KfK 3586
September 1983

The Density Dependence of the Effective Alpha-Nucleon Force and Isoscalar Transition Rates of Nuclei

D. K. Srivastava, H. Rebel
Institut für Kernphysik

Kernforschungszentrum Karlsruhe

KERNFORSCHUNGSZENTRUM KARLSRUHE
Institut für Kernphysik

KfK 3586

THE DENSITY DEPENDENCE OF THE EFFECTIVE ALPHA-NUCLEON FORCE AND ISOSCALAR TRANSITION RATES OF NUCLEI

D.K. SRIVASTAVA[+] AND H. REBEL

Kernforschungszentrum Karslruhe GmbH, Karlsruhe

[+] Bhaba Atomic Research Institute, Calcutta, India

Kernforschungszentrum Karlsruhe GmbH
ISSN 0303-4003

Abstract

Effects of the density-dependence of the α-particle - bound nucleon effective interaction on single-folding models of inelastic α-particle scattering are studied, in particular in view of corrections of the isoscalar transition rates extracted by implicit folding procedures. The l-dependent correction factors C_l are calculated and tabulated for applications.

DIE DICHTEABHÄNGIGKEIT DER ALPHA-NUKLEON KRAFT UND ISOSKALARE ÜBERGANGSRATEN

Die Auswirkung der Dichteabhängigkeit der effektiven Wechselwirkung zwischen α-Teilchen und Kernnukleonen auf Einfach-Faltungsmodelle der unelastischen α-Teilchenstreuung werden untersucht, insbesondere im Hinblick auf Korrekturen zu den isoskalaren Übergangsraten, die durch implizierte Faltungsmodell-Verfahren gewonnen werden. Die l-abhängigen Korrekturfaktoren C_l werden berechnet und im Anhang tabelliert.

Introduction

The procedures currently used [1] for extracting isoscalar multipole transition rates from inelastic α-particle scattering e.g. are implicitly or explicitly based on the assumption that the (real) form factor $U_{if}(r_\alpha)$ (coupling potential) can be understood as the result of a folding of the transition density $\rho_{if} = \langle i|\delta(\vec{r}-\vec{r}')|f\rangle$ with an effective α-particle-bound nucleon interaction $V_{eff}^{\alpha N}(\vec{r}, \vec{r}_\alpha)$. Explicit folding model approaches require a rather detailed specification of $V_{eff}^{\alpha N}$ which can be hardly done free from any arbitrariness, and the results derived may be affected by the particular choice of the effective interaction. This difficulty is avoided by implicit folding procedures, recently worked out in detail [2-4]. In a first step of the analysis of the measured cross sections the coupling potentials have been determined on a conventional optical-model basis, in some cases rather accurately by using flexible parametrizations of U_{if} (thus minimizing constraints in the radial shapes) and including multiple excitation by coupled-channel calculations [2,4]. In a further step the transition moments and transition radii calculated for the resulting potentials are translated into corresponding quantities of the nuclear density distributions by use of simple mathematical properties [5,6] (known as Satchler's theorem [6]) for the radial moments of folding integrals. The basic relation between the multipole moments e.g. of the potential q_{lm}^u and of the nucleon distribution q_{lm}^ρ is given by

$$\frac{q_{lm}^u}{J_u} = \frac{q_{lm}^\rho}{J_\rho} \tag{1.1}$$

with J_u and J_ρ being the volume integrals of the distributions

$$U(\vec{r}_\alpha) = \int \rho(\vec{r})\, V_{eff}^{\alpha N}(\vec{r}_\alpha, \vec{r})\, d\vec{r} \tag{1.2}$$

and $\rho(\vec{r})$, respectively. It is mathematically correct as long as the function $V_{eff}^{\alpha N}$ is central and *in*dependent from ρ.

A density *in*dependent effective interaction, however, is only a first approximation. Refined analyses of α-particle scattering have revealed the importance of density dependence, in particular when probing deeper into the nucleus with energies above 100 MeV. Quite recently Srivastava [10] has extended the concept of an implicit folding model analysis to a double folding basis and considered the consequences of the density dependence of the nucleon-nucleon interaction. The values of the deduced transition rates appear to be considerably affected, in particular for higher multipolarity transitions. In the present paper we consider the problem from the point of view of a single folding approach with a density dependent effective interaction $V_{eff}^{\alpha N}(\vec{r}_\alpha,\vec{r},\rho)$ and study the limits and corrections of the procedure proposed in ref. 2-4.

2. Basic considerations

We introduce the density dependence of the effective α-bound-nucleon interaction by a saturation factor $g(\rho)$, writing

$$V_{eff}^{\alpha N}(\vec{r}_\alpha,\vec{r},\rho) = V_{\alpha N}^{o}(|\vec{r}-\vec{r}_\alpha|) \cdot g(\rho) \qquad (2.1)$$

Several alternative analytical forms for $g(\rho)$ are proposed (see ref. 11), and even the question which local density (the density to be used in 2.1)is not answered in a unique way (see ref. 9). Following previous studies [11] we use here

$$g(\rho) = [\, 1 - \gamma\, \rho^{2/3}(\vec{r}) \,] \qquad (2.2)$$

with the assumption that the saturation of the effective interaction is due to the nucleon density at the site of the bound-nucleon interacting with the α-particle (positioned at $\vec{r}_\alpha$). In the context of explicit folding model studies of elastic α-particle scattering it has been argued [9] that this kind of local density (suggested by the relatively short range of V_{eff}) is not so well justified and that there are also contributions from the nucleon density $\rho(r_\alpha)$ at the site of the projectile. We think,

however, that such refinements have only small influence on the integral quantities (radial moments) we are extracting by the implicit procedures [4-6].

