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A Computer Program for Statistical Analysis of Near-Real-Time-Accountancy (NRTA) Data

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A Computer Program for Statistical Analysis of Near-Real-Time-Accountancy(NRTA) Data

by

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Abstract

The computer program PROSA (Program for Statistical Analysis of NRTA Data) is a tool to decide on the basis of statistical considerations if, in a given sequence of materials balance periods, a loss of material might have occured or not. The evaluation of the material balance data is based on statistical test procedures. In PROSA three truncated sequential tests are applied to a sequence of material balances.

The manual describes the statistical background of PROSA and how to use the computer program on an IBM-PC with DOS 3.1.

PROSA: Ein Computerprogramm zur statistischen Bewertung realzeitnaher Bilanzdaten

Zusammenfassung

Das Computerprogramm PROSA ist ein Entscheidungswerkzeug. Es ermöglicht eine auf statistischen Überlegungen basierende Aussage, ob in einer Folge von Bilanzperioden sich ein Materialverlust ereignet haben könnte. Die Auswertung der Bilanzdaten erfolgt mit Hilfe statistischer Tests. In der vorgelegten PROSA-Version werden drei gestützte sequentielle Tests angewendet.

Dieses Handbuch beschreibt den statistischen Hintergrund von PROSA und wie man das Computerprogramm auf einem IBM-PC mit DOS-Betriebssystem anwendet.

Acknowledgement

We want to take the opportunity to thank Morton Canty from KFA Jülich for his helpful comments on this manual of PROSA. Furthermore, we are grateful to Tanja Schmitt and Brigitte Gräble who did much useful programming work.

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1.OVERVIEW

The Computer program PROSA has been developed as a tool to apply truncated sequential statistical tests to a sequence of materials balance results, the origin of which is a model facility or an existing plant. PROSA is a tool to decide, on the basis of statistical considerations, if in a given sequence of materials balance periods a loss of material might have occurred or not. The evaluation of the materials balance data is based on statistical test prodecures.

One essential aspect in designing statistical tests for nuclear materials accounting data is their expected performance in detecting losses of nuclear material. Performance measures embody the concepts of loss-detection probability and loss-detection time. The performance of a special test has to be studied undera variety of loss patterns. These patterns have to be selected according to reasonable assumptions. One is interested in the analysis of the performance of different test procedures for selected loss patterns.

In Prosa, the three statistical tests:

- Truncated Sequential CUMUF Test
- CUSUM Test with Power-One Thresholds
- CUSUM Test with Page's Thresholds

are selected / AVE84, AVE85,BEE83b/.

These three test procedures are the result of several years of statistical research and, at the moment, are the most promising ones, as far as the detection probability for a loss of material and the timeliness of detection of a loss is concerned.
The program PROSA has been developed for evaluating accountancy data from reprocessing facilities. However it is also able to evaluate accountancy data of all kinds of facilities as long as they posess a particular, but fairly general structure.

The evaluation of a given data set can be performed with a desired false alarm probability α. This enables sensitivity studies for given data sets.

In order to use PROSA it is not necessary to understand all of the statistical details. But it is important that the user is aware of the measurement model of

the plant under consideration. The measurement model is the basis for the statistical tests performed on a given sequence of materials balance results.
This manual describes the statistical background of PROSA and how to use the computer program on an IBM-PC with DOS 3.1. The most important point for the user to know is the correct input of the plant data; i.e. inventories, transfers, measurement uncertainties, MUF-sequence.

2. MULTIPLE BALANCE MODEL

Let us assume a discrete number of balance periods $k = 1,2,.....,n$ for a well defined class of material. For each period k we calculate the difference between book and physical inventory, which is defined as /BEE83 b/

$$MUF_k = I_{k-1} + T_k - I_k \quad (2.1)$$

where T_k is defined as

$$T_k = R_k - S_k. \quad (2.2)$$

In Eqs. (2.1) and (2.2) the symbols have the following meaning:

I_{k-1}: beginning inventory of period k
I_k : ending inventory of period k and beginning inventory of period k + 1
R_k : increase in inventory during balance period k
S_k : decrease in inventory during balance period k.

The concept of multiple balances is primarily used for detection of possible nuclear materials losses in a bulk handling facility. The detection has to be timely and have a sufficiently high probability. The true MUF_k values are zero in the ideal situation of no losses and no measurement errors. In actual practice, however, nonzero MUF_k's may occur for a number of reasons, e.g.
(a) measurement errors (b) loss of material.
Measurement errors in our model are represented as random variables in determining the materials balance.

We assume that I_k, R_k and S_k are random variables that can be written as

$$I_k = E(I_k) + ZI_k + SI_k.$$

$E(I_k)$ is the true value of inventory, ZI_k is the random error of measurement and SI_k is the systematic measurement error. Furthermore,we define

$$T_k = R_k - S_k = E(T_k) + ZT_k + ST_k$$

for all k, where $E(T_k)$ are the true values, ZT_k the random measurement errors and ST_k the systematic measurement errors.

A further assumption is that all measurement errors are stochastically independent.

The variances for period k are defined as

$$\mathrm{var}(I_k) = \mathrm{var}(ZI_k) + \mathrm{var}(SI_k) \qquad \text{and}$$

$$\mathrm{var}(T_k) = \mathrm{var}(ZT_k) + \mathrm{var}(ST_k) \ .$$

For two periods i and j we define the covariance of MUF_i and MUF_j as

$$\sigma_{ij} = \mathrm{cov}(MUF_i, MUF_j) \ .$$

All the variance and covariance calculations may be summarized in the variance-covariance matrix Σ, also called dispersion matrix, of the sequence MUF_1, MUF_2,......,MUF_n:

$$\Sigma = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ & \ddots & \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{pmatrix} \tag{2.3}$$

The matrix Σ is the condensed form of the measurement model of the facility under consideration. It is an essential component of the statistical analysis of the MUF sequence.

Given a sequence of nonzero MUF values we have to decide whether the values are due to measurement errors or loss. In our case we use the theory of statistical hypothesis testing to decide on the basis of a given sequence of MUF values whether the situation of no loss or loss of nuclear material pertains. Loss of material may occur in a variety of patterns and we have to take into account that the actual loss pattern is unknown.

We assume two hypotheses for the mean values of the random variables MUF_k. If there is no loss of material, all materials balances have zero mean. This situation is described by the null hypothesis:

$$H_0 : E(MUF_k) = 0 \text{ for all periods } k = 1,2,....,n. \qquad (2.4)$$

A loss of material can take place in one or more balance periods.
Taking this into account, we formulate the alternative hypothesis:

$$H_1 : \quad E(MUF_k) = m_k \neq 0$$

$$\text{with } \Sigma m_k > 0. \qquad (2.5)$$

Hypothesis H_1 means that a loss of material occurred in at least one balance period k. In our considerations we are not restricted to a fixed number of inventory periods.

