



KfK 4447
August 1989

Uncertainty Analyses for the Atmospheric Dispersion Submodule of UFOMOD with Emphasis on Parameter Correlations

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KfK 4447

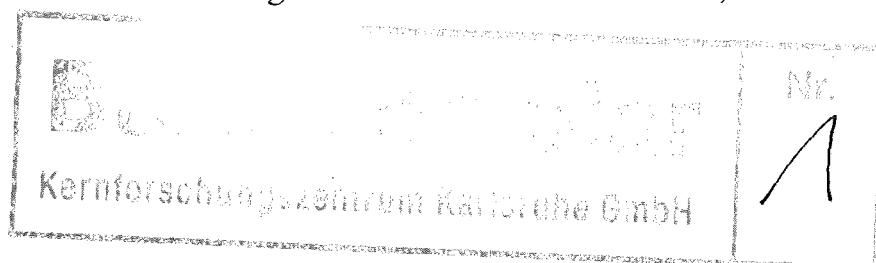
**UNCERTAINTY ANALYSES FOR THE
ATMOSPHERIC DISPERSION SUBMODULE
OF UFOMOD WITH EMPHASIS
ON PARAMETER CORRELATIONS**

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Work has been performed with support of the
Commission of the European Communities
Radiation Protection Programme
Contract No. BI6/F/128/D
and with support of the
International Bureau of Kernforschungsanlage Jülich GmbH
within the bilateral German - Yugoslav cooperation
in scientific research and development

Kernforschungszentrum Karlsruhe GmbH, Karlsruhe



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Kernforschungszentrum Karlsruhe GmbH
Postfach 3640, 7500 Karlsruhe 1

ISSN 0303-4003

This report refers to uncertainty analyses of the atmospheric dispersion and deposition submodule (trajectory model) of the new program system UFOMOD, version NE 87 /1, whose most important input parameters are linked with probability distributions derived from expert judgement.

Special emphasis is put in this report on some helpful theoretical investigations concerning:

- different types of correlation between uncertain model parameters,
- test procedures to find out the most sensitive uncertain model parameters,
- the percentage contribution of parameter variations to changes in consequence values.

For the mentioned submodule of the new program system UFOMOD the most important parameters are stable in their rankings regardless of the underlying distributions (*predefined by experts* or all uniformly distributed, respectively). For the less important parameters the values of the sensitivity measures (PRCC values) vary for *predefined* or *uniform* distributions, respectively. But even then in most cases the ranking is the same.

Increasing sample sizes ($n = 80,100$) lead to more precision in sensitivity calculations, i.e. lead to a *decrease* of the critical 'white noise level' (garbage level) of absolute PRCC values.

R²-values (coefficients of determination) in conjunction with the corresponding PRCC values visualize the percentage contribution of each uncertain model parameter (or groups of uncertain parameters) to uncertainty in consequences. This is important, because a large absolute PRCC value is not in every case an indication for a considerable amount of responsibility for uncertainty in consequences.

The restriction to pure model parameters and revised parameter variations in the atmospheric dispersion and deposition submodule of the new UFOMOD code system lead to significantly smaller uncertainty bands than for the old UFOMOD / B3 code. Variations in deposition velocities and the mixing height parameters play the most important role for uncertainties in the activity concentration values.

Unsicherheitsanalysen für den Teilmodul 'Atmosphärische Ausbreitung' von UFOMOD unter besonderer Berücksichtigung von Parameterkorrelationen

Diese Untersuchung bezieht sich auf den atmosphärischen Ausbreitungsteilmodul (Trajektorienmodell) des neuen Programmsystems UFOMOD, Version NE 87 / 1. Den wichtigsten Parametern liegen Wahrscheinlichkeitsverteilungen zugrunde, die in Zusammenarbeit mit Experten erstellt wurden.

Besondere Beachtung finden in diesem Bericht einige nützliche theoretische Untersuchungen zu

- verschiedenen Arten von Korrelationen zwischen unsicheren Modellparametern,
- statistischen Testprozeduren, um die sensitivsten unsicheren Modellparameter zu identifizieren,
- den prozentualen Beiträgen von Parameterschwankungen zur Gesamtvariation der Ergebnisse.

Für den erwähnten Teilmodul des neuen Programmsystems UFOMOD bleiben die sensitivsten Modellparameter stabil in ihrer Rangreihenfolge, gleichgültig ob für ihre Schwankungen die *von Experten vorgegebenen* oder *rechteckige* Verteilungen verwendet werden. Für die weniger wichtigen Parameter schwanken die Werte der Sensitivitätsmaße (PRCC Werte), je nach der zugrunde liegenden Verteilungssituation (*vordefiniert* bzw. *insgesamt rechteckverteilt*). Aber selbst dann bleibt in den meisten Fällen die Rangreihenfolge unverändert.

Zunehmende Stichprobenumfänge ($n=80,100$) führen zu größerer Genauigkeit der Sensitivitätsberechnungen, d.h. sie bedingen eine *Abnahme* des kritischen 'white noise levels' von absoluten PRCC Werten, unterhalb dessen alle Werte rein zufällig sind.

R^2 -Werte (Bestimmtheitsmaße) in Verbindung mit den entsprechenden PRCC Werten verdeutlichen den prozentualen Beitrag jedes einzelnen unsicheren Modellparameters (oder von einer Gruppe unsicherer Parameter) zur Gesamtunsicherheit in den Ergebnisvariablen. Das ist wichtig, da große absolute PRCC Werte nicht in jedem Fall eine Hauptverantwortlichkeit für die Ergebnisunsicherheiten beweisen.

Die Beschränkung auf reine Modellparameter und überarbeitete Wertebereiche führen im atmosphärischen Ausbreitungsteilmodul des neuen Programmsystems UFOMOD, Version NE 87 / 1 zu deutlich schmalere Konfidenzbändern als in der entsprechenden Untersuchung für den alten UFOMOD / B3 Code. Schwankungen in den Ablagerungsparametern und in der Mischungsschichthöhe sind hauptsächlich verantwortlich für die Variationen in den Aktivitätskonzentrationen.

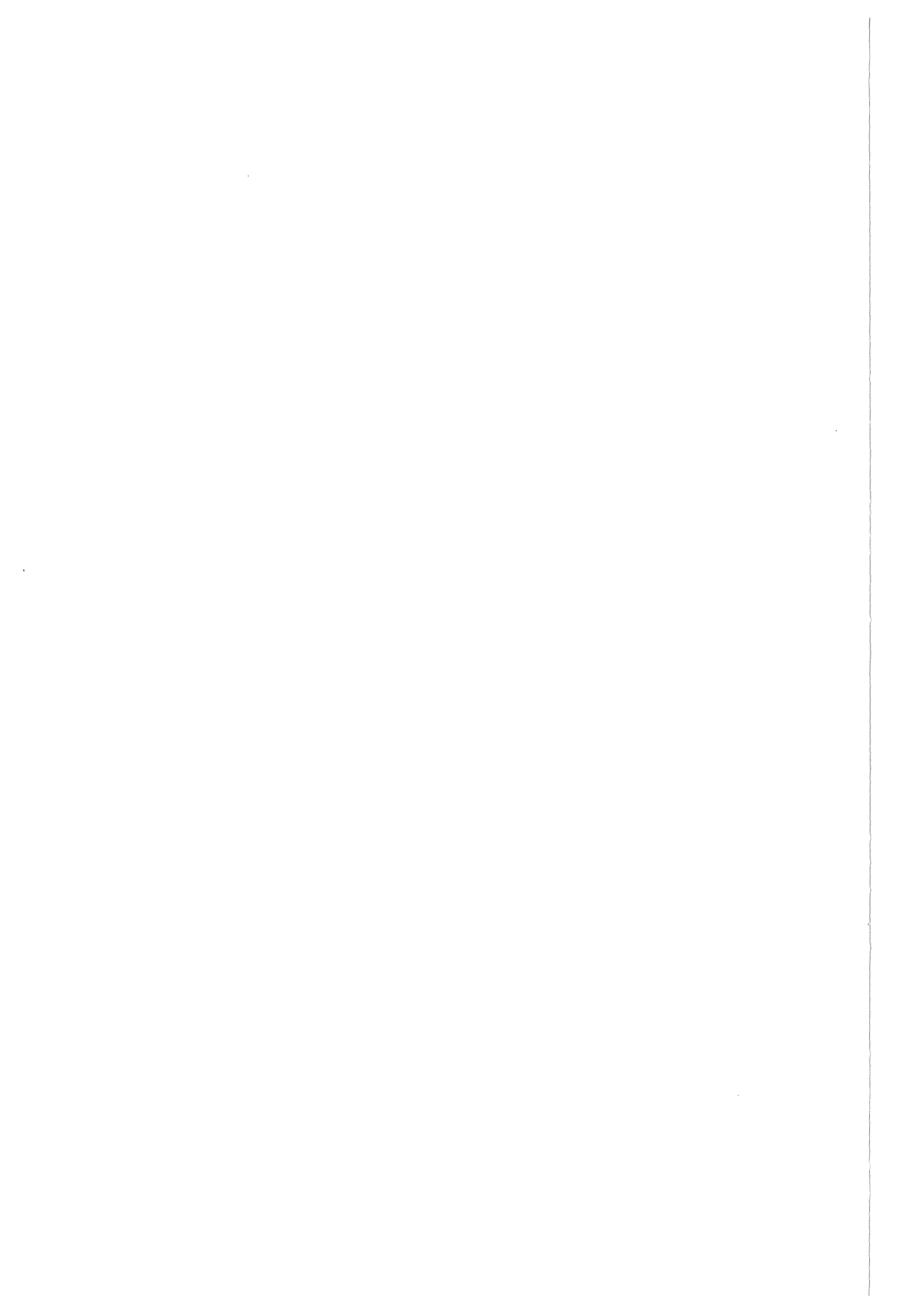
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1. Introduction

Risk studies for installations of the nuclear fuel cycle have been carried out in the USA (e.g. WASH-1400, ZION, LIMERICK, INDIAN POINT), in the United Kingdom (e.g. SIZEWELL, HINKLEY POINT)) and in West Germany (e.g. GERMAN RISK STUDY, RISK ORIENTED ANALYSIS OF THE SNR 300) to quantify and compare accident consequences and their frequencies.

In applying accident consequence analyses (ACA) codes to specific sites, it is of considerable importance to understand the nature and magnitude of uncertainties that are associated with the various models and parameters that are used in the code and the effects of these uncertainties on predicted consequences. This is not only a prerequisite for safety goal comparative studies but also facilitates the identification of modelling weakpoints and thus areas for further improvements and supporting research and development activities.

Although the main goal of uncertainty analysis is the quantification of the uncertainties in the assessed consequences, it fades into sensitivity analysis whenever the effect of each single parameter (or a group of parameters) on the total uncertainty is being considered.

Carefully designed procedures are to be used to determine the impact of uncertain parameters in individual submodels on the predictions of accident consequence assessments. Some general aspects of the role and importance of uncertainty and sensitivity analyses are described in a previous paper [18].

The accuracy of the description of uncertainties of the model parameters depends on the information available, finally condensed in a probability distribution for each parameter. The construction of these distributions may be based on expert opinion procedures and/or on experimental data.

And additionally, following [1], the estimated distribution of the consequence variables can only be meaningful in a probabilistic sense if the model parameters have meaningful probability distributions associated with them. For the determination of those parameters that contribute significantly to sensitivity, the form of the distribution is not as important as the representation of each parameter over its physically possible range and the possible inter-correlations between parameters.

Following [35], the examination of the uncertainty in large accident consequence assessment models is a very complex undertaking and is reasonably performed in a sequential manner. The analysis should first involve the individual components of the system, and then, at a later stage the model should be examined in its entirety. In the first stage, much effort is directed at understanding and simplifying the individual components in the model. In the

second stage, effort is directed at pulling this understanding together for use in an integrated analysis.

The program system UFOMOD [15] has been completely restructured and remodeled; it replaces the the accident consequence assessment program UFOMOD for the German Risk Study, Phase A [5].

The first module of the program system UFOMOD models the dispersion of radioactive material in the atmosphere and the processes of removal of material from the atmosphere leading to deposition on ground. Thereby the meteorological conditions prevailing during the release and time of travel of the radioactive plume are taken into account, which determine the dispersion and the direction in which the activity is transported. Removal mechanisms by dry and wet deposition are included. Dependent on the release characteristics plume rise due to the buoyancy of the released activity and the influence of the turbulent building wake are modelled.

Various atmospheric dispersion models ranging from straight line Gaussian to puff - trajectory models are available for use in UFOMOD. The investigations described in this report refer to the segmented plume model MUSEMET [51], which calculates Gaussian type concentration distributions along trajectories.

Before starting the uncertainty and sensitivity analyses, a detailed discussion of the parameter variations in the atmospheric dispersion model took place together with the experts of the National Radiological Protection Board (NRPB), UK. It led to a common list of parameters to be considered and to a revision of their distribution functions and correlations in comparison to uncertainty investigations with UFOMOD / B3 performed before [23]. One of the main agreements was to restrict to pure model parameters and to leave out quantities describing the source term (like thermal energy) or measured values (like wind speed or wind direction). Therefore, the list was reduced to twenty parameters given in Table 2 and Table 3 of Chapter 2.

As source term an unit release (1 Ci) of I-131 and Cs-137 in three hourly subsequent phases is chosen. The release height is assumed to be 10 meters.

The following aspects of accident consequence assessments are investigated: The concentration fields in the air near ground (1 m height) and on ground surface considering the variability of the averaged¹ concentration values at three distances: D1 (.875 km), D2 (4.9 km) and D3 (27 km).

In contrast to the investigations with UFOMOD/B3, where the variations of the concentration and dose values under the plume centerline are representative of all the azimuthal

¹ averaged over 144 weather sequences which represent the weather of the two years 1982/83

results due to the straight-line Gaussian modelling, all concentration values calculated with a trajectory model must be used to construct the CCFDs and their confidence bounds.

Appropriate techniques to propagate parameter uncertainties through accident consequence assessment models like UFOMOD consist of performing stochastic calculations using Monte Carlo simulations. Due to [45], for these simulations a number of vectors are sampled from the distribution functions. The various modules of the ACA codes are run repeatedly for different model parameter vectors. *Random sampling* techniques require a large number of runs to ensure that all combinations of parameter values are considered. *Stratified sampling* techniques, e.g. Latin Hypercube Sampling (LHS), aim at optimizing the sample selection in order to ensure that all relevant parameter values and their combinations are included in the calculations, even for a relatively small number of runs.

Chapter 3.1 briefly describes the IMAN / CONOVER procedure for inducing a special type of correlation between model parameters.

Investigating correlated model parameters one may ask which type of correlation is to be used (e.g. correlation measured on original **raw** data or based only on the order of observations (**rank** data)) and how much information is lost by using the data only to determine relative magnitudes. 'Differences' between raw and rank correlations can be measured and they depend on the sample size, the number of correlated model parameters and the type of underlying distributions. The effect of substitution of a given complex distribution by an appropriate linearized distribution is shortly described in Chapter 3.2. A formal discussion is presented in Appendix A.2.

The estimation of confidence bounds is indicated in Chap 3.3.

The identification of important contributors to variations in consequences is done by the use of a sensitivity measure, the so-called partial (rank) correlation coefficient, PCC or PRCC. Both sensitivity measures, PCC or PRCC, respectively, are measures that quantify the relation between the uncertainty in consequences and those of model parameters. When a nonlinear relationship is involved it is often more revealing to calculate PCCs between parameter *ranks* than between the *actual* values for the parameters. The numerical value of the PRCCs can be used for hypothesis testing to quantify the confidence in the correlation itself, i.e. by statistical reasons one can determine which PRCC values indicate really an importance (significance) of a parameter or which PRCC values are simply due to 'white noise'. This is described in Chapter 3.4 or more explicitly in Appendix A.3. Moreover, as it is pointed out in Appendix A.4, it is possible to calculate the percentage contribution of each uncertain model parameter to uncertainty in consequences by use of so-called *coefficients of determination* (R^2).

The last step in performing uncertainty analyses is to present and interpret the results of the analyses. Chapter 3.5 condenses the information obtained from the uncertainty analysis

for the atmospheric dispersion and deposition submodule of the program system UFOMOD, version NE 87/1 and gives a guideline to understand the detailed figures and tables in the Appendices B and C.

2. General Features of the Submodule

Following [51] any accident consequence assessment (ACA) assuming the release of radionuclides from nuclear installations must be based on predictions of the distribution of the radioactive material throughout the environment. Accidental releases into the atmosphere are the most severe ones in terms of the radiological consequences. Therefore, modelling the atmospheric dispersion and deposition is of essential importance in an ACA. Once the material is released, the effluent particles and gases form a plume which is transported in the downwind direction and which expands horizontally and vertically due to diffusion conducted by turbulent eddies in the atmosphere.

During the dispersion the effluent material may be removed from the plume by several mechanisms. Gravitational settling and contact with the ground, vegetation and structure in urban areas are referred to as dry deposition. Wet deposition may result from precipitation formation processes within the cloud, leading to removal by rainout, or from interaction between falling rain drops and the dispersing material, referred to as washout. Additionally, radioactive decay reduces the activity in the plume. Depending on the release characteristics special features may have influence on the dispersion and deposition, for example the effect of plume rise due to the buoyancy or momentum of the released activity and the behaviour of plumes released into building wakes. Both phenomena affect the concentration distributions and, hence, the consequences arising in the vicinity of a nuclear installation.

To simulate the very complex processes whereby material is dispersed in and deposited from the atmosphere and to calculate the resulting distributions of activity concentrations in the air and on the ground, a large number of models of different physical complexity has been developed. Due to its simplicity the straight-line Gaussian model [52] is the one most commonly used in practical applications of atmospheric dispersion modelling. Also in most of the computer programs developed for ACAs in the last ten years, e.g. CRAC [54], CRAC2 [55], MARC [10] and UFOMOD [5], this model has been implemented to describe the atmospheric dispersion and deposition. The model is derived from a simplified theoretical treatment of dispersion which assumes that the atmospheric flow- and turbulence fields are homogeneous and stationary [62]. Especially, the model does not allow for changes of wind direction during the release and during the subsequent dispersion of the released material through the atmosphere, processes everybody can see in the nature observing a smoke plume coming out off a chimney. Therefore, to increase the applicability and acceptance of ACAs, much effort has been invested during the development of the new ACA program system UFOMOD [15] to substitute the straight-line Gaussian plume model by more realistic atmospheric dispersion models.

Recently, a benchmark study has been carried out at the Institut für Neutronenphysik und Reaktortechnik (INR) of the Kernforschungszentrum Karlsruhe (KfK)

- to quantify the characteristic physical features of various dispersion models [48], comprising the straight-line Gaussian plume model and Gaussian-type trajectory plume and puff models, Eulerian grid-point models, and a Lagrangian random walk model,
- to identify those models which can be applied in ACA codes under the demands of reasonable computer time and availability of meteorological input data [50],
- to quantify the implications of different concepts of dispersion modelling by comparing the results of an ACA after the application of a straight-line Gaussian plume model and improved dispersion models which take into account the changes of wind directions [50].

The study demonstrated that with respect to the demands

- high flexibility,
- user friendliness,
- acceptable computing time
- availability of meteorological input data

only Gaussian-like trajectory models are applicable in ACAs. Although these models use the Gaussian formalism to calculate concentration fields, the ability to consider changes of wind direction during the release and the dispersion led to more realistic consequence assessments compared to the straight-line Gaussian model.

The conclusion from the benchmark was to apply trajectory models in ACAs. In general, the range of validity of these models is limited to the region near to the site, since in most cases the meteorological data are available only from the site or a meteorological station representative for it. A Gaussian dispersion over more than some 10 kilometres based on these data is hard to defend, especially in topographical structured areas, and even over flat land, this type of dispersion has never been proven at longer distances. Therefore, long-range dispersion models are needed to describe the transport of radioactive material over large areas up to thousands of kilometres. This led to the completely novel concept of atmospheric dispersion modelling in the new program system UFOMOD, which distinguishes between near range (≤ 5 km) and far range (≥ 50 km) atmospheric dispersion models [51].

In the near range (≤ 50 km) modified versions of the atmospheric dispersion models MUSEMET [60] and RIMPUFF [46] are used, at present.

The segmented plume model MUSEMET (KFA, Jülich, F.R.G., [60]) and the puff model RIMPUFF (RISO National Laboratory, Denmark, [46]) are both Gaussian-like trajectory models. The investigations described in this report are based on calculations with MUSEMET.

Based on the source term characteristics and the meteorological conditions, the atmospheric dispersion models in UFOMOD calculate normalized time-integrated concentrations pat-

terns in the air near to the ground and on the ground surface. Thereby, the models distinguish between different dry and wet deposition characteristics which depend on the physical and chemical form of the isotopes released. The spatial concentration fields are transferred to subsequent modules of UFOMOD to calculate distribution functions of air concentrations, contaminated areas, organ doses and health effects together with areas and numbers of persons affected by countermeasures which are taken to reduce the exposure and thus the health implications in the population.

2.1 Parameter selection

2.1.1 Parameters contributing to uncertainty in this analysis

The following list gives the name and the meaning of the parameters:

| | |
|-----------------|--|
| σ_{y0} | initial horizontal plume width in the wake of the reactor building |
| σ_{z0} | initial vertical plume width in the wake of the reactor building |
| $h_m(S)$ | mixing height for stability class S ($S \in \{A,B,C,D,E,F\}$) |
| $\sigma_y(S)$ | vertical plume diffusion for stability class S |
| $\sigma_x(S)$ | horizontal plume diffusion for stability class S |
| $wp(S)$ | wind profile exponent for stability class S |
| $v_d(AE)$ | dry deposition of aerosols |
| $v_d(IO)$ | dry deposition of elementary iodine |
| $\Lambda_i(AE)$ | washout coefficients of aerosols for three rain intensities i ($i \in \{0-1 \text{ mm/s}, 1-3 \text{ mm/s}, > 3 \text{ mm/s}\}$) |
| $\Lambda_i(IO)$ | washout coefficients of elementary iodine for three rain intensities i ($i \in \{0-1 \text{ mm/s}, 1-3 \text{ mm/s}, > 3 \text{ mm/s}\}$) |

2.1.2 Factorization of parameters

For the purpose of clearness all uncertain parameters have been split into two factors:

$$Par = w \cdot Par_{50} \quad [1]$$

the first of them being a random variable w with a suitable frequency distribution, and the second one being the reference value for instance the median or 50%-quantile Par_{50} . or, when the frequency distribution $f(w)$ is constructed from experimental data another factorization is useful:

$$Par = w' \cdot Par_{modal} \quad [2]$$

In this case the *modal*(most frequent) values Par_{modal} characterize the distributions $f(w')$.

For example , the original $wp(DC=A)$ values vary between 0.065 and 0.135. This corresponds to Table 2 in the following manner:

$$wp(DC = A) = w' \cdot wp(DC = A)_{modal} \quad [3]$$

2.1.3 Correlation of parameters

Suppose there are physical mechanisms by which the different uncertain parameters are linked in nature. Then these mechanisms have to be considered as a source of uncertainty of more than one parameter value. There are groups of parameters, say $P(S_i)$, representing only one physical quantity, which depend on one class variable, say S . Choose for example the parameter $\sigma_x(S)$ with six stability classes $S \in \{A,B,C,D,E,F\}$.

The parameters $P(S_i)$ may be equally influenced by a common cause (which is not S_i). Then the uncertainty factors $w(S_i)$ have to be completely correlated for all S_i .

If the parameters $P(S_i)$ are stochastically influenced, the $w(S_i)$ have to be completely uncorrelated.

2.2 Ranges and distributions

2.2.1 Initial plume width in the building wake, σ_{y0}, σ_{z0}

Wind-tunnel studies of building wakes [16], [17] show tracer concentrations varying by a factor of three from minimum to maximum. The Gaussian plume formula gives

$$\bar{c} \sim \frac{\dot{Q}}{u \cdot \sigma_{y0} \cdot \sigma_{z0}} \quad [4]$$

for the average concentration \bar{c} in the wake. When the source rate \dot{Q} and the transport velocity u are kept constant, fluctuations of the concentrations only can be generated by variations of the lateral (horizontal and vertical) dimensions of the wake volume. Therefore the range of variation of the initial plume width σ_{y0} and σ_{z0} can be derived from $\frac{c_{max}}{c_{min}} \sim 3$.

The factorized distributions for σ_{y0} and σ_{z0} are chosen to be uncorrelated, symmetric and triangular with $w_{\min} = .5$, $w_{\text{modal}} = 1.$, $w_{\max} = 1.5$ (see Figure 1).

2.2.2 Mixing height, $h_m(S)$

Variations from the mean mixing layer heights can be described sufficiently well by a mean value $\pm 50\%$. As distribution the density of Figure 1 is used.

This is derived from mixing height measurements at different stabilities S and from theoretical considerations [33], [9], [59].

It is assumed that mixing height variations have 100% correlation at different stability classes S, i.e. a variation mechanism common to all stabilities is assumed. Such mechanisms exist. For example, smaller radiation fluxes result in lower mixing heights, for unstable cases as well as for stable cases. This can be shown by the summer to winter differences and the land to water differences in the mixing heights.

2.2.3 Horizontal and vertical plume diffusion parameters, $\sigma_y(S)$, $\sigma_z(S)$

The diffusion parameters depend on distance from the source and the atmospheric stability S. For simplicity the variation of σ_y and σ_z shall have no direct dependence on x, therefore, by definition, a fixed representative distance $x = 1000$ m has been chosen, where the uncertainty ranges of $\sigma_y(S)$ and $\sigma_z(S)$ are defined by the neighbouring stability class values $\sigma_{y,z}(S - 1) = \sigma_{\min}(S)$ and $\sigma_{y,z}(S + 1) = \sigma_{\max}(S)$.

The result is a triangular distribution $h_{y,z}(w, S)$ for w:

$$\sigma_{y,z}(S) = w \cdot \sigma_{y,z}(S)_{\text{modal}} \quad [5]$$

2.2.3.1 Plume diffusion parameters, $\sigma_z(S)$

The values for w are defined by

$$w_{\text{modal}} = \frac{\sigma_{y,z}(S)_{\text{modal}}}{\sigma_{y,z}(S)_{\text{modal}}} \equiv 1 \quad [6]$$

$$w_{\min, \max} = \frac{\sigma_{y,z}(S \pm 1)}{\sigma_{y,z}(S)_{\text{modal}}} \quad [7]$$

If $S = A$ or $S = F$ then w_{\max} or w_{\min} are defined by extrapolation:

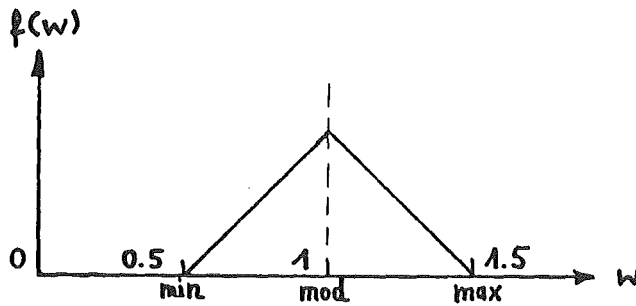


Figure 1. Symmetrical triangular distribution $f(w)$

$$w_{y,z \max}(A) = \frac{\sigma_{y,z \text{ mod}}(A)}{\sigma_{y,z \text{ mod}}(B)} = \frac{\sigma_{y,z \text{ mod}}(A)}{\sigma_{y,z \text{ min}}(A)} = \frac{1}{w_{y,z \text{ min}}(A)} \quad [8]$$

$$w_{y,z \text{ min}}(F) = \frac{\sigma_{y,z \text{ mod}}(F)}{\sigma_{y,z \text{ mod}}(E)} = \frac{\sigma_{y,z \text{ mod}}(F)}{\sigma_{y,z \text{ max}}(F)} = \frac{1}{w_{y,z \text{ max}}(F)} \quad [9]$$

2.2.3.2 Plume diffusion parameters, $\sigma_y(S)$

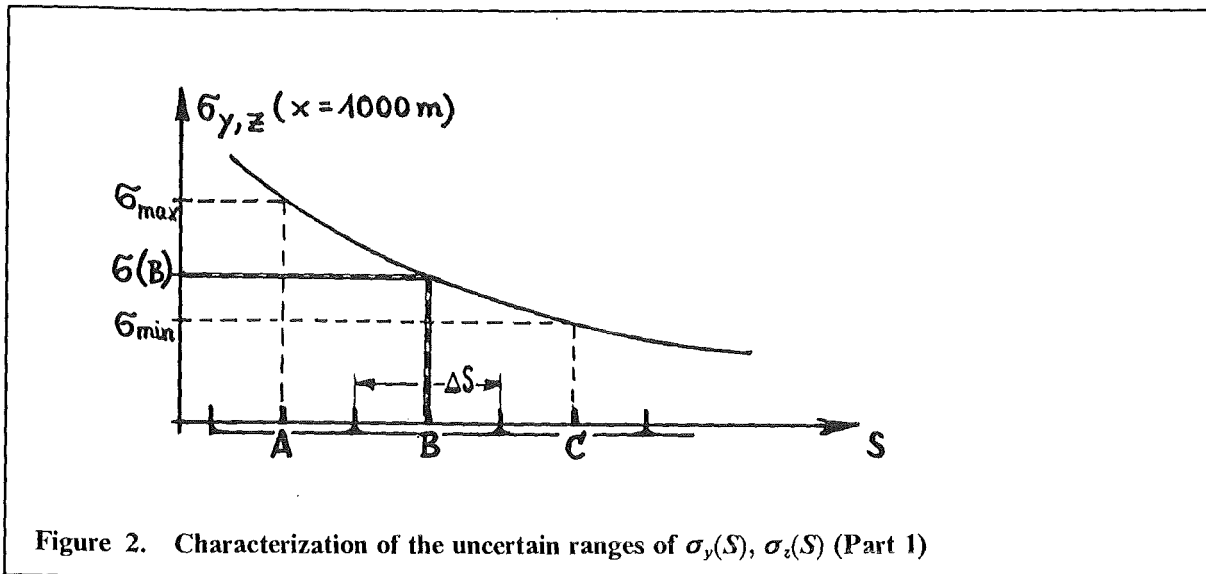
Since $\sigma_y(S)$ is not a monotone function of stability S in the case of the 'Karlsruhe-Jülich'-parameters the stable cases $S=E, F$ have to be treated different from the non-stable cases $S=A, B, C, D$ (see Figure 3).

Eq. [6] and Eq. [7] are used to define the triangular distribution of w . The marginal values like $w_{\min}(D)$, $w_{\max}(A)$, $w_{\min}(E)$, $w_{\max}(F)$ are defined by extrapolation like in Eq. [8] and Eq. [9].

The resulting numbers for the w -distributions of the σ -parameters corresponding to three different plume heights are given in Table 1.

2.2.3.3 Correlation of variations of σ -parameters for different classes S

The plume σ -parameters describe the plume width generated by turbulent motions in the air. When sampling times get long (for example: 1 h) the horizontal plume width is dominated by very low frequency motions, i.e. wind direction changes. The variation of the amplitude of these wind direction changes represents the variation of $\sigma_y(S)$.



The variability of this amplitude is *not dependent* on the small-scale turbulence which defines the stability class and acts like a stochastic influence. Therefore, the variation of $\sigma_y(S_1)$ is *not correlated* to the variation of $\sigma_y(S_2)$ $S_1 \neq S_2$, and there are six independent distribution with density functions $f_i(w_i)$, $i = 1, \dots, 6$ for $\sigma_y(S)$.

The variation of the amplitude of vertical turbulent motions generates the variation of $\sigma_z(S)$ at a given stability class S . Since mechanisms for enhanced vertical turbulent motions (like roughness length, density of hills in a given terrain) act systematically for all stability classes S it is assumed that there is a *correlated contribution* to the variation of $\sigma_z(S)$ for different classes S . Of course, there are also stochastic influences on $\sigma_z(S)$. So, both correlated and uncorrelated influences superpose.

As a consequence, the correlation is chosen as follows: 50% for each pair $(\Delta\sigma_z(S_i), \Delta\sigma_z(S_j))$ with $i \neq j$.