Introducing $V_{eff}^{\alpha N}$ (2.1) into the folding formula (1.2) and separating the density independent part of $V_{eff}^{\alpha N}$ leads to some kind of "effective density"

$$\rho'(\vec{r}) = \rho(\vec{r})[1-\gamma\ \rho^{2/3}(\vec{r})] \tag{2.3}$$

for which effective moments $q_{lm}^{\rho'}$, volume integrals $J_{\rho'}$, etc. can be defined, replacing q_{lm}^{ρ} and J_{ρ} in the above theorem (eq. 1.1). The volume integrals and ms radii of $\rho(r)$ and $\rho'(r)$ are related [12] by

$$J_{\rho'} = A - \gamma K \tag{2.4a}$$

and

$$\langle r^2\rangle_{\rho'} = \langle r^2\rangle_{\rho} + [(\gamma K/A)/(1-\gamma K/A)]\ [\langle r^2\rangle_{\rho} - \langle r^2\rangle_{x}] \tag{2.4b}$$

where

$$K = \int \rho^{5/3}(\vec{r})\, d\vec{r} \tag{2.4c}$$

and

$$\langle r^2\rangle_{x} = \int r^2\ \rho^{5/3}(\vec{r})\, d\vec{r} \,/\, K \tag{2.4d}$$

For density-independent interactions ($\gamma \equiv 0$) $J_{\rho'}$ and $\langle r^2\rangle_{\rho'}$ reduce, of course, to J_{ρ} and $\langle r^2\rangle_{\rho}$. But also in the case of a homogeneous density distribution, the density dependence of the effective interaction can be absorbed by renormalization factor of the density-independent strength, so that the relations

$$J_u = A \cdot J_v \tag{2.5}$$

and

$$\langle r^2\rangle_u = \langle r^2\rangle_{\rho} + \langle r^2\rangle_v\ , \tag{2.6}$$

known from the density independent folding model, are again valid. Since the density in the nuclear interior is very

similar for all nuclei, an implicit folding model analysis of elastic α-particle scattering, using the relation

$$\Delta\langle r^2\rangle_u^{AA'} = \Delta\langle r^2\rangle_\rho^{AA'} \tag{2.7}$$

is well able to provide nuclear matter radii *differences* for neighboring nuclei [13]. Introducing the "defect moments"

$$M_{lm} = \int r^l \, Y_{lm} \, \rho^{5/3} \, (\vec{r}) \, d\vec{r} \tag{2.8}$$

we extend the considerations to the multipole moments by

$$\frac{q^{\rho'}_{lm}}{J_{\rho'}} = \frac{q^{\rho}_{lm} - \gamma \, M_{lm}}{A - \gamma \, K} \tag{2.9}$$

Similarily the transition radius of the multipole transition l is

$$R^2_{tr}(l) = R^2_{tr}(l,\rho') + \frac{1}{3}(2l+3)\langle r^2\rangle_v$$

In order to get some feeling about the corrections, tab. 1 presents some numerical values for the relevant quantities, calculated for a (permanently) deformed Fermi distribution

$$\rho(r) = \rho_o \; / \; [1 + \exp((r-c)/a)]$$

with $$c(r) = c_o \, (1 + \beta_2 \, Y_{20} + \beta_4 \, Y_{40})$$

and adopting the parameter values for the ^{56}Fe nucleus [14]

$A = 56$ $\qquad$ $\beta_2 = 0.24$

$c_o = 3.98$ fm $\qquad$ $\beta_4 = 0.03$

$a = 0.53$ fm

$\langle r^2\rangle_\rho$	q^ρ_{20}	q^ρ_{40}	K	$\langle r^2\rangle_x$	M_{20}	M_{40}
[fm]	[fm^2]	[fm^4]	[fm^{-2}]	[fm^2]	[fm^0]	[fm^2]
3.70	63.7	241.7	12.7	10.5	12.9	42.6

Tab. 1 Integral quantities of interest for a typical Fermi distribution

(in this case, the comments correspond to *intrinsic* multipole moments).

3. Reduced transition moments of a Fermi distribution

Numerical calculations of the corrections of the theorem (1.1) due to the density dependence require a specification of the radial shape of $\rho(\vec{r})$ and a deformation prescription since we are discussing integral properties, it is reasonable to expect that the results are practically independent of the form of the density distribution as long as this form is leptodermous. In ref. 12 the relevant quantities K, $\langle r^2 \rangle_x$ for a (spherical) Fermi distribution ($\kappa = 1$)

$$\rho(r) = \rho_o \left[1 + \exp\left((r-c)/a\right)\right]^{-\kappa} \tag{3.1}$$

are given in terms of $\varepsilon = a/c$. In first order in ε the relation (2.6) is improved by

$$\langle r^2 \rangle_u = \langle r^2 \rangle_\rho + \langle r^2 \rangle_v + \frac{6}{5} \cdot c \cdot a \cdot \alpha \cdot A_1 \tag{3.2a}$$

with

$$\alpha = \gamma \rho_o^{2/3} / (1 - \gamma \rho_o^{2/3}) \tag{3.2b}$$

$$A_1 \quad (5/3) = 0.759 \text{ (see ref. 12)}$$

showing that the correction is linear in c and a.