The basic problem is to find test procedures that enable a decision between H_0 and H_1. A further problem is to find test procedures with a given probability of Type I error α (false alarm probability) and with a small probability of Type II error (decision for H_0 if H_1 is true, i.e. there is a loss and we do not detect it). And an even further problem is to find test procedures which indicate a loss almost immediately after it has happened.

The statistical test procedures that are applied in this study assume that the materials accounting data are not falsified.

3. NRTA TEST PROCEDURES

Sequential test procedures are used to evaluate a given sequence $MUF_1, \dots, MUF_n$ of materials balance data. The sequential testing procedures in PROSA evaluate a given sequence of normally distributed MUF_i values and give a decision between the two hypotheses (2.3) and (2.4):

H_0: no loss of material and $E(MUF_i) = 0$ for all $i = 1, 2, \dots, n$

H_1: $E(MUF_k) = m_k \neq 0$

with $\Sigma m_k > 0$.

The sequential tests in PROSA are truncated versions, this means they give a decision at the end of the n-th balance period or earlier. We use three sequential test procedures in PROSA, all of which are evaluated with the same selected false alarm probability α.

In the following the three sequential test procedures are described.

3.1 Test Based on MUF'S

A. Truncated Sequential CUMUF Test

CUMUF is defined as the cummulative sum of the material balance results MUF_i:

$$CUMUF_i = MUF_1 + \dots + MUF_i \qquad i = 1,2,\dots\dots,n . \qquad (3.1)$$

The test is performed as follows /BEE83 a/:

for $i = 1, 2, \dots, n-1$:

$$CUMUF_i \begin{cases} > s_i, \text{ reject } H_0 \\ \leq s_i, \text{ no decision and go to the next period.} \end{cases}$$

for i = n:

$$CUMUF_n \begin{cases} \leqq s_n, \text{ reject } H_1 \\ > s_n, \text{ reject } H_0. \end{cases}$$

The significance thresholds s_1, s_2, ..., s_n are determined by a Monte Carlo simulation to give a given false alarm probability α. In our case we select

$$s_i = \text{var}(CUMUF_i)^{1/2} \cdot U_{1-\alpha'}$$

where U is the inverse standard normal distribution function. The value α' corresponds to the total false alarm probability α.

3.2 Tests Based on the MUF-Residuals

The materials balance equations MUF_i are stochastically dependent random variables. With a linear transformation it is possible to transform the sequence MUF_1, ..., MUF_n to a sequence of stochastically independent random variables $MUFR_1$, ..., $MUFR_n$. There are numerous possibilities for this tranformations. We selected the transformation given by:

$$MUFR_i = MUF_i - E(MUF_i \mid MUF_1, \ldots, MUF_{i-1})$$

for $i = 2, \ldots, n$ with $MUFR_1 = MUF_1$. The values $MUFR_i$ are called MUF residuals because they describe the difference between the estimate for the mean of MUF_i based on the last i-1 results and the realization of MUF_i.

The transformation can be described as a n x n matrix T with

$$(MUFR_1,\ldots,MUFR_n) = (MUF_1,\ldots,MUF_n) \cdot T \tag{3.2}$$

$T \cdot \Sigma \cdot T^t$ is a diagonal matrix, the dispersion matrix of the MUFR-vector.

For the hypotheses we get:

(H_0): $E(MUFR_i) = 0$ for $i = 1, 2, \dots, n$. (3.3)

under the alternative hypothesis H_1 positive or negative values for the sum of the means of MUFRi are possible, and this is an important difference to MUF_i's. Therefore, we have a two-sided hypothesis:

(H_1): $E(MUFR_i) \neq 0$ for at least one i. (3.4)

B. Power-One Test

The power-one test was proposed by Robbins as a procedure that accepts H_1 with probability one when it is true and testing can continue indefinitely. For this test we use the cumulative sum of the standardized MUFRi variables:

$$T_i = \sum_{j=1}^{i} \frac{MUFR_j}{var(MUFR_j)^{1/2}} \qquad (3.5)$$

The test procedure is defined as follows /ROB70, SEL 83/:

for $i = 1, 2, \dots, n-1$:

$$|T_i| \begin{cases} > b_i, & \text{reject } H_0 \\ \leqq b_i, & \text{no decision and go to the next period} \end{cases}$$

for $i = n$:

$$|T_n| \begin{cases} > b_n, & \text{reject } H_0 \\ \leqq b_n, & \text{reject } H_1. \end{cases}$$

The parameters b_i are calculated as

$$b_i = [(i+m) \cdot (-2\, ln(a) + ln(1 + \frac{i}{m}))]^{1/2}$$

where a is determined by simulation to obtain a specific false alarm probability α and m is a parameter which influences the distribution of the false alarms.

C. CUSUM-Test with Page Thresholds

The test was proposed by Page and uses the following statistics:

$S_0 = 0$

$T_0 = 0$

$S_i = \max\{0, S_{i-1} + MUFR_i - k\}$

$T_i = \min\{0, T_{i-1} + MUFR_i + k\}$

for i = 1, 2, ... , n where k is a fixed real number. The test procedure called Page's test is defined as follows, where h is a real number /PAG54, SEL82/:

for i = 1, 2, ... , n-1.

1. $S_i > h$ or $T_i < -h$, reject H_0

2. $S_i \leqq h$ and $T_i \geqq -h$, no decision and go to the next period

for i = n:

1. $S_n > h$ or $T_n < -h$, reject H_0

2. $S_n \leqq h$ and $T_n \geqq -h$, reject H_1.

The parameters h and k are determined by simulation to garantee a false alarm probability α for the n balance periods. In our case we selected k = 0.

4. MODEL FACILITY

The concept of a model facility is a helpful tool to study the performance of NRTA in plants which are designed but not yet built / Ave85/. The model facility consists of four parts:

- the inventory
- the measurement model for the inventory
- the transfers
- the measurement model for the transfers.

In this report we concentrate on a particular model reprocessing facility because it has been used for a great deal of NRTA analysis. But it is principally possible to study other facilities with this model as well.
Balance periods for a well defined class of material are established with simulated data of a model facility. The facility is divided into five inventory blocks. The input consists of a certain number of batches in a given time period. Two kinds of output are allowed. Again, for the output we allow a certain number of batches for a given time period. We assume that the facility is operated in steady state which means constant inventory in all plant components. One or several time periods (e.g. days) may be summarized to one accounting period.

We assume that all terms in the material balance equation are based on measured values and we assume two kinds of measurement errors:

random errors:	these errors have a normal distribution with mean 0 and a given variance, these errors may differ from one measurement to the next
systematic errors:	these errors have a normal distribution with mean 0 and a given variance; they do not change for several balance periods.

