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SAMPLING FOR THE VERIFICATION OF MATERIALS BALANCES

by

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Abstract

The results of a theory for verification of nuclear materials balance data are presented. The sampling theory is based on two diversion models where also a combination of models is taken into account.
The theoretical considerations are illustrated with numerical examples using the data of a highly enriched uranium fabrication plant.

Das Ziehen von Stichproben zur Verifikation von Materialbilanzen

Zusammenfassung

Eine Theorie zur Verifikation nuklearer Materialbilanzen wird in ihren Ergebnissen dargestellt. Die stichprobentheoretischen Untersuchungen werden anhand zweier Materialentwendungsmodelle durchgeführt, wobei die Entwendungsmodelle auch kombiniert werden.
Die theoretischen Überlegungen werden mit einem numerischen Beispiel illustriert, wobei die Daten einer Fabrikationsanlage für hoch angereichertes Uran verwendet werden.

Contents

1. Introduction

In a previous paper /1/ the state of the theory of the verification of nuclear materials accountancy data was presented. Data verification procedures for Model A (falsification of all batch data in case of falsification) and for Model B (falsification of only a part of the batch data) both for attribute and for variable sampling, and combined materials balance and data verification test procedures were applied to the data of a highly enriched uranium fabrication plant in order to determine the efficiency of the safeguards system, namely the probability of detecting a diversion of a given quantitiy of nuclear material for fixed false alarm probability and verification effort.

Due to the lack of analytical formulae for Model B and the variable sampling case, the previous paper suffered from two deficiencies. First, the optimal sample sizes needed for the distribution function of the D-statistic for the data verification could not be determined for Model B, thus, the attribute formulae were used. Second, it was not possible to analyze the use of the two different measurement methods available, namely destructive and nondestructive analysis. In the meantime, the optimization problem of Model B and the variable sampling case has been solved /2/, therefore, it was considered reasonable to analyze the available data once more with the better tools.

In this paper, only the data verification aspects are discussed as the MUF-test as well as the combined (D,MUF)-test have not been improved from the theoretical point of view. Furthermore, only the variable sampling case is considered, as all seals are controlled, and as all measurement methods contain random measurement errors which cannot be neglected. In the second chapter those plant data are presented in short which are relevant to the data verification procedures. In the third chapter the new theoretical results and, together with them, their application to the plant data are given in order to avoid repetitions.

The numerical calculations contained in this paper are performed with the help of computer codes which have been developed in the framework of a diploma work at the Hochschule der Bundeswehr München /3/. Fig. 1-1 gives an overview of the structure of the study. Because of their size these codes have not been reproduced here, they can, however, be obtained upon request from the authors of this study. In this diploma work, also the combined (D,MUF)-test, in other words, the efficiency of the whole materials accountancy data verification system has been determined numerically for the inventory period data given in /1/.

Structure for the Study of Data Verification

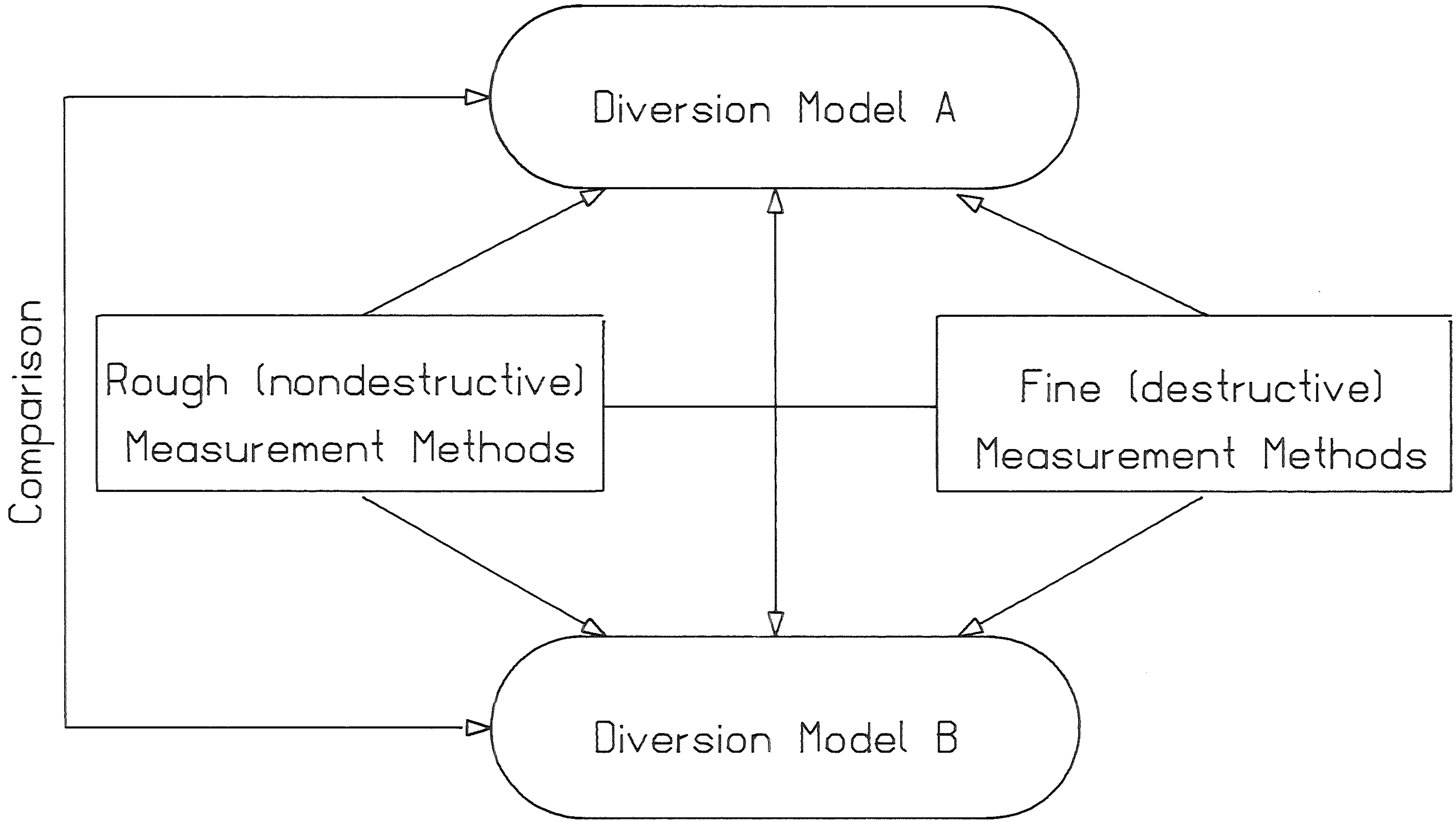


Figure 1-1

2. Plant Data Used for the Numerical Calculations

The theory that will be outlined in the subsequent chapter is applied to inventory data of a nuclear material fabrication plant. The plant under consideration is the NUKEM fabrication plant in Hanau, Federal Republic of Germany. As this plant has been described in major detail in earlier papers (/1/,/4/), here we only give a condensed description of the facility.

The main production activity of the NUKEM plant in terms of the flow of highly enriched uranium is the fabrication of fuel elements for material testing reactors and for pebble bed high temperature reactors. There exists an accounting system that has been run since 1975 on the basis of an electronic data banking system. This system enables the plant operator to produce at any time physical inventory listings for all material on storage. As there does not exist a stationary production state in the NUKEM plant, it is not possible to give representative figures for throughput and inventory. Instead, in /1/ a concrete inventory period, lasting from October 1977 to April 1978, had been selected. In this paper, again these data will provide the basis for the numerical calculations. We will consider only the verification of the inventory data because it represents an especially important part of safeguards: Flow measurement data sometimes can be verified by comparing skipper and receiver data, but there is no alternative that can replace inventory data verification with the help of independent measurements.

The physical inventories are stratified according to chemical, physical and geometrical viewpoints; only the U-235 data are considered. In Table 2-1 the slightly adjusted data of the initial physical inventory of the inventory period mentioned above are listed.

In Table 2-2 the relative standard deviations of the rough, i.e., nondestrucive measurement methodes are shown, including the time necessary to verify the data of a single batch. We assume that the plant operator and the inspector use the same instruments, or at least the same type of instruments which means that both their measurements have the same uncertainties.

In Table 2-3 the relative standard deviations for the fine, i.e., destructive measurement methods are listed. We assume that these methods can be applied in classes 4,5 and 6 of the initial physical inventory, as classes 1 and 7 represent initial products. In class 8 there are different kinds of material and not a single measurement method to verify these data. In classes 2 and 3 we have seal checks and we do not break the seals if they are all right.

Table 2-1: Physical Inventory Data from Beginning Inventory.

Class	Material	Total Isotopic Weight [kg U-235]	Number of Items	Average Isotopic Weight per Item [kg U-235]
1	UF_6	384.8	53	7.26
2	MTR, RHF Elements	28.9	53	0.55
3	HTR Elements	379.9	*) $3.8*10^5$	.001
4	Fuel Plates	79.5	4498	.016
5	Fuel Rods	10.3	147	.07
6	Pure Metals	108.8	30	3.63
7	Intermediate Products	334.1	2539	.132
8	Waste, Heterogeneous Scrap, Liquids	8.2	76	.132
	Total	1334.5		

*) 380 batches with 1000 items per batch.

Table 2-2: Relative Standard Deviations (SD) for rough (nondestructive) measurement methods.

Class	Measurement method	Random error SD (%)	Systematic error SD (%)	Time needed to verify one item (min)
1	Seal check	-	-	3
2	Seal check	-	-	0.5
3	Seal check	-	-	0.5
4[1)]	γ-scanner	0.2	0.1	4
5[1)]	γ-scanner	0.3	0.2	4
6	Sb-Be	5	2	6
7	Sb-Be	5	2	6
8	Sb-Be	20	20	6

1) The standard deviations in classes 4 and 5 seem to be a little optimistic, but because the quantities are small compared to the total amount of material our results should be still valid.

Table 2-3: Relative Standard Deviations (SD) for fine (destructive) measurement methods.

Class	Random error SD (%)	Systematic error SD (%)	Inspection effort [1)]
4	0.2	0.1	1
5	0.1	0.05	1
6	0.1	0.05	1

1) This column indicates only that we assumed the same inspection effort for each of the destructive measurement methods.

3. Verification Effort Optimization

According to the model agreement /5/ an inspector, sent by the safeguards authority to the plant under consideration, verifies on a random sampling basis the materials balance data reported by the plant operator. Formally, he performs a statistical test in such a way that he tests the null hypothesis H_0 - no data falsification - against the alternative hypothesis H_1 - falsification of the data - corresponding to a certain amount of nuclear material.

In the following, we first consider the case that the operator uses - if at all - only one class specific data falsification procedure and that the inspector uses correspondingly only one measurement method for the verification. Thereafter, we extend these considerations to the case that both, operator and inspector, use two different falsification and measurement methods.

3.1 Tests with one Class Specific Measurement Method

As already mentioned, we have to consider fine and rough verification methods with the help of which fine and rough falsifications of data shall be detected. Fine methods are provided by chemical or so-called destructive assays (DA); rough methods are so-called nondestructive assays (NDA) which make use of the radiation of nuclear material.

In this section we assume that the operator performs - if at all - either a fine or a rough falsification of the materials accountancy data, and furthermore, that the inspector knows this which means that he uses either the fine or the rough measurement method for the verification of the data reported to him.

Let us introduce now the following class specific entities which describe the problem to be analyzed in the following:

(3.1) $K=\{1...k\}$ set of material classes

A_i set of batches in the i-th class ($|A_i|=N_i$),

ε_i effort (time or money) for the inspector's measurement of the material content of one batch in the i-th class,

A_i^x set of batches in the i-th class the data of which are verified by the inspector ($A_i^x \subseteq A_i$, $|A_i|=n_i$),

μ_i class specific falsification of one batch datum in the i-th class

A_i^y set of batches in the i-th class the data of which are falsified.

Furthermore, let Y_{ij} be the random variable describing the measurement result of the operator for the material content of the j-th batch of the i-th class, i=1...k, j=1...N_i. It is written as

$$(3.2)\qquad Y_{ij} = T_{ij} + e_{0ij} + d_{0i} \text{ for } i=1...k,\ j=1...N_i,$$

where T_{ij} ist the true U-235 content before any falsification, e_{0ij} the random measurement error, and d_{0i} the class specific systematic measurement error. We assume that the measurement errors are independent and normally distributed random variables with zero mean values and known variances:

$$(3.3)\qquad E(e_{0ij})=E(d_{0i})=0,\ i=1...k,\ j=1...N_i;$$
$$var(e_{0ij})=\sigma^2_{0ri},\ i=1...k;\ j=1...N_i;$$

$$cov(e_{0ij},e_{0i'j'})=0,\ i,i'=1...k;\ j\neq j';\ j,j'=1...N_i;$$
$$var(d_{0i})=\sigma^2_{0si},\ i=1...k;$$

$$cov(d_{0i},d_{0i'})=0,\ i\neq i';\ i,i'=1...k;$$

$$cov(e_{0ij},d_{0i'})=0,\ i\neq i';\ i,i'=1...k,\ j=1...N_i.$$

Let us assume now that the inspector verifies n_i of the N_i batch data with the help of independent measurements. Let X_{ij}, i=1..k, j=1...n_i be the random variable describing the measurement result of the inspector for the material content of the j-th batch of the i-th class (for simplicity we assume that after a random selection procedure the batches are rearranged in such a way that the first n_i batch data are verified). If no data are falsified, we get for the null hypothesis H_0 (no data falsification)

$$(3.4)\qquad X_{ij} = T_{ij} + e_{Iij} + d_{Ii},\ i=1\ldots k,\ j=1\ldots n_i,$$

where e_{Iij} is the random measurement error, and d_{Ii} the systematic measurement error of the inspector. Again we assume that the measurement errors are independent and normally distributed random variables with zero mean values and known variances (which may be different from those of the operator):

$$(3.5)\qquad \begin{aligned} &E(e_{Iij})=E(d_{Ii})=0,\ i=1\ldots k,\ j=1\ldots n_i \\ &var(e_{Iij})=\sigma^2_{Iri},\ i=1\ldots k;\ j=1\ldots n_i; \\ &cov(e_{Iij},e_{Ii'j'})=0,\ i,i'=1\ldots k;\ j\neq j';\ j,j'=1\ldots n_i; \\ &var(d_{Ii})=\sigma^2_{Isi},\ i=1\ldots k; \\ &cov(d_{Ii},d_{Ii'})=0,\ i\neq i';\ i,i'=1\ldots k; \\ &cov(e_{Iij},d_{Ii'})=0,\ i'\neq i';\ i,i'=1\ldots k,\ j=1\ldots n_i. \end{aligned}$$

Under the alternative hypothesis H_1, that the batch data of the set A_i^Y are falsified by the amount μ_i, we have

$$(3.6)\qquad X_{ij}=\begin{cases} T_{ij}-\mu_i+e_{Iij}+d_{Ii} & \text{for } j\in A_j^x\cap A_j^y \\ T_{ij}+e_{Iij}+d_{Ii} & \phantom{\text{for }} j\notin A_j^x\cap A_j^y \end{cases}$$

In the following we specify the falsification strategies. We consider two models which we call Models A and B. It should be noted, however, that these two models do by no means exhaust all falsification possibilities.

3.1.1 Model A

We call Model A that set of falsification strategies where all N_i batch data of the i-th class are falsified by the class specific amount μ_i. This means

$$(3.7)\qquad |A_i^Y|=N_i \text{ or } r_i=N_i \text{ for } i=1\ldots k.$$

Let us assume that the operator intends to divert the total amount M of nuclear material by means of data falsification. This means that he has to observe for the single falsifications μ_i the boundary condition

(3.8) $$M = \sum_{i \in K} N_i \mu_i$$

Let us assume furthermore, that the inspector has the total effort C at his disposal. This means that he has to observe for the sample series n_i the boundary condition

(3.9) $$C = \sum_{i \in K} n_i \varepsilon_i$$

The problem of the inspector consists in optimizing the probability of detection $1-\beta(n,\mu)$, where $\underline{n}'=(n_1 \ldots n_k)$, $\underline{\mu}'=(\mu_1 \ldots \mu_k)$, for a given false alarm probability α, with respect to $\underline{n}$ under the boundary condition (3.9) for any set $\underline{\mu}$ subject to the boundary condition (3.8). In other words, he has to solve the following minimax-problem

(3.10) $$\max_{\underline{n}} \min_{\underline{\mu}} (1-\beta(\underline{n},\underline{\mu})),$$

with the boundary conditions (3.8) and (3.9).

As all measured results entering the decision procedure of the inspector are disturbed by measurement errors, the data may be evaluated with a <u>test procedure</u>. The inspector is not interested in estimating the true values T_{ij}, but only in the true differences between the operator's and his data. Therefore he will construct the test with the help of the differences

(3.11) $$Z_{ij} = Y_{ij} - X_{ij}, \quad i=1\ldots k, \quad j=1\ldots n_i$$

which are according to our assumptions independent and normally distributed with known variances

(3.12) $$\text{var}(Z_{ij}) = \sigma^2_{Ori} + \sigma^2_{Iri} + \sigma^2_{Osi} + \sigma^2_{Isi}, \quad \begin{matrix} i = 1,2,..,k \\ j = 1,2,..,n; \end{matrix}$$

If one treats the sample series n_i, $i=1..k$, as continuous variables, then the solution of the problem (3.10), and also the solution of the optimization problem

$$\min_{\underline{\mu}} \max_{\underline{n}} (1-\beta(\underline{n},\underline{\mu})),$$

is given by the following set of formulae /2/:

(3.13a) $$n_i^* = \frac{C}{\sum_e N_e \sigma_{re} \sqrt{\varepsilon_e}} \cdot \frac{N_i \sigma_{ri}}{\sqrt{\varepsilon_i}}$$

(3.13b) $$\mu_i^* = \frac{M}{\sigma^2(C)} \cdot \frac{1}{C} \cdot ((\sum_e N_e \sigma_{re} \sqrt{\varepsilon_e}) \sigma_{ri} \cdot \sqrt{\varepsilon_i} + CN_i \sigma_{si}^2)$$

$i = 1,2,\ldots,k$

(3.13c) $$1-\beta_A^* = \Phi(\frac{M}{\sigma(C)} - U_{1-\alpha})$$

where

(3.13d) $$\sigma^2(C) = \frac{1}{C} (\sum_e N_e \sigma_{re} \cdot \sqrt{\varepsilon_e})^2 + \sum_e N_e^2 \sigma_{se}^2,$$

(3.13e) $$\sigma_{ri}^2 = \sigma_{Ori}^2 + \sigma_{Iri}^2, \quad \sigma_{si}^2 = \sigma_{Osi}^2 + \sigma_{Isi}^2$$

and where $\Phi(.)$ is the normal distribution function and U. its inverse. The optimal test procedure is the so-called D statistic

(3.14) $$D = \sum_{i\in K} N_i (\sum_j Z_{ij})/n_i^*$$

which has been proposed earlier by Stuart /6/ who gave heuristic arguments for its use.

It should be noted that the solution (3.13) of the optimization problem (3.10) includes the solution of one further optimization problem which has not been mentioned explicitly, namely the determination of the best test procedure in the sense of the Lemma of Neyman and Pearson /7/.

From (3.13c) and (3.13d) we get the effort C_A^* which is necessary for achieving the guaranteed probability of detection $1-\beta_A^*$ which we call here simply $1-\beta$:

$$(3.15) \qquad C^* = \frac{(U_{1-\alpha} + U_{1-\beta^*})^2 \cdot (\sum_i N_i \sigma_{ri} \cdot \sqrt{\varepsilon_i})^2}{M^2 - U_{1-\alpha}^2 (\sum_i N^2 \sigma_{si}^2)}$$

In case of destructive analysis we have $\varepsilon_i=\varepsilon$ for i=1...k, therefore we get with $n=C/\varepsilon$ from (3.12)

$$n_i^* = \frac{n}{\sum_e N_e \sigma_{re}} N_i \sigma_{ri}$$

$$(3.16) \qquad \mu_i^* = \frac{M}{\sigma^2(n)} ((\sum_e N_e \sigma_{re}) \sigma_{ri} + N_i \sigma_{si}^2)$$

$$1-\beta_A^* = \Phi(\frac{M}{\sigma(n)} - U_{1-\alpha})$$

$$\sigma(n) = \frac{1}{n} (\sum_i N_i \sigma_{ri})^2 + \sum_i N_i^2 \sigma_{si}^2$$

$$n^* = \frac{(U_{1-\alpha}-U_{1-\beta})^2 (\sum_i N_i \sigma_{ri})^2}{M^2-U_{1-\alpha}^2 (\sum N_i^2 \cdot \sigma_{si}^2)}$$

3.1.2 Model B

We call Model B that set of falsification strategies, where only $r_i(\leq N_i)$ batch data of the i-th class are falsified by the class specific amount μ_i:

$$(3.17) \qquad |A_i^y|=r_i \text{ for } i=1...k.$$

If the operator intends to divert the total amount M of nuclear material by means of data falsification, then he has to observe for the single falsifications μ_i and for the sample series r_i the boundary condition

$$(3.18) \qquad M = \sum_{i\in K} \mu_i r_i$$

In this case the optimization problem of the inspector is

(3.19) $$\max_{\underline{n},\underline{r}} \min_{\underline{\mu}} (1-\beta(\underline{n},\underline{r},\underline{\mu}))$$

where $\underline{r}'=(r_1...r_k)$, and where $\underline{n},\underline{r}$ and $\underline{\mu}$ are subject to the boundary conditions (3.9) and (3.18).

Contrary to the case of Model A it is not possible to give a complete analytical solution for this problem. If one takes the test statistic

(3.20) $$D = \sum_i N_i (\sum_j Z_{ij})/n_i$$

which was proven to be optimal in case of Model A, also as test statistic for Model B, then one can solve the limited problem (3.19). If one treats the sample series n_i and r_i, $i=1...k$, as continuous variables, then the solution, which is also solution of the problem

$$\max_{\underline{\mu}} \min_{\underline{n},\underline{r}} (1-\beta(\underline{n},\underline{r},\underline{\mu}))$$

is under the assumption

(3.21) $$M^2/U^2_{1-\alpha} >> (\sum_i N_i\ \sigma_{ri} \cdot \sqrt{\varepsilon_i})^2/C + \sum_i N_i^2\ \sigma^2_{si}$$

given by the following set of formulae /2/:

(3.22a) $$n_i^* = \frac{C}{\sum_e N_e \sigma_{re} \sqrt{\varepsilon_e}} \cdot \frac{N_i \sigma_{ri}}{\sqrt{\varepsilon_i}}$$

(3.22b) $$r_i^* = N_i/2$$

(3.22c) $$\mu_i^* = \frac{2M}{\sum_e N_e \sigma_{re}} \sigma_{ri}, \qquad i = 1,2,\ldots,k$$

(3.22d) $$1-\beta^* = \Phi((M - U_{1-\alpha} \sigma_{DO}^*)/\sigma_{D1}^*)$$

Where σ_{DO}^{*2} and σ_{D1}^{*2} are given by

(3.22e) $$\sigma_{DO}^{*2} = (\sum_{i\in K} N_i \sigma_{ri} \sqrt{\varepsilon_i})^2/C + \sum_{i\in K} N_i^2 \sigma_{si}^2$$

(3.22f) $$\sigma_{D1}^{*2} = (\sum_{i\in K} N_i \sigma_{ri} \sqrt{\varepsilon_i})^2 (1 + M^2/(\sum_i N_i \sigma_{ri})^2)/C + \sum_{i\in K} N_i^2 \sigma_{si}^2$$

and where σ_{ri}^2 and σ_{si}^2 are again given by (3.13e).