Here we extend the calculations to the multipole transition moments of a deformed Fermi distribution with an angular dependent half-way radius

$$c = c_o \left(1 + \Sigma \alpha_{1m} Y_{1m}(\hat{r})\right) \tag{3.3}$$

The reduced transition density, written in the form of a higher order (t) vibrational model, is

$$\rho_1(r) = \sum_t \rho_1^{(t)}(r) \langle I || Q_1^{(t)} || I' \rangle \tag{3.4}$$

with $\langle I || Q_1^{(t)} || I' \rangle$ the reduced matrix elements ("deformation parameters") of the transition operators (built by the α_{1m}), and

$$\rho_1^{(t)} = \frac{1}{t!} c_o^t \frac{\partial^t}{\partial c_o^t} \tag{3.5}$$

We are interested in the reduced transition moments which are rather directly related to the reduced transition probabilities. We have to calculate

$$q_1^{\rho'} = \int \rho_1'(r) r^{1+2} dr = q_1 - \gamma M_1 \tag{3.6}$$

Introducing the moments of the derivative expressions (3.5) up to t = 2 for distributions of the type (3.1) with $\kappa = 1$ and $\kappa = 5/3$,

$$y_l(\kappa) = c_o \int \frac{\partial \rho^{(\kappa)}}{\partial c_o} \, r^{l+2} dr \tag{3.7a}$$

$$z_l(\kappa) = \frac{c_o^2}{2} \int \frac{\partial^2 \rho^{(\kappa)}}{\partial c_o^2} \, r^{l+2} \, dr \tag{3.7b}$$

we write

$$q_l^{\rho\prime} = \langle I||Q_l^{(1)}||I'\rangle \cdot y_l(1) + \langle I||Q_l^{(2)}||I'\rangle \cdot z_l(1)$$

$$- \gamma \rho_o^{2/3} \left(\langle I||Q_l^{(1)}||I'\rangle \cdot y_l(5/3) + \langle I||Q_l^{(2)}||I'\rangle \cdot z_l(5/3)\right) \tag{3.8a}$$

Following ref. 15 $y_l(\kappa)$ and $z_l(\kappa)$ can be expanded in power series in $\varepsilon = a/c$. We adopt following definitions:

$$I_l = \frac{\langle I||Q_l^{(2)}||I'\rangle/2}{\langle I||Q_l^{(1)}||I'\rangle + (l+2)/2 \cdot \langle I||Q_l^{(2)}||I'\rangle} \tag{3.9a}$$

$$H_l(\kappa) = 1 + \sum_{n=o}^{l} (-1)^{n+1} \frac{(l+2)!}{(l-n+1)!} A_{n+1}(\kappa) \varepsilon^{n+1} (1-(n+1)I_l) \tag{3.9b}$$

$$+ (-1)^l (l+2)! \, A_{l+2}(\kappa) \, \varepsilon^{l+2}$$

with [15]

$$A_1(1)=o, \; A_2(1) = \pi^2/6, \; A_3(1)=o \qquad \text{etc.}$$

$$A_1(5/3)=0.759, \; A_2(5/3)=1.517, \; A_3(5/3)=1.301 \quad \text{etc.}$$

With these definitions the moment $q_l^{\rho\prime}$ can be written

$$q_l^{\rho\prime} = \rho_o c_o^{l+3} \{\langle I||Q_l^{(1)}||I'\rangle + \tfrac{1}{2}(l+2)\langle I||Q_l^{(2)}||I'\rangle \cdot \{H_l(\kappa=1) - \gamma \rho_o^{2/3} H_l(\kappa=5/3)\} \tag{3.8b}$$

The volume integral of the effective density (eq.2.4a) can calculated [12] by a function $F(\varepsilon) \cdot (1-\gamma\rho_o^{2/3}) = J_{\rho'}/J_\rho$

$$F(\varepsilon) = 1 + B_1 \varepsilon + B_2 \varepsilon^2 + B_3 \varepsilon^3 + \ldots$$

$$B_1 = 3A_1(5/3)\alpha \qquad B_2 = [\pi^2 - 6A_2(5/3)]\alpha$$

$$B_3 = 2 \cdot [3A_3(5/3) - A_1(5/3)\pi^2]\alpha \tag{3.9c}$$

Using the expansions $H_1(\kappa)$ and $F(\varepsilon)$ we modify eq. 1.1

$$\frac{q_1^u}{J_u} = \frac{q_1^\rho}{J_\rho} \cdot C_1 \qquad (3.10)$$

with the correction function

$$C_1 = \left[1 - \gamma\rho_o^{2/3} \frac{H_1(\kappa=5/3)}{H_1(\kappa=1)} \right] / \left(F(\varepsilon)\cdot(1-\gamma\rho_o^{2/3})\right) \qquad (3.11)$$