Fig. 4.1 illustrates the structure of the model facility and Tab. 4.1 gives an example for the data set of the facility in Fig. 4.1.

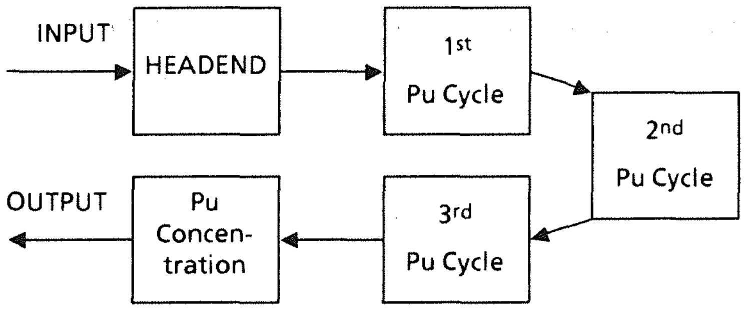


Figure 4.1
Plutonium-chemical separation process of a reprocessing facility used as a basis for the numerical model.

Table 4.1
PLUTONIUM INVENTORY OF 1000t MODEL REPROCESSING FACILITY AND MEASUREMENT UNCERTAINTIES

	Plutonium Inventory (kg)	Relative Standard Deviations Random	Systematic
Headend	196.5	0.010	0.010
1st plutonium-cycle	7.6	0.010	0.010
2nd plutonium-cycle	50.0	0.005	0.005
3rd plutonium-cycle	143.0	0.005	0.005
Plutonium concentration	62.5	0.005	0.005
Total	450.6		

PLUTONIUM CONTENT OF INPUT AND OUTPUT BATCHES AND CORRESPONDING MEASUREMENT UNCERTAINTIES[a)]

	Plutonium/ Batch (kg)	Relative Standard Deviations Random	Systematic
Input	16.73	0.010	0.010
Product Output	25.00	0.002	0.002
Waste Output	0.20	0.250	0.250

a) Based on three input batches, two product batches, and one waste output batch/day and a plant operation of 200 days/yr.

5. STATISTICAL TEST PROCEDURE FOR A GIVEN MUF-SEQUENCE

In the following we illustrate the procedure performed by PROSA to evaluate a given sequence of materials balances. First, a sequence of material balances MUF_1, MUF_2, ... , MUF_n is needed, assumed to be accountancy results from the facility under consideration. An important component is the measurement model Σ, which in our case will be calculated with a model facility. Furthermore, a false alarm probability α has to be assumed to calculate the test thresholds based on standardized random variables for the three test procedures.
The given MUF_i values are standardized by PROSA and tested with our three test procedures. PROSA indicates if an alarm has occurred. In case of an alarm PROSA gives the balance period of the first alarm.
Let us now assume that we have a sequence of 5 materials balances for Pu with the following MUF_i's:

MUF_1 = 2.1 kg Pu, MUF_2 = 0.5 kg Pu, MUF_3 = 0.8 kg Pu, MUF_4 = 1.7 kg Pu, MUF_5 = 0.5 kg Pu.

Furthermore, we assume that these MUF_i's came from the model facility in Tab. 5.1.

Table 5.1:

PARAMETERINPUT FOR A BLOCK-MODEL OF A REPROCESSING FACILITY

THROUGHPUT (kg) :
250.

NUMBER OF WORKING DAYS :
5

BALANCE INTERVAL IN DAYS :
1

INTERVAL OF RECALIBRATIONS :
0

HEAD-END:	PU-INVENTORY (kg) 196.50	RELATIVE STANDARD (RANDOM) 0.010	DEVIATION (SYSTEMATIC) 0.000 [1)]
1. PU-CYCLE:	PU-INVENTORY (KG) 7.6	RELATIVE STANDARD (RANDOM) 0.010	DEVIATION (SYSTEMATIC) 0.000

1) In a steady state operating facility the systematic error of the inventories is irrelevant.

2. PU-CYCLE:	PU-INVENTORY (KG)	RELATIVE STANDARD (RANDOM)	DEVIATION (SYSTEMATIC)
	50.0	0.005	0.000

3. PU-CYCLE:	PU-INVENTORY (KG)	RELATIVE STANDARD (RANDOM)	DEVIATION (SYSTEMATIC)
	134.0	0.005	0.000

PU-CONCENTRATION:	PU-INVENTORY (KG)	RELATIVE STANDARD (RANDOM)	DEVIATION (SYSTEMATIC)
	62.5	0.005	0.000

INPUT:	PU/BATCH (KG)	BATCHES PER DAY	RELATIVE STANDARD (RANDOM)	DEVIATION (SYSTEMATIC)
	16.73	3.	0.010	0.010

PRODUCT:	PU/BATCH (KG)	BATCHES PER DAY	RELATIVE STANDARD (RANDOM)	DEVIATION (SYSTEMATIC)
	25.00	2.	0.002	0.002

WASTE:	PU/BATCH (KG)	BATCHES PER DAY	RELATIVE STANDARD (RANDOM)	DEVIATION (SYSTEMATIC)
	0.200	1.	0.250	0.250

The model facility is the basis for calculating the dispersion matrix Σ. This is done by PROSA. PROSA performs a standardization of Σ in which all the diagonal elements are 1. In our case we get

$$\Sigma = \begin{pmatrix} 1.0000 & -0.4975 & 0.0014 & 0.0014 & 0.0014 \\ -0.4975 & 1.0000 & -0.4975 & 0.0014 & 0.0014 \\ 0.0014 & -0.4975 & 1.0000 & 0.4975 & 0.0014 \\ 0.0014 & 0.0014 & -0.4975 & 1.0000 & 0.4975 \\ 0.0014 & 0.0014 & 0.0014 & 0.4975 & 1.0000 \end{pmatrix}$$

In the next step we select the false alarm probability $\alpha = 0.05$. Based on Σ and α, PROSA calculates with Monte Carlo Simulations the thresholds for the three test procedures which are used to decide if the given MUF sequence indicates a loss of Pu or not. Now we are ready to perfom the tests with MUF_1, MUF_2, MUF_3, MUF_4, MUF_5.

The results are shown in Figures 5.1 - 5.3. It can be seen that, in this case, there is no alarm, i.e. no indication of a loss.