From (3.22d) we get the effort C_B^x which is necessary for achieving the guaranteed probability of detection $1-\beta_B^*$ which we call here simply

1-β: For $\alpha<0.5$ and $\beta<0.5$ we get

$$C_B^* = - \frac{A^2(B(K^2-L^2)(K^2H-L^2)-M^2(K^2H+L^2))}{B^2(K^2-L^2)^2-2M^2B(K^2+L^2)+M^4} +$$

(3.23a)

$$+ \frac{A^2\ 2\ M\ K\ L\ \sqrt{M^2\ H+B\ (K^2H-L^2)(H-1)}}{B^2(K^2-L^2)^2-2\ M^2\ B(K^2+L^2)+M^4}$$

where the quantities A,B,D,H,K and L are defined by

(3.23b) $$A = \sum_{i\in K} N_i\ \sigma_{ri}\ \sqrt{\varepsilon_i},$$

(3.23c) $$B = \sum_{i\in K} N_i^2\ \sigma_{si}^2,$$

(3.23d) $$D = \sum_{i\in K} N_i\ \sigma_{ri},$$

(3.23e) $$H = 1 + M^2/D^2,$$

(3.23f) $$K = U_{1-\beta}$$

and

(3.23f) $$L = U_{1-\alpha}.$$

(The capital letters A,B,D,H and K have already been used in a different meaning, but there should be no confusion, as they are used in the meaning given here only as arguments of β and in connection with C.)

In case of destructive analyses we have $\varepsilon_i=\varepsilon$ for i=1...k, therefore we get with $n=C/\varepsilon$ from (3.22):

$$n_i^* = \frac{n}{\sum_e N_e \sigma_{re}} N_i \sigma_{ri} , \quad i = 1,2,..,k \tag{3.24}$$

r_i^*, μ_i^*, $i=1,..,k$, and $1-\beta_B^*$ are the same as (3.22b,c and d),

$$\sigma_{D0}^{*2} = (\sum_{i\in K} N_i \sigma_{ri})^2/n + \sum_{i\in K} N_i^2 \sigma_{si}^2$$

$$\sigma_{D1}^{*2} = \sigma_{D0}^{*2} + M^2/n$$

and furthermore, for n^* we get the same expression as that given by (3.23a), where A is replaced by D.

3.1.3 Comparison

The guaranteed probability of detection for Models A and B can according to formulae (3.13c) and (3.22d) be written as

$$1-\beta_{A/B}^* = \Phi\left(\frac{M- \sqrt{A^2/C + B}\; U_{1-\alpha}}{\sqrt{A^2 H/C + B}}\right) \tag{3.25a}$$

where H is given by

(3.25b) $$H = \begin{cases} 1 & \\ & \text{for Model} \\ 1 + M^2/D^2 & \end{cases} \begin{matrix} A \\ \\ B \end{matrix}$$

and where A,B and D are given by (3.23b,c and d). Both probabilities (3.25a) are monotonically increasing functions of the effort C with the limiting probability

(3.26) $$\lim_{C\to\infty} (1-\beta_A^*) = \lim_{C\to\infty} (1-\beta_B^*) = \Phi(M/\sqrt{B} - U_{1-\alpha})$$

For given values of the amount M of material to be diverted and inspection effort C the operator can influence the guaranteed probability of detection only via the choice of the Models. As the Φ-function is a monotonely increasing function of its argument, we have

$$1-\beta_A^* \lesseqgtr 1-\beta_B^* \text{ if and only if } M \lesseqgtr \sigma_{DO}^* \, U_{1-\alpha}$$

It should be noted, however, that Model A is taken only if the argument of the Φ-function in (3.24a) is negative, i.e., if the probability of detection is smaller than 0.5. If we assume this to be an irrelevant case, then always Model B will be taken by the operator.

On the other hand it should be kept in mind, that the solution for Model B holds only under the assumption (3.21), i.e., under the assumption

$$M >> \sqrt{A^2/C+B} \, U_{1-\alpha}.$$

This means, that if this solution holds, then Model B is better for the operator than Model A. In general terms, we can interpret these results as follows: If the operator intends to divert only a small amount of material, then he will use Model A because such a small falsification might be covered by the measurement errors. If he intends to divert large amounts, he "plays vabangue", he falsifies only a few data by relatively large amounts and hopes that they will not be chosen for verification by the inspector.

In Figures 3-1 through 3-6 and 3-7 through 3-12 examples for the probabilities of detection $1-\beta_A$ and $1-\beta_B$ as functions of the amount M to be diverted and fixed verification effort C for the initial inventory (data given in Table 2) with fine and rough measurements are given. One observes that the change of the Model, which is better from the operator's point of view, occurs at $1-\beta=0.5$; the corresponding value of M depends on the value of C. If one compares the figures which belong to different C values, one gets a qualitative idea for those regions of values of C, where the probability of detection changes significantly, in other words, where an increase of the verification effort still is justified.

Let us still consider the question of the choice of the best Model from the point of view of the operator, if the amount M of material to be diverted and the probability of detection, defining H according to (3.25b). One can show that for given values of

$$K=U_{1-\beta}>0.5, \quad L=U_{1-\alpha}>0.5$$

the value of C_B^* is always larger than that of C_A^* which means that also under the boundary condition of a given probability of detection $1-\beta$ the operator will chose Model B.

In Figures 3-13 through 3-15 and 3-16 through 3-18 for the initial inventory data given by Table 2-1 examples for the inspection efforts C_A and C_B as functions of the amounts M to be diverted with fixed probability of detection are given. If one compares the figures which belong to different C values, one gets a qualitative idea for those regions of values of the probability of detection, where the effort changes significantly.

In Tables 3-1 and 3-2 for the initial inventory data given by Table 2-1 sample series, amounts to be diverted and standard derivations for Models A and B for fine and for rough measurements are given for fixed values of M, C_A, C_B and $1-\beta$.

Table 3-1: Solutions for n_i^*, μ_i^* and the corresponding standard deviations using Tab. 2.1, Model A and B and destructive analysis
$M=35$, $\beta=0.05$, $C_A=0.293$, $C_B=4.06$

Class	4	5	6
σ_r	3.2 E-5	2.1 E-4	1.815 E-1
σ_s	1.6 E-5	1.4 E-4	7.26 E-2
n^* (Mod. A)	8.312 E-3	1.604 E-3	2.83 E-1
n^* (Mod. B)	1.152 E-1	2.223 E-2	3.921
μ^* (Mod. A)	1.908 E-4	1.25 E-3	1.429
μ^* (Mod. B)	3.975 E-4	2.608 E-3	2.254

Model A: σ_{DO} = 10.63

Model B: σ_{DO} = 3.546, σ_{D1} = 17.73

Table 3-2: Solutions for n_i^*, μ_i^* and corresponding standard deviations using Tab. 2.1, Model A and B and nondestructive analysis
$M=35$, $\beta=0.05$, $C_A=1230$, $C_B=1860$

Class	4	5	6	7	8
σ_r	4.525 E-5	2.97 E-4	2.567 E-1	9.334 E-3	3.734 E-2
σ_s	2.263 E-5	1.98 E-4	1.027 E-1	3.734 E-3	3.734 E-2
n^* (Mod. A)	1.651	3.186 E-1	30.	14.12	15.91
n^* (Mod. B)	2.496	4.818 E-1	30.	21.36	25.57
μ^* (Mod. A)	2.708 E-6	1.436 E-1	1.111 E-1	1.143 E-2	3.469 E-2
μ^* (Mod. B)	9.180 E-5	6.025 E-4	5.207 E-1	1.893 E-2	7.574 E-2

Model A: σ_{DO} = 10.63
Model B: σ_{DO} = 10.54, σ_{D1} = 10.73

Figures 3-1 to 3-12:

Detection probability 1-β as a function of amount of falisification M for fixed inspection effort.

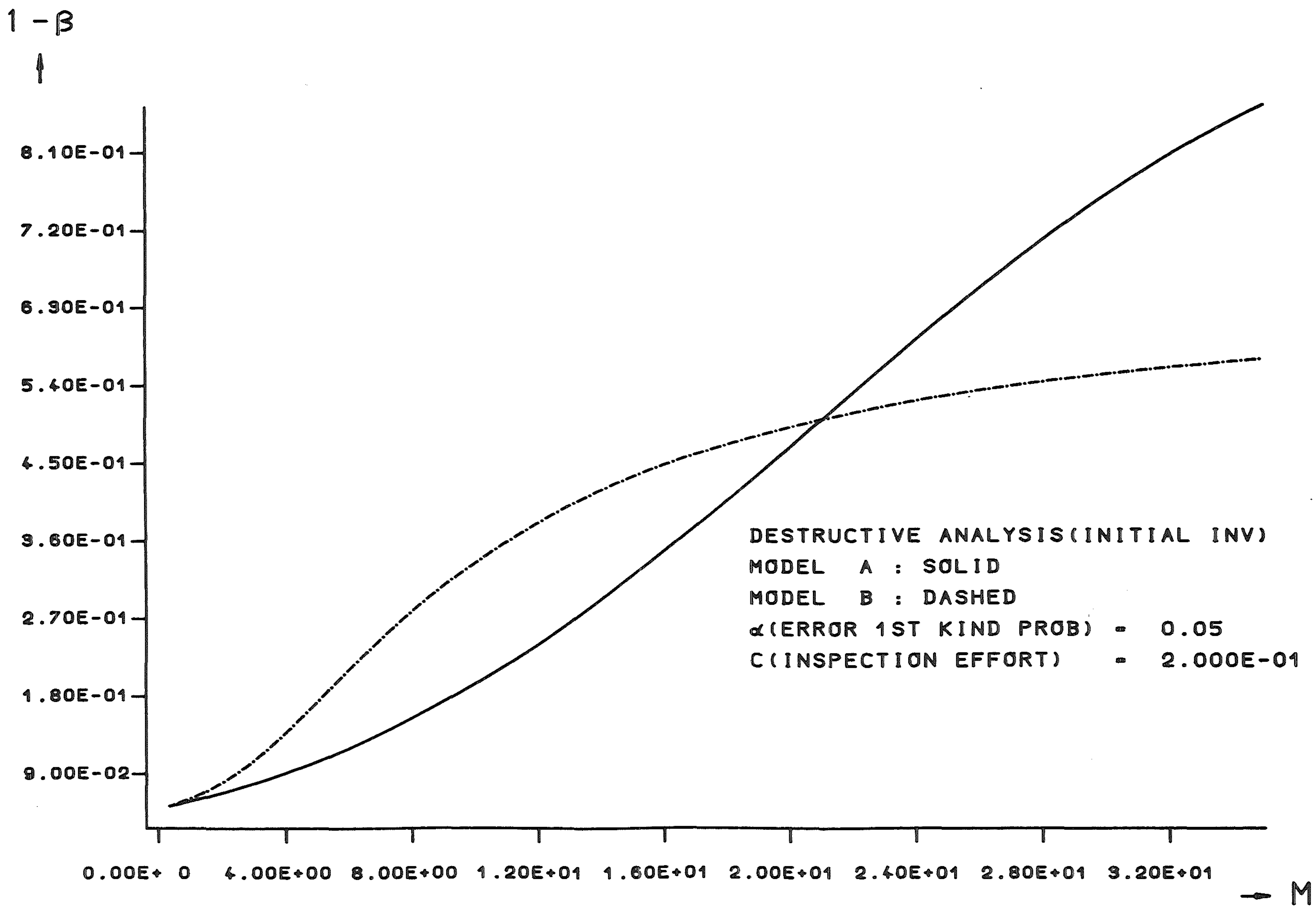

1 - β
8.10E-01
7.20E-01
6.30E-01
5.40E-01
4.50E-01
3.60E-01
2.70E-01
1.80E-01
9.00E-02
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M
DESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 2.000E-01

Figure 3-1

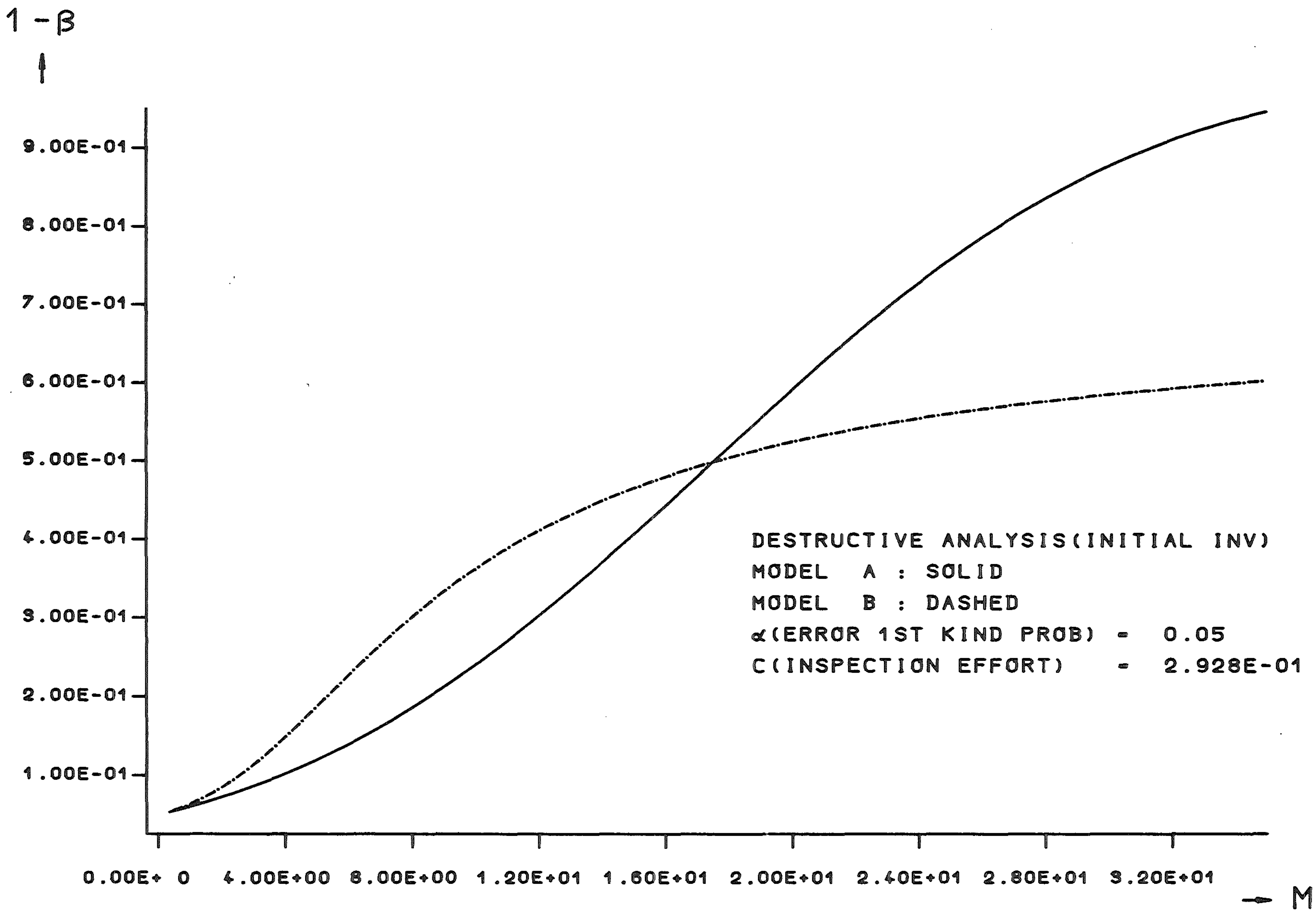

1 - β
9.00E-01
8.00E-01
7.00E-01
6.00E-01
5.00E-01
4.00E-01
3.00E-01
2.00E-01
1.00E-01
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M
DESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 2.928E-01

Figure 3-2

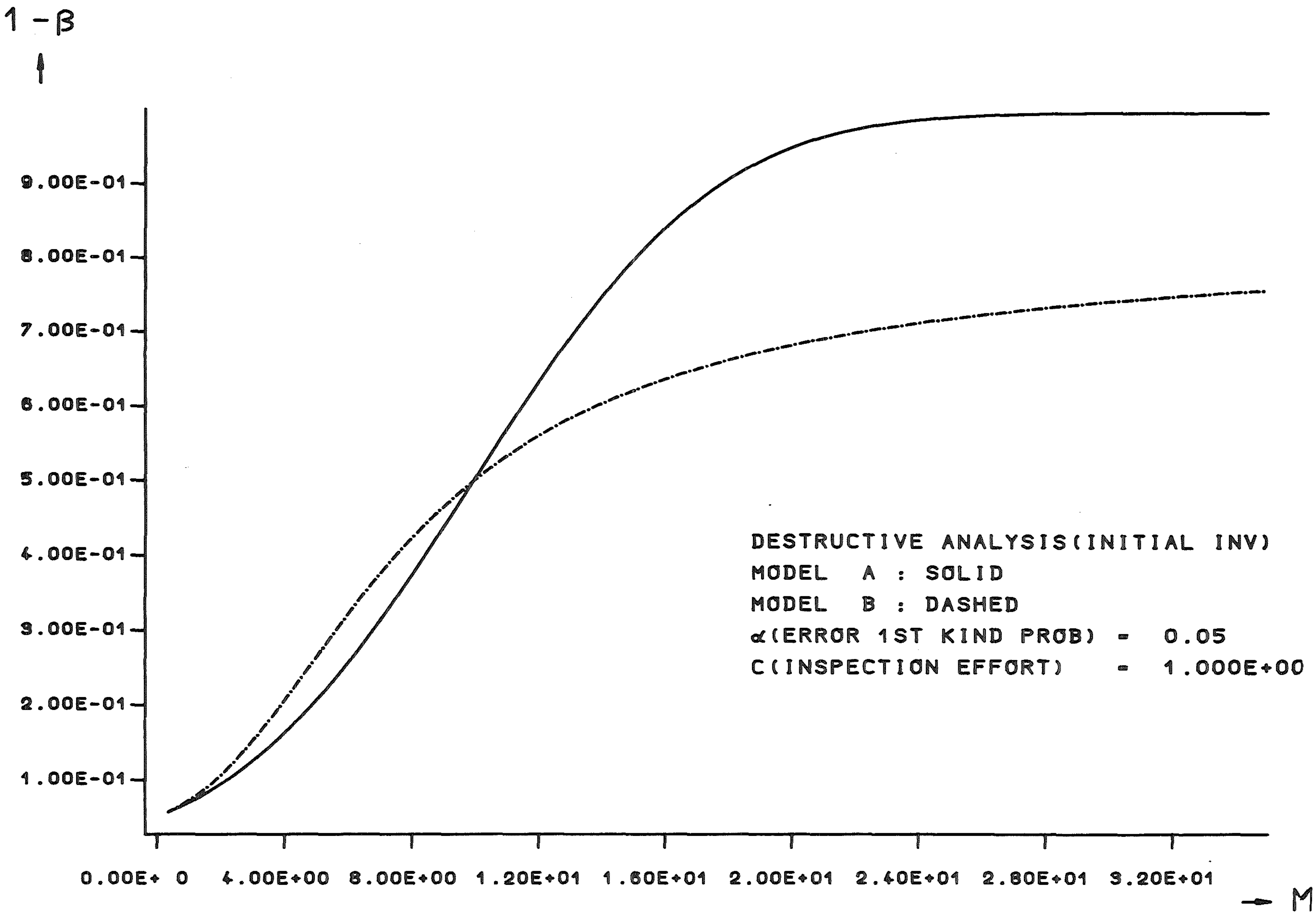

1 -β
9.00E-01
8.00E-01
7.00E-01
6.00E-01
5.00E-01
4.00E-01
3.00E-01
2.00E-01
1.00E-01
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M
DESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 1.000E+00

Figure 3-3

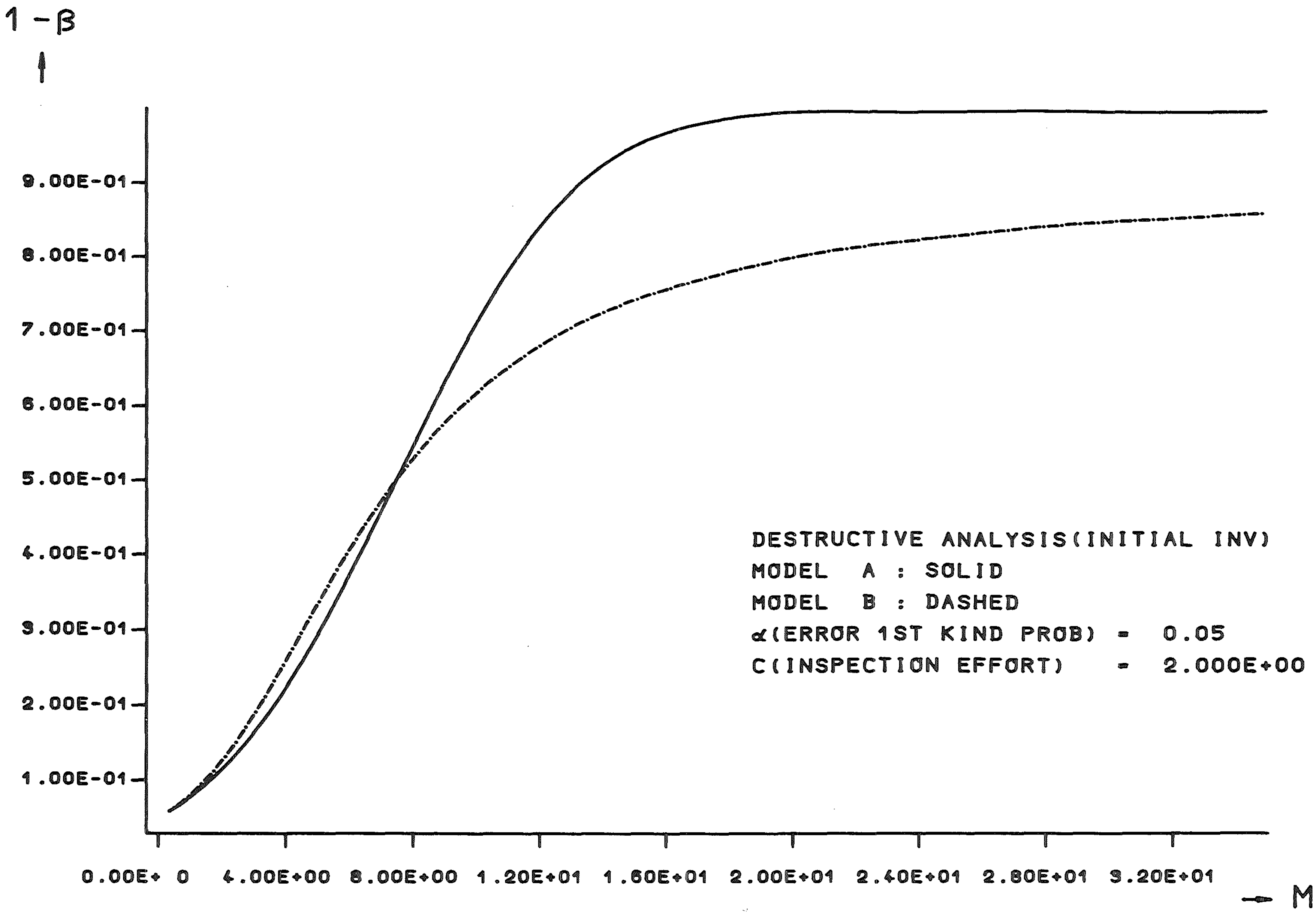

1 -β
9.00E-01
8.00E-01
7.00E-01
6.00E-01
5.00E-01
4.00E-01
3.00E-01
2.00E-01
1.00E-01
DESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 2.000E+00
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M

