

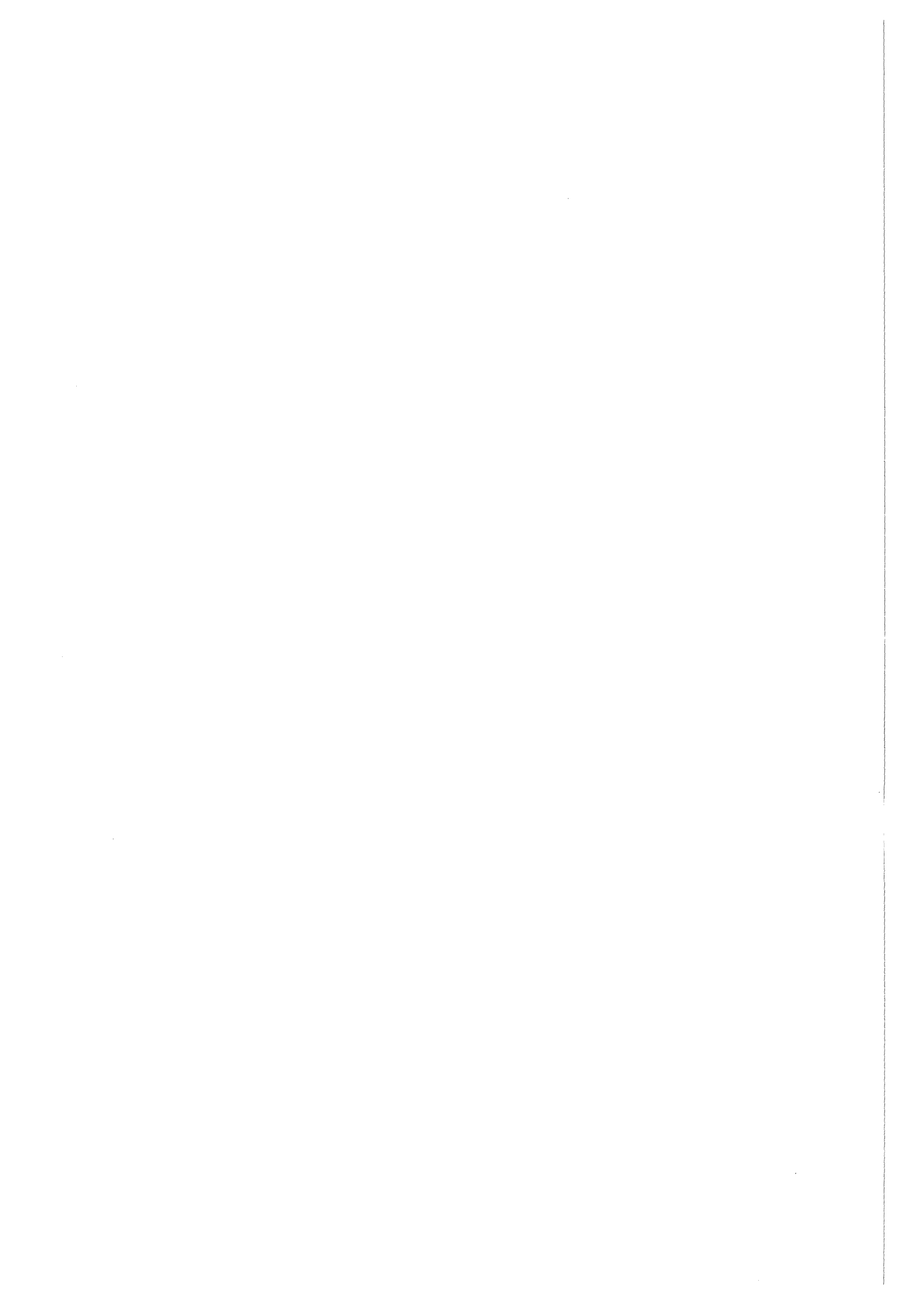
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SODIUM

A Code for Calculating Thermophysical Properties of the Sodium in the Liquid and Gaseous States

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IN THE LIQUID AND GASEOUS STATES

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Abstract

The Code SODIUM calculates the following thermal properties for the liquid or gaseous sodium :

- the density on the saturation line ,
- the pressure and its derivatives ,
- the energy-density and its derivatives ,
- the surface tension of the liquid ,
- the vapor quality in the two-phase mixture ,
- the viscosity and
- the thermal conductivity .

The thermal properties are returned either as functions of the density and temperature, or as pressure-temperature dependent properties.

The present paper describes the experimental data and mathematical methods used in calculating the thermal properties viscosity , energy density and the surface tension of the liquid. The paper also summarizes the equations used in calculating the properties pressure , heat capacity and thermal conductivity. In presenting the code the paper gives a short description of the involved routines, describes the variables occurring in the I/O-procedures of the code and shows two examples of calculations.

SODIUM. Ein Programmpaket zur Berechnung der thermischen Eigenschaften des Natriums in den flüssigen und gasförmigen Zuständen.

Zusammenfassung :

Der Code SODIUM berechnet folgende thermischen Eigenschaften für flüssiges und gasförmiges Natrium :

- den Dichteverlauf der Sättigungslinie ,
- den Druck und seine Ableitungen ,
- die Energiedichte und ihre Ableitungen ,
- die Oberflächenspannung der Flüssigkeit ,
- die Dampf-Zusammensetzung in der Zweiphasen-Mischung ,
- die Viskosität und
- die Wärmeleitfähigkeit .

Die thermischen Eigenschaften kann man sowohl als Funktionen der Dichte und der Temperatur erhalten als auch als Funktionen des Druckes und der Temperatur .

Die Messdaten und Rechenverfahren - die beim Aufbau der thermischen Eigenschaften Viskosität, Energiedichte und Oberflächenspannung benutzt wurden - werden im Detail beschrieben, die Eigenschaften Druck, Wärmekapazität und Wärmeleitfähigkeit werden dagegen nur skizzenhaft vorgestellt. Die Vorstellung des Codes umfasst einen kurzen Umriss der Rechenprogramme, die diese Zustandsdaten dem Benutzer verfügbar machen und die Beschreibung des Datenzuganges beim Code. Zwei Rechenbeispiele sollen den Umgang mit dem Code erleichtern.

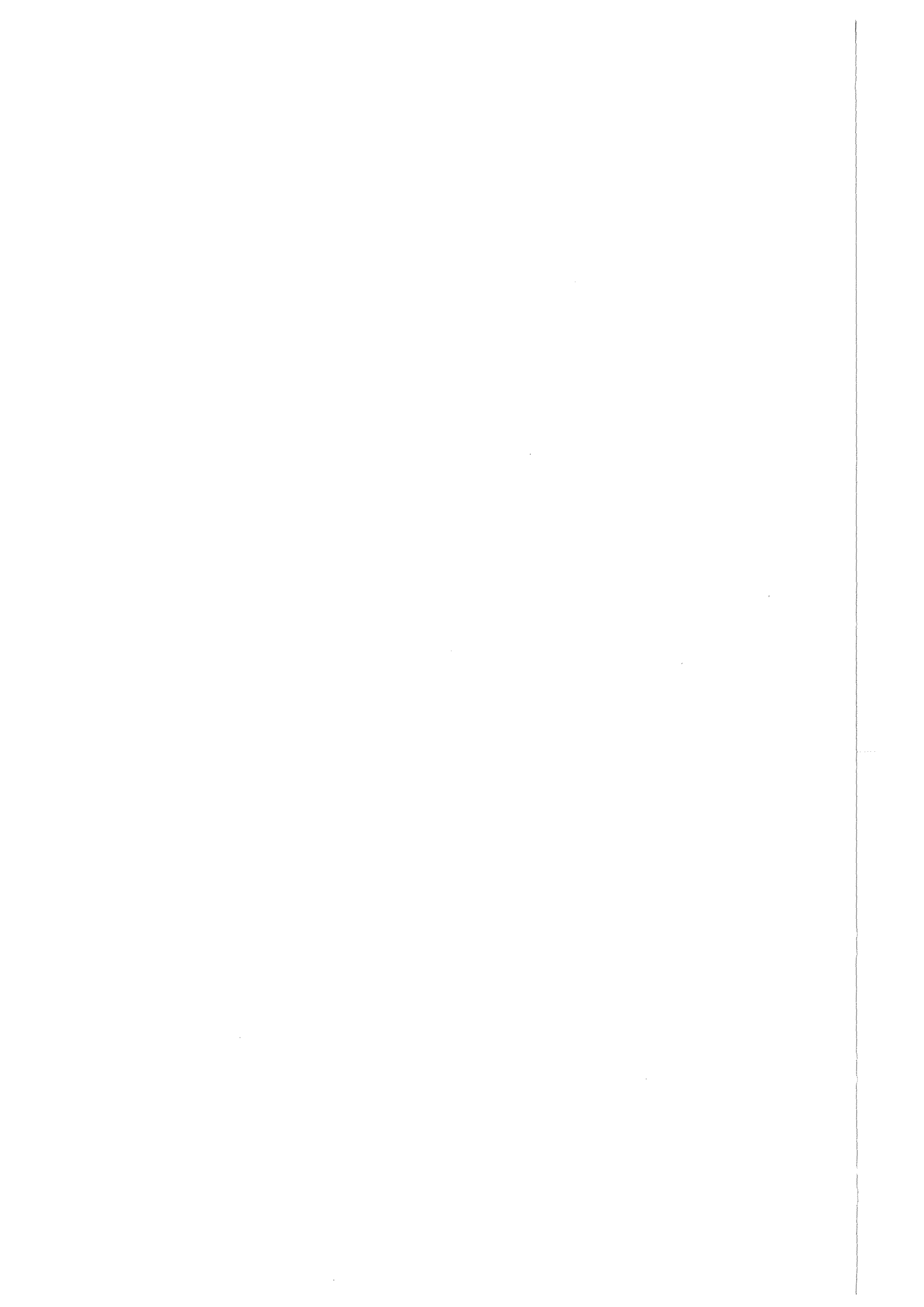


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1.0 Glossary.

ALF , α	0.11 , critical exponent for CV
BET , β	0.325 , critical exponent for the density
CV , C_V	heat capacity at constant volume $\left(\frac{J}{kg \cdot K}\right)$
CP , C_P	heat capacity at constant pressure $\left(\frac{J}{kg \cdot K}\right)$
CVL , CV(1)	$C_V(\rho_L)$, heat capacity of the saturated liquid
CVV , CV(2)	$C_V(\rho_V)$, heat capacity of the saturated vapor
CS(1) , $\tilde{C}_{V,L}$	$C_V(\rho_L - 0)$ heat capacities on the border
CS(2) , $\tilde{C}_{V,V}$	$C_V(\rho_V + 0)$ of the two-phase mixed state
EPS	$T/T^x - 1$, 'distance' to the saturation line
GAM , γ	1.24 , critical exponent for $\partial P / \partial \rho$
GIS , g	surface tension of the liquid (J/m^2)
H	density of the enthalpy (J/kg)
HR	$\partial H / \partial \rho$, density derivative of H
K	index of the sodium-state :
= 1, 2, 3	liquid , vapor , mixed
mixed state	$T_M < T < T_c$ with $\rho_V < \rho < \rho_L$
μ	$\frac{u}{e_u - 1}$
P	pressure (J/m^3)
PC , P_c	critical pressure (25.646 J/m^3)
PI , Π	$\frac{p^x}{RGAS \cdot T}$, reduced vapor pressure
SIG	$T \cdot \Pi' / \Pi$ reduced derivative of the vapor pressure
PS , P^x	vapor pressure
PT	$\partial P / \partial T$, temperature derivative of the pressure
PR	$\partial P / \partial \rho$, density derivative of the pressure
PR# , P_ρ^x	$\frac{\partial P / \partial \rho}{RGAS \cdot T}$
PT# , P_T^x	$\frac{\partial P / \partial T}{RGAS \cdot \rho}$
QT , Q_T	thermal conductivity $\left(\frac{W}{m \cdot K}\right)$
QTC	thermal conductivity at the critical point $\left(5 \cdot \frac{W}{m \cdot K}\right)$

QTL , QTS(1) ,	thermal conductivities of the
QTV , QTS(2)	saturated states
RH , ρ	density (kg/m^3)
RHC , ρ_c	critical density (230. kg/m^3)
RHL , RHS(1)	ρ_L , density of the saturated liquid
RHV , RHS(2)	ρ_V , density of the saturated vapor
RHM , ρ_M	melting density of the liquid (927.4775 kg/m^3)
RKS , r	$\frac{T}{\rho} \frac{d\rho}{dT}$, reduced derivative of the saturation line
RGAS	the gas-law constant of the sodium (361.65 $\frac{J}{kg \cdot K}$)
s	$\ln(\rho_c) / \ln(\rho)$
S.L.	saturation line
T	temperature (K)
TB , T_B	boiling point (1154 K)
TC , T_c	critical temperature (2508 K)
TM , T_M	melting point (371 K)
TR , T^x	saturation temperature corresponding to the density ρ
U	density of the internal energy (J/kg)
UR	$\partial U / \partial \rho$, density derivative of U
u	$0.2 \frac{T_c - T^x}{T - T^x}$
V , v	$1/\rho$, specific volume
X , x	$1 - T/T_c$, 'distance' to the critical point
XI , ξ	$\frac{m(V)}{m(L) + m(V)}$, vapor quality in the mixed state
YT , Y_T	dynamic viscosity ($\frac{m}{kg \cdot s}$)
YTC	viscosity at the critical point ($6.5 \cdot 10^{-5} \frac{m}{kg \cdot s}$)
Z , z	$\frac{P \cdot V}{RGAS \cdot T}$, factor of reality
ZV	factor of reality of the saturated vapor

Abbreviations, valid only in the description of supercritical states:

a	$y \cdot A$
Δf	$f(w) - f(1)$
G_j, H_j	polynomial-coefficients in the description of z
w	$2 \cdot S(1) + (2 \cdot B - a) \cdot \Delta w$
S	$B \cdot w^2 - A \cdot w \cdot y + 1$
S'	$2 \cdot B \cdot w - a$
σ	$4 \cdot B - a^2$
Y, y	T_c/T , reduced temperature
W, w	ρ/ρ_c , reduced density
ζ	$6 \cdot a \cdot B / \sqrt{\sigma}$



2.0 Main features of the code system SODIUM.

SODIUM is a code system providing the user with different thermal properties of this metal in the liquid and gaseous states. The code system calculates these thermal properties either as functions of ρ and T (SODIUM) or in the (P,T)-dependence (SODINV). All the occurring variables and the supplied properties are given in S.I.-units. In the calculations the code system depends on the property - descriptions given in the papers /1/ and /2/ .

The routine SODIUM returns - for a given temperature and density - the following thermal properties :

- the pressure , P and its derivatives
- $PT = \partial P / \partial T$
- $PR = \partial P / \partial \rho$
- the density of the internal energy , U and its derivatives
- $CV = \partial U / \partial T$
- $UR = \partial U / \partial \rho$
- the thermal conductivity , Q_T
- the viscosity , Y_T
- the vapor quality in the two-phase mixture , ξ and
- the surface tension of the liquid , GIS .

The routine SODINV returns for a given P and T as a rule the same thermal properties as SODIUM. The exceptions are:

- instead of P SODINV calculates the density ρ ,
- instead of U SODINV returns the enthalpy-density , H ,
- instead of C_V SODINV returns the constant- P specific heat C_P .

SODINV fails to calculate the thermal properties in the liquid-vapor mixed state, because the properties ρ , C_P are not, or not unequivocally defined in this part of the (P,T)-plane.

The code system has the following range-limitations : On calling the temperature must be higher than 370 K and the density less than 930 kg/m^3 , i.e. the sodium must be liquid or gaseous. Figure 1 displays the range of validity of the code system in the temperature-specific volume - plane ($v = 1/\rho$). The shape of the saturation line in this figure is only roughly correct.

The experimental data used for the code SODIUM depend mainly on the measured properties recommended in the paper of Golden and Tokar /3/. This means, that

- vapor pressure, liquid density and PVT-data were taken as measured by the NRL-group Ewing, Miller, Spann, Steinkuller, Stone, Williams and
- the heat capacities were developed from the C_P -values measured by Ginnings, Douglas and Ball /4/.

In the Figure 1 a dashed horizontal line indicate the upper temperature-limit of these measurements. A complete list of the reports mentioned above is given in /1/.

In extrapolating the properties to the critical point the hypothesis of the Universality of the Critical Exponent was used (see e.g. in /5/) .

The following figures show some thermal properties - calculated by SODIUM in the sub-region , in wich P exceeds the atmospheric pressure ($T \geq T_B$, $\rho \geq \rho_V(T_B)$) - as temperature-specific-volume dependent surfaces :

Figure 6 shows the factor of reality and
Figure 7 the pressure ,
Figure 13 , Figure 14 and Figure 15 show different views of the heat capacity ,
Figure 17 shows the thermal conductivity and
Figure 23 the viscosity ,
Figure 39 and Figure 40 display different sides of the density of the internal energy,
Figure 41 and
Figure 42 show the (T,v)-surfaces of the enthalpy- resp. of the entropy-density.
Figure 43 is the Mollier-surface of the Sodium.

Historical notes : The first version of the code - named originally KANAST - was completed in the year 1975. This first version, KANAST-1 had many common features with the present code, as well in the range of validity of the code as in the thermal properties calculated by it. Unfortunately in the following year Bhise and Bonilla /6/ presented a new and measured critical temperature, $T_c = 2503$ K , which was much lower than the estimated $T_c = 2850$ K used in KANAST-1. So it was necessary to revise the code accordingly.

The "critically" corrected , second version, KANAST-2 was finished in 1982. This version calculated already - in the liquid and vapor states - the pressure with its derivatives , C_V and the thermal conductivity . In the main part of the subcritical domain the saturation density , the pressure derivative $\partial P/\partial T$, the heat capacity and the thermal conductivity were taken from widely spaced function tables.

A first version of the (P,T)-code SODINV - KANAPT - was also completed in 1982. Both codes were written in the FORTRAN 66 - language and mixed freely REAL*8 and REAL*4 constants and functions in the calculations.

The third version of KANAST, KANAST-3 - finished in 1986 - included also a tentative description for the density of the internal energy . The language changed from FORTRAN 66 to FORTRAN 77 and the REAL*4 words were dropped.

This last version of KANAST was in some respect worse than its predecessor ; as well the pressure as the density of the internal energy showed an unphysical step on the low - density part of the critical isotherm.

Figure 2 - this figure has been copied from /1/ - shows the reduced pressure as calculated by the code KANAST-2 , Figure 3 displays the same property calculated by KANAST-3 . The step in the energy density - produced by KANAST-3 - can be seen from both sides on the Figure 4 and the Figure 5 .

The code system SODIUM has been created, mainly, to avoid these shortcomings of KANAST-3 , but also to include the new properties viscosity and the surface tension of the liquid. SODIUM abandons tabulating key properties - as , e. g. the saturation density - and the advantages of the FORTRAN 77 - language are fully used. To ease the development and the checking of SODIUM a stand-alone sodium-property-code , ZUNGE (s. "Appendix D. The code ZUNGE") was written in the on-line language SPEAKEASY /7/ . All of the fittings , smoothings and numerical integrations - mentioned somewhere in this paper - were performed by ZUNGE.

3.0 Calculating the saturation line .

In describing the thermal properties it is usefull to divide the (ρ, T) - validity-domain of the sodium into the following five sub-regions (see also Figure 1) :

- the saturation line,
- the compressed liquid,
- the overheated vapor,
- the two-phase mixed state and
- the supercritical gas.

The saturation line $\{\rho_L(T), \rho_V(T)\}$ is defined as the densities ρ_V and ρ_L , confining a region in which vapor and liquid remain in equilibrium with each other , i.e. :

$$\text{for } T \leq T_c \quad \text{and} \quad \rho_V(T) \leq \rho \leq \rho_L(T) \quad [3.1]$$

$$P(\rho, T) \equiv P^x(T)$$

The saturation line divides the subcritical domain into the parts

liquid	$\rho > \rho_L(T)$
vapor	$\rho < \rho_V(T)$
mixed state	$\rho_V(T) < \rho < \rho_L(T)$.

In the low-temperature part ($T < 1700$ K) the description of the saturation line has been changed : instead of tabulating the density functions as done in KANAST , SODIUM returns to the polynomials described in /1/ and /2/ . This means for the cold , saturated liquid :

$$\rho_L(T) \text{ [kg/m}^3 \text{]} \equiv 1011.65 - .220523 \cdot T - 1.92252 \cdot 10^{-5} \cdot T^2 + 5.63797 \cdot 10^{-9} \cdot T^3 \quad [3.2]$$

In the cold vapor the saturated density is calculated according to the Eq.

$$\rho_V(T) = \frac{P^x}{RGAS \cdot T \cdot z_V} \quad , \quad z_V(T) = e^{\tilde{Z}(T)} \quad \text{with} \quad [3.3]$$

$$\tilde{Z}(T) = \sum_{j=1}^5 A_{kj} \cdot T^{j-1} \quad \text{if} \quad T_{k-1} < T \leq T_k \quad , \quad k = 1, \dots, 4$$

The coefficients A_{kj} - listed in Table 1 - and the coefficients a_{kj} of the function -

$$\frac{d \ln z_V}{d \ln T} = \sum_{j=1}^5 a_{kj} \cdot T^{j-1}$$

- given in /2/ are related to each other through the following relation (s. "Appendix A. Integrating a property described by a set of polynomials.") :

$$\begin{aligned}
A_{0l} &\equiv 0 && \text{for } \forall l \\
A_{kl} &\equiv \frac{a_{k,l-1}}{l-1} && , \quad l=2, \dots, 5, \quad k=1, \dots, 4 \\
A_{k1} &\equiv A_{k-1,1} + \sum_{l=2}^5 (A_{k-1,l} - A_{kl}) \cdot T_{k-1}^{l-1} && \quad k=1, \dots, 4.
\end{aligned} \tag{3.4}$$

The near-critical description of the saturation line ($T > 1700$ K)

$$\begin{aligned}
\rho_L(T) &= \rho_c \cdot \left(1 + 2 \cdot x^\beta + x \cdot \sum_{j=1}^4 E_{Lj} \cdot x^{j-1} \right) \\
\rho_V(T) &= \rho_c \cdot \left(1 - 2 \cdot x^\beta + x \cdot \sum_{j=1}^4 E_{Vj} \cdot x^{j-1} \right)
\end{aligned} \tag{3.5}$$

with $\rho_c = 230 \text{ kg/m}^3$, $x \equiv 1 - \frac{T}{T_c}$ and $\beta = 0.325$

remains practically unchanged , only some of the E_V -s have been modified slightly (compare /2/ and the Table 2) to get a moore smooth density-transition at the switching point , $T = 1700$ K .

T^x , the inverse function of the saturation line is also needed by SODIUM for calculating different thermal properties . T^x - the saturation temperature - is defined as the temperature , at which a given density equals to the saturated density of the liquid or to the saturated density of the vapor , i. e.

$$\rho \equiv \begin{cases} \rho_L(T^x) & \text{for } \rho \geq \rho_c \\ \rho_V(T^x) & \text{for } \rho < \rho_c \end{cases} \tag{3.6}$$

SODIUM calculates T^x by using newly developed approximations . For liquid densities ($\rho \geq \rho_c$) T^x is :

$$T^x(\rho) = \sum_{j=1}^6 A_{L,kj} \cdot T^{j-1} \quad \text{if } \rho_{k-1} < \rho \leq \rho_k, \quad k=1, \dots, 7 \tag{3.7}$$

and for vapor densities ($\rho \leq \rho_c$) T^x is described as :

$$T^x(\rho) = \sum_{j=1}^6 A_{V,kj} \cdot s^{j-1} \quad \text{if } s_{k-1} < s \leq s_k, \quad k=1, \dots, 9 \tag{3.8}$$

with the density-variable

$$s \equiv \frac{\ln(\rho_c)}{\ln(\rho)} \tag{3.9}$$

The coefficients $A_{L,kj}$, $A_{V,kj}$ are tabulated in Table 3 resp. Table 4 . The temperature-deviations of these new approximations are less than 0.001 K in the whole density-range.

4.0 Calculating the pressure .

In the calculation of the pressure SODIUM also returned to the descriptions given in /1/ resp. /2/ , instead of using - as in KANAST - tabulated functions for the pressure derivatives.

In the two-phase mixture the pressure is the density independent vapor pressure , P^x :

$$\ln P^x = 9.6164 - \frac{12153.}{T} - 0.195 \cdot \ln T , \quad [4.1]$$

P^x in M Pascals , T in K .

The only existing pressure derivative is the temperature-derivative of P^x .

On the saturation line the pressure equals also to the vapor pressure , P^x , but here it depends also on the density . The T-dependence of the saturation density connects the two pressure derivatives in this region by the Eq. .

$$\frac{dP^x}{dT} \equiv \frac{\partial P}{\partial T}(\rho, T) + \frac{\partial P}{\partial \rho}(\rho, T) \frac{d\rho}{dT} , \quad [4.2]$$

so it is enough to define one of these derivatives , the remaining can be calculated by the Eq. [4.2] .

In the low temperature part of the saturation line the temperature-derivative is used as a primary function . In the liquid , this property - calculated in /1/ via the velocity of sound - is approximated with a set of polynomials :

$$\frac{\partial P_L}{\partial T}(T) = RGAS \cdot \rho_L(T) \cdot \sum_{j=1}^6 A_{kj} \cdot T^{j-1} \text{ if } T_{k-1} < T \leq T_k , k = 1, \dots, 5 . [4.3]$$

Table 6 list the coefficients A_{kj} . In the vapor ,

$$\frac{\partial P_V}{\partial T}(T) = RGAS \cdot \rho_V(T) \cdot \sum_{j=1}^4 A_{kj} \cdot T^{j-1} \text{ if } T_{k-1} < T \leq T_k , k = 1, \dots, 4 [4.4]$$

uses the old A_{kj} -s , described in in /2/ (or in Table 7) .

In the high temperature part of the saturation line the calculation remains as it was : the density derivatives are set to

$$\frac{\partial P}{\partial \rho}(T) = \frac{p_0}{\rho_c} \cdot \rho(T) \cdot x^\gamma \cdot \sum_{j=1}^5 E_j \cdot x^{j-1} \text{ with } x = 1 - \frac{T}{T_c} [4.5]$$

and the temperature derivatives are calculated via Eq. [4.2] . The coefficients of the description are

$$p_0 = 7.15089 \frac{M \text{ Joule}}{kg} \quad \text{and} \quad \gamma = 1.24 [4.6]$$

and the E_j -s are given in Table 8 (s. also /2/) .

In the compressed liquid and in the high-density part of the saturated vapor $\rho \geq 10 \text{ kg/m}^3$ the code calculates the pressure according to the equation :

$$P(\rho, T) = P^x(T^x) + (T - T^x) \cdot \frac{\partial P}{\partial T}(T^x) \quad [4.7]$$

(s. Eq. (74) in /1/) . $T^x = T^x(\rho)$ is here the inverse function of the saturation line (s. Eq. [3.6]) and

$$\frac{\partial P}{\partial T}(T^x) \equiv \frac{\partial P}{\partial T} [\rho(T^x), T^x] \quad [4.8]$$

is the T-derivative of the pressure of the saturated liquid resp. in the saturated vapor .

In the low-density-part of the overheated vapor ($\rho < 10 \text{ kg/m}^3$) Eq. [4.7] does not suffice to calculate the whole pressure and supplementary , (ρ , T) - dependent terms are needed. The complete formulas for the pressure and its T-derivative in this region are (s. Eq. (82)-(86) and (90) in /1/) :

$$P(\rho, T) = P^x(T^x) + (T - T^x) \cdot \frac{\partial P}{\partial T}(T^x) + (T - T^x) \cdot RGAS \cdot \rho \cdot G(s) \cdot \mu(u) \quad [4.9]$$

and

$$\frac{\partial P}{\partial T}(\rho, T) = \frac{\partial P}{\partial T}(T^x) + RGAS \cdot \rho \cdot G(s) \cdot \mu(u) \cdot [u + \mu(u)] .$$

G in this Eq. depends - via s (s. Eq. [3.9]) only on the density :

$$G(s) = [s \cdot (0.32 - s)]^2 \cdot \eta(s) \quad \text{with}$$

$$\eta = \sum_{j=1}^4 A_{kj} \cdot s^{j-1} \quad \text{if} \quad s_{k-1} < s \leq s_k \quad , \quad k = 1, 2, 3 \quad [4.10]$$

- for the coefficients see Table 5 - μ is the function

$$\mu(u) = \frac{u}{e^u - 1} \quad [4.11]$$

with a the temperature-dependence hidden in the variable

$$u = 0.2 \frac{T_c - T^x}{T - T^x} \quad [4.12]$$

At critical and supercritical temperature the pressure and its derivatives are calculated as described in /1/ and /2/ :

$$P = RGAS \cdot \rho \cdot T \cdot z$$

$$\frac{\partial P}{\partial T} = RGAS \cdot \rho \left(z - y \frac{\partial z}{\partial y} \right)$$

$$\frac{\partial P}{\partial \rho} = RGAS \cdot T \left(z + w \frac{\partial z}{\partial w} \right) \quad [4.13]$$

with $y = T_c/T$ and $w = \rho/\rho_c$

z - the factor of reality - is the following , van-der-Waals-like function :

$$z = 1 + w \cdot y \cdot \left[\frac{A}{S} - \sum_{j=1}^5 [G_{kj} + (y-1) \cdot H_{kj}] \cdot w^{j-1} \right] \quad [4.14]$$

if $w_{k-1} < w \leq w_k$, $k = 1, \dots, 9$,

with $S = 1 - w \cdot y \cdot A + w^2 \cdot B$

The constants A and B are

$$A = 0.814159413 \quad , \quad B = 1.297860659 \quad [4.15]$$

and the coefficients G and H are tabulated in Table 9 resp. in Table 10 .

The surface of the factor of reality calculated by SODIUM in the sub-region with pressures exceeding one atmosphere displays Figure 6. The corresponding surface of the pressure can be seen on the Figure 7 .



5.0 Calculating the heat capacity .

SODIUM calculates the heat capacity by

1. defining the heat capacity on the baseline

$$\rho_B(T) = \begin{cases} \rho_L(T) - 0 & \text{for } T < T_c \\ \rho_c & \text{for } T \geq T_c \end{cases} \quad [5.1]$$

2. and using the density derivative

$$\frac{\partial C_V}{\partial \rho} = -\frac{T}{\rho^2} \cdot \frac{\partial^2 P}{\partial T^2} \quad [5.2]$$

to calculate $C_V(\rho, T)$ along the T - isotherm (s. /1/) .

3. At subcritical temperatures the heat capacity-steps , occuring at the intersections of the saturation line

$$\tilde{C}_{V,L} - C_{V,L} \equiv C_V(\rho_L - 0) - C_V(\rho_L) \quad \text{resp.} \quad [5.3]$$

$$\tilde{C}_{V,V} - C_{V,V} \equiv C_V(\rho_V + 0) - C_V(\rho_V)$$

must also be taken in account.

The heat capacity on the path $\rho_B(T)$ is defined as :

$$C_{V,B}(T) = \begin{cases} C_{PL}(T) + c_{(-)} \left[\left(1 - \frac{T}{T_c}\right)^{-\alpha} - d(T) \right] & \text{for } T < T_c \\ c_{(+)} \left(\frac{T}{T_c} - 1 \right) & \text{for } T \geq T_c \end{cases} \quad [5.4]$$

$C_{PL}(T)$ in this equation is the CP-polynomial of the liquid sodium after Ginnings , Douglas and Ball (/4/)

$$C_{PL}(T) \equiv (4.50751 - T \cdot 2.30472 \cdot 10^{-3} + T^2 \cdot 1.27973 \cdot 10^{-6}) \cdot RGAS \quad ,$$

the constants are

$$\alpha = 0.11$$

$$c_{(-)} = 5.53 \cdot RGAS$$

$$c_{(+)} = 2.20 \cdot RGAS$$

and $d(T)$ is a polynomial

$$d(T) \equiv 1.00626 + T \cdot 2.0137 \cdot 10^{-5} + T^2 \cdot 3.116 \cdot 10^{-8}$$

At subcritical temperatures and beside the baseline SODIUM calculates the heat capacity as follows (/1/) :

- the heat capacity-step at the saturated liquid is

$$C_V(\rho_L, T) = \tilde{C}_{V,L}(T) - \frac{r_L^2}{T} \frac{\partial P}{\partial \rho}(\rho_L, T) \quad [5.5]$$

with

$$\tilde{C}_{V,L}(T) \equiv C_V(\rho_L \rightarrow 0, T) = C_{V,B}(T) ,$$

- in the compressed liquid it is

$$C_V(\rho, T) = C_V(\rho_L, T) \quad [5.6]$$

since here the derivative $\partial C_V / \partial \rho$ vanishes ,

- in the mixed , two phase state one has

$$C_V(\rho, T) = \tilde{C}_{V,L}(T) + T \frac{d^2 P^x(T)}{dT^2} \left[\frac{1}{\rho} - \frac{1}{\rho_B(T)} \right] \quad [5.7]$$

since here the pressure , P^x is density-independent ,

- the heat capacity-step at the saturated vapor is

$$C_V(\rho_V, T) = \tilde{C}_{V,V}(T) - \frac{r_V^2}{T} \frac{\partial P}{\partial \rho}(\rho_V, T) \quad [5.8]$$

with

$$\tilde{C}_{V,V}(T) \equiv C_V(\rho_V \rightarrow 0, T) = \tilde{C}_{V,L}(T) + T \frac{d^2 P^x(T)}{dT^2} \left[\frac{1}{\rho_V} - \frac{1}{\rho_B(T)} \right]$$

- and in the overheated vapor the heat capacity is developed as

$$C_V(\rho, T) = C_V(\rho_V, T) + \int_{\rho_V(T)}^{\rho} d\rho \cdot \frac{\partial C_V}{\partial \rho} . \quad [5.9]$$

The property " r " , occuring in the Eq. [5.5] and Eq. [5.8] is the logarithmic derivative of the saturation line :

$$r = \frac{T}{\rho} \frac{d\rho}{dT} . \quad [5.10]$$

In the supercritical state the heat capacity is based on the critical density (/1/ , Eq. 119) :

$$C_V(\rho, T) = C_V(\rho_c, T) + RGAS \cdot y^2 \left[2 \sum_{j=1}^5 \frac{w^j - 1}{j} H_j - A^2 \cdot \Delta I \right] . \quad [5.11]$$

$w = \rho / \rho_c$, $y = T / T_c$ are here the reduced density resp. temperature , the H-s are the coefficients used in Eq. [4.14] and ΔI is

$$\Delta I = \frac{1}{\sigma} \left[\frac{4\zeta}{\sqrt{\sigma}} \arctan \left(\frac{\Delta w \sqrt{\sigma}}{w} \right) + \Delta \varphi \right] . \quad [5.12]$$

$\Delta \varphi$ in this Eq. is the (w,1) - difference of the function

$$\varphi(w) = \frac{1}{B \cdot S} \left[\zeta \cdot S' - \sigma + a \frac{S' - \sigma \cdot w}{S} \right] . \quad [5.13]$$

"Appendix B. Calculating the difference of the expression $\varphi (w)$." evaluates $\Delta \varphi$.

In the **overheated vapor** the calculation begins with the heat capacity of the saturated vapor :

$$C_V(\rho, T) = C_V(\rho_V, T) + \Delta C_V(\rho, T)$$

Since it is not possible , to simplify the derivative

$$\frac{\partial C_V}{\partial \rho} = -\frac{T}{\rho^2} \cdot \frac{\partial^2 P}{\partial T^2} = -RGAS \frac{G(s)}{\rho} T \frac{\partial[\mu \cdot (\mu + u)]}{\partial T}$$

(s. /1/ , Eq. 82) enough , to allow an integration in analytical form , the C_V -departure

$$\begin{aligned} \Delta C_V(\rho, T) &= \int_{\rho_V(T)}^{\rho} d\rho \cdot \frac{\partial C_V}{\partial \rho} \\ &= RGAS \int_{T^x}^T \frac{dt}{t} r_V(t) \frac{T \cdot G(s)}{T-t} \mu [\mu + u] [2(\mu - 1) + u] \end{aligned} \quad [5.14]$$

(/1/ , Eq. 94) must be integrated numerically. The Figure 8 shows shapes of heat capacities , integrated along some isotherms in the overheated vapor . The saturation temperature $T^x = 100$ K on this figure corresponds to a vapor "density" of $\approx 3 \cdot 10^{-48} \text{ kg/m}^3$.

KANAST used to express these numerically-integrated C_V -s with the following description:

$$\begin{aligned} \Delta C_V(\rho, T) &= \frac{\overline{\Delta C_V}(T)}{e^{2F^o(q, T)} + 1} , \quad q = \frac{T^x}{T} \\ F^o(q, T) &= Y0(T) + Y1(T) \cdot q + Y3(T) \frac{[q - YZ(T)]^3}{q(1-q)} \end{aligned} \quad [5.15]$$

with the restrictions

$$|F^o| \leq 4 \quad \text{and} \quad q(1-q) \geq 0.0001$$

All the functions in this expression $\overline{\Delta C_V}$, $Y0$, ... , YZ were polynomials of the form :

$$\overline{\Delta C_V}(T) = \sum_{j=1}^4 A_{kj} \cdot T^{j-1} \quad \text{if} \quad T_{k-1} < T \leq T_k , \quad k=1, \dots, 6 \quad [5.16]$$

Eq. [5.15] was a poor approximation for the integrated C_V -s , the ΔC_V - deviations exceeded 5 % in some (ρ, T) - regions of the vapor .

