_1-\mu_1}{\sigma_1^2}+\frac{x_2-\mu_2}{\sigma_2^2}}{\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}+\frac{1}{\sigma_3^2}}\right)^2+\frac{(x_1-\mu_2)^2}{\sigma_1^2}\cdot\left(1-\frac{1}{\sigma_1^2}\cdot\frac{1}{\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}+\frac{1}{\sigma_3^2}}\right)+$$

$$+\frac{(x_2-\mu_2)^2}{\sigma_2^2}\cdot\left(1-\frac{1}{\sigma_2^2}\cdot\frac{1}{\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}+\frac{1}{\sigma_3^2}}\right)-\frac{2}{\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}+\frac{1}{\sigma_3^2}}\cdot\left(\frac{x_1-\mu_1}{\sigma_1^2}\cdot\frac{x_2-\mu_2}{\sigma_2^2}\right)$$

$$=\left[\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}+\frac{1}{\sigma_3^2}\right]\cdot(x_3-a)^2+\frac{1}{1-\rho^2}\cdot\left(\frac{(x_1-\mu_1)^2}{\sigma_1^2+\sigma_3^2}-2\rho\cdot\frac{x_1-\mu_1}{\sqrt{\sigma_1^2+\sigma_3^2}}\cdot\frac{x_2-\mu_2}{\sqrt{\sigma_2^2+\sigma_3^2}}+\frac{(x_2-\mu_2)^2}{\sigma_2^2+\sigma_3^2}\right),$$

where a is a constant, and where

$$\rho:=\frac{\operatorname{cov}(X_1,X_2)}{\sqrt{\operatorname{var}(X_1)}\cdot\sqrt{\operatorname{var}(X_2)}}=\frac{\sigma_3^2}{\sqrt{\sigma_1^2+\sigma_3^2}\cdot\sqrt{\sigma_2^2+\sigma_3^2}}$$

is the correlation between X_1 and X_2, we get

$$f_{X_1,X_2}(x_1,x_2)=(2\pi)^{-\frac{3}{2}}\cdot\frac{\frac{1}{\sigma_1\cdot\sigma_2\cdot\sigma_3}}{\sqrt{\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}+\frac{1}{\sigma_3^2}}}\cdot\exp\left[-\frac{1}{2}\cdot\frac{1}{1-\rho^2}\cdot\left[\frac{(x_1-\mu_1)^2}{\sigma_1^2+\sigma_3^2}-2\rho\cdot\frac{x_1-\mu_1}{\sqrt{\sigma_1^2+\sigma_2^2}}\cdot\frac{x_2-\mu_2}{\sqrt{\sigma_2^2+\sigma_3^2}}+\frac{(x_1-\mu_2)^2}{\sigma_2^2+\sigma_3^2}\right]\right].$$

As can be seen immediately, this can be written as

$$f_{X_1,X_2}(x_1,x_2) = (2\pi)^{-\frac{3}{2}} \cdot (\det(\underline{\underline{\Sigma}}))^{-1} \cdot \exp(-\frac{1}{2} \cdot (\underline{x}-\underline{\mu})' \cdot \underline{\underline{\Sigma}}^{-1} (\underline{x}-\underline{\mu})) \quad ,$$

where the covariance matrix $\underline{\underline{\Sigma}}$, defined by

$$\underline{\underline{\Sigma}} := \begin{pmatrix} \mathrm{var}(X_1) & \mathrm{cov}(X_1,X_2) \\ \mathrm{cov}(X_1,X_2) & \mathrm{var}(X_2) \end{pmatrix}$$

is given by

$$\underline{\underline{\Sigma}} := \begin{pmatrix} \sigma_1^2+\sigma_3^2 & \sqrt{\sigma_1^2+\sigma_2^2} \cdot \sqrt{\sigma_2^2+\sigma_3^2} \cdot \rho \\ \sqrt{\sigma_1^2+\sigma_2^2} \cdot \sqrt{\sigma_2^2+\sigma_3^2} \cdot \rho & \sigma_2^2+\sigma_3^2 \end{pmatrix} \quad ,$$

and where the vectors $\underline{x}$ and $\underline{\mu}$ are given by

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} , \quad \underline{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} .$$