Figure 3-4

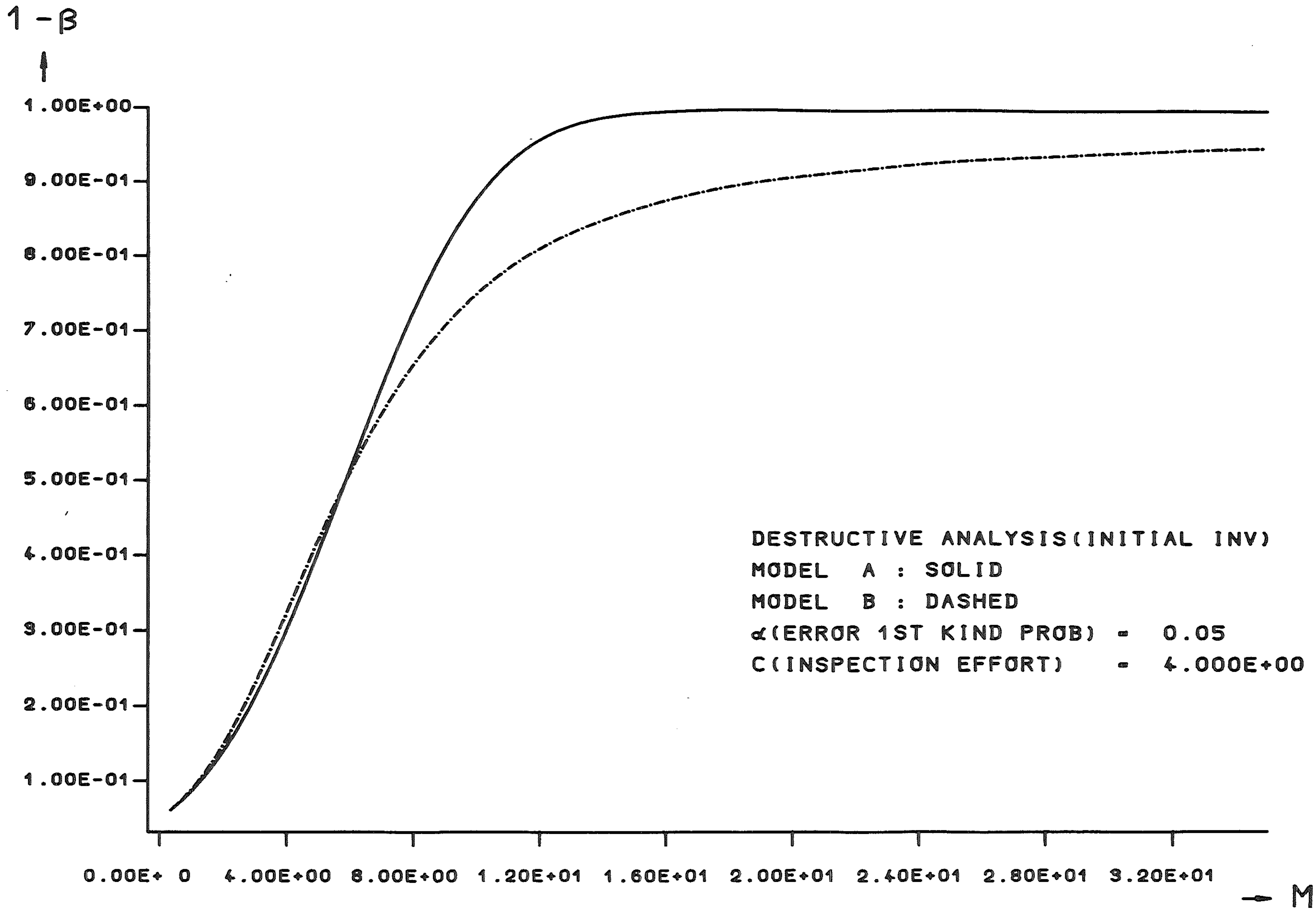

1 -β
1.00E+00
9.00E-01
8.00E-01
7.00E-01
6.00E-01
5.00E-01
4.00E-01
3.00E-01
2.00E-01
1.00E-01
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M
DESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 4.000E+00

Figure 3-5

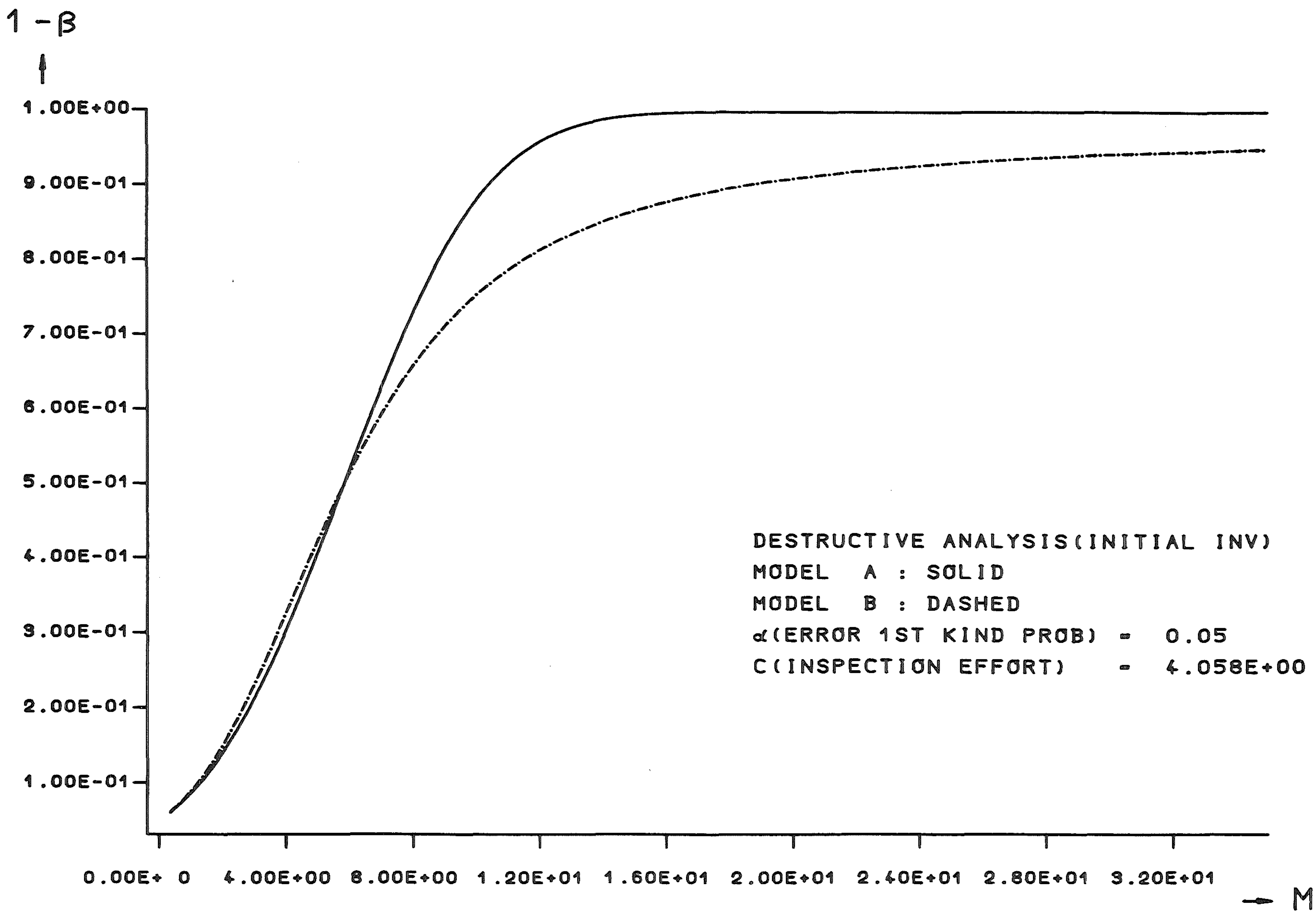

1 - β
1.00E+00
9.00E-01
8.00E-01
7.00E-01
6.00E-01
5.00E-01
4.00E-01
3.00E-01
2.00E-01
1.00E-01
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M
DESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 4.058E+00

Figure 3-6

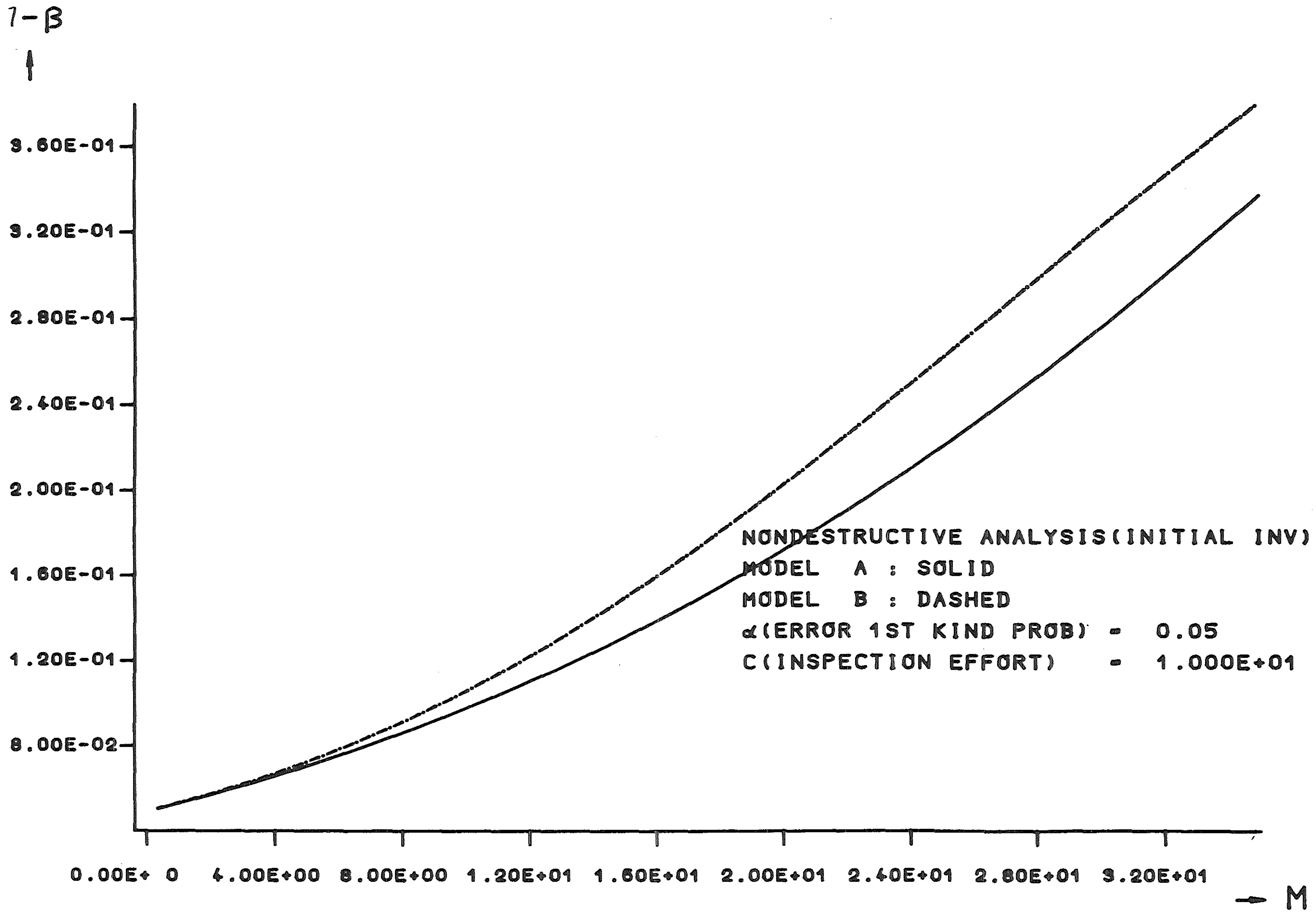

1-β
3.60E-01
3.20E-01
2.80E-01
2.40E-01
2.00E-01
1.60E-01
1.20E-01
8.00E-02
NONDESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 1.000E+01
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M

Figure 3-7

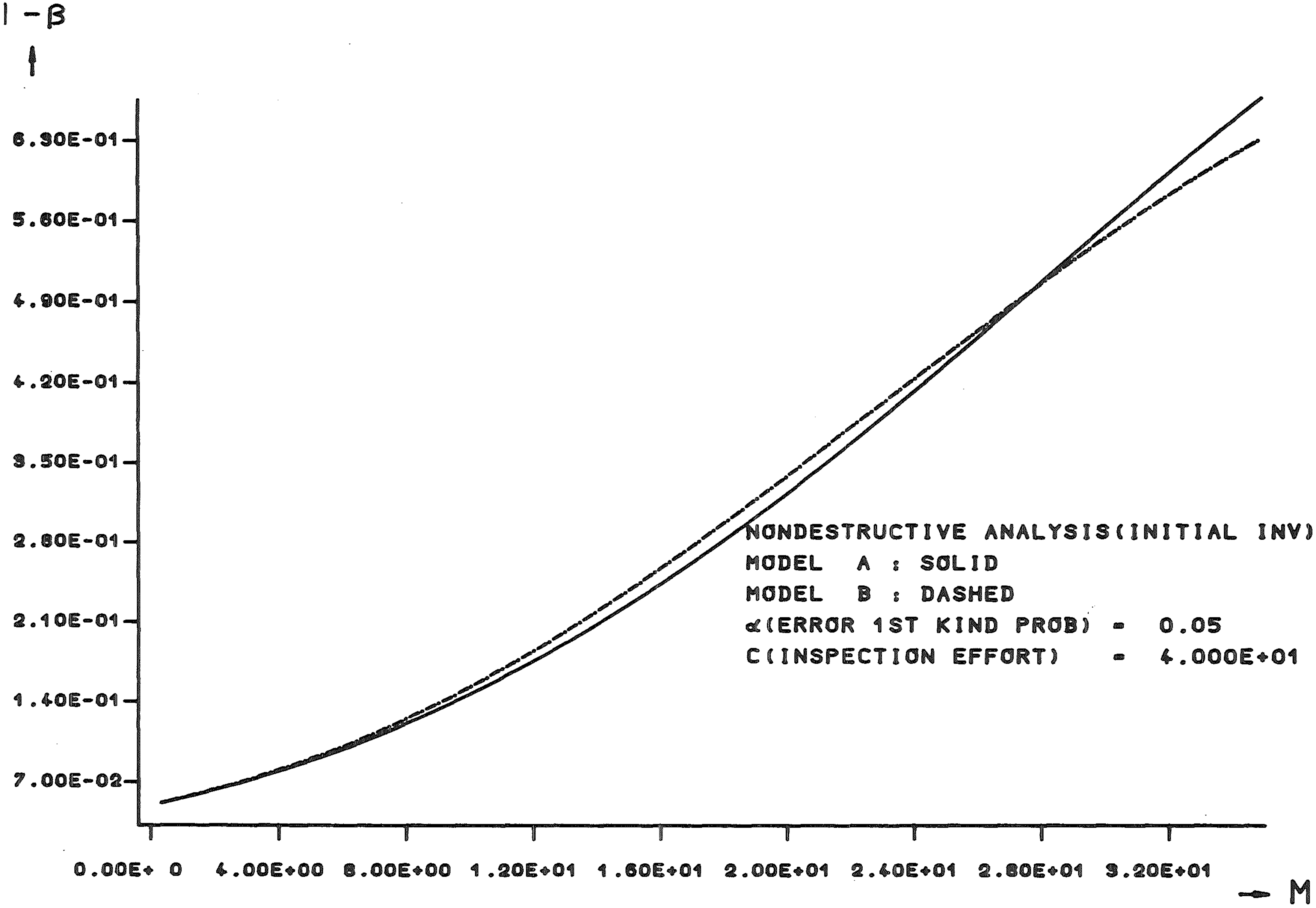

1 - β
6.30E-01
5.60E-01
4.90E-01
4.20E-01
3.50E-01
2.80E-01
2.10E-01
1.40E-01
7.00E-02
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M
NONDESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 4.000E+01

Figure 3-8

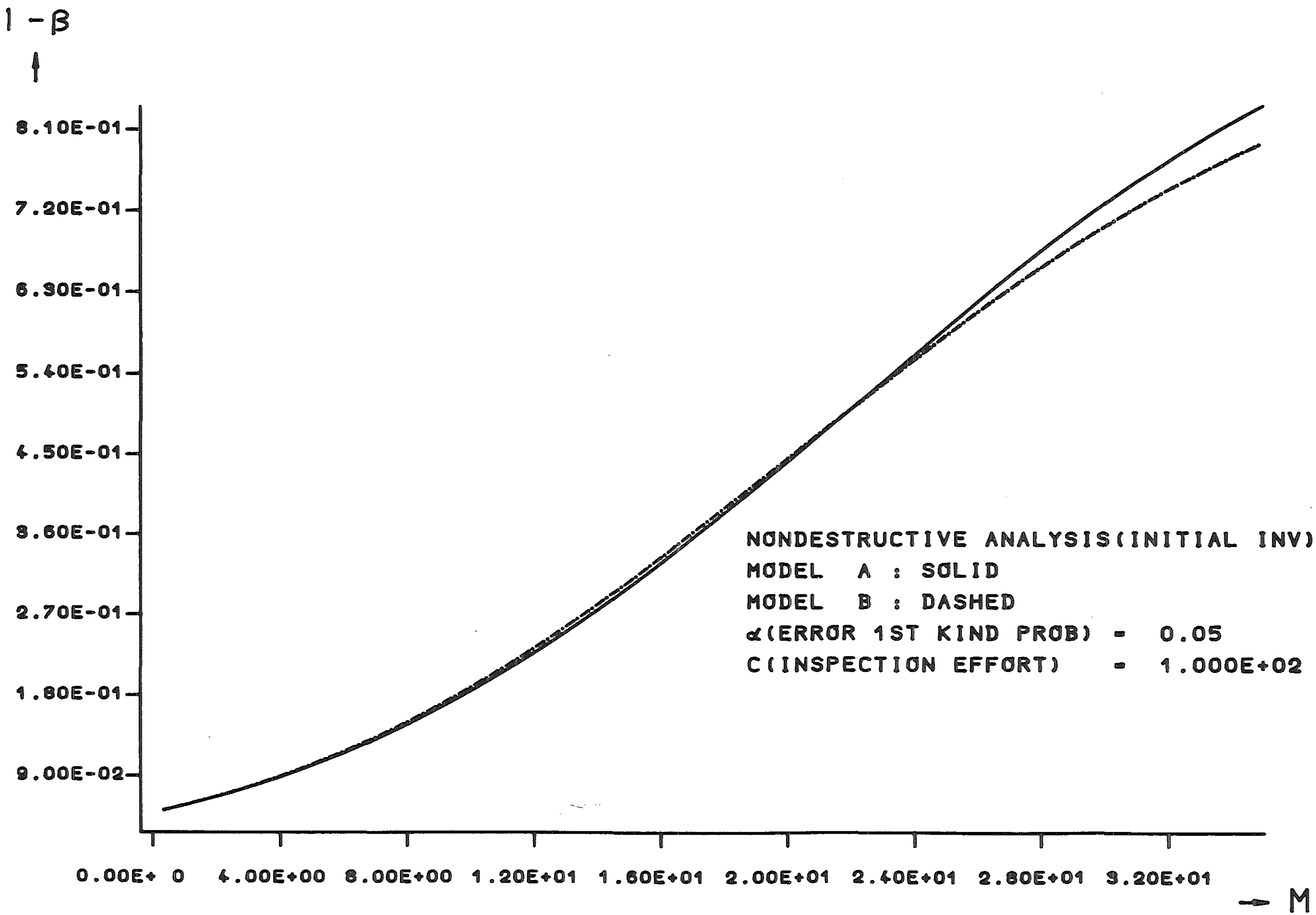

1 - β
8.10E-01
7.20E-01
6.30E-01
5.40E-01
4.50E-01
3.60E-01
2.70E-01
1.80E-01
9.00E-02
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M
NONDESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 1.000E+02

