# Upgrading the Lyon Cluster Ion Accelerator by a Radiofrequency Quadrupole 

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# Nachrüstung des Lyoner Clusterionen-Beschleunigers mit einem Radio-frequenz-Quadrupol 

## Zusammenfassung

Der Entwurf eines RFQ mit variabler Endenergie, der geeignet ist, Clusterionen vom elektrostatischen Clusterionen-Beschleuniger in Lyon nachzubeschleunigen in den Massenbereichen 1 bis 25 u bzw. 1 bis 50 u auf kinetische Energien von $1.32-2.5 \mathrm{MeV}$ bzw. $2.64-5.0 \mathrm{MeV}$ im Dauerstrich- bzw. Pulsbetrieb, wird vorgestellt. Zusätzlich wird ein Strahlführungssystem, das den elektrostatisch vorbeschleunigten Strahl mit Hilfe von zwei elektrostatischen Quadrupoltriplets an den $R E Q$ anpaßt, beschrieben. Auch ohne RFQ dient dieses Strahlführungssystem der Verbesserung der Strahlparameter auf dem Target, wie z. B. der Teilchenstromdichte oder der Strahldivergenz. Eine Abschätzung der Kosten des Projektes führt zu etwa DM 345000 .- bzw. FF 1200 000.- zuzüglich Mehrwertsteuer.

## Upgrading the Lyon Cluster-Ion Accelerator by a Radiofrequency Quadrupole

## Abstract

The design is presented of an $R F Q$ with variable final energy suitable to post-accelerate cluster ions from the Lyon electrostatic cluster-ion accelerator in the mass ranges from 1 to $25 u$ and 1 to $50 u$ up to kinetic energies of $1.32-2.5 \mathrm{MeV}$ and $2.64-5.0 \mathrm{MeV}$ for cw and pulsed operation, respectively. Furthermore, a beam line is described which matches the electrostatically preaccelerated beam to the RFQ by use of electrostatic quadrupole triplets. When used without RFQ this beam line serves to improve beam parameters on the target, such as the particle flux density or beam divergence. The estimated costs of this project are about DM $345000 .-$ or EF 1200 000.- without VAT.

## Introduction

The Lyon cluster-ion accelerator was initated 15 years ago within the frame of a collaboration between the Institut de Physique Nucléaire de Lyon and KfK as part of a project conducted by $K f K$ and aimed at the development of intense hydrogen cluster-ion beams for nuclear fusion /1/. After completion of the fusion-oriented work in 1978, and in particular during the past few years, this accelerator has seen a growing experimental activity (for references see /2/). In order to widen the available range of beam parameters, and to improve the performance, a possible upgrading is studied which includes, as the most important part, an $R F Q$ as a post-accelerator and a new beam line to transport the beam from the exit of the acceleration gap to the target or the entrance of the post-accelerator.

## RFQ parameters

A first example of a $4-\operatorname{rod}-\lambda / 2-R F Q$ accelerating cluster ions for basic research is discussed in /2/. Compared to this design the basic parameters were modified in order to comply with budget constraints. The final energy was halved to 2.5 MeV in ew operation which shortens the RFQ to about 3 m length. Using a standard 20 kW FM transmitter a kinetic energy of $100 \mathrm{keV} / \mathrm{u}$ is possible within the mass range of 1 to $25 u$ in ow operation, or within 1 to 50 u in the pulsed mode. The latter case corresponds to a final energy of 5 MeV . Since the input cluster-ion beam is already pulsed, the pulsed operation of the $R E Q$ does not mean a serious restriction. An overview of the major parameters is given in Table 1. Figure 1 displays

Table 1: RFQ parameters

| Initial kinetic energy per u | keV |  |
| :--- | ---: | ---: |
| Final kinetic energy per u | keV | 10 |
| Maximum total kinetic energy | MeV | $2.5(5)$ |
| Maximum ion mass | u | $25(50)$ |
|  |  | 2.96 |
| Length of structure | m | 4 |
| Clear aperture between electrodes | mm | 281 |