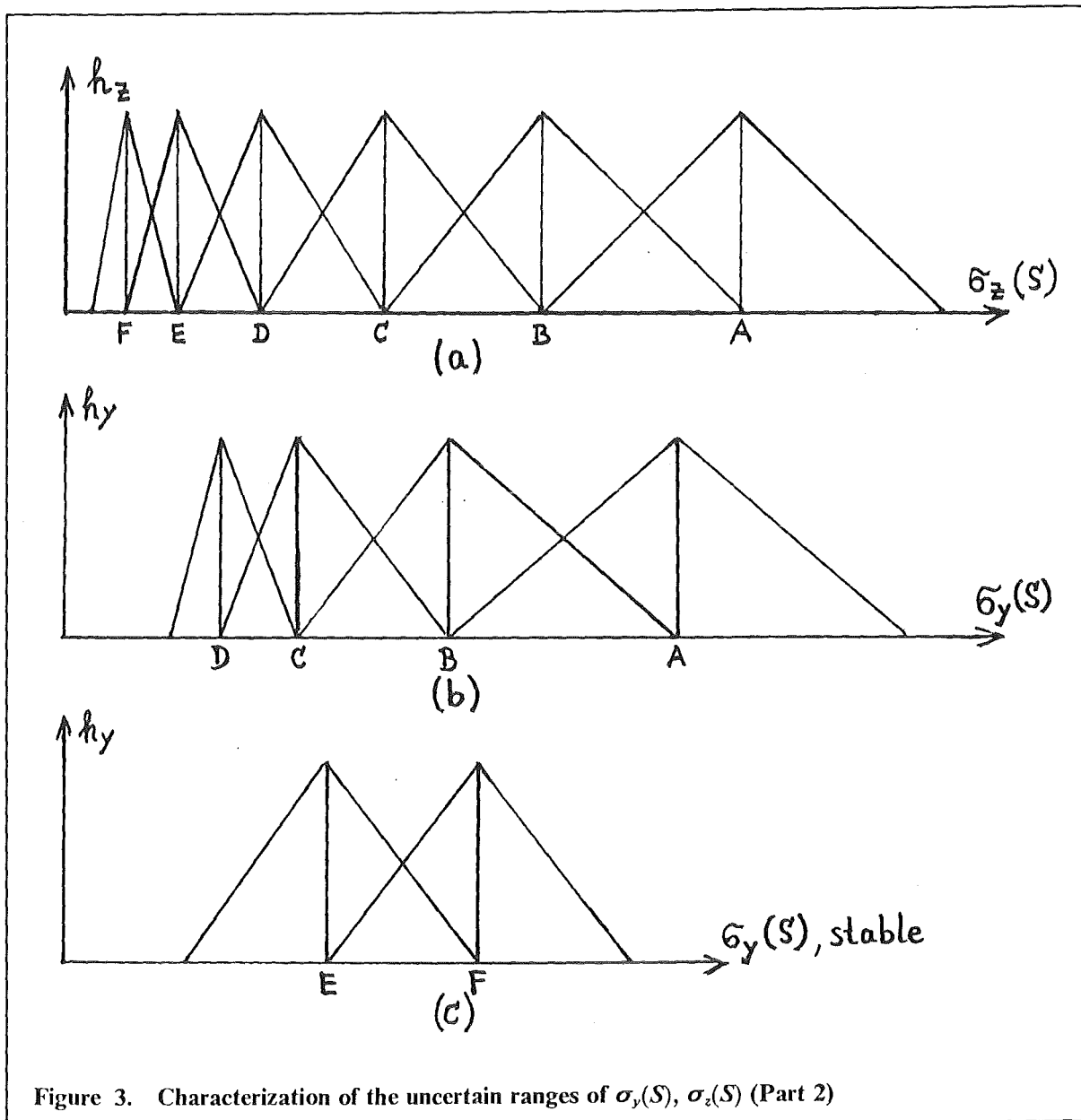


Figure 3. Characterization of the uncertain ranges of $\sigma_y(S)$, $\sigma_z(S)$ (Part 2)

| Height (m) | Stability Category | Special w-distribution values for σ -parameters | | | | | |
|------------|--------------------|--|----------------------------|----------------------|----------------------|----------------------------|----------------------|
| | | $w_{\min}(\sigma_y)$ | $w_{\text{mod}}(\sigma_y)$ | $w_{\max}(\sigma_y)$ | $w_{\min}(\sigma_z)$ | $w_{\text{mod}}(\sigma_z)$ | $w_{\max}(\sigma_z)$ |
| 50 | A | 0.54 | 1 | 1.84 | 0.39 | 1 | 2.56 |
| | B | 0.67 | 1 | 1.84 | 0.60 | 1 | 2.56 |
| | C | 0.83 | 1 | 1.48 | 0.60 | 1 | 1.67 |
| | D | 0.83 | 1 | 1.21 | 0.57 | 1 | 1.66 |
| | E | 0.79 | 1 | 1.26 | 0.42 | 1 | 1.75 |
| | F | 0.79 | 1 | 1.26 | 0.42 | 1 | 2.38 |
| 100 | A | 0.29 | 1 | 3.41 | 0.44 | 1 | 2.29 |
| | B | 0.48 | 1 | 3.41 | 0.62 | 1 | 2.29 |
| | C | 0.78 | 1 | 2.08 | 0.61 | 1 | 1.61 |
| | D | 0.78 | 1 | 1.29 | 0.58 | 1 | 1.64 |
| | E | 0.48 | 1 | 2.06 | 0.47 | 1 | 1.71 |
| | F | 0.48 | 1 | 2.06 | 0.47 | 1 | 2.14 |
| 200 | A | 0.62 | 1 | 1.62 | 0.38 | 1 | 2.62 |
| | B | 0.56 | 1 | 1.62 | 0.34 | 1 | 2.62 |
| | C | 0.90 | 1 | 1.79 | 0.48 | 1 | 2.95 |
| | D | 0.90 | 1 | 1.12 | 0.52 | 1 | 2.08 |
| | E | 0.51 | 1 | 1.95 | 0.60 | 1 | 1.91 |
| | F | 0.51 | 1 | 1.95 | 0.60 | 1 | 1.67 |

Table 1. Special w-distribution values: Min-, mode-, and max-values of the w-distribution of the σ -parameters for three different plume heights

2.2.4 Wind profile exponent, $w_p(S)$

The wind profile exponent w_p describes the increase of wind velocity with height by the equation

$$u(z) = u(10m) \cdot \left(\frac{z}{10m} \right)^{w_p(S)} \quad [10]$$

which is an approximation of the log-lin profile laws of surface boundary layer similarity theory [38] and [8]. These profiles are dependent on atmospheric stability S and on roughness length z_0 . If stability S and the wind velocity $u(z')$ are kept constant ($z' = 200m$, $z' \geq z(\text{SBL})$, surface boundary layer height) then an increase of z_0 causes all wind velocities $u(z)$ with $z < z'$ to decrease. This corresponds to an increase of the wind profile exponent w_p .

So one source of uncertainty of w_p is the variation of roughness length. A roughness length variation in the interval from 0.1 m to 1.0 m produces a w_p -variation of $\pm 30\%$ in the case of a non-stable stability class, and $\pm 10\%$ for the other classes. This results from calculations with the log-lin profile formulae [8] and from [38]. The variations are 100% correlated concerning different stability classes S . Another contribution is the random variation of w_p itself within the stability class intervals. This is about $\pm 20\%$ and is uncorrelated with respect to stability class.

A complete consideration of all uncertainty contributions to the wind profile exponent would result in six independent uncertainty contributions $f_S(w_S)$ with $S = (A,B,C,D,E,F)$. For $S = (A,B,C,D)$ the corresponding w_S would be correlated with about 60% and the range would be $(w_{\min}, w_{\text{mod}}, w_{\max}) = (0.65, 1, 1.35)$. For $S = (E,F)$ the correlation would be about 40% and the range would be $(w_{\min}, w_{\text{mod}}, w_{\max}) = (0.75, 1, 1.25)$. $(w_{\min}, w_{\text{mod}}, w_{\max})$ again refers to triangular distributions.

In a former uncertainty analysis of the atmospheric submodel of the old UFOMOD code it was shown that the contribution of w_p to uncertainty in relation to others is not very important [23]. Therefore a very detailed treatment of the outlined uncertainty contributions of w_p would be a waste of computer time.

It is more adequate to take simplified w -distributions. Therefore the w_p -uncertainty is represented by *only one w -distribution* for all stability classes (i.e. 100% correlation). The range is $(w_{\min}, w_{\text{mod}}, w_{\max}) = (0.65, 1, 1.35)$.

This is a little bit pessimistic point of view with regard to uncertainty, and without major importance for the analysis.

2.2.5 Dry deposition velocities, v_d

The ranges of deposition velocity variations are quite large; data spread over more than one decade. Measured data appear to be normally distributed on a logarithmic scale [57]. Therefore lognormal distributions describe the variation of v_d . Some quantiles of the distribution of the uncertainty factors w of v_d are given for aerosols by

$$w_{10} = \frac{1}{5.5}, w_{50} = 1, w_{90} = 5.5$$

and for iodine by

$$w_{10} = \frac{1}{3}, w_{50} = 1, w_{90} = 3$$

The w-variables for aerosols and elementary iodine are uncorrelated, i.e. $v_d(AE)$ and $v_d(IO)$ can vary independently.

2.2.6 Washout coefficients, Λ

Similar arguments are valid concerning the variation of the washout coefficients. Lognormal distributions $f(w)$ of the uncertainty factors w of the Λ -parameters describe the variation of washout coefficients. Some quantiles of the distribution of the uncertainty factors w of Λ are given for aerosols, iodine and all rain intensities by

$$w_{10} = \frac{1}{5}, w_{50} = 1, w_{90} = 5$$

The relative variations of Λ are independent of rain intensity and thus 100% correlated with respect to different rain intensities.

The general form of the washout coefficient is

$$\Lambda = a \cdot I^b$$

where I is the rainfall rate in mm/h, a and b are model parameters. The influence of a variation of b is not specific to the material washed out; therefore a correlated contribution originates from the variation of the parameter a . A 50% correlation is assumed between the aerosol and iodine washout coefficient variations.

| No. | Parameter | Additional characteristics | Reference value | Distribution | Range of variation | | | Correlation of parameters |
|-----|---------------|----------------------------|-------------------------------------|--------------|--------------------|-----------|-----------|------------------------------------|
| | | | | | w_{min} | w_{mod} | w_{max} | |
| 1 | σ_{y0} | | BW/4.3 | triangular | 0.5 | 1 | 1.5 | no correlation |
| 2 | σ_{z0} | | BH/2.15 | | 0.5 | 1 | 1.5 | |
| 3 | $h_m(S)$ | DC = A | 1600 m | triangular | 0.5 | 1 | 1.5 | 100% between all stability classes |
| | | DC = B | 1200 m | | | | | |
| | | DC = C | 800 m | | | | | |
| | | DC = D | 600 m | | | | | |
| | | DC = E | 300 m | | | | | |
| | | DC = F | 200 m | | | | | |
| 4 | $\sigma_y(S)$ | DC = A | $\sigma_y(x,S)$ KA-JÜ z = 50m | triangular | 0.54 | 1 | 1.84 | no correlation |
| 5 | | DC = B | | | 0.67 | 1 | 1.84 | |
| 6 | | DC = C | | | 0.83 | 1 | 1.48 | |
| 7 | | DC = D | | | 0.83 | 1 | 1.21 | |
| 8 | | DC = E | | | 0.79 | 1 | 1.26 | |
| 9 | | DC = F | | | 0.79 | 1 | 1.26 | |
| 10 | $\sigma_z(S)$ | DC = A | $\sigma_z(x,S)$ KA-JÜ z = 50m | triangular | 0.39 | 1 | 2.56 | 50% between all stability classes |
| 11 | | DC = B | | | 0.60 | 1 | 2.56 | |
| 12 | | DC = C | | | 0.60 | 1 | 1.67 | |
| 13 | | DC = D | | | 0.57 | 1 | 1.66 | |
| 14 | | DC = E | | | 0.42 | 1 | 1.75 | |
| 15 | | DC = F | | | 0.42 | 1 | 2.38 | |
| 16 | wp | DC = A | 0.10 | triangular | 0.65 | 1 | 1.35 | 100% between all stability classes |
| | | DC = B | 0.13 | | | | | |
| | | DC = C | 0.16 | | | | | |
| | | DC = D | 0.22 | | | | | |
| | | DC = E | 0.35 | | | | | |
| | | DC = F | 0.55 | | | | | |

Table 2. Parameter distribution table

| No. | Parameter | Additional characteristics | Reference value | Distribution | Range of variation | | | Correlation of parameters |
|-----|---------------------|----------------------------|-----------------|-----------------|--------------------|----------|----------|---------------------------|
| | | | | | w_{10} | w_{50} | w_{90} | |
| 17 | $v_d(AE)^{1)}$ | | 0.55 E-3 | lognormal *) | 1/5.5 | 1 | 5.5 | no correlation |
| 18 | $v_d(IO)^{1)}$ | | 1.00 E-2 | | 1/3 | 1 | 3 | |
| 19 | $\Lambda_{AE}^{2)}$ | 0-1 mm | 0.34 E-4 | lognormal *) | 1/5 | 1 | 5 | **) |
| | | 1-3 mm | 1.17 E-4 | | | | | |
| | | > 3 mm | 3.29 E-4 | | | | | |
| 20 | $\Lambda_{IO}^{2)}$ | 0-1 mm | 0.42 E-4 | lognormal *) | 1/5 | 1 | 5 | **),***) |
| | | 1-3 mm | 1.06 E-4 | | | | | |
| | | 1-3 mm | 2.31 E-4 | | | | | |

Note:
DC = Diffusion category
*) lognormal distribution truncated at 10th and 90th percentile
**) 100% with respect to different rain intensities
***) 50% between $\Lambda_i(AE)$ and $\Lambda_i(IO)$
¹⁾ Units for $v_d(AE)$, $v_d(IO)$ are [m/s] ²⁾ Units for Λ_{AE} , Λ_{IO} are [1/s]

Table 3. Parameter distribution table (cont'd)

2.2.7 Parameters not considered in this analysis

This analysis is restricted to the investigation of uncertain model predictions coming from model parameters of the atmospheric dispersion and deposition submodule, only.

This means especially, that variations of the following (input) variables are *not* considered in the uncertainty analysis:

- release of radionuclides
 - all variables concerning source-terms, including
 - thermal energy
 - release height
- meteorological situation
 - wind direction
 - wind velocity
 - stability class
 - rain intensity
- plume rise and 'lift-off'-modelling

The parameters of the plume rise model and the 'lift-off'-criterion are kept fixed in this analysis. The reasons are:

- The general influence of a variation of plume rise parameters has already been shown in a former uncertainty analysis of the atmospheric model of the DRS (Phase A) - code [23]. It turns out that there are effects only at distances close to the source ($r \leq 0.5 - 1\text{km}$)
- Presently there is no information about the amount of thermal energy released from the source. However, uncertainty related to plume rise strongly depends on thermal energy.

3. Uncertainty Analysis

The preceding chapter described to some extent ranges, distributions and correlations of the model parameters, respectively.

Prior to the actual analysis performed with the program system UFOMOD it is necessary to define specific vectors of the uncertain model input parameters to be used in each run of UFOMOD. The selection of these sets of specific parameter values is done by a suitable *sampling scheme*. With *one* parameter set each run produces *one* complementary cumulative distribution function (CCFD). From all runs a family of curves results, which visualizes the variability of the CCFDs of consequences. Confidence bands can be derived together with sensitivity measures, which determine what causes this variability in consequences.

Important questions are, how to construct CCFD curves and confidence bands, how to calculate sensitivity measures and how many UFOMOD-runs are necessary to get reliable uncertainty and sensitivity results?

Uncertainty analysis methods may need much computer runs and time if there are a lot of model parameters and the accident consequence code is long-running. Therefore, one hand the designer of a sampling scheme should aim at a low number of runs, on the other hand the number of runs should be large enough to get stable and trustworthy results.

3.1 The sampling scheme

From the various possible sampling strategies the Latin hypercube sampling (LHS) approach was selected. LHS is a modified random sampling with stratified samples and is found to have very good sampling characteristics when compared to other methods (see [35] and [47] (Vol. 3 K-5)).

The sampling procedure forces the value of each model parameter to be spread across its entire range. In random sampling it is possible by chance to choose only a portion of the range of model parameters, leaving out another part of the possible range that could greatly influence the consequence variables. The intent of LHS is to make more efficient use of computer runs than random sampling even for *smaller* sample sizes. For *large* sample sizes there is little difference between the two techniques.

A Latin hypercube sample of size n stratifies the range of each model parameter into " n " nonoverlapping intervals on the basis of equal probability. Randomly a value is selected from each of these intervals. Let X_i ($i=1,\dots,k$) be the model parameters. The n values

obtained for X_1 are paired at random with the n values obtained for X_2 . These n pairs are combined in a random manner with the n values for X_3 to form n triples. The process is continued until a set of n k -tuples is formed.

There may exist "spurious" correlations between model parameter values within a Latin hypercube sample, due to the random pairing of the model parameter values in the generation of the sample. This is most likely when n is small in relation to k . Such correlations can be avoided by modifying the generation of the sample through use of a technique introduced by R.I. Iman and W.J. Conover [30]. This technique preserves the fundamental nature of LHS, but replaces the random pairing of model parameter values with a pairing that keeps all of the pairwise rank² correlations among the k model parameters close to zero.

The Iman/Conover-technique can also be used to induce a desired rank correlation structure among the model parameters. The procedure is distribution free and allows exact marginal distributions to remain intact. This is used for the UFOMOD - LHS - design. For some mathematical details see [30] and [23].

In producing correlated model parameters one may ask which type of correlation is to be used (e.g. correlation measured on raw or ranked data).

A correlation coefficient computed on raw data may lose meaning and interpretation with data from non-normal populations or in the presence of outliers, as is pointed out in [34]. **Rank correlations** can be quite meaningful in modelling situations where the model parameters are monotonically related and not necessarily normally distributed. Additionally, it may make more sense to talk about monotone relationships, and hence rank correlations, because of the unusual behaviour of the Pearson correlation coefficient in certain joint probability distributions (In some cases there is an unusual lower bound on the correlation value which is greater than -1).

Following [24], another aspect is: When actual measurements are impossible or not feasible to obtain but relative positions can be determined, rank order statistics make full use of all the available information. The question is however, how much information is lost by using the data only to determine relative magnitudes. An approach to a judgement concerning the potential loss of efficiency is to determine the correlation between the variate values and their assigned ranks. If the correlation is high, we would feel intuitively more justified in the replacement of actual values by ranks for the purpose of analysis. The hope is that inference procedures based on ranks alone will lead to conclusions which seldom differ from a corre-

² The rank order statistic for a random sample is any set of constants which indicate the order of observations. The actual magnitude of any observation is used only in the determination of its relative position in the sample array and is thereafter ignored in any analysis based on rank order statistics.

sponding inference based on actual variate values. But nevertheless one may ask to which extent influences are to be expected if an model parameter is substituted by its rank statistic. Is it reasonable to expect that these results will depend on the type of distributions? So, one should try to calculate correlations between variables and their ranks for each type of distribution and to check the effect on the consequence variables. If the mentioned correlation is near to one, there should be more confidence in the obtained results.

3.2 Correlation considerations

The interest is now concentrated on formal coherence of variate values and their ranks for different distribution types. It can be measured by the population correlation coefficient between ranks and variate values, i.e. between X_i and $Y = R(X_i)$, where $R(X_i) = \text{rank of } X_i$

$$r = \frac{E(X_i Y_i) - E(X_i)E(Y_i)}{\sigma_{X_i} \sigma_{Y_i}} \quad [11]$$

From [61], the correlation between raw values and corresponding ranks is rather high for some commonly used distributions:

normal distribution $r = .98$ Eq. [12]

uniform distribution $r = 1$ Eq. [13]

Gamma distribution with parameter m Eq. [14]

$r = .78$ ($m = 1/2$) (Chi square distribution)

$r = .87$ ($m = 1$) (Exponential distribution)

$r = .90$ ($m = 2$)

$r = .95$ ($m = 4$)

In Appendix A is shown:

For lognormal distributions $r = .69$, for triangular distributions from $r = .98$ to $r = .99$ (depending on the mode). For standardized logtriangular distributions the value of r depends on b , the mode value of the distribution, and is for $b = .5$ about $r = .98$. The uniform distribution shows a 'stable' behaviour, i.e. a perfect correlation between raw and rank values.

In general, correlations between raw and rank values differ and depend on some distribution parameters and the sample size in the LHS - design. The main intention is to get raw and rank correlations created by the design as close as possible. This actually means to get a

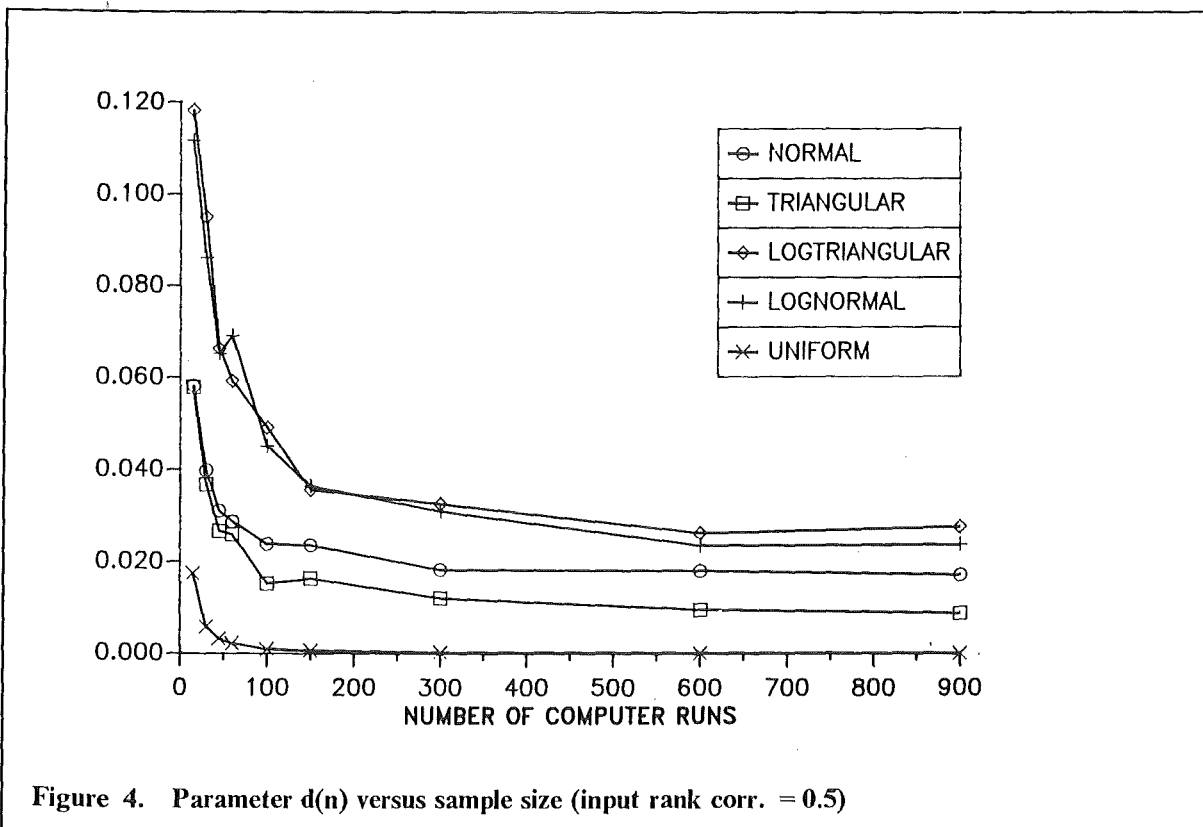
small value for a 'correlation stable' measure, $d(n)$, the 'mean difference'³ between two (rank and raw) correlation matrices (calculated element pairwise),

$$d(n) = \frac{1}{q} \sum_{m=1}^q d(n, s_m) \quad [15]$$

q is number of different used random seeds, s_m , and

$$d(n, s_m) = \frac{2}{k(k-1)} \sum_{i=2}^k \sum_{j=1}^{i-1} |r_{ij}(n, s_m) - \rho_{ij}(n, s_m)| \quad [16]$$

$r_{ij}(n, s_m)$ and $\rho_{ij}(n, s_m)$ represent the raw and rank correlation between variables i and j , respectively. The number n represents the sample size.



The 'difference' between rank and raw correlation matrices strongly decreases if the ratio of sample size, n , and the number of correlated variables, k , is small. Increasing the ratio n/k beyond 10 has no influence on the 'stability measure' $d(n)$. This might be important for the number of necessary computer runs to create a trustworthy design.

³ It was assumed that all parameters are correlated with each other. The number of correlation pairs is $k(k-1)/2$

New distributions can be introduced to substitute the lognormal distribution, especially because of the 'instabilities' detected during the former uncertainty/sensitivity analyses [23] of the old UFOMOD code. If it is possible to substitute e.g. the lognormal distribution by an appropriate linearized distribution the benefit could be significant. Such substitutions would have the same effect as increasing the sample size. With other words, the same 'correlations stability', i.e. variable $d(n)$, is obtained with a smaller number of necessary computer runs.

For example from Figure 4, to get $d(n)=0.04$, about 150 runs are needed for lognormal distributions, but only about 30 runs are necessary for triangular distributions.

In Appendix C the effect of using uniform distributions for all model parameters is demonstrated.

3.3 Estimation of confidence bounds

The next task is to run the accident consequence code with the sampled input parameter values from the LHS-design.

The following distinctions are necessary:

- There are stochastic variations e.g. in weather conditions or wind directions. Each run of UFOMOD therefore produces one frequency distribution (CCFD) of consequences.
- Due to lack of knowledge about the actual model parameter values there is an uncertainty in these results. This can quantitatively be expressed by confidence intervals of the frequency distribution of consequences.

CCFD curves are generated by considering the probability of equaling or exceeding each consequence level on the x-axis. To construct a CCFD keep in mind 144 weather sequences with different probabilities, say $PWET(L)$ ($L=1,\dots,144$), and 72 azimuthal sectors of 5° each, are considered. For each radius (distance) there exist 144×72 point values with the probability $PWET(L)/72$. The 144×72 consequence values are sorted into 90 classes (which correspond for instance to nine decades of consequence values on a logarithmic x-scale). Each class has its own probability of occurrence given by summing up the probabilities of the members of the class. Adding the probabilities of the classes stepwise from the right to the left will give the CCFD.

To get confidence curves for each consequence level so-called p-quantiles are calculated from the number n_0 of associated probability values at this consequence level x.

Example:

Suppose $n_0 = 40$ UFOMOD - runs, i.e. there are 40 CCFDs and - corresponding for each consequence level x - 40 probability points. To get a (p %) - confidence the following procedure has been adopted:

For each consequence level x find the (p %) - smallest probability value of n_0 ordered values. For all individual consequence levels these selected probability points are connected to obtain the estimated (p %) - confidence curve.

Particularly for the 5 % (95 %) - confidence curves connect the $p \times n_0$ -th numbers from the bottom in the ordered list of n_0 probability points, i.e. in our example connect the 2-th and the 38-th values from the bottom, respectively. Mean and median curves can be created in a similar manner.

□

Remark:

Following [1] the 5 % - and 95 % - quantiles can be taken to be confidence intervals on the *mean* curve. The 5 % - and 95 % - quantile estimation is possible by using two methods:

- direct calculation of the empirical 5 % - and 95 % - quantiles at each consequence level (see example above), and
- using ± 1.645 times sigma based on the assumption of a normal distribution at each consequence level.⁴

[1] points out that the two methods give substantially similar estimates.

□

⁴ The Central Limit Theorem states that for large sample sizes, n , the distribution of $(\bar{X} - \mu)/(\sigma/\sqrt{n})$ is approximately normal, so that holds true

$$P\left(-z_{1-\alpha/2} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < z_{1-\alpha/2}\right) \cong 1 - \alpha$$

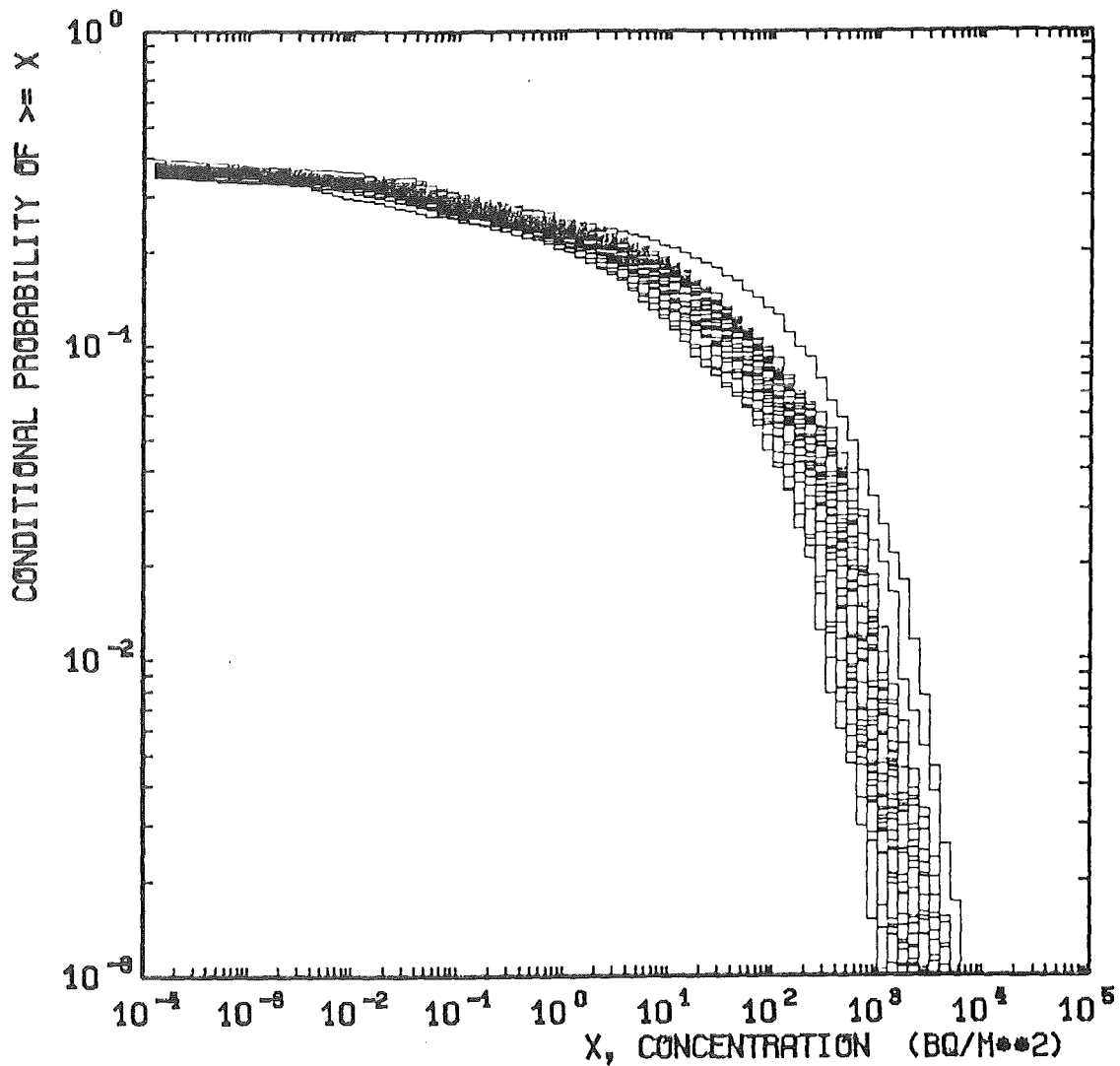
or

$$P\left(\bar{X} - z_{1-\alpha/2} \cdot \sigma/\sqrt{n} < \mu < \bar{X} + z_{1-\alpha/2} \cdot \sigma/\sqrt{n}\right) \cong 1 - \alpha$$

This statement forms the basis for the confidence interval for the average of n distributions at each consequence level. $z_{1-\alpha/2}$ is the $(1 - \alpha/2)$ - quantile of the standardized normal distribution.

For instance $\alpha = 0.1$, then $z_{0.05} = -1.6449$ and $z_{0.95} = 1.6449$.

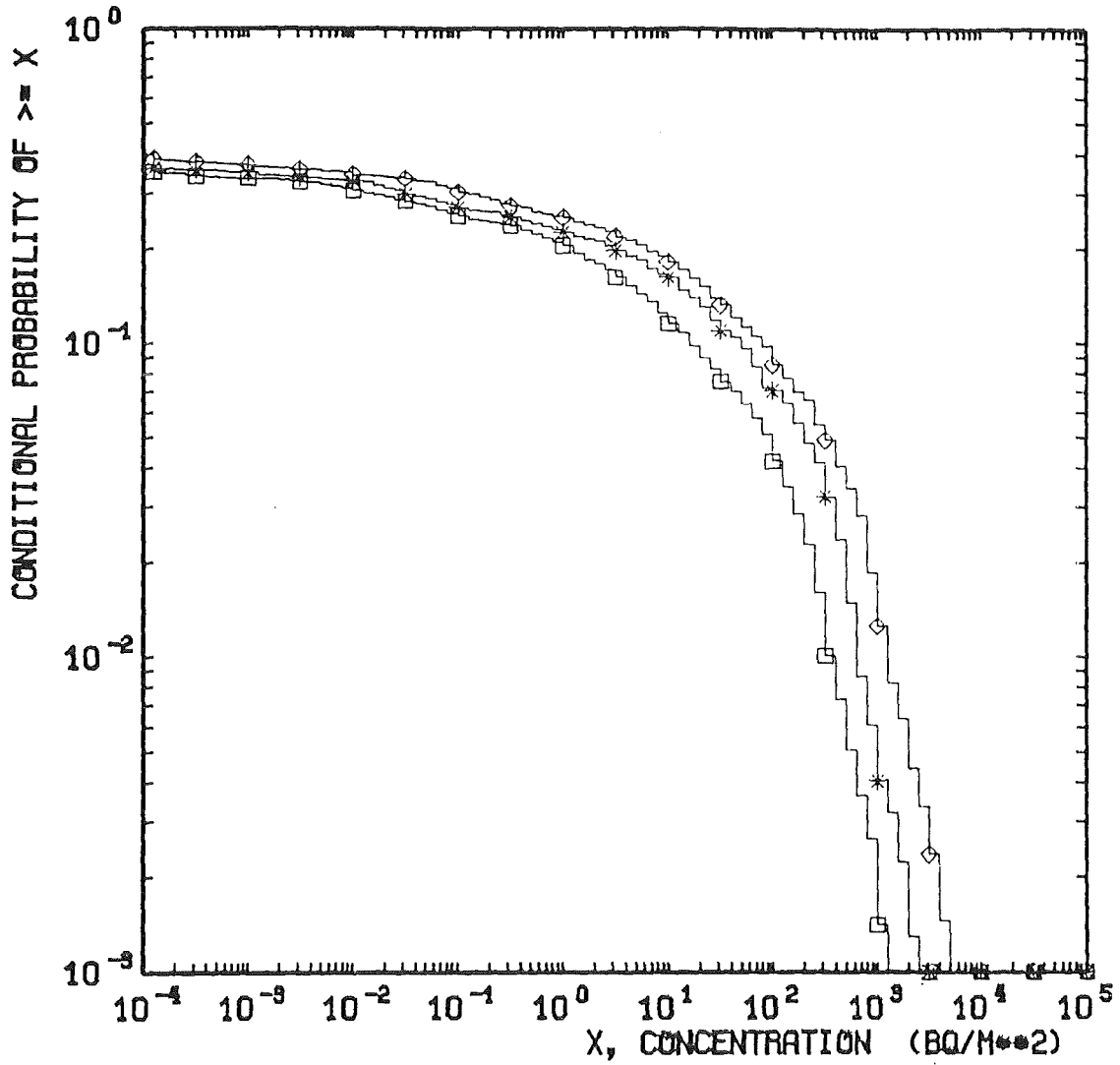
UFOMOD Uncertainty Analysis (1987)



Concentration on ground surface
Nuclide: Cs - 137
Distance: 0.875 km

Figure 5. Complementary cumulative frequency distributions (CCFDs) of activity concentrations on ground surface: Each CCFD (assuming release has occurred) corresponds to one of the 40 runs in a Latin hypercube sample of size 40.