For the new version of the code the C_V -departures on 40 isotherms were integrated numerically. The resulting $\Delta C_V(\rho, T)$ -s were brought in a reduced form :

$$\Delta C_V(\rho, T) = \overline{\Delta C_V}(T) \cdot [F(q, T) - 1] \quad \text{with} \quad [5.17]$$

$$\overline{\Delta C_V}(T) = C_V(\rho_V, T) - C_V(\rho = 0, T) \quad \text{and} \quad q = \frac{T^x}{T}$$

SODIUM uses the C_V -difference $\overline{\Delta C_V}(T)$ (s. Figure 9) as a tabulated function . At temperatures, not corresponding to a table value , a cubic spline procedure is used to calculate $\Delta C_V(T)$.

Figure 10 shows $F (q , T)$, the surface of the reduced C_V -departure. SODIUM calculates $F (q , T)$ as follows :

- The surface is tabulated as a set of contour lines

$$q_i \equiv q_i(T) \quad \text{with} \quad F(q_i, T) \equiv C_i \quad , \quad i = 1, \dots, 12 .$$

The Figure 11 shows 10 of the 12 contour-levels , Figure 12 displays the contour-lines themselves .

- Using again - if needed - a cubic spline procedure allows to find for every T' a set of q'_i -s , with

$$F(q'_i, T') = C_i \quad , \quad i = 1, \dots, 12$$

- In a last step SODIUM uses the function

$$\{ (0.0, 0.0) , (q'_0, 0.0) , (q'_1, C_1) , \dots , (q'_{12}, C_{12}) , (q'_{13}, 1.0) , (1.0, 1.0) \}$$

$$\text{with} \quad q'_0 = \frac{2}{3} q'_1$$

$$\text{and} \quad q'_{13} = \frac{2 q'_{12} + 1}{3}$$

to calculate - with the spline procedure - the value $F (q' , T')$.

Part of the heat capacity-surface calculated by the code SODIUM is shown on the Figure 13 - as seen from the side of the compressed liquid , on the Figure 14 - as seen from the side of the overheated vapor and on the Figure 15 - as seen from supercritical temperatures . These figures use a C_V -value of $12 \cdot \text{RGAS}$ on the critical isotherm , instead of the correct value , $C_V = \infty$.

6.0 Calculating the thermal conductivity .

In the cold part of the saturation line the thermal conductivities are described with polynomials :

$$Q_{T,L} = Q_{T,c} \cdot \sum_{j=1}^4 D_L(k,j) \cdot T^{j-1} \quad , \quad k = 1, \dots, 3 \quad [6.1]$$

respectively

$$Q_{T,V} = Q_{T,c} \cdot \sum_{j=1}^4 D_V(k,j) \cdot T^{j-1} \quad , \quad k = 1, \dots, 4 \quad [6.2]$$

In the near-critical saturated states the following functions are used:

$$\begin{aligned} Q_{T,L}(T) &= Q_{T,c} \cdot \left(1 + 2 \cdot x^\beta + x \cdot \sum_{j=1}^4 D_L(4,j) \cdot x^{j-1} \right) \\ Q_{T,V}(T) &= Q_{T,c} \cdot \left(1 - 2 \cdot x^\beta + x \cdot \sum_{j=1}^4 D_V(5,j) \cdot x^{j-1} \right) \end{aligned} \quad [6.3]$$

with $x = 1 - T/T_c$ and $\beta = 0.325$.

The critical value of the thermal conductivity is according to /1/

$$Q_{T,c} = 5.0 \frac{W}{m \cdot K} \quad [6.4]$$

The other constants in these equations are given in the Table 11 and Table 12 .

These thermal conductivity descriptions are essentially the same , as those presented in /1/ . The only difference is , that the near-critical description , Eq. [6.4] begins instead of 1280 K at a higher temperature (1700 K) to ensure a moore smooth transition at this point. A comparison between the old and new saturated thermal conductivity-functions can bee seen on the Figure 16 .

In the mixed state the thermal conductivity is calculated by using a simple model to describe the structure of this mixture of liquid and vapor components (s. /1/) , resulting in the following Q_T -function in this (ρ , T) - domain :

$$\frac{1}{Q_T} = \frac{1-\eta}{Q_{T,V}} + \frac{\eta}{Q_{T,V} + \eta^2(Q_{T,L} - Q_{T,V})} \quad [6.5]$$

(s. /1/ , Eq. 64) , with the mixture parameter

$$\eta = \sqrt[3]{\frac{\rho - \rho_V}{\rho_L - \rho_V}} \quad [6.6]$$

In the remaining areas of the (ρ , T) - domain , namely

- in the compressed liquid ,
- in the overheated vapor and
- in the supercritical gas

the thermal conductivity depends - for the sake of simplicity - only on the density (s. /1/):

$$Q_T(\rho, T) \equiv \begin{cases} Q_{T,L}(T^x) & \text{for } \rho \geq \rho_c , \\ Q_{T,V}(T^x) & \text{for } \rho < \rho_c . \end{cases} \quad [6.7]$$

Figure 17 shows part of the thermal conductivity-surface - calculated by SODIUM - in the T-V-dependence.

7.0 Calculating the dynamic viscosity.

Using two recently published viscosity measurements and extrapolating the measured values a temperature-density-dependent description for the viscosity of the sodium was constructed .

The viscosity data of the liquid sodium are measured by Shpil'rain and Fomin /9/. Besides the measured viscosities the paper presents also the following least-squares-equation for the data :

$$\log_{10} Y_{T,L} = -1.68144 + \frac{234.655}{T} - 0.42961 \cdot \log_{10} T , \quad Y_T \text{ in } \frac{g}{cm \cdot s} . \quad [7.1]$$

The temperature-range of the measurements is 160 - 800 C ° , but the authors compare their formula with the $Y_{T,L}(T)$ -data of other groups and they state a good agreement in the extended range of 160 - 1200 C ° .

Figure 18 presents the viscosity-function of Shpil'rain and Fomin . It also shows the $Y_{T,L}(T)$ recommended by Golden and Tokar in /3/ . At low temperatures ($T < 2250$ K) SODIUM uses Eq. [7.1] in a reduced form :

$$\log_{10} \left(\frac{Y_{T,L}}{Y_{T,c}} \right) = D_L(1,1) + \frac{D_L(1,2)}{T} - D_L(1,3) \cdot \log_{10} T . \quad [7.2]$$

In calculating the viscosity of the cold saturated vapor SODIUM uses the data measured by Timrot and Varava , /12/ . Their $Y_{T,V}$ -values can be seen on the Figure 20 along with the older data of Stepanov et al. , /10/ , cited in /3/ .

Boris Stefanov /11/ estimates the errors of /10/ and /12/ and recommends the data of Timrot and Varava . The Figure 20 and the Figure 19 presents also the the viscosity function which he calculated for the monatomic sodium vapor. Some other monatomic vapor-viscosities are shown on the Figure 19 . This figure shows also a monatomic viscosity , calculated from the heat capacity and thermal conductivity via the conductivity-relation :

$$Q_T = a \cdot C_V \cdot Y_T , \quad a = 2.4 \text{ for monatomic vapor} \quad [7.3]$$

(s. e.g. /8/) .

SODIUM calculates $Y_{T,V}$ at low temperatures ($T < 1950$ K) by using a linearized version of the data given in /12/ :

$$Y_{T,V} = D_V(1,1) + D_V(1,2) \cdot T . \quad [7.4]$$

The viscosities at the near-critical part of the saturation line were calculated by extrapolating the functions of /9/ and /12/ . For this end it was assumed , that in the vicinity of the critical point the viscosity and the thermal conductivity behaves in a similar way. Consequently , to describe the near-critical part of the saturated viscosity the following functions were chosen (compare Eq. [6.3] or the Eq.s 56 and 57 in /1/) :

$$\begin{aligned}
Y_{T,L}(T) &= Y_{T,c} \cdot \left(1 + 1.5 \cdot x^\beta + x \cdot \sum_{j=1}^4 D_L(2j) \cdot x^{j-1} \right) , \\
Y_{T,V}(T) &= Y_{T,c} \cdot \left(1 - 1.5 \cdot x^\beta + x \cdot \sum_{j=1}^4 D_V(2j) \cdot x^{j-1} \right) .
\end{aligned}
\tag{7.5}$$

Fitting the coefficients $Y_{T,c}$, D_L and D_V to achieve smooth and monotonous transitions at the respective switching points $T(L) = 2250$ K , $T(V) = 1950$ K resulted in the following value for the critical viscosity :

$$Y_{T,c} = 6.5 \cdot 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{sec}} .
\tag{7.6}$$

Figure 21 displays the sodium viscosities on the whole extent of the saturation line . As a comparison , an estimated saturated-viscosity-function of Tsai and Olander /13/ is also shown. All the coefficients needed in the viscosity-calculations are given in the Table 13 respective in the Table 14 .

The viscosity in the mixed , two-phase state was constructed via the momentum-transfer-resistance. The simplified model - used for the thermal conductivity - was also employed here to describe the density-structure of the mixture. An equation for the momentum transfer - similarly as the Eq. [6.5] for the heat transfer - was then evaluated for the specific momentum-conductance , the viscosity :

$$\frac{1}{Y_T} = \frac{1-\eta}{Y_{T,V}} + \frac{\eta}{Y_{T,V} + \eta^2(Y_{T,L} - Y_{T,V})} .
\tag{7.7}$$

For η in this Eq. see Eq. [6.6] . Examples of the volume-dependence of this relation shows the viscosity-volume-chart , Figure 22 for some subcritical isotherms .

In the remaining states of the (T, ρ)-domain the sodium viscosity was simply set to a function depending solely on the density (see also Eq. [6.7]) :

$$Y_T(\rho, T) \equiv \begin{cases} Y_{T,L}(T^\times) & \text{for } \rho \geq \rho_c \\ Y_{T,V}(T^\times) & \text{for } \rho < \rho_c \end{cases} .
\tag{7.8}$$

T^\times in this equation is the saturation temperature (s. Eq. [3.6]).

The resulting final (v , T) - surface of the viscosity can be seen on the Figure 23 .

8.0 Calculating the density of the internal energy.

U , the internal energy density can be calculated by integrating a derivative of this property on a given line $L(\rho, T)$ in the (ρ, T) -Plane.

In a state-point (ρ_f, T_f) which lies either

- in the liquid state ,
- in the mixed state or
- in the supercritical state

SODIUM calculates $U(\rho_f, T_f)$ as follows:

1. the energy density of the melting liquid is set to zero :

$$\begin{aligned} U(\rho_M, T_M) &= 0.0 \\ \rho_M &= 927.478 \text{ kg/m}^3 \\ T_M &= 371. \text{ K} \end{aligned}$$

2. then the energy density difference between the temperatures T_f , T_M - calculated on the density path

$$\rho_B(T) = \begin{cases} \rho_L(T) & \text{for } T < T_c \\ \rho_c & \text{for } T \geq T_c \end{cases} \quad [8.1]$$

- $\Delta U(\rho_x, T_f, \rho_M, T_M)$, is added ,

3. finally , the energy density difference - calculated following an isotherm from the point (ρ_x, T_f) to the given point - $\Delta U(\rho_f, T_f, \rho_x, T_f)$ is added :

$$\begin{aligned} U(\rho_f, T_f) &= U(\rho_M, T_M) + \\ &+ \Delta U(\rho_x, T_f, \rho_M, T_M) + \Delta U(\rho_f, T_f, \rho_x, T_f) \end{aligned} \quad [8.2]$$

If the state-point (ρ_f, T_f) lies in the overheated vapor , then a different path is used:

1. the energy of the saturated vapor is calculated as above at the density ρ_f (the corresponding temperature in this point is T^x) ,
2. the energy density difference , calculated along the isochore $\rho = \rho_f$ is then added :

$$\begin{aligned} U(\rho_f, T_f) &= U(\rho_M, T_M) + \\ &+ \Delta U(\rho_x, T^x, \rho_M, T_M) + \Delta U(\rho_f, T^x, \rho_x, T^x) + \Delta U(\rho_f, T_f, \rho_f, T^x) \end{aligned} \quad [8.3]$$

Calculating ΔU following the path ρ_B : On the the low-temperature part of this path sodium is a saturated liquid. The energy-increment in this state is given by the equation

$$dU_L = \left[\frac{\partial U}{\partial T} + \frac{\partial U}{\partial \rho} \cdot \frac{d\rho}{dT} \right]_L dT$$

Using the expression for $\partial U / \partial \rho$ (see e.g. /14/)

$$\frac{\partial U}{\partial \rho} = \frac{1}{\rho^2} \left(P - T \cdot \frac{\partial P}{\partial T} \right) \quad [8.4]$$

one get the following 'specific heat' for the saturated liquid :

$$\frac{\partial U_L}{\partial T} = C_V(\rho_L) + \left[\left(P - T \cdot \frac{\partial P}{\partial T} \right) \frac{1}{\rho^2} \frac{d\rho}{dT} \right]_L$$

This equation can be transformed in a more convenient form using the formula (eq. (19) in /1/) :

$$C_V(\rho_L - 0) - C_V(\rho_L) = \frac{T}{\rho^2} \frac{d\rho}{dT} \left(\frac{dP^x}{dT} - \frac{\partial P}{\partial T} \right) \Big|_L$$

The resulting specific heat is :

$$\frac{\partial U_L}{\partial T} = C_V(\rho_L - 0) + \frac{r}{\rho} \left(\frac{P^x}{T} - \frac{dP^x}{dT} \right) \Big|_L \quad [8.5]$$

(see for " r " Eq. [5.10]). The first term in this specific heat is the basic specific heat of the code system SODIUM in the subcritical region (s. "Calculating the heat capacity ." or /1/). The Figure 24 shows the T-dependence of this specific heat in the range

$$371. K < T < T_c$$

As other specific heats , this one too diverges approximating the critical temperature :

$$\frac{\partial U_L}{\partial T} \propto x^{-\delta} \quad \text{for} \quad x \rightarrow 0 \quad [8.6]$$

with $\delta = 0.68$

The value of the critical exponent δ was gained by using the log-log presentation of the function $\partial U/\partial T$ (s. Figure 25).

$\partial U/\partial T$ was then approximated with the following set of functions :

$$\frac{\partial U_L}{\partial T}(T) \equiv RGAS \cdot \sum_{j=1}^5 A_{kj} \cdot x^{\delta+j-1} \quad \text{if} \quad x_{k-1} \leq x \leq x_k, \quad k=1, \dots, 5 \quad [8.7]$$

The limiting x-values and the corresponding coefficients are described in the Table 15 .

The internal energy on the subcritical part of the path ρ_B is then the T-integral of this function . One can transform this T-integral into a set of polynomials (s. "Appendix A. Integrating a property described by a set of polynomials.") :

$$\Delta U(T_f, T_M) = RGAS \left(B_{k1} + \sum_{j=2}^6 B_{kj} \cdot x^{\delta+j-1} \right) \quad \text{if} \quad T_{k-1} \leq T_f \leq T_k \quad [8.8]$$

with the following coefficients :

$$B_{0l} \equiv 0 \quad \text{for} \quad \forall l$$

$$B_{kl} \equiv -T_c \cdot \frac{A_{k,l-1}}{\delta+l-1}, \quad l=2, \dots, 6, \quad k=1, \dots, 5 \quad [8.9]$$

$$B_{k1} \equiv B_{k-1,1} + \sum_{l=2}^6 (B_{k-1,l} - B_{kl}) \cdot x_{k-1}^{\delta+l-1} \quad \text{for} \quad k=1, \dots, 5$$

On the supercritical part of the path the heat capacity is (eq. (115) in /1/) :

$$C_V(\rho_c, T) = c_{(+)} \left(\frac{T}{T_c} - 1 \right)^{-\alpha} .$$

U is gained on this path by integrating the expression above

$$\Delta U(T_f, T_c) = T_c \cdot c_{(+)} \frac{x_f^{-\alpha}}{1-\alpha} .$$

Calculating ΔU along an isotherm in the mixed state: In the mixed state the pressure depends only on the temperature . So it is easy to integrate the energy density along an isotherm in this part of the (ρ , T)-Plane :

$$\begin{aligned} \Delta U(\rho_f, T_f, \rho_L, T_f) &= \int_{v_L}^{v_f} dv \frac{\partial U}{\partial v} = \int_{v_L}^{v_f} dv \left(T \cdot \frac{\partial P}{\partial T} - P \right) \\ &= \left(T \cdot \frac{dP^x}{dT} - P^x \right) \cdot (v_f - v_L) . \end{aligned}$$

P^x is the vapor Pressure and the v-s are the specific volumina at the temperature T_f .

Calculating ΔU along a subcritical isotherm in the compressed liquid: In the compressed liquid the code calculates the pressure according to the Eq. [4.7] . Since $\partial P / \partial T$ depends here (via $T^x(\rho)$) only on the density , the same holds for the U-derivative :

$$\frac{\partial U}{\partial \rho} = \frac{1}{\rho^2} \left(P - T \cdot \frac{\partial P}{\partial T} \right) = \frac{1}{\rho^2} \left[P^x(T^x) - T^x \cdot \frac{\partial P}{\partial T}(T^x) \right] .$$

This above function had been calculated in the whole density-range (s. Figure 26) and was approximated with the following set of polynomials :

$$\frac{\partial U}{\partial \rho}(\rho) \equiv RGAS \cdot \sum_{j=1}^4 A_{kj} \cdot \rho^{j-1} \quad \text{if} \quad \rho_{k-1} \leq \rho \leq \rho_k , \quad k=1, \dots, 6 \quad . \quad [8.10]$$

For the limiting ρ -values and the coefficients see the Table 16 . Similarly as the Eq. [8.7] , this Eq. also allows to calculate the internal energy using polynomial expressions :

$$\begin{aligned} \Delta U(\rho_f, \rho_x) &= RGAS \cdot [F(\rho_f) - F(\rho_x)] \\ &\quad \text{with} \\ F(\rho) &\equiv \sum_{j=1}^5 B_{kj} \cdot \rho^{j-1} \quad \text{if} \quad \rho_{k-1} \leq \rho \leq \rho_k \end{aligned} \quad [8.11]$$

with the set of coefficients :

$$\begin{aligned} B_{0l} &\equiv 0 && \text{for} \quad \forall l \\ B_{kl} &\equiv \frac{A_{k, l-1}}{l-1} , && l=2, \dots, 5 , \quad k=1, \dots, 6 \\ B_{k1} &\equiv B_{k-1,1} + \sum_{l=2}^5 (B_{k-1,l} - B_{kl}) \cdot x_{k-1}^{l-1} && k=1, \dots, 6 . \end{aligned} \quad [8.12]$$

Integrating $\partial U/\partial \rho$ along a supercritical isotherm: Using the pressure description in the supercritical state (Eq. [4.13]) one gets the following U-derivative in this region :

$$\frac{\partial U}{\partial \rho} = \frac{1}{\rho^2} \left[P(\rho, T) - T \cdot \frac{\partial P}{\partial T}(\rho, T) \right] = \frac{RGAS \cdot T_c}{\rho} \frac{\partial z}{\partial y} .$$

For $\partial z/\partial y$ the Eq. (114) and (108) in /1/ give :

$$\frac{\partial z}{\partial y} = w \cdot A \frac{1 + w^2 B}{S^2} - \sum_{j=1}^5 [G_j + (2y - 1) H_j] \cdot w^j ,$$

so for $T \geq T_c$ the U derivative is :

$$\frac{\partial U}{\partial \rho} = \frac{RGAS \cdot T_c}{\rho_c} \left[A \frac{1 + w^2 B}{S^2} - \sum_{j=1}^5 w^{j-1} [G_j + (2y - 1) H_j] \right] . \quad [8.13]$$

The energy-incrementum on an isotherm is the following integral :

$$\begin{aligned} \Delta U(\rho_f, T_f, \rho_c, T_f) &= \int_{\rho_c}^{\rho_f} d\rho \cdot \frac{\partial U}{\partial \rho} \\ &= RGAS \cdot T_c \int_1^w dw \left[A \frac{1 + w^2 B}{S^2} - \sum_{j=1}^5 w^{j-1} [G_j + (2y - 1) H_j] \right] \quad [8.14] \\ &= RGAS \cdot T_c \left[\Delta I_U(w) - \sum_{j=1}^5 \frac{w^j - 1}{j} [G_j + (2y - 1) H_j] \right] . \end{aligned}$$

ΔI_U is the (w,1)-difference of the function

$$I_U(w) = A \int^w dw \frac{1 + w^2 B}{S^2} . \quad [8.15]$$

This function can be transformed to (see also the Appendix I in /1/) :

$$\begin{aligned} I_U(w) &= A \left[\int^w \frac{dw}{S} + a \int^w \frac{dw \cdot w}{S^2} \right] \\ &= A \left[\left(1 + \frac{a^2}{\sigma}\right) \cdot J_1(w) + \frac{a}{\sigma} \frac{a \cdot w - 2}{S} \right] , \end{aligned}$$

$$\text{with } J_1(w) = \int^w \frac{dw}{S} = \frac{2}{\sqrt{\sigma}} \arctan \left(\frac{S'}{\sqrt{\sigma}} \right)$$

For $J_1(w)$ it holds :

$$\Delta J_1(w) = J_1(w) - J_1(1) = \frac{2}{\sqrt{\sigma}} \arctan \left(\frac{\Delta w \sqrt{\sigma}}{w} \right) .$$

The (w,1)-difference of the function $\psi(w) = \frac{a \cdot w - 2}{S}$ is :

$$\Delta\psi(w) = \Delta w \frac{4B - a \cdot B - a}{S(w) \cdot S(1)},$$

so the $l_U(w)$ -difference in Eq. [8.14] turns out to be :

$$\begin{aligned} \Delta l_U(w) &= A \left[\left(1 + \frac{a^2}{\sigma}\right) \Delta J_1 + \frac{a}{\sigma} \Delta\psi \right] \\ &= A \left[\left(1 + \frac{a^2}{\sigma}\right) \frac{2}{\sqrt{\sigma}} \arctan\left(\frac{\Delta w \sqrt{\sigma}}{w}\right) + \frac{a}{\sigma} \Delta w \frac{4B - a \cdot B - a}{S(w) \cdot S(1)} \right]. \end{aligned}$$

Integrating $\partial U/\partial T$ along an isochore : In the low-density-part of the overheated vapor ($\rho < 10 \text{ kg/m}^3$) the pressure-formula Eq. [4.9] calculates the following energy-density derivative :

$$\begin{aligned} \rho^2 \cdot \frac{\partial U}{\partial \rho} &= P(\rho, T) - T \cdot \frac{\partial P}{\partial T}(\rho, T) = \\ &= \left[P^x(T^x) - T^x \cdot \frac{\partial P}{\partial T}(T^x) \right] - RGAS \cdot \rho \cdot G(s) \cdot \mu [T^x + T \cdot (u + \mu - 1)] \quad [8.16] \end{aligned}$$

The first term of this derivative depends only on the density and the corresponding ΔU could be - as in the case of the compressed liquid - described by polynomials, but the second term depends on both variables ρ and T so it cannot be approximated with a polynomial-expression and the energy-contribution of this second term

$$\Delta \tilde{U}(\rho_f, \rho_v) = -RGAS \cdot \int_{T_f}^{T^x(\rho_f)} dt \frac{d\rho}{dT} \cdot \frac{G}{\rho} \cdot \mu [t + T \cdot (u + \mu - 1)]$$

must be integrated numerically .

Since in this low-density region the numerical integration could not be avoided, the more simple and direct route has been chosen for calculating ΔU : to integrate C_V on the isochores from the saturation point (ρ, T^x) till to the critical isotherm (ρ, T_c) in the whole region of the overheated vapor.

Figure 27 shows some of the calculated C_V -isochores, Figure 28 shows the shape of the corresponding energy-density functions.

The departure of the energy-density from the saturated values

$$\Delta U(\rho, T) = \int_{T^x}^T dt C_V(\rho, t) \quad [8.17]$$

has been calculated for some 250 isochores in the overheated vapor. Then the resulting surface was brought in a more convenient form :

- the variable ρ was replaced by the saturation temperature T^x
- the variable T was replaced by a reduced temperature

$$\tau \equiv \frac{T - T^x}{T_c - T^x}, \quad [8.18]$$

- the energy-departure was made dimensionless and a factor τ was split off:

$$\Delta U(\rho, T) = \overline{\Delta U}(T^x) \cdot \tau \cdot E(T^x, \tau) \quad [8.19]$$

with

$$\begin{aligned}\overline{\Delta U}(T^x) &= U(T^x, T_c) - U(T^x, T^x) = \\ &= \int_{T^x}^{T_c} dt C_V(T^x, t)\end{aligned}\quad [8.20]$$

$$\text{and} \quad E(T^x, \tau) = \frac{U(\rho, T) - U(\rho, T^x)}{\tau \cdot \overline{\Delta U}} \quad [8.21]$$

The function $\overline{\Delta U}$ is presented on the Figure 29 as a function of T^x and on Figure 30 as a function of ρ . Figure 29 shows also the approximation

$$\overline{\Delta U}(T^x) \approx \sum_{j=1}^4 A_{kj} \cdot T^{xj-1} \quad \text{if} \quad T_{k-1}^x \leq T^x \leq T_k^x, \quad k = 1, \dots, 6 \quad [8.22]$$

used in SODIUM. For the coefficients A_{kj} see the Table 17.

Figure 29 presents actually two $\overline{\Delta U}(T^x)$ curves, one of them ($\overline{\Delta U}_1(T^x)$, "calculated") is the result of the subtraction $U(T^x, T_c) - U(T^x, T^x)$, the other ($\overline{\Delta U}_2(T^x)$, "integrated numerically") results from the C_V -integration (s. Eq. [8.20]). In the low-density part of the vapor there are a marked difference between these two functions (s. e.g. Figure 30). The reason for this discrepancy lies in the inaccurate description of the property $\partial P/\partial T$ by the function $G(s)$ resp. $\eta(s)$ in these places (s. the Eq. [4.9] and the Eq. [4.10] or /1/, p. 35 - 36). As the Figure 31 shows (= Fig. 19 in /1/), the function used by the code is - in the region $0.1 < s < 0.25$ - quite different from the function, needed for the correcture of $\partial P/\partial T$. The incorrect description of $\partial P/\partial T$ leads then to inaccurate C_V -values in this part of the overheated vapor and integrating these C_V -s results in overestimating the energy densities at the low-T - border ($T = T_c - 0$) of the critical isotherm (s. the Figure 4 and the Figure 5).

Since the shape of $\partial P/\partial T$ of the saturated vapor is not fixed rigidly by measurements, an attempt was made to eliminate these discrepancies between the different η -s by modifying this $\partial P/\partial T$ in the concerned density region of the saturated vapor. Figure 32 shows the function, used in SODIUM and the two varied $\partial P/\partial T$ -functions. The correcting functions, corresponding these changed $\partial P/\partial T$ -s are depicted on the Figure 33 (for the decreased $\partial P/\partial T$ shape) resp. on the Figure 34 (for the increased shape). As one can see, varying the form of this pressure derivative doesn't even reduce the discrepancies between the different η -s.

The reduced surface $E(T^x, \tau)$ of the Eq. [8.21] can be seen from different views on the Figure 35 and Figure 36. Figure 37 shows the near-critical part of this property. SODIUM calculates this function by using a skeleton of 20×48 - values of the numerically integrated surface (s. Figure 38) as a functions-table. E-Values, not corresponding to one of the knots of this (τ, T^x) - grid are interpolated via a two-dimensional spline-procedure /15/.

The energy-density-surface calculated by SODIUM in the sub-region with pressures exceeding one atmosphere displays Figure 39 and Figure 40.

9.0 Calculating the vapor quality and the surface tension of the liquid.

The vapor quality is defined as

$$\xi = \frac{m(V)}{m(L) + m(V)} \quad [9.1]$$

SODIUM calculates this property in the two-phase mixture via the specific volume, $v = 1 / \rho$ (s. Eq. 60 in /1/):

$$\xi(\rho) = \frac{v - v_L}{v_V - v_L} \quad [9.2]$$

In the (saturated) liquid ξ is uniformly zero, in the (saturated) vapor ξ is uniformly one and in all the supercritical states ξ is set to -1.

The surface tension of the liquid is calculated by using a Van-der-Waals type equation:

$$g(T) \text{ [Joule/m}^2\text{]} = 0.240 \cdot \left[1 - \frac{T}{T_c} \right]^{1.132} \quad [9.3]$$

Goldman /16/ obtained the constants in this Eq. by comparing the measurements of 13 groups, made in the years 1954 - 1982 and fitting the formula to these data. The critical temperature, which he used in the calculations differs only slightly from the T_c used in SODIUM (2509.5 K \approx 2508 K).

In all non-liquid states SODIUM sets the surface tension to zero.

10.0 Using the code SODIUM.

Normally - for calculating the thermal properties in a given point (RH,T) - one calls:

`SODIUM(T,RH,P,PR,PT,U,UR,CV,XI,TR,QT,YT,GIS)`

In this call the user supplies the following properties:

- RH (> 0.0) the density (in kg/m^3) and
- T (> 0.0) the temperature (in K) .

The above call returns the following thermal properties in S.I.-units :

- the pressure (P), with its density- (PR) and temperature- (PT) derivatives ,
- the density of internal energy (U), with its density- (UR) and temperature- (CV) derivatives ,
- the vapor quality in the mixed state (XI) ,
- the saturation temperature at the density RH (TR) ,
- the thermal conductivity , (QT) ,
- the viscosity , (YT) and
- the surface tension of the liquid (GIS) .

XI = -1 in the critical and supercritical states. GIS is zero in all not-liquid states.

For a negative temperature , $T' < 0$

`SODIUM(T',RHC,P,PR,PT,U,UR,RGAS,XI,TR,QTC,YTC,GIS)`

returns some thermal constants of the sodium :

- the critical density (RHC) and temperature ($T' = TC$) ,
- the specific gas-law constant of the sodium (RGAS) ,
- the thermal conductivity at the critical point (QTC) and
- the viscosity at the critical point (YTC) .

To get the properties as (pressure,temperature)-functions one calls the routine SODINV as follows:

`SODINV(T,P,RH,PR,PT,H,HR,CP,XI,TR,QT,YT,GIS)`

In this call only P (in J/m^3) and T is needed from the user.

- RH , PR , PT , XI , TR , QT , YT and GIS denote the same properties as in the SODIUM calls .
- H is the density of the enthalpy , HR and CP are the density- resp. the temperature-derivatives of H .

Both SODIUM and SODINV terminate for temperatures below $T < 370$ K abnormally . SODIUM abends also for too high density-values ($RH > 930$ kg/m^3) . In the case of an abnormal end both of the routines return negative temperature-values .

Access to the code system : in the Kernforschungszentrum Karlsruhe the moduls of the code are in the unprotected dataset 'INR105.KATHER.LOAD', (volume = BAT00C) . For the thermal properties of the sodium the following moduls are needed :

- SODIUM ,
- CALORA ,

- DPCLIQ ,
 - DPCVAP
- and in the case of (P,T)-calculations also the modul
- SODINV .

The following job exemplifies a method of accessing the sodium routines via the keyword-parameter "USER" in the EXEC-statement :

```

//INR105B1 JOB (0abc,xyz,p9x9y),User,MSGLEVEL=(1,1)
//SODTES EXEC F7CLG,USER='INR105.KATHER.LOAD'
//C.SYSIN DD *
PROGRAM SODTES
C TESTPROGRAMM FUER DIE SODIUM-ZUSTANDSDATEN
IMPLICIT REAL*8 (A-H,O-$)
DATA ZER/0.DO/,T/1.5D+3/,RI/9.D-1/,RF/0.9D+3/,RFAC/1.05/
=,PFAC/1.D-5/,CVFAC/2.7651D-3/,UFAC/2.7651D-3/,QTFAC/0.2/
=YTFAC/15.3846D+3/,RHFAC/1.D-3/
NAMELIST /LAIST/ T,RI,RF,RFAC,PFAC,CVFAC,UFAC,QTFAC,YTFAC,RHFAC
TC= -1.D-0
RHC=0.1D-0
CALL SODIUM(TC,RHC,P,PR,PT,U,UR,RGAS,XI,TR,QTC,YTC,GIS)
WRITE(6,1000) TC,RHC,QTC,YTC,RGAS
WRITE(6,2000)
100 READ(5,LAIST)
IF(T .LT. 0.) STOP
IF(RI .LE. ZER .OR. RF .LE. RI .OR.
= RFAC .LE. 1) THEN
WRITE(6,3000) RI,RF,RFAC
GO TO 100
ENDIF
RH=RI
DO 101 J=1,300
IF(RH .GT. RF) RH=RF
CALL SODIUM(T,RH,P,PR,PT,U,UR,CV,XI,TR,QT,YT,GIS)
IF(T .LE. 0) THEN
T=-T
WRITE(6,4100) T,RH
GO TO 100
ENDIF
IF(MOD(J,50) .EQ. 1) THEN
WRITE(6,4000) T
WRITE(6,5000) RHFAC,PFAC,PFAC,PFAC,CVFAC,UFAC,QTFAC,YTFAC
WRITE(6,'()')
ENDIF

```



```

RHI=RH*RHFAC
PIS=P*PFAC
PTI=PT*PFAC
PRI=PR*PFAC
CVI=CV*CVFAC
QTI=QT*QTFAC
YTI=YT*YTFAC
UI=U*UFAC
WRITE(6,5000) RHI,PIS,PTI,PRI,CVI,UI,QTI,YTI
IF(RH .GE. RF)          GOTO 100
101 RH=RH*RFAC
    GO TO 100

C
1000 FORMAT(18H KRITISCHE DATEN :/
  =/7H TC =,G12.5,2H K
  =/7H RHC =,G12.5,8H KG/M**3
  =/7H QTC =,G12.5,8H W/(M*K)
  =/7H YTC =,G12.5,9H M/(KG*S)
  =/7H RGAS =,G12.5,9H J/(KG*K) )
2000 FORMAT(12HOEINHEITEN : /
  =/10X,17HRH IN KG/M**3 ,
  =/10X,17HP IN J/M**3 ,
  =/10X,17HPT IN J/(M**3*K) ,
  =/10X,17HPR IN J/KG ,
  =/10X,17HCV IN J/(KG*K) ,
  =/10X,17HU IN J/KG ,
  =/10X,17HQT IN W/(M*K) ,
  =/10X,17HYT IN M/(KG*S) . )
3000 FORMAT(/18H EINGABEFehler !!!,3X,16HRI , RF , RFAC =,3G13.5 )
4000 FORMAT(22H1ISOTHERM-TEMPERATUR =,F10.3,2H K /
  =/6X,4HRH *,11X,4HP *,11X,4HPT *,11X,4HPR *,11X,4HCV *,11X,4HU *
  =,11X,4HQT *,11X,4HYT *)
4100 FORMAT(36HDEFINITIONSBEREICH UEBERSCHRITTEN !
  =/10X,3HT =,G12.5,2H K,5X,4HRH =,G12.5,8H KG/M**3 )
5000 FORMAT(8G15.6)
    END
//G.SYSPRINT DD SYSOUT=*
//G.SYSIN DD *
&LAIST T=1500.,RHFAC=1.D0,PFAC=1.D0,CVFAC=1.D0,UFAC=1.D0,QTFAC=1.D0,
YTFAC=1.D0,RFAC=1.1515D0,RF=931., &END
&LAIST T=-100., &END
//

```

If it is not possible to employ the keyword-parameter "USER" in the EXEC-statement , then one can append the routines directly in the linkage step, as shown in the second example :

```

//INR105B2 JOB (0abc,xyz,p9x9y),User,MSGLEVEL=(1,1)
//SODTES EXEC F7CLG,PARM.L='LIST,MAP'
//C.SYSIN DD *
PROGRAM SODTES
C TESTPROGRAMM FUER DIE SODIUM-ZUSTANDSDATEN
IMPLICIT REAL*8 (A-H,O-$)
DATA ZER/0.DO/,T/1.5D+3/,RI/9.D-1/,RF/0.9D+3/,RFAC/1.05/
=,PFAC/1.D-5/,CVFAC/2.7651D-3/,UFAC/2.7651D-3/,QTFAC/0.2/
=,YTFAC/15.3846D+3/,RHFAC/1.D-3/
NAMELIST /LAIST/ T,RI,RF,RFAC,PFAC,CVFAC,UFAC,QTFAC,YTFAC,RHFAC
TC= -1.D-0
RHC=0.1D-0
CALL SODIUM(TC,RHC,P,PR,PT,U,UR,RGAS,XI,TR,QTC,YTC,GIS)
WRITE(6,1000) TC,RHC,QTC,YTC,RGAS
WRITE(6,2000)
100 READ(5,LAIST)
IF(T .LT. 0.) STOP
IF(RI .LE. ZER .OR. RF .LE. RI .OR.
= RFAC .LE. 1) THEN
WRITE(6,3000) RI,RF,RFAC
GO TO 100
ENDIF
RH=RI
DO 101 J=1,300
IF(RH .GT. RF) RH=RF
CALL SODIUM(T,RH,P,PR,PT,U,UR,CV,XI,TR,QT,YT,GIS)
IF(T .LE. 0) THEN
T=-T
WRITE(6,4100) T,RH
GO TO 100
ENDIF
IF(MOD(J,50) .EQ. 1) THEN
WRITE(6,4000) T
WRITE(6,5000) RHFAC,PFAC,PFAC,PFAC,CVFAC,UFAC,QTFAC,YTFAC
WRITE(6,'()')
ENDIF
RHI=RH*RHFAC
PIS=P*PFAC
PTI=PT*PFAC
PRI=PR*PFAC
CVI=CV*CVFAC
QTI=QT*QTFAC
YTI=YT*YTFAC
UI=U*UFAC
WRITE(6,5000) RHI,PIS,PTI,PRI,CVI,UI,QTI,YTI
IF(RH .GE. RF) GOTO 100
101 RH=RH*RFAC
GO TO 100
C