| Initial length of cell ( $\beta \lambda / 2$ ) | min | 6.4 |
| :---: | :---: | :---: |
| Maximum modulation of electrodes |  | 1.8 |
| Diameter of vacuum chamber | m | 0.5 |
| Frequency | MHz | 108 |
| Peak voltage | kV | 50(10) |
| Power | kW | 20 ( |
| Maximum field strength | MV/m | 2.3 ( |
| Duty cycle |  | 1.0 ( |
| Radial phase advance per cell |  | $13^{\circ}$ |
| Initial longitudinal phase after shaper |  | -30 ${ }^{\circ}$ |
| Final longitudinal phase |  | -20 ${ }^{\circ}$ |
| Final longitudinal phase spread |  | $\pm 22$ 。 |
| Final relative energy spread | \% | $\pm 1.1$ |
| Transmission | \% | 58 |
| Normalized acceptance | $\mathrm{d} \cdot \mathrm{mm}$ | 3 |

(Values in parentheses are valid for pulsed operation)
the longitudinal dependence of some parameters of interest.
A spectacular confirmation of the validity of the 4 -rod-RFQ principle was obtained very recently by the Frankfurt group. A 1.2 m long 202.5 MHz 4 -rod RFQ was installed at the HERA injection beam line at DESY and accelerated up to 35 mA of $\mathrm{H}^{-}$ions from 18 keV to 750 keV . The final energy spread was $\pm 15 \mathrm{keV}$ to be compared with $\pm 13 \mathrm{keV}$ at 20 mA as computed with PARMTEQ. The measured emittance was $1 \cdot \pi \cdot \mathrm{~mm} \cdot \mathrm{mrad}$ in pretty good agreement with the PARMTEQ result of $1 \cdot \pi \cdot m m \cdot m r a d ~(90 \%$ beam). After conditioning of the resonator tank the design value of 20 mA was reached within one hour. Despite the low duty factor of this machine (pulse length 0.25 ms , repetition rate 1 Hz ) the results show that the specific design of the electrodes and the resonator was successful and that the accelerator performance was quite insensitive to the operating conditions.

A new feature which is a prerequisite of the use as an accelerator for basic research purposes has been included. This is the possibility to vary the final energy. Up to now, RFQs have been fixed energy accelerators with, at most, very small variations of the final energy such as with the
proposed decelerator at CERN /3/. For the RFQ outlined in this report we take advantage of the dependence of both the resonance frequency and the shunt impedance, on the geometric parameters of the four-rod structure, as shown in figs. 2 and $3 / 4 /$. The resonance frequency can be appreciably tuned so that it seems possible to scan the FM range from about 80 MHz to 110 MHz , corresponding to an energy change by a factor of 1.89 . The maximum final energy of $2.5(5.0) \mathrm{MeV}$ would be obtained with 110 MHz , the minimum being $1.32(2.64) \mathrm{MeV}$ at 80 MHz . The shunt impedance and power would vary between 250 and $380 \mathrm{kOhm} / \mathrm{m}$ and $30(120)$ and $21(84) \mathrm{kW}$, respectively. The height $H$ of the main current loop (Fig. 3) was chosen as variable parameter. An experimental embodiment of its variation is currently investigated in order to prove the practical usefulness of this approach.

## Further perspectives

With the RFQ designed for $1-50 \mathrm{u}$ and $100 \mathrm{keV} / \mathrm{u}$ final energy with a $80-$ 110 MHz transmitter heavier cluster ions can be accelerated by choosing smaller values of frequency and final energy per mass unit. The scaling laws connecting the mass number $N$ with the frequency $f$ and final energy Euf are $\mathrm{N} \cdot \mathrm{f}^{2}=$ const and $\mathrm{N} \cdot \mathrm{E}_{\mathrm{uf}}=$ const, respectively. A rather cheap way to obtain the mass range of 1 to $10^{3} \mathrm{u}$ with an initial energy and a final energy of $0.5 \mathrm{keV} / \mathrm{u}$ and $5 \mathrm{keV} / \mathrm{u}$, respectively, would be to modify the transmitter and the structure such that they work between 17.9 and 24.6 MHz . The option for this modification may be included when the transmitter will be ordered. This mass range comprises cluster ions with, e.g. 1000 hydrogen atoms, 62 oxygen atoms, 17 nickel atoms or 9 silver atoms. For an analysis of the mass range 1 to $10^{6}$ u see ref. /2/.