UFOMOD Uncertainty Analysis (1987)



Concentration on ground surface
 Nuclide: Cs - 137
 Distance: 0.875 km

* : Ref.-Curve
 □ : 5% -Curve
 ◇ : 95% -Curve

Figure 6. Reference CCFD of activity concentrations: The empirical 5%-,95%- quantiles are given as estimated confidence bounds at discrete points of the x-axis.

It has been tested that different samples or an increase of sample size (for sample sizes larger than $n=40$) do not change the 5%-95%-confidence bands.⁵ As a typical example Figure 5 shows 40 estimated complementary cumulative frequency distributions for Cs-137 activity concentrations on ground surface at the distance of 0.875 km.

Figure 6 shows the corresponding estimated so-called *reference CCFD* (all uncertain input model parameters are at their point value (50%-quantile)) and the empirical 5%-95%-quantiles at each consequence level. The 5%-95%-'confidence curves' were generated by considering the probability of equaling or exceeding each consequence level appearing on the x-axis. For each consequence level the 5% and 95%-quantiles (or other values: mean, median etc.) were calculated from the 40 associated probability values. These probability estimates for individual consequence levels were then connected to obtain the empirical 5%-95%-confidence curves (see [1]).

So, the confidence bounds have to be interpreted as follows:

There is 90%-confidence that the conditional probability for the activity concentrations, x , on ground surface, is

- below the ordinate value at x of the 95%-curve, and
- above the ordinate value at x of the 5%-curve.

The width of the CCFD-confidence band is an indicator of the sensitivity of model predictions with respect to variations in parameters, which are imprecisely known.

3.4 Sensitivity analysis

Now, those uncertain input model parameters have to be identified which are important contributors to variations in consequences. Following [35], there are several methods for quantifying the relative importance of the uncertain model parameters to the output of the accident consequence model. Usually, each of the uncertain model parameters is ranked on the basis of its influence on the consequences. Some methods provide such an overall ranking while others (e.g. stepwise regression) are designed to select subsets consisting of only the most influential parameters.

⁵ In [35] is stated, that good results can be obtained even with $n = 4/3$ times the number of uncertain model parameters. For $n < k$ it seems appropriate to use the LHS - technique in a piecewise fashion on subsets of the k model parameters. For details see [30].

- Rankings beyond the first few most important uncertain parameters usually have little or no meaning in an absolute ordering, since only a small number of the total number of uncertain parameters actually turns out to be significant. This will be explained later in more detail.
- Sensitivity analysis in conjunction with any form of sampling or design is easiest to carry out *if a regression model is fitted* between the model consequences and the model parameter values. Such a regression model is inherent in the calculation of correlation coefficients. But, regression techniques are influenced by extreme observations and nonlinearities. Therefore it seems to be appropriate to transform the data.

A method which

- is regression based,
- ranks either all uncertain model parameters or only those within a subset, and additionally
- avoids sophisticated transformations

is the ranking on the basis of *partial rank correlation coefficients*.

Now, *regression analyses* define the mathematical relationship between two (or more) variables, while *correlations* measure the strength of the relationship between two variables.

But do all correlation numbers indicate a significant relationship between variables, i.e. is there an actual relationship or only one by chance ('white noise')? Up to which level ('**white noise'-level, critical value**) the correlation numbers are treated as garbage?

The numerical values of correlation coefficients or partial (rank) correlations coefficients can be used for significance testing of the correlation, or with other words, for hypothesis testing to quantify the confidence in the correlation itself. For details see Appendix A.3.

But to summarize the main results in advance:

To get **statistically stable results for sensitivity analyses** larger sample sizes than for confidence bounds calculations have to be chosen. The number of uncertain model parameters, which have a sensitivity measure value above the so-called 'white noise level' increase with sample size. For details see Appendix A and the sensitivity tables in Appendix C, which compare the results for $n=40, 80$ and 100 computer runs.

The **partial correlation coefficient (PCC)** is a measure that explains the linear relation between for instance a consequence variable and one or more uncertain model parameters with the possible linear effects of the remaining parameters removed. Following [25], when nonlinear relationships are involved, it is often more revealing to calculate PCCs between variable *ranks* than between the *actual values* for the variables. Such coefficients are known as **partial rank correlation coefficients (PRCCs)**. Specifically, the smallest value of each vari-

able is assigned the rank 1, the largest value is assigned the rank n (n denotes the number of observations). The partial correlations are then calculated on these ranks.

Remark:

One may ask:

- Why rank correlation for explaining sensitivity ?
- Why Pearson's product moment correlation and not Kendall's coefficient ?
- What's about standardized and stepwise regression coefficients ?
- What has to be done if there are different rankings with respect to different sensitivity measures ?

Rank correlations are more comprehensive if there are nonlinearities in the computational models. Some other arguments for using these measures have been given at the beginning of this section.

The 'concordance'-based sensitivity measure, Kendall's τ , can also be chosen for partial (rank) correlation. But, on the other hand, an advantage of using the extension of Spearman's product-moment based ρ is, that existing computer programs for finding Pearson's partial correlation coefficients may be used on the ranks instead of the data, and the partial rank correlation coefficients are obtained easily (for more details concerning similarities and discriminations of the two measures see [11], [24] and [7]).

Standardized (rank) regression coefficients (SRC, SRRC) have been calculated, too. There was nearly the identical importance ranking as in the PCC, PRCC - calculations.

Stepwise regression calculations have not been carried out. But in [35] there is a summary of comparisons of some results achieved with different regression-based sensitivity measures.

In [32] and [35] a way is shown to measure agreement on the selection of the most important model parameters by computing the ordinary correlation coefficient on scores based on the sum of the reciprocals of the assigned ranks (so-called Savage scores).

The next step is to pick out the relevant sensitivity information out of the bulk of hidden messages within the CCFDs.

There are various possible ways to condense the extensive data:

- Estimate fractiles, the estimated mean values etc. of the n CCFDs *at certain consequence levels*. There will be possibly divergent 'importance rankings' for different consequence values.
- Estimate *one* fractile, *one* estimated mean value etc. for each of the n consequence curves.

The second procedure is used for the UFOMOD - uncertainty and sensitivity analyses. To find the most important contributors to uncertainty in the consequences partial rank correlation coefficients (PRCCs) are used.

Importance ranking is done by taking *absolute* values of the PRCC values. The model parameter associated with the largest absolute PRCC value is called the **most important** one responsible for uncertainty in consequences and gets **importance rank 1**.

This differs from the definition of *ranks of sample values*, where the smallest values has rank 1, the next smallest has rank 2 and so on.

Example:

On the basis of 40 UFOMOD - runs with LHS, the most important uncertain parameters including their PRCC and *importance rank* for each consequence (e.g.: Cs-137 activity concentrations on ground surface at the distance of 0.875 km) are identified. By statistical reasons (as explained before), a parameter is significant with confidence 95%, if the absolute value of the corresponding PRCC is greater than .43 (for $n=40$). The absolute value describes the strength of the input-output dependency, while the (+,-)-sign indicates increasing (decreasing) model consequences for increasing uncertain parameter values. Wet and dry deposition velocity of aerosols, Λ_{AE} , and $v_d(AE)$, are the most important sources of variation for the activity concentration with PRCC-values of .97 and .96, respectively. Increasing Λ_{AE} and $v_d(AE)$ lead to a strong increase of activity concentration (see Appendices).

□

In addition to evaluating the influence of each uncertain model parameter on the model consequences, the calculation of PCCs or PRCCs provide a good indicator of the 'fit of the analysis' to the model behaviour: the **coefficient of determination, R^2** , which is a measure of how well the linear regression model based on PCCs (or the corresponding standardized regression coefficients) can reproduce the actual consequence values. Or, in other words, it reflects the fraction of the variance in model consequences which can be explained by regression, i.e. it is possible to calculate the *percentage contribution* of each uncertain model parameter to variations in consequences. R^2 varies between 0 and 1 and is the square of the corresponding PCC. The closer R^2 is to unit, the better is the model performance.

To clarify this, let us observe a hypothetical system of about twenty uncertain model parameters. Take each parameter separately, omitting all other 19 parameters, and calculate the R^2 - value. Assume model parameter X has importance rank 1 and parameter Y has importance rank 20 with respect to the consequence. It is expected (at least when all parameters are uncorrelated) that the greatest amount of the observed variation is accounted for by the most important model parameter (in the linear regression model). The second

most important model parameter has a smaller R^2 - value, and so on. Thus, the R^2 - values should describe a monotonous nonincreasing function of importance ranks.

However, assume now, only X and Y are correlated and have rank 1 or 20, respectively. The corresponding R^2 - value for the unimportant Y may be significantly greater than even e.g. the parameter with rank 2! But an unimportant parameter cannot be responsible for more variation than the most important ones.

Therefore, in the case of correlated model parameters, some of the calculated R^2 - values (from separately taken parameters) give misleading effects. So, one has to be very careful in interpreting these coefficients of determination when there are correlations present within the group of model parameters.

3.5 Results

This chapter summarizes the main conclusions for the UFOMOD/NE87 submodule uncertainty and sensitivity investigations. For details the reader is referred to the Appendices B and C.

The following results of accident consequence assessments are investigated:

- The activity concentrations of I-131 and Cs-137 in the air near ground (1-m height) and on ground surface considering the variability of the averaged⁶ concentrations values at three distances D1 (0.875 km), D2 (4.9 km) and D3 (27 km).

Due to weather conditions each UFOMOD run produces *one* frequency distribution (CCFD) of consequences.

In Chap 3.3 the way of construction of CCFDs and corresponding confidence curves has been indicated. The calculation of sensitivity measures has been outlined in Chap 3.4.

⁶ (averaged over 144 weather sequences which represent the weather of the two years 1982 / 83)

| DISTANCE [km] | IODCG [Bq/m ²] | IODCA [Bq/m ³] | CAECG [Bq/m ²] | CAECA [Bq/m ³] |
|--|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 0.875 | 3 10 ⁴ | 3 10 ⁶ | 4 10 ³ | 4 10 ⁶ |
| 4.9 | 1 10 ³ | 1 10 ⁵ | 3 10 ² | 3 10 ⁵ |
| 27 | 1 10 ² | 6 10 ³ | 3 10 ¹ | 1 10 ⁴ |
| WIDTH | ≤ 0.2 decade | ≤ 1.0 decade | ≤ 1.0 decade | ≤ 1.0 decade |
| Note: IODCG = I-131 activity concentration on ground surface IODCA = I-131 activity concentration in air near ground (1-m height) CAECG = Cs-137 activity concentration on ground surface CAECA = Cs-137 activity concentration in air near ground (1-m height) | | | | |

Table 4. 99 th percentiles of activity concentrations: This table indicates the 99 th percentiles of activity concentration values and the variability of corresponding consequences

The restriction to pure model parameters and revised variations in the atmospheric dispersion and deposition submodule of UFOMOD version NE87/1 (see Chapter 2) leads to significantly smaller uncertainty bands than for the old UFOMOD / B3 code (see for comparison [23]). The number of necessary UFOMOD - runs to get reliable uncertainty and sensitivity results depends on different arguments:

- As mentioned before, even small samples or an increase of sample size (for sample sizes larger than $n=40$) do not change the confidence bands. I.e. the sample of size $n=40$ is already sufficient and adequate to estimate the distribution of results. Otherwise different samples would provide a hint of the variability of results due to sampling.
- By statistical reasons sensitivity analyses need larger sample sizes than for confidence bound calculations. The so-called 'white noise level' decreases with increasing sample size.

The variability of the CCFDs is fairly small. This is shown by the width of the corresponding confidence bands which is not larger than one decade.

The confidence bounds are considerably smaller than those calculated in the earlier analysis, where about 1.5 decades (width) resulted at a distance of about 1 km (for details see [23]). This is mainly due to the fact, that a near ground release without thermal energy and its variation is considered. In addition, the variations of dry and wet deposition parameters are somewhat reduced and unreasonable correlations are removed. At far distances, where the plume extends vertically in the whole mixing layer, the difference of the confidence bounds become smaller.

| Consequence variable | Particularity | Important parameters | Importance ranking | Range of PRCC | Range of R^2 in (%) | Range of PRCC U | Range of R^2 in (%) U |
|--|---------------|----------------------|--------------------|---------------|-----------------------|-----------------|-------------------------|
| IODCGD1 | | $v_d(IO)$ | 1 | .98 | (.79, .83) | (.96, .98) | (.75, .83) |
| | | $\sigma_z(E)$ | 2 | (-.61, -.69) | (.16, .19)* | (-.49, -.61) | (.12, .19)* |
| IODCGD2 | | $v_d(IO)$ | 1 | (.79, .95) | (.36, .44) | | |
| | | h_m | 2 | (-.67, -.68) | (.08, .22) | | |
| | 40 | Λ_{IO} | 2 | .83 | | | |
| | 40U | h_m | 1 | | | -.82 | 31 |
| | 100U | h_m | 1 | | | -.73 | 28 |
| IODCGD3 | | h_m | 1 | (-.94, -.96) | (.38, .54) | (-.91, -.95) | (.47, .51) |
| | | $v_d(IO)$ | 2 | (.92, .95) | (.32, .44) | (.88, .91) | (.28, .34) |
| | | Λ_{IO} | 3 | (.52, .74) | (.03, .09) | (.35, .55) | (.02, .04) |
| IODCAD1 | | $v_d(IO)$ | 1 | (-.91, -.94) | (.36, .51) | (-.96, -.98) | (.68, .72) |
| | | $\sigma_z(E)$ | 2 | (-.61, -.75) | (.40, .44)* | (-.58, -.79) | (.22, .28)* |
| | 80 | σ_{z0} | 2 | -.72 | .7 | | |
| IODCAD2 | | $v_d(IO)$ | 1 | (-.95, -.97) | (.76, .86) | (-.96, -.98) | (.82, .85) |
| | | h_m | 2 | (-.61, -.77) | (.03, .07) | (-.73, -.84) | (.08, .11) |
| IODCAD3 | | $v_d(IO)$ | 1 | (-.94, -.97) | (.48, .55) | (-.97, -.98) | (.63, .65) |
| | | h_m | 2 | (-.93, -.97) | (.37, .40) | (-.95, -.96) | (.27, .33) |
| Note: *) The R^2 - values are calculated for the total group of correlated σ_z (S) - parameters. | | | | | | | |

Table 5. Most important parameters for uncertainties in I-131 activity concentrations: This table indicates the most important parameters (including ranking, range of PRCC - and R^2 - values) for the variability in consequences for different sample sizes (40, 80, 100) and different assumptions on distribution types (U means all parameters are uniformly distributed)

Table 4 shows decreasing concentration values from near to far distances. The smallest concentrations are for aerosols on ground surface.

Table 5 and Table 6 try to summarize the main sensitivity information given in Appendix C.

The most important parameters are listed together with some 'ranges' ⁷ for PRCC- and R²-values. Some particularities are detected and appear in the tables.

| Consequence variable | Particularity | Important parameters | Importance ranking | Range of PRCC | Range of R ² in (%) | Range of PRCC U | Range of R ² in (%) U |
|--|---------------|----------------------|--------------------|---------------|--------------------------------|-----------------|----------------------------------|
| CAECGD1 | | Λ_{AE} | 1 | (.89, .97) | (44, 56) | | |
| | | $v_d(AE)$ | 2 | (.89, .96) | (35, 40) | (.95, .97) | (46, 48) |
| CAECGD2 | | Λ_{AE} | 1 | (.98, .99) | (91, 93) | (.98, .99) | (88, 91) |
| | | $v_d(AE)$ | 2 | (.82, .92) | (3, 7) | (.86, .90) | (2, 8) |
| CAECGD3 | | Λ_{AE} | 1 | (.98, .99) | (90, 94) | (.94, .97) | (70, 77) |
| | | $v_d(AE)$ | 2 | (.83, .89) | (3, 6) | (.77, .87) | (6, 16) |
| CAECAD1 | | $\sigma_z(F)$ | 1 | (-.94, -.95) | (88, 94) | (-.95, -.96) | (90, 93) |
| | | $\sigma_z(E)$ | 2 | (-.78, -.86) | | (-.85, -.90) | |
| | 80 | σ_{z0} | 2 | -83 | 10 | | |
| CAECAD2 | | $\sigma_z(F)$ | 1 | (-.91, -.93) | (74, 89) | (-.81, -.93) | (69, 70) |
| | | h_m | 2 | -.76 | 9 | (-.69, -.84) | (8, 12) |
| | 80 | $\sigma_z(E)$ | 2 | -.67 | 89 * | | |
| CAECAD3 | | h_m | 1 | -.99 | (93, 96) | (-.98, -.99) | (83, 87) |
| | | $v_d(AE)$ | 2 | (-.74, -.83) | (1, 9) | (-.90, -.92) | (10, 16) |
| Note: *) The R ² - values are calculated for the total group of correlated $\sigma_z(S)$ - parameters. | | | | | | | |

Table 6. Most important parameters for uncertainties in Cs-137 activity concentrations: This table indicates the most important parameters (including ranking, range of PRCC - and R² - values) for the variability in consequences for different sample sizes (40, 80, 100) and different assumptions on distribution types

The reason for introducing the uniform distribution into the analysis is the simplicity of this distribution type.

If there is no detailed knowledge about the distribution behaviour and all parameter values seem to be equally likely then possibly this type of distribution may be appropriate. This special case is simulated by setting all parameter distributions to be uniform.

- For IODCGD1 there is a dominant influence of dry deposition which has a PRCC value of .98. From 73 % to 83 % of the uncertainty in the consequence variable can

⁷ Due to the fact, that PRCC and R²-values depend on the sample size, there exist three (not necessarily different) values, building the 'ranges' (column 5,6 and 7,8) in Table 5 and Table 6 of the corresponding PRCC and R²-values.

be explained by variations of the model parameter dry deposition, $v_d(IO)$, depending on the number of UFOMOD runs (40, 80, 100) and the distribution type.

- For the consequence variable IODCGD2 the dry deposition parameter, $v_d(IO)$, again seems to be the most important parameter responsible for variation in the consequence with sample size $n=40,80,100$. The second most important parameter is the mixing height parameter, h_m . The PRCC values are from -.67 to -.68. Only from 8 % to 22 % of the uncertainty in the consequence variable can be explained by variation in the mixing layer.

However, if all parameters are uniformly distributed, for the sample size $n=40,100$ (marked as 40U, 100U) the most important parameter becomes h_m . On the other hand, looking at the corresponding tables in Appendix C, the actual PRCC values differ only very slightly for $n=40$ (PRCC = -.80) and $n=40U$ (PRCC = -.82), respectively.

- For small distances from the source deposition and in some case vertical σ -parameters are dominant. For the second (4.9 km) and third (27 km) distance category the mixing layer becomes important.

Similar results with physically interpretable dependences are obtained for the air near ground.

Studying Table 5, Table 6 and Appendix C shows *for this particular modelling situation*:

- The most important parameters are stable in their rankings regardless of the underlying distributions (different and *predefined by experts* or all parameters uniformly distributed, respectively).
- For the less important parameters the PRCC values vary for the predefined or uniform distribution situation, but even then in most cases the ranking is the same.

To avoid improper generalizations:

For the atmospheric dispersion and deposition submodule of the new UFOMOD code system the distribution effect was not as important as it was expected.

It is supposed that a different model with different characteristics, behaviour and complexity may be more sensitive to changes in distributions.

But to verify this a lot of time consuming and possibly expensive computer runs for different distribution sets would be necessary.

4. Summary

The restriction to pure model parameters and revised parameter variations in the atmospheric dispersion and deposition submodule of the new UFOMOD code system led to significantly smaller uncertainty bands than for the old UFOMOD / B3 code. Variations in deposition velocities and the mixing height parameters play the most important role for uncertainties in the activity concentration values.

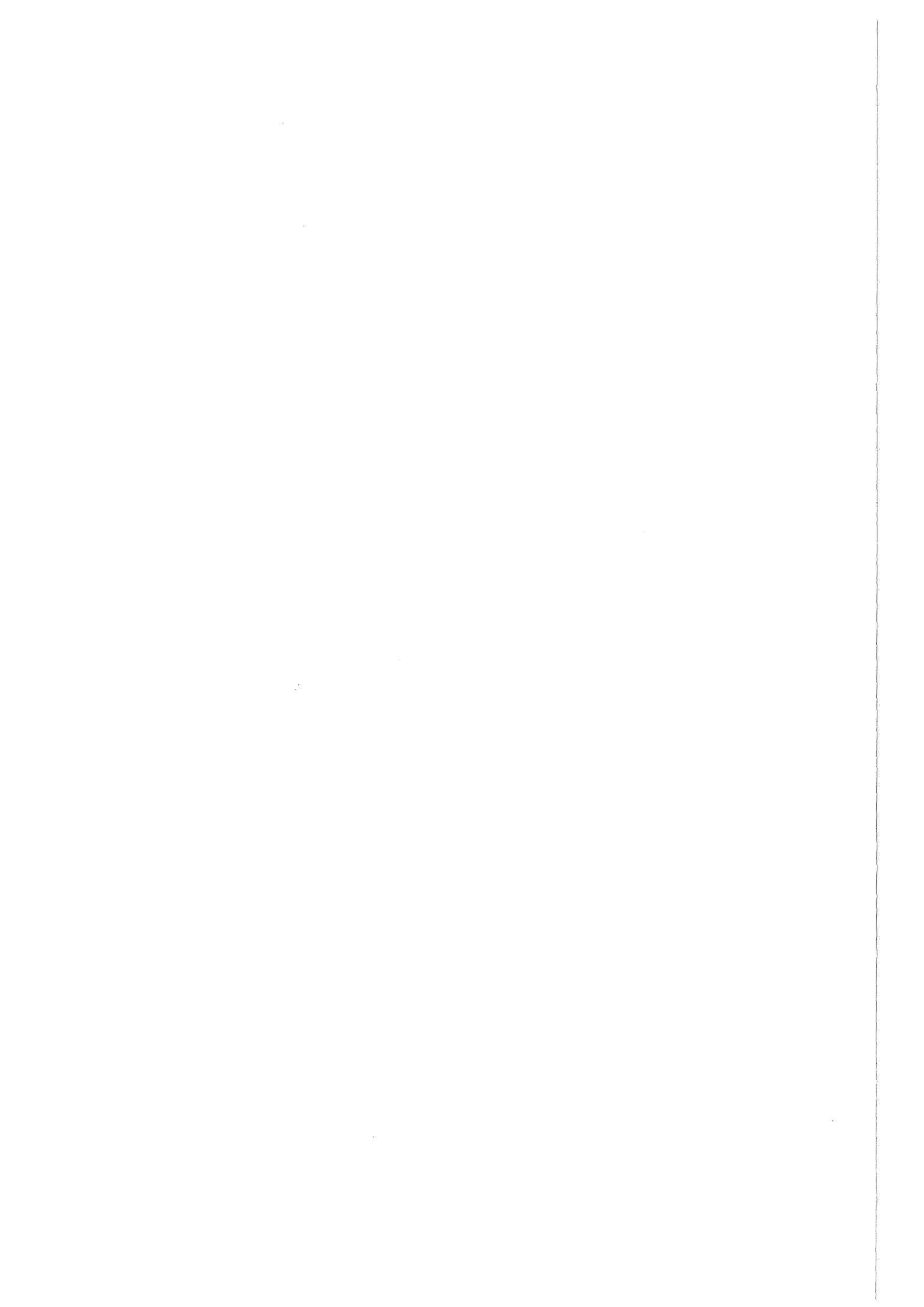
Special emphasis was put in this report on some helpful theoretical investigations concerning:

- different types of correlation between uncertain model parameters,
- test procedures to find out the most sensitive uncertain model parameters,
- the percentage contribution of parameter variations to changes in consequence values.

For mentioned submodule of the new program system UFOMOD the most important parameters are stable in their rankings regardless of the underlying distributions (*predefined by experts* or all uniformly distributed, respectively). For the less important parameters the PRCC values vary for *predefined* or *uniform* distributions, respectively. But even then in most cases the ranking is the same.

Increasing sample sizes ($n = 80,100$) led to more precision in sensitivity calculations, i.e. led to a *decrease* of the critical 'white noise level' (garbage level) of absolute PRCC values.

R^2 -values (coefficients of determination) in conjunction with the corresponding PRCC values visualized the percentage contribution of each uncertain model parameter (or groups of uncertain parameters) to uncertainty in consequences. This was important, because a large absolute PRCC value was not in every case an indication for a considerable amount of responsibility for uncertainty in consequences.



More Details, Figures and Tables

Appendix A.1 gives some details concerning the LHS-procedure and the IMAN / CON-OVER-method for inducing rank correlations.

Appendix A.2 illustrates some results concerning the correlation between variate values and ranks.

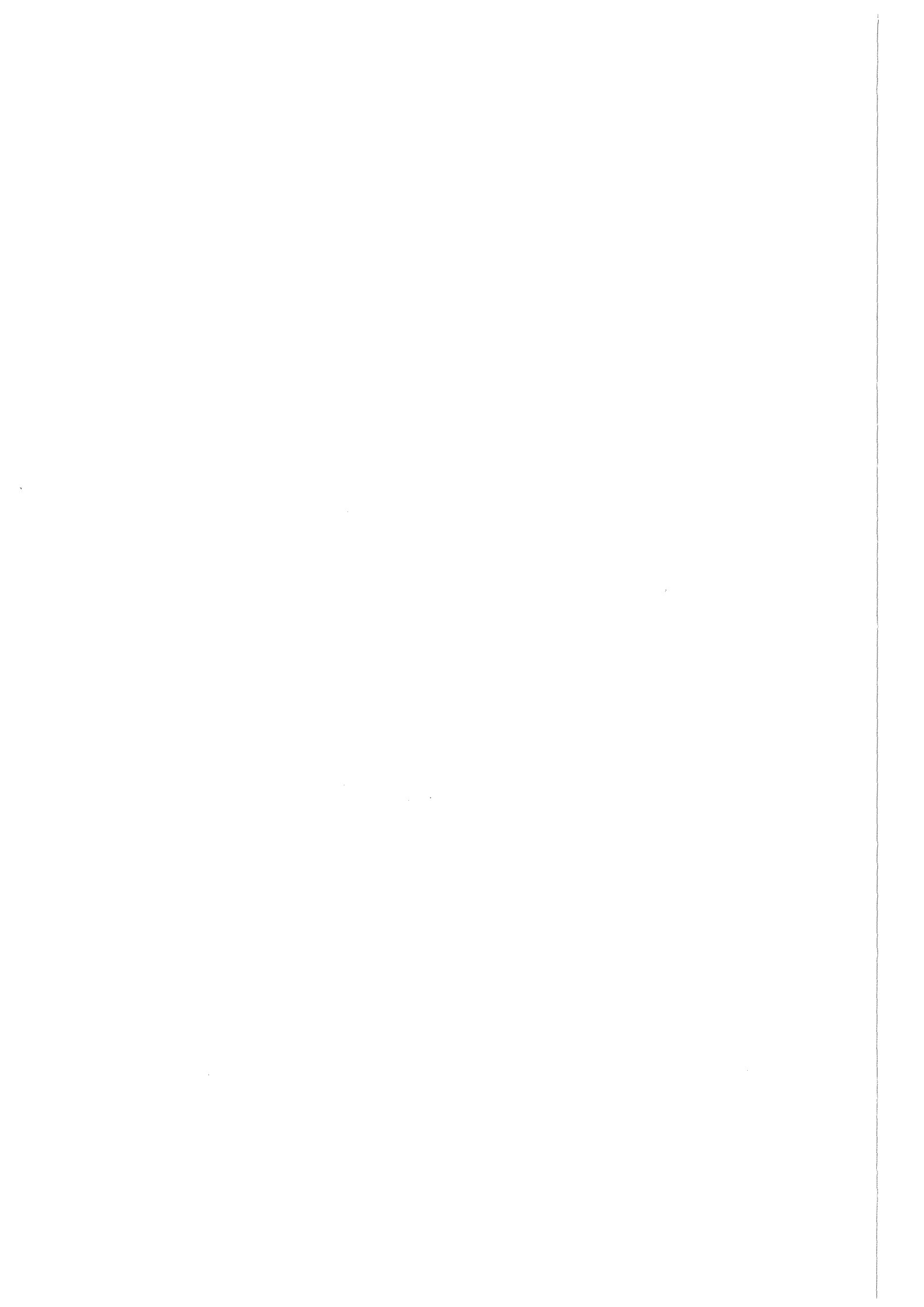
Appendix A.3 describes the partial (rank) correlation coefficient and some significance testing problems.

Appendix A.4 gives some remarks concerning the coefficient of determination, R^2 .

Appendices B and C comprise a detailed set of figures for uncertainty and sensitivity analyses, respectively. If necessary some legends to understand abbreviations are added. The figures and tables are given in the following sequence:

- **UNCERTAINTY** (CCFDs and confidence curves)
 - **Activity concentrations** (Iodine, Aerosols) on ground surface and in the air near ground at three distance intervals

- **SENSITIVITY** (Tables of PRCC values)
 - Comparison of concentration runs for $n=40, 80, 100$



Appendix A. Some Mathematical Details

A.1 The IMAN/CONOVER - procedure

This paragraph follows some results presented in [30].

Let C be a (k,k) -rank correlation matrix supplied by the user. Use the Cholesky-factorization to find a lower triangular matrix P , such that $PP' = C$. For a sample of size n form a (n,k) -matrix R , whose k columns are unique random mixes of van der Waerden scores (see [24]):

$$\{\Phi^{-1}(i/(n+1)), i = 1, \dots, n\} ,$$

where Φ^{-1} is the inverse of the standardized normal distribution.

Let T represent the correlation matrix of R . Use the Cholesky-factorization on T to find a lower triangular matrix Q , such that $QQ' = T$. Next, one wishes to find a matrix S , such that $STS' = C$ or $SQQ'S' = C$, respectively, for which one solution is $S = PQ^{-1}$. The matrix $R^* = RS$ has a correlation matrix exactly equal to the original correlation matrix C . This comes out by the following

Theorem (see [63])

Let $X = (X_1, X_2, \dots, X_k)$ be a k -dimensional random vector with expectation $(\mu_1, \mu_2, \dots, \mu_k)$ and correlation matrix K . For a linear transformation $Y = PX$ we have: The correlation matrix of vector Y is $D = PKP'$.

□

If $X_i = R_i$ are the columns of the matrix R of van-der-Waerden scores, $T = QQ'$ is the correlation matrix of R and $P = (Q^{-1})'$, then

$$D = Q^{-1}T(Q^{-1})' = I$$

That is, the column vectors $R_i(Q^{-1})'$ have the correlation matrix I . By Wilks' theorem, the column vectors of $R^* = R(Q^{-1})'P'$ have the correlation matrix $C = PP'$.

Finally, it only remains to generate the (n,k) -matrix of model parameters values, according to any desired method or distribution, as if the k random model parameters were independent of each other. Then the values of the parameter in each column are arranged so, that they have the same (rank) order as the corresponding column in the matrix R^* . Therefore $M \sim C$, i.e. the sample rank correlation matrix of the model parameter vectors, C , will be

the same as the sample rank correlation matrix, say M , of R^x . This is formally explained in [40] or [44].

A.2 Correlation between variate values and ranks

The rank-order statistics for a random sample are any set of constants which indicate the order of the observations. The actual magnitude of any observation is used only in the determination of its relative position in the sample array and thereafter ignored in any analysis based on rank - order statistics. Thus any statistical procedures based on rank - order statistics depend only on the relative magnitudes of the observations.

If the rank- order statistics for a random sample X_1, X_2, \dots, X_N are denoted by

$$\mathbf{R}(X_1), \mathbf{R}(X_2), \dots, \mathbf{R}(X_N)$$

where \mathbf{R} is any function such that $\mathbf{R}(X_i) \leq \mathbf{R}(X_j)$ whenever $X_i \leq X_j$. As with order statistics, rank - order statistics are invariant under monotone transformations, i.e. if $\mathbf{R}(X_i) \leq \mathbf{R}(X_j)$ then $\mathbf{R}[F(X_i)] \leq \mathbf{R}[F(X_j)]$ in addition to $F[\mathbf{R}(X_i)] \leq F[\mathbf{R}(X_j)]$ where F is any nondecreasing function.

A functional definition of the rank of any x_i in a set of different observations is provided by

$$\mathbf{R}(X_i) = \sum_{j=1}^N S(x_i - x_j) = 1 + \sum_{i \neq j} S(x_i - x_j) \quad [17]$$

where

$$S(u) = \begin{cases} 0 & \text{if } u < 0 \\ 1 & \text{if } u \geq 0 \end{cases} \quad [18]$$

When actual measurements are impossible or not feasible to obtain, but relative positions can be determined, rank - order statistics make full use of all the available information. However, if the fundamental data consist of variate⁸ values, how much information is lost by using the data only to determine relative magnitudes? One approach to a judgment concerning the potential loss of efficiency is to determine the correlation between variate values and their ranks.

⁸ In this paper statistical variable is called variate value, or variate.

If the correlation is high, we would feel intuitively more justified in the replacement of actual values by ranks for the purpose of analysis. The hope is that inference procedures based on ranks alone will lead to conclusions which seldom differ from a corresponding inference based on actual values.

The correlation coefficient between two random variables X and Y is

$$r = \frac{E(XY) - E(X)E(Y)}{\sigma_X \sigma_Y} \quad [19]$$

Assume that for a continuous population F_X we would like to determine the correlation between the random variable X and its rank $R(X)$. Theoretically, a random variable from an infinite population cannot have a rank, since values on continuous scale cannot be ordered. However, an observation X_i , of a random sample of size N from this population, does have a rank $R(X_i)$ as defined in Eq. [18]. The distribution of X_i is the same as the distribution of X and $R(X_i)$ and X_i are identically distributed though not independent.

Therefore, it is reasonable to define the population correlation coefficient between variate and their rank values as the correlation between X_i and $Y_i = R(X_i)$, or,

$$r_{V,R} = \frac{E(X_i Y_i) - E(X)E(Y_i)}{\sigma_X \sigma_{Y_i}} \quad [20]$$

Following [61], Eq. [20] can be rewritten as

$$r_{V,R}[X, R(X)] = \sqrt{\frac{12(N-1)}{N+1}} \frac{\left\{ E[F_X(X)] - \frac{1}{2} E(X) \right\}}{\sigma_X} \quad [21]$$

and⁹

$$\begin{aligned} r &= \lim_{N \rightarrow \infty} r_{V,R}[X, R(X)] \\ &= \frac{2\sqrt{3}}{\sigma_X} \left\{ E[X F_X(X)] - \frac{1}{2} E(X) \right\} \end{aligned} \quad [22]$$

Let's define

$$K = E[X F_X(X)] - \frac{1}{2} E(X) \quad [23]$$

Then Eq. [22] becomes

$$r = \lim_{N \rightarrow \infty} r_{V,R}[X, R(X)] = \frac{2\sqrt{3} K}{\sigma_X} \quad [24]$$

⁹ Treatment of ties in rank tests is not considered.