Figure 3-9

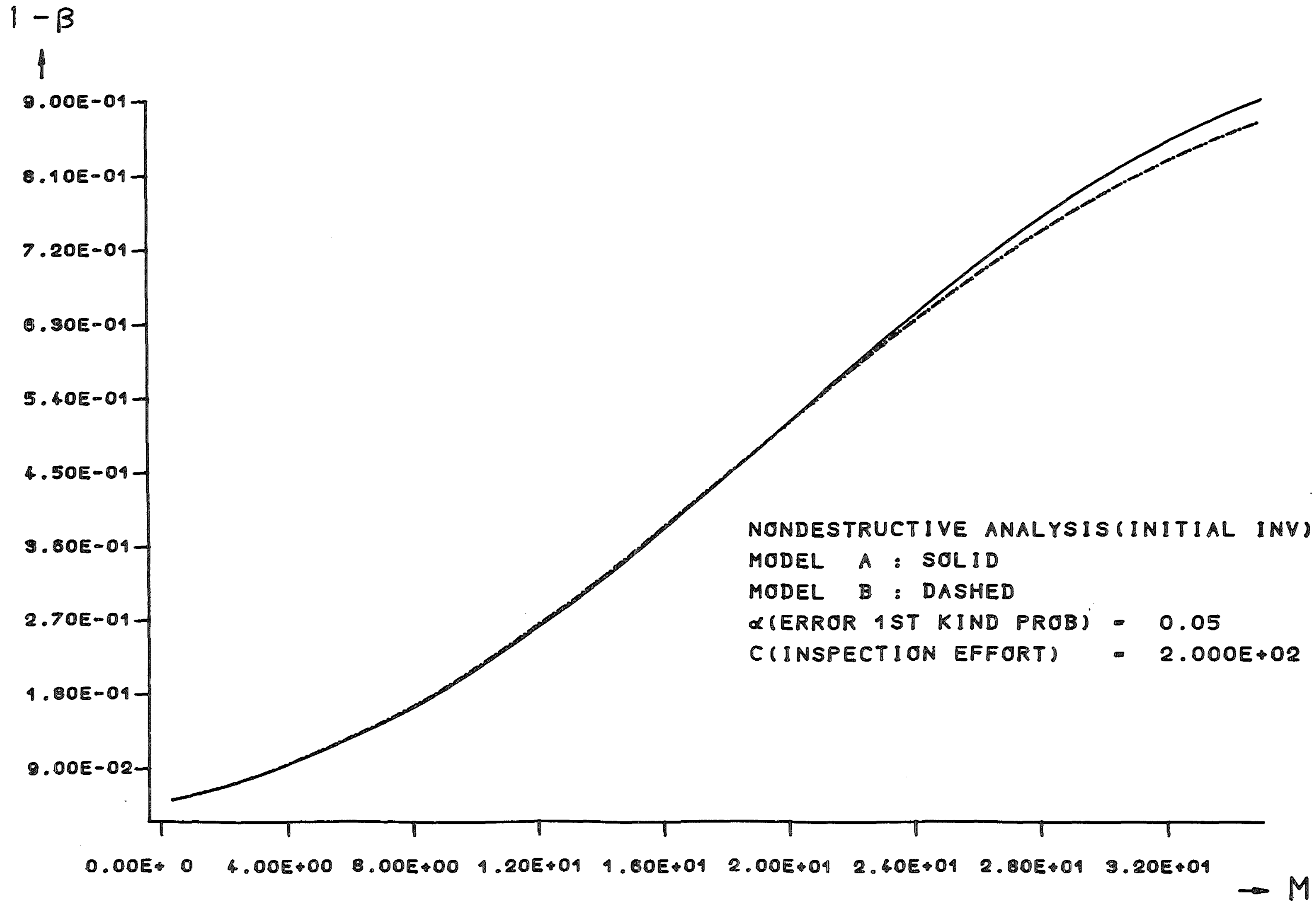
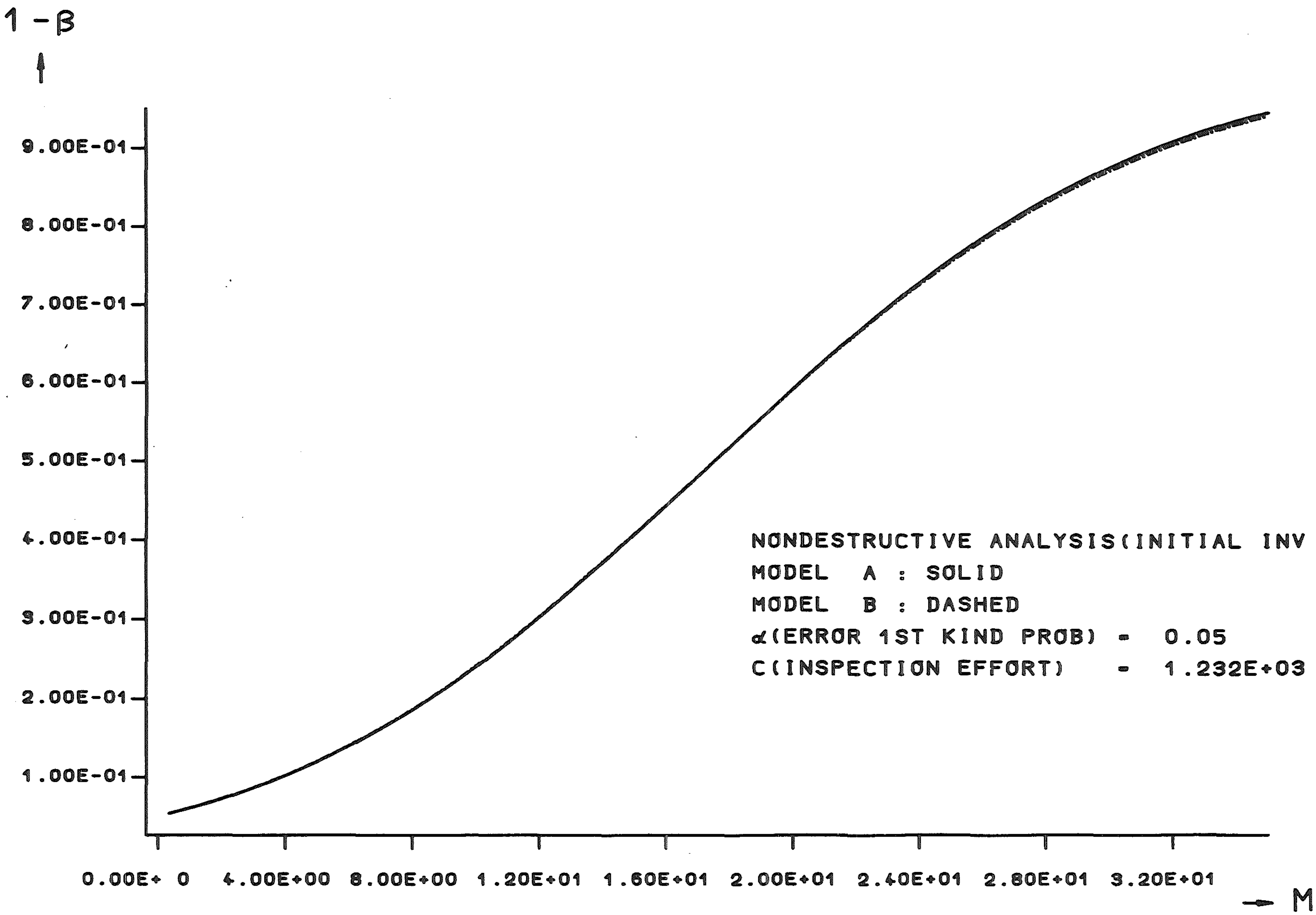

1 - β
9.00E-01
8.10E-01
7.20E-01
6.30E-01
5.40E-01
4.50E-01
3.60E-01
2.70E-01
1.80E-01
9.00E-02
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M
NONDESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 2.000E+02

Figure 3-10



1 - β
9.00E-01
8.00E-01
7.00E-01
6.00E-01
5.00E-01
4.00E-01
3.00E-01
2.00E-01
1.00E-01
NONDESTRUCTIVE ANALYSIS(INITIAL INV
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 1.232E+03
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M

Figure 3-11

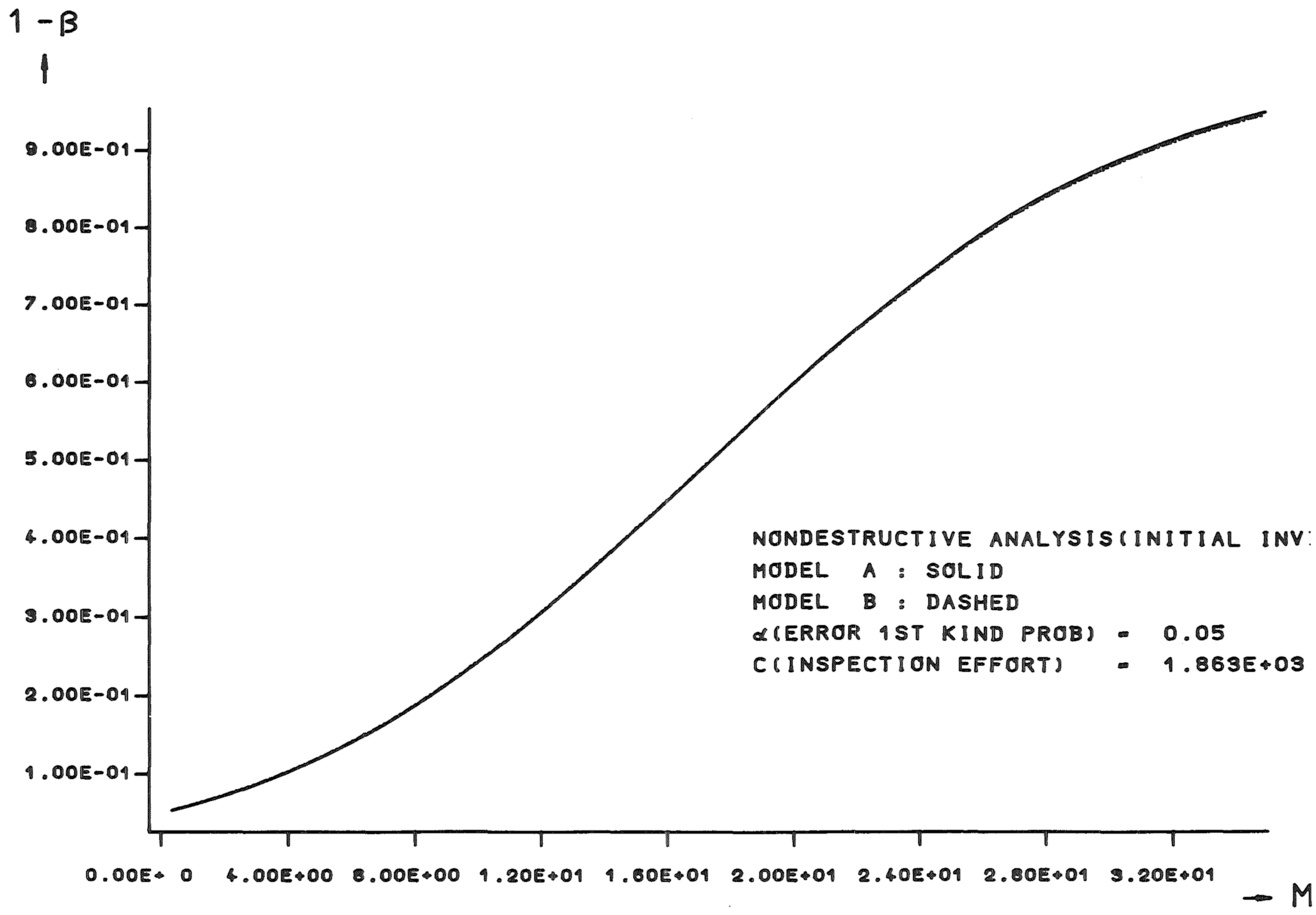

1 - β
9.00E-01
8.00E-01
7.00E-01
6.00E-01
5.00E-01
4.00E-01
3.00E-01
2.00E-01
1.00E-01
NONDESTRUCTIVE ANALYSIS(INITIAL INV
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
C(INSPECTION EFFORT) = 1.863E+03
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M

Figure 3-12

Figures 3-13 to 3-18:

Inspection efforts C_A and C_B as functions of the amount of falsification M with constant detection probabilities.

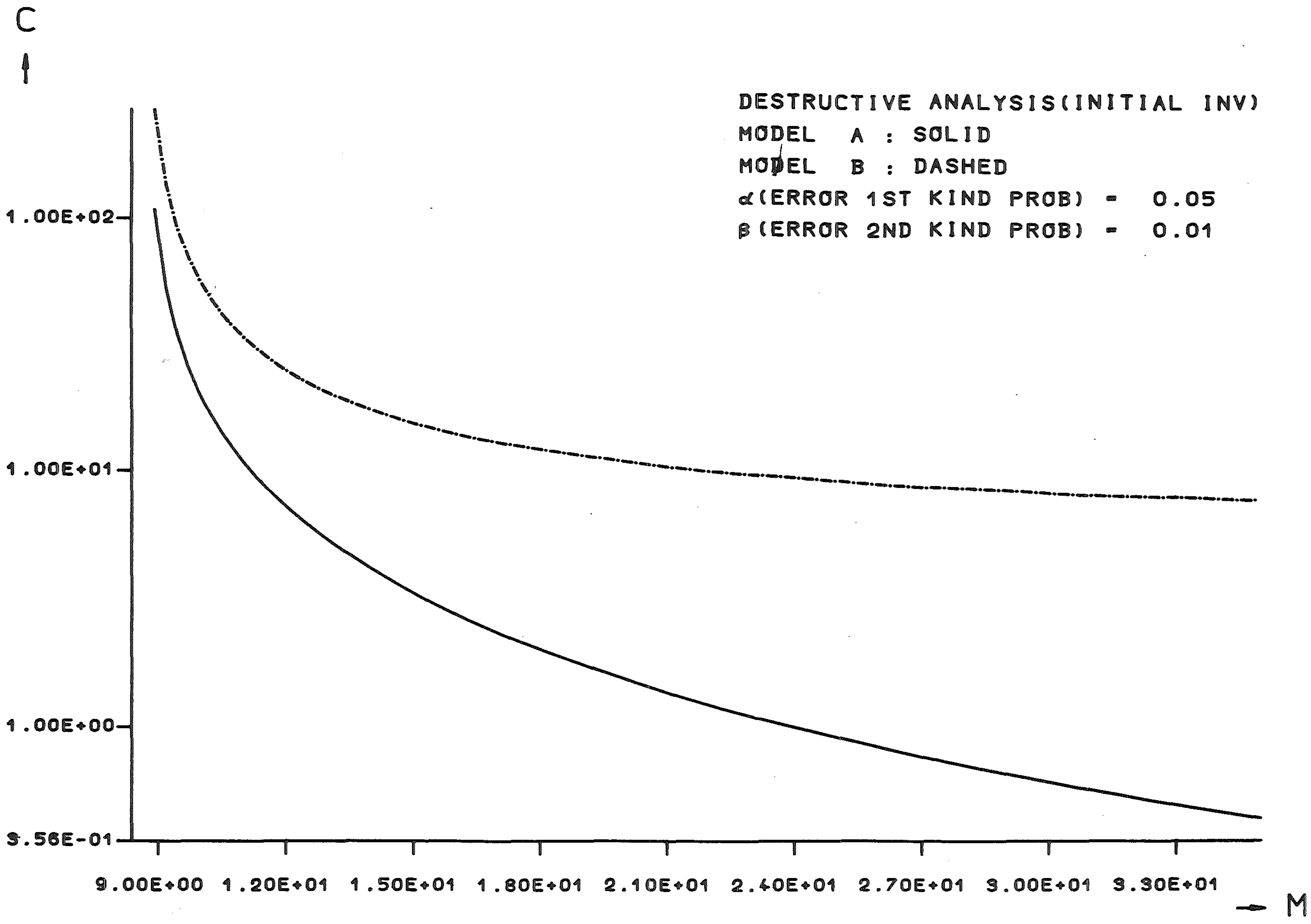

C
DESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
β(ERROR 2ND KIND PROB) = 0.01
1.00E+02
1.00E+01
1.00E+00
3.56E-01
9.00E+00
1.20E+01
1.50E+01
1.80E+01
2.10E+01
2.40E+01
2.70E+01
3.00E+01
3.30E+01
M

Figure 3-13

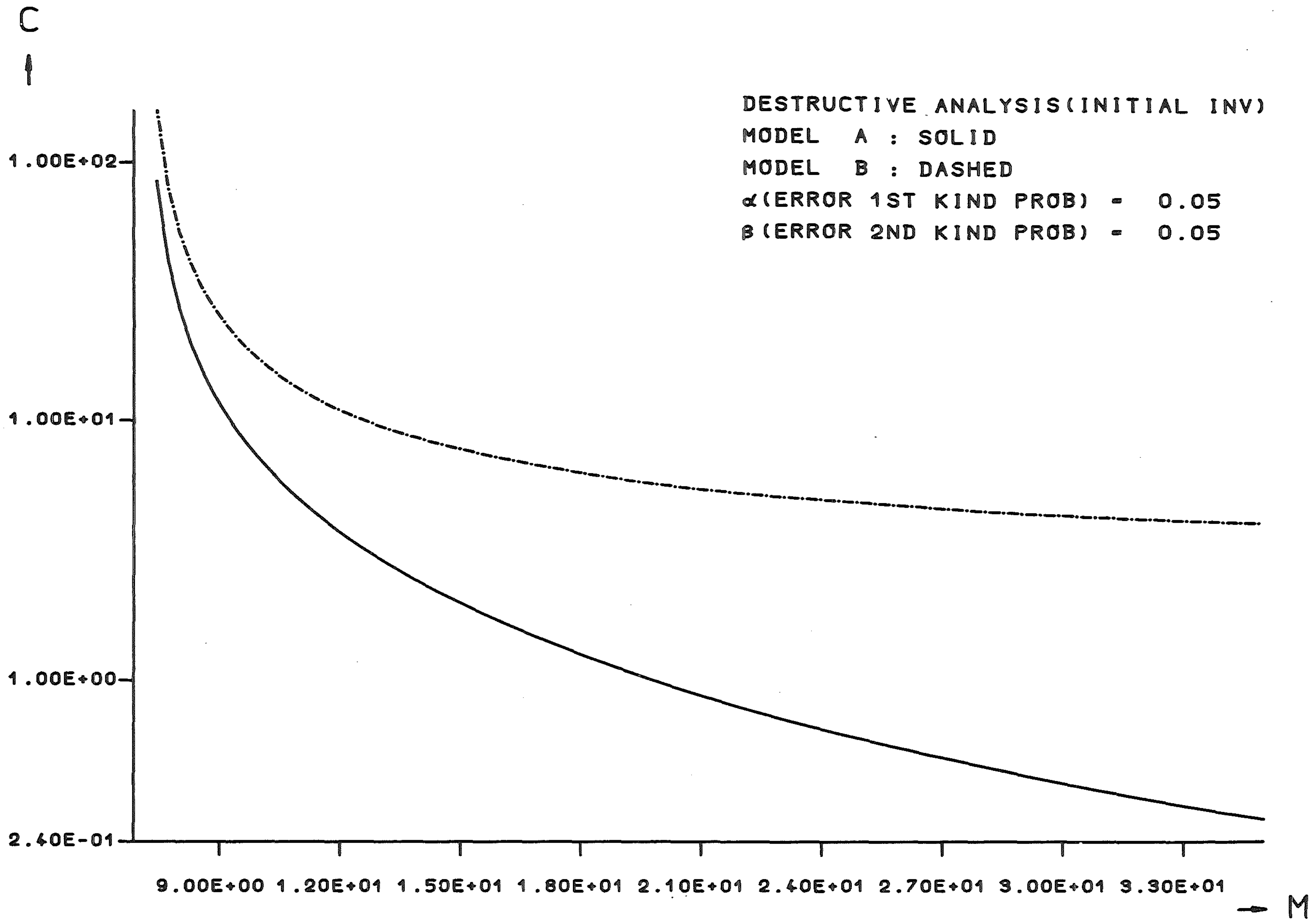

C
DESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
β(ERROR 2ND KIND PROB) = 0.05
1.00E+02
1.00E+01
1.00E+00
2.40E-01
9.00E+00 1.20E+01 1.50E+01 1.80E+01 2.10E+01 2.40E+01 2.70E+01 3.00E+01 3.30E+01
M

Figure 3-14

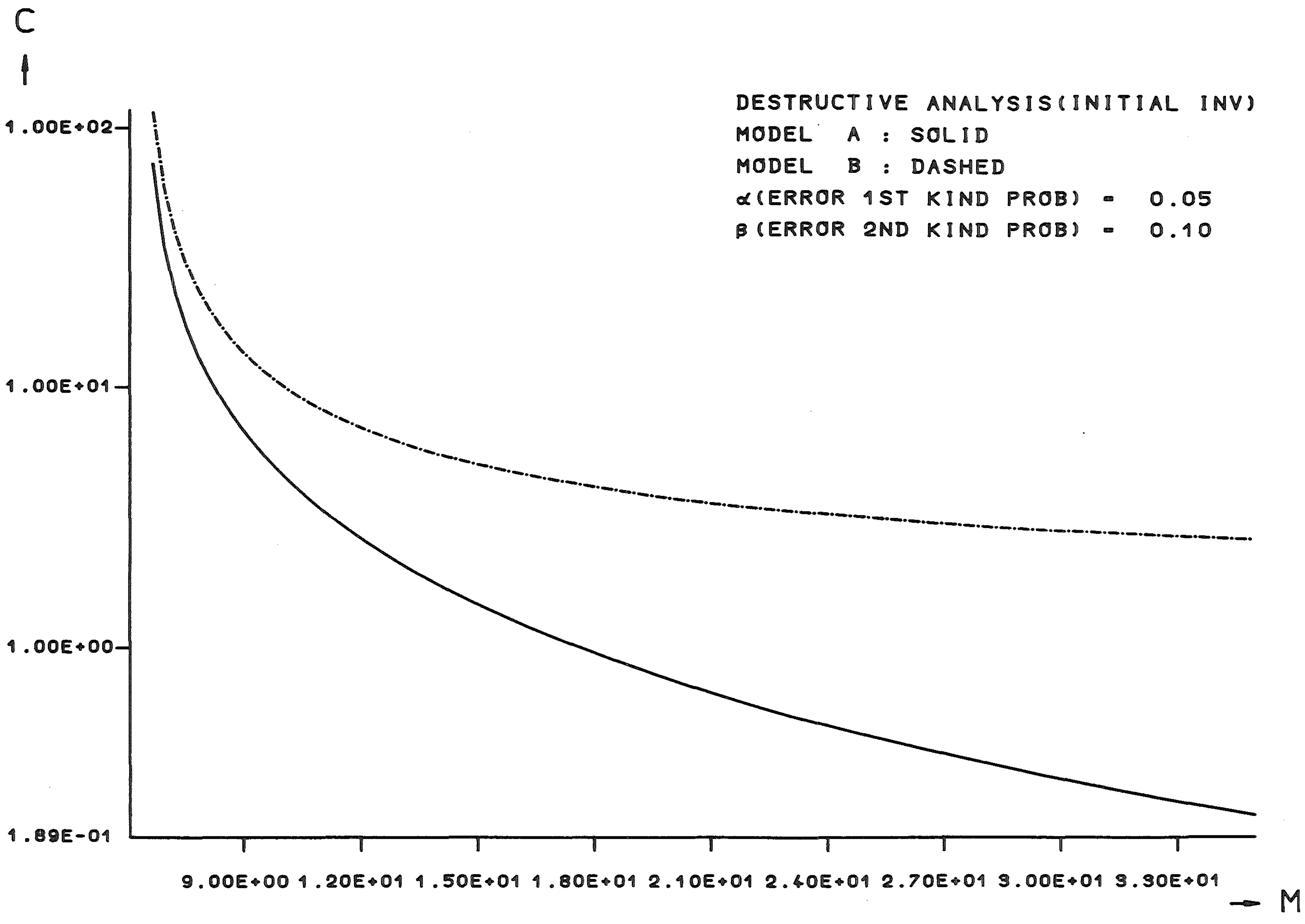

C
DESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
β(ERROR 2ND KIND PROB) = 0.10
1.00E+02
1.00E+01
1.00E+00
1.89E-01
9.00E+00 1.20E+01 1.50E+01 1.80E+01 2.10E+01 2.40E+01 2.70E+01 3.00E+01 3.30E+01
M