```

```

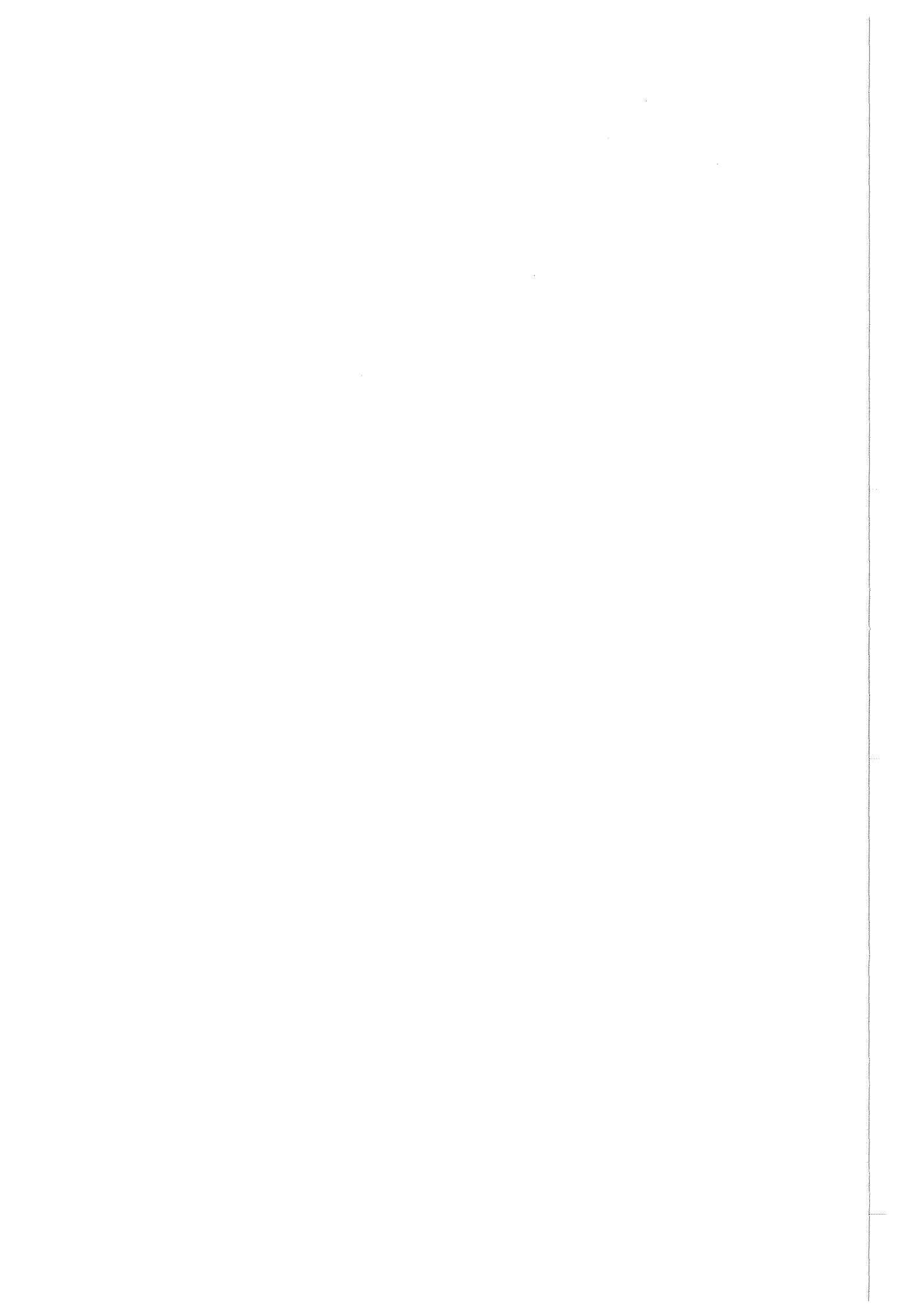
1000 FORMAT(18H KRITISCHE DATEN :/
    =/7H TC =,G12.5,2H K
    =/7H RHC =,G12.5,8H KG/M**3
    =/7H QTC =,G12.5,8H W/(M*K)
    =/7H YTC =,G12.5,9H M/(KG*S)
    =/7H RGAS =,G12.5,9H J/(KG*K) )
2000 FORMAT(12HOEINHEITEN : /
    =/10X,17HRH IN KG/M**3 ,
    =/10X,17HP IN J/M**3 ,
    =/10X,17HPT IN J/(M**3*K) ,
    =/10X,17HPR IN J/KG ,
    =/10X,17HCV IN J/(KG*K) ,
    =/10X,17HU IN J/KG ,
    =/10X,17HQT IN W/(M*K) ,
    =/10X,17HYT IN M/(KG*S) . )
3000 FORMAT(/18H EINGABEFehler !!!,3X,16HRI , RF , RFAC =,3G13.5 )
4000 FORMAT(22H1ISOTHERM-TEMPERATUR =,F10.3,2H K /
    =/6X,4HRH *,11X,4HP *,11X,4HPT *,11X,4HPR *,11X,4HCV *,11X,4HU *
    =,11X,4HQT *,11X,4HYT *)
4100 FORMAT(36HDEFINITIONSBEREICH UEBERSCHRITTEN !
    =/10X,3HT =,G12.5,2H K,5X,4HRH =,G12.5,8H KG/M**3 )
5000 FORMAT(8G15.6)
    END
/**
//L.BIKT DD DSN=INR105.KATHER.LOAD,DISP=SHR
//L.SYSPRINT DD SYSOUT=*
//L.SYSIN DD *
INCLUDE BIKT(SODIUM,CALORA,DPCLIQ,DPCVAP,SODINV)
ENTRY SODTES
/**
//G.SYSPRINT DD SYSOUT=*
//G.SYSIN DD *
&LAIST T=3000.,RHFAC=1.DO,PFAC=1.DO,CVFAC=1.DO,UFAC=1.DO,QTFAC=1.DO,
YTFAC=1.DO,RFAC=1.1515DO, &END
&LAIST T=-100., &END
//

```

Part of the output of both jobs are reproduced in "Appendix E. Results of the examples".

Time considerations : SODIUM needs some ≈ 0.3 msec CPU-time on the Siemens-7890 for a call , if the densities are in the "compressed"-region , $\rho > 10 \text{ kg/m}^3$. In the remaining density domain SODIUM needs ≈ 1.7 msec CPU-time for a call , mainly because of the need of calculating supplementary , (ρ , T)-dependent terms for the heat capacity and for the density of the internal energy in this part of the overheated vapor.

SODINV runs roughly three times more slowly , since it needs in the average three iterative steps (= SODIUM calls) to reach the correct density for a given pressure.



11.0 A short description of the routines of the code system.

SODIUM (T,RH,P,PR,PT,U,UR,CV,XI,TR,QT,YT,GIS)

is the controlling routine for the (ρ,T) - calculations . In the calculations SODIUM depends on the routines

CALORA
SATLIN
TVONRH
ZUBER
PINT
VISCON

collected in the module CALORA ,

DPCLIQ
DU1DRH

collected in the module DCPLIQ and

DPCVAP
DU2DT
HAMU
GEVONS
DELCEV
Valse
DATDUR
BORDUR
SVXDUR
SVYDUR
DERAN
MAPROD
FINTER

collected in the module DCPVAP.

CALORA (T,CVB,UB)

evaluates for a given temperature T the following caloric properties on the baseline :

$$CVB = C_V[\rho_B(T)] \quad \text{and} \quad UB = U[\rho_B(T)] \quad .$$

SATLIN (K,T,RH,PI,SIG,RHS,RKS,R3S,VS)

returns for a given T and state-index K vapor pressure and saturated density functions. These are :

$$PI = \frac{p^x}{RGAS \cdot T}$$

the reduced vapor pressure and

$$SIG = \frac{T \cdot \Pi'}{\Pi}$$

the reduced derivative of the vapor pressure.

All of the following variables RHS , RKS , R3S and VS are two-dimensional arrays :
RHS(1) and *RHS(2)* are the densities of the saturated liquid resp. of the saturated vapor
, *RHK(1)* and *RHK(2)* are the logarithmic density derivatives

$$RKS = \frac{T}{\rho} \frac{d\rho}{dT}$$

of the liquid and the vapor, $R3S(1)$ and $R3S(2)$ are the second density derivatives:

$$R3S = \frac{T}{RKS} \cdot \frac{dRKS}{dT}$$

$VS(1)$ and $VS(2)$ are the specific volumina of the saturated liquid resp. of the saturated vapor.

TVONRH (RH,TR)

calculates to a given density RH the saturation temperature TR.

ZUBER (T,RH,Z,ZW,ZY,DC,DU)

calculates thermal and caloric properties for a given temperature T and density RH in the supercritical region. Z, ZW and ZY are the factor of reality resp. the reduced pressure derivatives:

$$Z = \frac{P \cdot V}{RGAS \cdot T}$$

$$ZW = z + w \cdot \frac{\partial z}{\partial w} = P_{\rho}^x$$

$$ZY = z - y \cdot \frac{\partial z}{\partial y} = P_T^x$$

DC and DU are the C_V - resp. U-departures on the isotherm T from the critical density:

$$DC = \frac{C_V(\rho, T) - C_V(\rho_c, T)}{RGAS}$$

$$DU = \frac{U(\rho, T) - U(\rho_c, T)}{RGAS}$$

PINT (H,XA,XE,EXO)

returns definite integrals of polynomials H(j):

$$PINT = \int_{XA}^{XE} dx \sum_{j=1}^5 H(j) \cdot x^{j-1+EXO}$$

VISCON (T,K,QT,YT)

returns for a given T and a given state-index K the values of the reduced thermal conductivity and the reduced viscosity:

$$QT = \frac{Q_T}{Q_{T,c}} \quad , \quad YT = \frac{Y_T}{Y_{T,c}}$$

DPCLIQ (T,RH,RK,R3,PI,SIG,PT,PR,TDPT)

and

DPCVAP (T,RH,RK,R3,PI,SIG,PT,PR,TDPT)

calculate the pressure-derivatives

$$PT = P_T^x \quad , \quad PR = P_{\rho}^x \quad \text{and} \quad TDPT = \frac{T}{PT} \cdot \frac{dPT}{dT}$$

for the saturated liquid resp. saturated vapor .To this end not only the temperature T and the saturated density RH with their derivatives RK and R3 are needed but also the reduced pressure PI and the reduced vapor pressure derivative SIG .

DU1DRH (RHS,RH)

integrates $\partial U/\partial \rho$ along the isotherm T in the liquid starting at the saturation point :

$$DU1DRH = \int_{RHS}^{RH} d\rho \cdot \partial U/\partial \rho \quad .$$

The following routines are needed only for the vapor states :

HAMU (U,MUE,HVONU,JVONU)

calculates for a given $U = u$ the following auxiliary functions (s. Eq. [4.11]) :

$$\begin{aligned} MUE &= \mu \quad , \\ HVONU &= \mu \cdot (u + \mu) \quad , \\ JVONU &= HVONU \cdot u \cdot [u + 2 \cdot (\mu - 1)] \quad , \end{aligned}$$

GEVONS (S,ETA,GS,DLSGS)

returns for a given S the correcting functions

$$\begin{aligned} ETA &= \eta \quad , \quad GS = G \\ &\text{and the derivative} \\ DLSGS &= \frac{s}{G} \cdot \frac{dG}{ds} \end{aligned}$$

DELCEV (T,TR)

calculates the CV-departure from the saturated values in the overheated vapor at the temperature T and at the density temperature $T^x = TR$:

$$DELCEV = \overline{\Delta C_V(T)} \cdot \left[F\left(\frac{T^x}{T}, T\right) - 1 \right]$$

(s. Eq. [5.17]) using VALSE.

VALSE (N,X,F,MEM,XP,FP)

calculates the value FP of the function { FX , X , N-points } in the place XP using cubic splines. If MEM is less or equal zero then a set of new coefficients will be created , otherwise the old ones remain in use in the interpolation.

DU2DT (TR,T)

integrates $\partial U/\partial T$ along the isochore RH in the overheated vapor starting at the saturation point :

$$DU2DT = \int_{TR}^T dt \cdot C_V(RH, t) \quad .$$

DU2DT needs the following routines for the calculations :

DATDUR

- a block data subprogram - contains the data surface $E(T^x, T)$ (s. Eq. [8.21]) ,

BORDUR

calculates the derivatives of this surface

$$PE = \frac{\partial E}{\partial T^x} \quad , \quad QE = \frac{\partial E}{\partial \tau} \quad , \quad RE = \frac{\partial^2 E}{\partial \tau \partial T^x}$$

at the borderlines ,

SVXDUR

and

SVYDUR

provide the remainder of the derivatives of the surface ,

FINTER (DV,DW,A)

calculates the E-value $FINTER = E(T^x, \tau)$ in a distance

$$DV = T^x - T^x_N \quad , \quad DW = \tau - \tau_M \quad ,$$

to the nearest grid-point (T^x_N, τ_M) using A , the 4 x 4 - matrix of the coefficients of the bicubic splines /15/ , valid in this grid-point.

There are some mathematical routines needed by these calculations :

DERAN (N,Z,F,F1)

returns the derivative F1 of function (F,Z) supplied as a set of (N,N)-numbers,

MAPROD (N,A,B,U,IQ)

multiplies the (N,N) matrices A and B to U :

$$U = A \cdot B \quad \text{if} \quad IQ = 0 \quad ,$$

$$U = \tilde{A} \cdot B \quad \text{if} \quad IQ = 1 \quad ,$$

$$U = A \cdot \tilde{B} \quad \text{if} \quad IQ = 2 \quad .$$

SODINV (T,P,RH,PR,PT,H,HR,CP,XI,TR,QT,YT,GIS)

direct the calculations in the (P,T) - case . SODINV divides the (P , T) validity-domain in four quadrants by the lines

$$T = T_c \quad , \quad P = P(\rho_c)$$

and inserts estimated $\rho_0(P, T)$ -s for each of these subregions. SODINV then calls SODIUM to calculate the actual pressure , P_0 and $\partial P / \partial \rho$ in the state-point (ρ_0, T) and corrects the estimated density using the newtonian-procedure :

$$\rho_{j+1} \approx \rho_j + \frac{P - P_j}{\left. \frac{\partial P}{\partial \rho} \right|_j} \quad , \quad \dots \quad j = 0, \dots, 13 \quad .$$

Beside of these main routines of the package there are some other ones which are needed only in testing the code-system :

SODTES

is needed for testing SODIUM and

INVTES

tests the code SODINV .

SURFACE

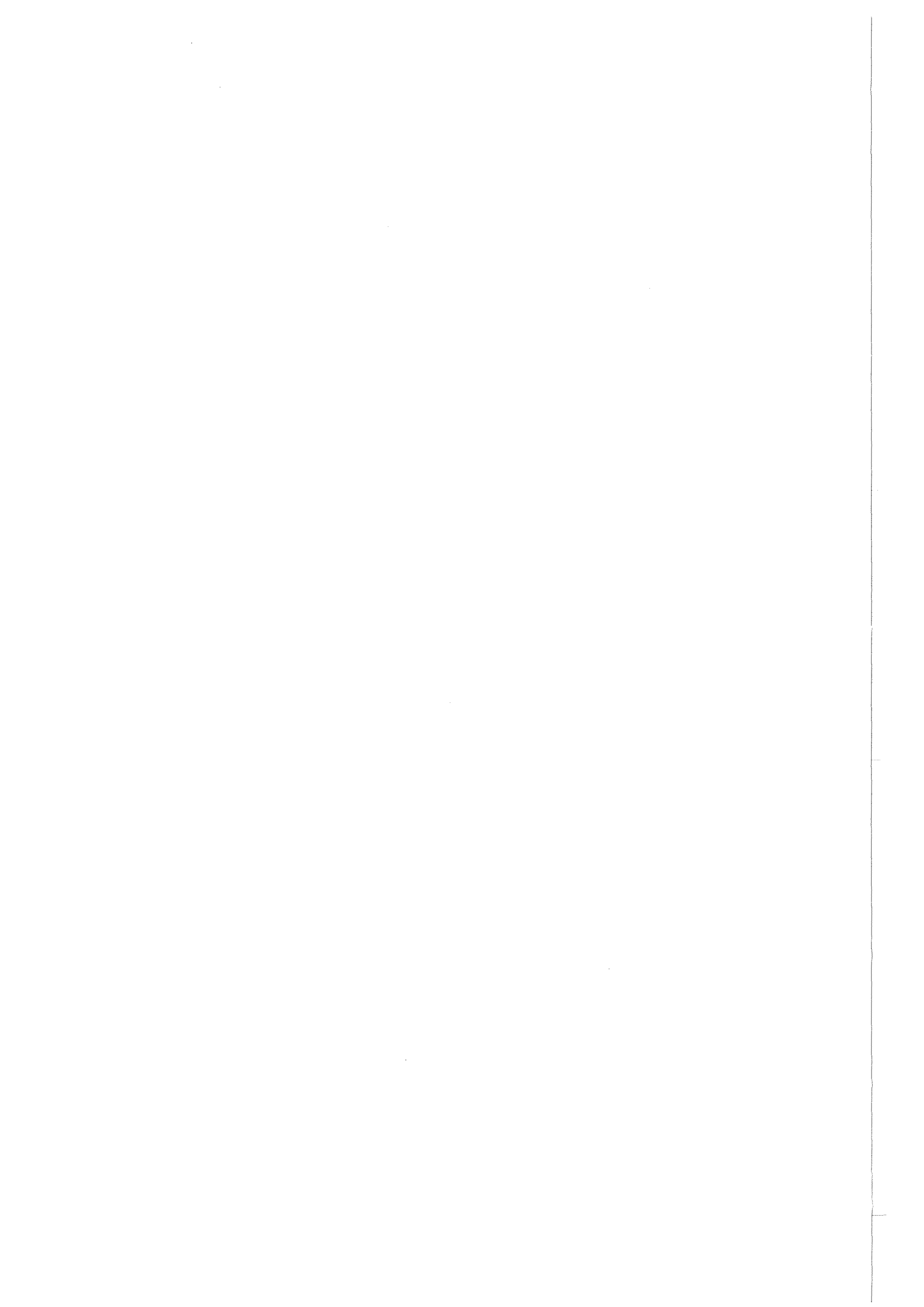
calculates whole surfaces of the Sodium-properties and

MOLLIE

calculates the Mollier-surface of the Sodium (s. Figure 43).
There exists also a routine

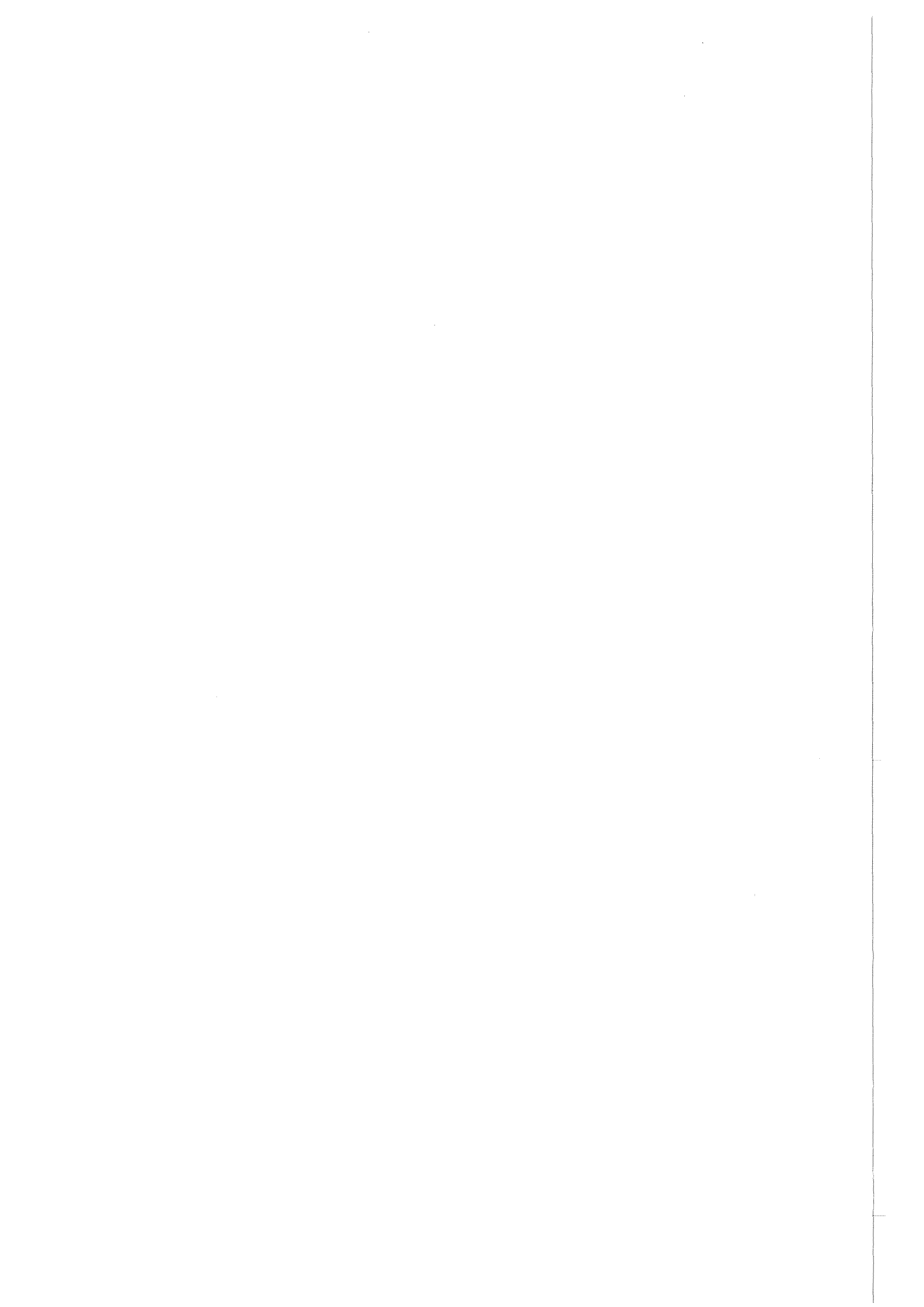
TVONP (P)

to invert the vapor pressure function of the sodium .



12.0 References.

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- /16/ J. H. Goldman, Surface Tension of Sodium, Journal of Nuclear Materials , **126** , p. 86-88, 1984.



13.0 Figures.

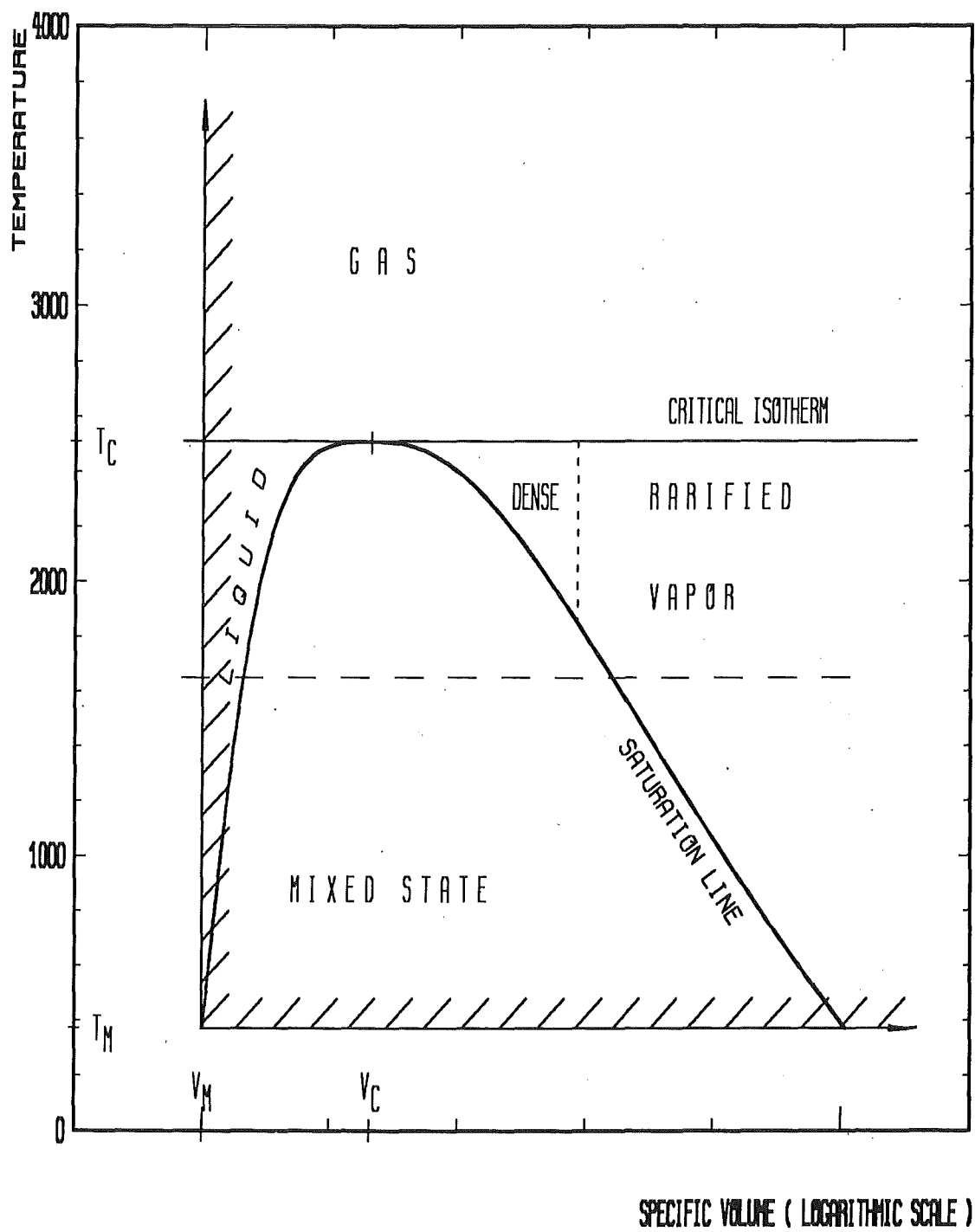


Figure 1. SODIUM. RANGE OF VALIDITY OF THE THERMAL PROPERTIES.

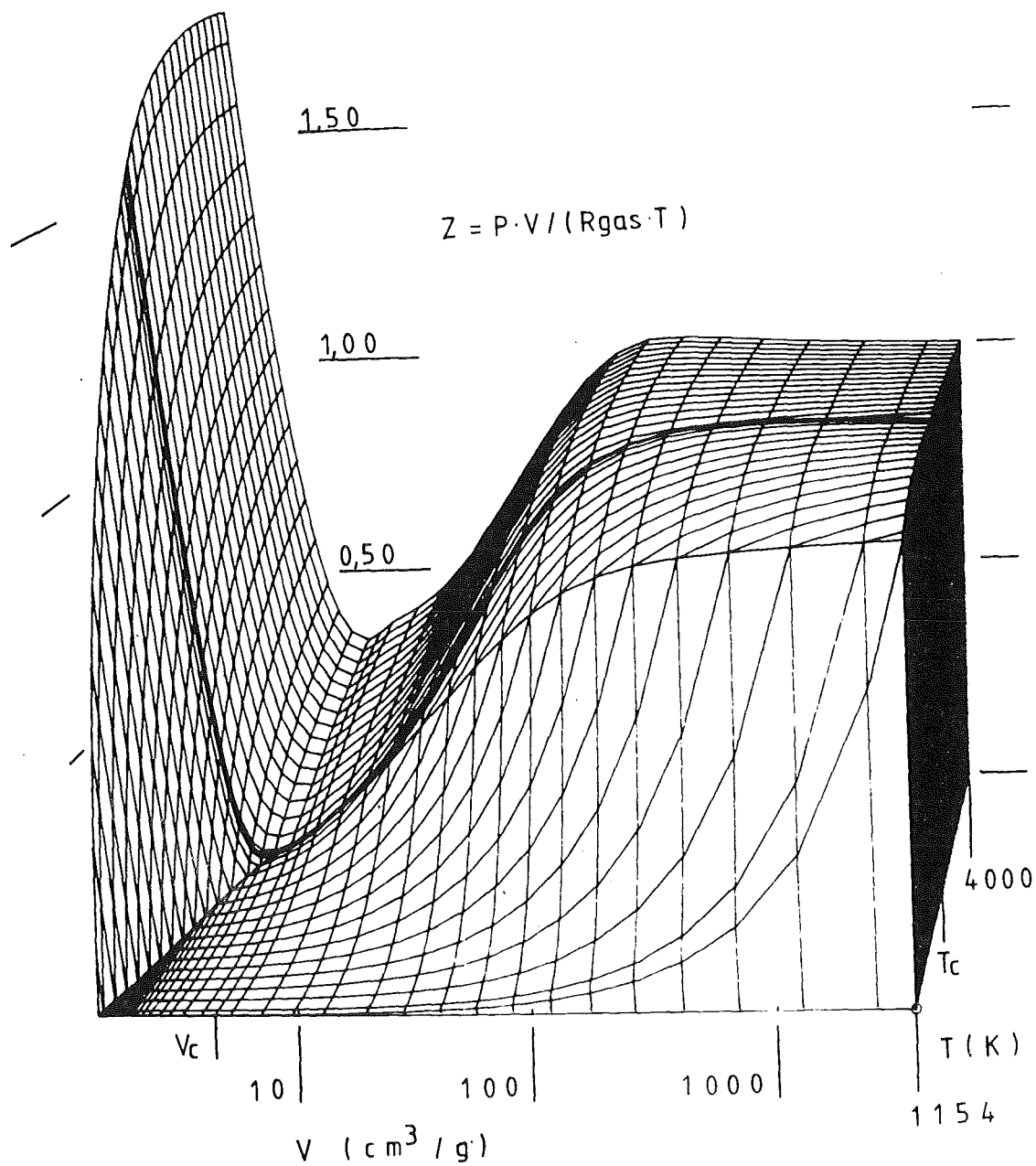
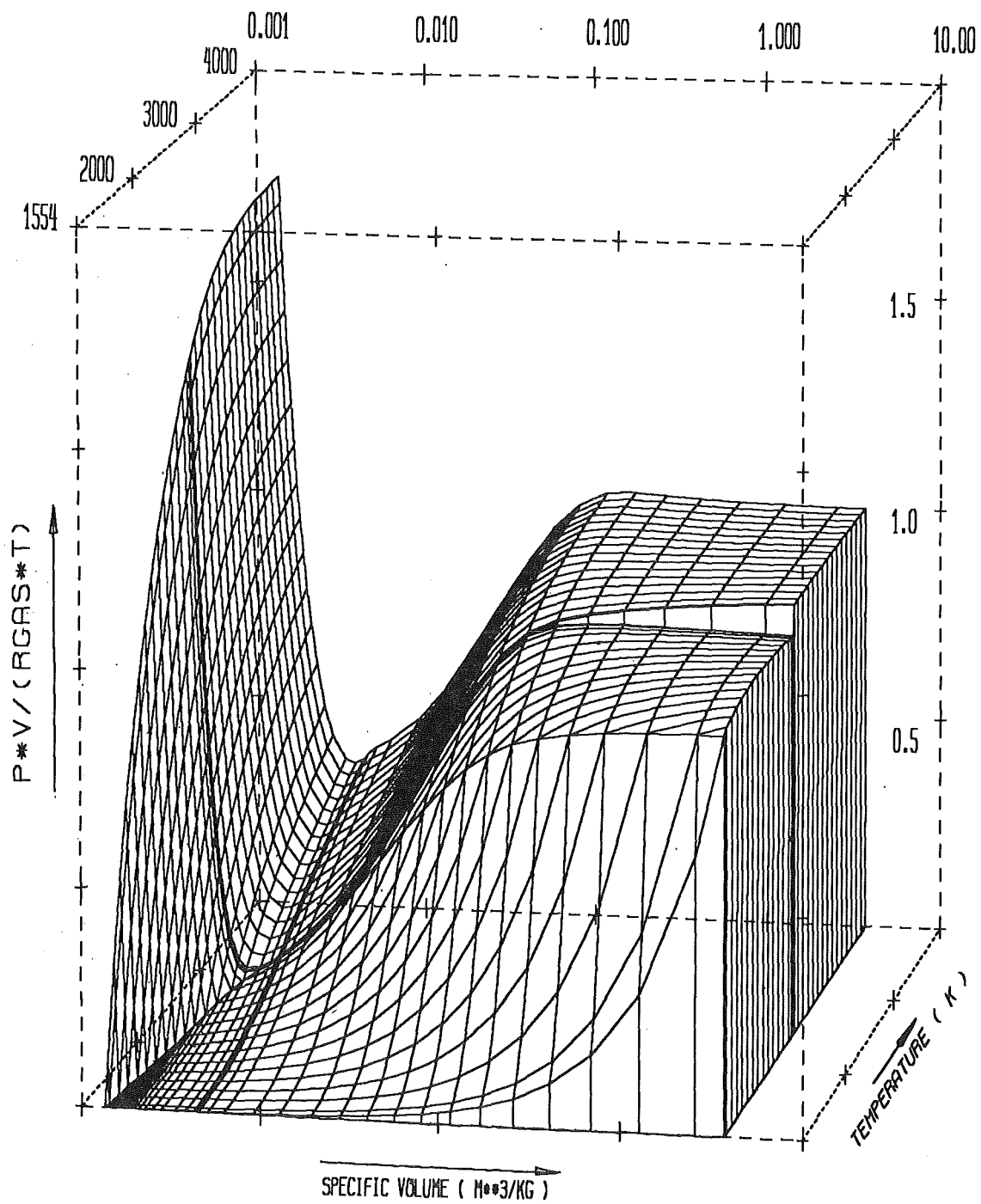


Figure 2. KANAST-2. THE (V,T) - SURFACE OF THE FACTOR OF REALITY .



KIK
TNR

Figure 3. KANAST-3. THE (V,T) - SURFACE OF THE FACTOR OF REALITY .

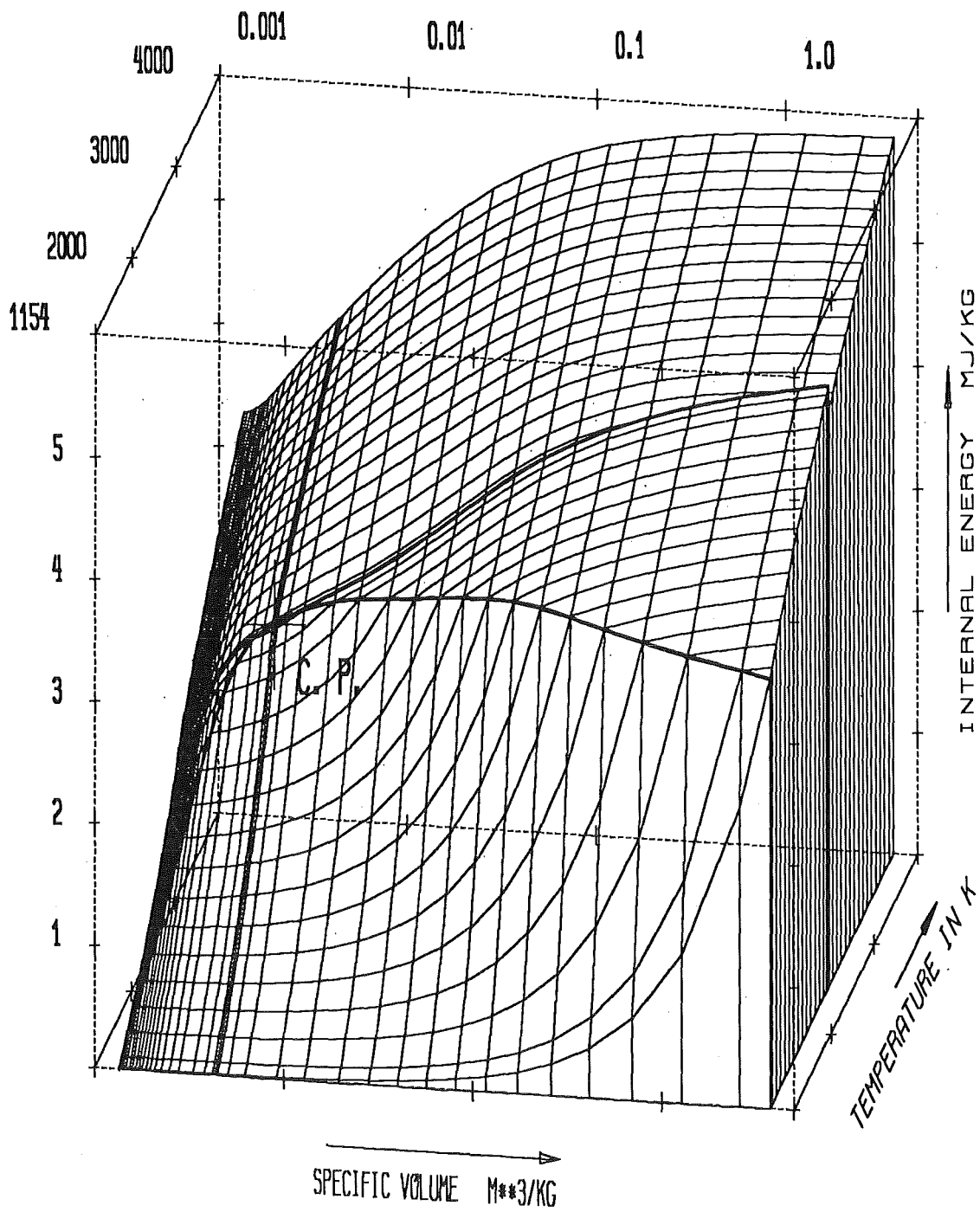


Figure 4. KANAST-3. THE DENSITY OF THE INTERNAL ENERGY AS A (T,V)-SURFACE.