If $f(x)$ represents density probability function, then

$$F(x) = \int_{-\infty}^x f(t) dt \quad [25]$$

and

$$E(x) = \int_{-\infty}^{\infty} tf(t) dt \quad [26]$$

Substituting Eq. [25] and Eq. [26] in Eq. [23]

$$\begin{aligned} K &= \int_{-\infty}^{\infty} xf(x) dx \int_{-\infty}^x tf(t) dt - \frac{1}{2} \int_{-\infty}^{\infty} xf(x) dx \\ &= \frac{1}{2} \int_{-\infty}^{\infty} x(2F(x) - 1) dx \end{aligned} \quad [27]$$

Defining (see [61]),

$$\begin{aligned} \Delta &= 4 \int_{-\infty}^{\infty} x \left[2F(x) - \frac{1}{2} \right] dF(x) \\ &= 2 \int_{-\infty}^{\infty} x(2F(x) - 1) dF(x) \end{aligned} \quad [28]$$

we have

$$\Delta = 4K \quad [29]$$

and

$$\begin{aligned} \Delta &= 2xF^2(x) \Big|_{-\infty}^{\infty} - 2 \int_{-\infty}^{\infty} F^2(x) dx \\ &\quad - 2xF(x) \Big|_{-\infty}^{\infty} + 2 \int_{-\infty}^{\infty} F(x) dx \\ &= 2 \int_{-\infty}^{\infty} F(x)(1 - F(x)) dx \end{aligned} \quad [30]$$

Eq. [30] can be rewritten as

$$\begin{aligned} \Delta &= 2 \int_{-\infty}^{\infty} \int_{-\infty}^x dF(y) [1 - F(x)] dx \\ &= 2 \int_{-\infty}^{\infty} \int_{-\infty}^x (x - y) dF(y) dF(x) \end{aligned} \quad [31]$$

To calculate parameter r from Eq. [24] , it is necessary to calculate the standard deviation and mean of the each distribution.

A.2.1 Normal distribution

For this case

$$dF(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)dx \quad [32]$$

$$dF(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right)dy \quad [33]$$

and following [61]:

$$\Delta = \frac{2}{\sqrt{\pi}} \quad [34]$$

Then, substitution Eq. [34] in Eq. [24] , gives finally correlation between variate values and ranks for normal distribution

$$r = \sqrt{\frac{3}{\pi}} = .98 \quad [35]$$

A.2.2 Lognormal distribution

For this case is

$$dF(x) = \frac{1}{x\sqrt{2\pi}} \exp\left(-\frac{\log^2 x}{2}\right) dx \quad [36]$$

$$dF(y) = \frac{1}{y\sqrt{2\pi}} \exp\left(-\frac{\log^2 y}{2}\right) dy \quad [37]$$

From Eq. [31] is

$$\Delta = \int_0^\infty \int_0^x \exp\left(-\frac{\log^2 x + \log^2 y}{2}\right) \frac{x-y}{xy} dx dy \quad [38]$$

By transformations

$$\begin{aligned} \log x &= r \cos \theta \\ \log y &= r \sin \theta \end{aligned} \quad [39]$$

with the Jacobian

$$J = r \exp[r(\cos \theta + \sin \theta)] \quad [40]$$

the integral from Eq. [38] becomes

$$\begin{aligned} \Delta &= \frac{1}{\pi} \int_0^\infty \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \exp\left(-\frac{r^2}{2} + r \cos \theta\right) r dr d\theta \\ &\quad - \frac{1}{\pi} \int_0^\infty \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \exp\left(-\frac{r^2}{2} + r \sin \theta\right) r dr d\theta \end{aligned} \quad [41]$$

Further

$$\begin{aligned} \Delta &= \frac{1}{\pi} \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} I_1(\theta) \exp\left(\frac{\cos^2 \theta}{2}\right) d\theta \\ &\quad - \frac{1}{\pi} \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} I_2(\theta) \exp\left(\frac{\sin^2 \theta}{2}\right) d\theta \end{aligned} \quad [42]$$

where

$$I_1(\theta) = \int_0^\infty \exp\left[-\frac{(r - \cos \theta)^2}{2}\right] r dr \quad [43]$$

$$I_2(\theta) = \int_0^\infty \exp\left[-\frac{(r - \sin \theta)^2}{2}\right] r dr \quad [44]$$

and after some short transformations,

$$I_1(\theta) = \exp\left(-\frac{\cos^2 \theta}{2}\right) + \frac{\sqrt{2\pi}}{2} \cos \theta \left[1 + \operatorname{erf}\left(\cos \frac{\theta}{\sqrt{2}}\right)\right] \quad [45]$$

$$I_2(\theta) = \exp\left(-\frac{\sin^2 \theta}{2}\right) + \frac{\sqrt{2\pi}}{2} \sin \theta \left[1 + \operatorname{erf}\left(\sin \frac{\theta}{\sqrt{2}}\right)\right] \quad [46]$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-x^2) dx$$

So, substituting expressions Eq. [45] and Eq. [46] in Eq. [42]

$$\Delta = \frac{1}{\sqrt{2\pi}} \int \left[\cos \theta \exp\left(\frac{\cos^2 \theta}{2}\right) - \sin \theta \exp\left(\frac{\sin^2 \theta}{2}\right) \right] d\theta + \delta \quad [47]$$

we finally get

$$\Delta = 2\sqrt{e} \operatorname{erf}\left(\frac{1}{2}\right) + \delta \quad [48]$$

where

$$\begin{aligned} \delta &= \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \left[\cos \theta \exp\left(\frac{\cos^2 \theta}{2}\right) \operatorname{erf}\left(\frac{\cos \theta}{\sqrt{2}}\right) \right] d\theta \\ &\quad - \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \left[\sin \theta \exp\left(\frac{\sin^2 \theta}{2}\right) \operatorname{erf}\left(\frac{\sin \theta}{\sqrt{2}}\right) \right] d\theta \\ &\equiv \delta_1 + \delta_2 \end{aligned} \quad [49]$$

Substituting $\theta = \frac{\pi}{2} - \varphi$ in Eq. [49] for δ_1 , we get

$$\delta_1 = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin \varphi \exp\left(\frac{\sin^2 \theta}{2}\right) \operatorname{erf}\left(\frac{\sin \varphi}{\sqrt{2}}\right) d\varphi \quad [50]$$

Then, Eq. [49] becomes

$$\delta = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \Psi(\theta) d\theta - \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \Psi(\theta) d\theta \quad [51]$$

However, due to

$$\Psi(\theta_1) = \Psi(\theta + \pi) = \Psi(\theta)$$

we finally have

$$\delta = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \Psi(\theta) d\theta - \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \Psi(\theta) d\theta \equiv 0$$

Substituting this result in Eq. [24] , we finally get

$$r = \frac{\sqrt{3}}{2} \frac{\Delta}{\sigma} = \frac{\sqrt{3}}{2} 2\sqrt{e} \operatorname{erf}\left(\frac{1}{2}\right) \frac{1}{\sqrt{e(e-1)}}$$

$$r = .69 \quad [52]$$

This results shows the correlation between rank and raw values is larger for the normal than for the lognormal distribution. The correlation value for the normal distribution is about 40 % larger than for the lognormal case. This is in correspondence to the experiences gained during (the UFOMOD uncertainty analyses see for example Figure 4 and the corresponding curves for normal and lognormal distributions).

A.2.3 Triangular distribution

$$dF(x) = \begin{cases} \frac{2(x-a) dx}{(c-a)(b-a)} & a \leq x \leq b \\ \frac{2(c-x) dx}{(c-a)(c-b)} & b \leq x \leq c \end{cases} \quad [53]$$

From Eq. [30]

$$\Delta = \frac{2}{15} (b^2 - b + 2) \quad [54]$$

and

$$\sigma^2 = \frac{1}{18} (b^2 - b + 1) \quad [55]$$

Substituting Eq. [54] and Eq. [55] into Eq. [24] we get the desired parameter r for triangular distribution in the following form

$$r = r(b) = \frac{\sqrt{6}}{5} \frac{b^2 - b + 2}{\sqrt{b^2 - b + 1}} \quad [56]$$

From Eq. [56] it follows that the values for parameter r depend on the mode of the distribution (i. e. parameter b). The first derivation of function $r(b)$ gives

$$r'(b) = \frac{\sqrt{6}}{10} \frac{b(1-b)(1-2b)}{(b^2 - b + 1)^{\frac{3}{2}}} \quad [57]$$

From Eq. [57] it follows that function $r(b)$ has extreme values for $b = 0$, $b = 1$ and for $b = 0.5$. Further examinations of this function may show that for $b = 0$, and $b = 1$, function $r(b)$ has a minimum. For $b = 0.5$, function $r(b)$ has a maximum.

Substituting these values for parameter b in Eq. [56] we get

$$r(0) = 0.98, \quad r(1) = 0.98, \quad \text{and} \quad r\left(\frac{1}{2}\right) = 0.99 \quad [58]$$

In Figure 7 the graph of $C(b)$ is shown with respect to Eq. [56]

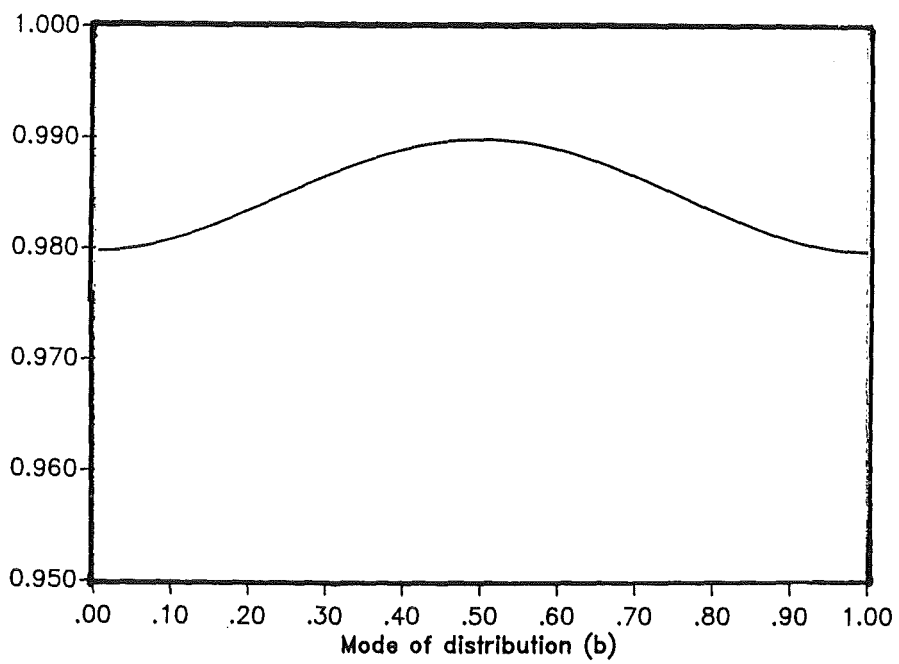


Figure 7. Correlation between variate values and ranks for triangular distribution

A.2.4 Logtriangular distribution

Let's now observe the probability density function of the standardized logtriangular distribution

$$dF(x) = \begin{cases} \frac{2 \log x dx}{x \log b} & 1 \leq x \leq b \\ \frac{2(1 - \log x) dx}{x(1 - \log b)} & b \leq x \leq e \end{cases} \quad [59]$$

After some algebra, Eq. [30] gives

$$\Delta(b) = 4 \frac{\log^3 b(2b - e - 1) + \log^2 b(-15b - 11e + 14)}{\log^2 b(1 - \log b)^2} + \frac{\log b(37b - 25) - 12b + 12}{\log^2 b(1 - \log b)^2} \quad [60]$$

and

$$\sigma^2(b) = \frac{\log^3 b(-e^2 + 1) + \log^2 b(-7e^2 + b^2 + 16e - 10)}{2 \log^2 b(1 - \log b)^2} + \frac{\log b(-b^2 + 16eb - 16b - 16e + 17) - 8b^2 + 16b - 8}{2 \log^2 b(1 - \log b)^2} \quad [61]$$

Substituting the last two expressions for Δ and $\sigma(b)$ we get

$$r(b) = \frac{\sqrt{3}}{2} \frac{\Delta(b)}{\sigma(b)} \quad [62]$$

The explicit analytic form of $r(b)$ and its first derivation would need too much space. Nevertheless, the graph of function defined by Eq. [62] is presented in Figure 8 .

It should be noticed that satisfactory results for function $r(b)$, presented in Figure 8 were achieved only with quadro precision.

Finally, in Figure 9 some available results for correlation between variate values and their ranks are presented. The assessment of parameter r for the Gamma distribution can be found in [61].

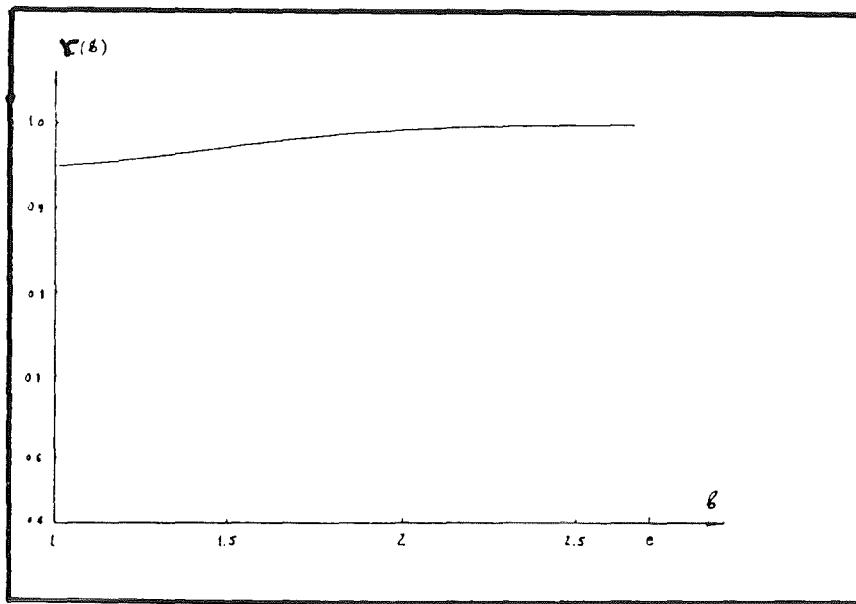


Figure 8. Correlation between variate values and ranks for logtriangular distribution

| DISTRIBUTION | CORRELATION BETWEEN RAW AND RANK VALUES | | |
|-------------------------------|---|------|------------|
| 1. Normal distribution | *) | 0.98 | see Eq. 35 |
| 2. Lognormal distribution | | 0.69 | see Eq. 52 |
| 3. Triangular distribution | | | see Eq. 58 |
| 4. Logtriangular distribution | | | see Eq. 62 |
| 5. Uniform distribution | *) | 1 | |
| 6. Gamma distribution | *) | | see Eq. 14 |
| *) see [61] | | | |

Figure 9. Correlation between variate values and ranks for some types of distributions

A.3 Partial correlation coefficients

A.3.1 Definition

This paragraph follows some results presented in [25].

Sensitivity analysis in conjunction with Latin hypercube sampling is based on the construction of regression models. The observations

$$(X_{1i}, X_{2i}, \dots, X_{ki}, Y_i) \quad i = 1, \dots, n$$

are used to construct models of the form

$$Y_{est} = b_0 + \sum_q b_q Z_q$$

subject to the constraint that

$$\Sigma(Y - Y_{est})^2$$

be minimized. b_0 , B_q are constants and each Z_q is a function of X_1, \dots, X_k .

An important property of least squares regression is that

$$\Sigma(Y - Y_m)^2 = \Sigma(Y - Y_{est})^2 + \Sigma(Y_{est} - Y_m)^2$$

where Y_m is the mean of the Y -values.

The R^2 - value (**coefficient of determination**) for a regression falls between 0 and 1 and is defined by

$$R^2 = \frac{\Sigma(Y_{est} - Y_m)^2}{\Sigma(Y - Y_m)^2}$$

The closeness of an R^2 - value to 1 provides an indication of how successful the regression model is in accounting for the variation in Y .

For a regression model of the form

$$Y_{est} = b_0 + b_1 Z$$

with an R^2 - value of r^2 , the number $sign(b_1)|r|$ is called the correlation coefficient between Y and Z , where $sign(b_1) = 1$ if $b_1 \geq 1$, and $sign(b_1) = -1$ if $b_1 < 1$. This number provides a measure of linear relationship between these two variables. When more than one independent variable is under consideration, *partial correlation coefficients* are used to provide a measure of the linear relationships between Y and the individual independent variables. The *partial correlation coefficient* between Y and an individual variable Z_p is obtained from the use of a sequence of regression models. The following two regression models are constructed:

$$Y'_{est} = a_0 + \sum_{q \neq p} a_q Z_q \quad \text{and}$$

$$Z'_{est} = c_0 + \sum_{q \neq p} c_q Z_q .$$

Then, the results of the two preceding regressions are used to define the new variables $Y - Y'_{est}$ and $Z_p - Z'_p$. By definition, the **partial correlation coefficient** between Y and Z_p is

the simple correlation coefficient between $Y - Y'_{est}$ and $Z_p - Z'_p$. Therefore, the partial correlation coefficient provides a measure of the linear relationship between Y and Z_p with the linear effects of the other variables removed.

Example:

Sometimes the apparent correlation between two variables may be due in part to the direct influence on both of the other variables: Y and X_1 are correlated, but are both influenced by a variable X_2 . The influence of X_2 on Y and X_1 must be removed. *Simple linear regression* of Y resp. X_1 on X_2 gives:

$$Y' = \beta_0 + \beta_1 X_2, \quad X'_1 = \gamma_0 + \gamma_1 X_2$$

Define new variables $(Y - Y')$ and $(X_1 - X'_1)$. The simple correlation (based on the Pearson product moment correlation) between the 'residuals' $(Y - Y')$ and $(X_1 - X'_1)$ is called the **partial correlation coefficient between Y and X_1 , given X_2** (i.e., the linear influence of X_2 on both Y and X_1 removed), and is denoted by $r_{1Y.2}$:

$$r_{1Y.2} = \frac{r_{1Y} - r_{12}r_{Y2}}{\sqrt{(1 - r_{12}^2)(1 - r_{Y2}^2)}} \quad [63]$$

r_{1Y} , r_{12} , r_{Y2} are simple Pearson product moment correlations of the corresponding variables. For more details see [35], [25], [29], [37] and [56].

□

A.3.2 Significance tests

Following [11], the well-known Pearson product-moment correlation formula can be used to estimate Pearson's partial correlation coefficient. Spearman's rank correlation ρ has also been extended to measure partial rank correlation.

Partial correlation coefficients (PRCs) are correlation coefficients on conditional distributions. The distribution of the partial correlation coefficients depends on the multivariate distribution function of the underlying variables. Therefore PRCs may not be directly used as test statistics in nonparametric tests.

Starting from some well-known theorems, we may nevertheless do some approximative tests and analyses.

Step 1:

Find the distribution of the sampling correlation coefficient for random variables (X, Y) with bivariate normal distribution.

Theorem (Pitman's test): (see [41])

Let $u_i = (x_i, y_i)$ ($i=1, \dots, n$) be a random sample from a bivariate normal distribution with correlation r . Let r_s be the sample correlation coefficient (Pearson's product moment coefficient):

$$r_s = \frac{\sum_i (y_i - y_m)(x_i - x_m)}{\left[\sum_i (y_i - y_m)^2 \sum_i (x_i - x_m)^2 \right]^{\frac{1}{2}}} \quad [64]$$

Let $r = 0$ then

$$T_s = r_s \sqrt{\frac{(n-2)}{(1-r_s^2)}} \quad [65]$$

is distributed as Student' t with $(n-2)$ degrees of freedom.

□

Theorem: (see [42] or [53])

Let (z_1, \dots, z_k) be a random sample from a k -dimensional normal distribution and $r_{ij, u_1, \dots, u_p} = 0$ where r_{ij, u_1, \dots, u_p} is the partial correlation coefficient) of order p ($p=k-2$). u_1, \dots, u_p are $p=k-2$ numbers from $\{1, \dots, k\}$ which are different from i and j . That means the *partial* correlation between Z_i and Z_j is tested, say, while the indirect correlation due to Z_{u_1}, \dots, Z_{u_p} is eliminated. Let $r_{s; ij, u_1, \dots, u_p}$ be the sample partial correlation coefficient) of order p ($p=k-2$). Take n samples from the vector z , then

$$T_s = r_{s; ij, u_1, \dots, u_p} \sqrt{\frac{(n-2-p)}{(1-r_{s; ij, u_1, \dots, u_p}^2)}} \quad [66]$$

is distributed as Student' t with $(n-2-p)$ degrees of freedom.

□

Step 2:

Try to find adequate approximate formulas for non-normal situations.

Let $w_i = (u_i, v_i)$ ($i = 1, \dots, n$) be a random sample from a bivariate distribution with correlation r . Let r_s be the sample correlation coefficient. Transform the sample values (u_1, \dots, u_n) and (v_1, \dots, v_n) into their order statistics $(u_{(1)}, \dots, u_{(n)})$ and $(v_{(1)}, \dots, v_{(n)})$. Then do an *expected normal scores transformation*: Replace the order statistics of the (u, v) -variables by the expected value of the corresponding order statistics of standard normal variates (X, Y) . Then r_s transforms approximately to ψ_s :

$$r_s \sim \psi_s = \frac{\sum_i E(x_{(i)})E(y_{(i)})}{\sqrt{\sum_i E^2(x_{(i)}) \sum_i E^2(y_{(i)})}} \quad [67]$$

(This is clear from the hint that for a $N(0,1)$ -distributed variable X one has $\sum E(X_{(i)}) = 0$ because of $E(X_{(i)}) = -E(X_{(n-i+1)})$.)

ψ_s can be used for an expected normal scores test of the hypothesis that U and V are uncorrelated.

[11] explains the role of the expected normal scores as well defined numbers which replace the unpleasant behaviour connected with using the order statistics from normal variables themselves. The procedure is based only on the ranks of the observations and is therefore a *rank test*.

Fisher and Yates (see [6]) suggested the analogue to Pitman's test using the exact normal scores instead of the the original data and applied the usual parametric procedures to these expected normal scores as a nonparametric procedure.

Step 3:

Give the significance test procedure.

The procedure is as follows:

The 'null' hypothesis reads: "No *partial* correlation exists between Y (the consequence variable) and X_i (one of the uncertain model parameters)", while the indirect influence due to the other model parameters is eliminated.

Then, for a sample of size n , the partial sample rank correlation, $\rho_{s; Y_i, u_1, \dots, u_p}$, between Y and X_i has to be calculated. ρ_s is then compared with the quantiles of the distribution of the test statistic. The comparison is made at a certain prescribed level of significance, α .

The 'null' hypothesis of *no* correlation is rejected, if the correlation value ρ_s leads to $|\rho_s| \geq T_{\alpha/2,n}$, the **critical value**, where $T_{\alpha/2,n}$ is a quantile of the test statistic's distribution.

$$T_{\alpha/2,n} \sim \frac{t_{\alpha/2,n-k}}{\sqrt{n-k+t_{\alpha/2,n-k}^2}} \quad [68]$$

$t_{\alpha/2,n-k}$ is the $(1 - \alpha/2)$ -quantile of the t-distribution with $n-k$ degrees of freedom (compare [31] or [43]). Eq. [68] is easily derived from Eq. [66].

Example:

For $k = 20$ uncertain input model parameters and $\alpha = 0.05$ significance level, the partial rank correlation value (PRCC), ρ , is significant, if its absolute value is greater than 0.43 (40 runs), 0.25 (80 runs) or 0.16 (100 runs), respectively.

□

A.4 Remarks to R^2 - values

Here some additional hints for motivation of the *coefficient of determination*, R^2 , are given.

The *total variation* of the consequence variable, Y , is defined as $\Sigma(Y - Y_m)^2$, i.e. the sum of squares of the deviation of values of Y from the mean Y_m .

$$\Sigma(Y - Y_m)^2 = \Sigma(Y - Y_{est})^2 + \Sigma(Y_{est} - Y_m)^2$$

The first term on the right is called the *unexplained variation* while the second term is called the *explained variation* (by a regression model), so called because the deviations $(Y_{est} - Y_m)$ have a defined pattern while the deviations $(Y - Y_{est})$ behave in a random or unpredictable manner.

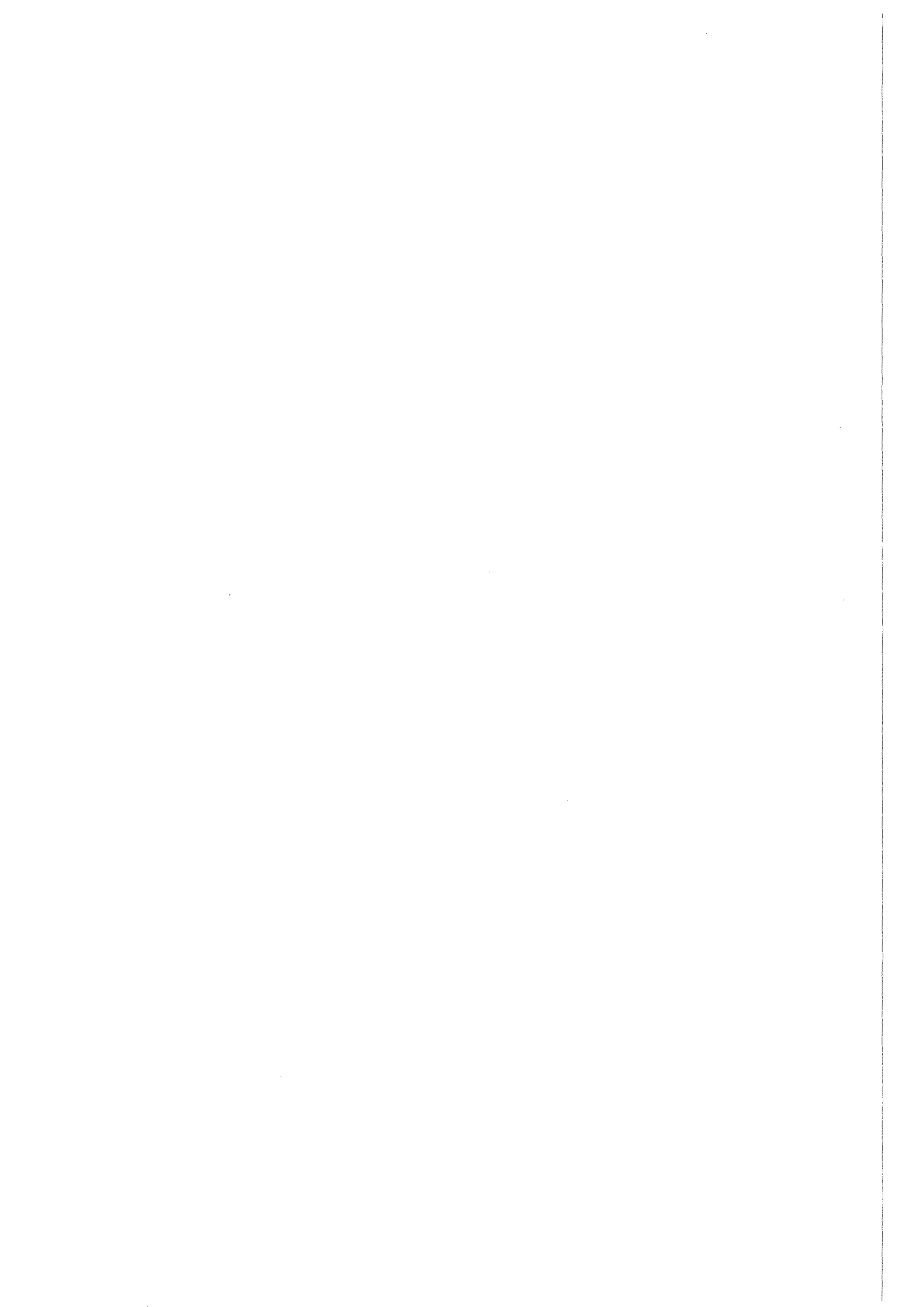
The ratio of explained variation to the total variation is called the *coefficient of determination*, R^2

$$R^2 = \frac{\Sigma(Y_{est} - Y_m)^2}{\Sigma(Y - Y_m)^2}$$

In this report all R^2 - values R^2_s are normalized by R^2_t .

$$R^2 = \left(\frac{R_s^2}{R_t^2} \right) \times 100,$$

where R_s^2 , R_t^2 are calculated by the SANDIA - PRCSRC-code (see [37]) and the R_t^2 - values are calculated with *all* (i.e. the complete set of) model parameters.



Appendix B. Uncertainty Analyses (Figures)

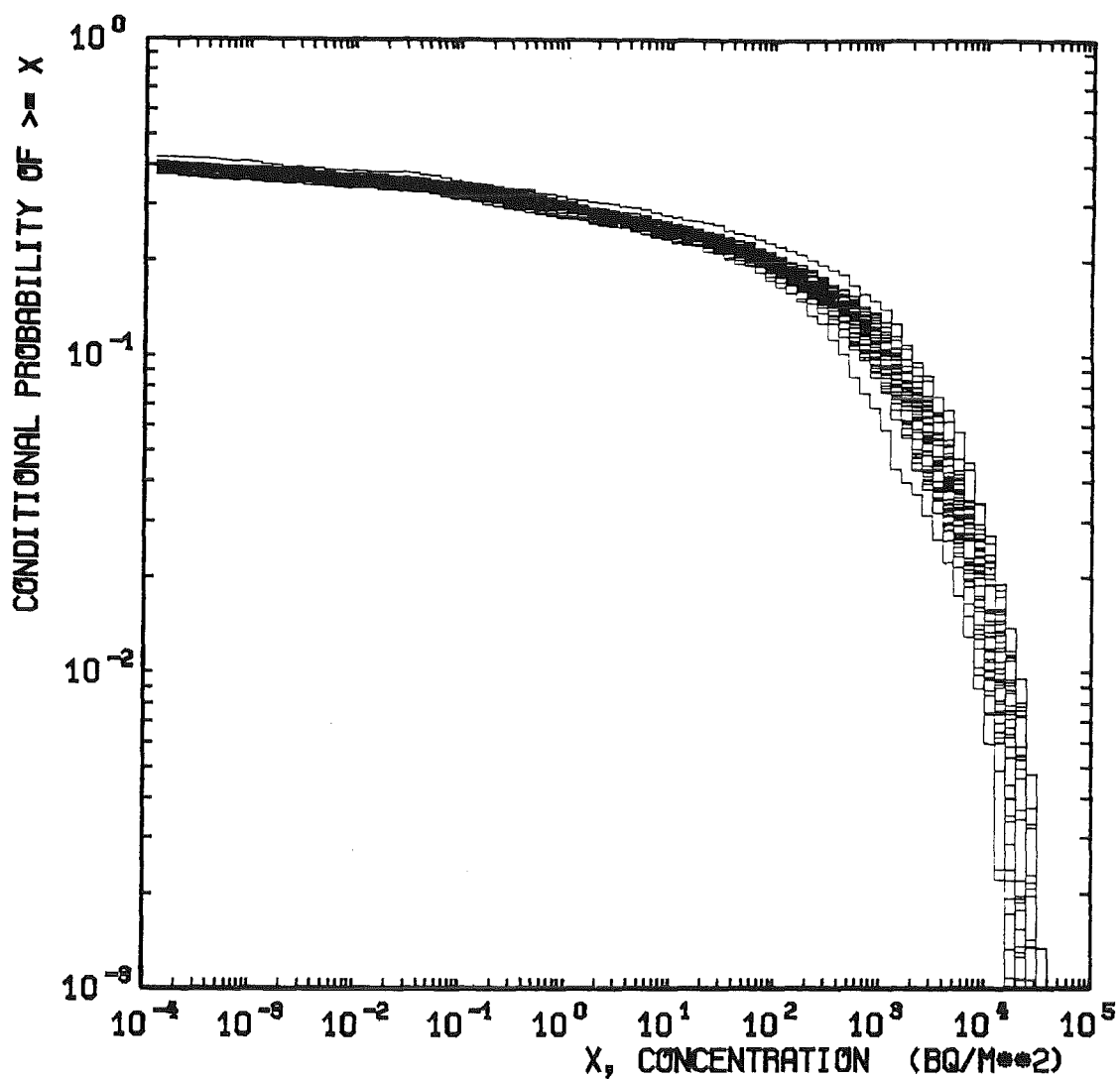
B.1 Activity concentrations

In this section CCFDs and the corresponding confidence curves are shown for activity concentrations (I-131, Cs-137) at three distance intervals on ground surface and in the air near ground.