Since the total energy of 5 MeV is about the limit for RFQs, at least at present, further increases would call for other accelerating schemes. In this context, low-velocity superconducting linear accelerating structures which appear to be useful and available from $\beta=1 \%$ to higher values /5/ might be of help. Even further acceleration by circular accelerators is conceivable /6/.

A schematic of the existing beam line is displayed in fig. 4. The beam is accelerated vertically, then bent into the horizontal direction by a cylindrical electrostatic deflector (74 ) and a bending magnet (16 ). The purpose of the magnet is to provide mass selection. Apart from the weak focussing due to the deflector and the magnet in the radial plane and some edge focussing through the tilted pole faces of the magnet (see below), the beam line has not contained focussing elements so far. To improve the beam transport, the effect of quadrupole triplets added to the beam line was analysed (fig. 5). The objective of the first triplet is to optimize the beam transport across the deflector and the magnet, while the second triplet serves to match the phase space occupied by the beam to a desired condition on the target or to the acceptance of an RFQ post-accelerator.

Electrostatic quadrupoles are chosen for two reasons. Firstly, as the velocity of 10 keV -protons is $1.38 \times 10^{6} \mathrm{~m} / \mathrm{s}$ corresponding to $\beta=0.46 \%$, an electric field strength of $13.8 \mathrm{kV} / \mathrm{cm}$ is equivalent to a magnetic flux density of $1 \mathrm{~T}(E=\mathrm{vxB})$. Thus, an electrostatic field is still rather effective. Secondly, the first triplet has to be placed in a quite restricted space inside the high vacuum, and electrostatic quadrupoles lend themselves more readily to comply with these restrictions.

## Method of beam optics calculation

Since the emittance of the cluster ion beam at the exit of the acceleration gap is not known a rather large value was assumed for the calculation, namely $\pi \cdot 5 \mathrm{~mm} .5 \mathrm{mrad}$. Then the non-linear optics and tracking code MARYLIE /7/ was used to trace 20 rays and to look at the phase space plots at 6 selected planes intersecting the design orbit perpendicularly. These planes are located at the entrance, the center, and the exit of the electrostatic deflector, at the exit of the bending magnet, and at 0.1 m and 1.05 m downstream of the second triplet. The phase space plots in these planes permit to follow the transport of the beam through the individual beam line elements.

Since MARYLIE's catalogue of beam line elements contains neither the electrostatic deflector nor the electrostatic quadrupole, a separate program, MPDOME, was written to calculate the maps describing these beam line elements. More details on this procedure will be given in a further report.

The following results were obtained using only the linear part (transfer matrix) of the map.

Additional focussing components

Figure 6 displays the geometry of the quadrupole triplet used. The electrostatic potential $\psi$ is given by

$$
\psi=\left(U_{o} / r_{0}^{2}\right)\left(y^{2}-x^{2}\right)
$$

where $2 U_{0}$ is the voltage between rods of opposite polarity and $r_{0}$ is the distance between the axis and the pole tips. If $U_{0}<0$, the quadrupole is focussing in $x$-direction and defocussing in $y$-direction, and vice versa for $\mathrm{U}_{\mathrm{O}}>0$. The x -coordinate lies in the plane containing the beam line, while the $y$-coordinate is perpendicular to that plane. The central quadrupole is focussing in the $x$-direction, leading and trailing quadrupoles in the $y$ direction. It has already been taken into account that the cross-section of the electrodes is not hyperbolic, but cylindrical in order to make fabrication easier. According to $/ 8 /$, the radius of the cylinders is 1.16 ro giving a radial distance of the electrode axes from the quadrupole axis of $2.16 r_{0}$. Figure 7 shows the focal lengths in $x$ - and $y$-directions, respectively, for the $1-2-1$ triplet versus the parameter $U_{o} / r_{0}{ }^{2}$. The focal length is extracted from the transfer matrix (see, e.g., /9/). If the components of the phase space vector are ( $x, p_{x}, y, p_{y}, t, p_{t}$ ), then the matrix elements $m_{21}$ and $m_{43}$ are related to the focal lengths $f_{x}$ and $f_{y}$ by the relations

$$
f_{x}=-1 / m_{21} \quad \text { and } \quad f_{y}=-1 / m_{43} .
$$

The triplet is fed by two high-voltage supplies featuring positive and negative output polarities.