FIGURE 5.1: EXAMPLE OF A TRUNCATED SEQUENTIAL CUMUF TEST

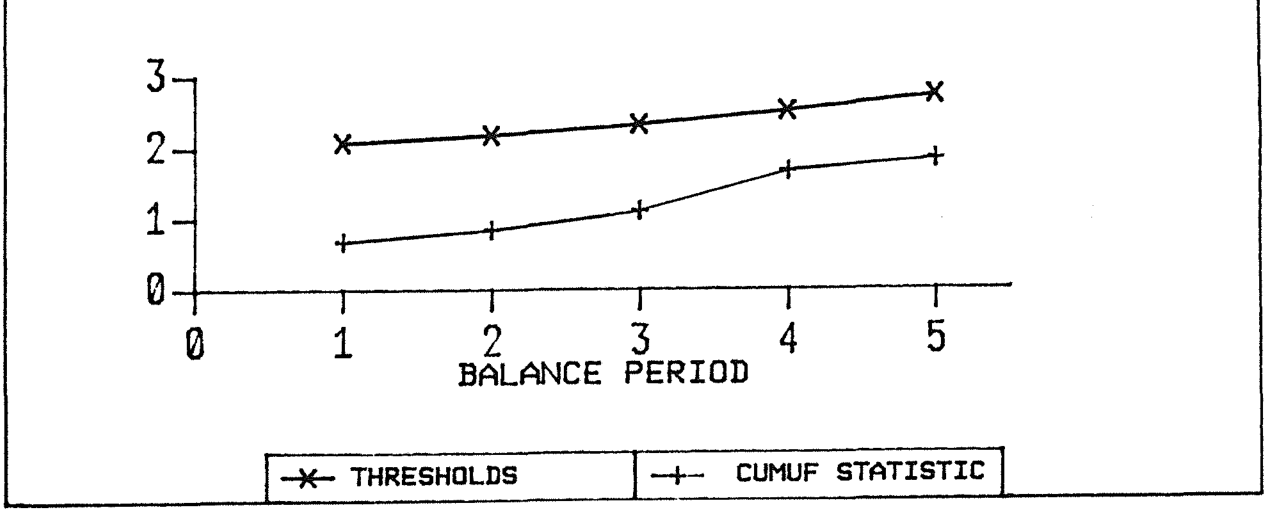


FIGURE 5.2: EXAMPLE OF A POWER ONE TEST

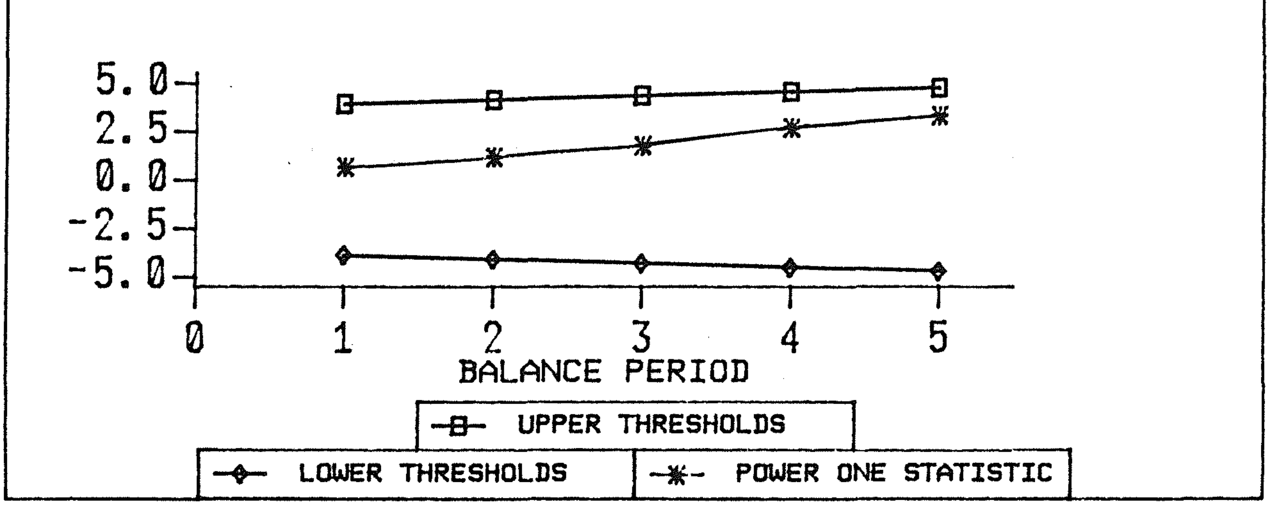
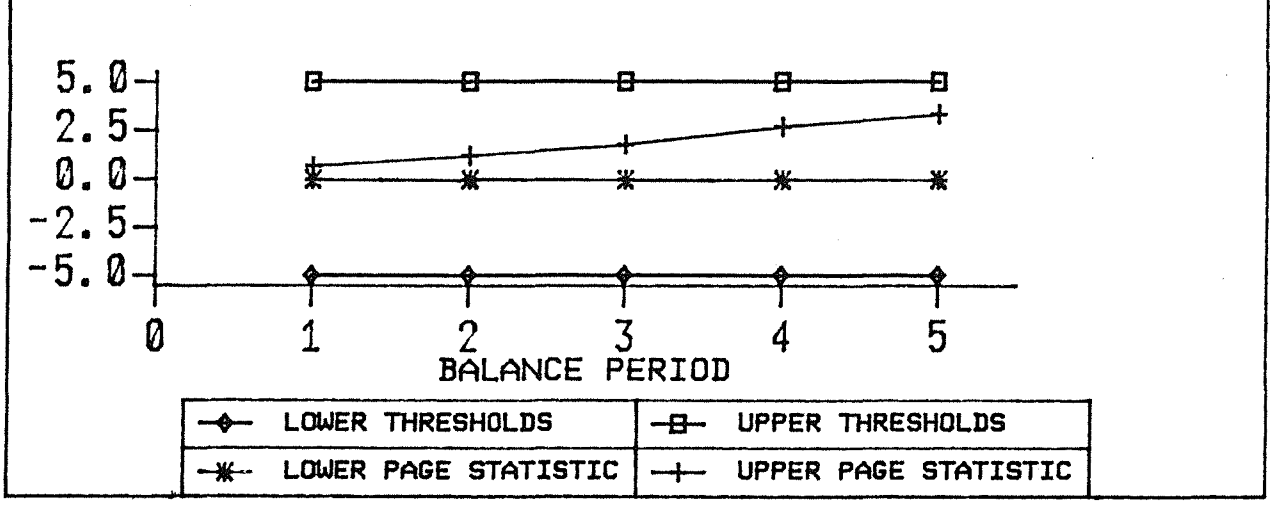


FIGURE 5.3: EXAMPLE OF A PAGE TEST



6. SOME REMARKS ON THE PRECISION OF MONTE CARLO SIMULATIONS

The application of Monte Carlo simulations is an essential part of PROSA. The calculation of the thresholds for the three test procedures is based on the desired false alarm probability α, the measurement model and the Monte Carlo experiment. Simulations are used because, for more than three balance periods it is impossible, e.g. for the CUMUF-test, to calculate analytically the n-dimensional integral /BEE83 b/.