Sequence of figures:

- Iodine
 - on ground surface
 - in the air near ground
- Aerosols
 - on ground surface
 - in the air near ground

UF0M0D Uncertainty Analysis (1987)

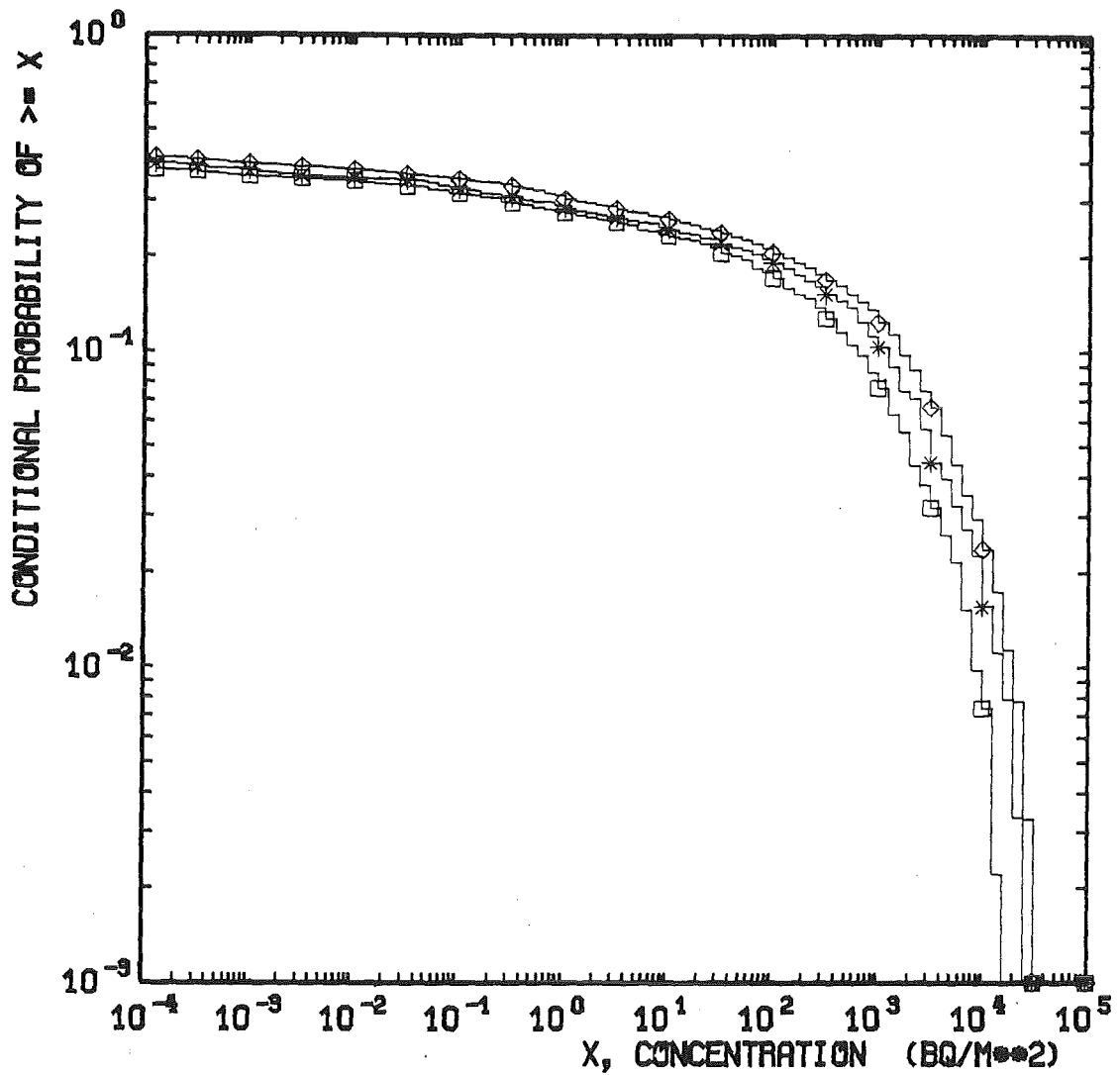


Concentration on ground surface
Nuclide: I - 131
Distance: 0.875 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFD) OF ACTIVITY CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMOD Uncertainty Analysis (1987)



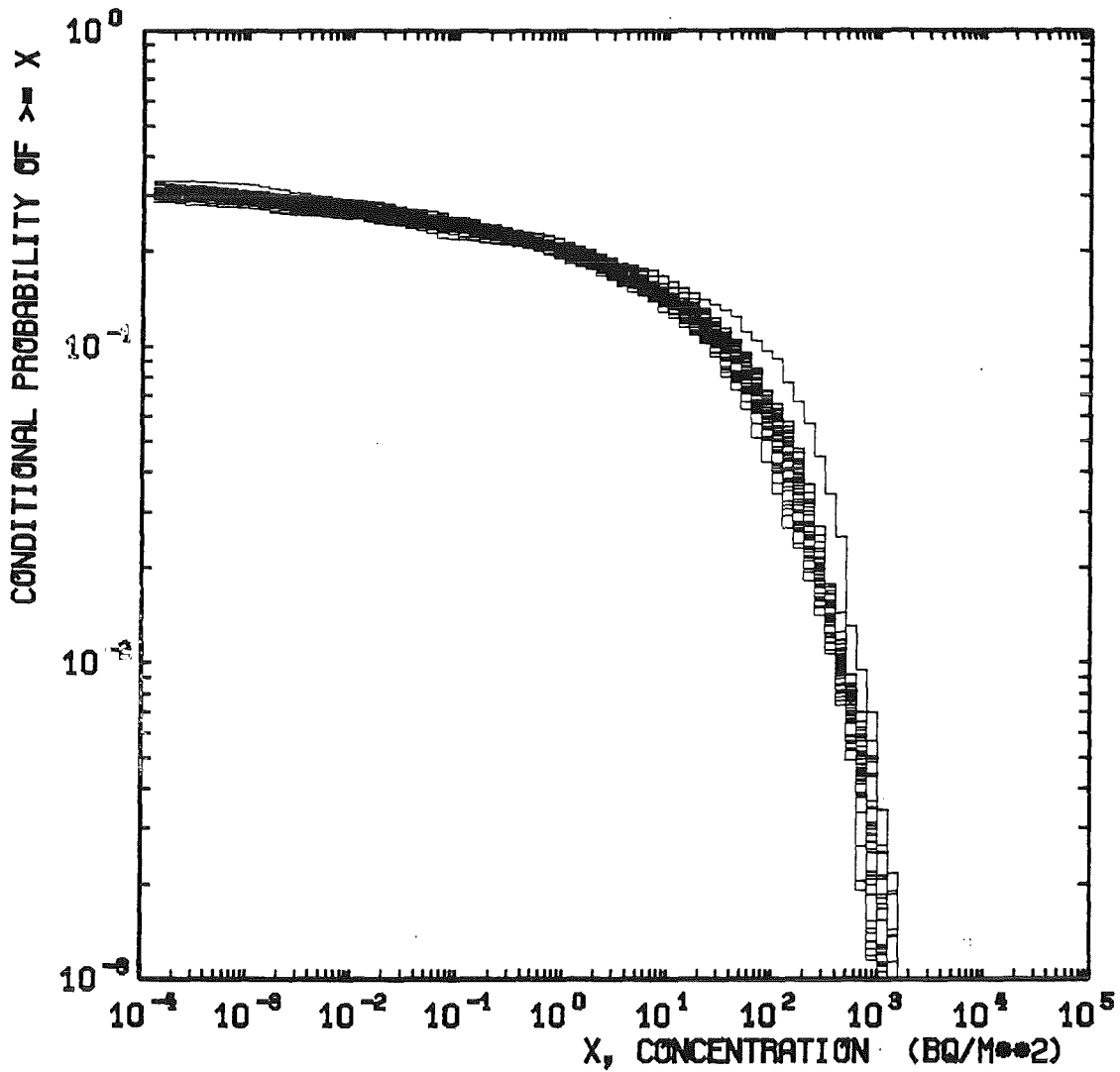
Concentration on ground surface
 Nuclide: I - 131
 Distance: 0.875 km

* : Ref.-Curve
 □ : 5% -Curve
 ◇ : 95% -Curve



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED)
 AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
 ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

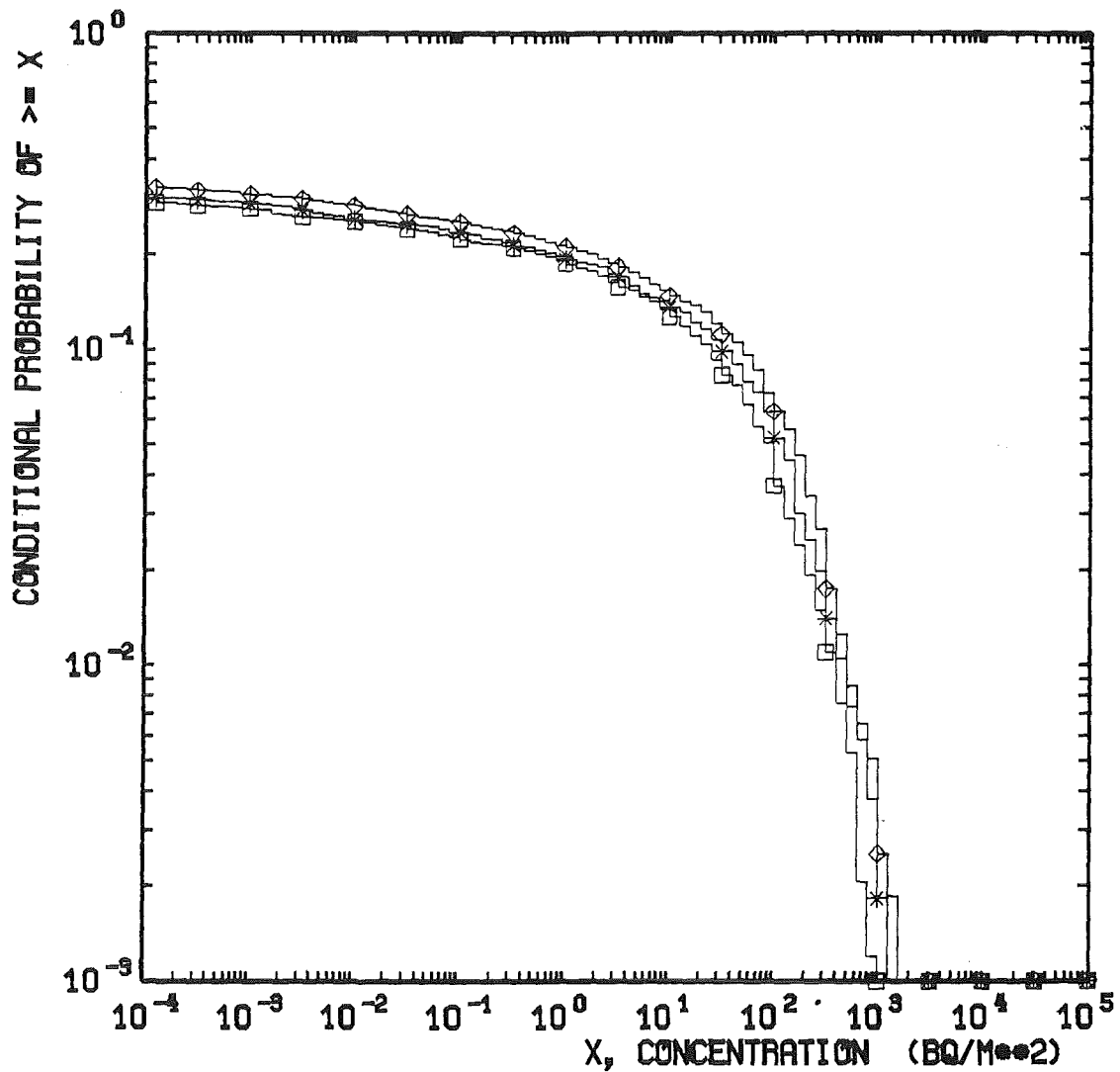


Concentration on ground surface
Nuclide: I - 131
Distance: 4.9 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFD) OF ACTIVITY CENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMOD Uncertainty Analysis (1987)



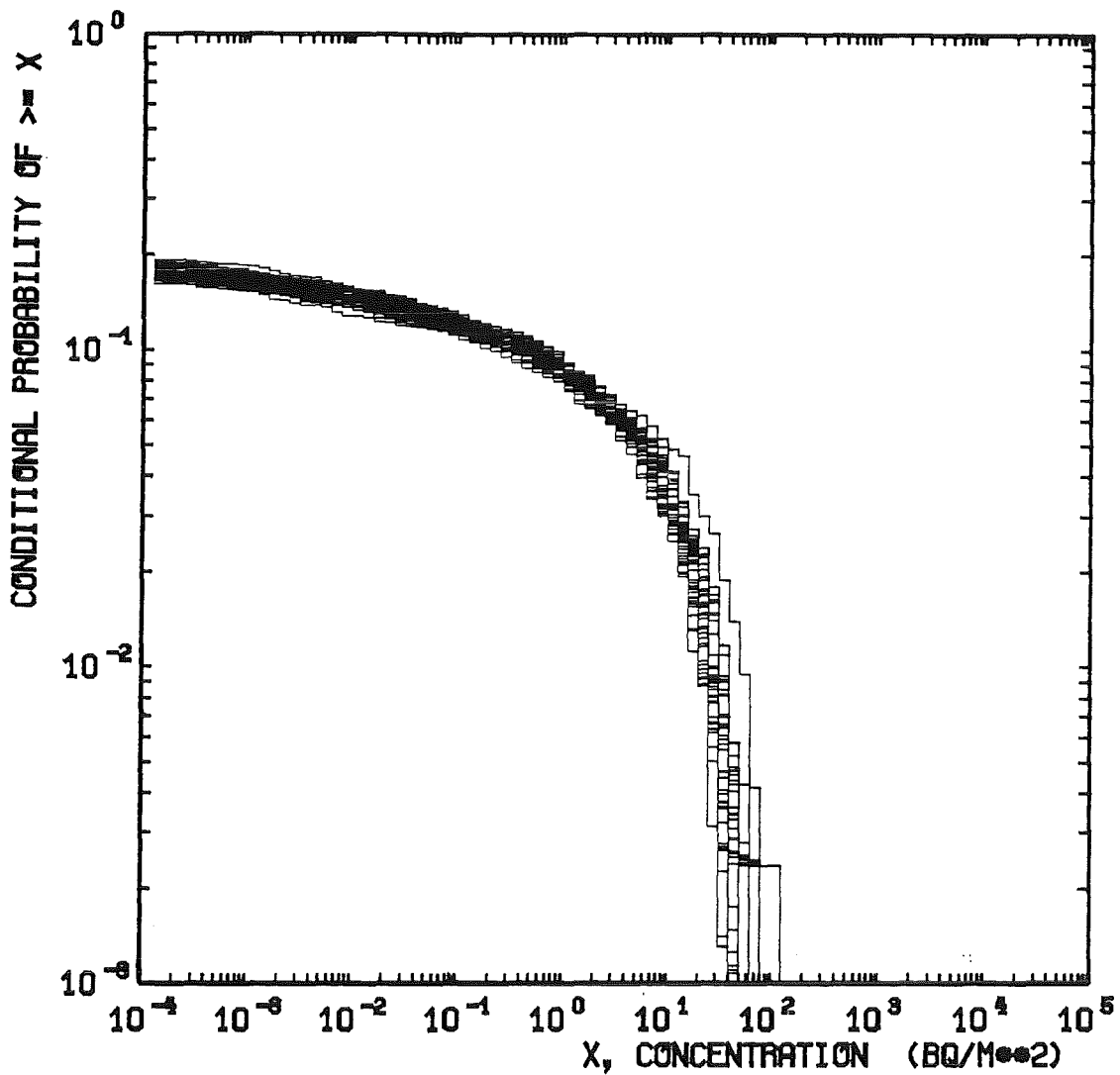
Concentration on ground surface
 Nuclide: I - 131
 Distance: 4.9 km

* : Ref.-Curve
 □ : 5% -Curve
 ◇ : 95% -Curve



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING RELEASER HAS OCCURRED)
 AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
 ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

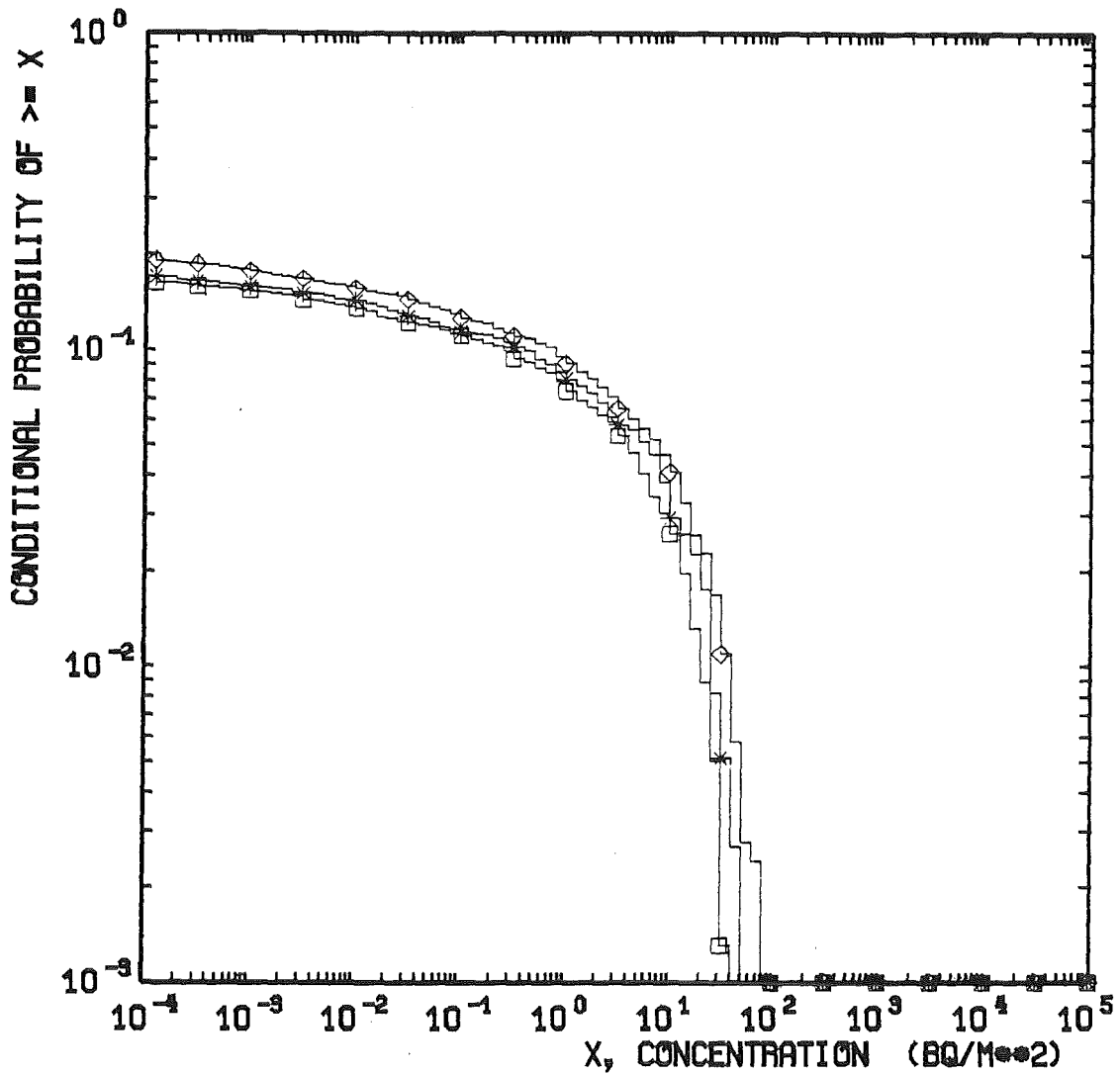


Concentration on ground surface
Nuclide: I - 131
Distance: 27 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFD) OF ACTIVITY CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMOD Uncertainty Analysis (1987)



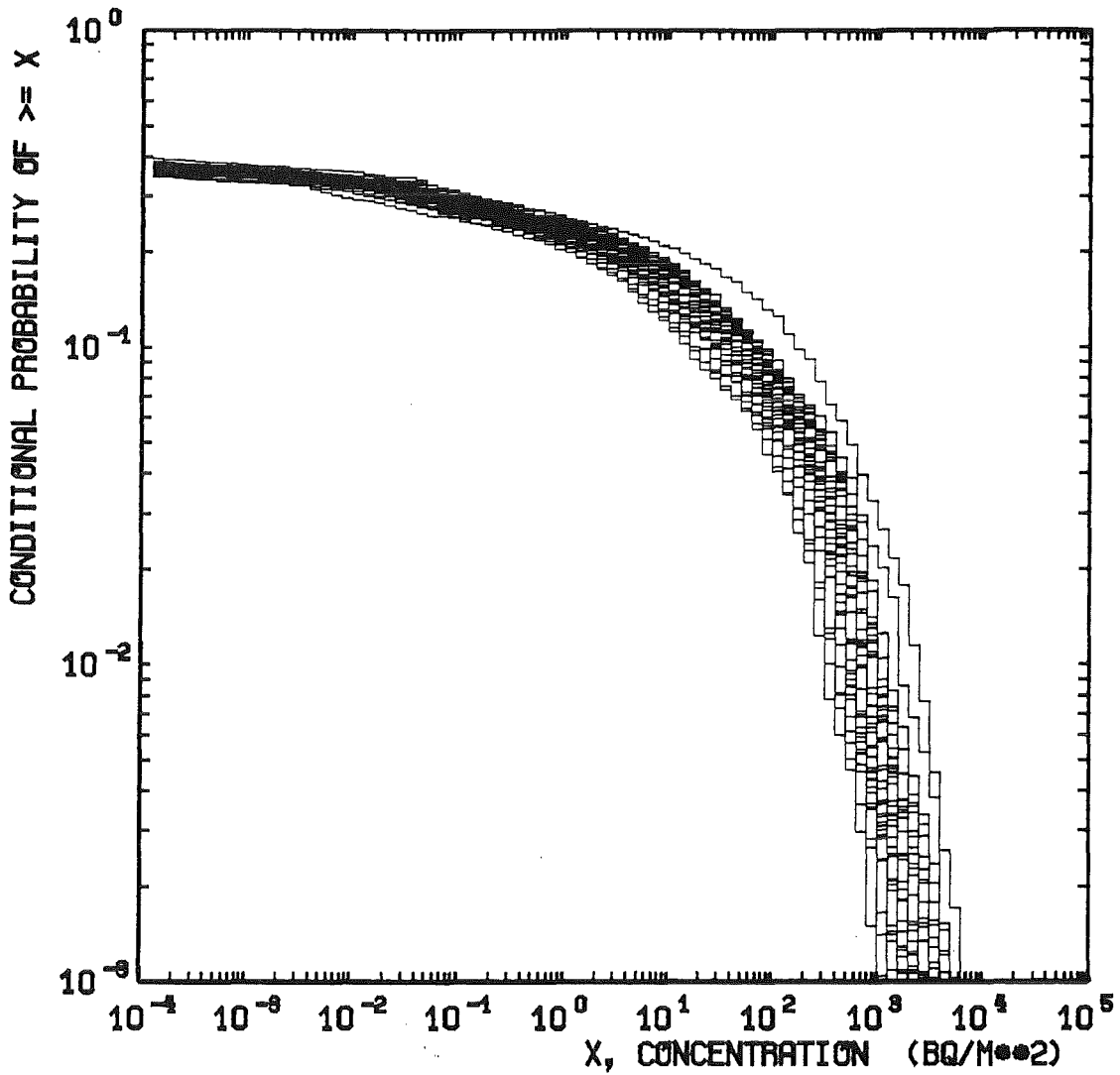
Concentration on ground surface
 Nuclide: I - 131
 Distance: 27 km

* : Ref.-Curve
 □ : 5% -Curve
 ◇ : 95% -Curve



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED)
 AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
 ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

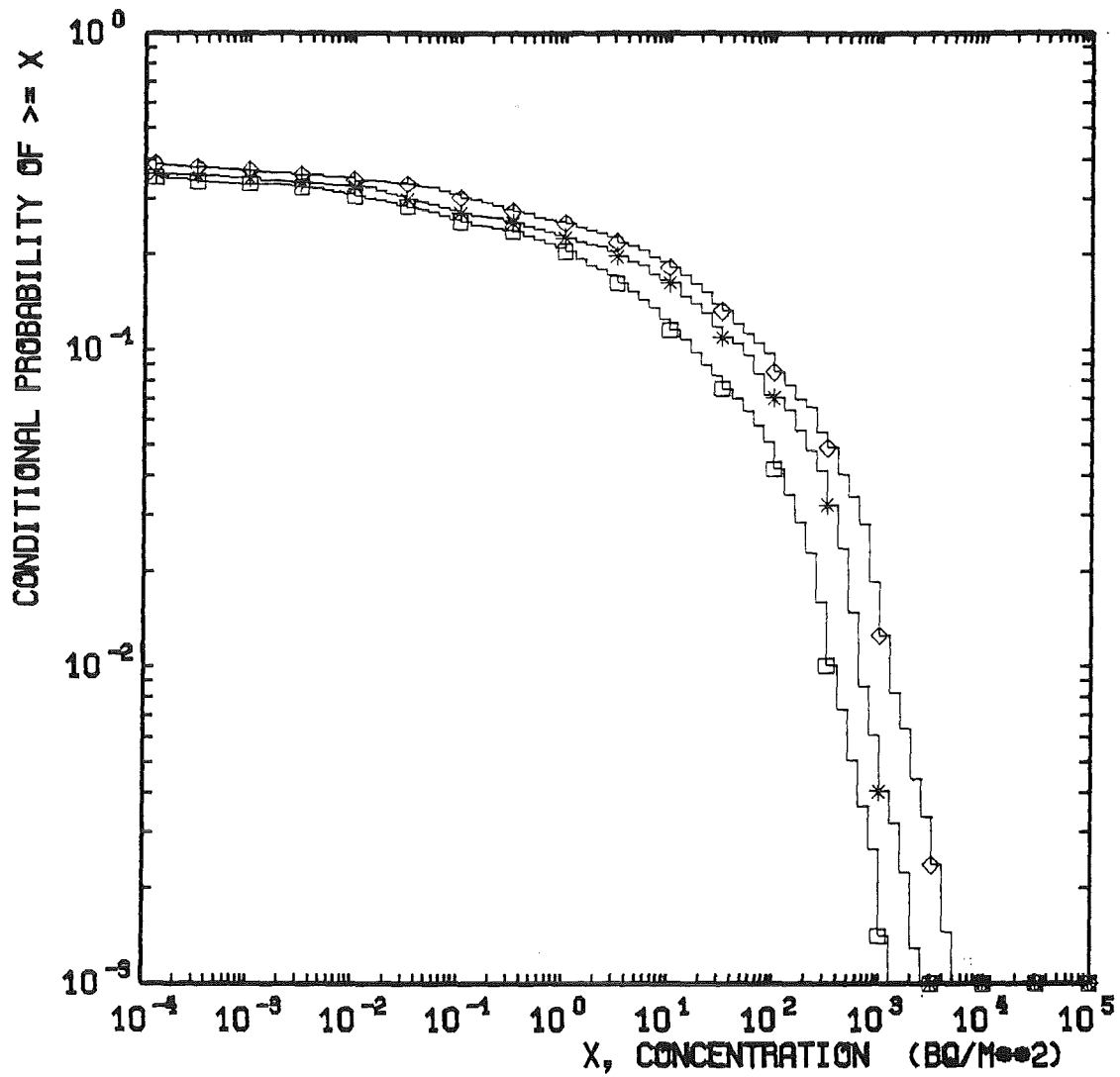


Concentration on ground surface
Nuclide: Cs - 137
Distance: 0.875 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFDs) OF ACTIVITY CENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMOD Uncertainty Analysis (1987)



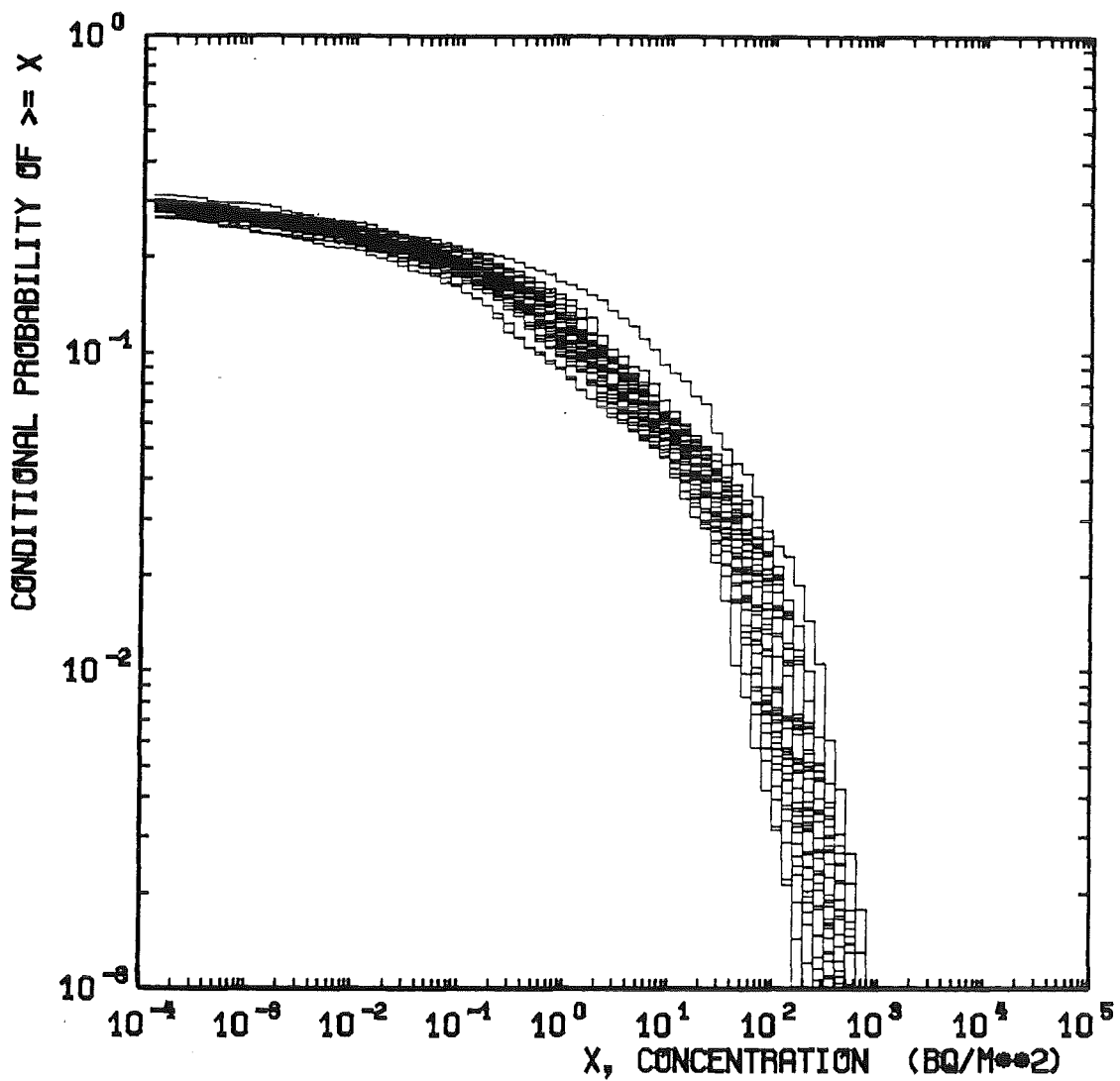
Concentration on ground surface
 Nuclide: Cs - 137
 Distance: 0.875 km

* : Ref.-Curve
 □ : 5% -Curve
 ◇ : 95% -Curve



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED)
 AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
 ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

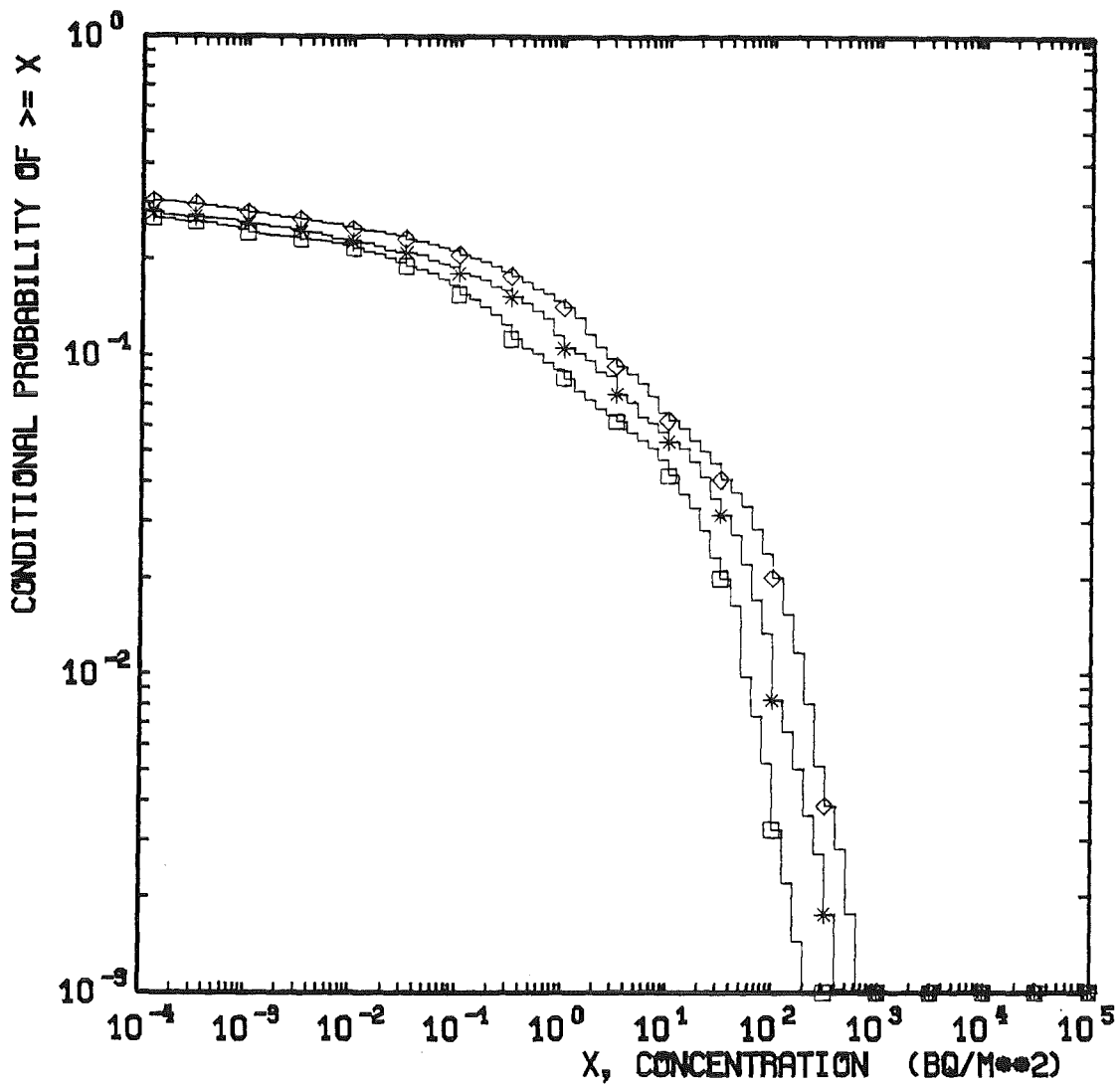


Concentration on ground surface
Nuclide: Cs - 137
Distance: 4.9 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFD) OF ACTIVITY CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMOD Uncertainty Analysis (1987)