C_1 reduces to 1 , if $\gamma = o$ (density independent) or $\varepsilon = o$ (homogenous distribution with sharp edge). To first order in deformation (t=1) and second order in ε

$$C_1 = 1 + (1-1)\cdot A_1\ (\kappa=5/3)\ \alpha\cdot\varepsilon + (1-1)\alpha\varepsilon^2\left[\left(\frac{\pi^2}{6} - A_2(\kappa=5/3)\right)(1+4) - 3A_1^2(\kappa=5/3)\alpha\right] \qquad (3.12)$$

which is independent from the "deformation"

Using this expression in Fig.1, C_1 is plotted for $\alpha = 1.34$ ($\gamma = 1.9$ fm^2 and $\rho_o = 0.165$ fm^{-3}) with $\varepsilon = 0.54/(1.12\ A^{1/3})$ fm.

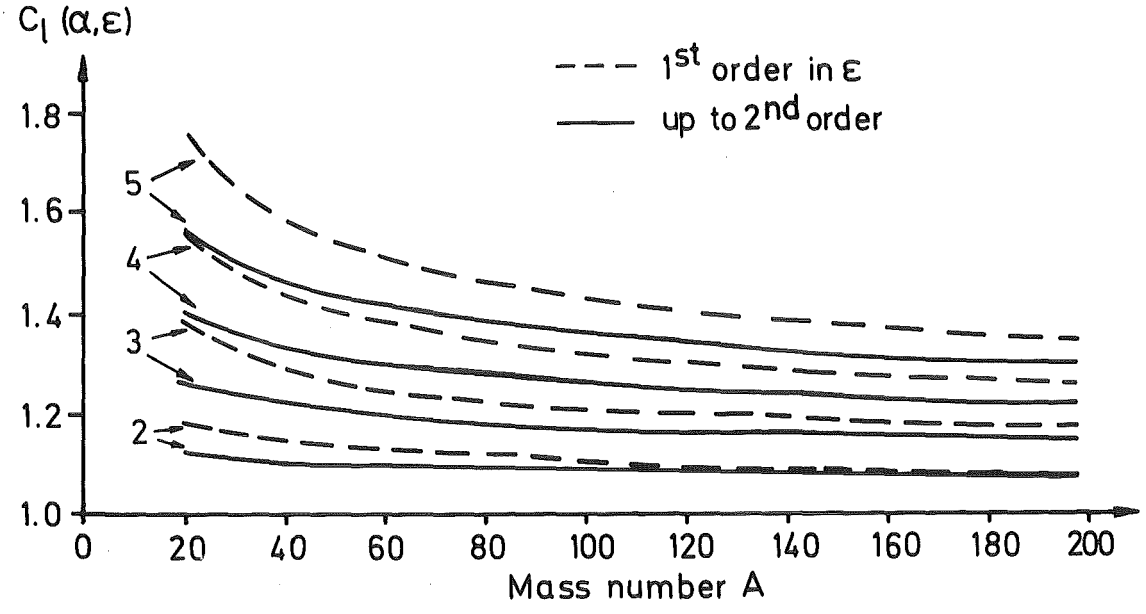

Fig. 1 Correction of the normalized multipole moments of a deformed Fermi distribution due to density dependence

It is interesting to note that the second order tries to cancel the effect.

4. Discussion

Implicit folding model procedures [1-5] have considerably improved the reliability of the results extracted from experimental data and have replaced the crude procedure previously used [16]. However there are only few results with realistic error estimates, and some results suffer on constraints or on the neglect of multiple excitation in the primary (extended) optical model analysis. It has been observed [1], that the extracted moments tend to have slightly larger values when compared to different studies. For light nuclei, in particular for N = Z nuclei, we have to expect that the isoscalar rates do not differ from electromagnetic rates. In fact most of the published results, especially for higher multipolarities, are improved in this sense by the C_1 correction. Remaining discrepancies may be ascribed to other reasons, e.g. the limits of a first order DWBA when applied for hexadecapole transitions. It should be also emphasized that the results may be influenced by the constraints due to the particular choice of the functional form for the radial shape of the transition potentials, increasingly with higher multipolarities. There are only few studies [2,4] which minimize this influence. Similarly, the correction function C_1 is expected to become less accurate for larger l-values as the tails of the transition densities are not very well approximated by derivatives of a Fermi distribution. Due to this fact, that an accurate calculation of the corrections requires a good knowledge of the densities, the implicit folding procedures loose somehow their primary elegance.