As a consequence of the geometry chosen the focal lengths are different in the same sense over the whole interval. This is a desired feature since the beam waist should be in the center of the deflector, for the x-direction, and in the magnet, for the y-direction (cf fig. 5). Thus, the triplet must be oriented in such a way that the short focal length applies to the x direction, and the long one to the y-direction.

The following figs. 8 - 10 show phase space plots in the 6 planes of section. The upper plot of a column displays $x$ and $p_{x}$ as abscissa and ordinate, respectively, while the center and the lower plots represent ( $y, p_{y}$ ) and ( $t, p_{t}$ ) in a similar way. From the two ellipses shown in a plot that at the orthogonal position gives the initial conditions in the cathode. The evolution of the emittance can thus always be compared with the initial situation. The plots are normalized to the maximum values of the coordinates as given in the table on top. Three typical cases of beam transport are analysed, namely, the "neutral" beam transport, where the shape of the phase space ellipses at the target close to the initial one, the case "parallel on target", where a small divergence angle on the target is favoured at the expense of a larger cross-section, and the case "focus on target", where a small spot on the target is favoured at the expense of a larger divergence angle. The settings of the triplets are given in table 2.

Table 2: Triplet settings

|  | $U_{0} / \mathrm{ro}^{2}$ |  |  |  |  |  |
| :--- | :--- | :--- | ---: | :--- | :--- | :---: |
|  | neutral | parallel | focus |  |  |  |
| triplet 1 | $2.5 \times 10^{7}$ | $2.5 \times 10^{7}$ | $2.5 \times 10^{7}$ | $\mathrm{~V} / \mathrm{m}^{2}$ |  |  |
| triplet 2 | $6.9 \times 10^{6}$ | $5.38 \times 10^{6}$ | $1.24 \times 10^{7}$ | $\mathrm{~V} / \mathrm{m}^{2}$ |  |  |
| (electro- |  | $5.24 \times 10^{6}$ | $1.38 \times 10^{7}$ |  |  |  |
| static) |  |  |  |  |  |  |
| triplet 2 | 10.0 | 7.8 | 18.0 | $\mathrm{~T} / \mathrm{m}$ |  |  |
| (magnetic) | -10.0 | -7.6 | -20.0 |  |  |  |

(The first row of triplet 2 holds for the central quad, the second row for leading and trailing quads.)

For $r_{0}=0.01 \mathrm{~m}$ we obtain $U_{0} \leq 2.5 \mathrm{kV}$. In the last rows of table 2 the values of the gradient of the magnetic flux density are given for the case that the second triplet is a magnetic one. Apart from purely computational reasons for inserting a magnetic triplet there, the actual location would not preclude using a magnetic triplet. The relationship between the gradients of the electric and the magnetic fields is obtained by equating the forces on a particle. Using the expressions

$$
\psi=\left(U_{0} / r_{0}{ }^{2}\right)\left(y^{2}-x^{2}\right)
$$

for the electrostatic, and

$$
A_{z}=\left(I_{0} / 2\right)\left(y^{2}-x^{2}\right)
$$

for the vector potential, and setting

$$
-\partial \psi / \partial x=v \partial A_{z} / \partial x,
$$

we find

$$
U_{0} / r_{0}^{2}=-v I_{0} / 2
$$

Where $v$ and $I_{0}$ are the particle velocity and the magnetic field gradient, respectively. Inserting $v=1.38 \times 10^{6} \mathrm{~m} / \mathrm{s}$ which holds for a 10 keV proton the electrostatic settings for triplet 2 in table 2 are computed. From fig. 8 we see that the width of the $x-p_{x}$-ellipse in $x$-direction is always less than $\pm 5 \mathrm{~mm}$ in planes $1-3$, indicating that the beam passes the deflector without exceeding its aperture. Correspondingly, the width of the $\mathrm{y}-\mathrm{p}_{\mathrm{y}}$-ellipse in planes 3 and 4 is less than $\pm 12.5 \mathrm{~mm}$, which means that the beam passes the magnet chamber without obstruction, too. The setting of the first triplet which determines the passage through the deflector and magnet, is kept constant for all three cases.

In the case of the "neutral" solution, the resulting phase space ellipses are pretty close to the initial ones.