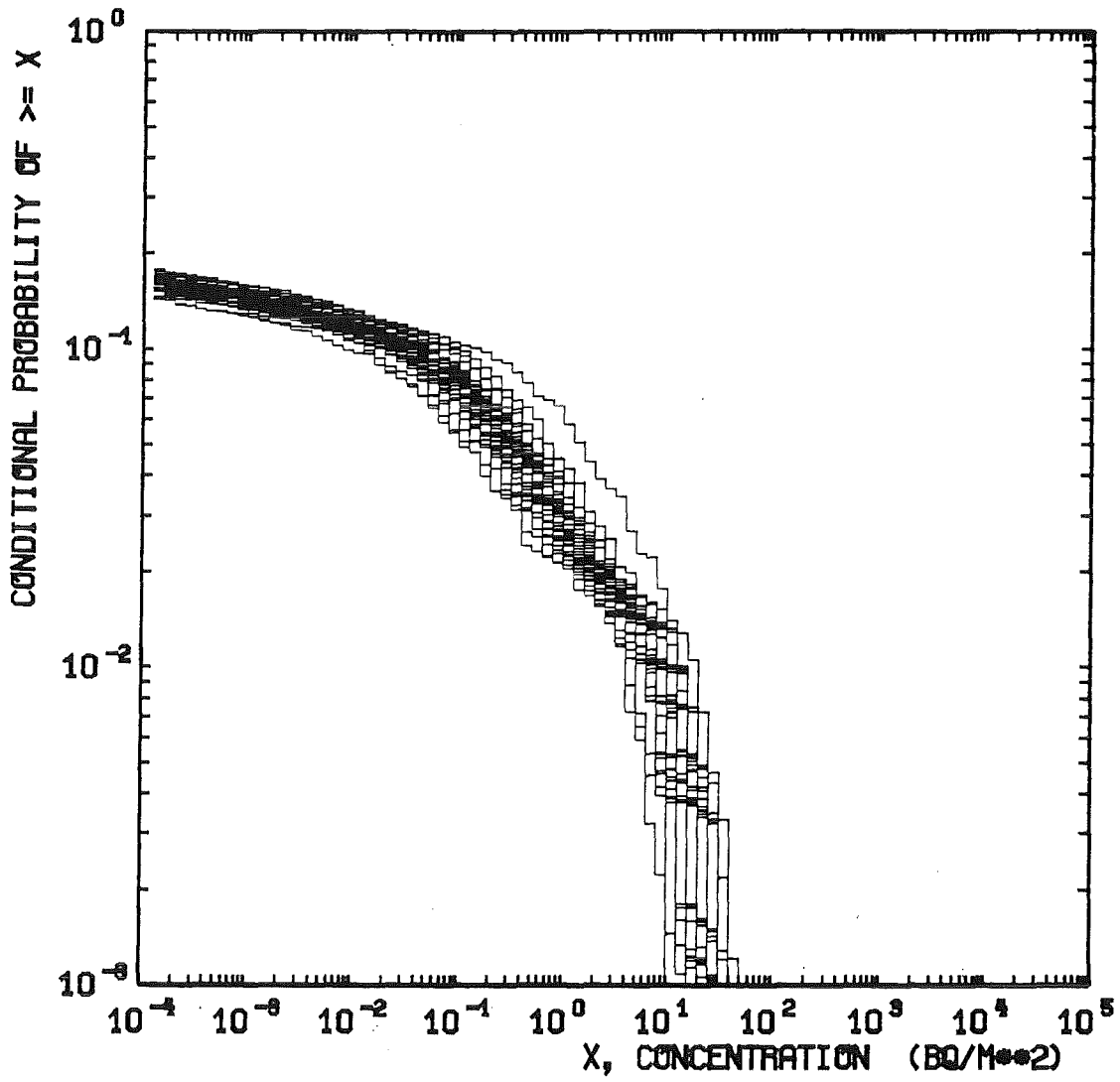
Concentration on ground surface
 Nuclide: Cs - 137
 Distance: 4.9 km

* : Ref.-Curve
 □ : 5% -Curve
 ◇ : 95% -Curve



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED)
 AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
 ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

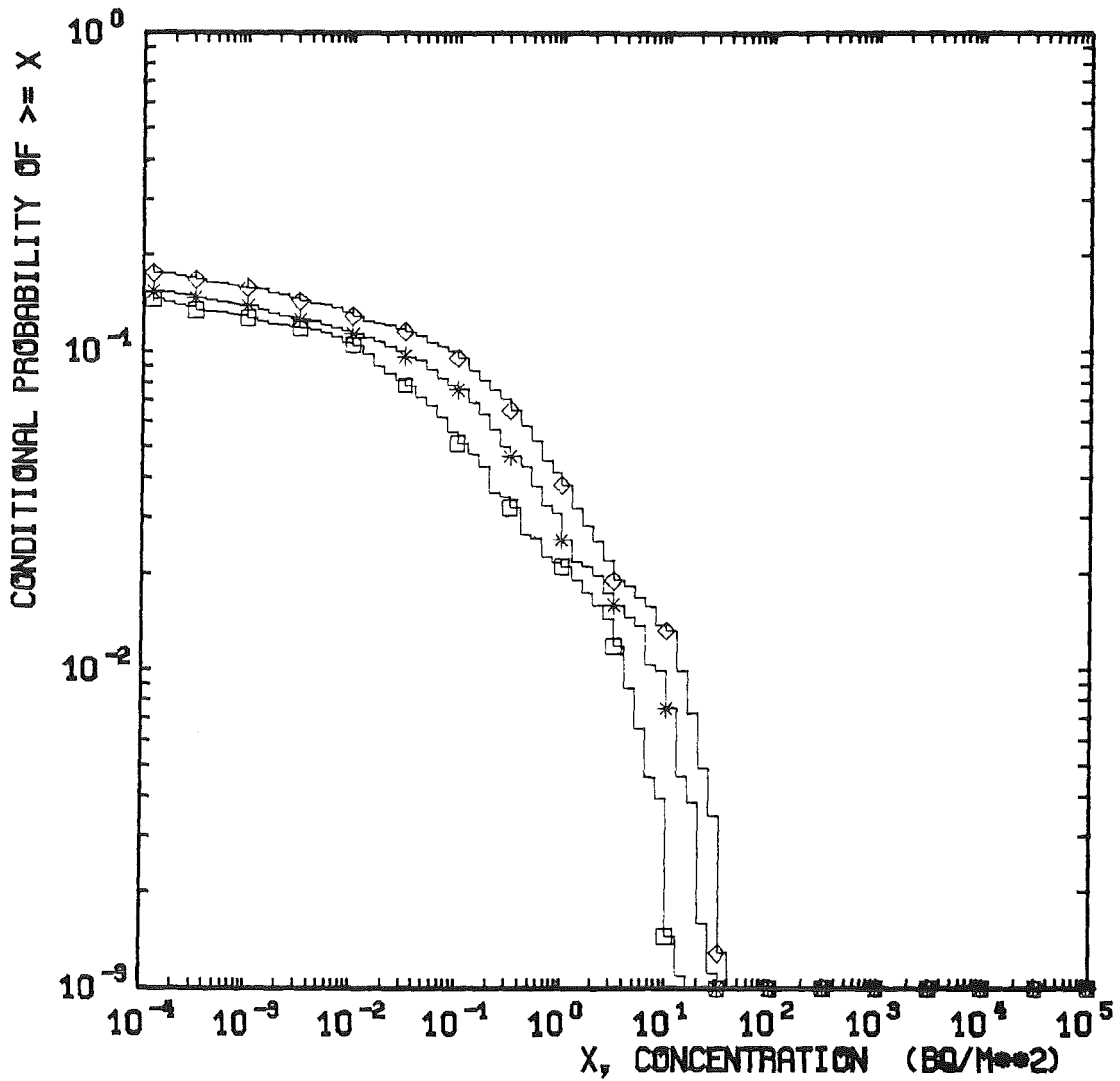


Concentration on ground surface
Nuclide: Cs - 137
Distance: 27 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFDs) OF ACTIVITY CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMOD Uncertainty Analysis (1987)



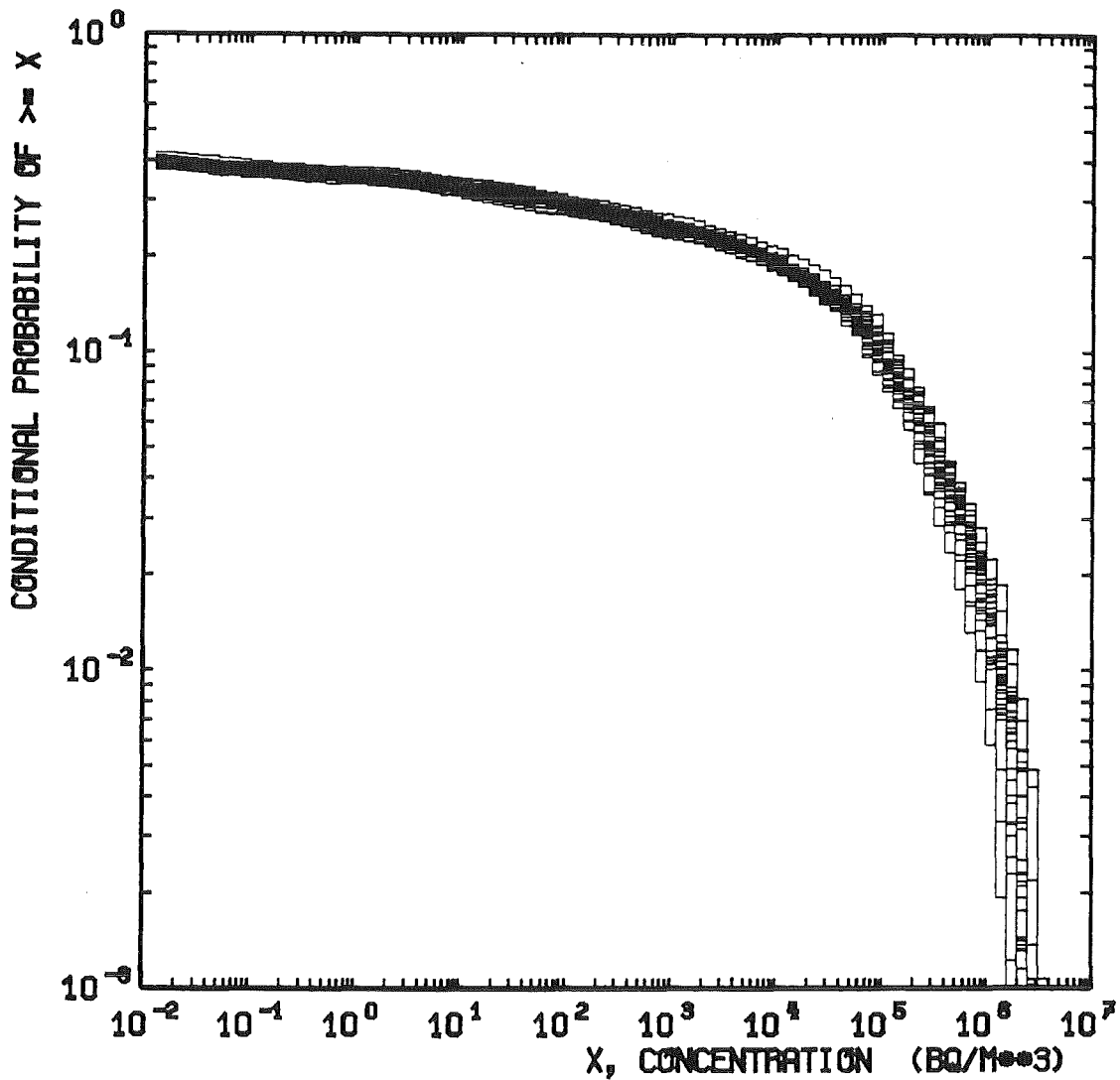
Concentration on ground surface
 Nuclide: Cs - 137
 Distance: 27 km

* : Ref.-Curve
 □ : 5% -Curve
 ◇ : 95% -Curve



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED)
 AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
 ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

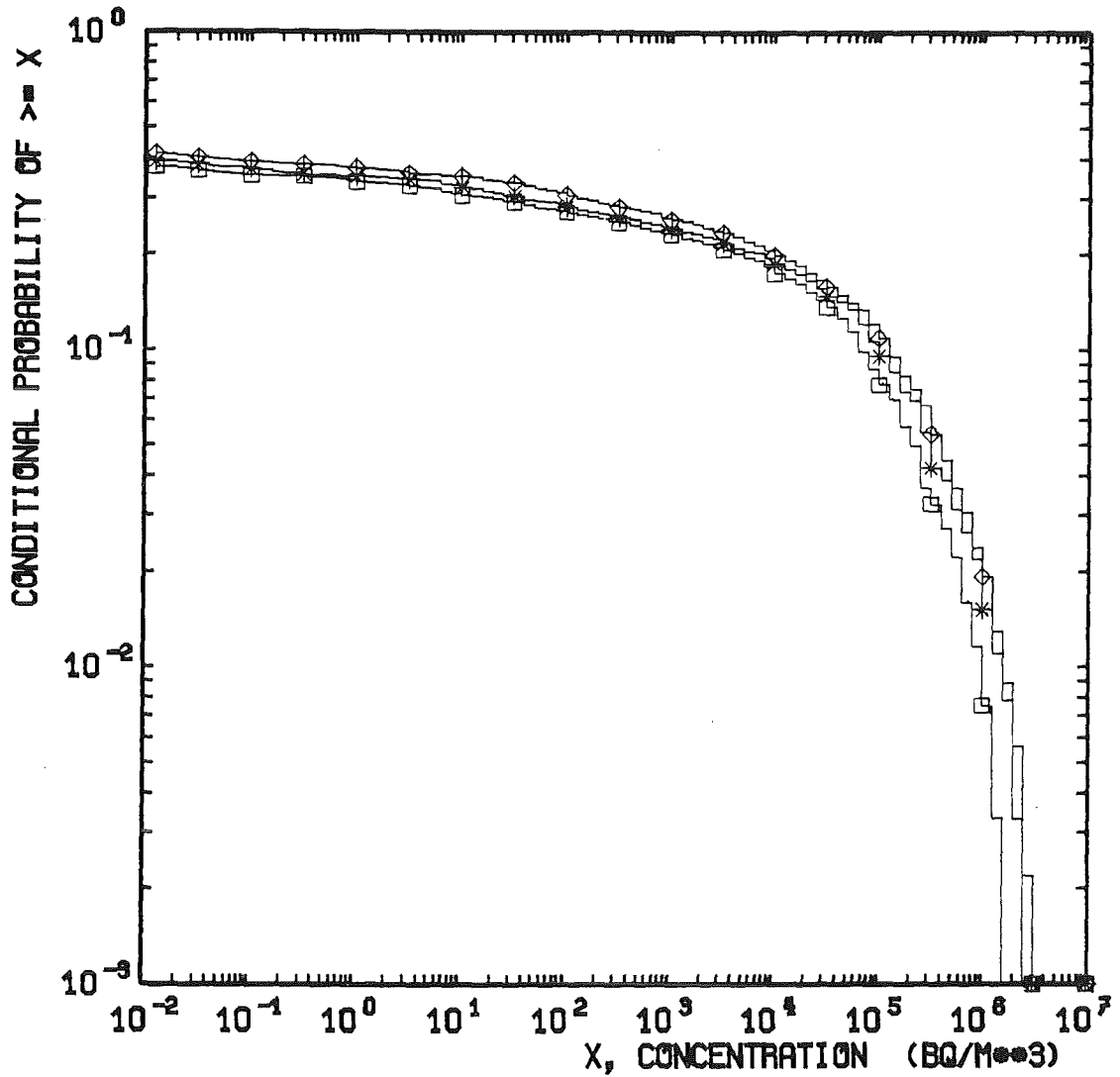


Concentration in the air near ground (1 m height)
Nuclide: I - 131
Distance: 0.875 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFD) OF ACTIVITY CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMOD Uncertainty Analysis (1987)

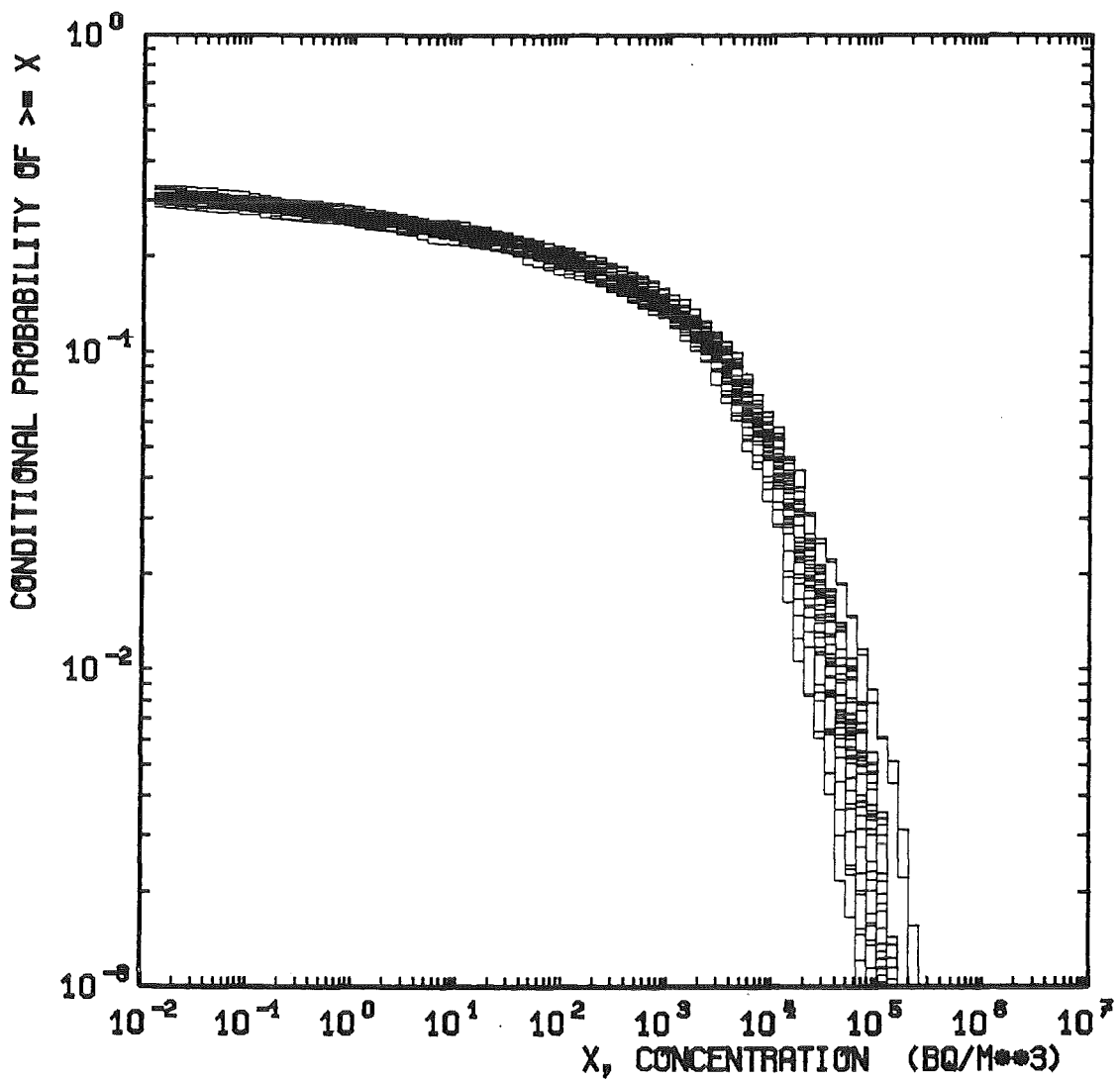


| | |
|--|----------------|
| Concentration in the air near ground (1 m height) | * : Ref.-Curve |
| Nuclide: I - 131 | □ : 5% -Curve |
| Distance: 0.875 km | ◇ : 95% -Curve |



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED) AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

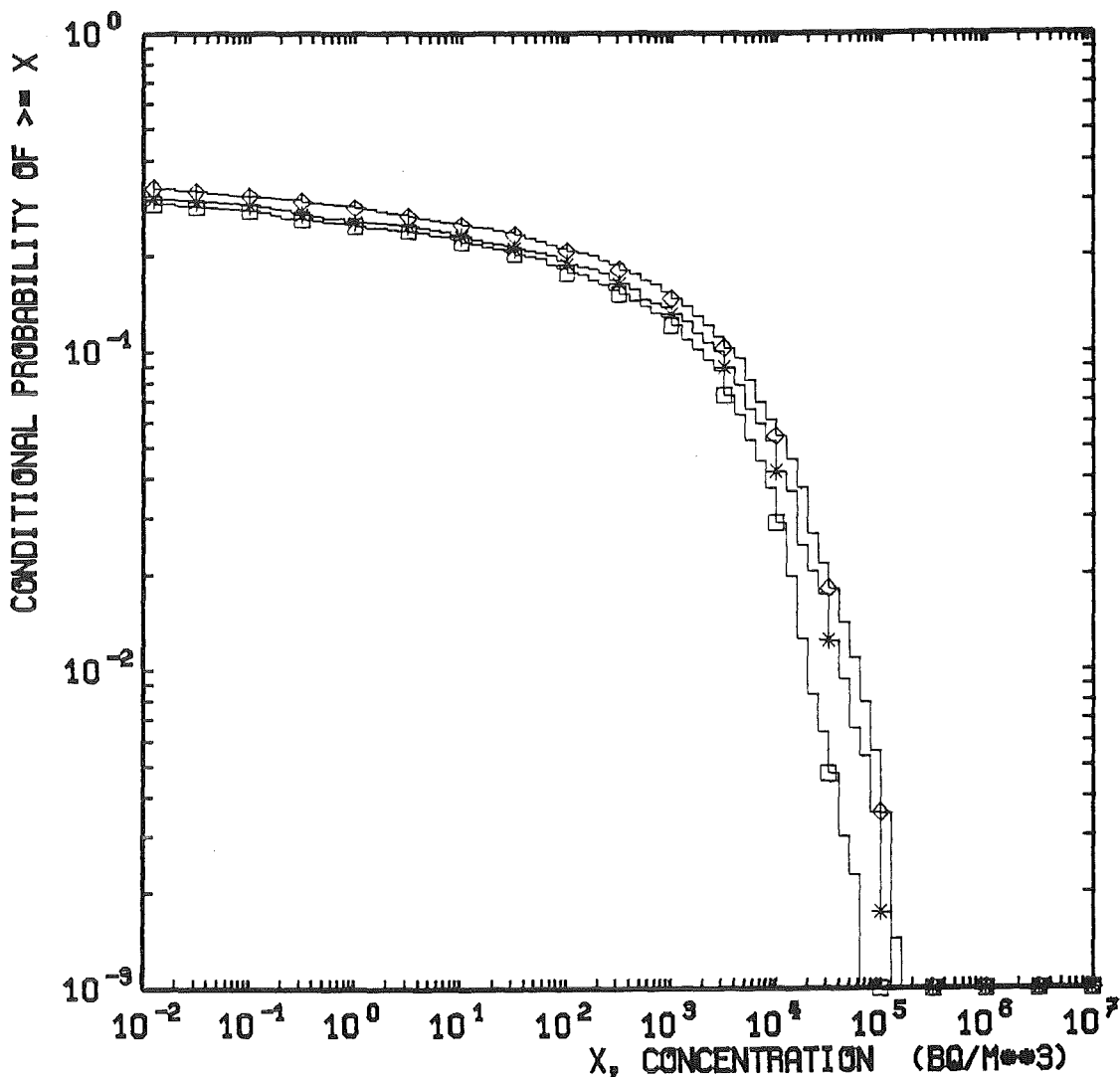


Concentration in the air near ground (1 m height)
Nuclide: I - 131
Distance: 4.9 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFD) OF ACTIVITY
CENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO
ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMØD Uncertainty Analysis (1987)

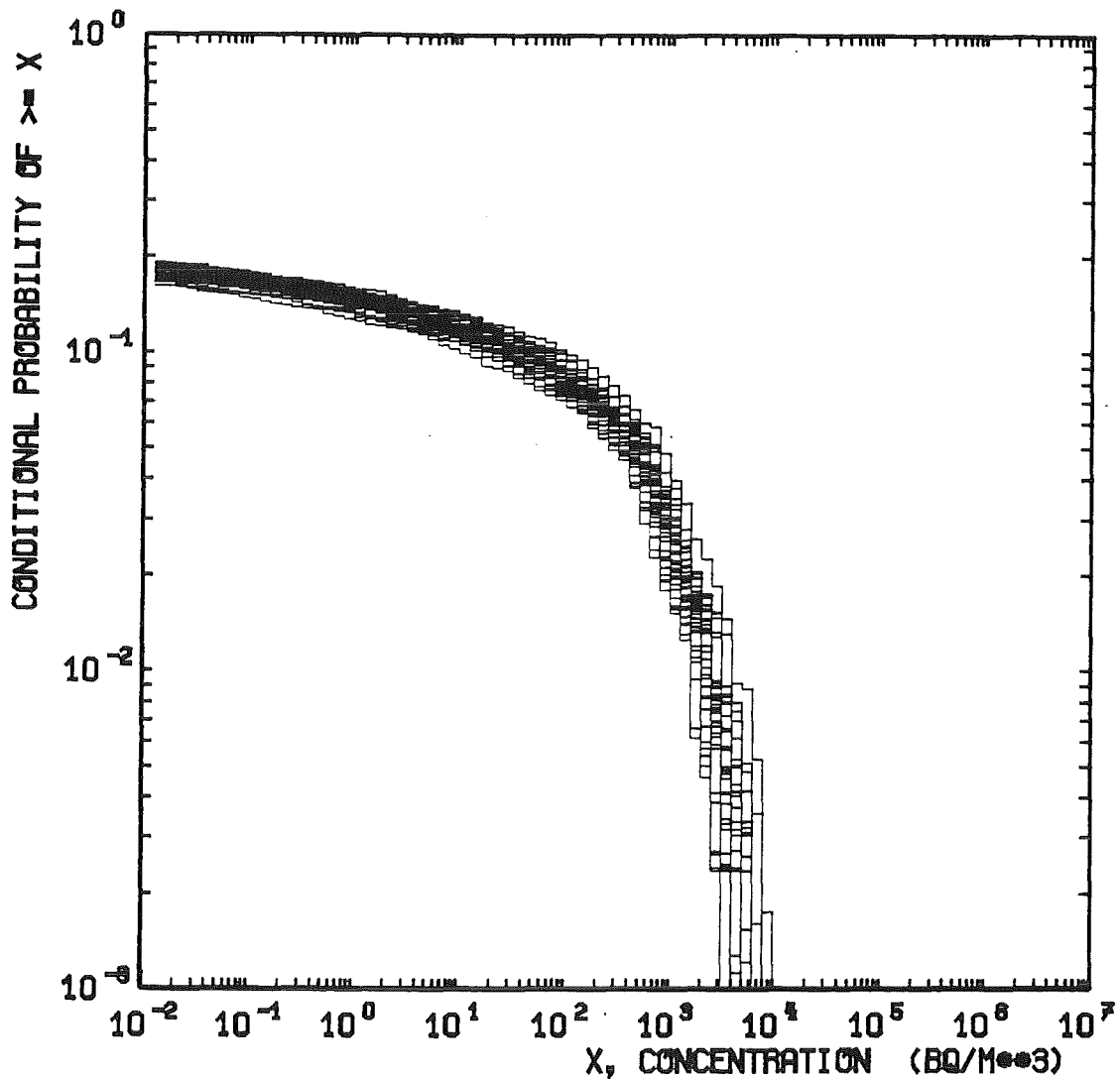


Concentration in the air near ground (1 m height) * : Ref.-Curve
 Nuclide: I - 131 □ : 5% -Curve
 Distance: 4.9 km ◇ : 95% -Curve



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING RELEASÉ HAS OCCURRED)
 AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
 ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

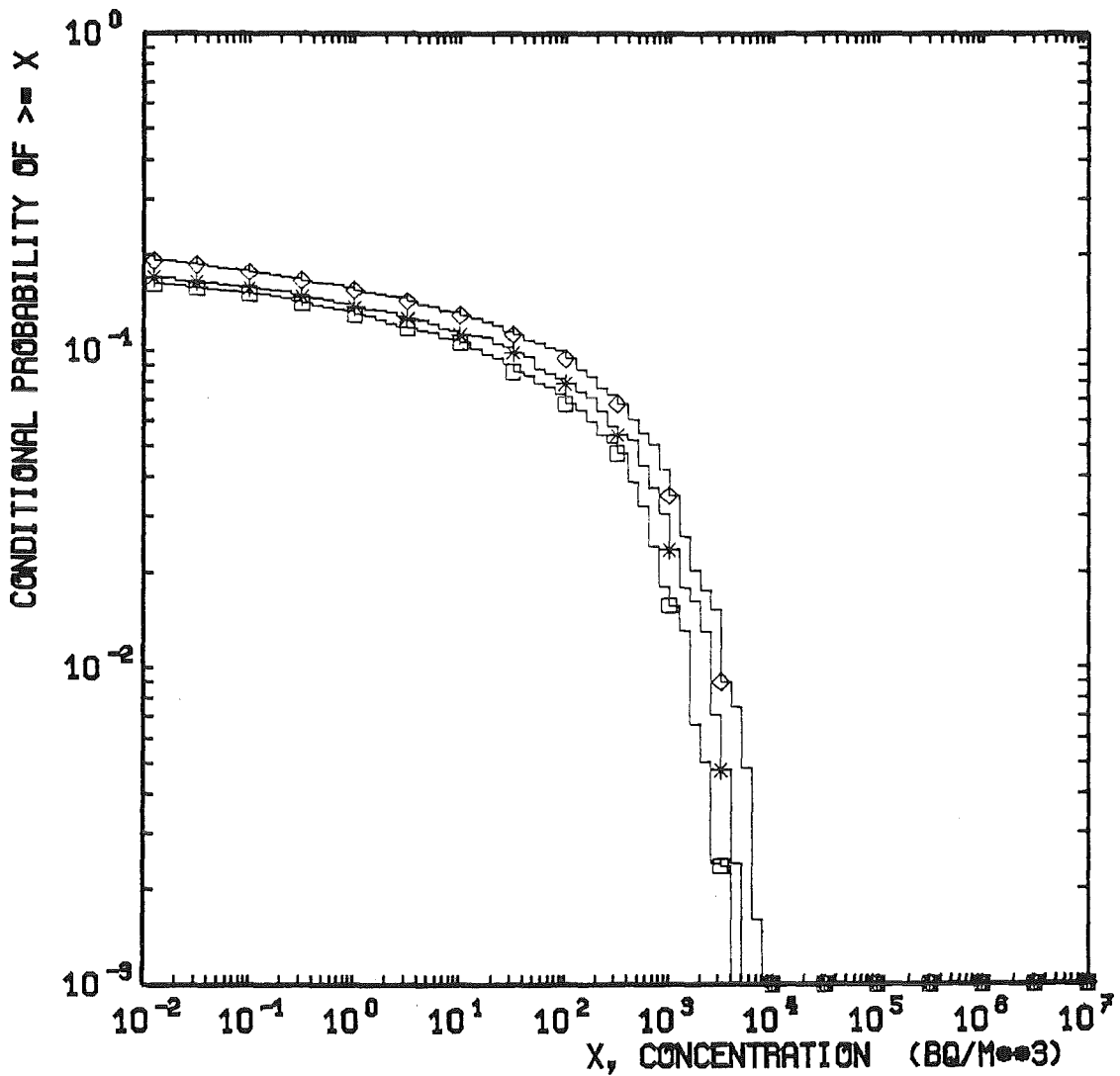


Concentration in the air near ground (1 m height)
Nuclide: I - 131
Distance: 27 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFDs) OF ACTIVITY CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMOD Uncertainty Analysis (1987)

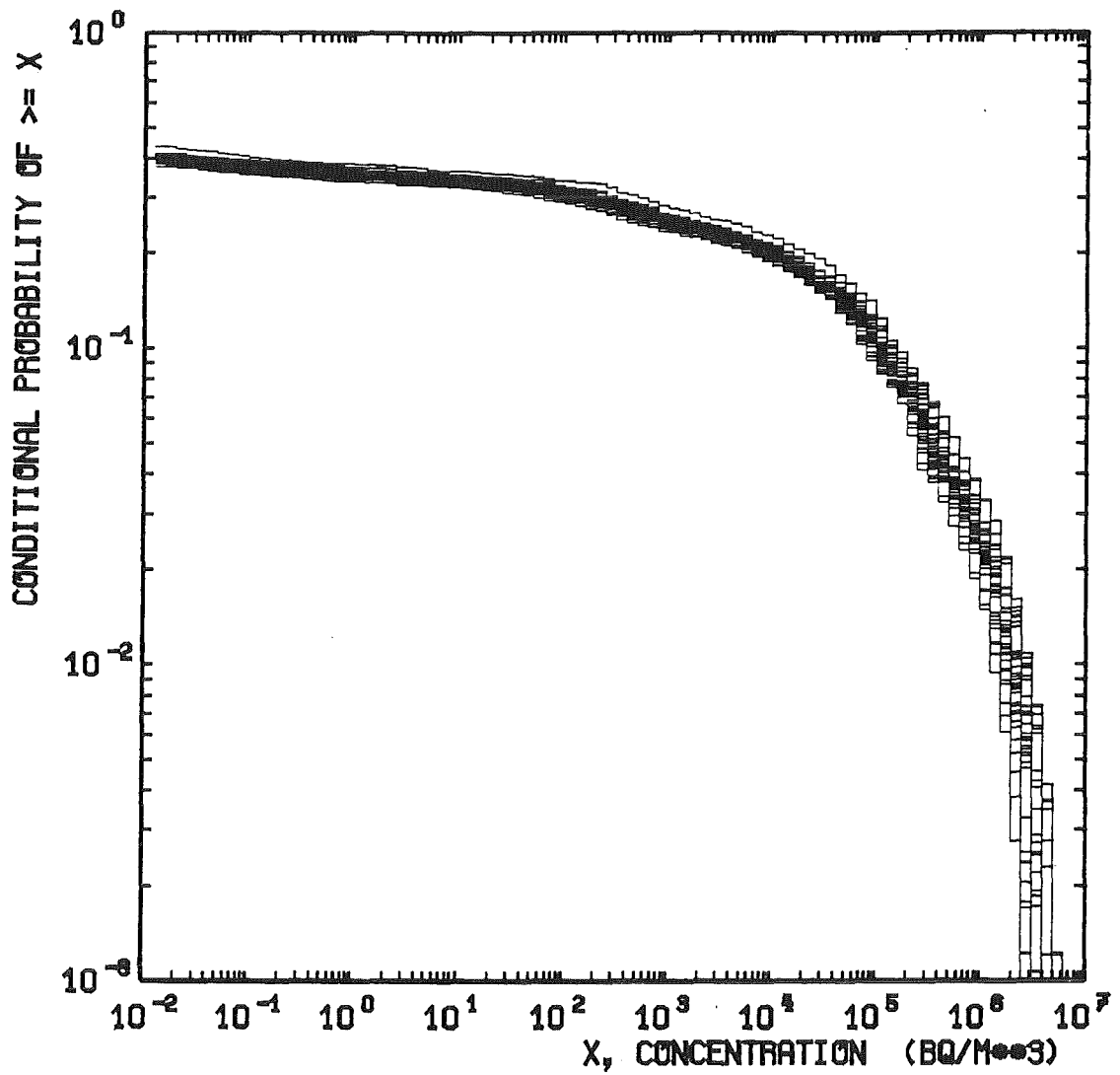


Concentration in the air near ground (1 m height) * : Ref.-Curve
Nuclide: I - 131 □ : 5% -Curve
Distance: 27 km ◇ : 95% -Curve