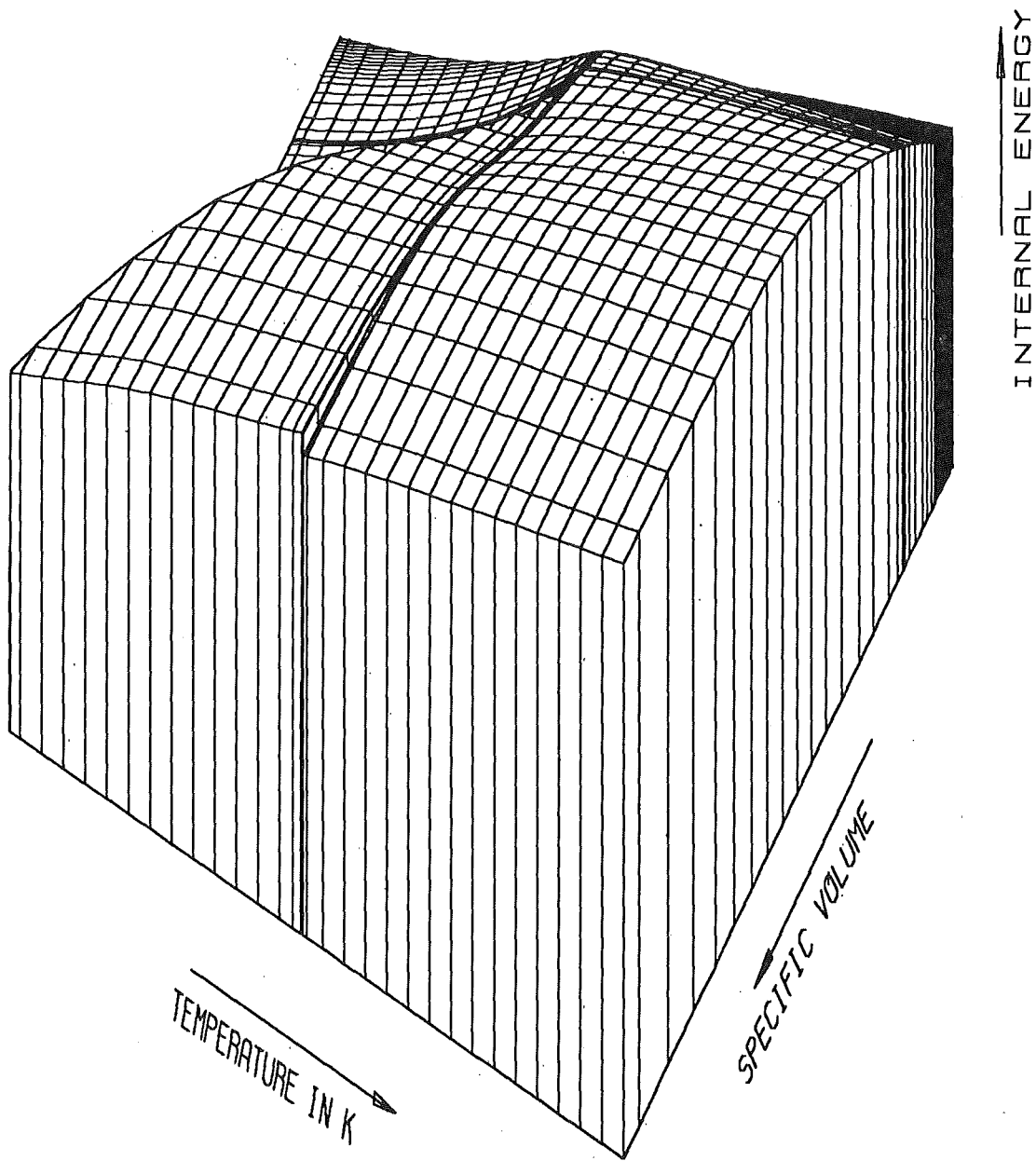


Figure 5. KANAST-3. THE DENSITY OF THE INTERNAL ENERGY AS A (T,V)-SURFACE.

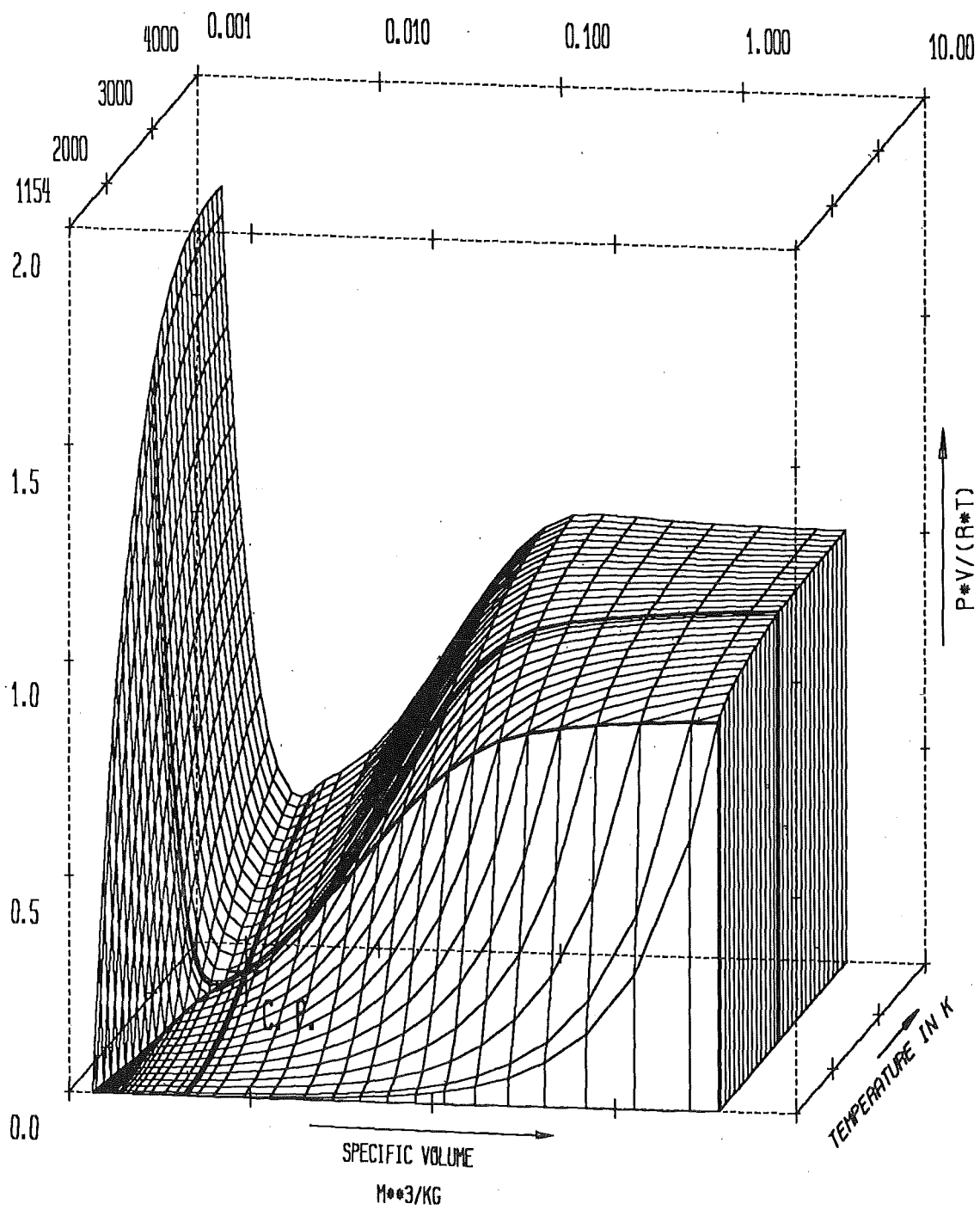


Figure 6. SODIUM. THE (V,T) - SURFACE OF THE FACTOR OF REALITY .

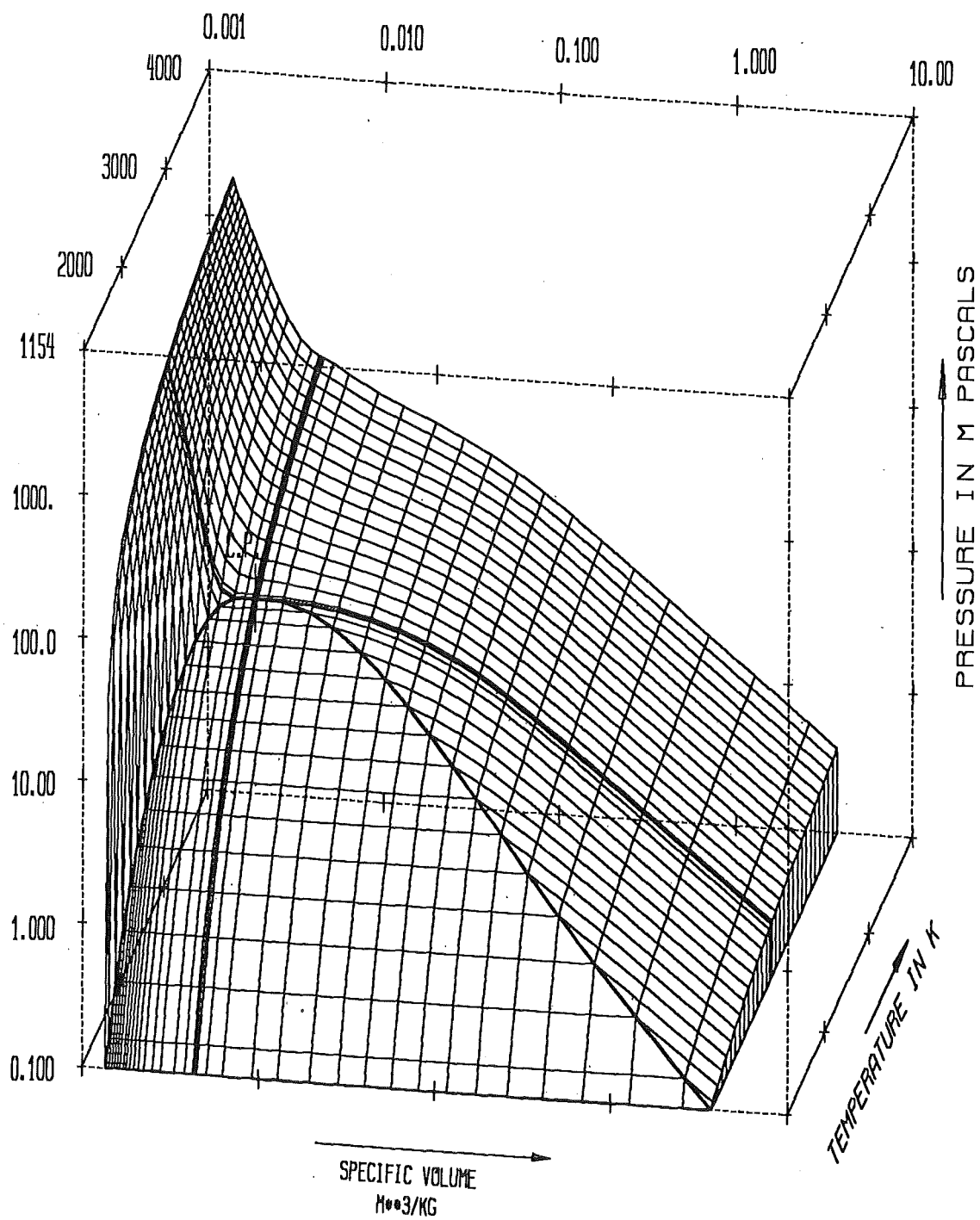


Figure 7. SODIUM. THE (V,T) - SURFACE OF THE PRESSURE .

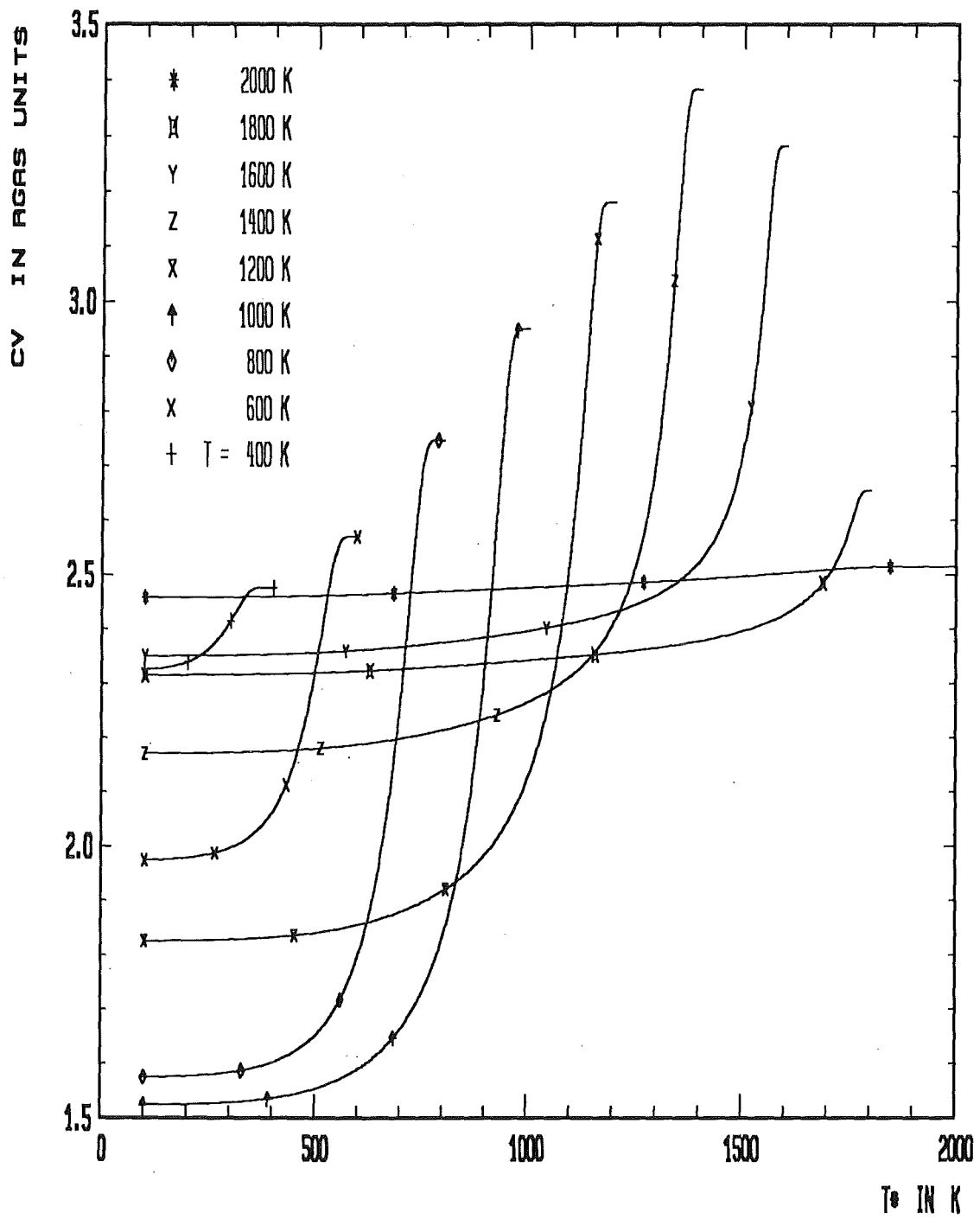


Figure 8. SODIUM. HEAT CAPACITY CV ON SOME ISOTHERMS IN THE OVERHEATED VAPOR .

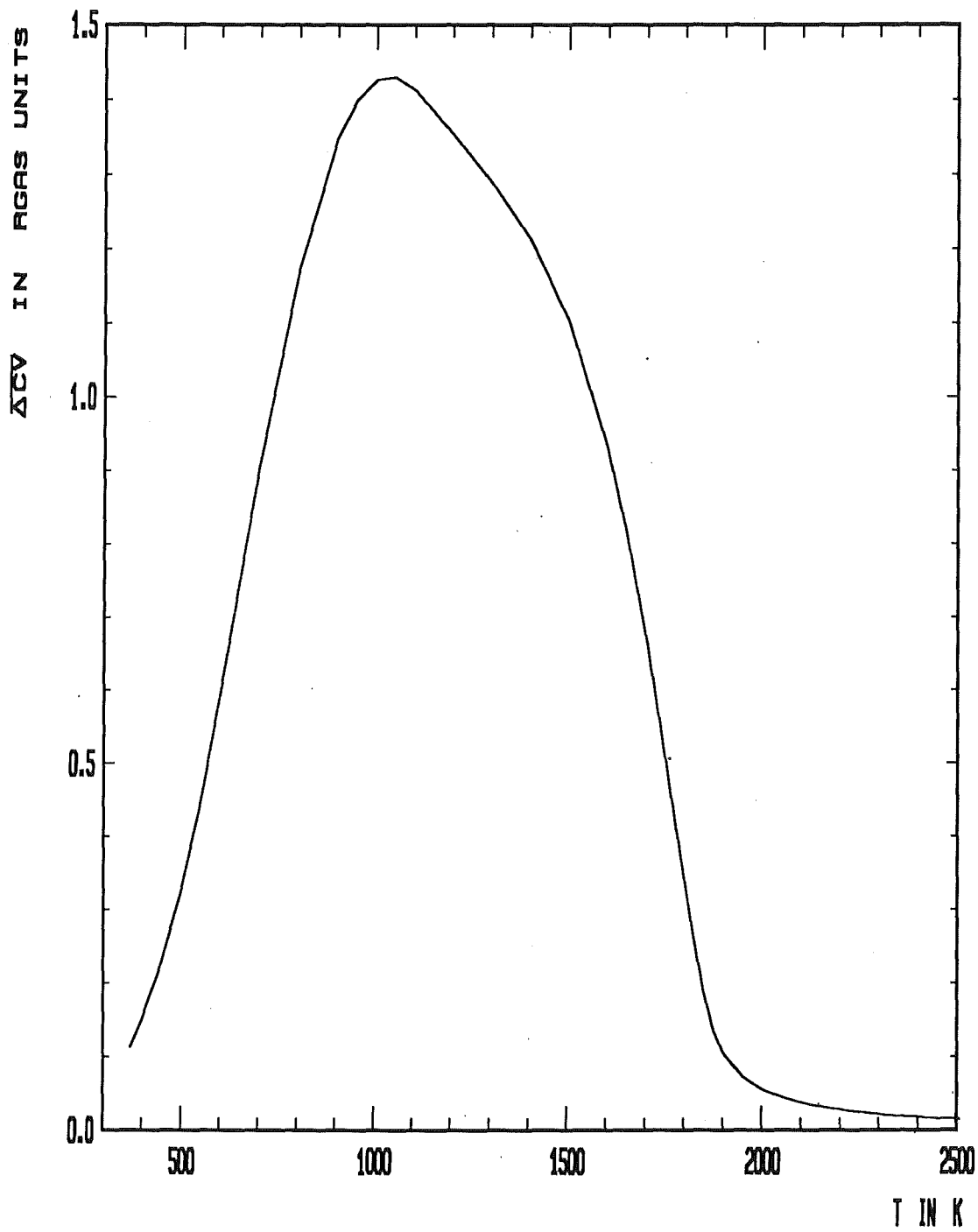


Figure 9. SODIUM. CV-DIFFERENCE BETWEEN THE SATURATED AND THE VACUUM STATES .

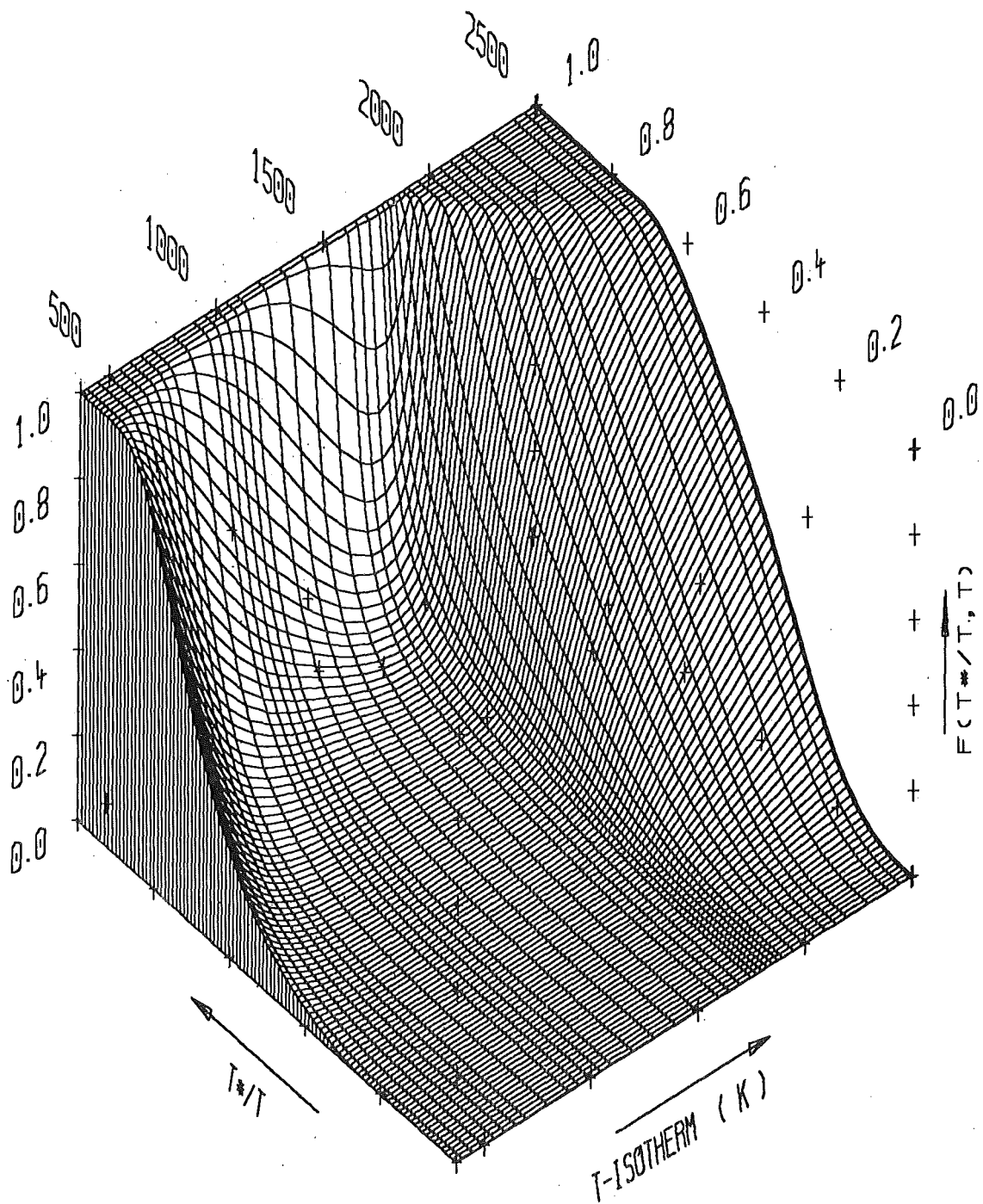


Figure 10. SODIUM. DENSITY-TEMPERATURE SURFACE OF THE CV-DEPARTURE (OVER-HEATED VAPOR).

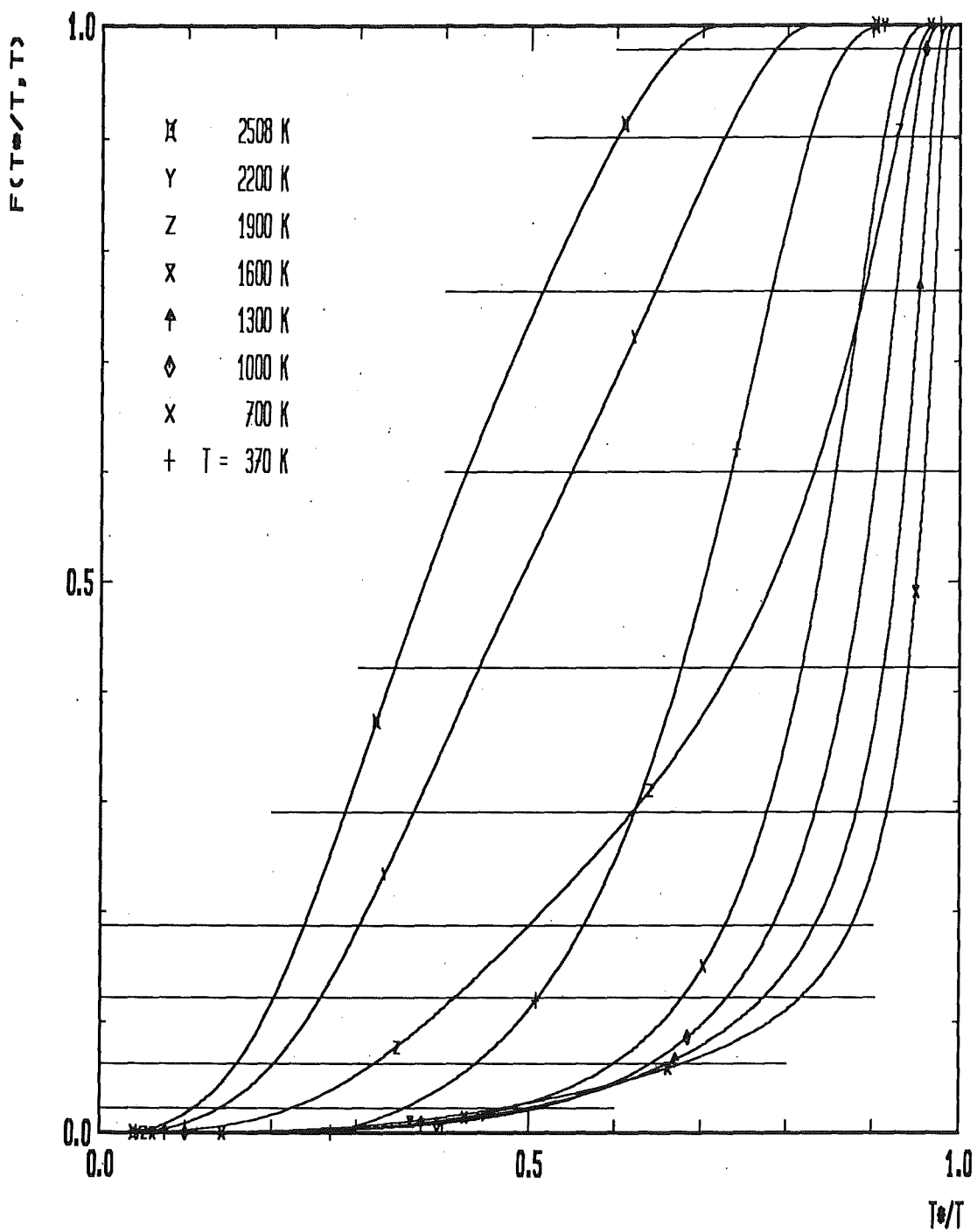


Figure 11. SODIUM. HEAT CAPACITY DEPARTURE IN THE OVERHEATED VAPOR .

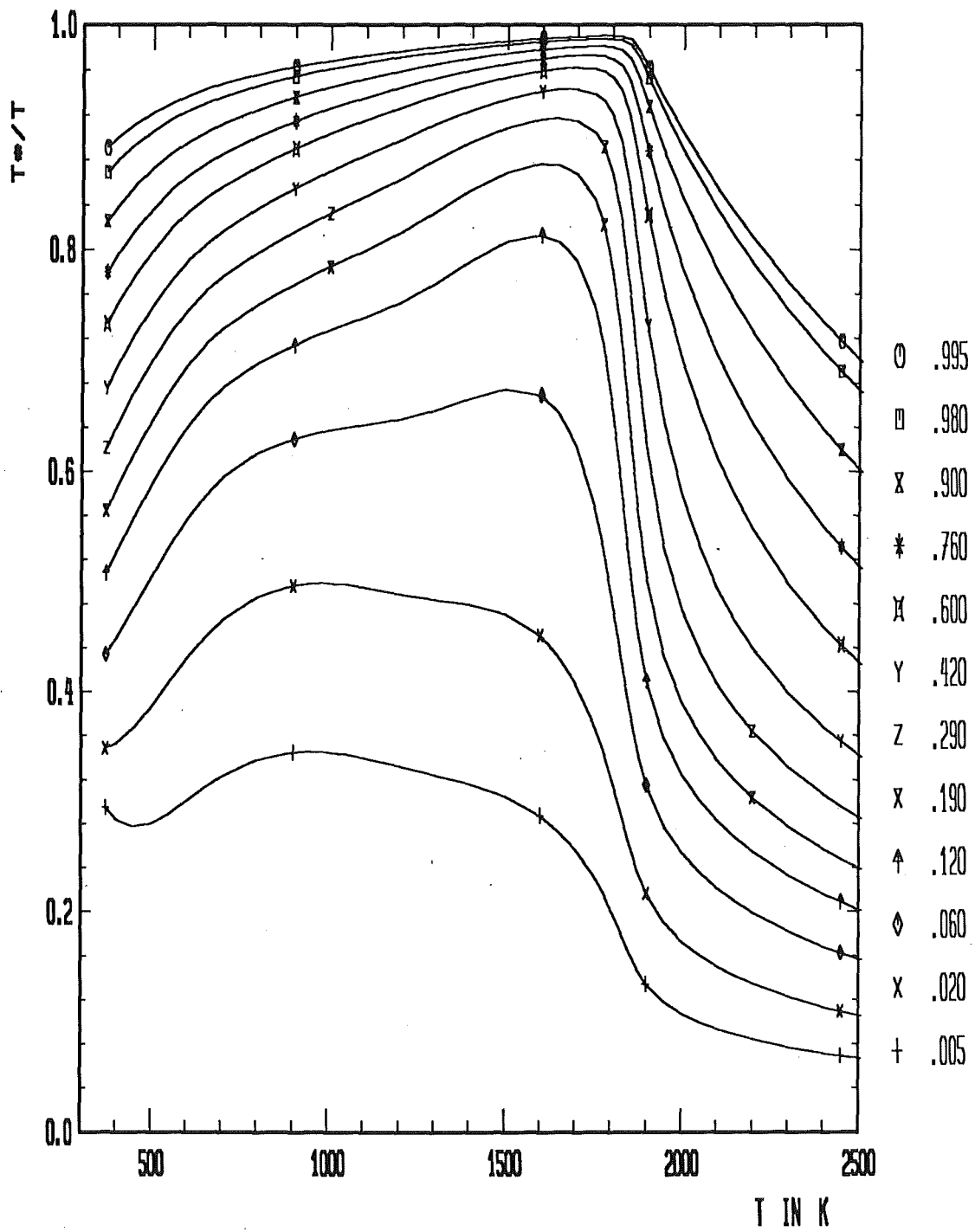


Figure 12. SODIUM. CONTOUR LINES OF THE SURFACE $F(T^*/T, T)$.

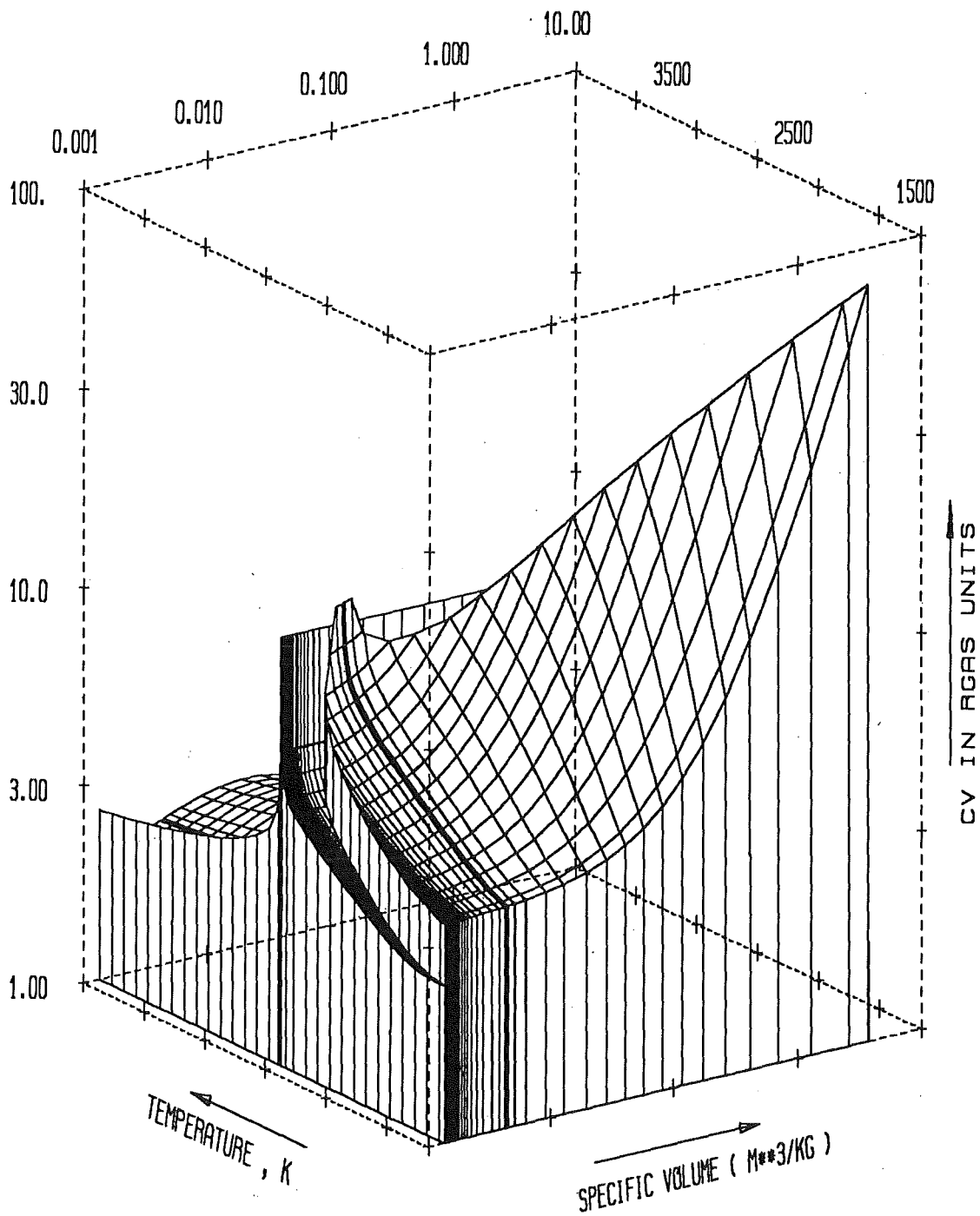


Figure 13. SODIUM. THE (V,T)-SURFACE OF THE HEAT CAPACITY CV (LIQUID SIDE).