Tab. 2 Intrinsic multipole moments of ^{56}Fe

$Q_{20}^{\rho'}$ [efm^2]	Q_{20}^{ρ} [efm^2]	$Q_{40}^{\rho'}$ [efm^4]	Q_{40}^{ρ} [efm^4]	Q_{20}^{fold} [efm^2]	Q_{40}^{fold} [efm^4]	Q_{20}^{elec} [efm^2]
+125±5	114±4	+615±100	+469±75	+100±4	+500±90	98±2

Table 2 presents a particular example of values of quadrupole and hexadecapole moments as found by 104 MeV α-particle scattering and resulting from different types of analysis. The $Q_{lo}^{\rho'}$ are the values derived by the implicit folding procedure in ref. 1, Q_{lo}^{ρ} are values corrected by the C_l factor showing improved agreement with electromagnetic result. The values Q_{lo}^{fold} result from an explicit folding model analysis [1] using a density-*in*dependent Gaussian effective interaction. This interaction, empirically found, describes very well the differential cross sections in the diffraction region, but fails for the backward angles due to the lack of saturation. In fact, it does not reproduce the phenomenologically found volume integral J_u of the real interaction. Approximately $J_u/J_v \simeq A\ (1-\gamma\rho_o^{2/3}) \approx 0.5 \cdot A$ is found. In contrast a density-dependent interaction as given by Friedman et al.[17] e.g. is able to resolve this discrepancy and reproduces the correct value of J_u within few percent.

In view of this inconsistency it is somwehat surprising that the rates and moments extracted by the explicit folding procedure with the density-*in*dependent interaction rather well agree with electromagnetic results (see also ref. 18). This is a consequence of the increased importance of the nuclear surface in inelastic scattering. In fact the density independent effective interaction applied in refs. 1 and 18 has been adjusted by elastic forward scattering on ^{40}Ca, thus effectively averaging over the density dependence in the sensitive region. Obviously, such an adequately adjusted interaction is working well in the transition region and provides results consistent with electromagnetic values, *even if its saturation properties are wrong*. Considering the density-dependent effective interaction of the type as given in ref. 17, the situation seems to be just reversed. Forcing V_{eff} by the factor $g(\rho)$ (eq. 2.2) to give the correct saturation leads to a too strong interaction at the surface (where $g(\rho)$ is small) as compared to the sucessfull density independent form of V_{eff} ($J_v^{\text{density d.}}/J_v^o > 1$ for $r > c_o$). This possible deficiency of $g(\rho)$ affects mainly the inelastic transition potentials, and indeed somewhat strange results (reduced rates) are found when applying density-dependent

effective interactions of the type (eq. 2.1) in inelastic folding model analyses. [19] Most recently [9] even in elastic scattering some inadequacy of the form factor $g(\rho)$ (eq.2.2) has been observed.

This uncertainty in the correct form of $g(\rho)$ may also influence the correction factor C_1. This remark includes some warning about the significance of differences of the neutron and proton components of the deformation as quoted in ref. 5 e.g. We conclude that comparative studies of electromagnetic and hadronic transition rates are of interest not only with respect of their nuclear structure information, but also by revealing details of the density dependence of the nucleus-nucleus interaction.

The authors would like to thank Dr. N.K. Ganguly and Dr. J. Albiński for encouraging discussions.

References

1 H. Rebel, Z. Phys. A277 (1976) 35

2 H. Rebel, R. Pesl, H.J. Gils and E. Friedman
Nucl. Phys. A368 (1981) 61

3 G.J. Wagner, P. Grabmayr and H.R. Schmidt
Phys. Lett. 113B (1982) 447

4 V. Corcalciuc, H. Rebel, R. Pesl and H.J. Gils
J. Phys. G: Nucl. Phys. 9 (1983) 177

5 R.S. Mackintosh, Nucl. Phys. A266 (1976) 379

6 G.R. Satchler, J. Math. Phys. 13 (1972) 1118

7 F. Friedman, H.J. Gils, H. Rebel and Z. Majka
Phys. Rev. Lett. 41 (1978) 1220

8 Z. Majka, H.J. Gils and H. Rebel
Z. Phys. A288 (1978) 139 - Acta Phys. Polonica B11 (1980) 227

9 H.J. Gils, Phys. Lett. B (submitted) - KfK 3555 (1983)

10 D.K. Srivastava, preprint May 1983

11 D.K. Srivastava, Phys. Lett. 122B (1983) 18

12 D.K. Srivastava, Phys. Lett. 113B (1982) 353

13 E. Friedman, H.J. Gils, H. Rebel and R. Pesl, Nucl. Phys.
A363 (1981) 137

14 H.J. Gils, H. Rebel, G. Nowicki, A. Ciocânel, D. Hartmann,
H. Klewe-Nebenius and K. Wisshak
Journ. Phys. G: Nucl. Phys. 3 (1975) 344

15 D.K. Srivastava, Phys. Lett. 112B (1982) 289

16 A.M. Bernstein, Adv. Nucl. Phys. 3 (1969) 325, eds. M. Barenger and E. Vogt, Plenum Press 1969

17 E. Friedman, H.J. Gils, H. Rebel and Z. Majka
Phys. Rev. Lett. 41 (1978) 1220

18 H.J. Gils and H. Rebel, Z. Phys. A274 (1975) 259

19 R. Pesl, H.J. Gils, H. Rebel, E. Friedman, J. Buschmann,
H. Klewe-Nebenius, S. Zagromski, KfK-3378 (1983) -
Z. Phys.A313 (1983) (in press)