The Monte Carlo simulation for determination of a sequence of thresholds to yield a desired α is based on the following theoretical consideration. We define a random variable Y_j for each simulation run j = 1,2,.....,MCR (MCR = number of Monte Carlo runs):

$$Y_j = \begin{cases} 0, & \text{if alarm} \\ 1, & \text{if no alarm.} \end{cases}$$

with mean

$$E(Y_j) = \alpha$$

for the theoretical but unknown false alarm probability α and in the case of no loss. The random variables Y_j are independet identically distributed Bernoulli variables. The "empirical false alarm probability" α_e for MCR runs is defined by

$$\alpha_e = \frac{1}{MCR} \sum_{j=1}^{MCR} Y_j \qquad (6.1)$$

and is binomially distributed.
According to the strong law of large numbers, we have

$$P(\lim_{MCR \to \infty} \alpha_e = \alpha) = 1, \qquad (6.2)$$

which means that α_e is a reasonable estimate for α. Using the central limit theorem, we see that for MCR $\geq$ 30, α_e is distributed normally with $E(\alpha_e) = \alpha$ and $var(\alpha_e) = \alpha\,(1-\alpha)/MCR$.

Eq. (6.2) does not say how close α_e to the real α is. But the Chebyschev inequality provides an estimate for the precision of the Monte Carlo estimation:

$$P(|\alpha_e - \alpha| \geq \varepsilon) \leq \frac{\alpha(1-\alpha)}{MCR\varepsilon^2} \leq \frac{1}{4 \cdot MCR \cdot \varepsilon^2} \tag{6.3}$$

for $\varepsilon > 0$. Inequality (6.3) can be changed to

$$P(\alpha_e - \varepsilon \leq \alpha \leq \alpha_e + \varepsilon) \geq 1 - \frac{1}{4MCR\varepsilon^2}. \tag{6.4}$$

Eq. (6.4) means that for MCR = 1000 we have a probability greater or equal to 0.9 that α differs from α_e to about 0.05. The inequality (6.3) may be improved if we have more information as to the true value of α.

7. ESSENTIALS OF PROSA 1.0

- PROSA is a computer program for a sequential statistical evaluation of a sequence of materials balance data to detect a possible loss of material; PROSA 1.0 is the first release of this software.

- PROSA is written in the language FORTRAN 77

- PROSA runs and is tested on

  a) IBM with operating system
  b) IBM PC/AT with DOS 3.1
  c) VAX 11/750 with operating system VAX/VMS 4.1
  d) Tektronix 4170 with CP/M

- PROSA allows two kinds of input for a facility measurement model

- PROSA allows as input the desired false alarm probability α

- PROSA consists of several modules. These modules have to be executed in specially defined steps.

- PROSA provides error messages if the data input is inconsistent

- A maximum of 30 materials balances can be handled by PROSA.

- For the PC Version of PROSA a configuration with 512K and a hard disk is strongly recommended.

8. PROSA FOR MODEL FACILITY DATA

The concept of model facilities is widely used in the case where the efficiency of NRTA tests has to be studied for plants in a planning stage.

The model facility consists of two parts:

I. information about plant inventory, input and output.

II. estimates of measurement uncertainties of inventory and transfers and for the correlation of the measurements.

Based on the model facility data, a sequence of MUF values can be simulated and the measurement model (dispersion matrix) calculated.
Furthermore, the detectability of selected loss patterns can be studied and sensitivity analyses for different α's can be performed.

8.1. Files of PROSA 1.0 for Model Facility Data

In this manual we concentrate our description of PROSA on a version for the IBM-PC/AT with DOS 3.1. We will explain step by step what program has to be executed and what are the results. All the data for the examples are given in kg.

PROSA 1.0 consists of the following executable (EXE) files:

ALPHA. EXE
MODEL. EXE
BMATRIX. EXE
THRESC. EXE
THRESR. EXE
THRESP. EXE
MUFIN. EXE
STATIST. EXE
LINEPR. EXE

Furthermore, a file MODEL.DAT is needed, in which the information about the model facility is given. This file is structured as in Tab. 5.1.

8.2 Example of a PROSA Run for Model Facility Data

a) ALPHA.EXE:

This file is the first to be executed. It asks for the number of materials balances which have to be considered and the desired false alarm probability alpha.

Example:

> ALPHA < Enter > [1)]

The question

"NUMBER OF MATERIAL BALANCES?"

on the screen has to be answered by a number between 1 and 30. < Enter >

The next prompt

"ENTER ALPHA"

has to be answered with a desired false alarm probability between 0. and 1. with a decimal number. < Enter >

ALPHA.EXE writes these parameters on the file ALPHA.DAT. Fig. 8.1 gives an illustration for the execution of ALPHA. EXE.

Figure 8.1:

```
NUMBER OF MATERIAL BALANCES ? 5

ENTER ALPHA Ø.Ø5

NUMBER OF MATERIAL BALANCES AND
ALPHA ARE WRITTEN ON DATA(ALPHA)
Execution terminated :  Ø
```

1) <Enter> means to hit the "enter" key on your computer.

b) MODEL.EXE:

To execute MODEL.EXE the file MODEL.DAT with the model plant information has to be available. This file must have a special structure and it is provided with the other PROSA-files. To study different types of facilities the file MODEL.DAT has to be edited and changed accordingly. MODEL.EXE calculates the standardized dispersion matrix of the model facility, the standardized variances of CUMUF and the variances of one material balance period.

Example:

> MODEL < Enter >

Now the facility model is displayed on the screen and the results of the calculations are written on the files:

COVAR.DAT
VSCMF.DAT
VMUF.DAT

Fig. 8.2 gives an example of the screen display.

Figure 8.2:

PROSA VERSION 1.0
PROGRAM FOR STATISTICAL ANALYSIS OF NRTA
COMPUTATION OF THE COVARIANCE-MATRIX

PARAMETERINPUT FOR A BLOCK-MODEL OF A REPROCESSING FACILITY

THROUGHPUT (KG)	:	250.00
NUMBER OF WORKING DAYS PER YEAR	:	5
BALANCE INTERVAL IN DAYS	:	1
INTERVAL OF RECALIBRATIONS	:	5

	PU-INVENTORY (KG)	RELATIVE STANDARD DEVIATION (RANDOM)	(SYSTEMATIC)
HEAD-END	196.500	0.010	0.000
1.PU-CYCLE	7.600	0.010	0.000
2.PU-CYCLE	50.000	0.005	0.000
3.PU-CYCLE	134.000	0.005	0.000
PU-CONCENTRATION	62.500	0.005	0.000

	PU/BATCH (KG)	BATCHES PER DAY	RELATIVE STANDARD DEVIATION (RANDOM)	(SYSTEMATIC)
INPUT	16.730	3.0000	0.010	0.010
PRODUCT	25.000	2.0000	0.002	0.002
WASTE	0.200	1.0000	0.250	0.250