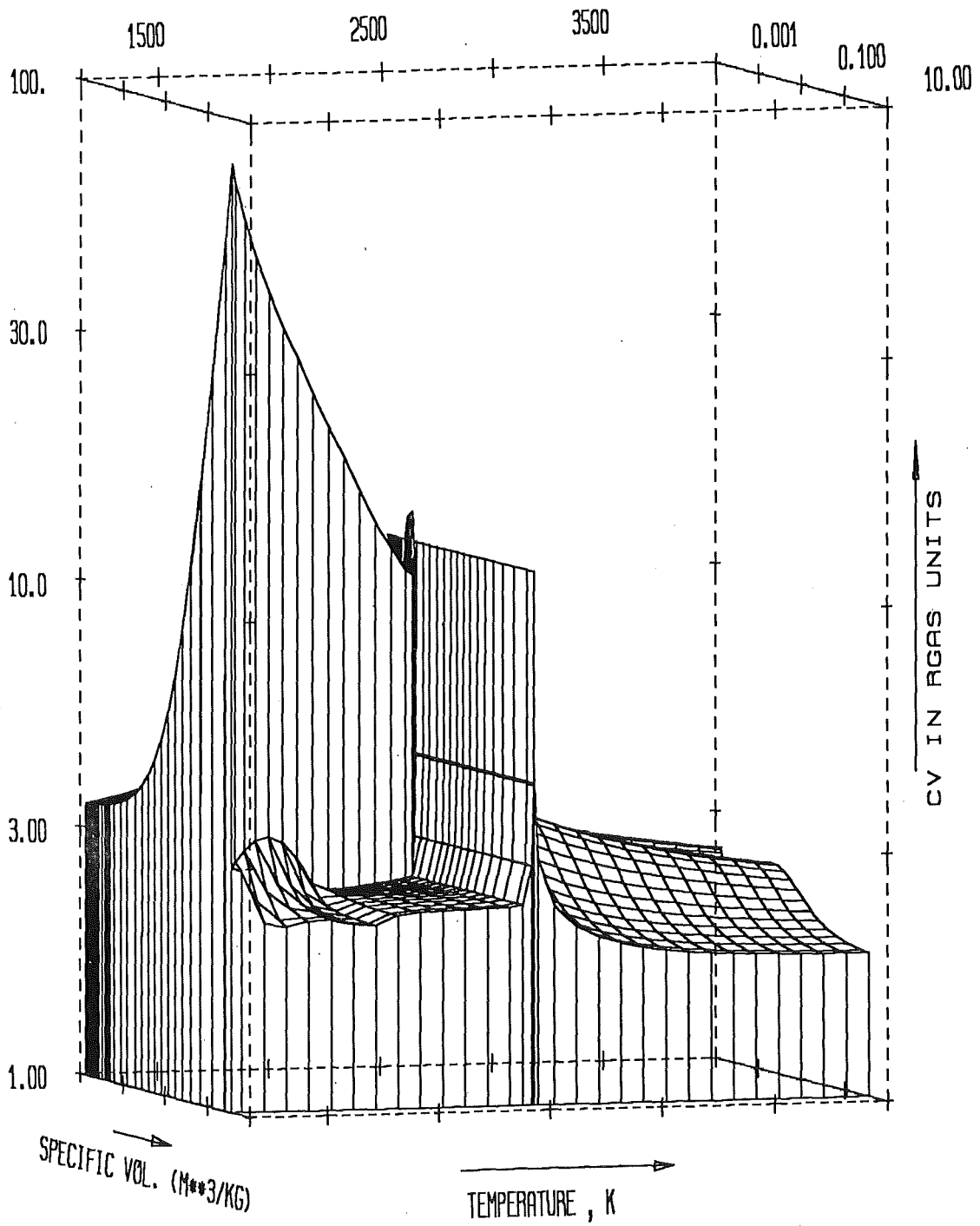


Figure 14. SODIUM. THE (V,T)-SURFACE OF THE HEAT CAPACITY C_V (VAPOR SIDE).

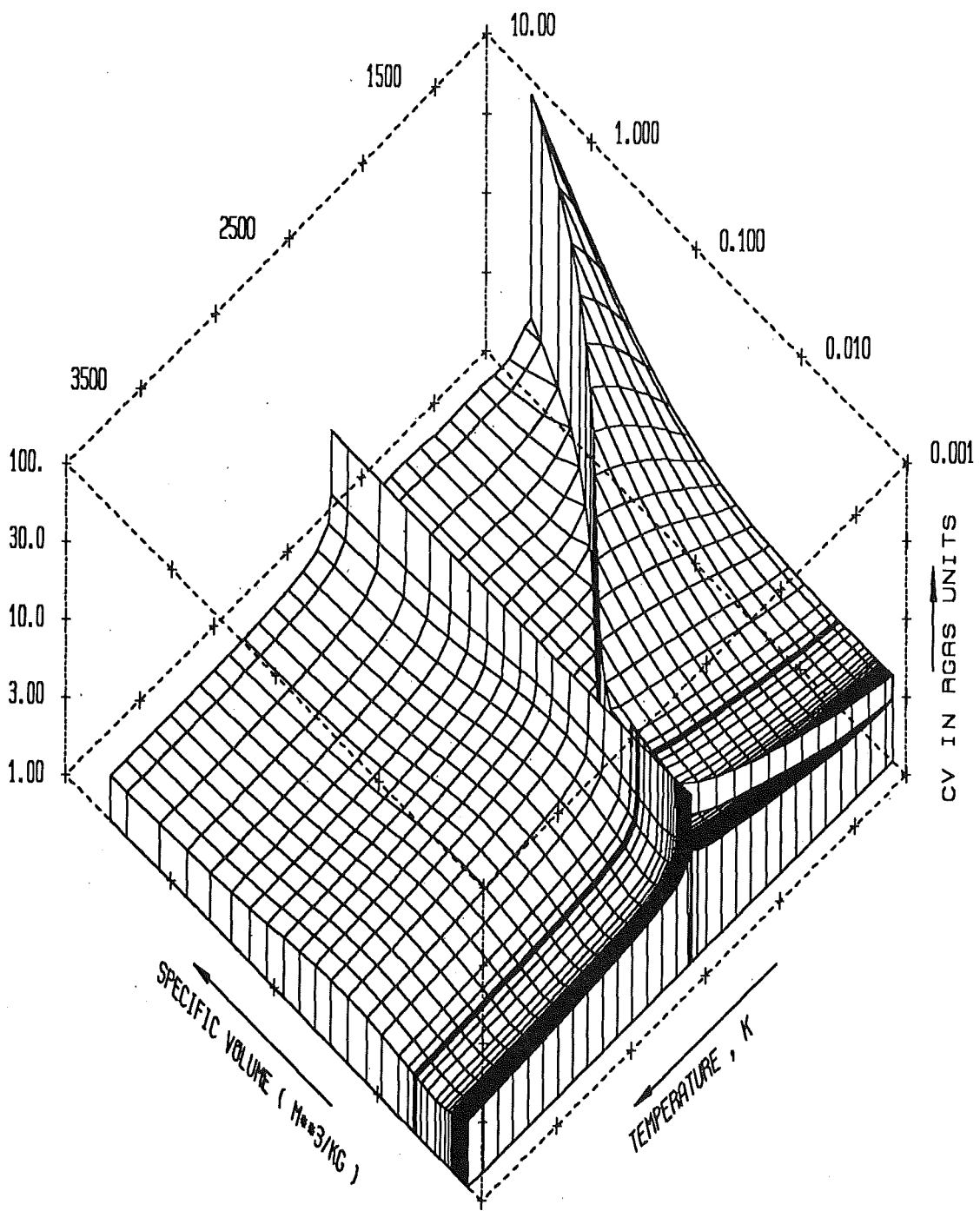


Figure 15. SODIUM. THE (V,T)-SURFACE OF THE HEAT CAPACITY C_v (GAS SIDE).

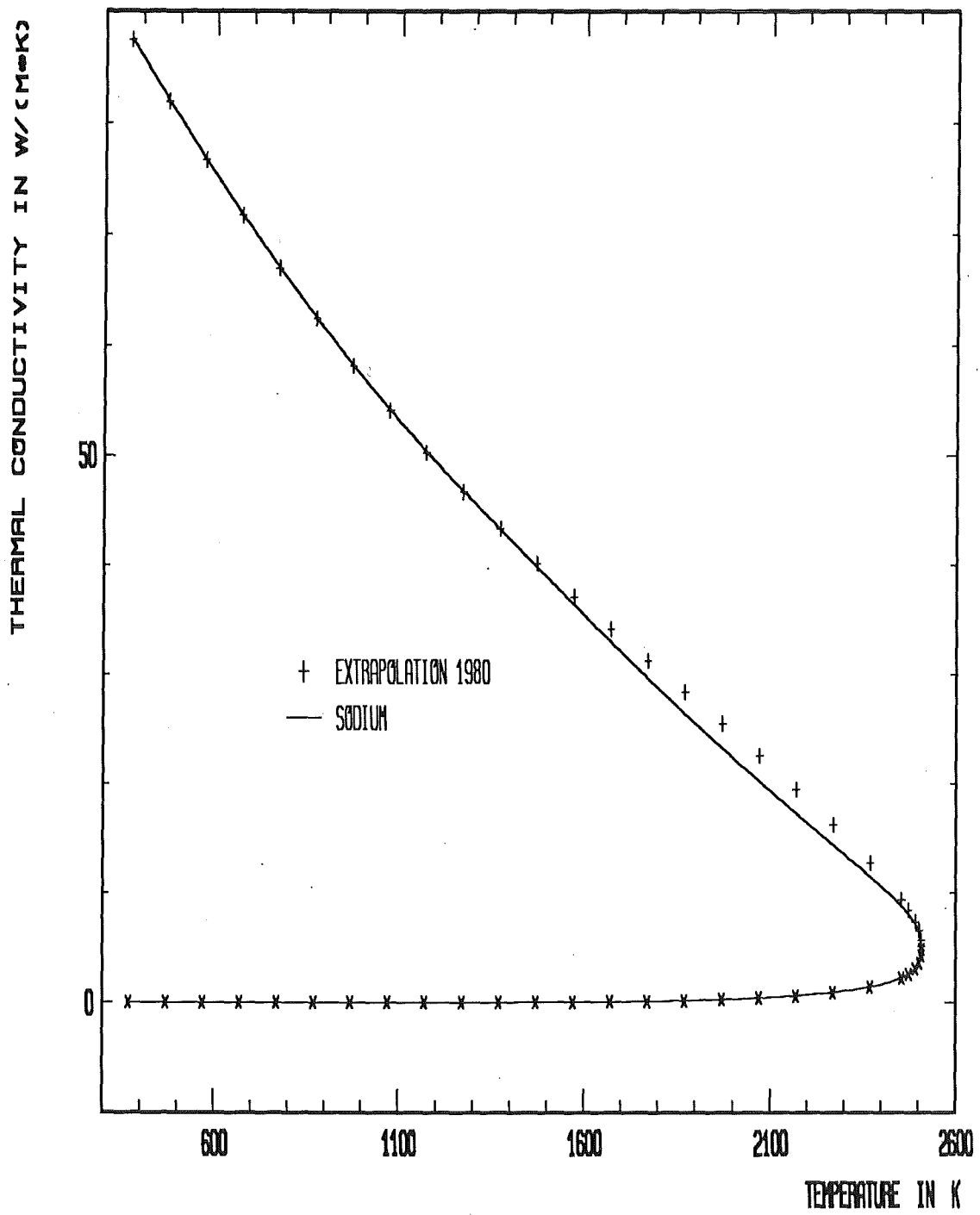


Figure 16. SODIUM. THERMAL CONDUCTIVITY ON THE SATURATION LINE.

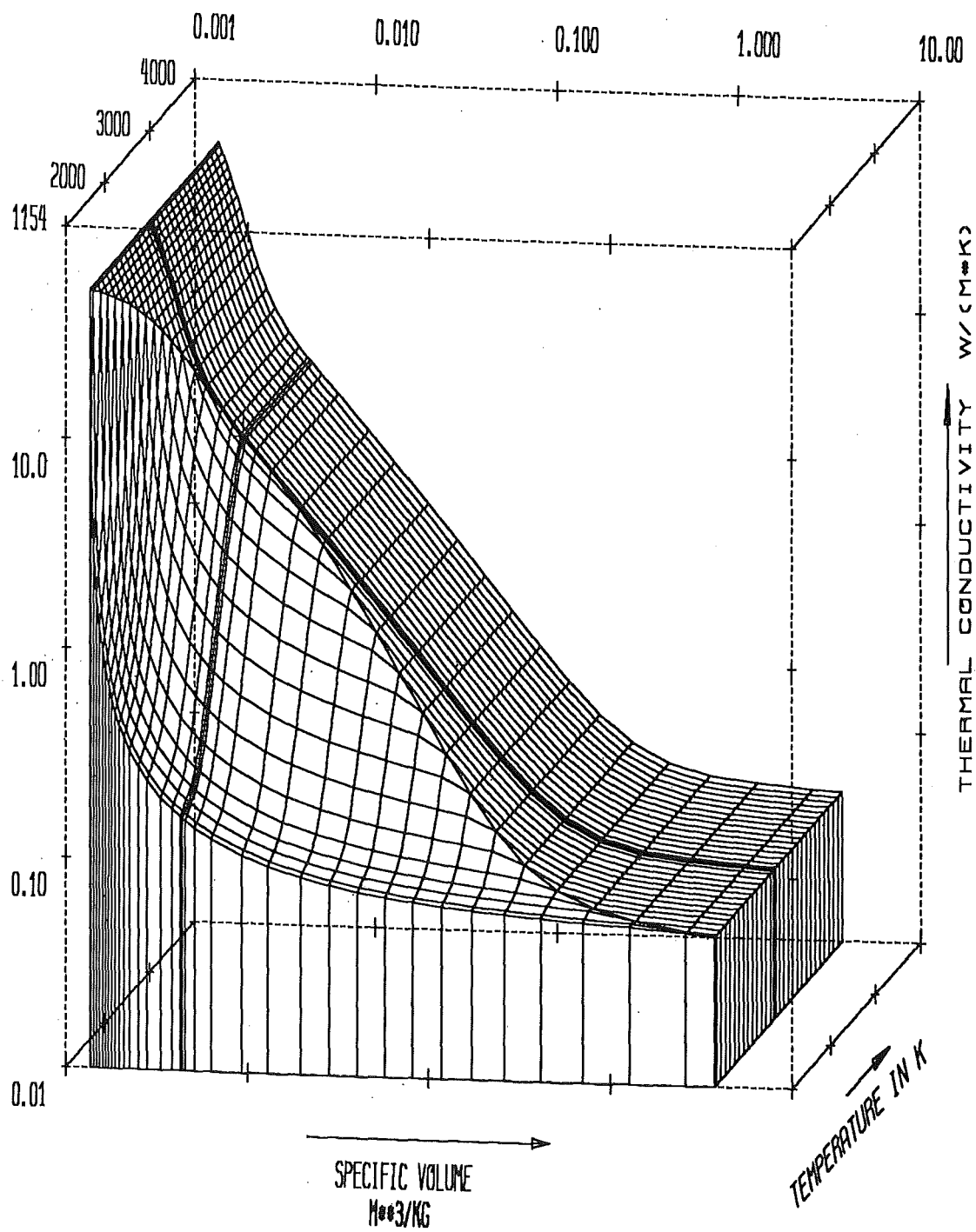


Figure 17. SODIUM. THE THERMAL CONDUCTIVITY AS A FUNCTION OF T AND V.

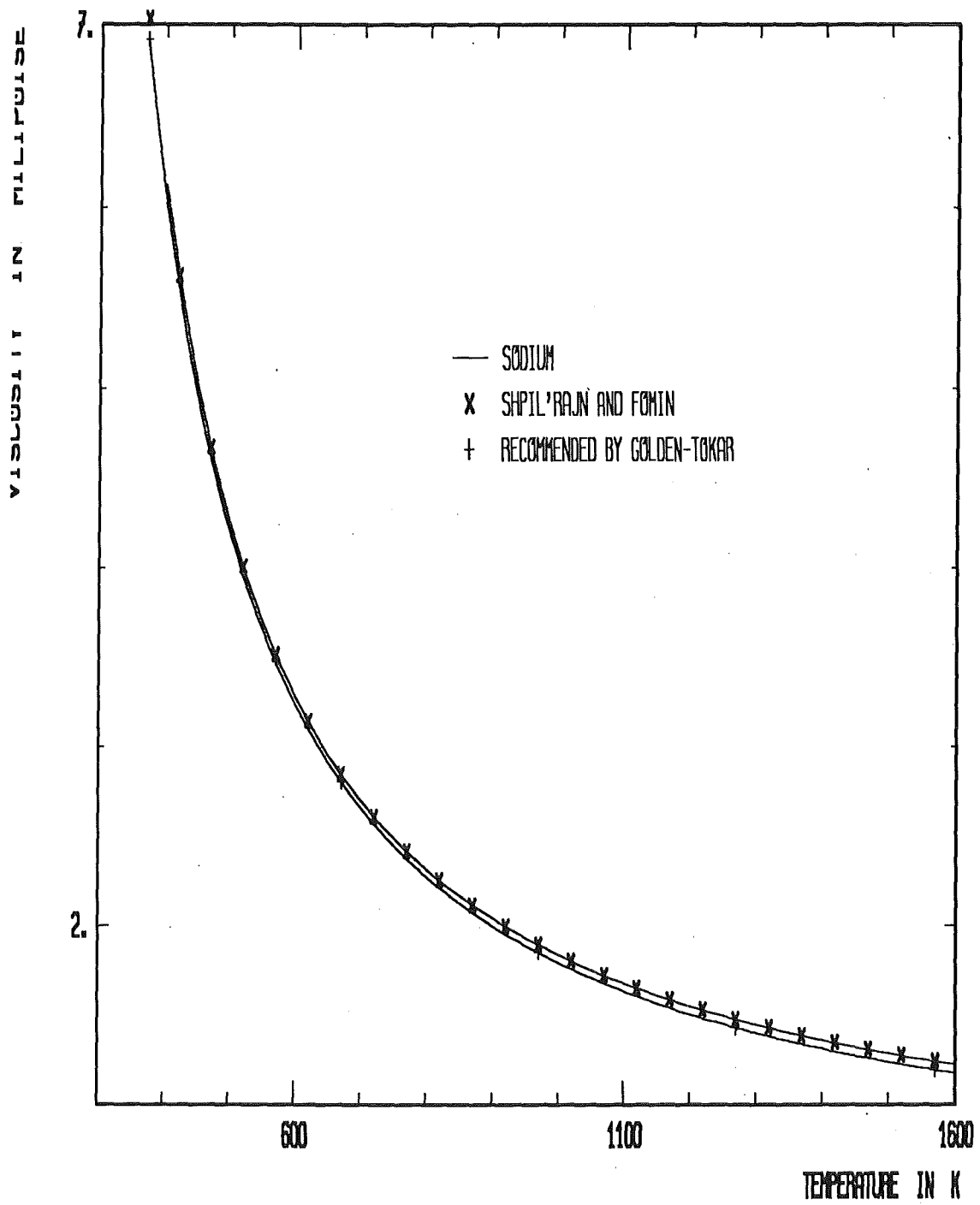


Figure 18. SODIUM. VISCOSITY OF THE SATURATED LIQUID.

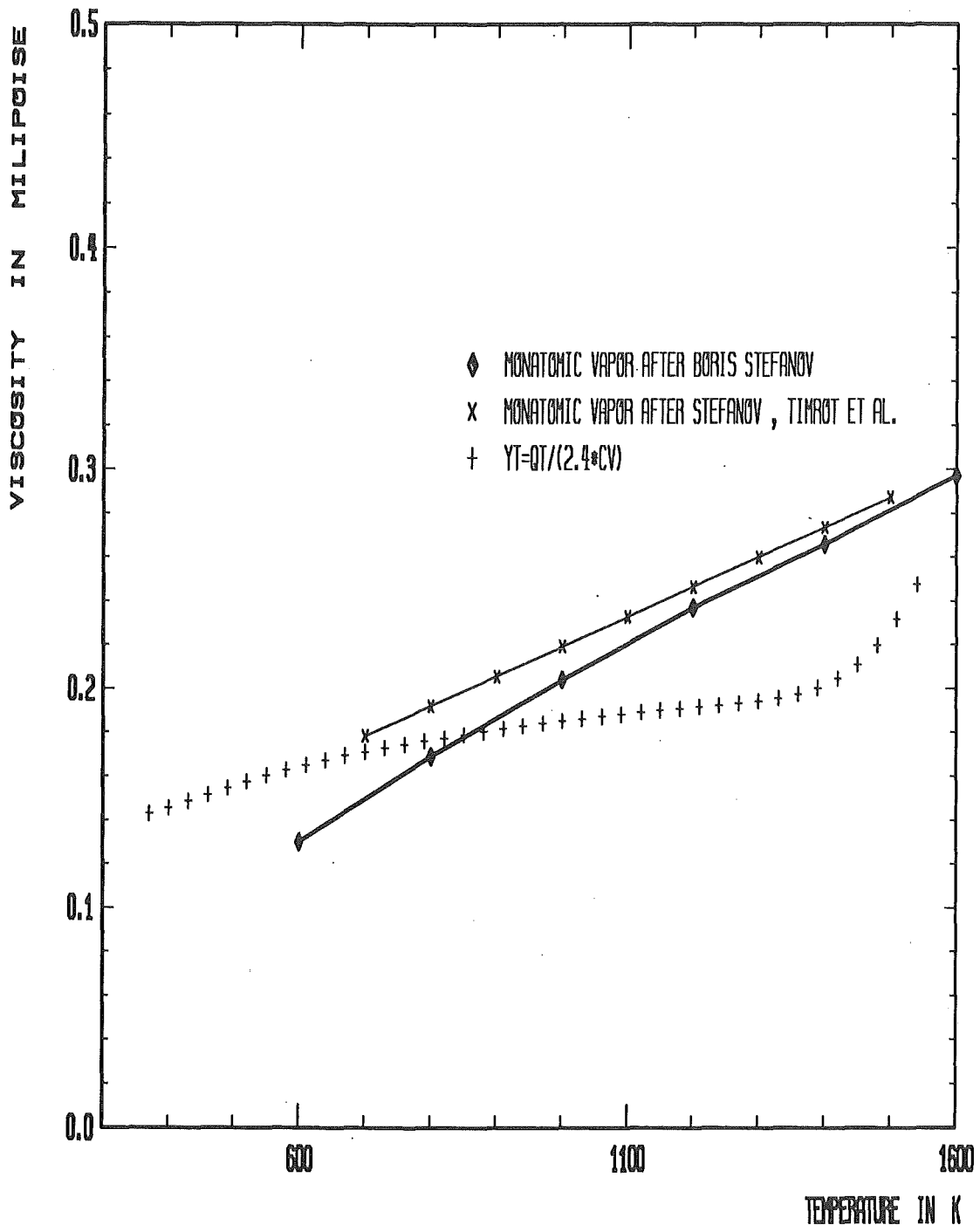


Figure 19. SODIUM. VISCOSITY OF THE MONATOMIC VAPOR.

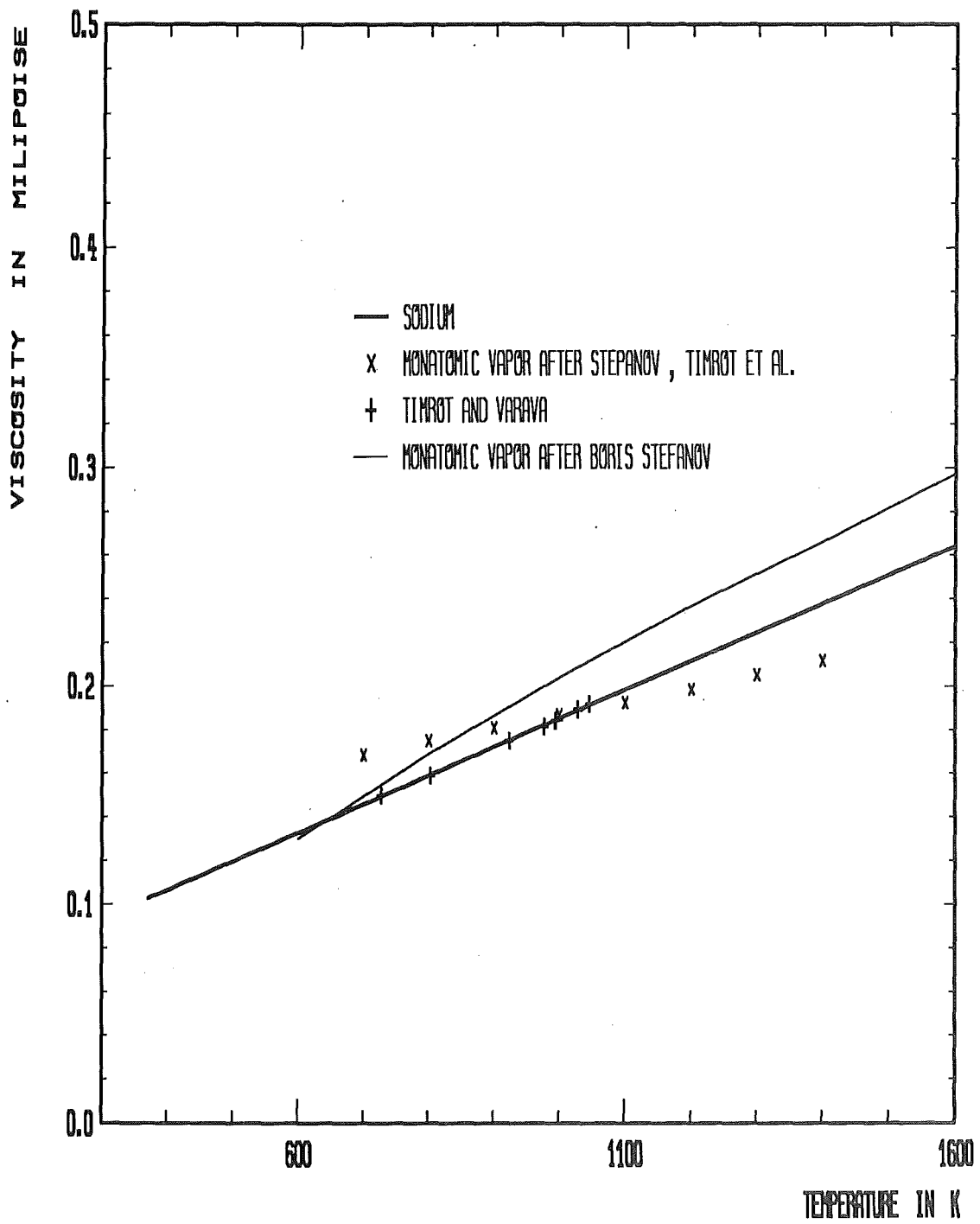


Figure 20. SODIUM. VISCOSITY OF THE SATURATED VAPOR.

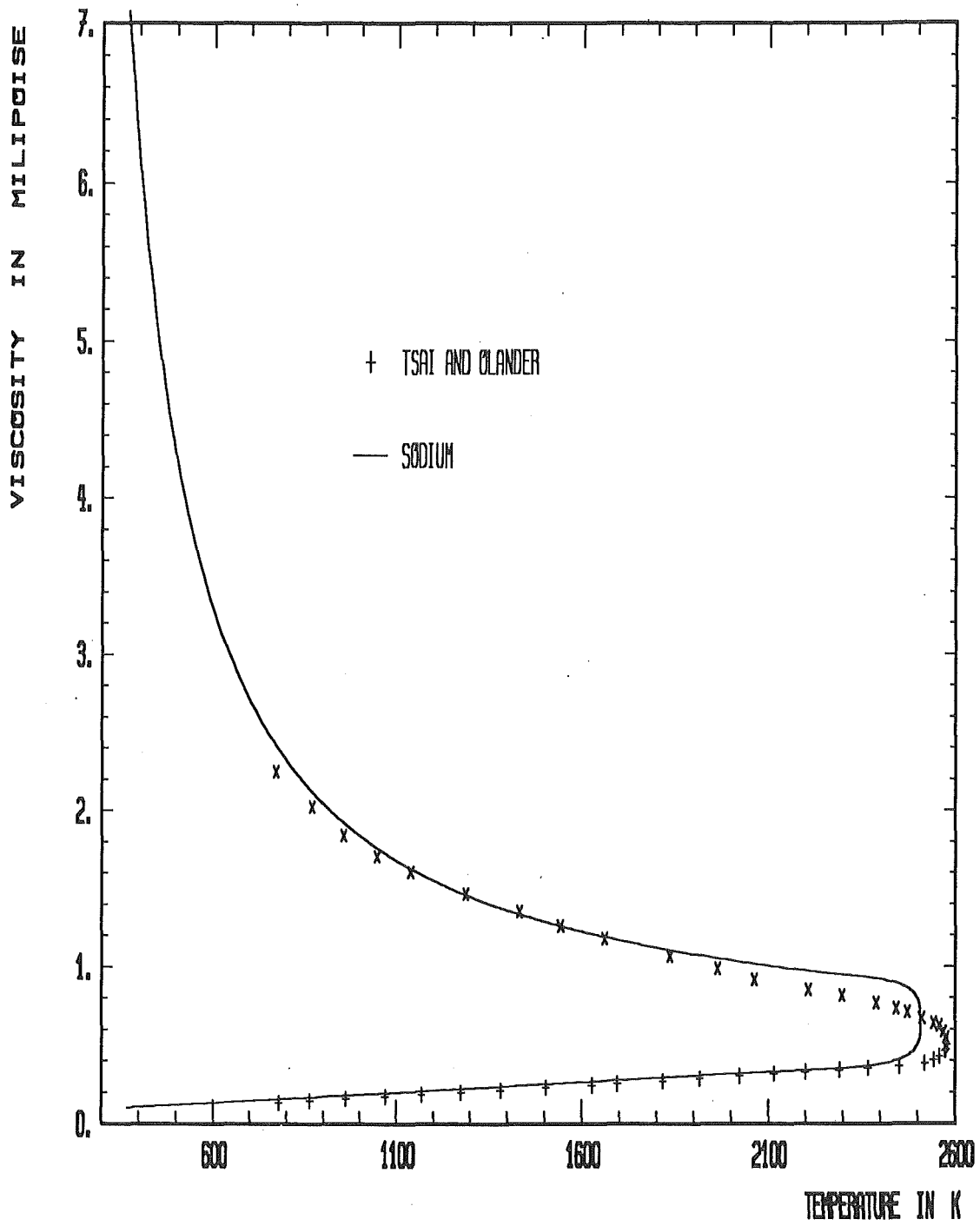


Figure 21. SODIUM. VISCOSITY ON THE SATURATION LINE.

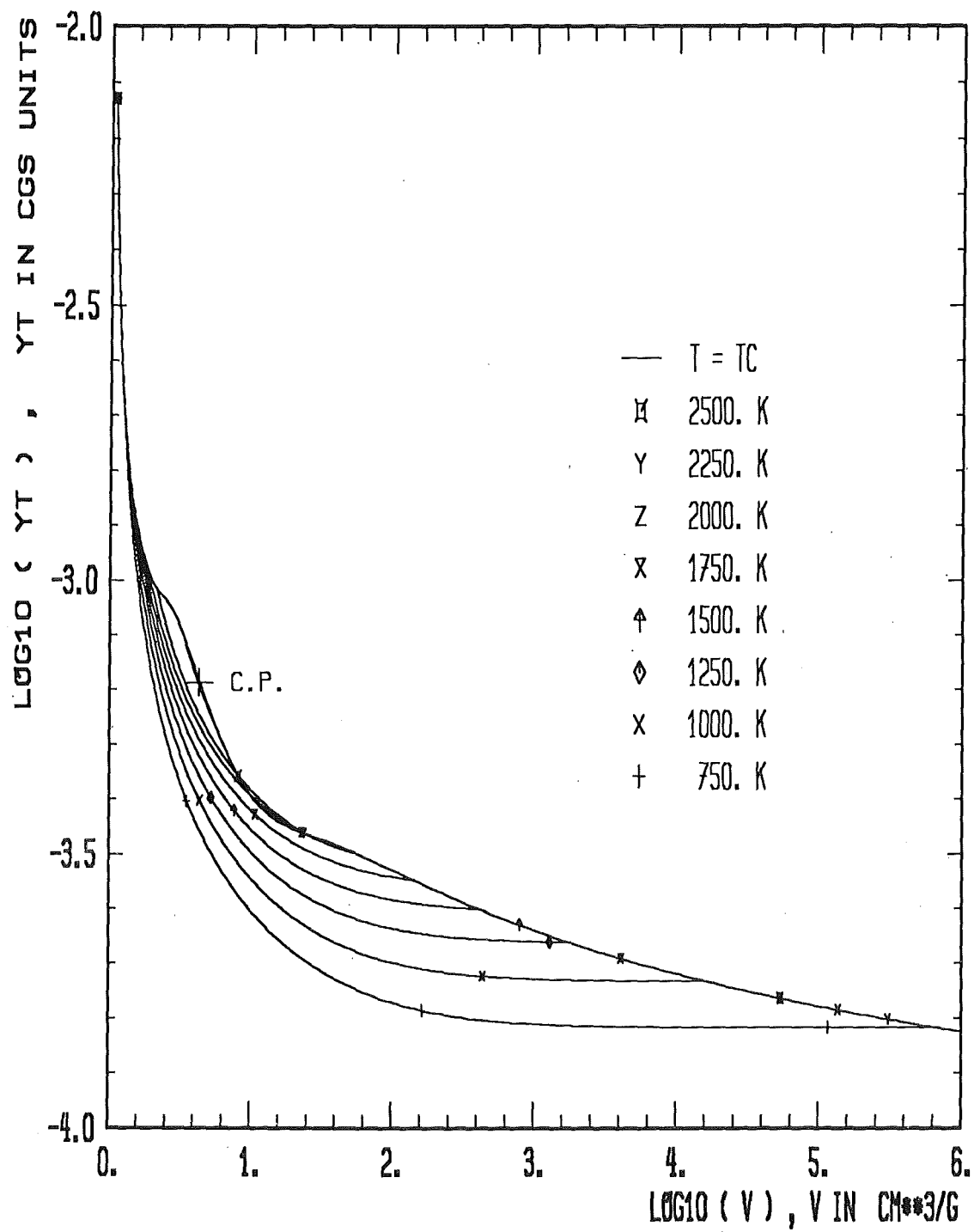


Figure 22. SODIUM. THE VISCOSITY - VOLUME CHART .

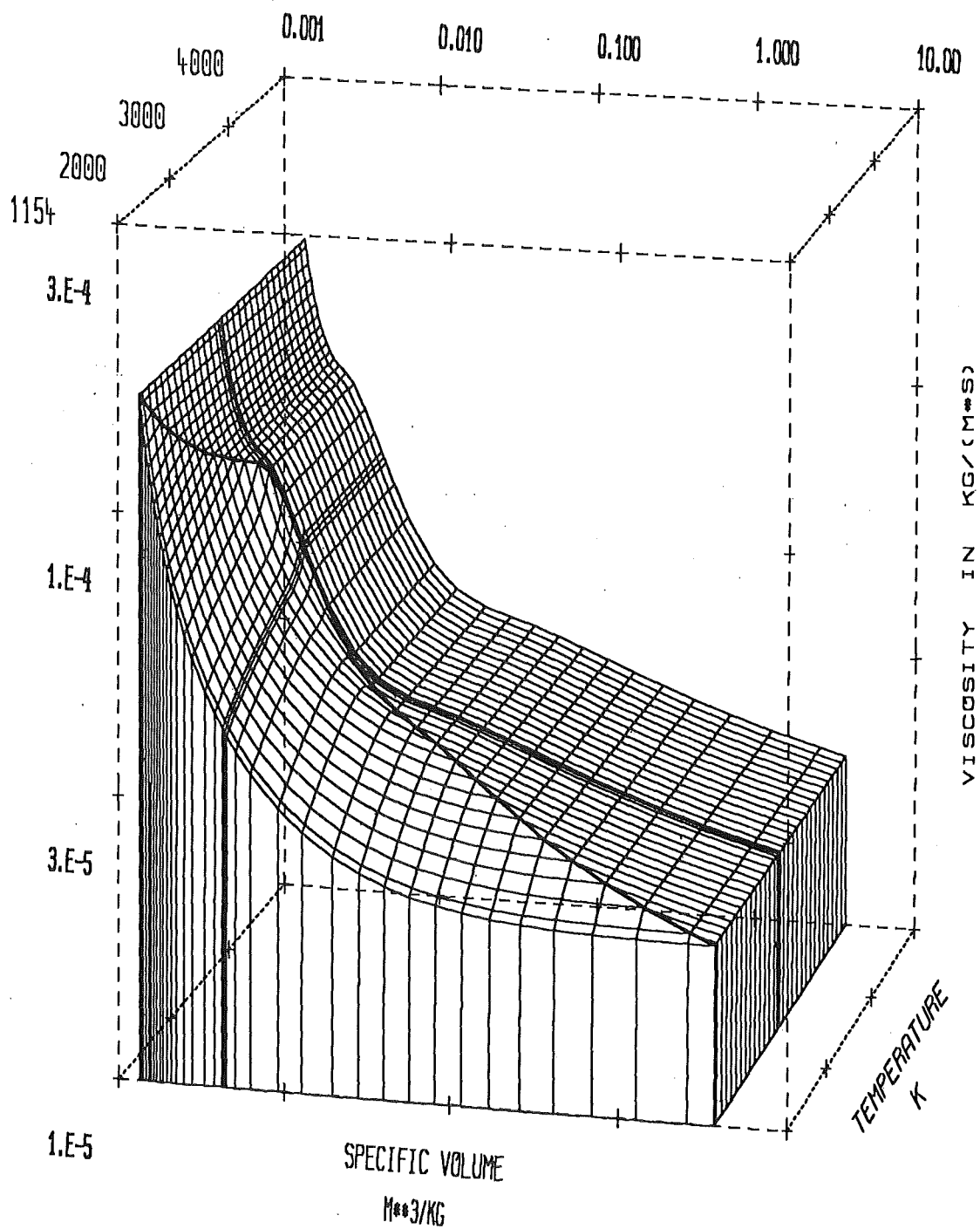


Figure 23. SODIUM. THE (V , T) - SURFACE OF THE VISCOSITY .