Appendix

We present numerical results based on a density distribution

$$\rho_o^\kappa \cdot f(\kappa) \text{ with}$$

$$f(\kappa) = 1/[1+\exp((r-c)/a]^\kappa$$

$$c_o = 1.115 \cdot A^{1/3} - 0.53 \cdot A^{-1/3} \text{ fm}$$

$$a = 0.57$$

$$\kappa = 1 \text{ and } \kappa = 5/3.$$

Choosing

$$\langle o||Q_1^{(1)}||I\rangle = \delta_{\lambda I}(-)^I \beta_1$$

$$\langle o||Q_1^{(2)}||I\rangle = \beta_1^2 \langle 11oo|Io\rangle / \sqrt{2}$$

with $\beta_1 = 1$

same quantities of interest and the correction factor $C_1 = (q_\rho,/J_\rho,)/(q_\rho/A)$ are tabulated for $1 = 2,3,4,5$, in addition to the radial moments $y(\kappa)$ and $z(\kappa)$. We note that the tabulated values result from numerical integration of the revelant expressions and allow to check the numerical precision of the algebraic approximations introduced in sect. 3. Tab. A 1 shows the error range for $A \in [20, 206]$ of the approximations represented by

$$\Delta y_1 = \frac{y_1(\text{algebr.}) - y_1(\text{exact.})}{y_1(\text{exact})} \cdot 100$$

e.g.. ΔC_1 refers to the error when C_1 is calculated by (3.12) up to high orders in ε.

1	$\Delta y_1(1)$	$\Delta y_1(5/3)$	$\Delta z_1(1)$	$\Delta z_1(5/3)$	ΔC_1
2	[0.01,1.66]	[o.22,0.42]	[0.07,2.68]	[0.25,0.47]	[0.53,2.35]
3	[0.01,3.21]	[0.27,0.51]	[0.09,4.65]	[0.33,0.47]	[0.51,2.07]
4	[0.01,6.00]	[0.32,0.48]	[0.11,7.77]	[0.40,0.41]	[0.51,1.84]
5	[0.015,10.58]	[0.37,0.48]	[0.12,12.81]	[0.47,0.58]	[0.52,1.64]

Tab. A 1 Range of errors in percent for $A \in [20,206]$ when using the power series expansions of sect. 3.

Tab. A2

Central density and normalized multipole moments ($\beta_1 = 1$)

Nucleus			ρ_o	q_2^ρ/A	q_3^ρ/A	q_4^ρ/A	q_5^ρ/A
-			fm^{-3}	fm^2	fm^3	fm^4	fm^5
10	NE	20	0.1505	2.5246	10.1649	43.637	198.79
12	MG	24	0.1525	2.8290	11.8463	52.772	249.27
14	SI	28	0.1541	3.1119	13.4722	61.902	301.20
16	S	32	0.1554	3.3781	15.0563	71.052	354.51
18	A	40	0.1574	3.8737	18.1424	89.569	465.94
20	CA	42	0.1578	3.9913	18.9006	94.249	494.77
22	TI	48	0.1589	4.3326	21.1556	108.470	584.08
24	CR	52	0.1595	4.5516	22.6447	118.098	645.87
26	FE	56	0.1600	4.7644	24.1217	127.816	709.18
28	NI	60	0.1605	4.9719	25.5908	137.653	774.26
30	ZN	66	0.1612	5.2742	27.7822	152.632	875.21
32	GE	72	0.1618	5.5669	29.9585	167.846	979.78
34	SE	76	0.1621	5.7573	31.4024	178.119	1051.49
38	SR	88	0.1630	6.3088	35.7090	209.578	1276.46
40	ZR	90	0.1631	6.3981	36.4236	214.911	1315.35
42	MO	96	0.1634	6.6620	38.5619	231.057	1434.32
44	RU	100	0.1637	6.8348	39.9841	241.948	1515.61
46	PD	106	0.1640	7.0896	42.1118	258.459	1640.36

Tab. A 2 (contd.)

Nucleus	ρ_o	q_2^ρ/A	q_3^ρ/A	q_4^ρ/A	q_5^ρ/A
-	fm^{-3}	fm^2	fm^3	fm^4	fm^5
48 CD 110	0.1642	7.2568	43.5275	269.586	1725.44
50 SN 116	0.1644	7.5037	45.6461	286.443	1855.76
52 TE 124	0.1648	7.8267	48.4641	309.238	2034.71
56 BA 136	0.1652	8.2972	52.6722	344.074	2313.83
60 ND 144	0.1654	8.6033	55.4778	367.713	2506.90
62 SM 148	0.1655	8.7542	56.8759	379.655	2605.52
78 PT 194	0.1666	10.4014	72.8693	522.413	3834.80
80 HG 200	0.1667	10.6059	74.9458	541.728	4007.63
82 PB 206	0.1668	10.8084	77.0206	561.194	4183.25