In the case of the "parallel on target" solution, the position and the incidence angle vary in the ranges $\pm 8 \mathrm{~mm}$ and $\pm 3.5 \mathrm{mrad}$, respectively, for the $x$-coordinate, and $\pm 9 \mathrm{~mm}$ and $\pm 2.9 \mathrm{mrad}$, respectively, for the y coordinate.

In the "focus on target" case the corresponding ranges for $x ; p_{x}$ and $y, p_{y}$ are ( $\pm 1.56 \mathrm{~mm}, \pm 16.4 \mathrm{mrad})$ and $( \pm 1.6 \mathrm{~mm}, \pm 14.8 \mathrm{mrad})$, respectively. This beam spot fits reasonably well the RFQ entrance with its 4 mm clearance. The corresponding emittances are about $81 \mathrm{~mm} \cdot \mathrm{mrad}$ and $75 \mathrm{~mm} \cdot \mathrm{mrad}$, respectively. They fit largely the acceptance of $650 \mathrm{~mm} \cdot \mathrm{mrad}$ calculated by dividing the normalized acceptance given in table 1 by $\beta=$ 0.0046 . It should be noted that in this case the last plane is shifted
towards the triplet, the distance between plane $\# 5$ and plane $\# 6$ then being 0.2 m instead of 0.95 m .

Figure 11 illustrates chromatic effects in the beam line. Comparison of fig. 11 with fig. 10 in the plane \#6 shows that a relative energy deviation of $10^{-4}$ leaves the position of the spot on the target almost unchanged while the range of impact angle is shifted by about 16 mrad from $\pm 16.4$ mrad to $0-33$ mrad. However, at the exit of the deflector (plane \#3), the beam exceeds the acceptable aperture of $\pm 5 \mathrm{~mm}$ by about 3 mm and is lost at about $50 \%$.

Thus, it depends on the emittance of the real beam whether or not this loss occurs. It could be mitigated by either increasing the aperture of the deflector providing a new outer electrode with a larger radius, or by dividing the deflector into two parts and putting quadrupoles in between to realize an achromatic system, or by a broad-band feedback system causing the deflection voltage to follow the acceleration voltage to better than $10^{-4}$.

Figure 12 serves to assess the influence of the pole face angle of the bending magnet. The pole faces are tilted by an angle of $4^{\circ}$ from the radial position towards a parallel configuration. However, the catalogue of beam line elements of MARYLIE contains either normal-entry bending magnets (tilt angle $0^{\circ}$ ) or parallel-faced bending magnets (tilt angle $=$ half of bending angle). All phase space plots were computed with the normal entry bend, except for those of fig. 12 which use the parallel-faced bend. The real situation, thus, is somewhere between figs. 12 and 10. Obviously, in a linear approximation, the influence of the pole face angle is small.

## Costs

Since the 4 -rod structure is comparably easy to manufacture the major costs are caused by the transmitter. A preliminary offer from a German manufacturer quotes DM 215 000.- without value added tax for a fixed frequency transmitter ( 108.5 MHz ). The option of varying the frequency between 80 and 110 MHz is estimated to cause an additional amount of DM 30 000.- without VAT. The modification of the transmitter to operate below 30 MHz is expected to require another DM 40 000.- without VAT. Estimating about DM 100 000.- for the manufacture of the structure, the tank and the vacuum
equipment, the order of DM 345000 .- or FF $1200000 .-$ without VAT seems to be realistic for the variable energy $R F Q$ without low frequency option.

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Fig. 1: Kinetic energy per mass unit u $T$, accelerating field strength Ez, phase $\phi$, and transmission $n$ versus position $z$ along the RFQ.



Fig. 2: Resonance frequency $f_{0}$ and shunt impedance $R_{S}$ versus the height of the current loop $H$ as illustrated in fig. 3.


Fig. 3: Geometry of current loop in the $4-\operatorname{rod}-\lambda / 2-\operatorname{RFQ}$ structure.


Fig. 4: Schematic of existing beam line.