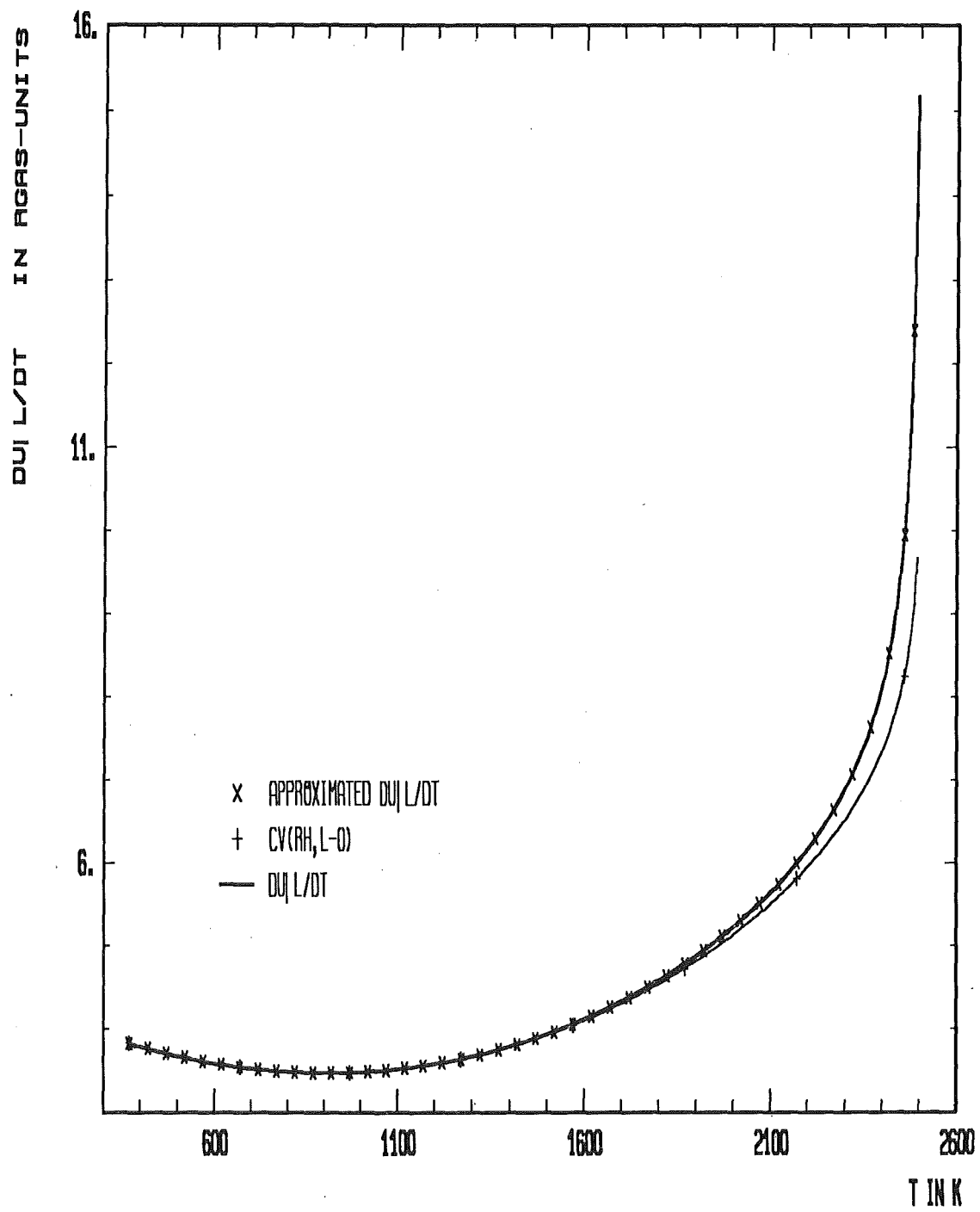


Figure 24. SODIUM, $DUJ L/DT$ AS A FUNCTION OF TEMPERATURE.

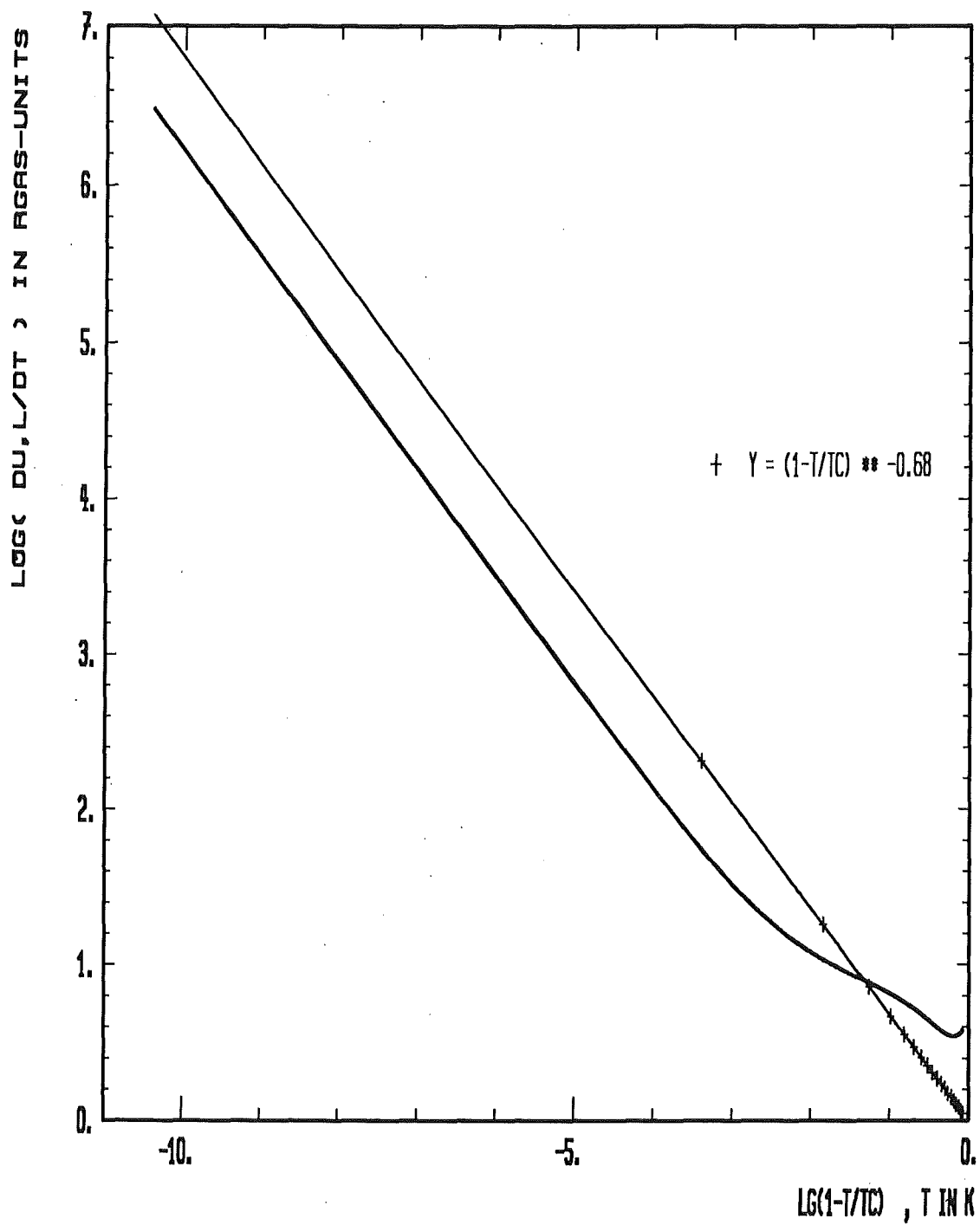
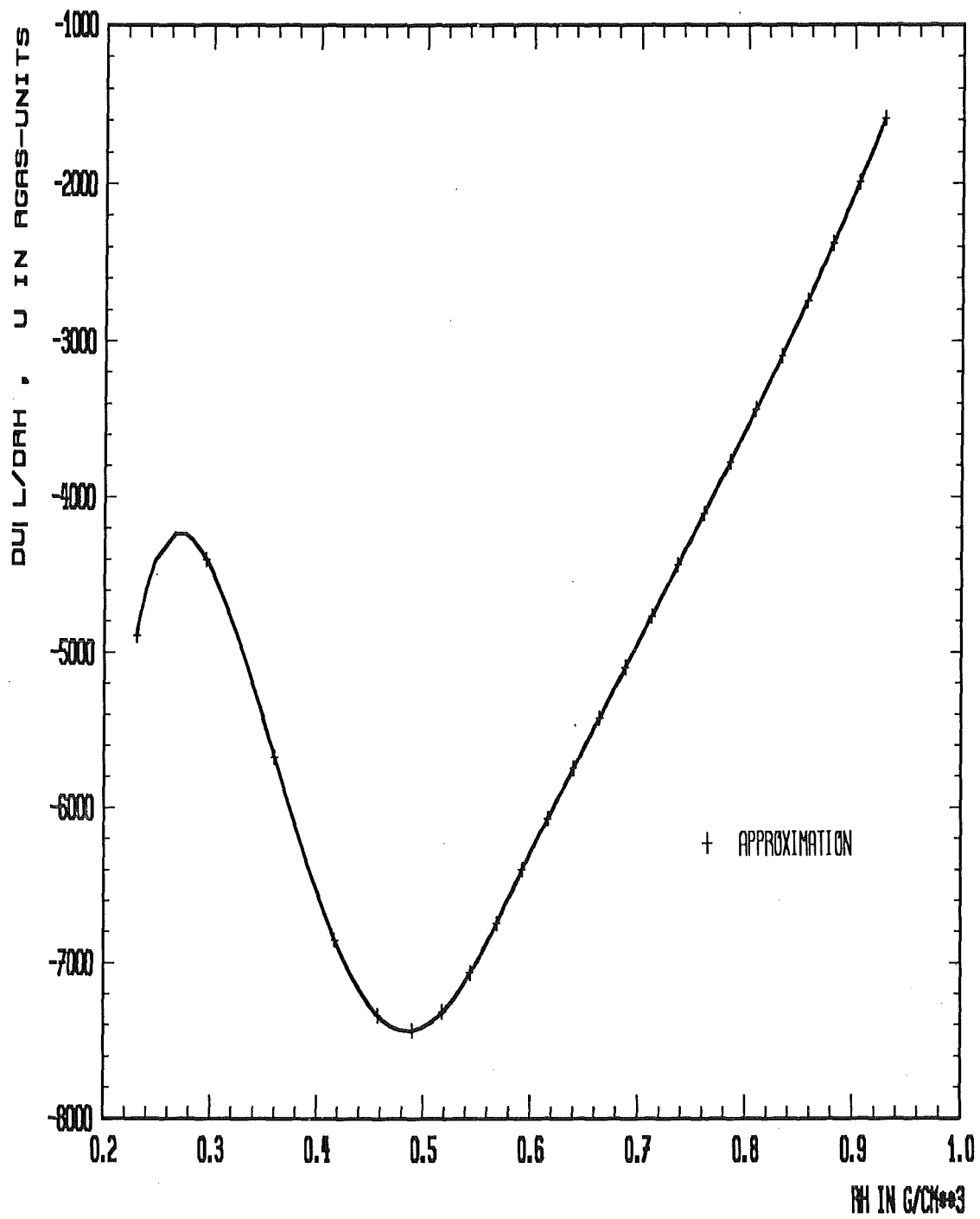
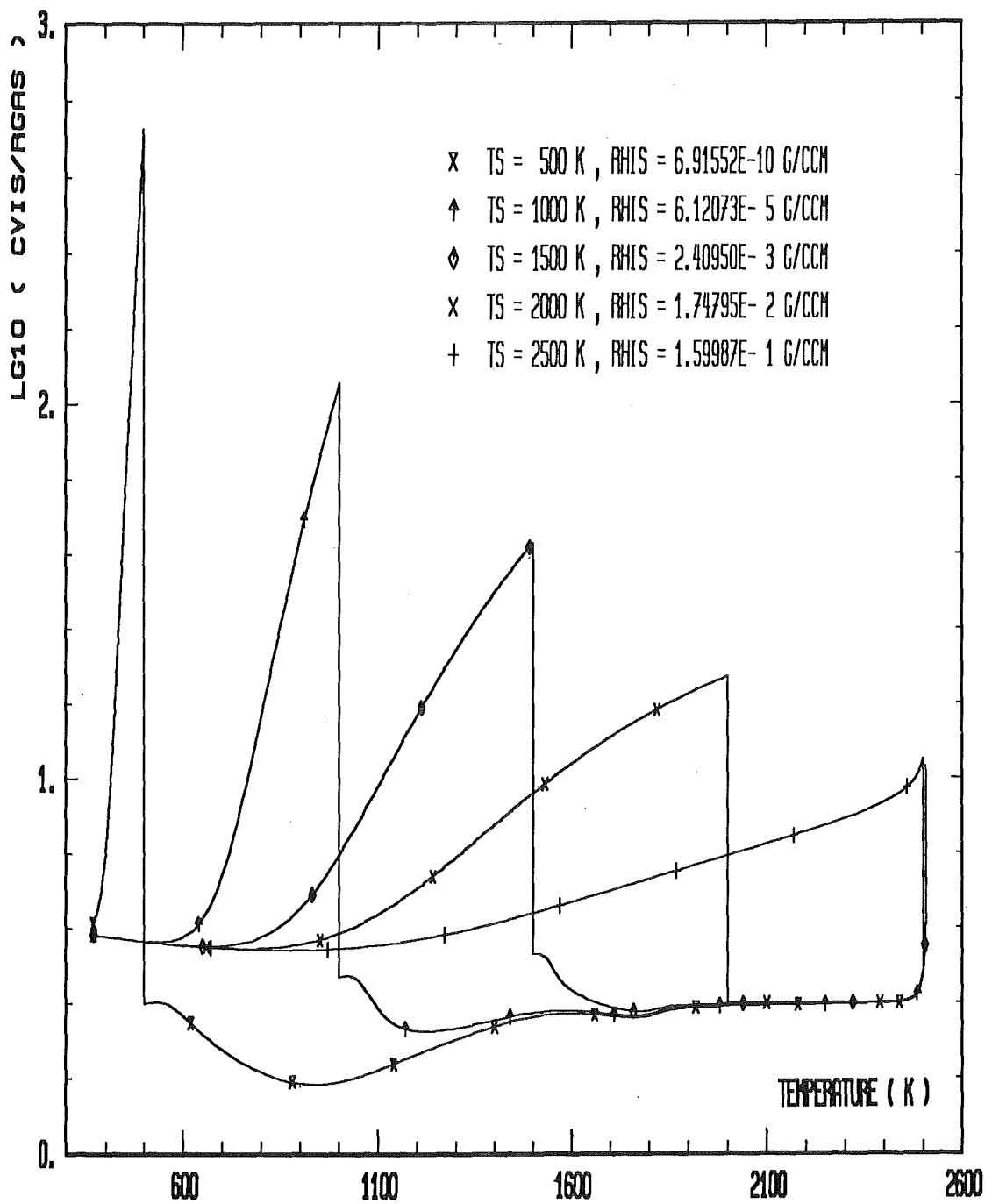


Figure 25. SODIUM . DU|L/DT AS A FUNCTION OF (1-T/TC.).



KJK
INR

Figure 26. SODIUM. DU|L/DRH AS A FUNCTION OF THE DENSITY.



KITZ
NMR

Figure 27. SODIUM. CV ON SOME ISOCORES IN THE SUBCRITICAL REGION.

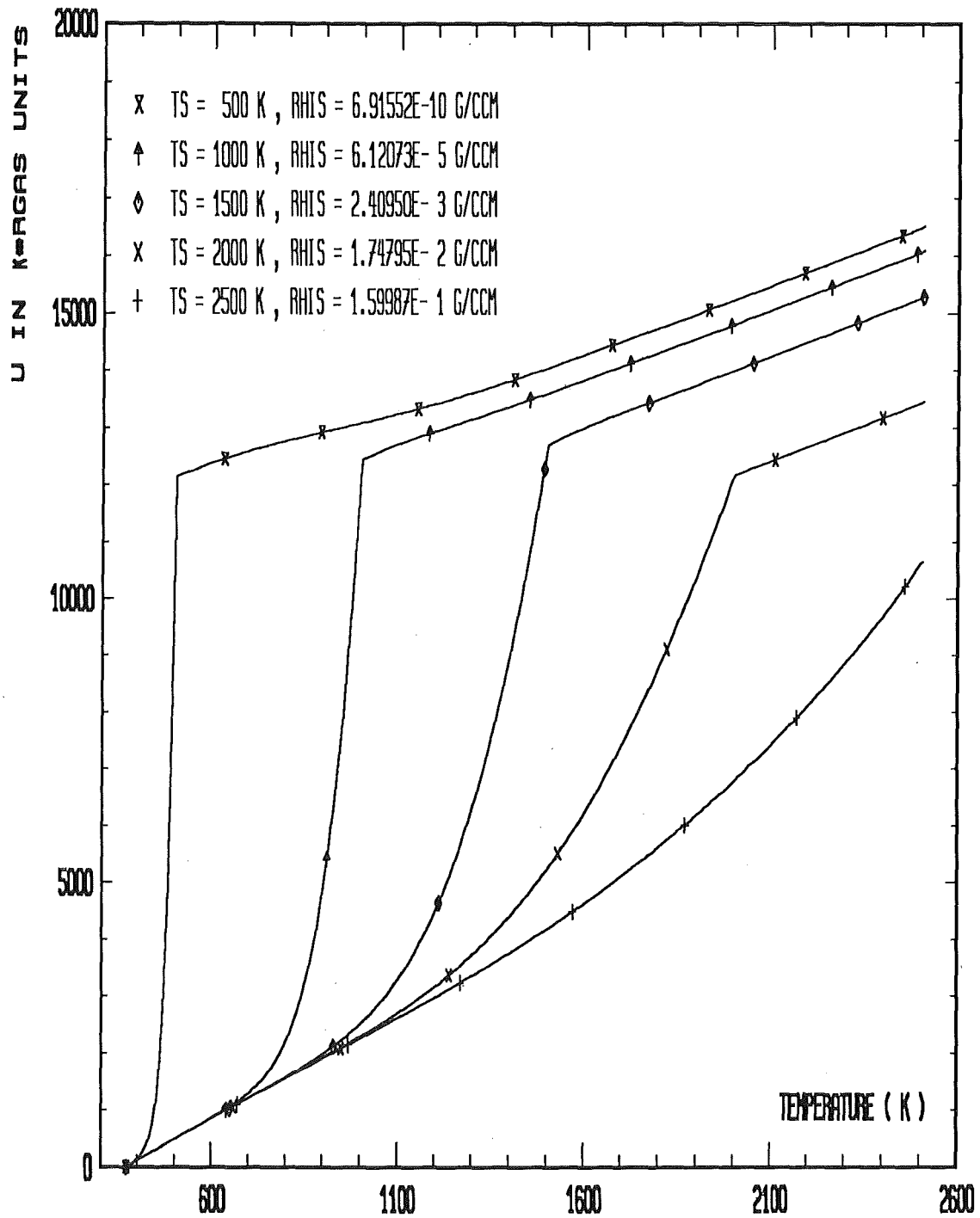


Figure 28. SODIUM. U ON SOME ISOCHORES IN THE SUBCRITICAL REGION.

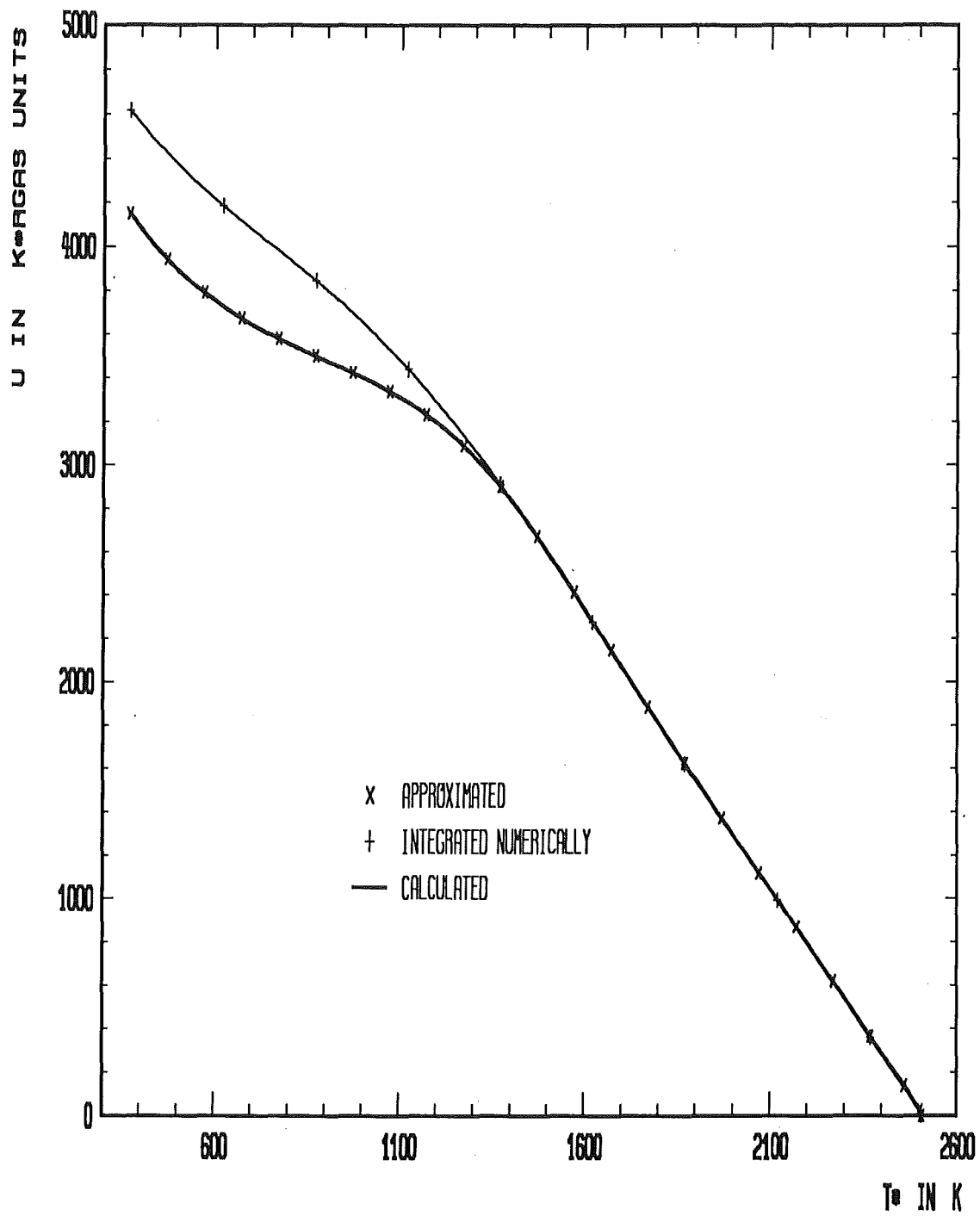
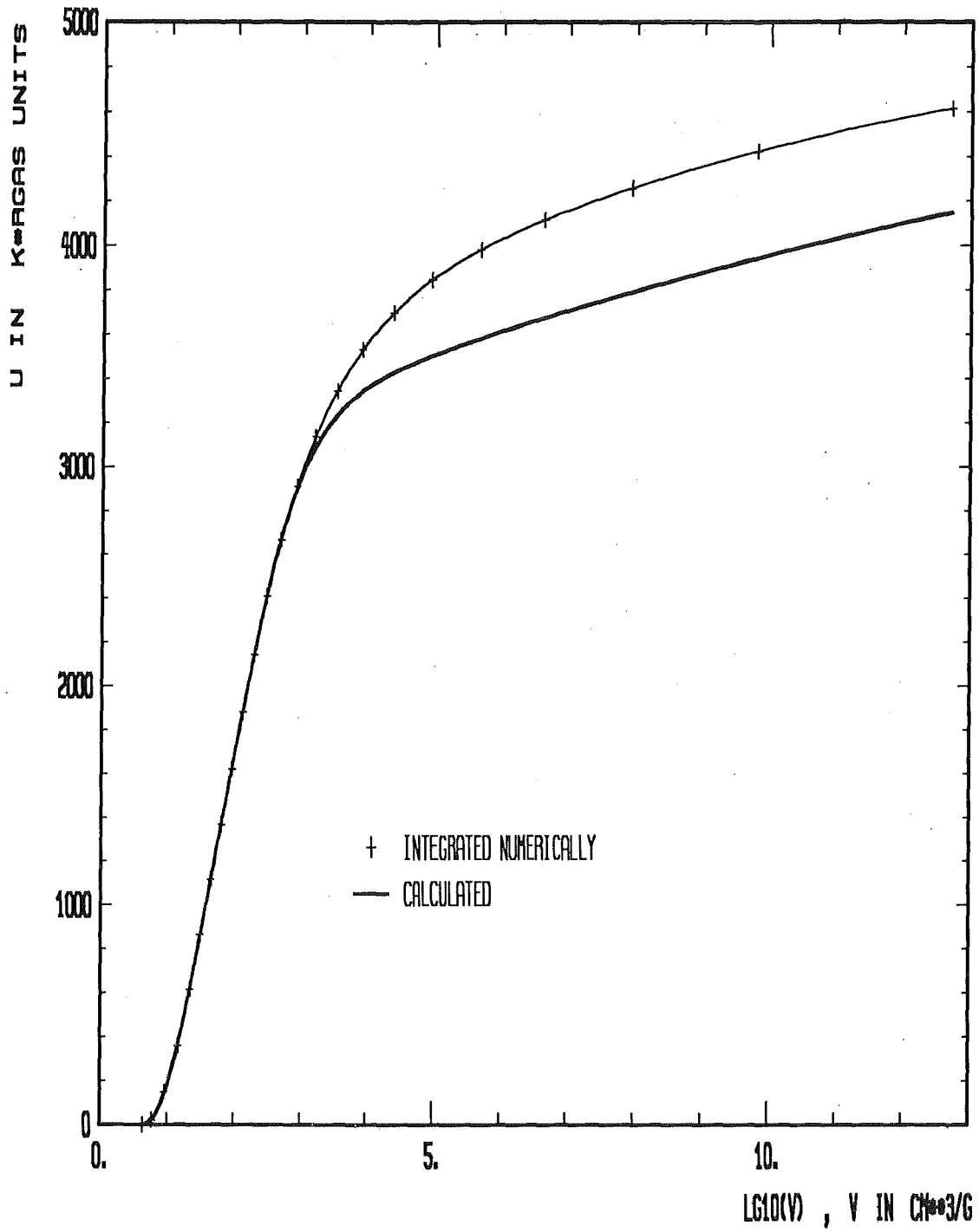


Figure 29. SODIUM. $U(T_C, \rho) - U(T^*, \rho)$ AS A FUNCTION OF THE SAT. TEMPERATURE T^* .



KfK
TNR

Figure 30. SODIUM. $U(TC, \rho) - U(T^*, \rho)$ AS A FUNCTION OF THE DENSITY.

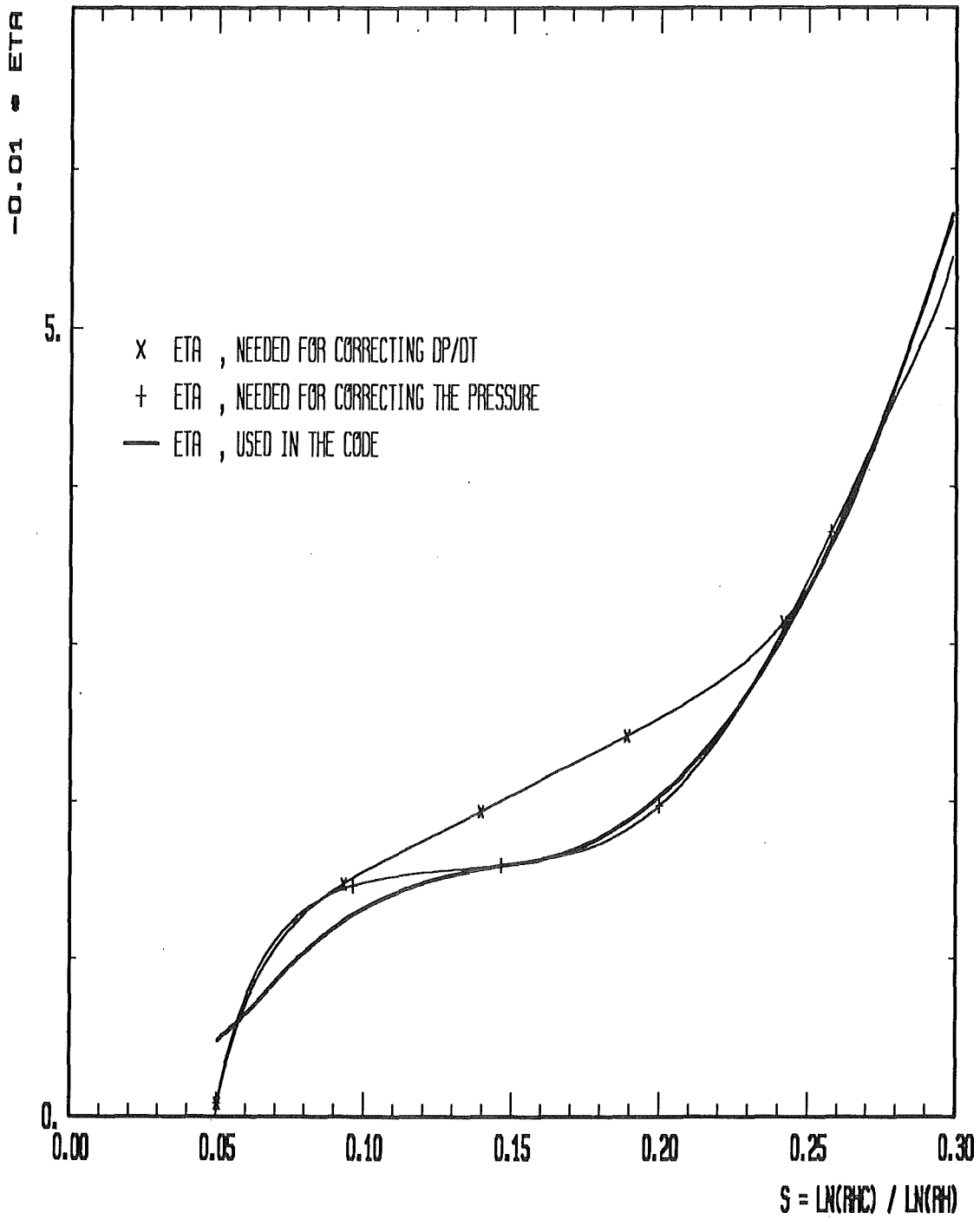


Figure 31. SODIUM. CORRECTING FUNCTION ETA IN THE OVERHEATED VAPOR.

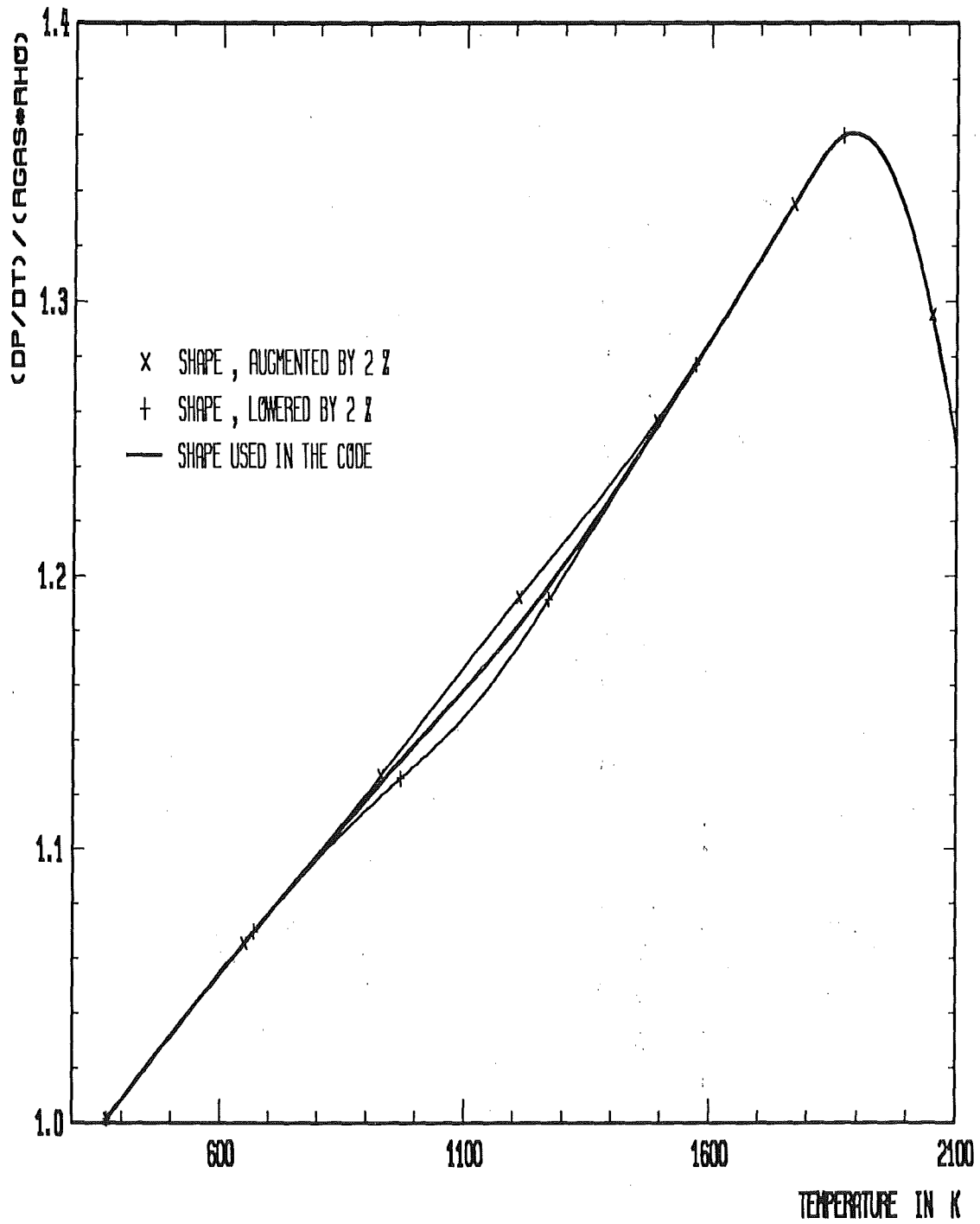


Figure 32. SODIUM. VARYING DP/DT IN THE SATURATED VAPOR.