A	$y_2(1)$	$z_2(1)$	$y_3(1)$	$z_3(1)$	$y_4(1)$	$z_4(1)$	$y_5(1)$	$z_5(1)$
20	.504918E+02	.740246E+02	.209298E+03	.347005E+03	.872738E+03	.165277E+04	.397596E+04	.812105E+04
24	.678962E+02	.103665E+03	.284510E+03	.504691E+03	.126053E+04	.250993E+04	.598248E+04	.128588E+05
28	.871325E+02	.130365E+03	.377221E+03	.690090E+03	.173524E+04	.355799E+04	.843346E+04	.188605E+05
32	.108099E+03	.172507E+03	.481801E+03	.902740E+03	.227308E+04	.480080E+04	.113443E+05	.262105E+05
40	.154940E+03	.254566E+03	.725096E+03	.140955E+04	.358270E+04	.789530E+04	.186376E+05	.452400E+05
42	.167635E+03	.277020E+03	.793825E+03	.155307E+04	.395844E+04	.879879E+04	.207804E+05	.509494E+05
48	.207967E+03	.348905E+03	.101547E+04	.202454E+04	.520055E+04	.118375E+05	.280357E+05	.705718E+05
52	.236684E+03	.400478E+03	.117752E+04	.237279E+04	.614108E+04	.141425E+05	.335854E+05	.858187E+05
56	.266807E+03	.454832E+03	.135082E+04	.274762E+04	.715771E+04	.166705E+05	.397139E+05	.102829E+06
60	.298312E+03	.511945E+03	.153545E+04	.314951E+04	.825921E+04	.194310E+05	.466554E+05	.121722E+06
66	.348099E+03	.602666E+03	.183302E+04	.380310E+04	.100757E+05	.240208E+05	.577636E+05	.153700E+06
72	.400819E+03	.699222E+03	.215701E+04	.451690E+04	.120849E+05	.291525E+05	.705441E+05	.190304E+06
76	.437556E+03	.766755E+03	.238659E+04	.502621E+04	.135370E+05	.328816E+05	.799135E+05	.217411E+06
88	.555170E+03	.984129E+03	.314240E+04	.671543E+04	.184428E+05	.455999E+05	.112328E+06	.312074E+06
90	.575826E+03	.102246E+04	.327812E+04	.702040E+04	.193420E+05	.479475E+05	.118382E+06	.329904E+06
96	.639549E+03	.114095E+04	.370194E+04	.797559E+04	.221813E+05	.553870E+05	.137695E+06	.387036E+06
100	.683479E+03	.122284E+04	.399841E+04	.864600E+04	.241948E+05	.606850E+05	.151561E+06	.428278E+06
106	.751501E+03	.134991E+04	.446386E+04	.970170E+04	.273960E+05	.691429E+05	.173878E+06	.496961E+06
110	.798249E+03	.143743E+04	.478802E+04	.104391E+05	.296545E+05	.751294E+05	.189798E+06	.542750E+06
116	.870430E+03	.157280E+04	.529495E+04	.115951E+05	.332274E+05	.846333E+05	.215268E+06	.619505E+06
124	.970457E+03	.176086E+04	.600955E+04	.132304E+05	.383455E+05	.983079E+05	.252304E+06	.731730E+06
136	.112841E+04	.205876E+04	.710410E+04	.158838E+05	.467940E+05	.121007E+06	.314681E+06	.922054E+06
144	.123887E+04	.226765E+04	.798880E+04	.177864E+05	.529507E+05	.137629E+06	.360994E+06	.106418E+07
148	.129561E+04	.237512E+04	.841763E+04	.187778E+05	.561889E+05	.146395E+06	.385617E+06	.114001E+07
194	.201786E+04	.375032E+04	.141366E+05	.320990E+05	.101548E+06	.269850E+06	.743951E+06	.225700E+07
200	.212119E+04	.394790E+04	.149892E+05	.340968E+05	.108340E+06	.289128E+06	.801525E+06	.243817E+07
206	.222654E+04	.414955E+04	.159062E+05	.361546E+05	.115600E+06	.309162E+06	.861749E+06	.262805E+07