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED)
AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

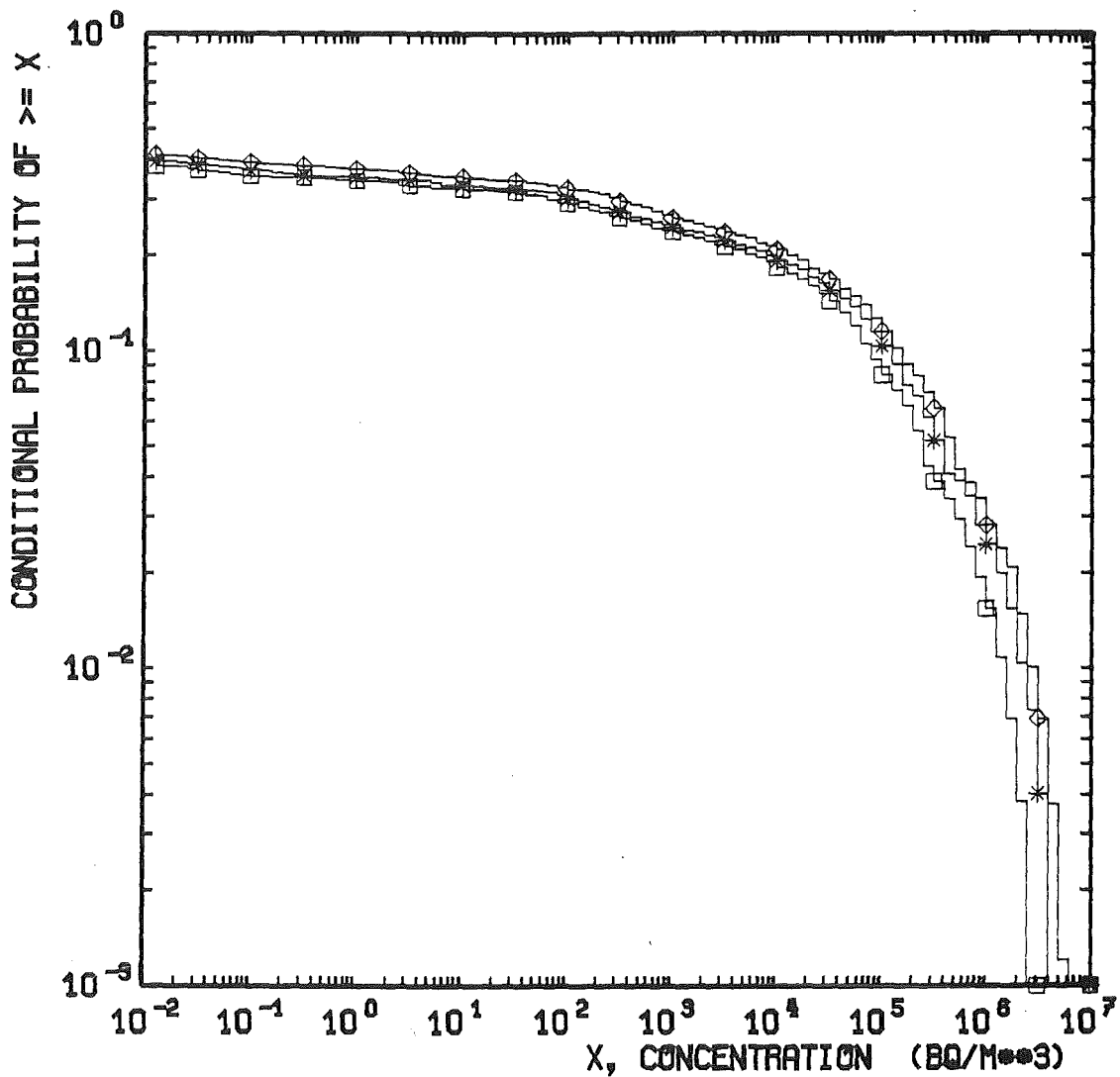


Concentration in the air near ground (1 m height)
Nuclide: Cs - 137
Distance: 0.875 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFDs) OF ACTIVITY CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMOD Uncertainty Analysis (1987)

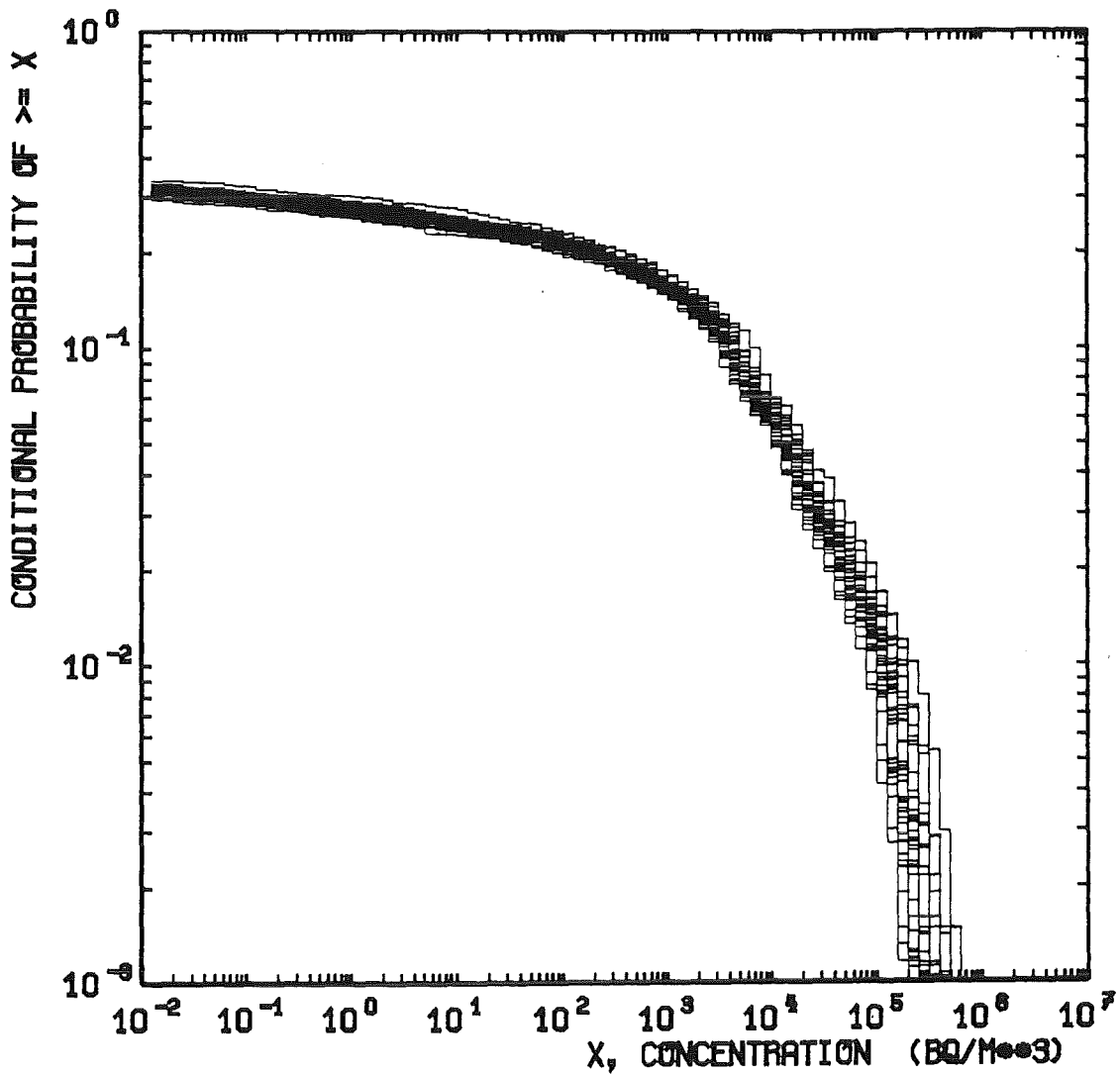


Concentration in the air near ground (1 m height) * : Ref.-Curve
 Nuclide: Cs - 137 □ : 5% -Curve
 Distance: 0.875 km ◇ : 95% -Curve



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED)
 AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
 ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

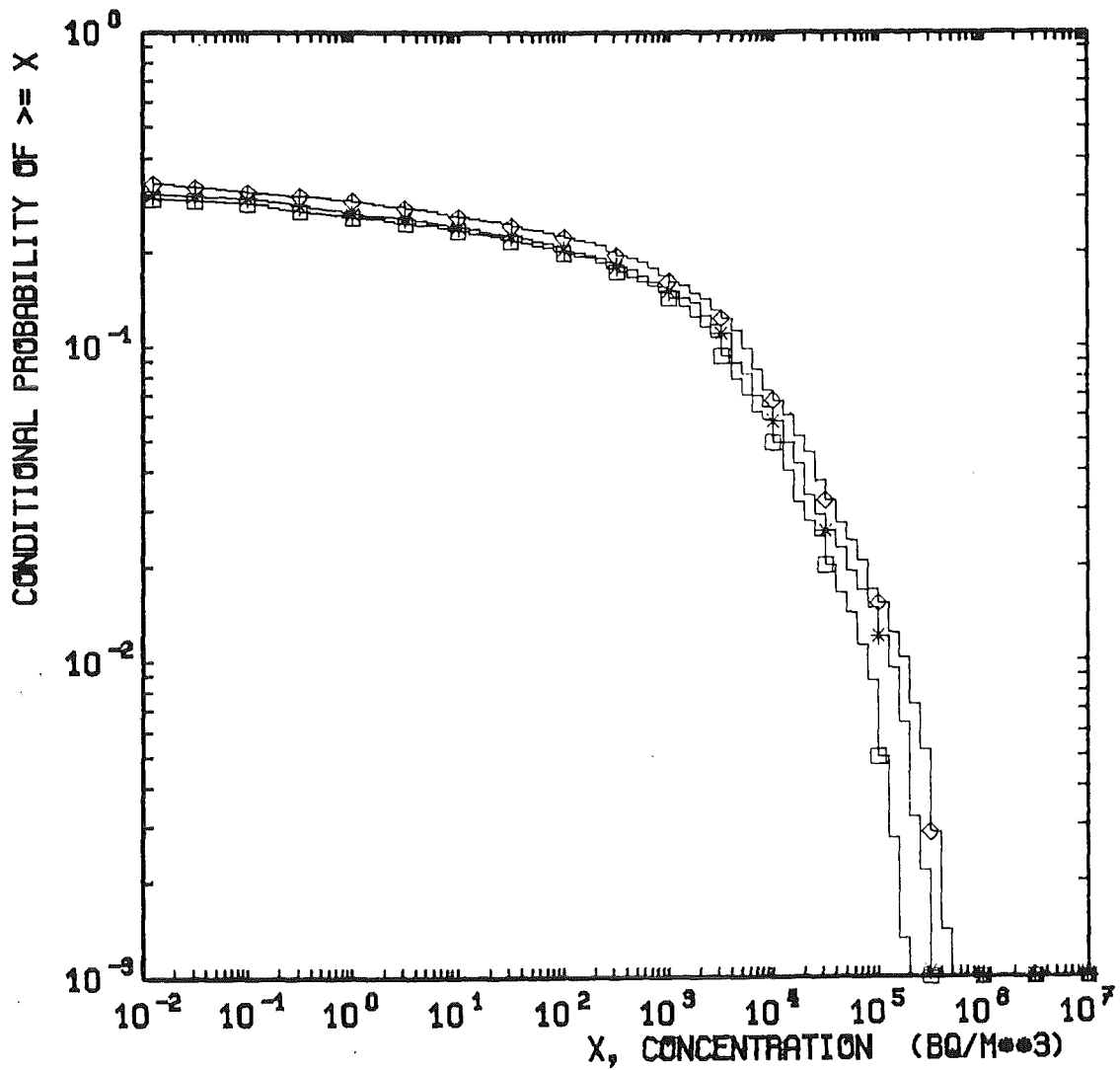


Concentration in the air near ground (1 m height)
Nuclide: Cs - 137
Distance: 4.9 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFD) OF ACTIVITY CENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

UFOMOD Uncertainty Analysis (1987)

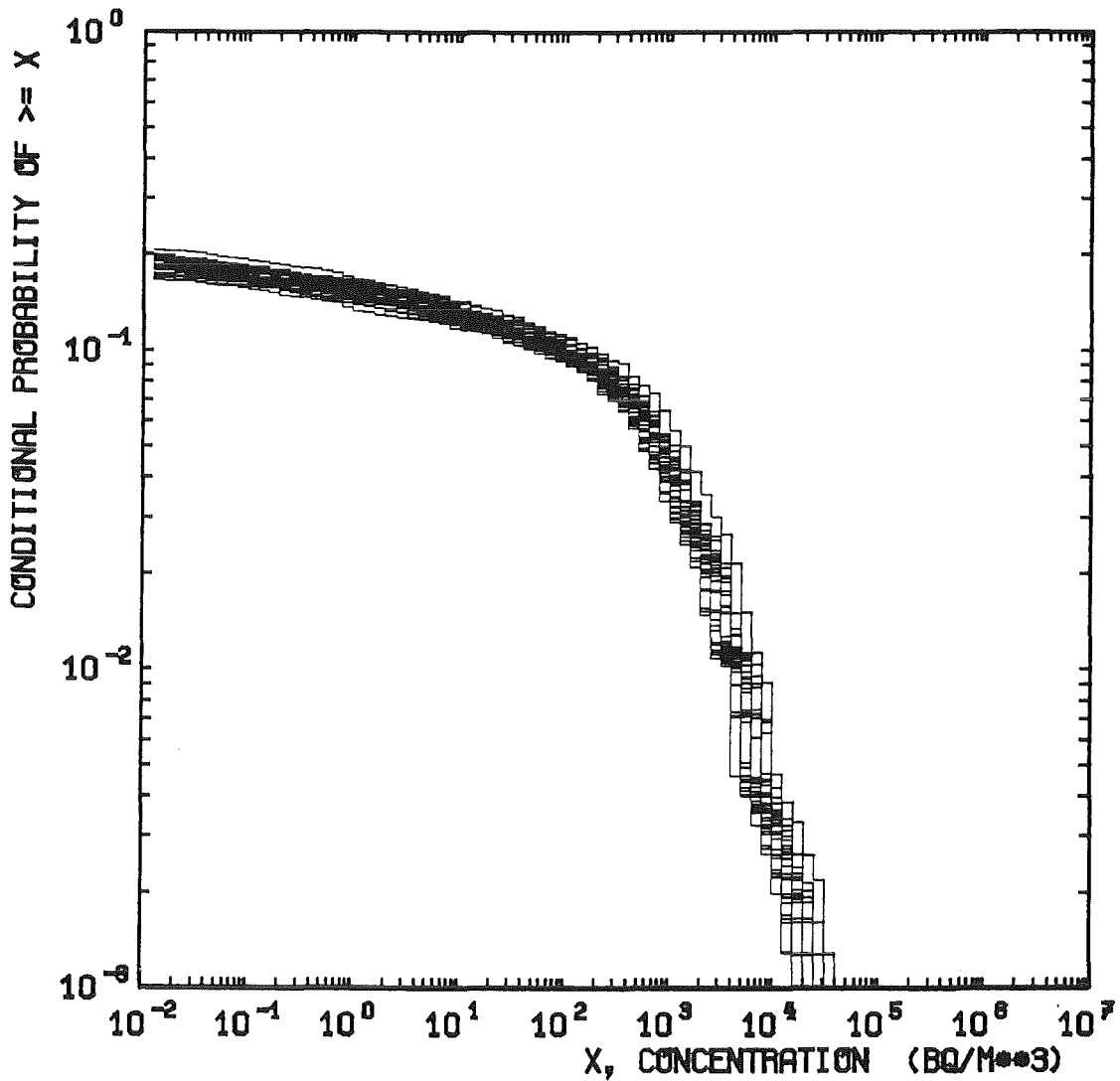


| | |
|--|----------------|
| Concentration in the air near ground (1 m height) | * : Ref.-Curve |
| Nuclide: Cs - 137 | □ : 5% -Curve |
| Distance: 4.9 km | ◇ : 95% -Curve |



REFERENCE CCDF OF THE CONCENTRATIONS (ASSUMING RELEASER HAS OCCURRED)
AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.

UFOMOD Uncertainty Analysis (1987)

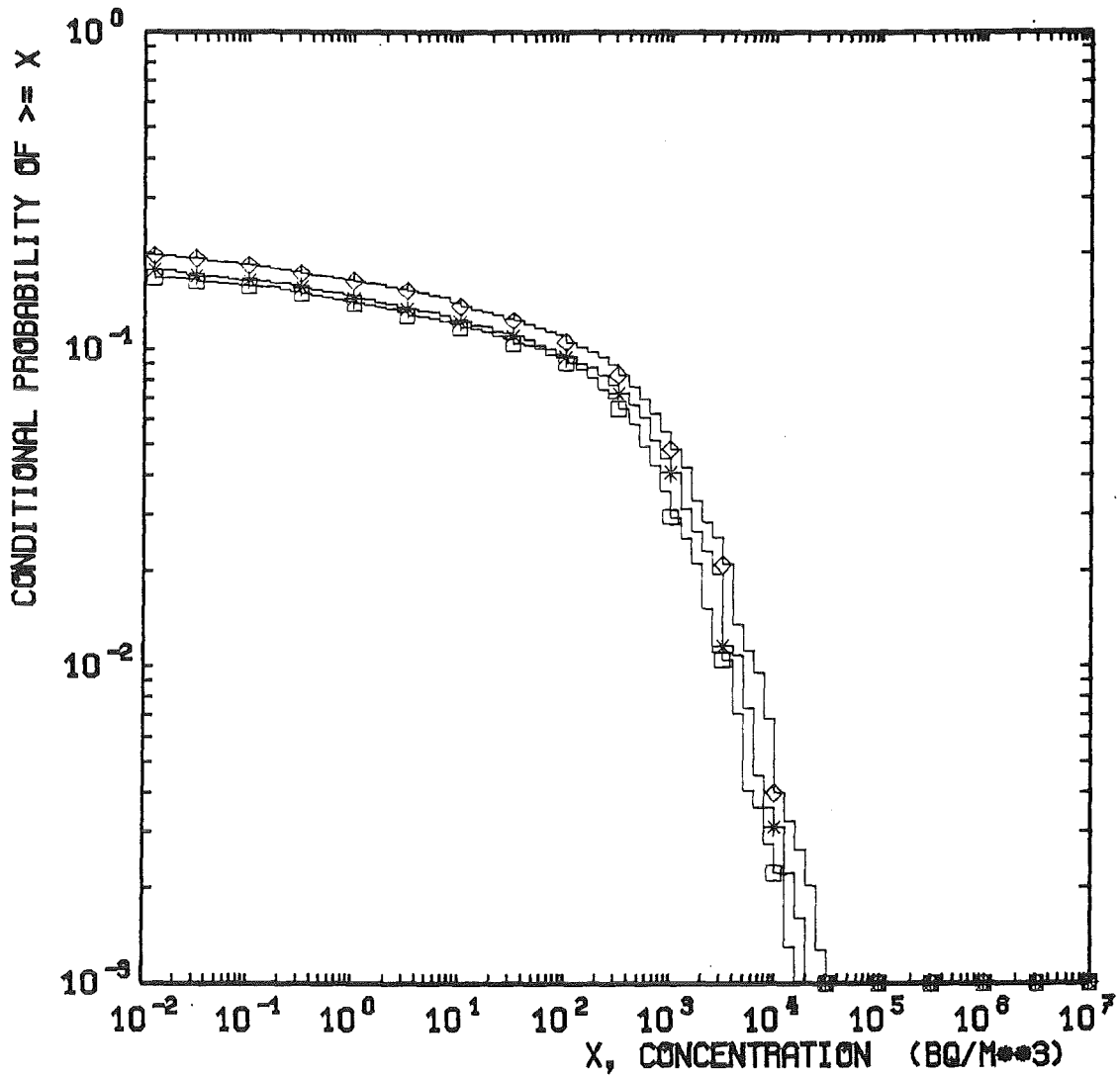


Concentration in the air near ground (1 m height)
Nuclide: Cs - 137
Distance: 27 km



COMPLEMENTARY CUMULATIVE FREQUENCY DISTRIBUTIONS (CCFDs) OF ACTIVITY CONCENTRATIONS (ASSUMING RELEASE HAS OCCURRED). EACH CCFD CORRESPONDS TO ONE OF THE 40 RUNS IN A LATIN HYPERCUBE SAMPLE OF SIZE 40.

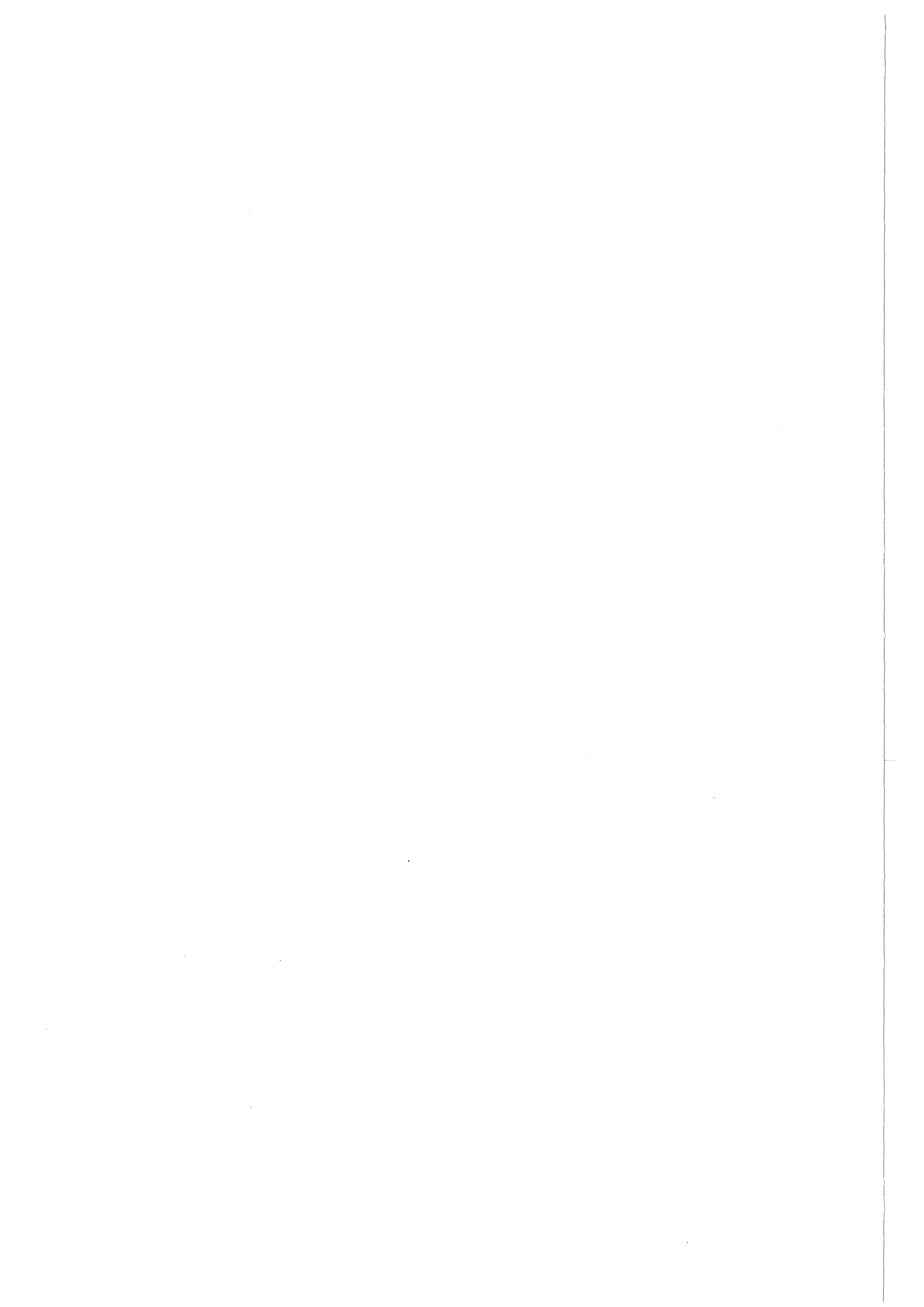
UFOMOD Uncertainty Analysis (1987)



Concentration in the air near ground (1 m height) * : Ref.-Curve
 Nuclide: Cs - 137 □ : 5% -Curve
 Distance: 27 km ◇ : 95% -Curve



REFERENCE CCFD OF THE CONCENTRATIONS (ASSUMING REALEASE HAS OCCURRED)
 AND THE EMPIRICAL 5% -, 95% - QUANTILES RESPECTIVELY ARE GIVEN AS
 ESTIMATED CONFIDENCE BOUNDS AT DISCRETE POINTS OF THE X - AXIS.



Appendix C. Sensitivity Analyses (Tables of PRCC values)

Legends for reading the PRCC - tables

The following list gives the name and the meaning of the parameters:

| | |
|----------------|--|
| σ_{y0} | initial horizontal plume width in the wake of the reactor building |
| σ_{z0} | initial vertical plume width in the wake of the reactor building |
| h_m | mixing height |
| $\sigma_y(S)$ | vertical plume diffusion for stability class S ($S \in \{A,B,C,D,E,F\}$) |
| $\sigma_z(S)$ | horizontal plume diffusion for stability class S |
| wp | wind profile exponent |
| $v_d(AE)$ | dry deposition of aerosols |
| $v_d(IO)$ | dry deposition of elementary iodine |
| Λ_{AE} | washout coefficients of aerosols |
| Λ_{IO} | washout coefficients of elementary iodine |

The following list gives the name and the meaning of the consequence variables:

| | | |
|---------|--|------------------|
| IODCGD1 | concentration of I-131 on ground surface | at D1 (0.875 km) |
| IODCGD2 | concentration of I-131 on ground surface | at D2 (4.9 km) |
| IODCGD3 | concentration of I-131 on ground surface | at D3 (27 km) |
| IODCAD1 | concentration of I-131 in air near ground | at D1 (0.875 km) |
| IODCAD2 | concentration of I-131 in air near ground | at D2 (4.9 km) |
| IODCAD3 | concentration of I-131 in air near ground | at D3 (27 km) |
| CAECGD1 | concentration of Cs-137 on ground surface | at D1 (0.875 km) |
| CAECGD2 | concentration of Cs-137 on ground surface | at D2 (4.9 km) |
| CAECGD3 | concentration of Cs-137 on ground surface | at D3 (27 km) |
| CAECAD1 | concentration of Cs-137 in air near ground | at D1 (0.875 km) |
| CAECAD2 | concentration of Cs-137 in air near ground | at D2 (4.9 km) |
| CAECAD3 | concentration of Cs-137 in air near ground | at D3 (27 km) |

C.1 Comparison of concentration runs (LHS; n = 40,80,100)

In this section PRCCs are shown for activity concentrations (I-131, Cs-137) at three distance intervals on ground surface and in the air near ground.

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TABLE ENTRIES REPRESENT THE VALUE OF THE PARTIAL RANK CORRELATION COEFFICIENT (AND ITS RANK) FOR EACH COMBINATION OF SELECTED INDEPENDENT AND SELECTED DEPENDENT VARIABLE, PROVIDED THAT THE ABSOLUTE VALUE OF THIS COEFFICIENT IS GREATER THAN $T(\text{ALPHA}) = 0.43$ (40 RUNS), 0.25 (80 RUNS) OR 0.21 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.05 SIGNIFICANCE LEVEL (E.G. THE CRITICAL VALUE IS $T(\text{ALPHA}) = 0.67$ (40 RUNS), 0.41 (80 RUNS) OR 0.36 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.001 SIGNIFICANCE LEVEL)

THE PERCENTAGE CONTRIBUTIONS TO UNCERTAINTY ARE GIVEN FOR EACH INDEPENDENT PARAMETER OR GROUPS OF INDEPENDENT PARAMETERS (SIGY, SIGZ, LD)

40U, 80U, 100U MEANS: IN THIS CASE ALL INDEPENDENT PARAMETERS ARE UNIFORMLY DISTRIBUTED

| #RUNS | IODCGD1 | | IODCGD1 | | IODCGD1 | | IODCGD1 | | IODCGD1 | | IODCGD1 | | |
|---------|---------|----------|---------|----------|----------|----------|---------|----------|----------|----------|---------|----------|----|
| | 40 | (%) | 40U | (%) | 80 | (%) | 80U | (%) | 100 | (%) | 100U | (%) | |
| SIGY0 | | | | | | | | | | | | | |
| SIGZ0 | | 2 | | | -0.52(4) | 2 | | | -0.64(4) | 2 | | | |
| HMIX | | | | | | | | | | | | 1 | |
| SIGY(A) | | # 2 | | # 1 | | # | | # 1 | | # 1 | | # | |
| SIGY(B) | | # | | # | | # | | # | | # | | # | |
| SIGY(C) | | # | | # | | # | | # | | # | | # | |
| SIGY(D) | | # | | # | | # | | # | | 0.34(5) | # | # | |
| SIGY(E) | | # | | # | | # | | # | | -0.24(7) | # | # | |
| SIGY(F) | | # | | # | | # | | # | | # | | # | |
| SIGZ(A) | | 118 | | 112 | | 119 | | 114 | | 116 | | 119 | |
| SIGZ(B) | | | | | | | | | | | | | |
| SIGZ(C) | | | | | | -0.25(6) | | | | | | | |
| SIGZ(D) | | | | | | -0.41(5) | | | | | | | |
| SIGZ(E) | | -0.61(2) | | -0.43(5) | | -0.69(2) | | -0.42(3) | | -0.26(6) | | -0.45(3) | |
| SIGZ(F) | | -0.52(3) | | -0.60(2) | | -0.56(3) | | -0.49(2) | | -0.69(2) | | -0.61(2) | |
| WP | | | | -0.47(4) | 4 | | | | | | | 0.23(5) | |
| VD(AER) | | | | | | | | | | | | | |
| VD(10D) | | 0.98(1) | 83 | 0.98(1) | 83 | 0.98(1) | 79 | 0.96(1) | 79 | 0.98(1) | 82 | 0.96(1) | 75 |
| LD(AER) | | | | | | | | | | | | | |
| LD(10D) | | 2 | | 0.50(3) | 2 | | | 1 | | | | 0.28(4) | |

AD-UFOMOD SENSITIVITY ANALYSIS (LHS-DESIGN - NEW UFOMOD CODE SYSTEM) CONCENTRATIONS PART 2 OF 12

TABLE ENTRIES REPRESENT THE VALUE OF THE PARTIAL RANK CORRELATION COEFFICIENT (AND ITS RANK) FOR EACH COMBINATION OF SELECTED INDEPENDENT AND SELECTED DEPENDENT VARIABLE, PROVIDED THAT THE ABSOLUTE VALUE OF THIS COEFFICIENT IS GREATER THAN $T(\text{ALPHA}) = 0.43$ (40 RUNS), 0.25 (80 RUNS) OR 0.21 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.05 SIGNIFICANCE LEVEL (E.G. THE CRITICAL VALUE IS $T(\text{ALPHA}) = 0.67$ (40 RUNS), 0.41 (80 RUNS) OR 0.36 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.001 SIGNIFICANCE LEVEL)

THE PERCENTAGE CONTRIBUTIONS TO UNCERTAINTY ARE GIVEN FOR EACH INDEPENDENT PARAMETER OR GROUPS OF INDEPENDENT PARAMETERS (SIGY, SIGZ, LD)

40U, 80U, 100U MEANS: IN THIS CASE ALL INDEPENDENT PARAMETERS ARE UNIFORMLY DISTRIBUTED

| #RUNS | IODCGD2 | | IODCGD2 | | IODCGD2 | | IODCGD2 | | IODCGD2 | |
|---------|----------|-----|----------|-----|----------|-----|----------|-----|----------|----------|
| | 40 | (%) | 40U | (%) | 80 | (%) | 80U | (%) | 100 | (%) |
| SIGYO | | | | | | | | | | |
| SIGZO | .80(3) | 6 | | 2 | .52(4) | 3 | .39(4) | 4 | .43(4) | 4 |
| HMIX | -.80(4) | 1 | -.82(1) | 31 | -.68(2) | 8 | -.71(2) | 23 | -.67(2) | 22 |
| SIGY(A) | # | 2 | # | 5 | # | 1 | # | 1 | # | 1 |
| SIGY(B) | # | | # | | # | | # | | # | |
| SIGY(C) | # | | # | | # | | # | | # | |
| SIGY(D) | .47(8) | # | # | | # | | # | | # | |
| SIGY(E) | # | | # | | # | | # | | # | |
| SIGY(F) | # | | # | | # | | # | | # | |
| SIGZ(A) | | 18 | | 8 | | 125 | | 8 | | 13 |
| SIGZ(B) | | | | | | | | | | |
| SIGZ(C) | -.54(6) | | | | | | | | | |
| SIGZ(D) | | | | | -.43(5) | | -.29(6) | | | -.24(5) |
| SIGZ(E) | | | | | -.43(6) | | | | | |
| SIGZ(F) | -.63(5) | | | | -.42(7) | | | | -.31(5) | |
| WP | | | | 1 | | | | | | |
| VD(AER) | | | | | | | .37(5) | | -.25(6) | |
| VD(10D) | .95(1) | 43 | .78(2) | 22 | .87(1) | 36 | .75(1) | 30 | .79(1) | 44 |
| LD(AER) | .47(7) | | -.48(4) | | | | | | | |
| LD(10D) | .83(2) | | .73(3) | | .62(3) | | .60(3) | | .60(3) | |