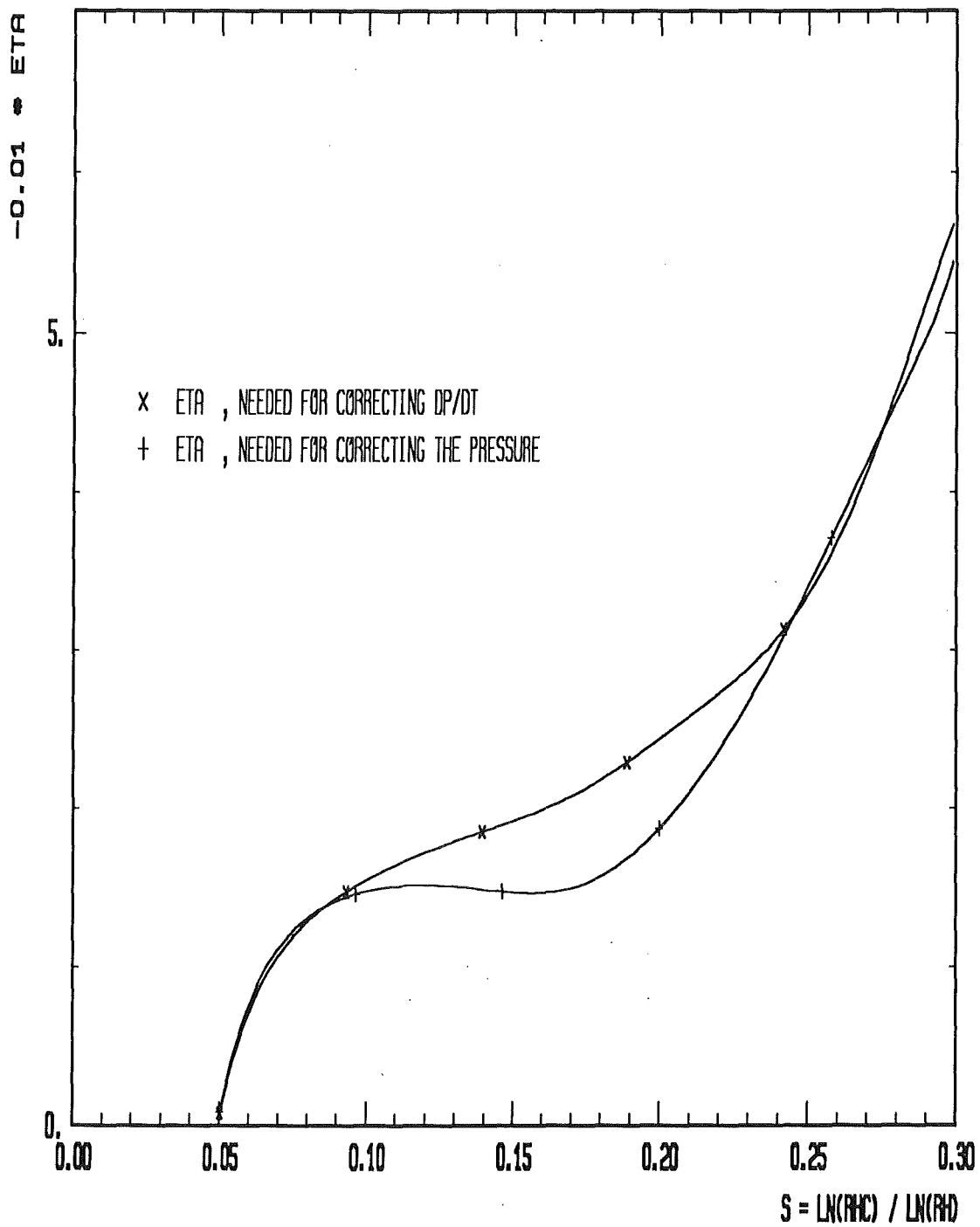


Figure 33. SODIUM. ETA SHAPES , RESULTING FROM A LOWERED DP/DT-PATTERN.

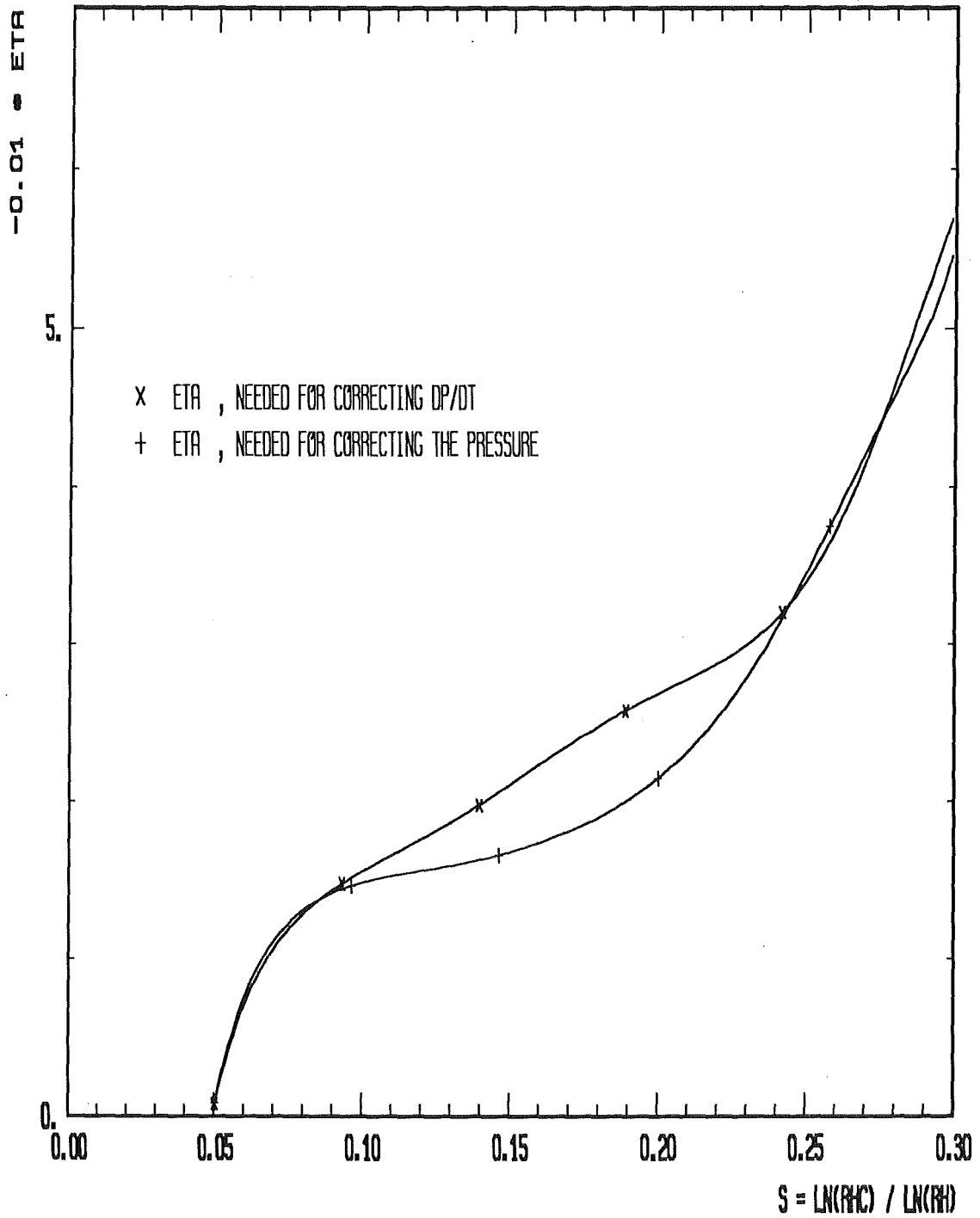


Figure 34. SODIUM. ETA SHAPES , RESULTING FROM AN INCREASED DP/DT-PATTERN.

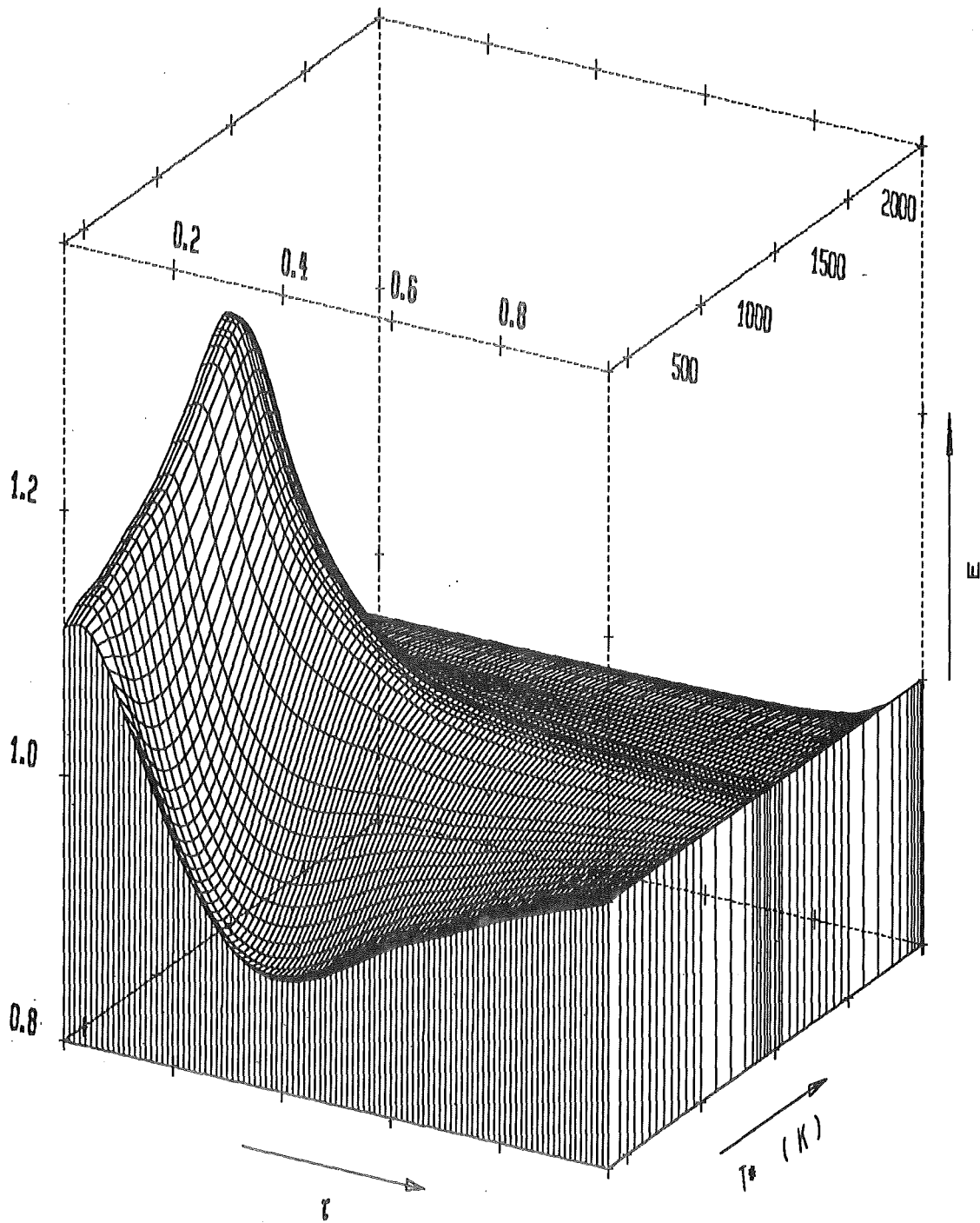
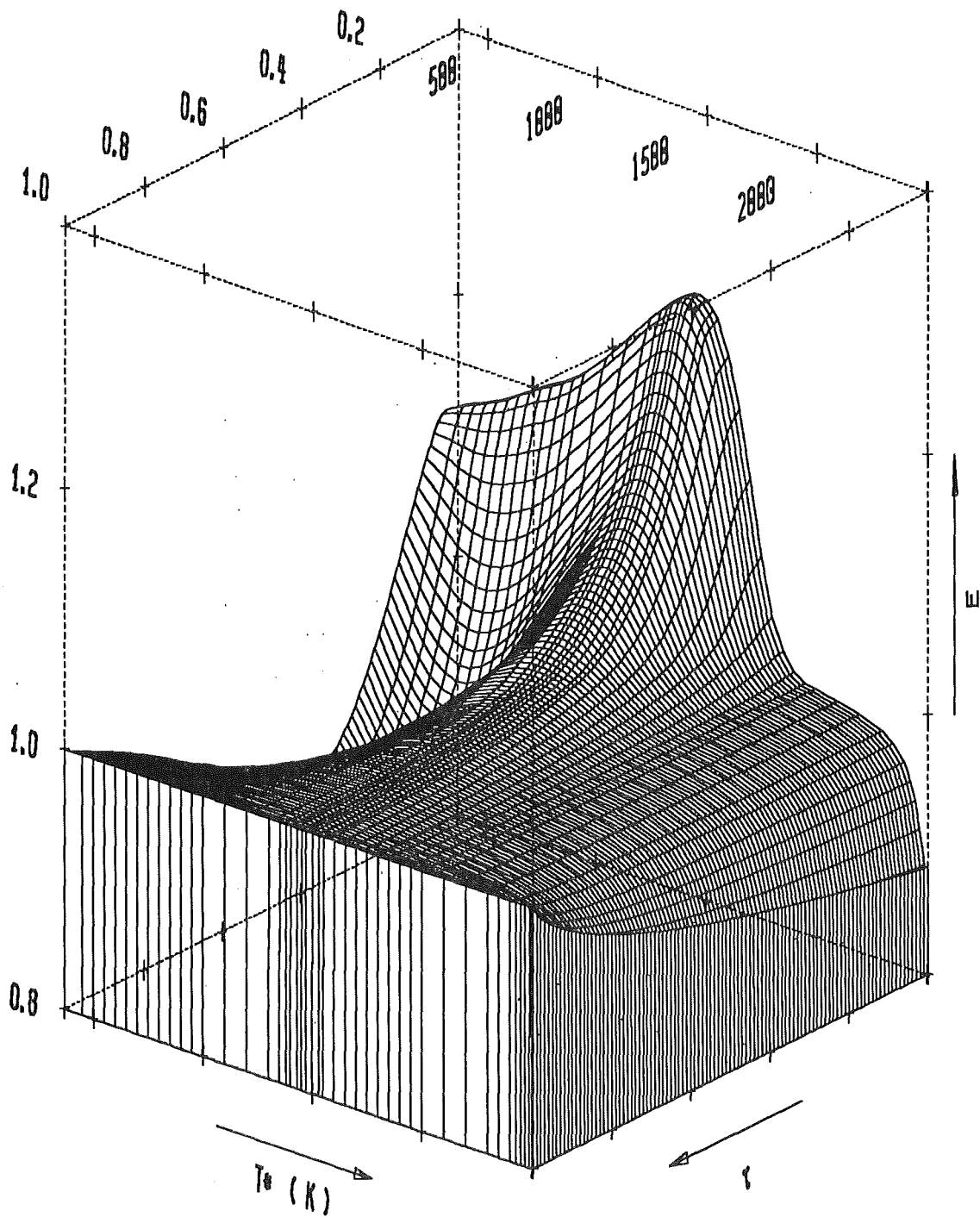


Figure 35. SODIUM. U-DEPARTURE FROM THE U.SAT IN THE VAPOR. COLD SIDE .



KFK
SYM

Figure 36. SODIUM. U-DEPARTURE FROM THE U.SAT IN THE VAPOR. HOT SIDE .

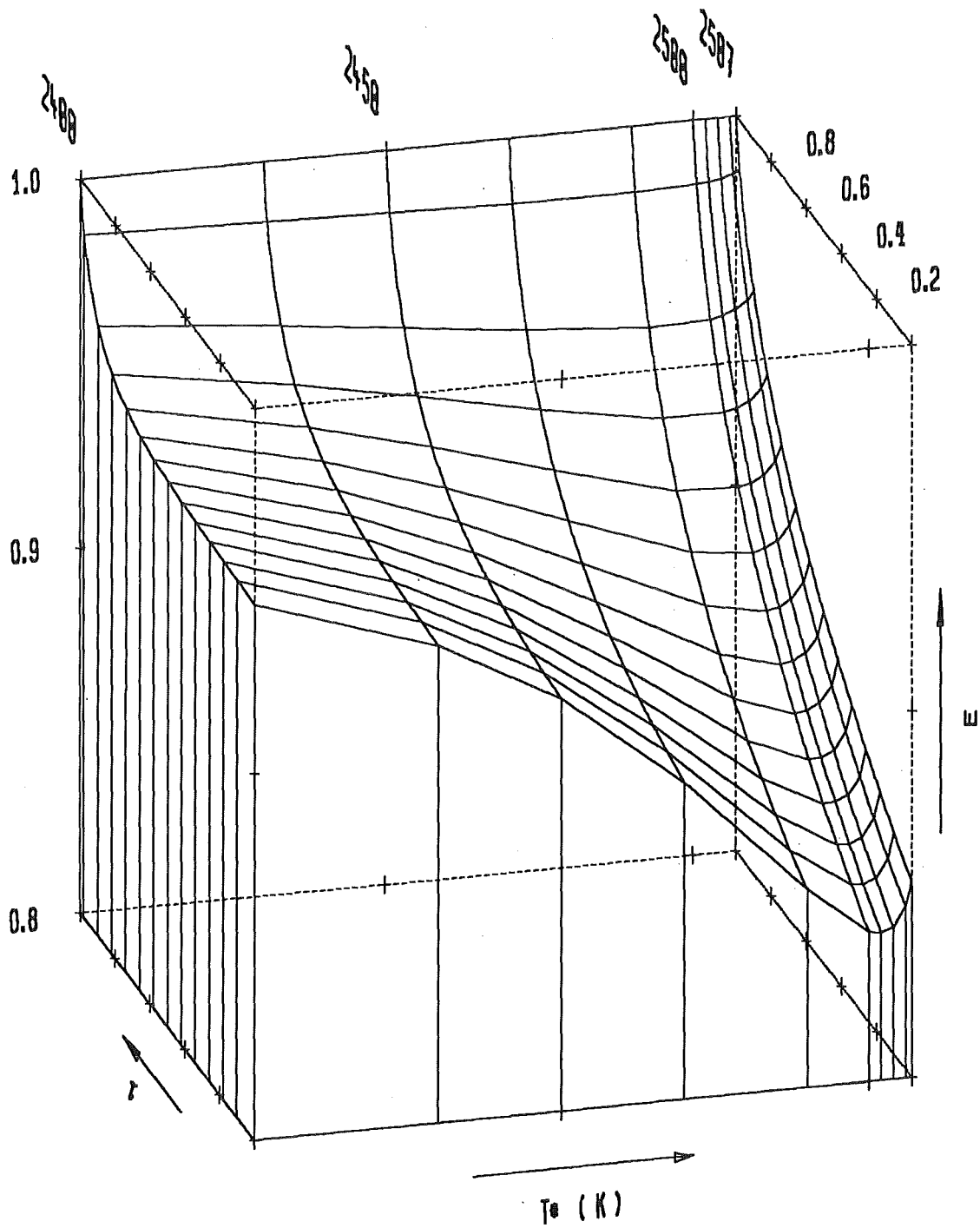


Figure 37. SODIUM. U-DEPARTURE FROM THE U.SAT IN THE VAPOR. NEARCRITICAL FURROW .



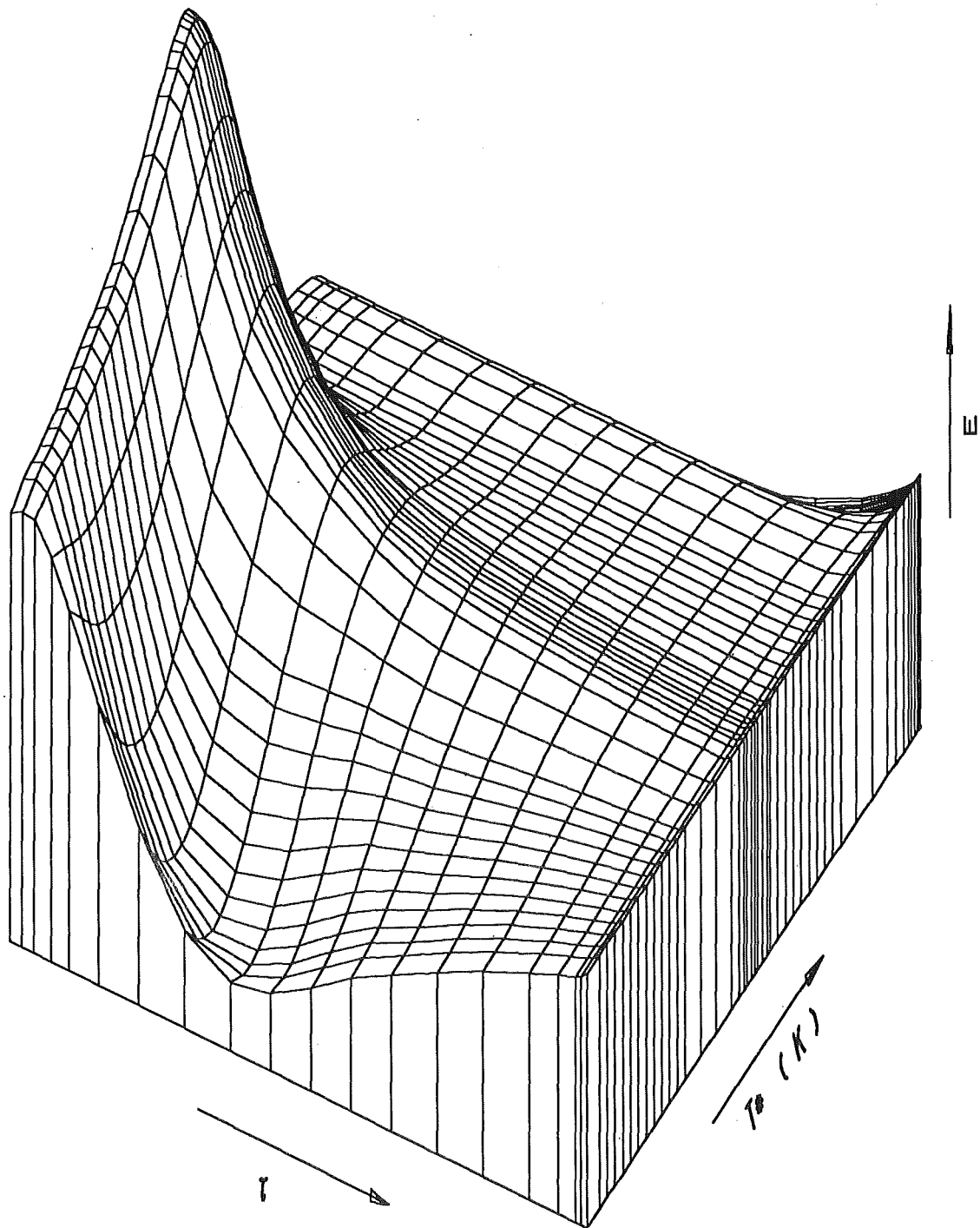


Figure 38. SODIUM. BASIC SURFACE OF E FOR CALCULATING THE INTERNAL ENERGY

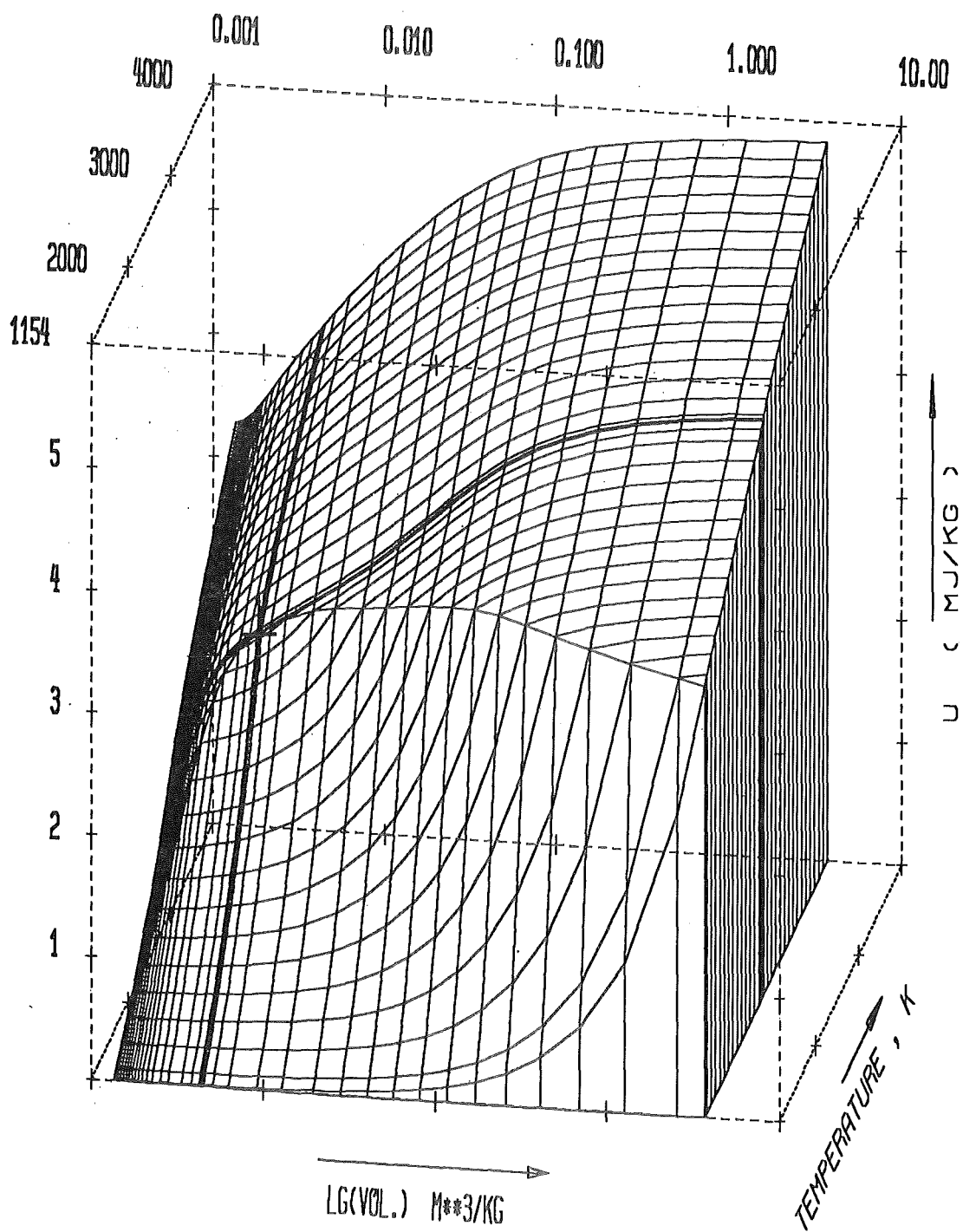


Figure 39. SODIUM. THE (T,V)-SURFACE OF THE INTERNAL ENERGY (COLD SIDE) .

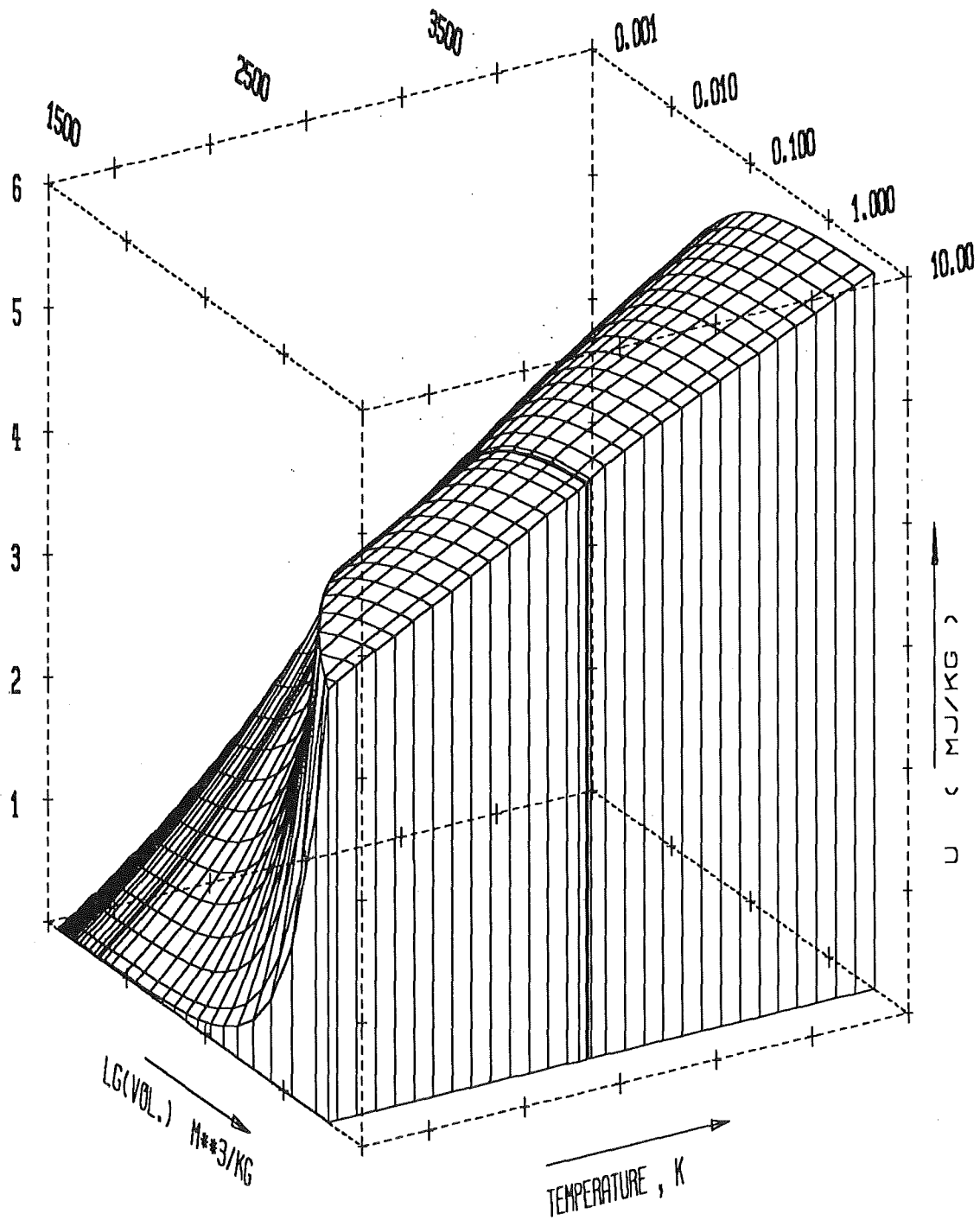


Figure 40. SODIUM. THE (T,V)-SURFACE OF THE INTERNAL ENERGY (VAPOR SIDE) .

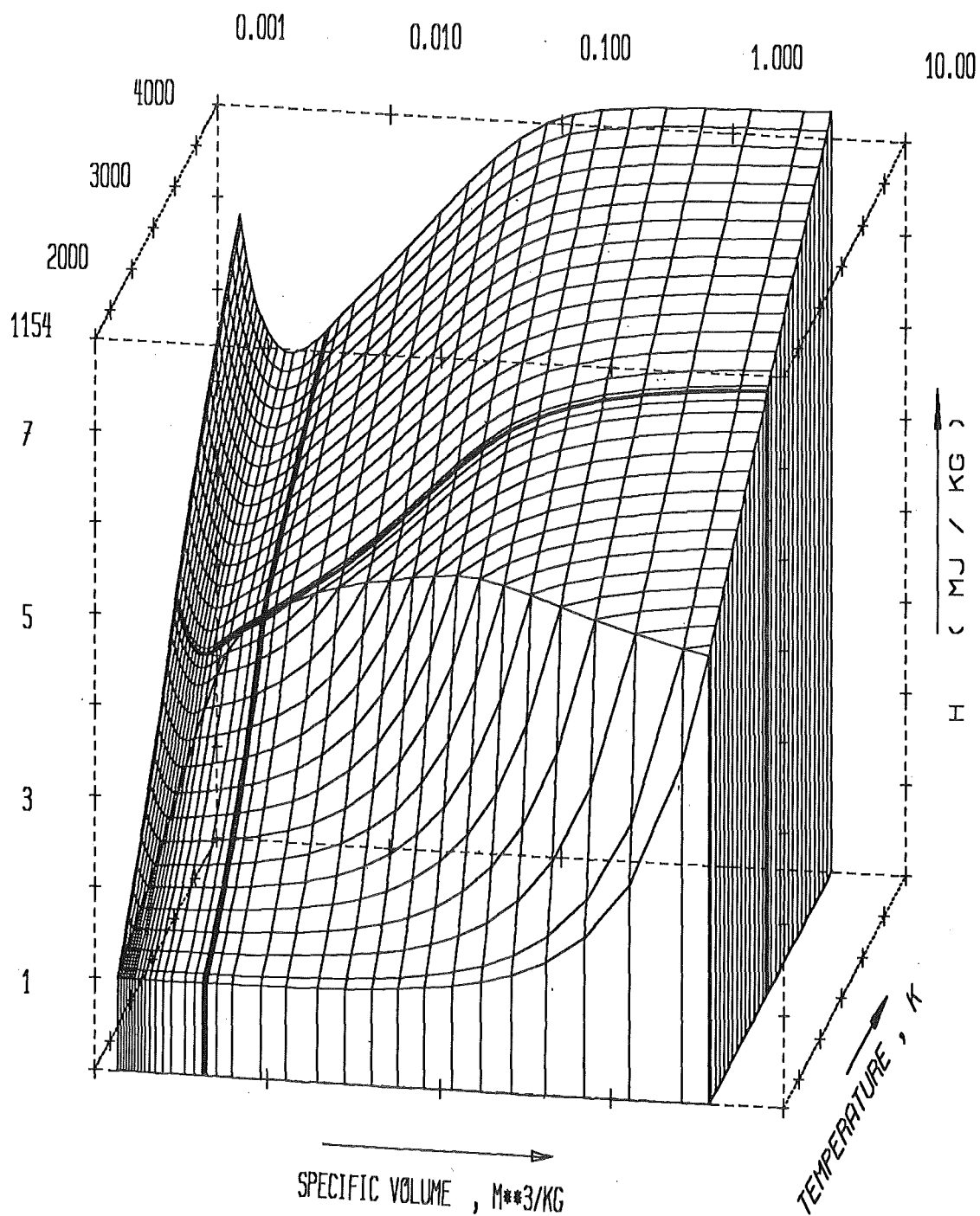


Figure 41. SODIUM. THE (V , T) - SURFACE OF THE ENTHALPY .

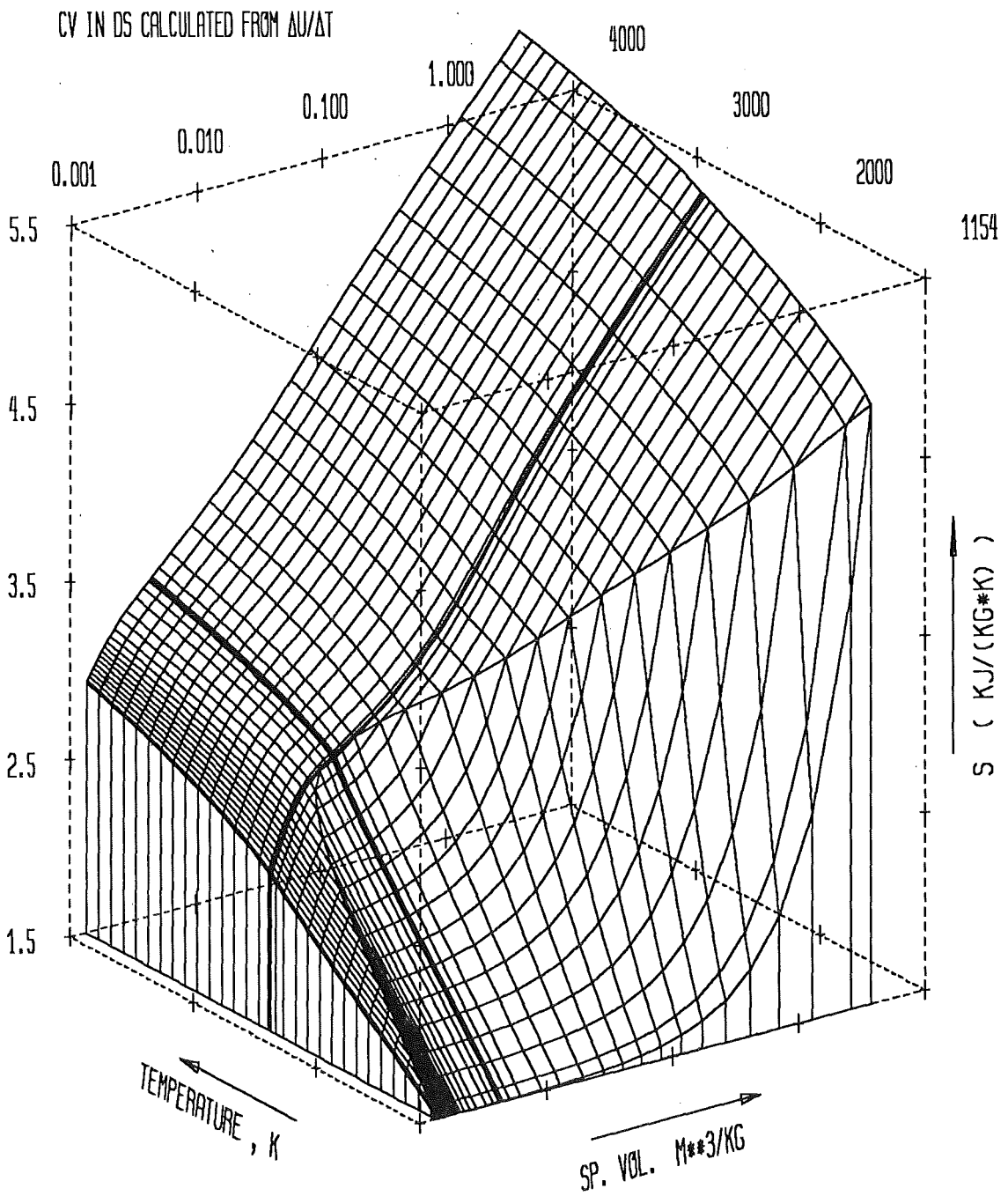


Figure 42. SODIUM. THE (V , T) - SURFACE OF THE ENTROPY .