Tab A 3 Radial moments $y_1(1)$ and $z_1(1)$

A	$y_2(5/3)$	$z_2(5/3)$	$y_3(5/3)$	$z_3(5/3)$	$y_4(5/3)$	$z_4(5/3)$	$y_5(5/3)$	$z_5(5/3)$
20	.256489E+02	.463874E+02	.849775E+02	.161435E+03	.295706E+03	.720120E+03	.108184E+04	.292653E+04
24	.357633E+02	.658418E+02	.123976E+03	.271047E+03	.450039E+03	.112720E+04	.171377E+04	.477836E+04
28	.473218E+02	.882902E+02	.170803E+03	.379848E+03	.644295E+03	.164503E+04	.253500E+04	.723794E+04
32	.602697E+02	.113021E+03	.223454E+03	.508175E+03	.878984E+03	.228100E+04	.356525E+04	.103739E+05
40	.901513E+02	.172556E+03	.358546E+03	.824465E+03	.147979E+04	.395474E+04	.632796E+04	.189432E+05
42	.984240E+02	.188955E+03	.396825E+03	.916125E+03	.165877E+04	.443227E+04	.717816E+04	.216146E+05
48	.125101E+03	.242009E+03	.525790E+03	.122169E+04	.226830E+04	.615812E+04	.101466E+05	.310119E+05
52	.144398E+03	.280516E+03	.618607E+03	.145108E+04	.273735E+04	.745970E+04	.124949E+05	.385108E+05
56	.164876E+03	.321471E+03	.721620E+03	.170120E+04	.325827E+04	.893485E+04	.151581E+05	.470603E+05
60	.186510E+03	.364825E+03	.832870E+03	.197217E+04	.383202E+04	.105682E+05	.181525E+05	.567306E+05
66	.221084E+03	.434248E+03	.101527E+04	.241797E+04	.479743E+04	.133247E+05	.232994E+05	.734508E+05
72	.258140E+03	.508826E+03	.121642E+04	.291132E+04	.589035E+04	.166623E+05	.292795E+05	.930019E+05
76	.284208E+03	.561347E+03	.136099E+04	.326679E+04	.669220E+04	.187725E+05	.337544E+05	.107692E+06
88	.368715E+03	.731984E+03	.184528E+04	.446168E+04	.946339E+04	.267948E+05	.496857E+05	.160318E+06
90	.383705E+03	.762290E+03	.193340E+04	.467966E+04	.998007E+04	.282958E+05	.527255E+05	.170404E+06
96	.430182E+03	.850342E+03	.221054E+04	.536608E+04	.116269E+05	.330891E+05	.625397E+05	.203048E+06
100	.462416E+03	.921621E+03	.240597E+04	.585084E+04	.128068E+05	.365306E+05	.696790E+05	.226858E+06
106	.512610E+03	.104334E+04	.271517E+04	.661882E+04	.147023E+05	.420700E+05	.813173E+05	.265771E+06
110	.547286E+03	.109366E+04	.293204E+04	.715813E+04	.160512E+05	.460189E+05	.897156E+05	.293915E+06
116	.601094E+03	.120285E+04	.327349E+04	.800821E+04	.182049E+05	.523342E+05	.103306E+06	.339555E+06
124	.676138E+03	.135523E+04	.375901E+04	.921866E+04	.213247E+05	.615017E+05	.123348E+06	.407047E+06
136	.795632E+03	.159808E+04	.455236E+04	.112001E+05	.265515E+05	.769003E+05	.157744E+06	.523271E+06
144	.879818E+03	.176930E+04	.512484E+04	.126321E+05	.304107E+05	.882965E+05	.183711E+06	.611278E+06
148	.923246E+03	.185706E+04	.544420E+04	.133815E+05	.324550E+05	.943410E+05	.197643E+06	.658573E+06
194	.148436E+04	.300069E+04	.949971E+04	.236126E+05	.617549E+05	.181341E+06	.407546E+06	.137532E+07
200	.156566E+04	.316045E+04	.101177E+05	.251672E+05	.663951E+05	.195164E+06	.442203E+06	.149418E+07
206	.164877E+04	.333592E+04	.107557E+05	.267730E+05	.712319E+05	.209585E+06	.478665E+06	.161934E+07

Tab A 4 Radial moments $y_1(5/3)$ and $z_1(5/3)$

Tab. A 5

Correction quantities

NUCLEUS	K/A	$J_{\rho'}/A$	C_2	C_3	C_4	C_5
-	fm^{-2}	-	-	-	-	-
10 NE 20	0.172	0.674	1.078	1.151	1.213	1.266
12 MG 24	0.178	0.661	1.081	1.155	1.221	1.278
14 SI 28	0.184	0.650	1.082	1.158	1.226	1.285
16 S 32	0.189	0.641	1.082	1.159	1.229	1.291
18 A 40	0.197	0.626	1.083	1.160	1.232	1.297
20 CA 42	0.198	0.623	1.083	1.160	1.232	1.298
22 TI 48	0.203	0.614	1.082	1.160	1.233	1.299
24 CR 52	0.206	0.609	1.082	1.160	1.233	1.300
26 FE 56	0.208	0.605	1.081	1. 159	1.232	1.300
28 NI 60	0.210	0.601	1.081	1.158	1.232	1.300
30 ZN 66	0.213	0.595	1.080	1.157	1.231	1.300
32 GE 72	0.216	0.590	1.080	1.156	1.229	1.299
34 SE 76	0.218	0.587	1.079	1.156	1.229	1.298
38 SR 88	0.222	0.579	1.078	1.153	1.226	1.295
40 ZR 90	0.222	0.577	1.078	1.153	1.225	1.295
42 MO 96	0.224	0.574	1.077	1.152	1.224	1.293
44 RU 100	0.225	0.572	1.077	1.151	1.223	1.292
46 PD 106	0.227	0.569	1.076	1.150	1.222	1.291

Table A 5 (contd.)

NUCLEUS	K/A	$J_{\rho'}/A$	C_2	C_3	C_4	C_5
-	fm^{-2}	-	-	-	-	-
48 CD 110	0.228	0.567	1.076	1.149	1.221	1.290
50 SN 116	0.229	0.564	1.075	1.148	1.219	1.288
52 TE 124	0.231	0.561	1.074	1.148	1.217	1.286
56 BA 136	0.234	0.556	1.073	1.145	1.215	1.283
60 ND 144	0.235	0.553	1.073	1.144	1.213	1.281
62 SM 148	0.236	0.552	1.072	1.143	1.212	1.280
78 PT 194	0.242	0.540	1.069	1.137	1.203	1.269
80 HG 200	0.243	0.538	1.069	1.137	1.203	1.269
82 PB 206	0.244	0.537	1.068	1.136	1.202	1.268