Figure 3-15

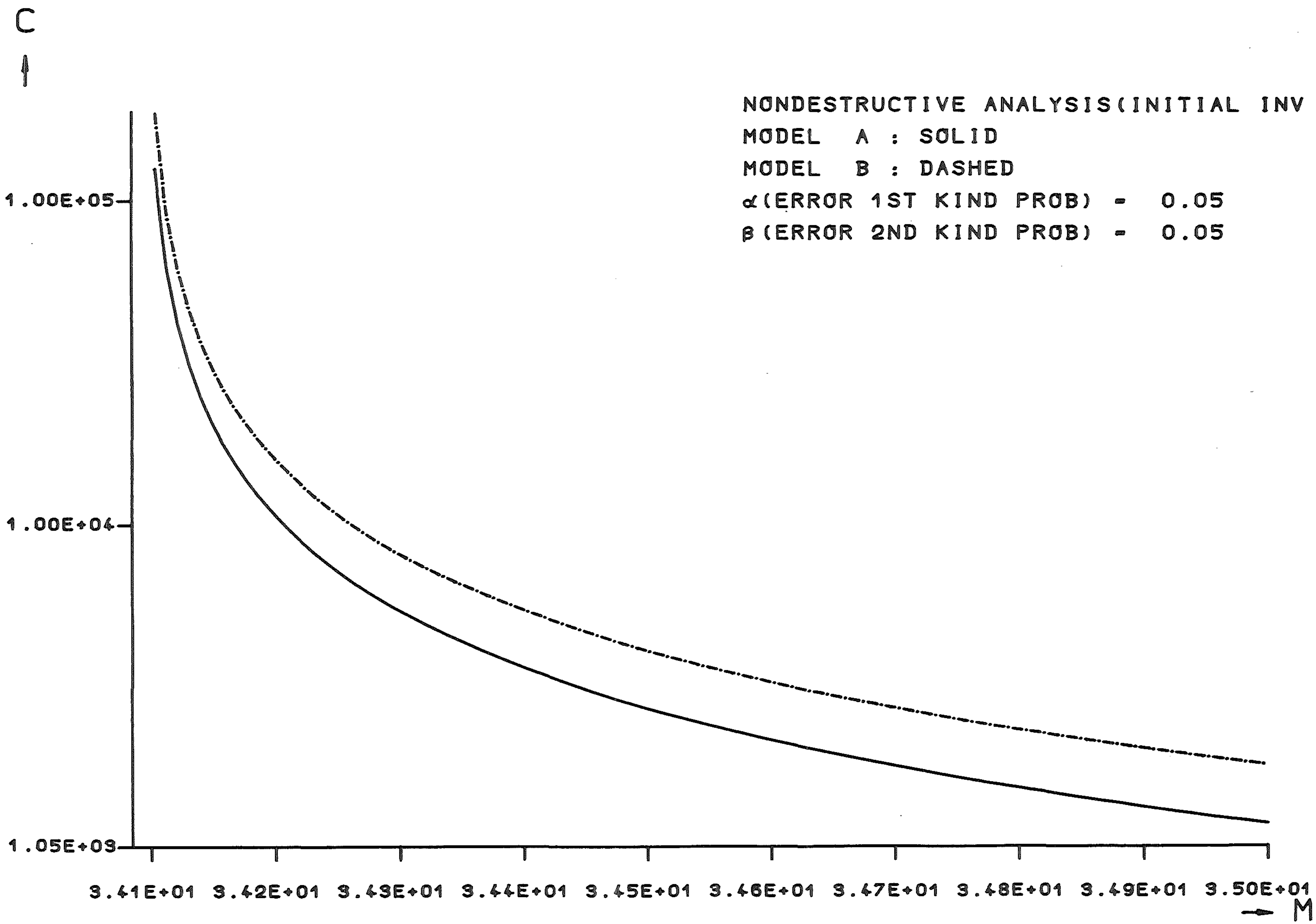

C
NONDESTRUCTIVE ANALYSIS(INITIAL INV
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
β(ERROR 2ND KIND PROB) = 0.05
1.00E+05
1.00E+04
1.05E+03
3.41E+01
3.42E+01
3.43E+01
3.44E+01
3.45E+01
3.46E+01
3.47E+01
3.48E+01
3.49E+01
3.50E+01
M

Figure 3-16

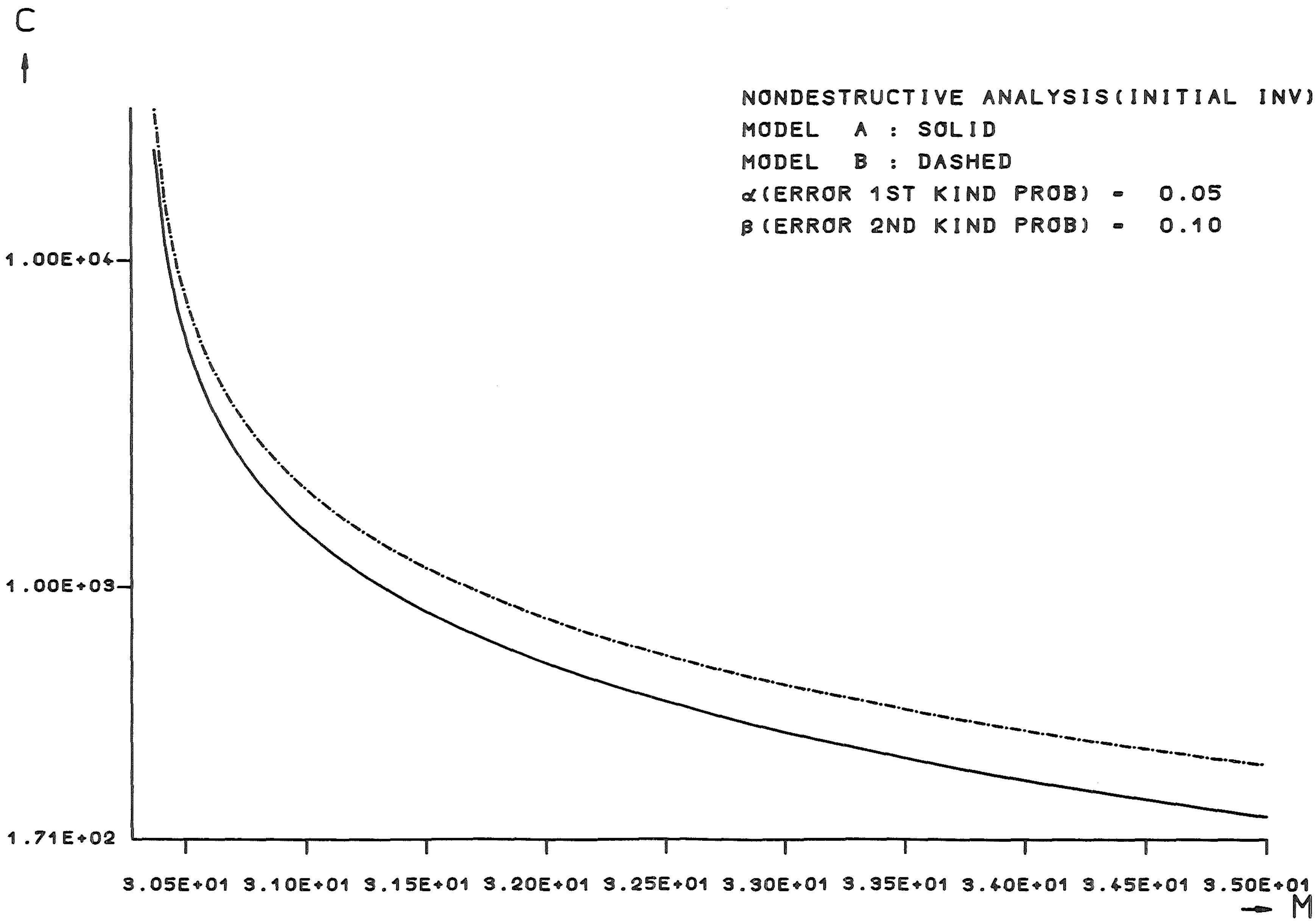

C
NONDESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
β(ERROR 2ND KIND PROB) = 0.10
1.00E+04
1.00E+03
1.71E+02
3.05E+01 3.10E+01 3.15E+01 3.20E+01 3.25E+01 3.30E+01 3.35E+01 3.40E+01 3.45E+01 3.50E+01
M

Figure 3-17

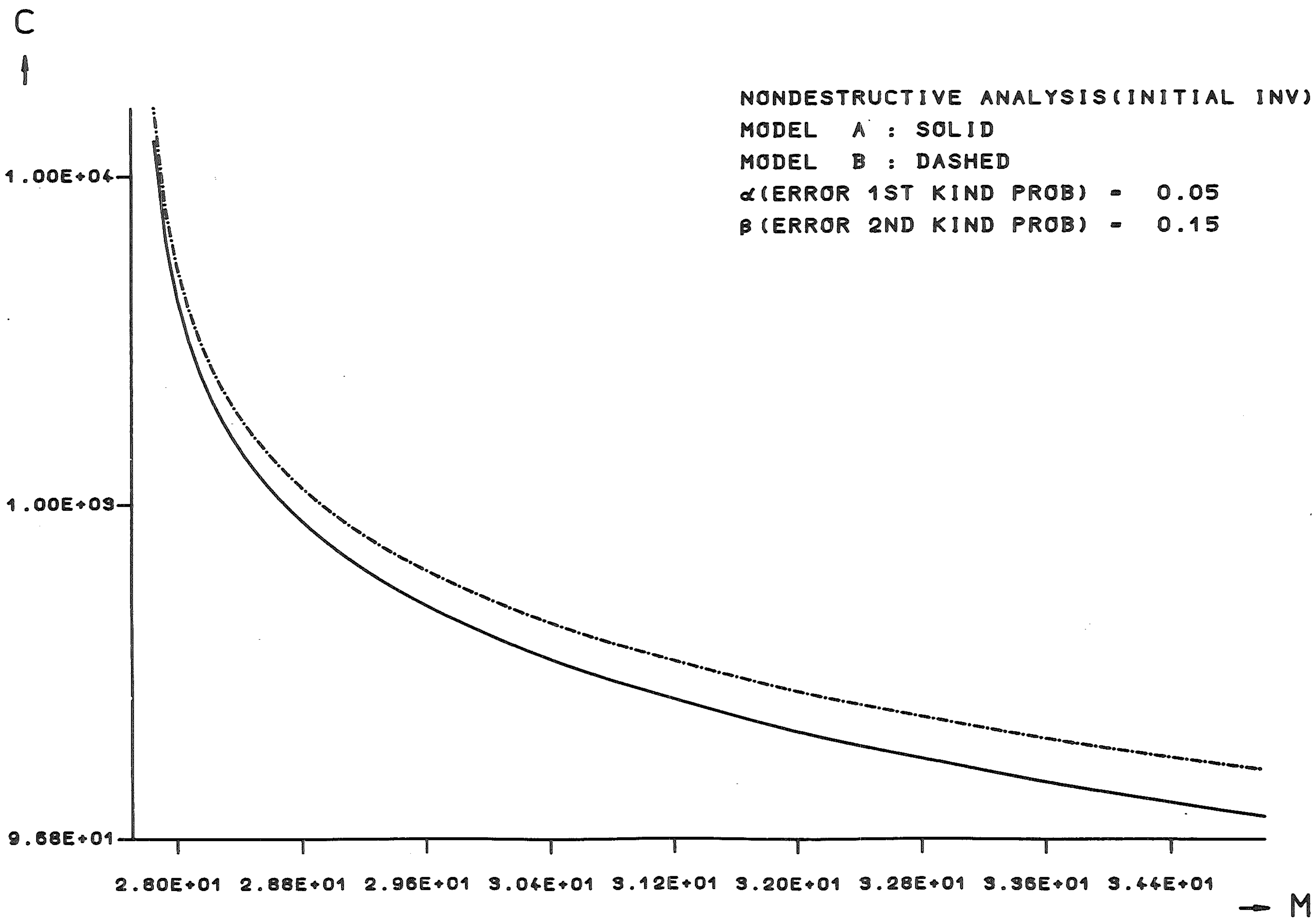

C
NONDESTRUCTIVE ANALYSIS(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.05
β(ERROR 2ND KIND PROB) = 0.15
1.00E+04
1.00E+03
9.68E+01
2.80E+01 2.88E+01 2.96E+01 3.04E+01 3.12E+01 3.20E+01 3.28E+01 3.36E+01 3.44E+01
M

Figure 3-18

3.2 Tests with two Class Specific Measurement Methods

In this section we assume that the operator will - if at all - falsify the material accountancy data by a total amount M of material, which is composed of two amounts M_1 and M_2 which correspond to rough and to fine falsifications. We assume, furthermore, that the data are falsified by class specific amounts in a rough resp. fine way, and that the composition of the total falsification is chosen by the operator in a way which is optimal for him.

Let us introduce again the following class specific entities which describe this problem:

(3.27)

$K = \{1...k\}$	set of material classes,
K_1	subset of K in which fine falsification and verification takes place,
K_2	subset of K in which rough falsification and verification takes place,
A_i	set of batches in i-th class ($\lvert A_i\rvert = N_i$),
$\varepsilon_i^{(1)}$	effort for fine measurement for one batch in i-th class
$\varepsilon_i^{(2)}$	effort for rough measurement for one batch in i-th class
$A_i^{x(1)}$	set of batches in i-th class the data of which are verified with fine method ($A_i^{x(1)} \subseteq A_i$, $\lvert A_i^{x(1)}\rvert = n_i^{(1)}$),
$A_i^{x(2)}$	set of batches in i-th class the data of which are verified with rough method ($A_i^{x(2)} \subseteq A_i$, $\lvert A_i^{x(2)}\rvert = n_i^{(2)}$),
$\mu_i^{(1)}$	class specific fine falsification of one batch in i-th class,
$\mu_i^{(2)}$	class specific rough falsification of one batch in i-th class,
$A_i^{y(1)}$	set of batches in i-th class which are falsified finely,
$A_i^{y(2)}$	set of batches in i-th class which are falsified roughly.

As again the inspector is not interested in estimating the true values T_{ij}, the test will be based on the differences

(3.28) $$Z_{ij}^{(1)} = Y_{ij} - X_{ij}^{(1)} \quad , \; j \in A_i^{x(1)} \quad , \; i = 1,2,\ldots,k, \; 1 = 1,2$$

where Y_{ij} is given by (3.2). Let $X_{ij}^{(1)}$ resp. $X_{ij}^{(2)}$ be the result of the destructive (fine) respectively nondestructive (rough) measurement of the inspector of the material content of the j-th batch of the i-th class. Under the assumption that the operator does not falsify data we have

(3.29) $$X_{ij}^{(1)} = T_{ij} + e_{Iij}^{(1)} + d_{Ii}^{(1)} \quad , \; j \in A_i^{x(1)} \quad , \quad i = 1,2,\ldots,k, \; j = 1,2$$

where T_{ij} is the true material content $e_{0ij}^{(1)}$ the random measurement error and $d_{0i}^{(1)}$ the class specific systematic measurement error. We assume that the measurement errors are independent and normally distributed random variables with zero mean values and known variances:

(3.30)
$$E(e_{Iij}^{(1)}) = E(d_i^{(1)}) = 0 \; , \quad i = 1,2,\ldots,k, \quad j = 1,2,\ldots, n_i, \; 1 = 1,2$$

$$\mathrm{var}(e_{Iij}^{(1)}) = \sigma_{Iri}^{(1)^2} \; , \quad i = 1,2,\ldots,k, \quad j \in A_i^{x(1)} \; , \; 1 = 1,2$$

$$\mathrm{cov}(e_{Iij}^{(1)} \; , \; e_{Ii'j'}^{(1')}) = 0 \; , \quad i \neq i', \; j \neq j \text{ or } 1 = 1'$$

$$\mathrm{var}(d_{Ii}^{(1)}) = \sigma_{Isi}^{(1)2} \; , \quad i = 1,2,\ldots,k$$

$$\mathrm{cov}(d_{Ii}^{(1)} \; , \; d_{Ii'}^{(1')}) = 0 \quad i \neq i' \text{ or } 1 \neq 1'$$

$$\mathrm{cov}(d_{Ii}^{(1)} \; , \; e_{Iij}^{(1')}) = 0, \quad i = 1,2,\ldots,k \; ; \; j = 1,2,\ldots,n_i, \; 1,1' = 1,2$$

Under the alternative hypothesis H_1, that the batch data of the set A_i^y are falsified by the amount $\mu_i^{(m)}$, m=1,2 we have

(3.31)
$$X_{ij}^{(l)} = \begin{cases} T_{ij} - \mu_{ij}^{(m)} + e_{Iij}^{(l)} + d_{Ii}^{(l)} & \text{for } j \in A_i^{x(l)} \cap A_i^{y(m)} \\ T_{ij} + C_{Iij}^{(l)} + d_{Ii}^{(l)} & \phantom{\text{for }} j \in A_i^{X(l)} \cap (A_i^{y(1)} \cap A_i^{y(2)}) \end{cases}$$

$$l = 1,2; \quad m = 1,2.$$

Again, we consider Models A and B.

3.2.1 Model A

We consider the case that the operator falsifies - if at all - all N_i batch data in the i-th class by a class specific amount $M_i^{(l)}$, l=1,2; i=1,2...k, i.e., we consider Model A. As we assume that fine and rough falsifications of one batch datum cannot occur at the same time,

$$A_i^{y(1)} \cap A_i^{y(2)} = \emptyset \text{ for } i=1...k.$$

The operator can falsify the batch data of a given class either finely of roughly. Table 3-3 shows all possibilities which result from this assumption.

Table 3-3: Falsification possibilities of the operator with respect to the initial inventory data given by Table 2-1. G means rough, F fine falsification.

Possibility	Class 1	2	3	4	5	6	7	8
A1	G	G	G	G	G	G	G	G
A2	G	G	G	F	G	G	G	G
A3	G	G	G	G	F	G	G	G
A4	G	G	G	G	G	F	G	G
A5	G	G	G	F	F	G	G	G
A6	G	G	G	G	F	F	G	G
A7	G	G	G	F	G	F	G	G
A8	G	G	G	F	F	F	G	G

Let us assume that the operator intends to divert the total amount M of nuclear material by means of data falsification. This means that he has to observe for the single falsifications $\mu_i^{(1)}$, 1=1,2 the boundary condition

(3.32) $$M = \sum_{i \in K_1} N_i \mu_i^{(1)} + \sum_{i \in K_2} N_i \mu_i^{(2)} = M_1 + M_2$$

where M_1 resp. M_2 is the total fine resp. rough falsification. The verification effort of the inspector is composed of the effort C_1 for fine measurements, and the effort C_2 for rough measurements,

(3.33) $$C_l = \sum_{i \in K_l} \varepsilon_i^{(l)} \cdot n_i^{(l)} \quad , \quad l = 1,2.$$

As the effort C_1 is given in monetary, the effort C_2 in inspection time units, C_1 and C_2 cannot be combined to one single effort.

In order to solve the problem of optimizing the overall probability of detection $1-\beta(n^{(1)},n^{(2)},\mu^{(1)},\mu^{(2)})$ with respect to $n^{(1)}$ and $n^{(2)}$ under the boundary condition (3.33) for any sets $\mu^{(1)}$ and $\mu^{(2)}$, subject to the boundary condition (3.32), i.e. in order to solve the problem

(3.34) $$\max_{\underline{n}^{(1)},\underline{n}^{(2)}} \quad \min_{\underline{\mu}^{(1)},\underline{\mu}^{(2)}} \quad (1 - \beta\,(\underline{n}^{(1)}, \underline{n}^{(2)}, \underline{\mu}^{(1)}, \underline{\mu}^{(2)})),$$

with the boundary conditions (3.32) and (3.33), one could in principle proceed as outlined in section 3.1.1, namely to determine first the best test in the sense of the Lemma of Neyman and Pearson. As this would lead us to <u>one single test</u> and as the fine and the rough measurement data of the inspector are available at different times, we proceed here in a different way. We construct <u>two best tests</u> for the comparison of the operator's fine and rough measurement data with the boundaries of given values of M_l, and C_l $l=1,2$. Because of the independence of the two test statistics the total guaranteed probability of no detection, β_{tA}^*, is given by

(3.35a) $$\beta_{tA}^* = \beta_{FA}^* \; \beta_{GA}^* \;,$$

where β_{FA}^* and β_{GA}^* are the single guaranteed probabilities of no detection. The total no false alarm probability is

(3.35b) $$1-\alpha_t = (1-\alpha_1)\;(1-\alpha_2)$$

where α_l, l=1,2 are the single false alarm probabilities which we will choose $\alpha_1=\alpha_2=0.05$ in the numerical examples.

The optimal sample series n_i^{l*} of the inspector and the optimal single falsifications μ_i^{l*} of the operator i=1...k, l=1,2 are then again given by the set (3.13) of formulae where all relevant quantities get the index l=1,2. The same holds for the efforts C_l necessary for achieving a guaranteed probability of detection; they are given by formula (3.15) for l=1,2.

It should be noted that there exist further reasonable possibilities for constructing test procedures for the two sets of data $Z_{ij}^{(l)}$, l=1,2, which have been discussed in /8/, which will, however, not be used here.

3.2.2 Model B

Let us now consider Model B i.e., that case where $r_i^{(1)}$ batch data of the i-th class are falsified by the amount $\mu_i^{(1)}$, and where $r_i^{(2)}$ batch data of the i-th class are falsified by the amount of $\mu_i^{(2)}$, i=1...k. Also in this case one batch datum cannot be falsified finely and roughly at the same time,

$$A_i^{y(1)} \cap A_i^{y(2)} = \emptyset \text{ for } i=1...k.$$

We know however, from formula (3.22b) that - in case of the twofold test procedure which we discussed before and which we will use again - the optimal values of the sample series r_i are given by $N_i/2$ for i=1...k which means that contrary to Model A here also both fine and rough falsifications are possible within one class.

If the operator intends to divert the total amount M of nuclear material by means of data falsification, then he has to observe for the single falsifications $\mu_i^{(l)}$ and $r_i^{(l)}$, i=1...k, l=1,2, the boundary condition

$$\text{(3.36)} \qquad M = \sum_{i \in K} (r_i^{(1)} \mu_i^{(1)} + r_i^{(2)} \mu_i^{(2)}) = M_1 + M_2$$

For the sample sizes $n_i^{(l)}$, i=1...k, l=1,2, we have again the two boundary conditions (3.33).

In order to solve the problem of optimizing the overall probability of detection $1-\beta(n^{(1)},n^{(2)},\mu^{(1)},\mu^{(2)},r^{(1)},r^{(2)})$ with respect to $n^{(1)}$ and $n^{(2)}$ under the boundary conditions (3.33) for any sets $\mu^{(1)},r^{(1)},\mu^{(2)},r^{(2)}$, subject to the boundary condition (3.36), i.e., in order to solve the problem

$$(3.37) \qquad \max_{\underline{n}^{(1)},\underline{n}^{(2)}} \min_{\underline{\mu}^{(1)},\underline{\mu}^{(2)},\underline{r}^{(1)},\underline{r}^{(2)}} (1 - \beta\,(\underline{n}^{(1)},\underline{n}^{(2)},\underline{\mu}^{(1)},\underline{\mu}^{(2)},\underline{r}^{(1)},\underline{r}^{(2)}))$$

with the boundary conditions (3.33) and (3.36), we proceed as in the foregoing section. We construct two best tests for the comparison of the operator's data with the inspector's fine and rough measurement data with the boundaries of given values of M_l and C_l, $l=1,2$, according to (3.33) and (3.36).

Contrary to <u>Model A</u> here the two test statistics are in general <u>not</u> independent as one operator's datum may be verified both by the inspector's fine and rough measurement. Therefore, a factorization of the total probability of no detection and of the total no false alarm probability in the sense of formulae (3.35) does not hold in general. In the following, we derive the exact expressions for the total detection and false alarm probabilities based on the D-statistics, and show at the hand of numerical examples that the dependence of the two statistics can be neglected in some cases.