## Beam line



## Quadrupole triplet



Fig. 6: Geometry of quadrupole triplet.


| Plane | 1 | Plane | 2 | Plane | 3 | Plane | 4 | Plane |  | PLANE | $\varepsilon$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| XMRX | 0.0050 | XMRX | 0.0050 | XMAX | 0.0050 | XMRX | 0.0050 | $X \max$ | 0.0057 | $X$ MAX | 0.0053 |
| PXP1RX | 0.0055 | PXMAX | 0.0030 | PXMAK | 0.00ss | PXMAX | 0.005s | PX:MAX | 0.0050 | PXMAX | 0.0050 |
| YMIAX | 0.0055 | YMAX | 0.0054 | Ymax | 0.0089 | YMAX | 6. 20.8 | YMAX | 0.0077 | Ymax | 0.0051 |
| PYMARX | 0.0050 | PYMAX | 0.0050 | PYMAX | 0.0050 | PYMAX | 0.0050 | PYMAR | 0.0050 | PYMAX | 0.0050 |
| TMAX | 0.0035 | TMAX | 0.1540 | TMAX | 0.2531 | TMAX | 0.2424 | TMAX | 0.2442 | TMAX | 0.2447 |
| PTMAX | 1.0000 | PTMAX | 1.0000 | ftmax | 1.0000 | PTMAX | 1.8000 | PTMRX | 1.0000 | PTMAX | 1.0000 |

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Fig. 8: Phase space plots in six control planes for the case of approximately identical initial and final conditions.


Fig. 9: Phase space plots in six control planes for the case of a beam having low divergence on the target.

| Plane | 1 | FLANE 2 |  | PLANE 3 |  | PLANE 4 |  | Plane 5 |  | Plane |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| XMAX | 0.0050 | $X \mathrm{MAX}$ | 0.0050 | XMRX | 0.0050 | XMRX | 0.8000 | XIARX | 0.0050 | XMAX | 0.0050 |
| PXMAX | 0.00s5 | PXMAX | 0.0080 | PXMAX | 0.0058 | PxMAX | 0.065s | Pxarax | 0.0154 | PXMAX | 0.0154 |
| YMAX | 0.0055 | YMRX | 0.0054 | YMAX | 0.0059 | YMAX | 0.8075 | YMAX | 0.0050 | YMAX | 0.0050 |
| PYMAX | 0.0050 | PrMAEX | 0.0050 | PYMAX | 0.0050 | PYMAX | 0.0050 | PYMAX | 0.0148 | PYMAX | 0.0148 |
| TMAX | 0.0038 | tMax | 0.1540 | TMAX | 0.2531 | TMAX | 0.2424 | tmax | 0.2485 | tmax | 0.8501 |
| PTMAX | 1.0000 | PTMRX | 1.0000 | PTMAX | 1.0000 | PTMAX | 1.0090 | PTMRX | 1.0000 | PTMPX | 1.0000 |


|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| - | - |  |  |  |  |

Fig. 10: Phase space plots in six control planes for the case of a beam which is focused on the target.

| Plane | 1 | PLANE | 2 | Plane | 3 | Plane | 4 | Plane | 5 | pliane | $E$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| XMRX | 0.0050 | XPARX | 0.0050 | XMAX | 0.0083 | XMRX | 0.0117 | XMRX | 0.0053 | Xminx | 0.0050 |
| PXiffx | 0.0055 | PXMPX | 0.0121 | PEMAX | 0.0125 | Primax | 0.0083 | PXMAX | 0.0330 | PXMAX | 0.0330 |
| YMAX | 0.0055 | YMAX | 0.0005 | YMAX | 0.0059 | YMRX | 0.0075 | YMRX | 0.0050 | Ymax | 0.0050 |
| PYIARX | 0.0050 | PYMRX | 0.0050 | PYMAX | 0.0050 | PYMAX | 0.0050 | PYMRX | 0.0148 | PYMAX | 0.0148 |
| TMAX | 0.5093 | TMAX | 0.9842 | TMAX | 1.2208 | TMAX | 1.4892 | TMAX | 1.7855 | TMAX | 1.3848 |
| PTMAX | 0.3001 | PTMAX | 0.0001 | PTMAX | 0.0001 | PTMAX | 0.0001 | PTMAX | 0.0001 | PTMAX | 0.30001 |

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Fig. 11: Chromatic effect. Influence of a $0.01 \%$ relative energy deviation on the phase space patterns.


Fig. 12: Influence of the pole face angle of the bending magnet on the phase space patterns.