TABLE ENTRIES REPRESENT THE VALUE OF THE PARTIAL RANK CORRELATION COEFFICIENT (AND ITS RANK) FOR EACH COMBINATION OF SELECTED INDEPENDENT AND SELECTED DEPENDENT VARIABLE, PROVIDED THAT THE ABSOLUTE VALUE OF THIS COEFFICIENT IS GREATER THAN $T(\text{ALPHA}) = 0.43$ (40 RUNS), 0.25 (80 RUNS) OR 0.21 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.05 SIGNIFICANCE LEVEL (E.G. THE CRITICAL VALUE IS $T(\text{ALPHA}) = 0.67$ (40 RUNS), 0.41 (80 RUNS) OR 0.36 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.001 SIGNIFICANCE LEVEL)

THE PERCENTAGE CONTRIBUTIONS TO UNCERTAINTY ARE GIVEN FOR EACH INDEPENDENT PARAMETER OR GROUPS OF INDEPENDENT PARAMETERS (SIGY, SIGZ, LD)

40U, 80U, 100U MEANS: IN THIS CASE ALL INDEPENDENT PARAMETERS ARE UNIFORMLY DISTRIBUTED

| #RUNS | IODCGD3 | | IODCGD3 | | IODCGD3 | | IODCGD3 | | IODCGD3 | | IODCGD3 | |
|---------|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|
| | 40 | (%) | 40U | (%) | 80 | (%) | 80U | (%) | 100 | (%) | 100U | (%) |
| SIGY0 | | 1 | | | | | | 1 | | | | |
| SIGZ0 | | | | 1 | .29(5) | 1 | | | .30(6) | | .23(5) | 1 |
| HMIX | -.95(1) | 38 | -.95(1) | 49 | -.94(1) | 47 | -.94(1) | 51 | -.96(1) | 54 | -.91(1) | 47 |
| SIGY(A) | | # 3 | | # 4 | | # 1 | | # 2 | | # | | # |
| SIGY(B) | | # | | # | | # | | # | | # | | # |
| SIGY(C) | | # | -.43(5) | # | | # | | # | | # | | # |
| SIGY(D) | | # | | # | | # | .25(7) | # | | # | | # |
| SIGY(E) | | # | | # | | # | | # | | # | | # |
| SIGY(F) | .47(6) | # | | # | | # | | # | | # | | # |
| SIGZ(A) | | 2 | | 8 | | 10 | | 4 | | 4 | | 4 |
| SIGZ(B) | | | | | | | | | | | | |
| SIGZ(C) | | | | | | | | | | | | |
| SIGZ(D) | | | | | | | .31(5) | | | | .23(6) | |
| SIGZ(E) | | | | | .26(6) | | .25(6) | | .35(5) | | | |
| SIGZ(F) | .56(5) | | | | .48(4) | | | | .43(4) | | .24(4) | |
| WP | | | | | | 1 | | | | | | |
| VD(AER) | .57(4) | | | | | | .33(4) | | | | | |
| VD(10D) | .95(2) | 38 | .91(2) | 28 | .92(2) | 32 | .90(2) | 32 | .95(2) | 44 | .88(2) | 34 |
| LD(AER) | | | -.51(4) | | | | | | | | | |
| LD(10D) | .74(3) | 9 | .55(3) | 12 | .52(3) | 3 | .38(3) | 4 | .69(3) | 5 | .35(3) | 2 |

AD-UFOMOD SENSITIVITY ANALYSIS (LHS-DESIGN - NEW UFOMOD CODE SYSTEM) CONCENTRATIONS PART 4 OF 12

TABLE ENTRIES REPRESENT THE VALUE OF THE PARTIAL RANK CORRELATION COEFFICIENT (AND ITS RANK) FOR EACH COMBINATION OF SELECTED INDEPENDENT AND SELECTED DEPENDENT VARIABLE, PROVIDED THAT THE ABSOLUTE VALUE OF THIS COEFFICIENT IS GREATER THAN $T(\text{ALPHA}) = 0.43$ (40 RUNS), 0.25 (80 RUNS) OR 0.21 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.05 SIGNIFICANCE LEVEL (E.G. THE CRITICAL VALUE IS $T(\text{ALPHA}) = 0.67$ (40 RUNS), 0.41 (80 RUNS) OR 0.36 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.001 SIGNIFICANCE LEVEL)

THE PERCENTAGE CONTRIBUTIONS TO UNCERTAINTY ARE GIVEN FOR EACH INDEPENDENT PARAMETER OR GROUPS OF INDEPENDENT PARAMETERS (SIGY, SIGZ, LD)

40U, 80U, 100U MEANS: IN THIS CASE ALL INDEPENDENT PARAMETERS ARE UNIFORMLY DISTRIBUTED

| #RUNS | IODCAD1 | | IODCAD1 | | IODCAD1 | | IODCAD1 | | IODCAD1 | | IODCAD1 | |
|---------|---------|-------|---------|-------|---------|-------|---------|-------|---------|------|---------|------|
| | 40 | (%) | 40U | (%) | 80 | (%) | 80U | (%) | 100 | (%) | 100U | (%) |
| SIGY0 | | | | | | | | | | | | |
| SIGZ0 | -0.60 | (3) | -0.50 | (6) | -0.72 | (2) | -0.46 | (3) | -0.56 | (4) | -0.32 | (4) |
| HMIX | | | | | | | | | | | | |
| SIGY(A) | | # 7 | | # 3 | | # 1 | | # | | # 1 | | # 1 |
| SIGY(B) | | # | | # | | # | | # | | # | | # |
| SIGY(C) | | # | | # | | # | | # | | # | | # |
| SIGY(D) | | # | | # | | # | | # | 0.34 | (6) | | # |
| SIGY(E) | | # | | # | | # | | # | | # | | # |
| SIGY(F) | | # | | # | | # | | # | | # | | # |
| SIGZ(A) | | 144 | | 128 | | 141 | | 124 | | 140 | | 122 |
| SIGZ(B) | | | | -0.47 | (7) | | | | | | | |
| SIGZ(C) | | | | | | -0.27 | (6) | | | | | |
| SIGZ(D) | | -0.44 | (5) | | -0.50 | (5) | | -0.39 | (5) | | -0.42 | (4) |
| SIGZ(E) | | -0.75 | (2) | | -0.79 | (2) | | -0.69 | (3) | | -0.58 | (2) |
| SIGZ(F) | | -0.58 | (4) | | -0.57 | (3) | | -0.59 | (4) | | -0.36 | (5) |
| WP | | | | | | | | | | | | |
| VD(AER) | | | | | | | | | | | | |
| VD(10D) | | -0.92 | (1) | | -0.98 | (1) | | -0.94 | (1) | | -0.96 | (1) |
| LD(AER) | | | | | | | | | | | | |
| LD(10D) | | 1 | | 0.54 | (4) | 1 | | 1 | | 1 | | 1 |

TABLE ENTRIES REPRESENT THE VALUE OF THE PARTIAL RANK CORRELATION COEFFICIENT (AND ITS RANK) FOR EACH COMBINATION OF SELECTED INDEPENDENT AND SELECTED DEPENDENT VARIABLE, PROVIDED THAT THE ABSOLUTE VALUE OF THIS COEFFICIENT IS GREATER THAN $T(\text{ALPHA}) = 0.43$ (40 RUNS), 0.25 (80 RUNS) OR 0.21 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.05 SIGNIFICANCE LEVEL (E.G. THE CRITICAL VALUE IS $T(\text{ALPHA}) = 0.67$ (40 RUNS), 0.41 (80 RUNS) OR 0.36 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.001 SIGNIFICANCE LEVEL)

THE PERCENTAGE CONTRIBUTIONS TO UNCERTAINTY ARE GIVEN FOR EACH INDEPENDENT PARAMETER OR GROUPS OF INDEPENDENT PARAMETERS (SIGY, SIGZ, LD)

40U, 80U, 100U MEANS: IN THIS CASE ALL INDEPENDENT PARAMETERS ARE UNIFORMLY DISTRIBUTED

| #RUNS | IODCAD2 | | IODCAD2 | | IODCAD2 | | IODCAD2 | | IODCAD2 | | IODCAD2 | |
|---------|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|
| | 40 | (%) | 40U | (%) | 80 | (%) | 80U | (%) | 100 | (%) | 100U | (%) |
| SIGY0 | | | | | | | | | | | | |
| SIGZ0 | .73(3) | 8 | | | | | .33(3) | 1 | .40(3) | 2 | .38(3) | 1 |
| HMIX | -.77(2) | 7 | -.84(2) | 11 | -.61(2) | 3 | -.73(2) | 9 | -.70(2) | 4 | -.78(2) | 8 |
| SIGY(A) | .47(5) | # 5 | # | # 1 | # | # 1 | # | # 1 | # | # 2 | -.28(4) | # 1 |
| SIGY(B) | # | # | # | # | # | # | # | # | # | # | # | # |
| SIGY(C) | # | # | # | # | # | # | # | # | -.22(5) | # | # | # |
| SIGY(D) | # | # | # | # | # | # | # | # | # | # | # | # |
| SIGY(E) | # | # | # | # | # | # | # | # | # | # | # | # |
| SIGY(F) | # | # | # | # | # | # | # | # | # | # | # | # |
| SIGZ(A) | | 6 | | 3 | | 8 | | 3 | | 3 | | 2 |
| SIGZ(B) | | | | | | | | | | | | |
| SIGZ(C) | | | | | | | | | | | | |
| SIGZ(D) | | | | | | | -.28(4) | | | | -.22(5) | |
| SIGZ(E) | | | | | -.38(4) | | | | | | | |
| SIGZ(F) | -.52(4) | | | | -.44(3) | | | | -.31(4) | | | |
| WP | | | | 2 | | | | | | | | |
| VD(AER) | | | | | | | | | | | | |
| VD(10D) | -.97(1) | 76 | -.98(1) | 84 | -.97(1) | 86 | -.96(1) | 82 | -.95(1) | 85 | -.97(1) | 85 |
| LD(AER) | | | | | | | | | | | | |
| LD(10D) | | 1 | | | | | | 1 | | | | |

AD-UFOMOD SENSITIVITY ANALYSIS (LHS-DESIGN - NEW UFOMOD CODE SYSTEM) CONCENTRATIONS PART 6 OF 12

TABLE ENTRIES REPRESENT THE VALUE OF THE PARTIAL RANK CORRELATION COEFFICIENT (AND ITS RANK) FOR EACH COMBINATION OF SELECTED INDEPENDENT AND SELECTED DEPENDENT VARIABLE, PROVIDED THAT THE ABSOLUTE VALUE OF THIS COEFFICIENT IS GREATER THAN $T(\text{ALPHA}) = 0.43$ (40 RUNS), 0.25 (80 RUNS) OR 0.21 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.05 SIGNIFICANCE LEVEL (E.G. THE CRITICAL VALUE IS $T(\text{ALPHA}) = 0.67$ (40 RUNS), 0.41 (80 RUNS) OR 0.36 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.001 SIGNIFICANCE LEVEL)

THE PERCENTAGE CONTRIBUTIONS TO UNCERTAINTY ARE GIVEN FOR EACH INDEPENDENT PARAMETER OR GROUPS OF INDEPENDENT PARAMETERS (SIGY, SIGZ, LD)

40U, 80U, 100U MEANS: IN THIS CASE ALL INDEPENDENT PARAMETERS ARE UNIFORMLY DISTRIBUTED

| #RUNS | IODCAD3 | | IODCAD3 | | IODCAD3 | | IODCAD3 | | IODCAD3 | | IODCAD3 | |
|---------|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|
| | 40 | (%) | 40U | (%) | 80 | (%) | 80U | (%) | 100 | (%) | 100U | (%) |
| SIGY0 | | | | | | | | | | | | |
| SIGZ0 | .51(4) | 2 | | | | | .35(4) | | | | | |
| HMIX | -.97(2) | 37 | -.95(2) | 33 | -.96(2) | 40 | -.96(2) | 30 | -.93(2) | 39 | -.95(2) | 27 |
| SIGY(A) | | # 3 | | # 1 | | # 1 | | # | | # 1 | | # |
| SIGY(B) | | # | | # | | # | | # | .21(6) | # | | # |
| SIGY(C) | | # | | # | -.29(6) | # | | # | | # | | # |
| SIGY(D) | | # | | # | | # | | # | | # | | # |
| SIGY(E) | | # | | # | | # | | # | | # | | # |
| SIGY(F) | | # | | # | | # | | # | | # | | # |
| SIGZ(A) | | 6 | | 4 | | 4 | | 3 | | 6 | | 3 |
| SIGZ(B) | | | | | | | | | | | | |
| SIGZ(C) | | | | | | | | | | | .23(5) | |
| SIGZ(D) | | | | | | | | | | | | |
| SIGZ(E) | .55(3) | | | | .36(4) | | | | .36(3) | | | |
| SIGZ(F) | .51(5) | | | | .44(3) | | | | .35(5) | | .34(4) | |
| WP | | | | 2 | | | | | | | | |
| VD(AER) | | | | | | | | | | | | |
| VD(IOD) | -.97(1) | 53 | -.97(1) | 63 | -.97(1) | 55 | -.98(1) | 65 | -.94(1) | 48 | -.98(1) | 65 |
| LD(AER) | | | | | | | | | | | | |
| LD(IOD) | | 1 | | 1 | -.32(5) | 1 | -.46(3) | 1 | -.26(5) | | -.39(3) | 1 |

TABLE ENTRIES REPRESENT THE VALUE OF THE PARTIAL RANK CORRELATION COEFFICIENT (AND ITS RANK) FOR EACH COMBINATION OF SELECTED INDEPENDENT AND SELECTED DEPENDENT VARIABLE, PROVIDED THAT THE ABSOLUTE VALUE OF THIS COEFFICIENT IS GREATER THAN $T(\text{ALPHA}) = 0.43$ (40 RUNS), 0.25 (80 RUNS) OR 0.21 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.05 SIGNIFICANCE LEVEL (E.G. THE CRITICAL VALUE IS $T(\text{ALPHA}) = 0.67$ (40 RUNS), 0.41 (80 RUNS) OR 0.36 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.001 SIGNIFICANCE LEVEL)

THE PERCENTAGE CONTRIBUTIONS TO UNCERTAINTY ARE GIVEN FOR EACH INDEPENDENT PARAMETER OR GROUPS OF INDEPENDENT PARAMETERS (SIGY, SIGZ, LD)

40U, 80U, 100U MEANS: IN THIS CASE ALL INDEPENDENT PARAMETERS ARE UNIFORMLY DISTRIBUTED

| #RUNS | CAECGD1 | | CAECGD1 | | CAECGD1 | | CAECGD1 | | CAECGD1 | | CAECGD1 | |
|---------|----------|-----|----------|-----|-----------|-----|----------|-----|----------|-----|----------|-----|
| | 40 | (%) | 40U | (%) | 80 | (%) | 80U | (%) | 100 | (%) | 100U | (%) |
| SIGY0 | | 1 | | 2 | | | | | | | | |
| SIGZ0 | | 1 | | | | | | | | | | |
| HMIX | | | | | | | | | | | | |
| SIGY(A) | | # 2 | .49(3)# | # 6 | | # 2 | .31(6)# | # 1 | | # 1 | | # 1 |
| SIGY(B) | .57(3)# | | | # | | # | | # | | # | | # |
| SIGY(C) | # | # | | # | -.29(3)# | | # | # | | # | | # |
| SIGY(D) | # | # | | # | -.27(5)# | | # | # | | # | | # |
| SIGY(E) | # | # | | # | .27(4)# | | # | # | | # | | # |
| SIGY(F) | # | # | | # | | # | # | # | | # | | # |
| SIGZ(A) | 7 | | 7 | | 3 | | 7 | | 3 | | 6 | |
| SIGZ(B) | | | | | | | .30(7) | | | | | |
| SIGZ(C) | | | | | | | | | | | | |
| SIGZ(D) | | | | | | | | | | | | |
| SIGZ(E) | | | | | | | -.39(5) | | -.32(4) | | -.34(4) | |
| SIGZ(F) | | | | | | | -.67(3) | | -.28(5) | | -.58(3) | |
| WP | | | | | | | | | | | | |
| VD(AER) | .96(2) | 40 | .95(1) | 35 | .89(2) | 35 | .97(1) | 44 | .92(1) | 39 | .96(1) | 41 |
| VD(10D) | | | -.45(4) | | | | | | | | | |
| LD(AER) | .97(1) | 156 | .95(2) | 146 | .89(1) | 144 | .97(2) | 148 | .92(2) | 154 | .95(2) | 148 |
| LD(10D) | | | | | | | | | | | | |

TABLE ENTRIES REPRESENT THE VALUE OF THE PARTIAL RANK CORRELATION COEFFICIENT (AND ITS RANK) FOR EACH COMBINATION OF SELECTED INDEPENDENT AND SELECTED DEPENDENT VARIABLE, PROVIDED THAT THE ABSOLUTE VALUE OF THIS COEFFICIENT IS GREATER THAN $T(\text{ALPHA}) = 0.43$ (40 RUNS), 0.25 (80 RUNS) OR 0.21 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.05 SIGNIFICANCE LEVEL (E.G. THE CRITICAL VALUE IS $T(\text{ALPHA}) = 0.67$ (40 RUNS), 0.41 (80 RUNS) OR 0.36 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.001 SIGNIFICANCE LEVEL)

THE PERCENTAGE CONTRIBUTIONS TO UNCERTAINTY ARE GIVEN FOR EACH INDEPENDENT PARAMETER OR GROUPS OF INDEPENDENT PARAMETERS (SIGY, SIGZ, LD)

40U, 80U, 100U MEANS: IN THIS CASE ALL INDEPENDENT PARAMETERS ARE UNIFORMLY DISTRIBUTED

| #RUNS | CAECGD3 | | CAECGD3 | | CAECGD3 | | CAECGD3 | | CAECGD3 | | CAECGD3 | |
|----------|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|
| | 40 | (%) | 40U | (%) | 80 | (%) | 80U | (%) | 100 | (%) | 100U | (%) |
| SIGY0 | | | | | | | | | | | | |
| SIGZ0 | | | | | | | | | | | | |
| HMIX | -.67(3) | | -.79(3) | 8 | -.47(3) | 1 | -.74(3) | 8 | -.36(3) | | -.70(3) | 6 |
| SIGY(A) | # | | # | 3 | # | | # | 1 | # | 1 | # | 1 |
| SIGY(B) | # | | # | | # | | # | | # | | # | |
| SIGY(C) | # | | .44(5) | # | -.29(5) | # | # | | # | | # | |
| SIGY(D) | # | | # | | # | | # | | # | | -.38(4) | # |
| SIGY(E) | # | | # | | # | | # | | # | | # | |
| SIGY(F) | # | | # | | # | | # | | # | | # | |
| SIGZ(A) | 3 | | -.49(4) | 4 | 2 | | -.28(5) | 1 | | | | 1 |
| SIGZ(B) | | | | | | | | | | | | |
| SIGZ(C) | | | | | | | | | | | | |
| SIGZ(D) | | | | | | | .33(4) | | | | | |
| SIGZ(E) | | | | | | | | | | | | |
| SIGZ(F) | | | | | .32(4) | | | | | | | |
| WP | -.54(4) | | | | | | .26(6) | | | | | |
| VD(AER) | .89(2) | 6 | .87(2) | 6 | .83(2) | 3 | .84(2) | 16 | .83(2) | 4 | .77(2) | 11 |
| VD(IOD) | | | | | | | | | | | | |
| LD(AER) | .99(1) | 192 | .97(1) | 177 | .98(1) | 190 | .94(1) | 170 | .99(1) | 194 | .94(1) | 172 |
| LD(IOD) | | | | | | | | | | | | |

AD-UFOMOD SENSITIVITY ANALYSIS (LHS-DESIGN - NEW UFOMOD CODE SYSTEM) CONCENTRATIONS PART 10 OF 12

TABLE ENTRIES REPRESENT THE VALUE OF THE PARTIAL RANK CORRELATION COEFFICIENT (AND ITS RANK) FOR EACH COMBINATION OF SELECTED INDEPENDENT AND SELECTED DEPENDENT VARIABLE, PROVIDED THAT THE ABSOLUTE VALUE OF THIS COEFFICIENT IS GREATER THAN $T(\text{ALPHA}) = 0.43$ (40 RUNS), 0.25 (80 RUNS) OR 0.21 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.05 SIGNIFICANCE LEVEL (E.G. THE CRITICAL VALUE IS $T(\text{ALPHA}) = 0.67$ (40 RUNS), 0.41 (80 RUNS) OR 0.36 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.001 SIGNIFICANCE LEVEL)

THE PERCENTAGE CONTRIBUTIONS TO UNCERTAINTY ARE GIVEN FOR EACH INDEPENDENT PARAMETER OR GROUPS OF INDEPENDENT PARAMETERS (SIGY, SIGZ, LD)

40U, 80U, 100U MEANS: IN THIS CASE ALL INDEPENDENT PARAMETERS ARE UNIFORMLY DISTRIBUTED

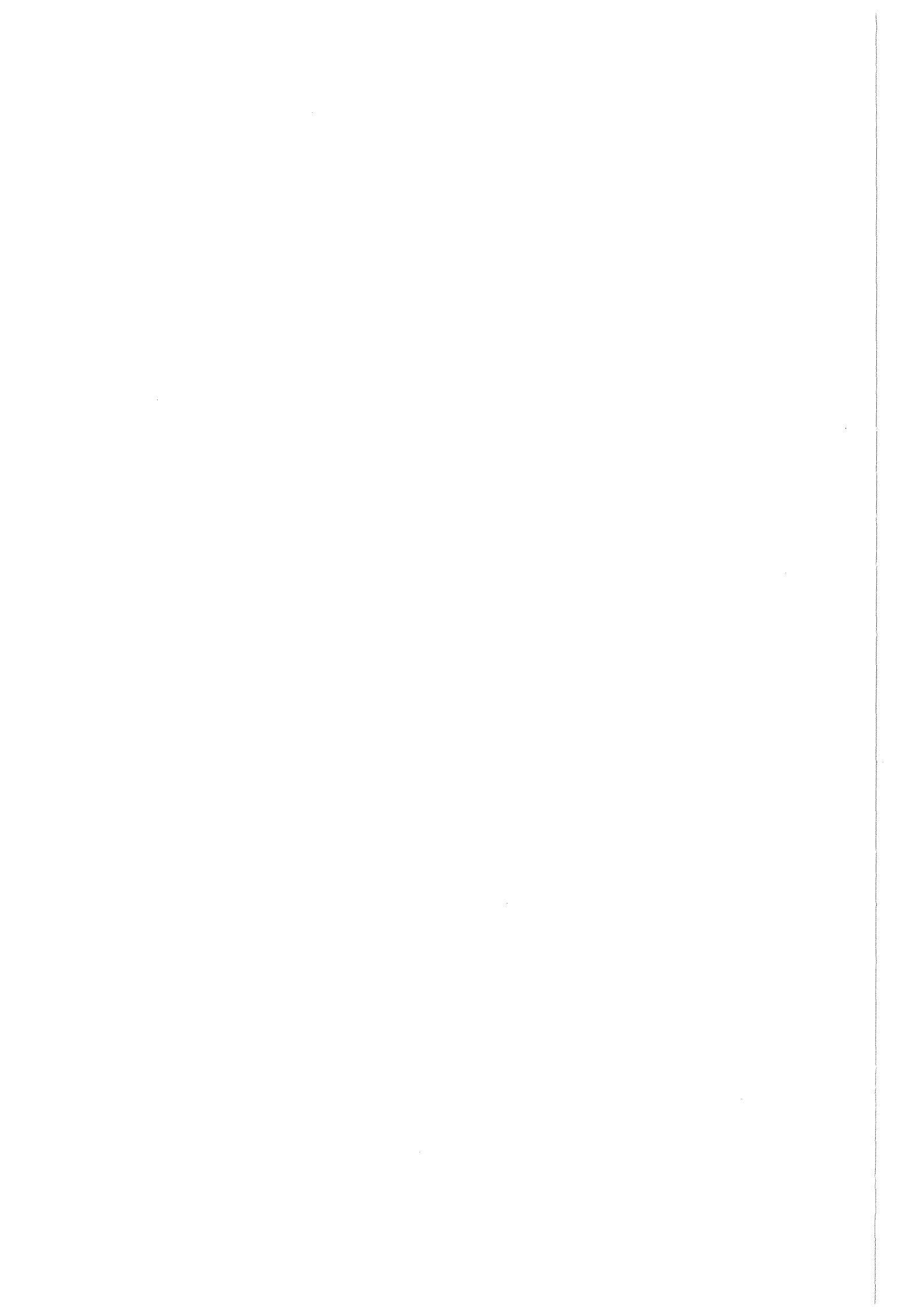
| #RUNS | CAECAD1 | | CAECAD1 | | CAECAD1 | | CAECAD1 | | CAECAD1 | |
|---------|---------|--------|---------|--------|---------|---------|---------|--------|---------|---------|
| | 40 | (%) | 40U | (%) | 80 | (%) | 80U | (%) | 100 | (%) |
| SIGY0 | | | | | | | | | | |
| SIGZ0 | -0.74 | (3) 5 | -0.77 | (3) 4 | -0.83 | (2) 10 | -0.84 | (3) 7 | -0.84 | (3) 6 |
| HMIX | | 1 | | | | | | | | |
| SIGY(A) | # | | # 2 | | # 1 | | # | | # 1 | |
| SIGY(B) | # | | # | | -0.25 | (6)# | # | | # | |
| SIGY(C) | # | | # | | # | | # | | # | |
| SIGY(D) | # | | # | | # | | # | | # | |
| SIGY(E) | # | | # | | # | | # | -0.27 | (6)# | # |
| SIGY(F) | # | | # | | # | | # | | # | |
| SIGZ(A) | | 191 | | 193 | | 188 | | 190 | 0.22 | (7)194 |
| SIGZ(B) | | | | | | | | | | |
| SIGZ(C) | | | | | | | | | | |
| SIGZ(D) | -0.47 | (4) | -0.67 | (4) | -0.43 | (4) | -0.66 | (4) | -0.63 | (4) |
| SIGZ(E) | -0.78 | (2) | -0.90 | (2) | -0.83 | (3) | -0.85 | (2) | -0.86 | (2) |
| SIGZ(F) | -0.95 | (1) | -0.95 | (1) | -0.94 | (1) | -0.95 | (1) | -0.95 | (1) |
| WP | | | | | | | | | | |
| VD(AER) | | 1 | -0.61 | (5) 1 | -0.42 | (5) | -0.61 | (5) 1 | -0.35 | (5) 1 |
| VD(10D) | | | | | | | | | | |
| LD(AER) | | 1 | | | | | | | | |
| LD(10D) | | | | | | | | | | |

TABLE ENTRIES REPRESENT THE VALUE OF THE PARTIAL RANK CORRELATION COEFFICIENT (AND ITS RANK) FOR EACH COMBINATION OF SELECTED INDEPENDENT AND SELECTED DEPENDENT VARIABLE, PROVIDED THAT THE ABSOLUTE VALUE OF THIS COEFFICIENT IS GREATER THAN $T(\text{ALPHA}) = 0.43$ (40 RUNS), 0.25 (80 RUNS) OR 0.21 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.05 SIGNIFICANCE LEVEL (E.G. THE CRITICAL VALUE IS $T(\text{ALPHA}) = 0.67$ (40 RUNS), 0.41 (80 RUNS) OR 0.36 (100 RUNS) RESPECTIVELY FOR ALPHA = 0.001 SIGNIFICANCE LEVEL)

THE PERCENTAGE CONTRIBUTIONS TO UNCERTAINTY ARE GIVEN FOR EACH INDEPENDENT PARAMETER OR GROUPS OF INDEPENDENT PARAMETERS (SIGY, SIGZ, LD)

40U, 80U, 100U MEANS: IN THIS CASE ALL INDEPENDENT PARAMETERS ARE UNIFORMLY DISTRIBUTED

| #RUNS | CAECAD2 | | CAECAD2 | | CAECAD2 | | CAECAD2 | | CAECAD2 | | CAECAD2 | |
|---------|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|----------|-----|
| | 40 | (%) | 40U | (%) | 80 | (%) | 80U | (%) | 100 | (%) | 100U | (%) |
| SIGY0 | | | | | | | | | | | | |
| SIGZ0 | | | | | | | | | | | | |
| HMIX | -.62(3) | 10 | -.84(2) | 8 | -.67(3) | 2 | -.71(2) | 12 | -.76(2) | 9 | -.69(2) | 12 |
| SIGY(A) | # | 4 | # | 7 | # | 1 | # | 3 | # | 2 | # | |
| SIGY(B) | # | | .44(7) | # | # | | # | | # | | # | |
| SIGY(C) | # | | .59(4) | # | # | | # | | -.26(5) | # | # | |
| SIGY(D) | # | | # | | # | | # | | # | | # | |
| SIGY(E) | .47(5) | # | .48(6) | # | # | | .35(5) | # | # | | # | |
| SIGY(F) | # | | # | | # | | # | | # | | # | |
| SIGZ(A) | | 174 | | 170 | | 189 | | 169 | | 187 | | 170 |
| SIGZ(B) | | | | | | | | | | | | |
| SIGZ(C) | | | | | | | | | | | | |
| SIGZ(D) | | | | | | | | | | | | |
| SIGZ(E) | -.48(4) | | -.54(5) | | -.67(2) | | -.52(3) | | -.59(3) | | -.39(4) | |
| SIGZ(F) | -.91(1) | | -.93(1) | | -.93(1) | | -.81(1) | | -.91(1) | | -.84(1) | |
| WP | | | | | | | | | | | | |
| VD(AER) | -.63(2) | 10 | -.83(3) | 10 | -.55(4) | 1 | -.51(4) | 4 | -.56(4) | 3 | -.63(3) | 10 |
| VD(10D) | .44(6) | | | | | | | | | | | |
| LD(AER) | | 1 | | 1 | | | | | | | | |
| LD(10D) | | | | | | | | | | | | |



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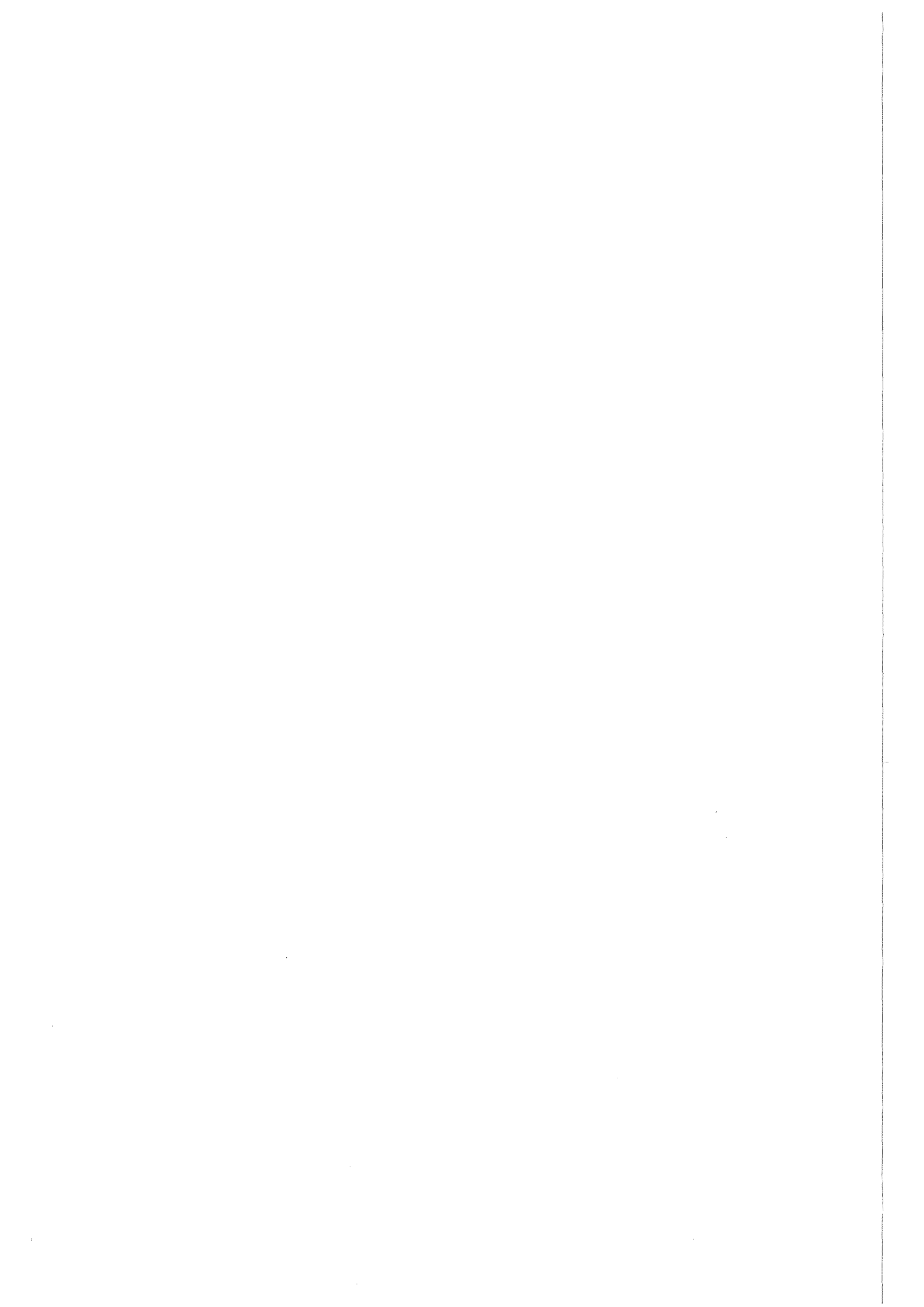
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