The D-statistics for the two tests are

$$(3.38) \qquad D_l = \sum_{i\in K} N_i \sum_j (Y_{ij} - X_{ij}^{(l)})/n_i^{(l)} \qquad , \ l = 1,2.$$

where Y_{ij} is given by (3.2) and $X_{ij}^{(l)}$ by (3.31). Let $\bar{n}_i$ be the number of batch data in the i-th class which are verified both by fine and rough measurements. If we assume that within one class no batch datum is verified twice as long as there are still data, which have not yet been verified, then we have

$$(3.39) \qquad \bar{n}_i = \begin{cases} n_i^{(1)} + n_i^{(2)} - N_i \ , & \text{if } N_i > n_i^{(1)} + n_i^{(2)} \\ 0 & , \text{otherwise} \end{cases}$$

$$i = 1,2,\dots,k.$$

With this definition we get for the covariance of D_1 and D_2 after some elementary calculations.

$$\text{(3.40a)} \qquad \mathrm{cov}(D_1,D_2) = \sum_{i\in K} \left(N_i^2 \cdot \frac{\bar{n}_i}{n_i^{(1)} + n_i^{(2)}} \sigma^2_{Ori} + N_i^2 \sigma^2_{Osi}\right)$$

If we call $\sigma_{DO}^{(l)2}$ and $\sigma_{D1}^{(l)2}$ the variances of D_l, l=1,2 under the null and under the alternative hypothesis H_0 and H_1, then the correlations of D_1 and D_2 under H_0 and H_1 are

$$\text{(3.40b)} \qquad \mathrm{cor}(D_1,D_2) = \begin{cases} \mathrm{cov}(D_1,D_2)/(\sigma_{DO}^{(1)} \cdot \sigma_{DO}^{(2)}) = \rho_o & \text{under } H_o \\ \mathrm{cov}(D_1,D_2)/(\sigma_{D1}^{(1)} \cdot \sigma_{D1}^{(2)}) = \rho_1 & \text{under } H_1 \end{cases}$$

With these definitions the common probability density function of D_1 and D_2 is under the null hypothesis H_0

$$\text{(3.41a)} \qquad f_o(x_1,x_2) = \frac{1}{2\pi} \frac{1}{\sigma_{DO}^{(1)} \sigma_{DO}^{(2)}} \frac{1}{\sqrt{1-\rho_o^2}} \exp\left(\frac{1}{2(1-\rho_o^2)} \left(\frac{x_1^2}{\sigma_{DO}^{(1)2}} - \frac{2\rho_o x_1 x_2}{\sigma_{DO}^{(1)}\sigma_{DO}^{(2)}} + \frac{x_2^2}{\sigma_{DO}^{(2)2}}\right)\right)$$

and under the alternative hypothesis H_1

$$\text{(3.41b)} \qquad f_1(x_1,x_2) = \frac{1}{2\pi} \frac{1}{\sigma_{D1}^{(1)} \sigma_{D1}^{(2)}} \cdot \frac{1}{\sqrt{1-\rho_1^2}} \cdot \exp\left(\frac{1}{2(1-\rho_1^2)} \left(\frac{(x_1-M_1)^2}{\sigma_{D1}^{(1)2}} - \frac{2\rho_1 (x_1-M_1)(x_2-M_2)}{\sigma_{D1}^{(1)} \cdot \sigma_{D1}^{(2)}} + \frac{(x_2-M_2)^2}{\sigma_{D1}^{(2)2}}\right)\right)$$

Therefore, the total no false alarm probability 1-α, which is defined as

$$(3.42a) \qquad 1-\alpha_t = \int_{-\infty}^{s_1} dx_1 \int_{-\infty}^{s_2} dx_2\; f_o(x_1,x_2)$$

where s_1 and s_2 are given by

$$(3.42b) \qquad s_e = U_{1-\alpha_e}\, \sigma_{DO}^{(e)} \quad , \quad e = 1,2 \quad ,$$

is explicitly given by the formula

$$(3.43) \qquad 1-\alpha_t = \frac{1}{2\Pi} \frac{1}{\sqrt{1-\rho_o^2}} \int_{-\infty}^{U_{1-\alpha_1}} dx_1 \int_{-\infty}^{U_{1-\alpha_2}} dx_2 \quad \exp\left(- \frac{x_1^2-2\rho_o x_1 x_2+x_2^2}{2(1-\rho_o^2)}\right)$$

Furthermore, the total probability of no detection β_t, which is defined as

$$(3.44) \qquad \beta_t = \int_{-\infty}^{s_1} dx_1 \int_{-\infty}^{s_2} dx_2 \quad f_1(x_1,x_2)$$

where s_1 and s_2 are again given by (3.42b), is explicitly given by the formula

(3.45)
$$\beta_t = \frac{1}{2\Pi} \cdot \frac{1}{\sqrt{1-\rho_1^2}} \int_{-\infty}^{\frac{\sigma_{DO}^{(1)} \cdot U_{1-\alpha_1} - M_1}{\sigma_{D1}^{(1)}}} dx_1 \int_{-\infty}^{\frac{\sigma_{DO}^{(2)} \cdot U_{1-\alpha_2} - M_2}{\sigma_{D1}^{(2)}}} dx_2 \ \exp\left(-\frac{x_1^2 - 2\rho x_1 x_2 + x_2^2}{2(1-\rho_1^2)}\right)$$

In Figures 3.19 through 3.21 the probability of detection with and without (i.e vanishing) correlations are given. In addition, in Figures 3.22 through 3.24 the correlations ρ_o and ρ_1 are shown. We see that for values of the total fine falsification M_1, which are not too small, we can neglect the dependence between the two test statistics D_1 and D_2, which, as already mentioned, exists only for

$$n_i^{(1)} + n_i^{(2)} > N_i \text{ for at least one } i=1,\ldots,k.$$

As a consequence, we proceed as outlined in the foregoing section. We write the total probability of no detection, β_{tB}^*, in the form

(3.46a) $$\beta_{tB}^* = \beta_{FB}^* \ \beta_{GB}^*$$

where β_{FB}^* and β_{GB}^* are the single guaranteed probabilities of no detection, and accordingly the total no false alarm probabilities $1-\alpha_1$ in the form

(3.46b) $$1-\alpha_1 = (1-\alpha_1)\ (1-\alpha_2),$$

where α_1, l=1,2 are the single false alarm probabilities which we will choose $\alpha_1=\alpha_2=0.05$ in the numerical examples.

Figures 3-19 to 3-21:

Influence of the correlation of D_1 and D_2 on the detection probability $1-\beta$.

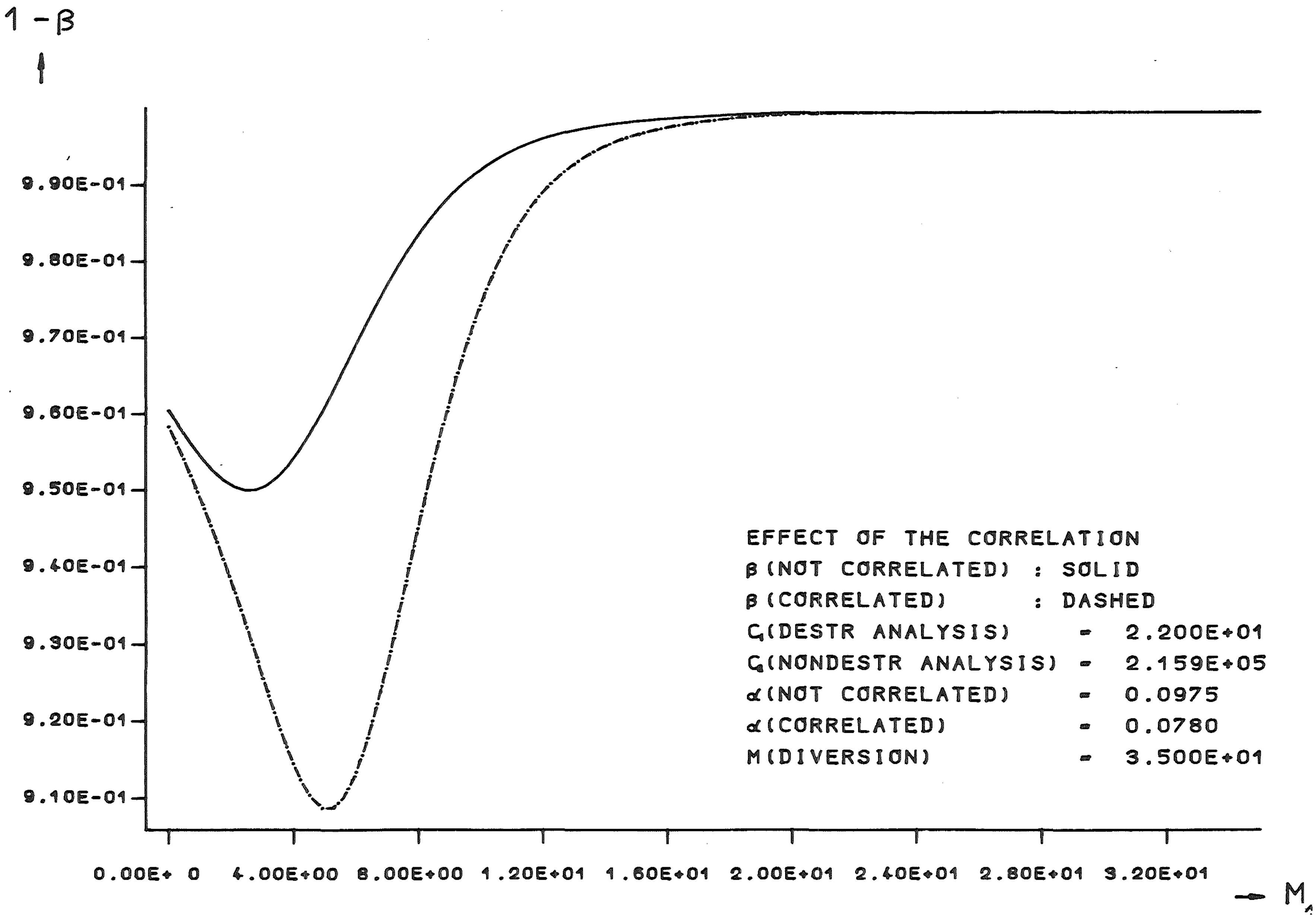

1 - β
9.90E-01
9.80E-01
9.70E-01
9.60E-01
9.50E-01
9.40E-01
9.30E-01
9.20E-01
9.10E-01
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M
EFFECT OF THE CORRELATION
β (NOT CORRELATED) : SOLID
β (CORRELATED) : DASHED
C(DESTR ANALYSIS) = 2.200E+01
C(NONDESTR ANALYSIS) = 2.159E+05
α(NOT CORRELATED) = 0.0975
α(CORRELATED) = 0.0780
M(DIVERSION) = 3.500E+01

Figure 3-19

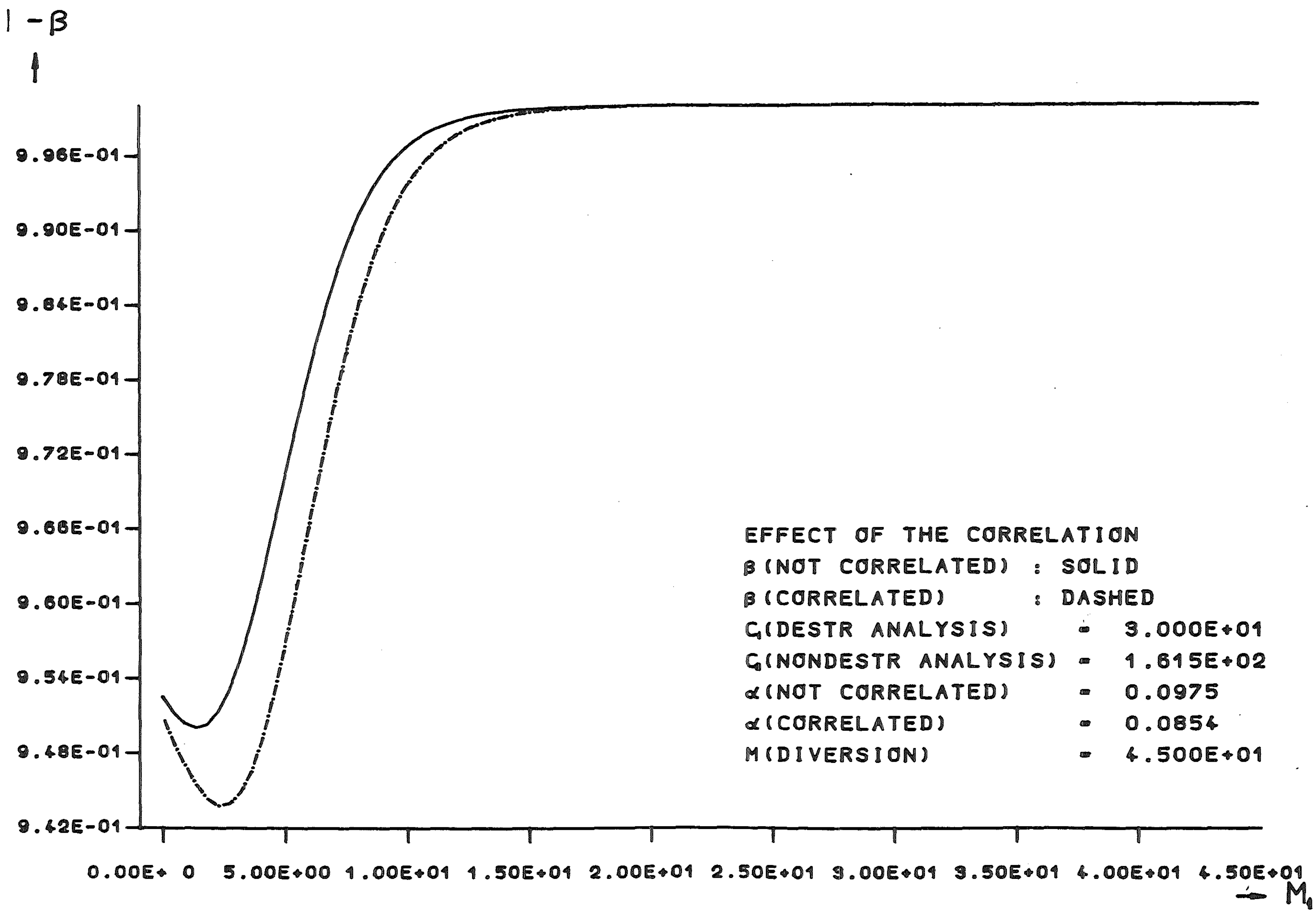
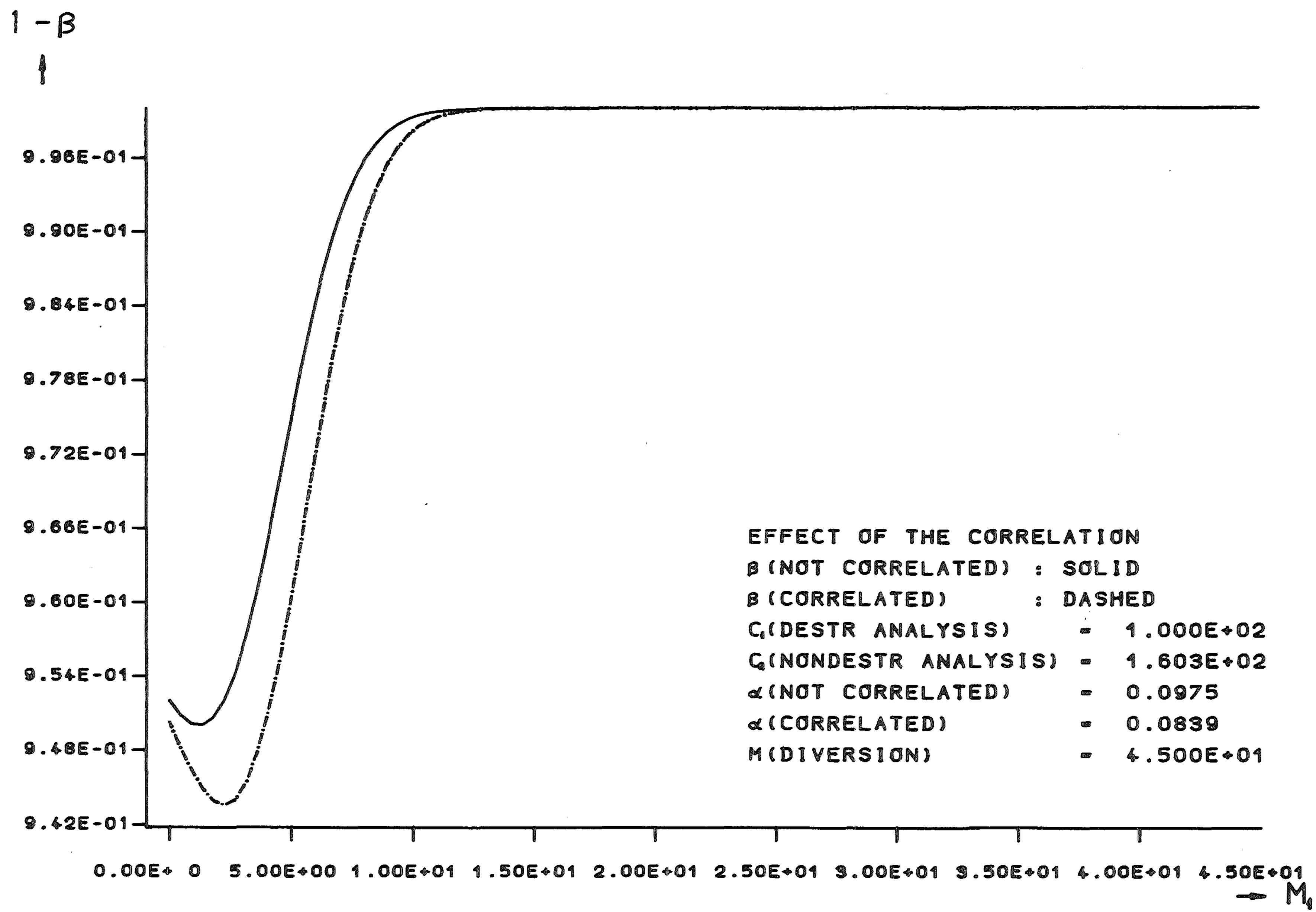

1 - β
9.96E-01
9.90E-01
9.84E-01
9.78E-01
9.72E-01
9.66E-01
9.60E-01
9.54E-01
9.48E-01
9.42E-01
EFFECT OF THE CORRELATION
β (NOT CORRELATED) : SOLID
β (CORRELATED) : DASHED
C1(DESTR ANALYSIS) = 3.000E+01
C1(NONDESTR ANALYSIS) = 1.615E+02
α(NOT CORRELATED) = 0.0975
α(CORRELATED) = 0.0854
M(DIVERSION) = 4.500E+01
0.00E+ 0
5.00E+00
1.00E+01
1.50E+01
2.00E+01
2.50E+01
3.00E+01
3.50E+01
4.00E+01
4.50E+01
M1

Figure 3-20

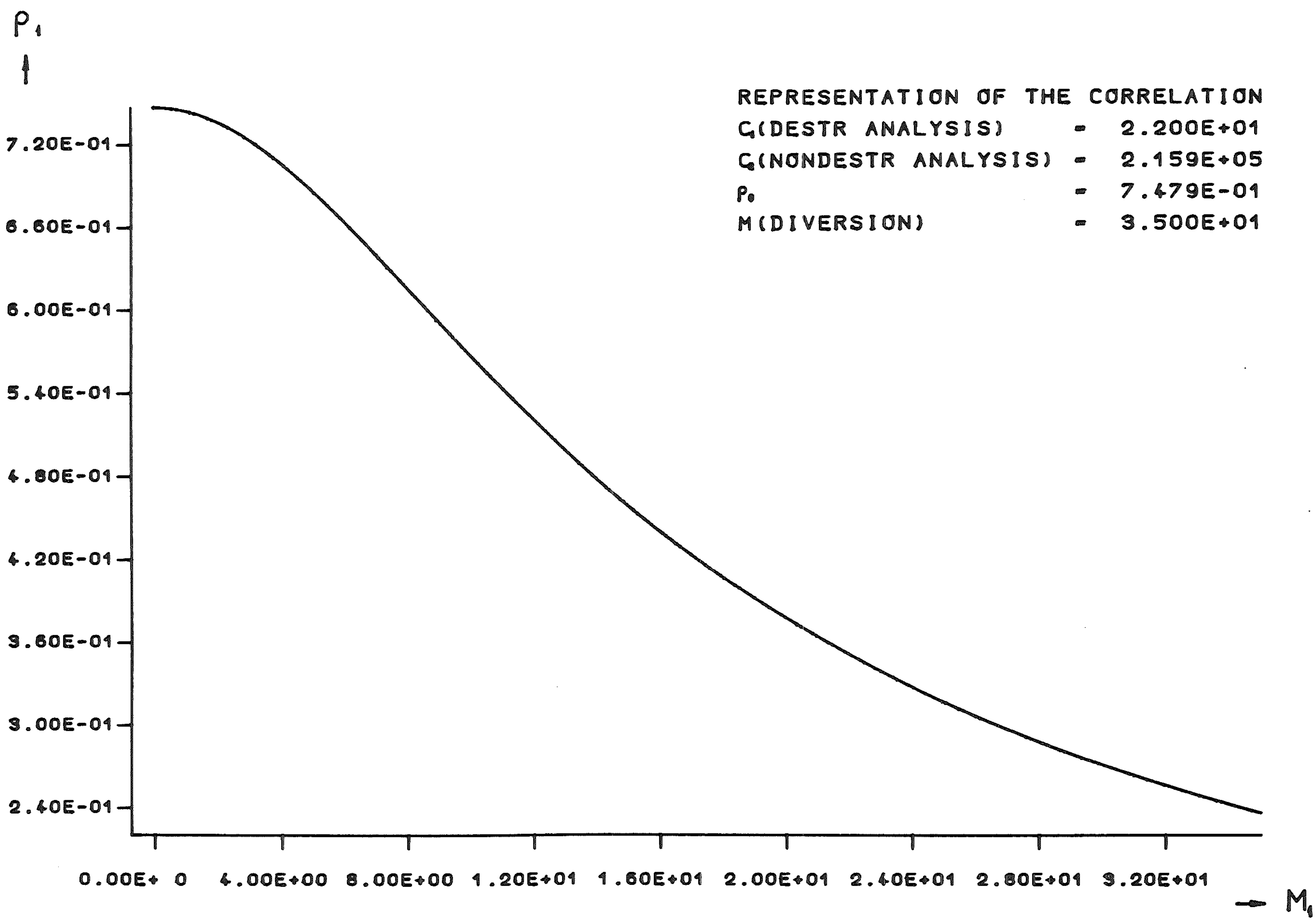


1 - β
9.96E-01
9.90E-01
9.84E-01
9.78E-01
9.72E-01
9.66E-01
9.60E-01
9.54E-01
9.48E-01
9.42E-01
EFFECT OF THE CORRELATION
β (NOT CORRELATED) : SOLID
β (CORRELATED) : DASHED
C1(DESTR ANALYSIS) = 1.000E+02
C2(NONDESTR ANALYSIS) = 1.603E+02
α (NOT CORRELATED) = 0.0975
α (CORRELATED) = 0.0839
M(DIVERSION) = 4.500E+01
0.00E+ 0
5.00E+00
1.00E+01
1.50E+01
2.00E+01
2.50E+01
3.00E+01
3.50E+01
4.00E+01
4.50E+01
M1

Figure 3-21

Figures 3-22 to 3-24:

Correlation ρ_1 of D_1 and D_2 under H_1 as a function of the amount of fine falsification M_1.