TOTAL INVENTORY(KG) = 450.600
TOTAL INPUT(KG) = 250.950
TOTAL PRODUCT(KG) = 250.000
TOTAL WASTE(KG) = 1.000

INVENTORY: VARIANCE OF RANDOM ERROR (KG**2) = 4.476057
VARIANCE OF SYSTEMATIC ERROR (") = 0.000000
TRANSFER : VARIANCE OF RANDOM ERROR (") = 0.091468
(PER DAY) VARIANCE OF SYSTEMATIC ERROR (") = 0.264404
VARIANCE OF MUF (") = 9.307985

STANDARDIZED DISPERSION MATRIX

ROW(1): 1.000 -0.452 0.028 0.028 0.028

ROW(2): -0.452 1.000 -0.452 0.028 0.028

ROW(3): 0.028 -0.452 1.000 -0.452 0.028

ROW(4): 0.028 0.028 -0.452 1.000 -0.452

ROW(5): 0.028 0.028 0.028 -0.452 1.000

c) BMATRIX. EXE

The execution of this program provides the transformation matrix T (see Eq.(3.2)) for independence transformation. Furthermore, the inverse matrix T^{-1} of the transformation matrix is calculated. These matrices are necessary for establishing the test statistics.

Example:

> BMATRIX < Enter >

The results are displayed on the screen and written into the files BMATRIX.DAT and CUMUF.DAT. An example of the screen display is shown in Fig. 8.3.

Figure 8.3:

PROSA VERSION 1.0
PROGRAM FOR STATISTICAL ANALYSIS OF NRTA
COMPUTATION OF TRANSFORMATION-MATRIX AND INVERSE

VARIANCE-COVARIANZ-MATRIX

1.0000	-0.4525	0.0284	0.0284	0.0284
-0.4525	1.0000	-0.4525	0.0284	0.0284
0.0284	-0.4525	1.0000	-0.4525	0.0284
0.0284	0.0284	-0.4525	1.0000	-0.4525
0.0284	0.0284	0.0284	-0.4525	1.0000

N: 5

TRANSFORMATION-MATRIX

1	1.000									
2	0.507	1.121								
3	0.255	0.636	1.150							
4	0.086	0.303	0.657	1.153						
5	-0.028	0.070	0.296	0.655	1.154					
6	0.000	0.000	0.000	0.000	0.000	0.000				
7	0.000	0.000	0.000	0.000	0.000	0.000	0.000			
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000		
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

d) THRESC. EXE

This program calculates the standardized thresholds for the CUMUF test based on the desired false alarm probability, measurement model and the selected number of Monte Carlo runs.

Example:

>THRESC < Enter >

Now the program asks for the desired number of Monte Carlo runs greater or equal to 100. It is suggested not to go beyond 1000 runs.

> 1000 < Enter >

The results are written on T CUMUF. DAT. In Fig. 8.4 the display on the screen is shown.

Figure 8.4:

```
PROSA  VERSION 1.Ø
PROGRAM FOR STATISTICAL ANALYSIS OF NRTA
TRUNCATED SEQUENTIAL CUMUF-TEST



INPUT OF MONTE-CARLO RUNS
( DEFAULT: MC     1ØØ ) 15Ø

ALPHA:    Ø.Ø5ØØØØØ
NUMBER OF PERIODS:  5
MC   :      15Ø
BEGIN OF ITERATION
 2 ITERATION: SALPHA =     Ø.14ØØØØØ
 3 ITERATION: SALPHA =     Ø.Ø533333

TRUNCATED SEQUENTIAL CUMUF-TEST
SALPHA  :   Ø.Ø533333
BALANCE PERIOD    THRESHOLD    CUM.FALSE ALARM PROBABILITY
     1             2.Ø724268        2
     2             2.1686788        3
     3             2.31417Ø8        3
     4             2.5ØØ3223        6
     5             2.7187946        8
THRESHOLD OF CUMUF IS WRITTEN ON TCUMUF.DAT
Execution terminated :  Ø
```

e) THRESR. EXE

The execution of this file calcutes analoguous to THRESC. EXE the standardized thresholds for the CUSUM test with the Power-One thresholds and writes the results on the file TROBIN. DAT. Fig. 8.5 gives an example of the display.

Figure 8.5:

```
PROSA  VERSION 1.0
PROGRAM FOR STATISTICAL ANALYSIS OF NRTA
CUSUM-TEST WITH POWER ONE THRESHOLD


INPUT OF MONTE-CARLO RUNS
NUMBER OF MONTE-CARLO RUNS MUST
BE GREATER OR EQUAL 100 ! 150

ALPHA:    0.0500000
NUMBER OF BALANCES:  5
MC   :      150
BEGIN OF FIRST ITERATION STEP
 1 ITERATION: SALPHA =     0.6466666
 2 ITERATION: SALPHA =     0.2133333
 3 ITERATION: SALPHA =     0.1066667
 4 ITERATION: SALPHA =     0.0333333
BEGIN OF SECOND ITERATION STEP
 2 ITERATION: SALPHA =     0.0733333
 3 ITERATION: SALPHA =     0.0733333
 4 ITERATION: SALPHA =     0.0333333
 5 ITERATION: SALPHA =     0.0466667


CUSUM-TEST WITH POWER 1 THRESHOLD
SALPHA              :      0.047
BALANCE PERIOD   LOWER THRESHOLD   UPPER THRESHOLD
      1          -3.9128909           3.9128909
      2          -4.1307673           4.1307673
      3          -4.3429518           4.3429518
      4          -4.5500236           4.5500236
      5          -4.7524600           4.7524600
BALANCE PERIOD   CUM. FALSE ALARM PROBABILITY
      1             0
      2             0
      3             3
      4             6
      5             7
THRESHOLD OF POWER ONE TEST IS WRITTEN ON TROBIN.DAT
Execution terminated :  0
```

f) THRESP. EXE

The execution of this file leads according to e) and f) to the standardized thresholds for the CUSUM test with the thresholds based on an idea of page the results are written on the file T PAGE.DAT. Fig. 8.6 gives an example of the display.

Figure 8.6:

```
PROSA  VERSION 1.0
PROGRAM FOR STATISTICAL ANALYSIS OF NRTA
CUSUM-TEST ACCORDING TO PAGE


INPUT OF MONTE-CARLO RUNS
NUMBER OF MONTE-CARLO RUNS MUST
BE GREATER OR EQUAL 100 ! 150

ALPHA:    0.0500000
NUMBER OF BALANCES:  5




MC    :      150
BEGIN OF FIRST ITERATION STEP
 1 ITERATION: SALPHA =    1.0000000
 2 ITERATION: SALPHA =    0.9666666
 3 ITERATION: SALPHA =    0.7200000
 4 ITERATION: SALPHA =    0.2400000
 5 ITERATION: SALPHA =    0.1066667
 6 ITERATION: SALPHA =    0.0533333


CUSUM-TEST ACCORDING TO PAGE
SALPHA             :     0.0533333
LOWER THRESHOLD  :    -5.0000000
UPPER THRESHOLD  :     5.0000000
BALANCE PERIOD  CUM.FALSE ALARM PROBABILITY
      1                    0
      2                    0
      3                    0
      4                    2
      5                    8
THRESHOLD OF PAGE TEST IS WRITTEN ON TPAGE.DAT
Execution terminated :  0
```

g) MUFIN. EXE

This program establishes the MUF sequence which is to be evaluated by the three test procedures. The result is written on the file MUF. DAT.