ρ_1
REPRESENTATION OF THE CORRELATION
C(DESTR ANALYSIS) = 2.200E+01
C(NONDESTR ANALYSIS) = 2.159E+05
ρ_0 = 7.479E-01
M(DIVERSION) = 3.500E+01
7.20E-01
6.60E-01
6.00E-01
5.40E-01
4.80E-01
4.20E-01
3.60E-01
3.00E-01
2.40E-01
0.00E+ 0
4.00E+00
8.00E+00
1.20E+01
1.60E+01
2.00E+01
2.40E+01
2.80E+01
3.20E+01
M_1

Figure 3-22

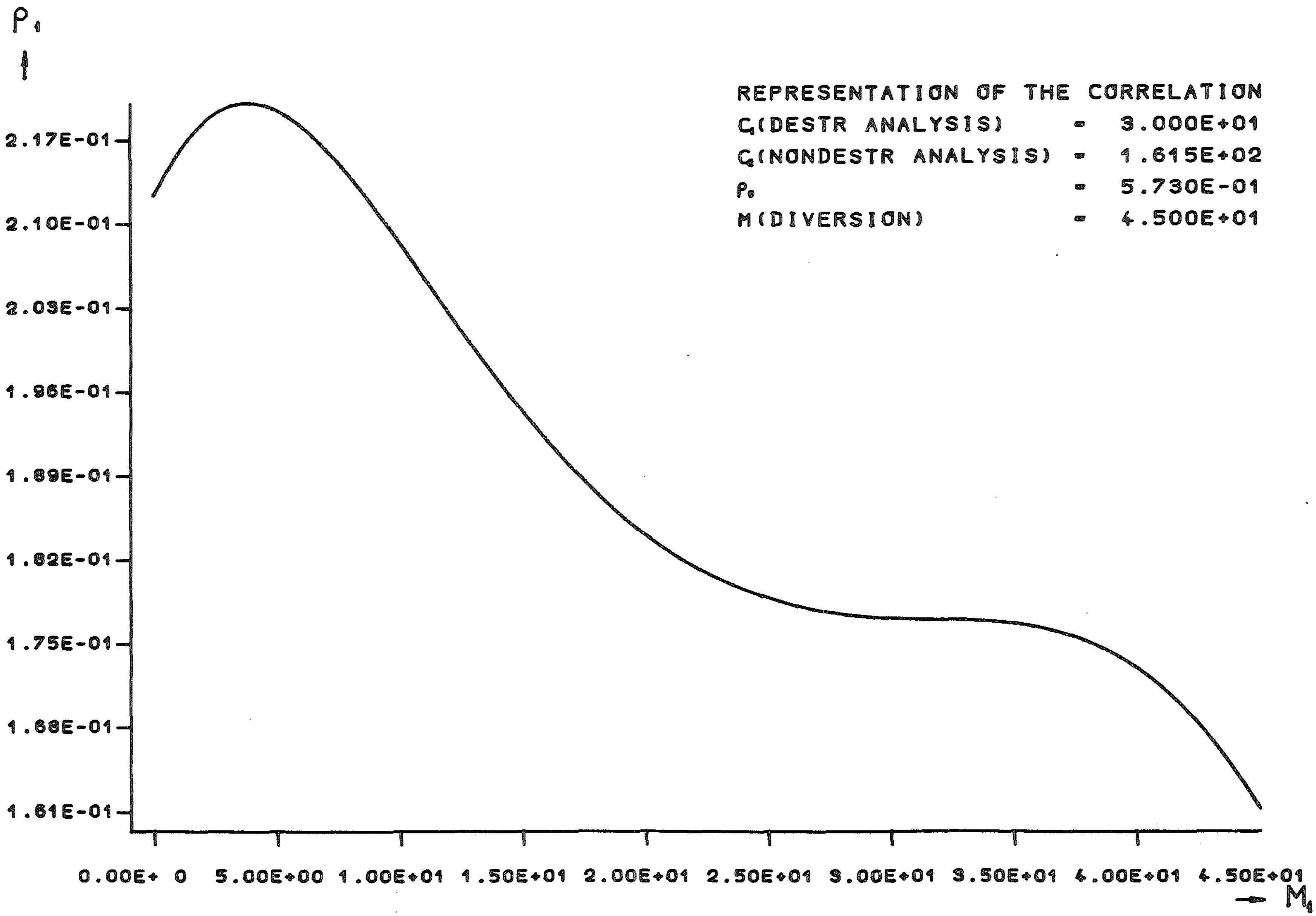

ρ1
REPRESENTATION OF THE CORRELATION
C1(DESTR ANALYSIS) = 3.000E+01
C1(NONDESTR ANALYSIS) = 1.615E+02
ρ0 = 5.730E-01
M(DIVERSION) = 4.500E+01
2.17E-01
2.10E-01
2.03E-01
1.96E-01
1.89E-01
1.82E-01
1.75E-01
1.68E-01
1.61E-01
0.00E+ 0
5.00E+00
1.00E+01
1.50E+01
2.00E+01
2.50E+01
3.00E+01
3.50E+01
4.00E+01
4.50E+01
M1

Figure 3-23

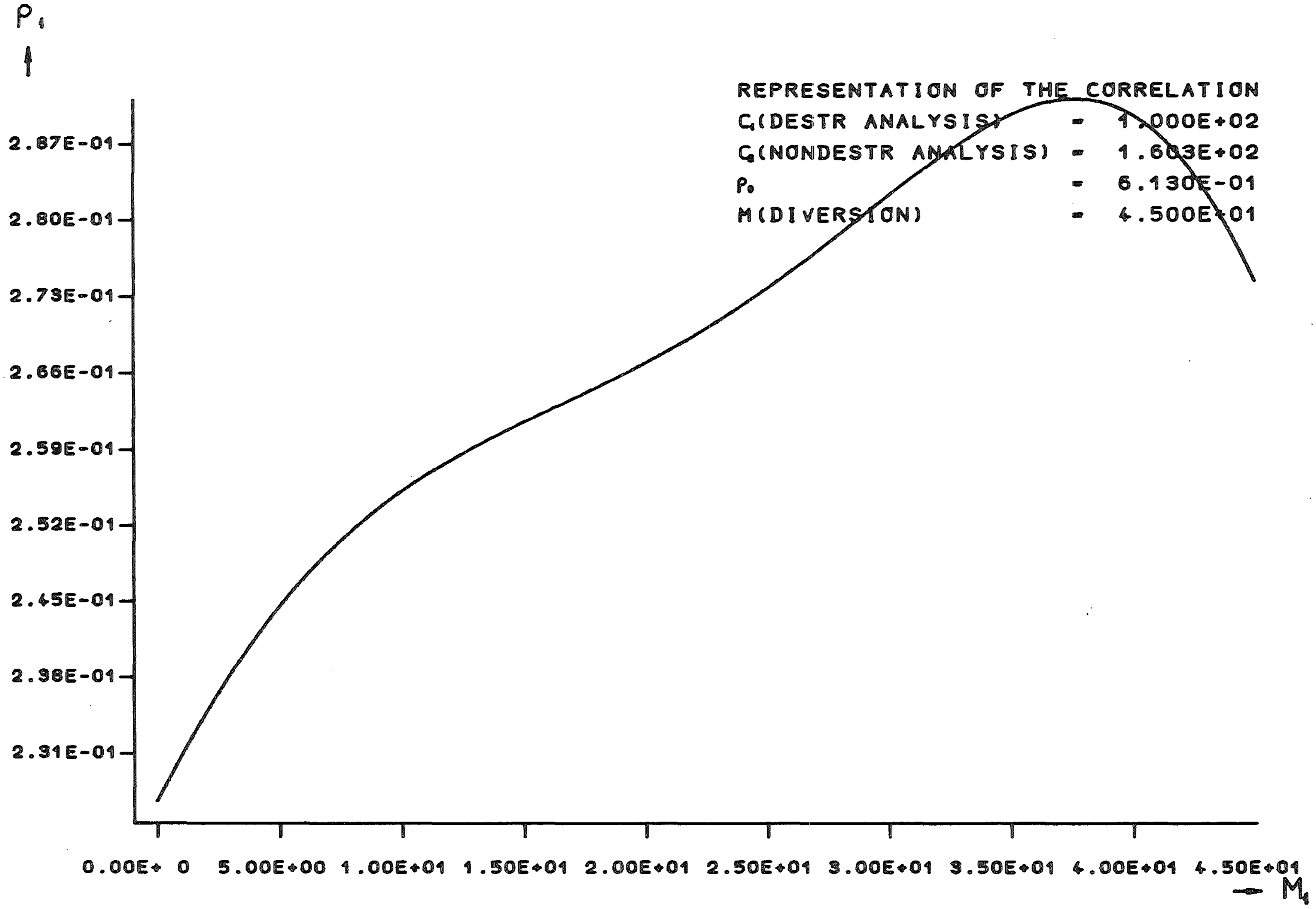

ρ₁
REPRESENTATION OF THE CORRELATION
C₁(DESTR ANALYSIS) = 1.000E+02
C₂(NONDESTR ANALYSIS) = 1.603E+02
ρ₀ = 6.130E-01
M(DIVERSION) = 4.500E+01
2.87E-01
2.80E-01
2.73E-01
2.66E-01
2.59E-01
2.52E-01
2.45E-01
2.38E-01
2.31E-01
0.00E+ 0
5.00E+00
1.00E+01
1.50E+01
2.00E+01
2.50E+01
3.00E+01
3.50E+01
4.00E+01
4.50E+01
M₁

Figure 3-24

The optimal sample sizes $n_i^{(*)}$ of the inspector and the optimal sample sizes r_i^{l*} and single falsifications μ_i^{l*}, i=1,...k, l=1,2 are then again given by the set (3.22) of formulae, where all relevant quantities get the index l=1,2. The same holds for the effort C necessary for achieving a guaranteed probability of detection, they are given by formula (3.23) for l=1,2.

3.2.3 Comparison

If one neglects the correlation between the two test statistics in Model B, then the total probability of no detection can both for Model A and B be written as

$$(3.47a) \qquad \beta^*_{tA,B} = \Phi\left(\frac{U_{1-\alpha_1}\sqrt{A_1^2/C_1+B_1} - M_1}{\sqrt{A_1^2\, H_1/C_1 + B_1}}\right)\,\Phi\left(\frac{U_{1-\alpha_2}\sqrt{A_2^2/C_2+B_2} - M_2}{\sqrt{A_2^2\, H_2/C_2 + B_2}}\right)$$

where H_l, l=1,2 is given by

$$(3.47b) \qquad H_l = \begin{cases} 1 & \text{for Model A} \\ 1 + M_l^2/D_l^2 & \text{for Model B} \end{cases}$$

and where

$$(3.47c) \qquad A_l = \sum_{i\in K_l} N_i\, \sigma_{ri}^{(l)} \sqrt{\varepsilon_i^{(l)}}$$

$$(3.47d) \qquad B_l = \sum_{i\in K_l} N_i^2\, {\sigma_{si}^{(l)}}^2$$

$$(3.47e) \qquad D_l = \sum_{i\in K_l} N_i \cdot \sigma_{ri}^{(l)} \quad .$$

Both probabilities of detection are monotonically increasing functions of the efforts C_l, l=1,2, with the limiting values

(3.48) $$\lim_{\substack{C_1\to\infty \\ C_2\to\infty}} \beta_{tA}^{*} = \lim_{\substack{C_1\to\infty \\ C_2\to\infty}} \beta^{*}_{tB} = \Phi(U_{1-\alpha_1} - M_1/\sqrt{B_1})\Phi(U_{1-\alpha_2} - M_2/\sqrt{B_2})$$

If the values of C_1 and C_2 are fixed, the operator can for given values of M_1 and M_2 influence the total probability of detection only by choosing Model A or B. With the abbreviations

(3.49) $$Z_l = U_{1-\alpha_2} \sqrt{A_l^2 / C_l + B_l} - M_l \ , \quad l = 1,2,$$

one sees immediately: For Z_1 and Z_2 greater zero, the operator will choose Model A, for Z_1 and Z_2 smaller zero, he will choose Model B. If Z_1 and Z_2 have different signs, then the operator chooses Model B, if the absolute value of the negative argument of the one Φ-function is larger than the absolute value of the positive argument of the other Φ-function, otherwise Model A.

If the total guaranteed probability of detection is given, and if it is larger than the limiting value given by (3.48), then there does not exist any pair of efforts (C_1,C_2) which can fulfill this. Lower boundaries for C_l l=1,2 are given by (3.23a) with

(3.50) $$L_l = \beta_t/\Phi(U_{1-\alpha_m} - M_m/\sqrt{B_m}), \quad l = 1,2, m = 3-l.$$

Under the assumption $L_l>0.5$, l=1,2 we calculate in the same way as in section 3.1.3 that the minimal values for the efforts C_l, l=1,2 are always larger for Model B than for Model A

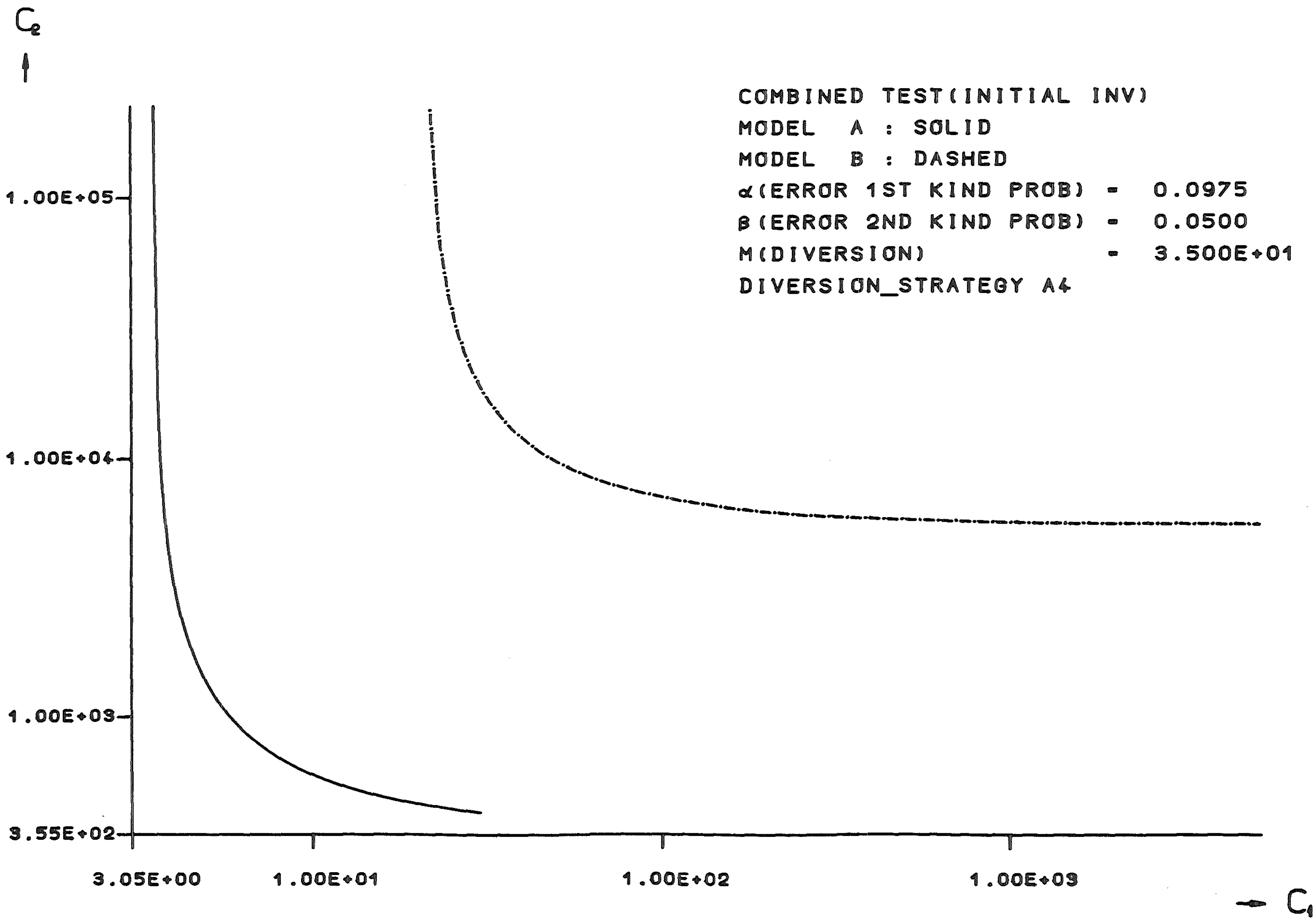
Figures 3.25 through 3.29 show optimal efforts C_1 and C_2 of the inspector for given total guaranteed probability of detection and given total amount M of material to be diverted via data falsification, both for Model A and B. Comparing Figures 3.25 through 3.27, one recognizes the influence of the diversion strategy (A4,A7,A8 of Table 3-3). Comparing Figures 3.27 through 3.29, one recognizes the influence of the value of the total diversion. One clearly recognizes, in addition the higher verification effort in case of Model B.

Tables 3-4 through 3.9 show for selected values of C_1, C_2 M and 1-β the optimal distribution (M_1,M_2) of the total falsification M, and furthermore, sample sizes and single falsifications for Model A. The comparison of Tables 3-4 through 3-9 shows the influence of the total falsification M.

Tables 3-10 through 3-12 show for selected pairs of values of C_1 and C_2, M and 1-β the optimal distribution (M_1,M_2) of the total falsification and furthermore, sample sizes and single falsifications for Model B. The comparison of these tables shows the influence of the total falsification M.

Figures 3-25 to 3-29:

Optimal efforts C_1 and C_2 for given guaranteed detection probability and amount of falsification M.



C2
COMBINED TEST(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.0975
β(ERROR 2ND KIND PROB) = 0.0500
M(DIVERSION) = 3.500E+01
DIVERSION_STRATEGY A4
1.00E+05
1.00E+04
1.00E+03
3.55E+02
3.05E+00
1.00E+01
1.00E+02
1.00E+03
C1

Figure 3-25

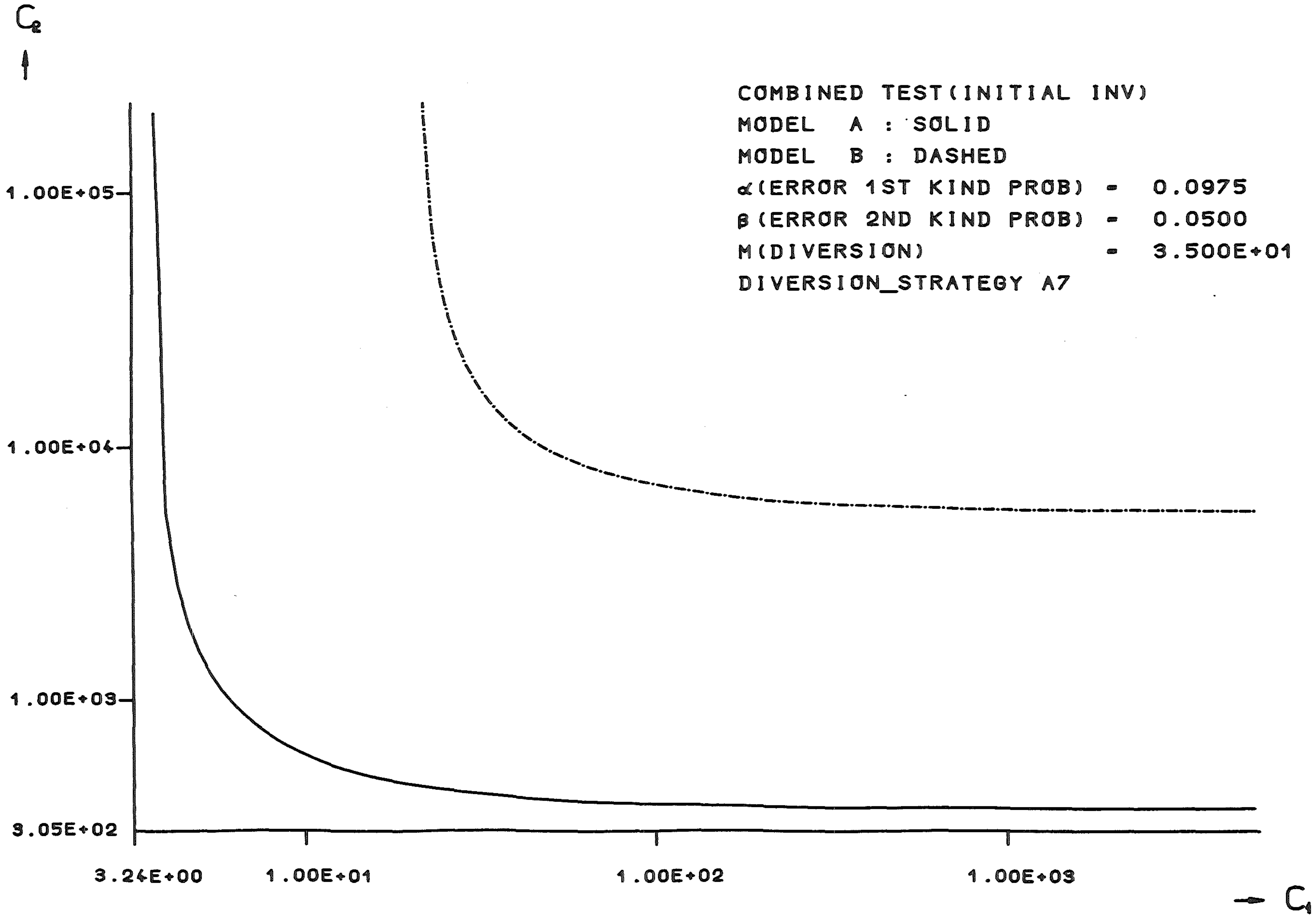
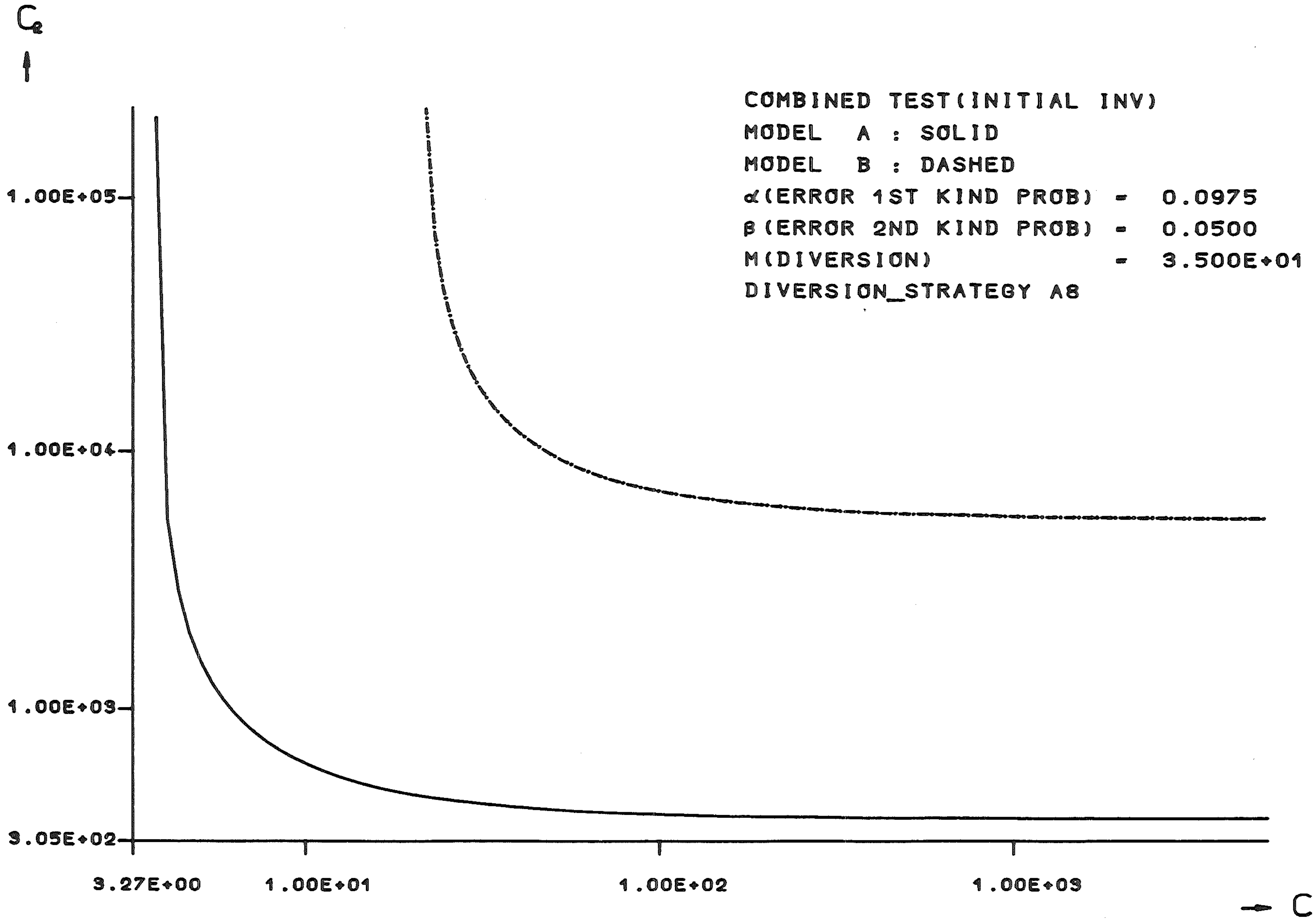