Example

> MUFIN < Enter >

The program asks for the number of materials balances considered.

"NUMBER OF MATERIAL BALANCES?:"

It has the to be answered by the corresponding figure and < Enter >.

The next question:

"NUMBER OF MUF-PERIODS?:"

has to be answered by the corresponding figure and < Enter. >.

The last question

"BEGINNING OF MUF-PERIODS?:"

has to be answered with number of the balance period of the first MUF-value and < Enter. >. In the next steps the program asks for the MUF's. After the last MUF-value the program displays all MUF-values to check if no errors have been made. In Fig. 8.7 the execution of the code is displayed.

Figure 8.7:

```
NUMBER OF MATERIAL BALANCES ?: 5

NUMBER OF MUF-PERIODS ?; 5

BEGINNING OF MUF-PERIODS ?: 1

MUF-VALUE OF THE PERIOD :  1? 2.1

MUF-VALUE OF THE PERIOD :  2? Ø.5

MUF-VALUE OF THE PERIOD :  3? Ø.8

MUF-VALUE OF THE PERIOD :  4? 1.7

MUF-VALUE OF THE PERIOD :  5? Ø.5

THIS IS YOUR MUF-VECTOR ?:
    1           2.1ØØ
    2           Ø.5ØØ
    3           Ø.8ØØ
    4           1.7ØØ
    5           Ø.5ØØ
ENTER YES /NO  y

MUF-VECTOR IS WRITTEN ON MUF.DAT
Execution terminated :  Ø
```

h) STATIST. EXE

The execution of this file applies the statistical tests for the given MUF-vector based on the model facility measurement model.

h) STATIST. EXE

The execution of this file applies the statistical tests for the given MUF-vector based on the model facility measurement model.

It asks for the file where the MUF/Loss vector, which has to be evaluated, is located.

Example

> STATIST < Enter >

The program asks for the input data set where the MUF vector is located. After the program ask for a name if a destruction data set to fix the results of the statistical analysis. In Fig. 8.8 an illustration is given.

Figure 8.8:

```
PROSA  VERSION 1.0
PROGRAM FOR STATISTICAL ANALYSIS OF NRTA
EVALUATION OF STATISTICS
ENTER NAME OF INPUT DATA SET :MUF/LOSS:MUF.DAT

            THRESHOLDS                   MUF
BALANCE     PAGE    ROBBIN   CUMUF      VEKTOR
PERIODS       5        5        5          5
    1       5.000    3.913    2.072    2.100
    2       5.000    4.131    2.169    0.500
    3       5.000    4.343    2.314    0.800
    4       5.000    4.550    2.500    1.700
    5       5.000    4.752    2.719    0.500


CUMUF-TEST
 STATISTICS DO NOT CROSS THE THRESHOLDS


POWER1-TEST
STATISTICS DO NOT CROSS THE THRESHOLDS


PAGE-TEST
STATISTICS DO NOT CROSS THE THRESHOLDS
ENTER DESTINATION DATA SET NAME:PROSA.DAT

STATISTICS ARE WRITTEN ON : PROSA.DAT
Execution terminated :  0
```

i) LINEPR. EXE

The execution of this program displays the results of the statistical analysis performed by STATIST.EXE and written e.g. on the file PROSA DAT. For each of the three tests a separate display is provided.

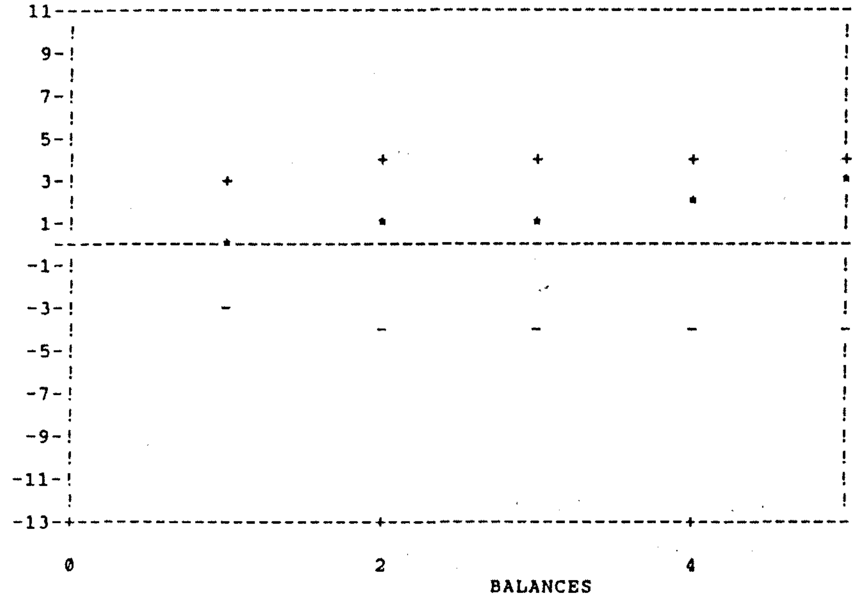
Example

> LINEPR <ENTER>

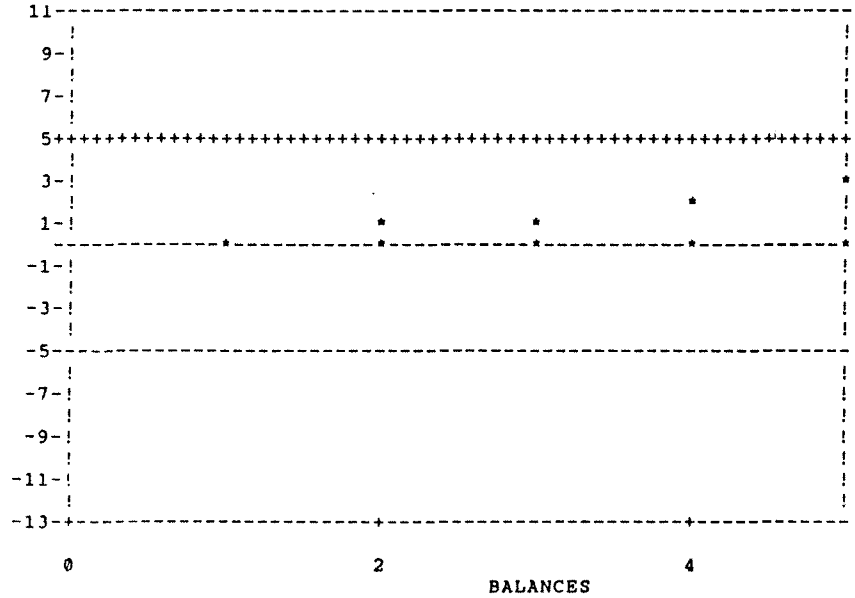
The program asks for the input data set on which the results of STATIST . EXE are written. The program stops after the display of each test. By hitting the "Return" key the execution can be continued. In Fig. 8.9 an example is illustrated.