C2
COMBINED TEST(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.0975
β(ERROR 2ND KIND PROB) = 0.0500
M(DIVERSION) = 3.500E+01
DIVERSION_STRATEGY A7
1.00E+05
1.00E+04
1.00E+03
3.05E+02
3.24E+00
1.00E+01
1.00E+02
1.00E+03
C1

Figure 3-26



C_e
COMBINED TEST(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.0975
β(ERROR 2ND KIND PROB) = 0.0500
M(DIVERSION) = 3.500E+01
DIVERSION_STRATEGY A8
1.00E+05
1.00E+04
1.00E+03
3.05E+02
3.27E+00
1.00E+01
1.00E+02
1.00E+03
C

Figure 3-27

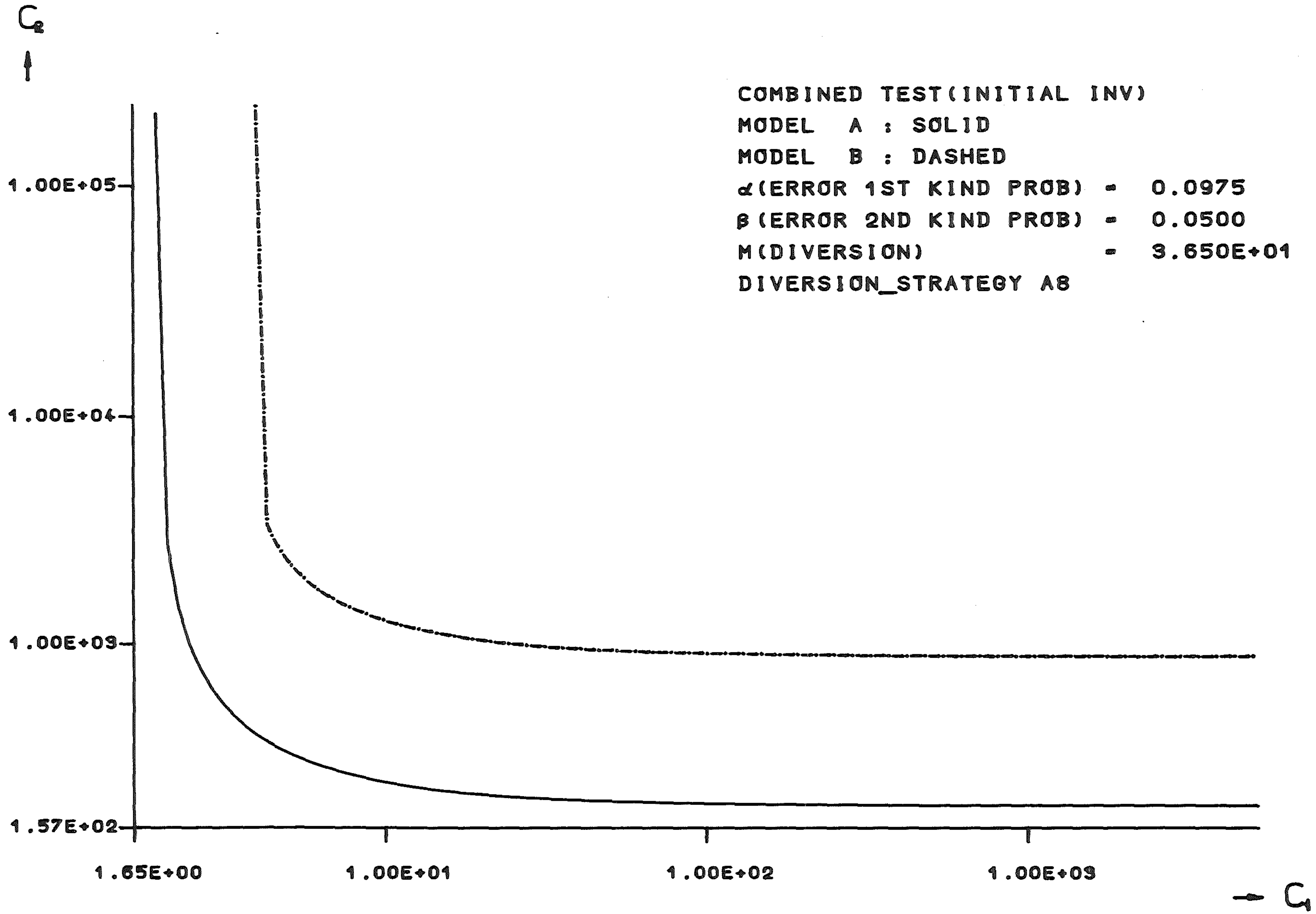

C2
COMBINED TEST(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.0975
β(ERROR 2ND KIND PROB) = 0.0500
M(DIVERSION) = 3.650E+01
DIVERSION_STRATEGY A8
1.00E+05
1.00E+04
1.00E+03
1.57E+02
1.65E+00
1.00E+01
1.00E+02
1.00E+03
C1

Figure 3-28

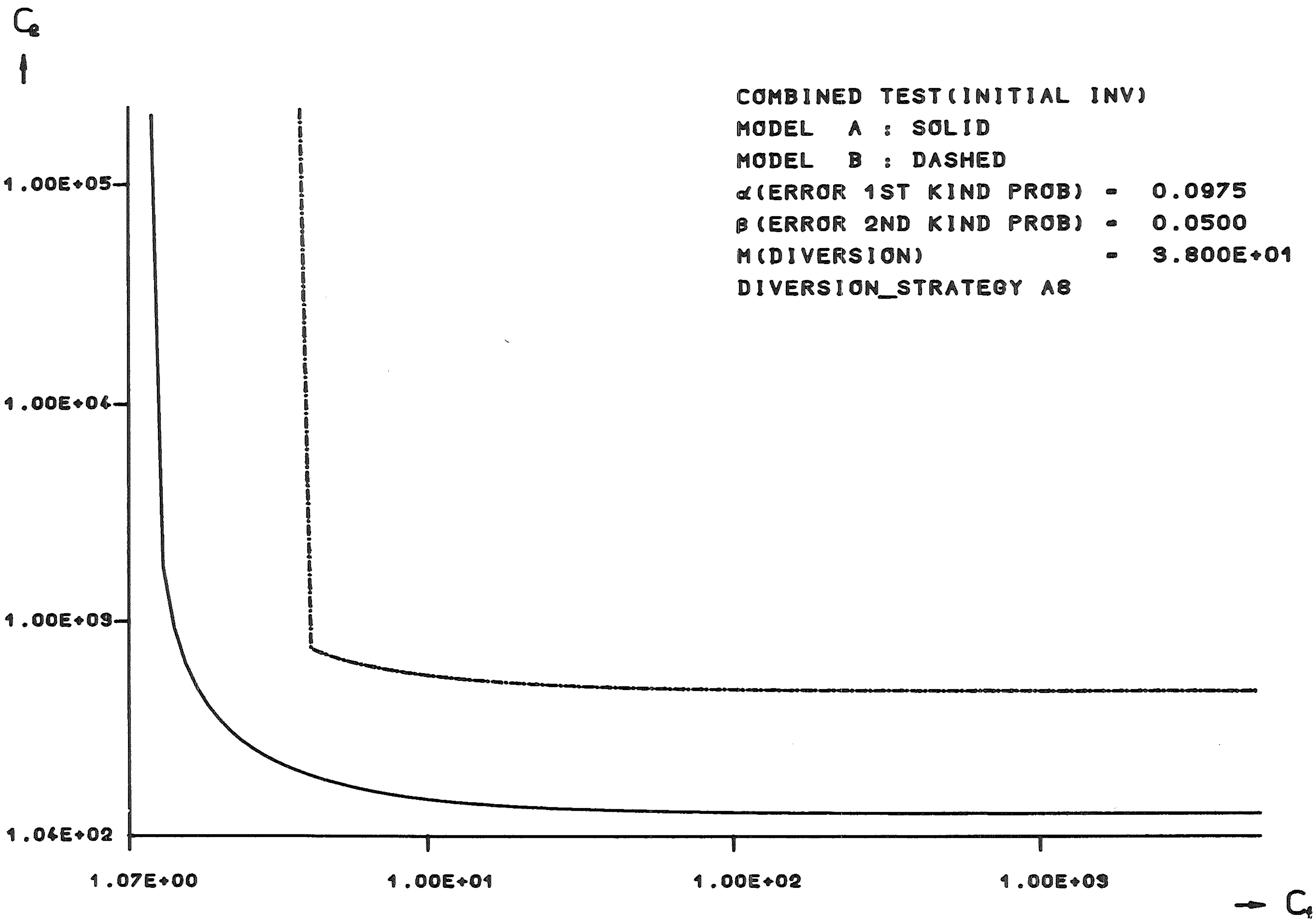

C2
COMBINED TEST(INITIAL INV)
MODEL A : SOLID
MODEL B : DASHED
α(ERROR 1ST KIND PROB) = 0.0975
β(ERROR 2ND KIND PROB) = 0.0500
M(DIVERSION) = 3.800E+01
DIVERSION_STRATEGY A8
1.00E+05
1.00E+04
1.00E+03
1.04E+02
1.07E+00
1.00E+01
1.00E+02
1.00E+03
C1

Figure 3-29

Tables 3-4 to 3-9:

Optimal distribution (M_1,M_2) of the total falsification M, sample sizes and single falsification for Model A and selected values C_1,C_2,M and $1-\beta_t$.

Tab. 3-4: $C_1 = 10.0$ $C_2 = 596.0$ $\beta_t = 0.05$

Class	4	5	6	7	8
$n^{(1)*}$			1.000 E+0		
$\mu^{(1)*}$			1.045 E-1		
$\sigma_r^{(1)}$			1.815 E-1		
$\sigma_s^{(1)}$			7.260 E-2		
$n^{(2)*}$	1.030 E+0	1.988 E-1		8.811 E+1	1.055 E+1
$\mu^{(2)*}$	3.788 E-6	2.152 E-5		1.149 E-2	3.516 E-2
$\sigma_r^{(2)}$	4.525 E-5	2.970 E-4		9.334 E-3	3.734 E-2
$\sigma_s^{(2)}$	2.263 E-5	1.980 E-4		3.734 E-3	3.734 E-2

$\sigma_{DO}^{(1)} = 2.776$ $M_1 = 3.14$

$\sigma_{DO}^{(2)} = 10.25$ $M_2 = 31.8$

Tab. 3-5: C_1 = 10.0 C_2 = 608.0 β_t = 0.05

Class	4	5	6	7	8
$n^{(1)*}$	2.854 E-1		9.715 E+0		
$\mu^{(1)*}$	7.871 E-6		1.064 E-1		
$\sigma_r^{(1)}$	3.200 E-5		1.815 E-1		
$\sigma_s^{(1)}$	1.600 E-5		7.260 E-2		
$n^{(2)*}$		2.042 E-1		9.051 E+1	1.084 E+1
$\mu^{(2)*}$		2.098 E-5		1.146 E-2	3.505 E-2
$\sigma_r^{(2)}$		2.970 E-4		9.334 E-3	3.734 E-2
$\sigma_s^{(2)}$		1.980 E-4		3.734 E-3	3.734 E-2

$\sigma_{DO}^{(1)}$ = 2.809 M_1 = 3.23

$\sigma_{DO}^{(2)}$ = 10.24 M_2 = 31.7

Tab. 3-6: $C_1 = 10.0$ $C_2 = 611.0$ $\beta_t = 0.05$

Class	4	5	6	7	8
$n^{(1)*}$	2.828 E-1	5.477 E-2	9.661 E+0		
$\mu^{(1)*}$	7.908 E-6	4.964 E-5	1.066 E-1		
$\sigma_r^{(1)}$	3.200 E-5	2.100 E-4	1.815 E-1		
$\sigma_s^{(1)}$	1.600 E-5	1.400 E-4	7.260 E-2		
$n^{(2)*}$				9.101 E+1	1.090 E+1
$\mu^{(2)*}$				1.146 E-2	3.504 E-2
$\sigma_r^{(2)}$				9.334 E-3	3.734 E-2
$\sigma_s^{(2)}$				3.734 E-3	3.734 E-2

$\sigma_{DO}^{(1)} = 2.815$ $M_1 = 3.24$

$\sigma_{DO}^{(1)} = 10.23$ $M_2 = 31.7$

Tab. 3-7: $C_1 = 10.0 \quad C_2 = 99.7 \quad \beta_t = 0.05$

Class	4	5	6	7	8
$n^{(1)*}$	2.838 E-1	5.477 E-2	9.601 E+0		
$\mu^{(1)*}$	7.149 E-6	4.487 E-5	9.638 E-2		
$\sigma_r^{(1)}$	3.200 E-5	2.100 E-4	1.815 E-1		
$\sigma_s^{(1)}$	1.600 E-5	1.400 E-4	7.260 E-2		
$n^{(2)*}$				1.484 E+1	1.777 E+0
$\mu^{(2)*}$				1.329 E-2	4.374 E-2
$\sigma_r^{(2)}$				9.334 E-3	3.734 E-2
$\sigma_s^{(2)}$				3.734 E-3	3.734 E-2

$\sigma_{DO}^{(1)} = 2.815 \qquad M_1 = 2.93$

$\sigma_{DO}^{(2)} = 11.84 \qquad M_2 = 37.0$

Tab. 3-8: $C_1 = 10.0$ $C_2 = 51.1$ $\beta_t = 0.05$

Class	4	5	6	7	8
$n^{(1)*}$	2.838 E-1	5.477 E-2	9.661 E+0		
$\mu^{(1)*}$	6.420 E-6	4.029 E-5	8.655 E-2		
$\sigma_r^{(1)}$	3.200 E-5	2.100 E-4	1.815 E-1		
$\sigma_s^{(2)}$	1.600 E-5	1.400 E-4	7.260 E-2		
$n^{(2)*}$				7.614 E+0	9.117 E-1
$\mu^{(2)*}$				1.513 E-2	5.214 E-2
$\sigma_r^{(2)}$				9.334 E-3	3.734 E-2
$\sigma_s^{(2)}$				3.734 E-3	3.734 E-2

$\sigma_{DO}^{(1)} = 2.815$ $M_1 = 2.63$

$\sigma_{DO}^{(2)} = 13.43$ $M_2 = 42.3$

Tab. 3-9: C_1 = 10.0 C_2 = 33.1 β_t = 0.05

Class	4	5	6	7	8
$n^{(1)*}$	2.838 E-1	5.477 E-2	9.661 E+0		
$\mu^{(1)*}$	5.799 E-6	3.640 E-5	7.818 E-2		
$\sigma_r^{(1)}$	3.200 E-5	2.100 E-4	1.815 E-1		
$\sigma_s^{(1)}$	1.600 E-5	1.400 E-4	7.260 E-2		
$n^{(2)*}$	4.9			4.930 E+0	5.902 E-1
$\mu^{(2)*}$				1.695 E-2	6.028 E-2
$\sigma_r^{(2)}$				9.334 E-3	3.734 E-2
$\sigma_s^{(2)}$				3.734 E-3	3.734 E-2

$\sigma_{DO}^{(1)}$ = 2.815 M_1 = 2.37

$\sigma_{DO}^{(1)}$ = 15.01 M_2 = 47.6

Tables 3-10 to 3-12:

Optimal distribution (M_1, M_2) of the total falsification M, sample sizes and single falsifications for Model B and selected values C_1, C_2, M and $1-\beta_t$.

Tab. 3-10: C_1 = 22.0 C_2 = 215000.0 β_t = 0.05

Class	4	5	6	7	8
$n^{(1)*}$	6.244 E-1	1.205 E-1	2.126 E+1		
$\mu^{(1)*}$	2.919 E-5	1.915 E-4	1.655 E-1		
$\sigma_r^{(1)}$	3.200 E-5	2.100 E-4	1.815 E-1		
$\sigma_s^{(1)}$	1.600 E-5	1.400 E-4	7.260 E-2		
$n^{(2)*}$	2.893 E+2	5.584 E+1	3.000 E+1	2.539 E+3	7.600 E+1
$\mu^{(2)*}$	8.506 E-5	5.582 E-4	4.825 E-1	1.754 E-2	7.018 E-2
$\sigma_r^{(2)}$	4.525 E-5	2.970 E-4	2.567 E-1	9.334 E-3	3.734 E-2
$\sigma_s^{(2)}$	2.263 E-5	1.980 E-4	1.027 E-1	3.734 E-3	3.734 E-3

$\sigma_{D0}^{(1)}$ = 2.488 $\sigma_{D0}^{(2)}$ = 10.36 M_1 = 2.56

$\sigma_{D1}^{(1)}$ 2.548 $\sigma_{D1}^{(2)}$ = 10.36 M_2 = 32.4

Tab. 3-11: $C_1 = 22.0$ $C_2 = 313.0$ $\beta_t = 0.05$

Class	4	5	6	7	8
$n^{(1)*}$	6.244 E-1	1.205 E-1	2.126 E+1		
$\mu^{(1)*}$	2.237 E-5	1.468 E-4	1.269 E-1		
$\sigma_r^{(1)}$	3.200 E-5	2.100 E-4	1.815 E-1		
$\sigma_s^{(1)}$	1.600 E-2	1.400 E-4	7.260 E-2		
$n^{(2)*}$	4.201 E-1	8.109 E-2	1.168 E+1	3.594 E+1	4.304 E+0
$\mu^{(2)*}$	9.975 E-5	6.546 E-4	5.658 E-1	2.057 E-2	8.230 E-2
$\sigma_r^{(2)}$	4.525 E-5	2.970 E-4	2.567 E-1	9.334 E-3	3.734 E-2
$\sigma_s^{(2)}$	2.263 E-5	1.980 E-4	1.027 E-1	3.734 E-3	3.734 E-2

$\sigma_{DO}^{(1)} = 2.488$ $\sigma_{DO}^{(2)} = 11.4$ $M_1 = 1.96$

$\sigma_{D1}^{(1)} = 2.523$ $\sigma_{D1}^{(2)} = 12.55$ $M_2 = 38.0$

Tab. 3-12: $C_1 = 22.0$ $C_2 = 162.0$ $\beta_t = 0.05$

Class	4	5	6	7	8
$n^{(1)*}$	6.244 E-1	1.205 E-1	2.126 E+1		
$\mu^{(1)*}$	1.678 E-5	1.101 E-4	9.515 E-2		
$\sigma_r^{(1)}$	3.200 E-5	2.100 E-4	1.815 E-1		
$\sigma_s^{(1)}$	1.600 E-5	1.400 E-4	7.260 E-2		
$n^{(2)*}$	2.173 E-1	4.193 E-2	6.039 E+0	1.859 E+1	2.225 E+0
$\mu^{(2)*}$	1.142 E-4	7.492 E-4	6.475 E-1	2.355 E-2	9.418 E-2
$\sigma_r^{(2)}$	4.525 E-5	2.970 E-4	2.567 E-1	9.334 E-3	3.734 E-2
$\sigma_s^{(2)}$	2.263 E-5	1.980 E-4	1.027 E-1	3.734 E-3	3.734 E-2

$\sigma_{DO}^{(1)} = 2.488$ $\sigma_{DO}^{(2)} = 12.3$ $M_1 = 1.47$

$\sigma_{D1}^{(1)} = 2.508$ $\sigma_{D1}^{(2)} = 14.87$ $M_2 = 43.5$

4. Conclusion

It is shown in the foregoing part that game theoretic considerations lead to a reasonable analysis to determine sample sizes for the verification of materials balaces. It is feasible to use two different measurement methods for the verification of operator's data. The formulae for inspector sample sizes can be easily implemented on a computer. Only two extreme diversion models have been used. Nevertheless, theoretical and numerical considerations give plausible assumptions about diversion strategies under certain parameter conditions. Especially the fact that the more general Model B can be treated leads to the conclusion that a distinction in attributed and variable sampling seems not necessary. The analysis enables several parameter studies.
For single verification methods a dependence between amount of falsification and inspection effort for a given probability of detection is presented. That means for a certain verification effort i.e. a limitation that is given in terms of time or money we can find an amount of falsification that is detected with an acceptable detection probability.

If we look at the situation where two measurement methods are used by the inspector to verify operator's data we can illustrate the relationship between the inspection efforts for both methods under a given detection probability. These areas can be isolated where reasonable combinations of inspection efforts should be. Furthermore, the dangerous areas for the inspector are demonstrated. That is these areas where a reduction of inspection effort for one method leads to necessity of large addition of the other method to attain a certain detection probability. This is a point of view that is very important for a inspection authority.

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