Figure 8.9:

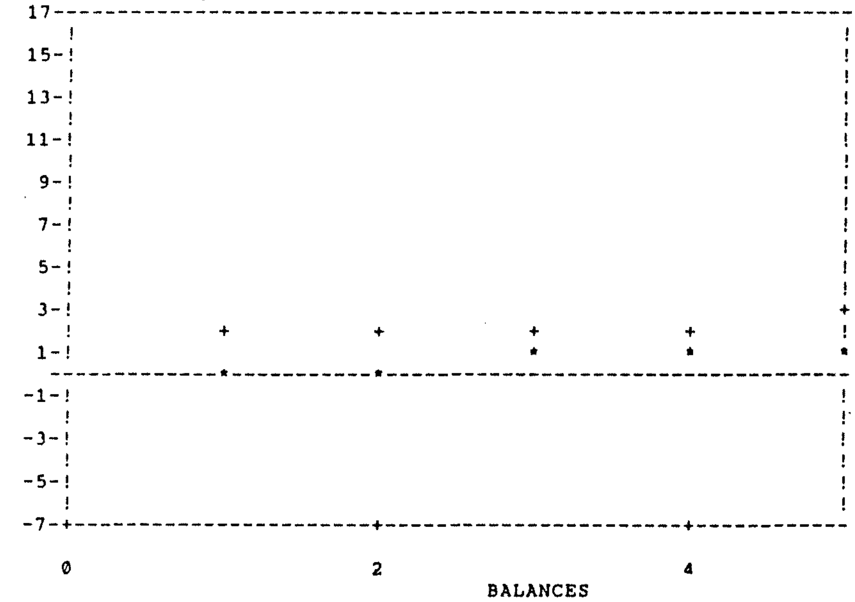
ENTER FILENAME
** CUSUM-TEST WITH POWER 1 TEST **



HIT RETURN KEY TO CONTINUE....
** CUSUM-TEST ACCORDING TO PAGE **



HIT RETURN KEY TO CONTINUE....
** TRUNCATED SEQUENTIAL CUMUF-TEST



HIT RETURN KEY TO CONTINUE....

9. PROSA FOR DATA OF A FACILITY WITH MEASUREMENT MODEL GIVEN BY A DISPERSION MATRIX

If data of real existing plants are to be evaluated by PROSA it is usually not possible to describe the measurement model of the considered facility by a steady state model. Therefore, PROSA allows as input any kind of measurement model represented by the dispersion matrix of the materials balance sequence in question. This allows a high flexibility in the application of PROSA.
The dispersion matrix consists of the variance and covariance terms of the MUF sequence under consideration and has to be calculated in advance.

9.1 Files of PROSA 1.0 for Dispersion Matrix Measurement Model

For an IBM-PC/AT or compatible with DOS 3.1 PROSA 1.0 consists of the following executable files:

ALPHA.EXE
COVIN.EXE
BMATRIX.EXE
THRESC.EXE
THRESR.EXE
THRESP.EXE
MUFIN.EXE
STATIST.EXE
LINEPR.EXE

It is obvious that COVIN.EXE replaces here the file MODEL.EXE in the version for model facility data.

9.2 Example of a PROSA Run with Dispersion Matrix

a) ALPHA.EXE:
This file is the first to be executed and is the same procedure as in 8.2.

b) COVIN.EXE:
The execution of this file allows an interactive input for the dispersion matrix. As the variance covariance matrix is symmetric, we restrict the input to the lower triangular. The program creates the complete dispersion matrix.

Example:

>COVIN <Enter>

Now the program asks for the number of considered balance periods. The prompt "ENTER NUMBER OF PERIODS:"
has to be answered with the corresponding number between 1 and 30. Now the input of the dispersion matrix elements has to be performed row by row for the lower triangular.
At the end of the last row the program asks if the input is correct. In the case "NO" the input has to be repeated. The program asks for the number of the line and the column of the element to be corrected. In the case "YES" the results are written on the files.

COVAR.DAT
VSCMF.DAT
VMUF.DAT.

Fig. 9.1 shows an example for the display on the screen. The files BMATRIX.EXE - DRUCK.EXE have to be executed as in to part 8.2.

Figure 9.1:

```
ENTER NUMBER OF PERIODS:5

INPUT OF COVARIANCE-MATRIX
ENTER  1 ELEMENT(S)
  0.000 1

ENTER  2 ELEMENT(S)
  1.000
  0.000   0.000 -0.345 1

ENTER  3 ELEMENT(S)
  1.000
 -0.345   1.000
  0.000   0.000   0.0000.028 -0.452 1

ENTER  4 ELEMENT(S)
  1.000
 -0.345   1.000
  0.028  -0.452   1.000
  0.000   0.000   0.000   0.000 0.028 0.028 -0.452 1

ENTER  5 ELEMENT(S)
  1.000
 -0.345   1.000
  0.028  -0.452   1.000
  0.028   0.028  -0.452   1.000
  0.000   0.000   0.000   0.000   0.000 0.028 0.028 0.028 -0.452 1

   1.000
  -0.345   1.000
   0.028  -0.452   1.000
   0.028   0.028  -0.452   1.000
   0.028   0.028   0.028  -0.452   1.000
 YOUR COVARIANCE MATRIX ? :YES OR NO:NO

 ENTER LINE AND COLUMN FOR THE ELEMENT :2 1

 REENTER THE ELEMENT  2  1 :-0.452

   1.000
  -0.452   1.000
   0.028  -0.452   1.000
   0.028   0.028  -0.452   1.000
   0.028   0.028   0.028  -0.452   1.000
 YOUR COVARIANCE MATRIX ? :YES OR NO:YES

 Execution terminated :  0
```

10. CONCLUDING REMARKS

All the information given in this manual describes the first release 1.0 of the computer program PROSA. However PROSA is not considered to be a static product. New sequential test procedures, better, i.e. faster, numerical algorithms are planned to be included in PROSA. Furthermore, it is intended to include in PROSA the calculation of detection probabilities for special loss patterns in a series of balance periods.

We would greatly appreciate both commets and criticisms on PROSA from its users.

11. REFERENCES

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