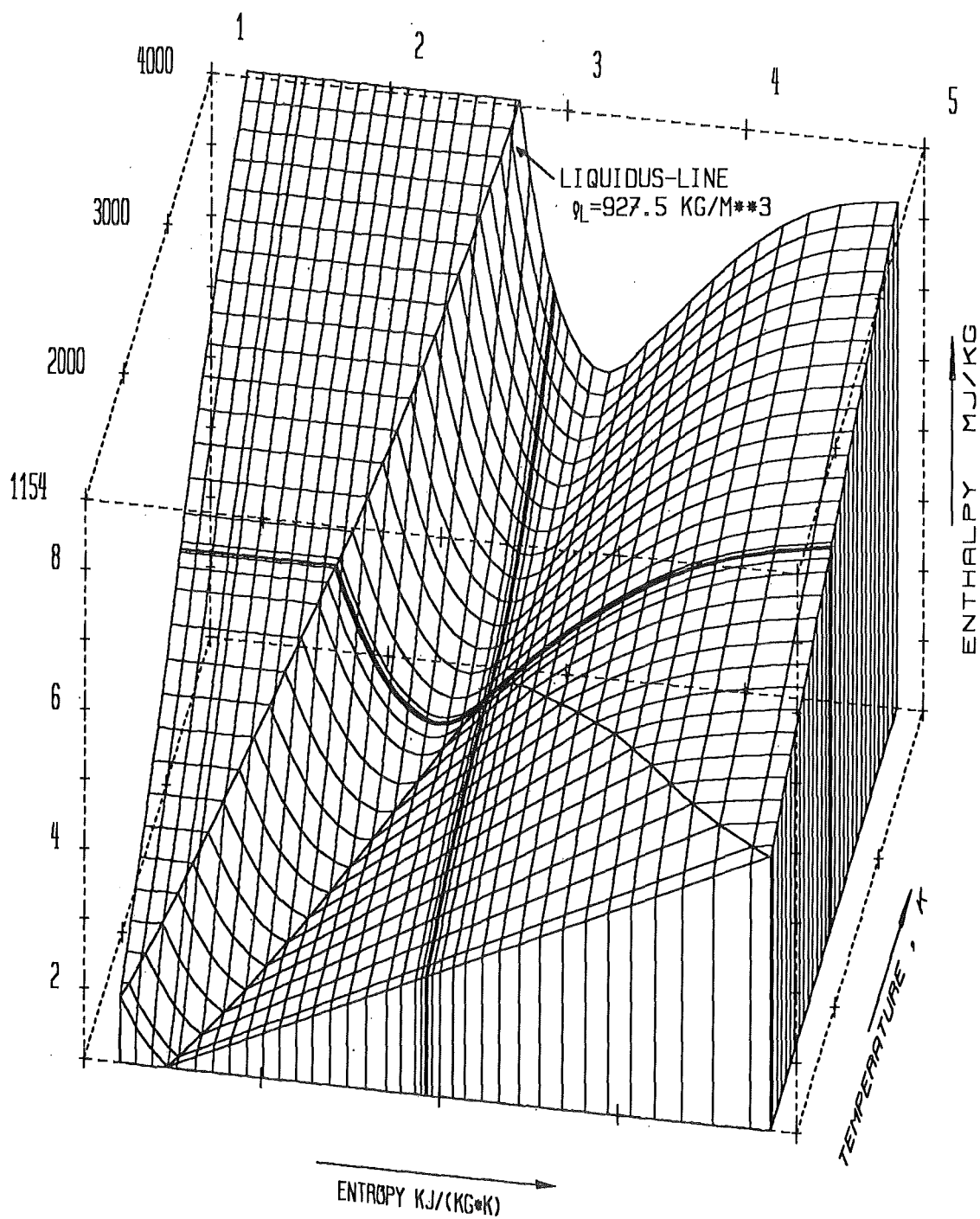


Figure 43. SODIUM. THE MOLLIER-SURFACE .

Appendix A. Integrating a property described by a set of polynomials.

The property is described as

$$\begin{aligned}
 F(T) &= f_k(T) \quad \text{if} \quad T_{k-1} \leq T \leq T_k, \quad k = 1, 2, \dots \\
 \text{with} \quad f_k(T) &\equiv \sum_{j=1}^5 a_{kj} \cdot x^{\delta+j-1}, \quad [A.1] \\
 \text{and} \quad x &\equiv 1 - T/T_c.
 \end{aligned}$$

The definite integral of this property can be transformed to :

$$U(T) - U(T_0) \equiv \int_{T_0}^T dt F(t) = \int_{T_0}^{T_1} dt f_1(t) + \dots + \int_{T_{k-1}}^T dt f_k(t) = I_1(T_1) + \dots + I_k(T)$$

with

$$I_k(T) \equiv \int_{T_{k-1}}^T dt f_k(t) \quad [A.2]$$

One can develop $I_k(T)$:

$$\begin{aligned}
 I_k(T) &= -T_c \cdot \int_{x_{k-1}}^x \sum_{j=1}^5 a_{kj} \cdot y^{\delta+j-1} dy = \\
 &= -T_c \cdot \sum_{j=1}^5 \int_{x_{k-1}}^x dy a_{kj} \cdot y^{\delta+j-1} = -T_c \cdot \sum_{j=1}^5 \frac{a_{kj}}{\delta+j} y^{\delta+j} \Big|_{x_{k-1}}^x
 \end{aligned}$$

With the coefficients

$$b_{kl} \equiv -T_c \cdot \frac{a_{k,l-1}}{\delta+l-1}, \quad l = 2, \dots, 6 \quad [A.3]$$

the integral turns to :

$$\begin{aligned}
 U(T) - U(T_0) &= \sum_{j=1}^{k-1} I_j(T_j) + I_k(T) = \\
 &= \sum_{j=1}^{k-1} \left(\sum_{l=2}^6 b_{jl} \cdot x_j^{\delta+l-1} - \sum_{l=2}^6 b_{jl} \cdot x_{j-1}^{\delta+l-1} \right) + \sum_{l=2}^6 b_{kl} \cdot x^{\delta+l-1} - \sum_{l=2}^6 b_{kl} \cdot x_{k-1}^{\delta+l-1}
 \end{aligned}$$

resp.

$$\begin{aligned}
 U(T) - U(T_0) &= \sum_{l=2}^6 -b_{1,l} \cdot x_0^{\delta+l-1} + \\
 &+ \sum_{j=1}^{k-1} \sum_{l=2}^6 (b_{j,l} - b_{j+1,l}) \cdot x_j^{\delta+l-1} + \sum_{l=2}^6 b_{1,l} \cdot x^{\delta+l-1}
 \end{aligned}$$

Using the following coefficients

$$\begin{aligned}
 b_{1,1} &\equiv - \sum_{l=2}^6 b_{1,l} \cdot x_0^{\delta+l-1} \\
 b_{2,1} &\equiv b_{1,1} + \sum_{l=2}^6 (b_{1,l} - b_{2,l}) \cdot x_1^{\delta+l-1} \\
 &\dots \\
 &\dots \\
 &\dots \\
 b_{k,1} &\equiv b_{k-1,1} + \sum_{l=2}^6 (b_{k-1,l} - b_{k,l}) \cdot x_{k-1}^{\delta+l-1} \\
 &\dots \\
 &\dots \\
 &\dots
 \end{aligned} \tag{A.4}$$

additively , one can transform the U -Integral into a set of polynomials :

$$U(T) = U(T_0) + b_{k,1} + \sum_{j=2}^6 b_{k,j} \cdot x^{\delta+j-1} \quad \text{if} \quad T_{k-1} \leq T \leq T_k \tag{A.5}$$

Appendix B. Calculating the difference of the expression $\varphi (w)$.

$$\varphi(w) = \frac{1}{B \cdot S} \left[\zeta \cdot S' - \sigma + a \frac{S' - \sigma \cdot w}{S} \right] \quad [B.1]$$

The abbreviations in this equation are :

$$a \equiv y \cdot A \quad , \quad \zeta \equiv 6 \frac{a \cdot B}{\sqrt{\sigma}} \quad , \quad \sigma \equiv 4B - a^2 \quad [B.2]$$

$$S \equiv B \cdot w^2 - a \cdot w + 1 \quad , \quad S' \equiv 2B \cdot w - a$$

In the following the difference-operator Δ means :

$$\Delta f \equiv f(w) - f(1)$$

The function Eq. [B.1] can be transformed to $\varphi(w) \equiv f/g$ with

$$f(w) \equiv \zeta \cdot S' - \sigma + \frac{a \cdot h}{S} \quad , \quad g(w) \equiv B \cdot S(w) \quad , \quad h(w) \equiv S' - \sigma \cdot w$$

Using the f/g - form to calculate the φ -difference one get :

$$\Delta \varphi = \frac{1}{g(1)} \left[\Delta f - \left(\frac{f}{g} \right) \Delta g \right] \quad [B.3]$$

The differences , needed in this equation are :

$$\Delta f = \zeta \cdot \Delta S' + \Delta \frac{a \cdot h}{S} = 2B \cdot \zeta \cdot \Delta w + \frac{a}{S} \left[\Delta h - \frac{h(1)}{S(1)} \Delta S \right]$$

and

$$\Delta g = B \cdot \Delta S = B \cdot \theta \cdot \Delta w$$

with

$$\theta(w) \equiv B \cdot \Delta w + 2B - a \quad [B.4]$$

$\Delta h = (2B - \sigma) \cdot \Delta w$ transforms Δf to

$$\Delta f = \left\{ 2B \cdot \zeta + \frac{a}{S} \left[2B - \sigma - \frac{h(1)}{S(1)} \theta \right] \right\} \cdot \Delta w$$

and one get for φ (s. Eq. [B.3]) :

$$g(1) \cdot \Delta \varphi = \left\{ 2B \cdot \zeta + \frac{a}{S} \left(2B - \sigma - \frac{h(1)}{S(1)} \theta \right) - \frac{f}{g} B \cdot \theta \right\} \Delta w$$

or

$$B \cdot S(1) \cdot S \frac{\Delta \varphi}{\Delta w} = \zeta \cdot [2B \cdot S - \theta \cdot S'] + a(2B - \sigma) + \theta \left[\sigma - a \left(\frac{h}{S} + \frac{h(1)}{S(1)} \right) \right] \quad [B.5]$$

Introducing

$$\omega(w) \equiv 2S(1) + (2B - a) \cdot \Delta w \quad [B.6]$$

gives

$$2B \cdot S - \theta \cdot S' \equiv \sigma - B \cdot \omega \quad [B.7]$$

Using the abbreviation

$$t \equiv \frac{a}{S} \quad [B.8]$$

and the relation

$$\frac{h}{h(1)} \equiv \frac{S' - \sigma \cdot w}{S'(1) - \sigma} \equiv 1 + \Delta w \frac{2B - \sigma}{S'(1) - \sigma}$$

the expression , containing the h/S- terms can be transformed to :

$$\begin{aligned} a \left(\frac{h}{S} + \frac{h(1)}{S(1)} \right) &= (S'(1) - \sigma) \cdot \left[\frac{S' - \sigma \cdot w}{S'(1) - \sigma} \cdot \frac{a}{S} + \frac{a}{S(1)} \right] \\ &= \Delta w \cdot (2B - \sigma) \cdot t + (S'(1) - \sigma) \cdot (t(1) + t) \end{aligned} \quad [B.9]$$

The Eq. [B.7] and Eq. [B.9] turns with some transformations the Eq. [B.5] to :

$$B \cdot S(1) \cdot S \frac{\Delta \varphi}{\Delta w} = \zeta(\sigma - B \cdot \omega) + (2B - \sigma) \cdot S(1) \cdot t + \theta [\sigma - (S'(1) - \sigma) \cdot (t(1) + t)]$$

or , with the abbreviation

$$b \equiv 2B - \sigma \quad [B.10]$$

$$\Delta \varphi = \Delta w \frac{\zeta(\sigma - B \cdot \omega) + b \cdot S(1) \cdot t + \theta [\sigma - (b - a)(t(1) + t)]}{B \cdot S(1) \cdot S} \quad [B.11]$$

Appendix C. Tables

\tilde{Z}			
$T \leq$	A(k,1)	A(k,2)	A(k,3)
	A(k,4)	A(k,5)	
334.5	0.0	0.0	0.0
	0.0	0.0	
900.	-4.50205874E-2	3.1101E-4	-6.6703E-7
	4.62373333E-10	-1.3446975E-13	
1400.	-9.01803003E-2	4.48651E-4	-7.9053E-7
	4.74670E-10	-1.15641E-13	
1700.	-1.90305993	5.81003E-3	-6.7302E-6
	3.3964E-09	-6.54095E-13	

Table 1. Polynomial coefficients of $\tilde{Z}(T)$.

the high temperature part of the saturation line				
	E (1)	E (2)	E (3)	E (4)
liquid	.569387	.578911	0.0	1.83755
vapor	1.38357611	-.332589774	0.0	-.183511941

Table 2. Coefficients in the description of the extrapolated part of the saturation line.

T^{\times} (liquid)			
$\rho \leq$	A(k,1)	A(k,2)	A(k,3)
	A(k,4)	A(k,5)	A(k,6)
301.455	2563.13595	662.140362	-17005.9531
	101550.625	-234815.183	176699.101
411.945	2859.68111	-4223.4978	15366.6252
	-6280.30208	-54281.0898	55186.0031
511.44	3455.75201	-11557.549	51554.7343
	-95791.6016	56706.404	0.0
562.477	1739.94994	1723.39144	12986.6914
	-45988.5547	32577.7246	0.0
608.899	-3762.78167	40771.5911	-90956.7305
	77025.0833	-22033.7812	0.0
858.158	5349.14987	-9979.9956	12080.4962
	-12960.9666	7607.69965	-2043.63682
1000.	3998.7677	-2600.66431	-4007.10352
	4523.36198	-1861.7168	0.0

Table 3. Polynomial coefficients of $T^{\times}(\rho)$ in the saturated liquid.

T^x (vapor)			
$s \leq$	A(k,1)	A(k,2)	A(k,3)
	A(k,4)	A(k,5)	A(k,6)
1.26902	5323.08321	-11256.8151	17677.4948
	-13640.0485	5199.46023	-795.17478
1.5312	3428.29668	-3706.99393	5620.65881
	-3994.0717	1333.50354	-174.257748
2.0109	2128.00063	527.589404	89.5214844
	-372.104848	144.526726	-17.7426773
3.10486	1519.53837	2081.02670	-1502.95190
	447.035905	-66.8347501	4.13665509
4.0199	5143.27072	-3479.1603	1906.49344
	-597.342204	92.9818535	-5.63748455
5.34602	3240.06391	-73.0786658	-326.292465
	97.830629	-11.6650338	5.16808534E-1
8.69186	4493.3435	-1441.45467	262.33691
	-27.3442416	1.52865285	-3.56319302E-2
14.3515	3127.98872	-644.43616	73.9352739
	-4.81667613	1.67401438E-1	-2.41660272E-3
25.0	2023.48317	-251.198836	17.1329955
	-6.59952561E-1	1.35161901E-2	-1.14688956E-4

Table 4. Polynomial coefficients of T^x (s) in the saturated vapor .

G (s)				
$s \leq$	A(k,1)	A(k,2)	A(k,3)	A(k,4)
0.071	0.	-294.391	-13180.0	0.
0.150	196.585	-6344.31	38950.6	-83405.9
0.320	-301.692	1272.06	2943.64	-34227.1
1.0	0.	0.	0.	0.

Table 5. Coefficients describing the function G (s).

P_{ρ}^{χ} (liquid)			
$T \leq$	A(k,1)	A(k,2)	A(k,3)
	A(k,4)	A(k,5)	A(k,6)
660.	4.47035844	-9.73850618E- 4	-1.05458824E- 6
	3.82598828E-10	3.79215526E-13	-1.93534495E-16
955.	4.44696294	-8.01949811E- 4	-1.56413699E- 6
	1.14467746E- 9	-1.96055737E-13	-1.81653257E-17
1132.	4.46889657	-8.77164532E- 4	-1.48756806E- 6
	1.14757097E- 9	-2.40401505E-13	0.0
1555.	3.33048867	2.29639251E- 3	-5.10350299E- 6
	3.39637081E- 9	-1.03288042E-12	1.17672369E-16
1700.	4.65205763	-1.77921518E- 3	-7.71588020E- 8
	2.97704824E-10	-7.79880717E-14	0.0

Table 6. Polynomial coefficients of P_{ρ}^{χ} (T) in the saturated liquid.

P_{ρ}^{χ} (vapor)				
$T \leq$	A(k,1)	A(k,2)	A(k,3)	A(k,4)
1100.	0.904096	2.87341E- 4	-7.28024E- 8	2.02252E-11
1820.	1.25073	-4.71509E- 4	4.50434E- 7	-8.87277E-11
2050	3.43495	-7.69492E- 3	6.39912E- 6	-1.53883E- 9
2508.	-25.6077	3.58686E- 2	-1.53757E- 5	2.08806E- 9

Table 7. Polynomial coefficients of P_{ρ}^{χ} (T) in the saturated vapor.

the high temperature part of P_{ρ}^{χ}					
	E (1)	E (2)	E (3)	E (4)	E (5)
liquid	1.0	-2.49586	4.09230	-9.91952	13.3423
vapor	1.0	-1.38740	117.942	-905.726	3852.37

Table 8. Coefficients in the description of the extrapolated part of P_{ρ}^{χ} .

G			
w ≤	G(k,1)	G(k,2)	G(k,3)
	G(k,4)	G(k,5)	
0.05	6.0334199	-266.45944	9335.6518
	-137699.05	774431.0	
0.2	1.9237665	57.225193	-538.34256
	2037.096	-2865.1897	
0.525	5.0795444	-10.290233	15.905257
	-14.822874	5.6403132	
0.85	4.4720136	-6.2228329	5.7459901
	-3.623567	1.0558672	
1.1	3.9783644	-3.7809708	1.2089292
	1.2780685E-1	-1.0833227E-1	
1.8	4.2712852	-4.7468967	2.3970671
	-5.1780325E-1	2.2297722E-2	
2.5	4.504422	-5.7703931	3.706177
	-1.1864015	1.4298775E-1	
3.2	-2.4280828	5.6205268	-3.4498913
	.84693365	-7.6921413E-2	
10.	13.563572	-14.603827	6.1541777
	-1.1831529	8.4273184E-2	

Table 9. The polynomial coefficients " G " in the formula for the supercritical pressures

H			
w ≤	H(k,1)	H(k,2)	H(k,3)
	H(k,4)	H(k,5)	
0.05	20.805816	-927.02882	33402.567
	-601791.79	3921787.2	
0.2	17.33203	-283.31725	1932.0445
	-6386.2301	8330.1056	
0.525	5.0562564	-46.151186	144.48637
	-198.40566	102.65837	
0.85	-2.0526223E-2	-4.0296012	12.06889
	-12.109581	3.9722104	
1.1	7.9808701	-43.630698	83.885431
	-68.887919	20.525174	
1.8	5.3896328	-14.130624	12.810811
	-4.8987166	6.9798362E-1	
2.5	3.8142766	-9.6062484	8.0301248
	-2.6875239	3.1932634E-1	
3.2	11.31246	-15.374629	8.0516051
	-1.8560865	1.6053695E-1	
10.	-10.102961	12.780187	-5.8143280
	1.1767982	-8.8127741E-2	

Table 10. The polynomial coefficients " H " in the formula for the supercritical pressures

$Q_{T,L}$				
$T \leq$	$D_L(k,1)$	$D_L(k,2)$	$D_L(k,3)$	$D_L(k,4)$
1280.	21.9594	-1.289934E-2	2.34558E-6	0.0
1490.	15.5088839	-1.20754936E-3	-4.11105759E-6	9.83993945E-10
1700.	16.3302297	-2.85554124E-3	-3.00788635E-6	7.37621266E-10
2508.	8.97741456	11.8539355	0.0	-9.25661142

Table 11. Coefficients in the description of the thermal conductivity of the saturated liquid.

$Q_{T,V}$				
$T \leq$	$D_V(k,1)$	$D_V(k,2)$	$D_V(k,3)$	$D_V(k,4)$
1280.	4.09852E-3	5.3905E-6	0.0	0.0
1440.	-2.50479043E-1	6.01102017E-4	-4.64673101E-7	1.20824384E-10
1590.	-3.17471512E-1	7.39894286E-4	-5.60566367E-7	1.42919535E-10
1700.	-4.60201694E-1	1.00943007E-3	-7.30280263E-7	1.78549779E-10
2508.	1.28153149	-8.66046586E-3	0.0	-4.98362427E-1

Table 12. Coefficients in the description of the thermal conductivity of the saturated vapor.

$Y_{T,L}$				
$T \leq$	$D_L(k,1)$	$D_L(k,2)$	$D_L(k,3)$	$D_L(k,4)$
2250.	1.50564664	234.655	-0.42961	
2508.	-3.13975	7.55527	0.0	46.4387

Table 13. Coefficients in the description of the viscosity of the saturated liquid.

$Y_{T,V}$				
$T \leq$	$D_V(k,1)$	$D_V(k,2)$	$D_V(k,3)$	$D_V(k,4)$
1950.	8.29964508E-2	2.02065996E-4	0.0	0.0
2508.	3.06453	-6.48698	0.0	14.9746

Table 14. Coefficients in the description of the viscosity of the saturated vapor.

$\partial U/\partial T$ on the baseline			
$x \leq$	A(k,1)	A(k,2)	A(k,3)
	A(k,4)	A(k,5)	
.019138756	.260570845	41.0873550	-2322.93912
	107233.247	-2030597.15	
.0929027113	.351884469	20.2832797	-198.890712
	1432.00711	-4587.34377	
.222488038	.518936966	12.3910479	-49.7571006
	118.823783	-125.016771	
.481658692	.764512764	8.04793140	-19.8762816
	24.2560637	-9.42119426	
.852073365	1.19899869	4.51599107	-8.93405372
	8.93750755	-1.24667011	

Table 15. polynomials of the description of the function $\partial U/\partial T$.

$\partial U/\partial \rho$ in the liquid				
$\rho \leq$	A(k,1)	A(k,2)	A(k,3)	A(k,4)
286.917407	-91015.9670	884.319420	-2.97610288	3.30136377E-3
389.617480	-37671.6942	314.797162	-9.44365293E-1	8.79905062E-4
498.812327	6372.82607	-20.0024849	-9.32652100E-2	1.56287854E-4
599.429782	87965.4140	-511.160202	8.94264577E-1	-5.06896371E-4
724.234849	-13662.7562	10.3334451	4.54767896E-3	-2.13217412E-6
927.710140	-35849.9885	98.0565010	-1.11172873E-1	4.88117183E-5

Table 16. polynomials of the description of the function $\partial U/\partial \rho$ in the liquid.

$\Delta U(T^x)$				
$T^x \leq$	A(k,1)	A(k,2)	A(k,3)	A(k,4)
760.	5620.92287	-5.92925212	6.21749218E-3	-2.54397356E-6
1310.	5768.9965	-6.13159176	5.97711442E-3	-2.21305212E-6
1700.	-6470.66401	21.5484023	-1.49447267E-2	3.07277200E-6
2390.	8747.11044	-5.93019371	1.60164707E-3	-2.49691696E-7
2502.	316806.23	-386.257521	1.58088581E-1	-2.17078315E-5
2508	-1886953.82	1510.57451	-3.02312761E-1	0.0

Table 17. Polynomials of the description of the internal energy departure between the lines $\{\rho = \rho_v, T\}$ and $\{\rho, T = T_c\}$.

Table 18. Low-temperature table of the first six contour lines $q_i(T)$.

$q_i(T)$, $i = 1, \dots, 6$						
T \ C	0.005	0.020	0.060	0.120	0.190	0.290
370.00	0.29484305	0.34881952	0.43427449	0.50829181	0.56469399	0.62137681
400.00	0.28456215	0.35258907	0.44929714	0.52763536	0.58531734	0.64200889
450.00	0.27761020	0.36727927	0.47731736	0.55957421	0.61774031	0.67338044
500.00	0.28009466	0.38731951	0.50558483	0.58935098	0.64693252	0.70091997
550.00	0.28860221	0.40911272	0.53238526	0.61640556	0.67291335	0.72503160
600.00	0.30027866	0.43054948	0.55686491	0.64046980	0.69568963	0.74590662
650.00	0.31231715	0.44950689	0.57760555	0.66047152	0.71437443	0.76277553
700.00	0.32294594	0.46497824	0.59413343	0.67620772	0.72890496	0.77592325
800.00	0.33799536	0.48601802	0.61652187	0.69797557	0.75058392	0.79737491
900.00	0.34500666	0.49645331	0.62910126	0.71408389	0.76860745	0.81588793
950.00	0.34591035	0.49854086	0.63354057	0.72117582	0.77686049	0.82440571
1000.0	0.34531694	0.49888013	0.63716690	0.72779087	0.78471892	0.83252044
1050.0	0.34337862	0.49767282	0.64006788	0.73397289	0.79222434	0.84027138
1100.0	0.34026338	0.49523373	0.64230908	0.73978550	0.79944407	0.84772039
1200.0	0.33265135	0.48940797	0.64736043	0.75279889	0.81523208	0.86358320
1300.0	0.32488816	0.48443786	0.65552571	0.76987870	0.83432848	0.88075097
1400.0	0.31655368	0.47961927	0.66620268	0.79004536	0.85364838	0.89639119
1500.0	0.30541357	0.47111845	0.67467727	0.80719845	0.86908700	0.90876919
1600.0	0.28764460	0.45137992	0.66943506	0.81320254	0.87703516	0.91655166
1650.0	0.27448454	0.43358272	0.65434853	0.80761337	0.87597386	0.91768783

Table 19. High-temperature table of the first six contour lines $q_i(T)$.

$q_i(T)$, $i = 1, \dots, 6$						
T \ C	0.005	0.020	0.060	0.120	0.190	0.290
1700.0	0.25704665	0.40779364	0.62389863	0.79041085	0.86801820	0.91514601
1750.0	0.23379497	0.37170700	0.57034862	0.75027124	0.84505813	0.90423894
1775.0	0.21941181	0.34916560	0.53302922	0.71479665	0.82196012	0.89167211
1800.0	0.20292767	0.32347381	0.48881693	0.66392894	0.78475333	0.86924265
1825.0	0.18441913	0.29489759	0.43942227	0.59618514	0.72667196	0.82974984
1850.0	0.16479287	0.26461958	0.38885565	0.51985380	0.64535661	0.76576073
1875.0	0.14713482	0.23692990	0.34547148	0.45414600	0.56115241	0.68592459
1900.0	0.13447299	0.21677361	0.31553704	0.41014320	0.50203780	0.61848849
1950.0	0.11839806	0.19080644	0.27836744	0.35776889	0.43277408	0.52971055
2000.0	0.10798256	0.17375203	0.25433288	0.32535784	0.39098545	0.47510758
2050.0	1.0028701E-1	0.16104246	0.23637290	0.30189056	0.36130912	0.43683053
2100.0	9.4184412E-2	0.15090655	0.22197005	0.28344800	0.33832470	0.40755273
2150.0	8.9130035E-2	0.14248019	0.20993959	0.26822618	0.31956701	0.38391239
2200.0	8.4819541E-2	0.13527669	0.19961274	0.25525152	0.30372092	0.36411758
2300.0	7.7744263E-2	0.12343209	0.18254564	0.23391384	0.27792078	0.33221560
2400.0	7.2079356E-2	0.11394512	0.16879695	0.21675311	0.25737908	0.30708086
2450.0	6.9626585E-2	0.10984071	0.16282713	0.20929502	0.24850224	0.29628464
2500.0	6.7375413E-2	0.10607701	0.15734151	0.20243284	0.24035994	0.28641361
2505.0	6.7160240E-2	0.10571748	0.15681692	0.20177607	0.23958185	0.28547184
2507.9	6.7036221E-2	0.10551027	0.15651455	0.20139744	0.23913342	0.28492919

Table 20. Low-temperature table of the last six contour lines $q_i(T)$.

$q_i(T)$, $i = 7, \dots, 12$						
T \ C	0.420	0.600	0.760	0.900	0.980	0.995
370.00	0.67536333	0.73361350	0.77995123	0.82543716	0.86934963	0.89114496
400.00	0.69503719	0.75135470	0.79556859	0.83851952	0.87960516	0.89987166
450.00	0.72425372	0.77717800	0.81799841	0.85709139	0.89401795	0.91207941
500.00	0.74941362	0.79903030	0.83675128	0.87245537	0.90583147	0.92204240
550.00	0.77115704	0.81768680	0.85262220	0.88535516	0.91567734	0.93031068
600.00	0.78976466	0.83342950	0.86582030	0.89595620	0.92372654	0.93708016
650.00	0.80454292	0.84588730	0.87645997	0.90470502	0.93048936	0.94279759
700.00	0.81654112	0.85648950	0.88568475	0.91233574	0.93638454	0.94777309
800.00	0.83695396	0.87472310	0.90148823	0.92529221	0.94628786	0.95608488
900.00	0.85459674	0.89024910	0.91472174	0.93595598	0.95430952	0.96276738
950.00	0.86263878	0.89721360	0.92057713	0.94061467	0.95777550	0.96564047
1000.0	0.87024318	0.90372650	0.92600415	0.94489849	0.96094167	0.96825735
1050.0	0.87744758	0.90982500	0.93103790	0.94883476	0.96383462	0.97064401
1100.0	0.88430850	0.91557500	0.93577592	0.95254155	0.96654357	0.97286849
1200.0	0.89849345	0.92700680	0.94484631	0.95940362	0.97144458	0.97686066
1300.0	0.91239898	0.93743390	0.95285135	0.96534409	0.97564070	0.98026623
1400.0	0.92449475	0.94632240	0.95963987	0.97037914	0.97920391	0.98316215
1500.0	0.93418426	0.95357980	0.96526402	0.97460268	0.98222305	0.98562749
1600.0	0.94104078	0.95914780	0.96976589	0.97808453	0.98476366	0.98771380
1650.0	0.94293856	0.96111500	0.97151836	0.97951959	0.98585093	0.98863149

Table 21. High-temperature table of the last six contour lines $q_i(T)$.

$q_i(T)$, $i = 7, \dots, 12$						
T \ C	0.420	0.600	0.760	0.900	0.980	0.995
1700.0	0.94291167	0.96214720	0.97274896	0.98066551	0.98677521	0.98943156
1750.0	0.93853961	0.96117890	0.97293683	0.98127772	0.98743947	0.99003291
1775.0	0.93249265	0.95892560	0.97213938	0.98115079	0.98757071	0.99020301
1800.0	0.92054225	0.95381530	0.96989777	0.98033732	0.98739156	0.99018010
1825.0	0.89683556	0.94221240	0.96414421	0.97779945	0.98642641	0.98965797
1850.0	0.85290396	0.91668850	0.94933875	0.96998168	0.98260086	0.98707759
1875.0	0.79080727	0.87462520	0.92067771	0.95128557	0.97023308	0.97659687
1900.0	0.73211837	0.83098150	0.88838815	0.92810684	0.95329916	0.96168565
1950.0	0.64258128	0.75784780	0.83085477	0.88440459	0.91995327	0.93207475
2000.0	0.57890169	0.69976840	0.78238243	0.84569937	0.88920361	0.90435732
2050.0	0.53132829	0.65208050	0.74063968	0.81107584	0.86083479	0.87845831
2100.0	0.49425069	0.61198810	0.70405609	0.77975577	0.83451156	0.85417220
2150.0	0.46430797	0.57769820	0.67158207	0.75118126	0.80996592	0.83132229
2200.0	0.43938704	0.54796820	0.64247276	0.72493807	0.78698626	0.80976269
2300.0	0.39953385	0.49879400	0.59225864	0.67823161	0.74506970	0.77004423
2400.0	0.36842341	0.45955420	0.55029222	0.63775877	0.70770484	0.73423543
2450.0	0.35513779	0.44268780	0.53174567	0.61944397	0.69048074	0.71760632
2500.0	0.34302982	0.42729480	0.51456379	0.60223878	0.67412176	0.70174303
2505.0	0.34187653	0.42582820	0.51291445	0.60057509	0.67253073	0.70019666
2507.9	0.34121214	0.42498330	0.51196332	0.59961470	0.67161157	0.69930300

Table 22. $E(T^x, \tau)$, 370 - 650

$E(T^x, \tau)$, $T^x = 370 - 650$							
$T^x \setminus \tau$	0.00	0.02	0.04	0.07	0.10	0.15	0.22
	0.30	0.38	0.45	0.52	0.59	0.66	0.74
	0.82	0.90	0.95	0.98	0.99	1.00	
370.	1.13939200	1.14453840	1.14337520	1.11976420	1.08523460	1.02199780	0.94077050
	0.87739300	0.85444148	0.86074543	0.88092019	0.90474383	0.92264243	0.94269215
	0.96208716	0.97877703	0.98807533	0.99348199	0.99555696	1.00000000	
450.	1.15199500	1.15534200	1.15032230	1.11296780	1.06445420	0.98844373	0.90711709
	0.85534127	0.84517983	0.85848585	0.88223876	0.90604695	0.92309414	0.94369788
	0.96273704	0.97904157	0.98811988	0.99341784	0.99548460	1.00000000	
500.	1.15574200	1.15972460	1.15335320	1.10753810	1.05146180	0.97013286	0.89082513
	0.84706679	0.84382015	0.86045229	0.88541168	0.90839914	0.92475414	0.94524295
	0.96374516	0.97955702	0.98836103	0.99351324	0.99554363	1.00000000	
550.	1.15921070	1.16401490	1.15627310	1.10333610	1.04121560	0.95623902	0.87991067
	0.84392370	0.84629641	0.86528552	0.89046581	0.91195417	0.92753108	0.94743745
	0.96511489	0.98020431	0.98861021	0.99355173	0.99553350	1.00000000	
600.	1.16352000	1.16937720	1.16093940	1.10193490	1.03494540	0.94749922	0.87489571
	0.84597130	0.85255110	0.87292922	0.89738099	0.91678111	0.93151771	0.95042250
	0.96701742	0.98117823	0.98907406	0.99373982	0.99564307	1.00000000	
650.	1.16903110	1.17547240	1.16595940	1.10244740	1.03201830	0.94337850	0.87532121
	0.85243426	0.86186575	0.88270130	0.90554977	0.92240677	0.93628534	0.95384349
	0.96914610	0.98221210	0.98950883	0.99385388	0.99567387	1.00000000	

Table 23. $E(T^x, \tau)$, 700 - 950

$E(T^x, \tau)$, $T^x = 700 - 950$							
$T^x \setminus \tau$	0.00	0.02	0.04	0.07	0.10	0.15	0.22
	0.30	0.38	0.45	0.52	0.59	0.66	0.74
	0.82	0.90	0.95	0.98	0.99	1.00	
700.	1.17552920	1.18260610	1.17264770	1.10542190	1.03249550	0.94394506	0.88089892
	0.86284159	0.87390014	0.89430108	0.91480475	0.92884931	0.94181262	0.95778621
	0.97164123	0.98348843	0.99011916	0.99410306	0.99581700	1.00000000	
750.	1.18283590	1.18977010	1.17877130	1.10938670	1.03534520	0.94865409	0.89073814
	0.87623431	0.88789187	0.90697392	0.92451206	0.93568926	0.94764739	0.96187938
	0.97418504	0.98473071	0.99065281	0.99425676	0.99586931	1.00000000	
800.	1.19012710	1.19702160	1.18570350	1.11476990	1.04097850	0.95741631	0.90424854
	0.89203652	0.90344167	0.92040261	0.93452139	0.94298730	0.95381027	0.96621963
	0.97692634	0.98613002	0.99132189	0.99453038	0.99602631	1.00000000	
850.	1.19699880	1.20310650	1.19086480	1.12074730	1.04898610	0.96956366	0.92044161
	0.90936819	0.91975172	0.93387693	0.94425635	0.95039957	0.95989962	0.97045745
	0.97955528	0.98741007	0.99187030	0.99468846	0.99608096	1.00000000	
900.	1.20278530	1.20910640	1.19771470	1.12934450	1.06021350	0.98490511	0.93879500
	0.92788557	0.93650599	0.94722087	0.95370480	0.95803591	0.96601887	0.97473985
	0.98225478	0.98878329	0.99252450	0.99495551	0.99623510	1.00000000	
950.	1.20987770	1.21625690	1.20519480	1.13983990	1.07396610	1.00239580	0.95826706
	0.94683921	0.95291364	0.95977444	0.96242685	0.96546152	0.97179448	0.97872904
	0.98471991	0.98997190	0.99302340	0.99509065	0.99627738	1.00000000	

Table 24. $E(T^x, T)$, 1000 - 1380

$E(T^x, T)$, $T^x = 1000 - 1380$							
$T^x \setminus T$	0.00	0.02	0.04	0.07	0.10	0.15	0.22
	0.30	0.38	0.45	0.52	0.59	0.66	0.74
	0.82	0.90	0.95	0.98	0.99	1.00	
1000.	1.21920620	1.22579370	1.21572640	1.15292280	1.08974170	1.02120530	0.97816721
	0.96579663	0.96861256	0.97137375	0.97055381	0.97264405	0.97733648	0.98258003
	0.98714218	0.99119647	0.99360176	0.99532526	0.99641440	1.00000000	
1100.	1.24146750	1.24728070	1.23787860	1.18007180	1.12193280	1.05802550	1.01611130
	1.00065740	0.99536544	0.98988431	0.98451205	0.98499228	0.98680419	0.98907735
	0.99116915	0.99315174	0.99444376	0.99559225	0.99653827	1.00000000	
1200.	1.26675820	1.27266940	1.26392690	1.20965280	1.15473510	1.09357670	1.05157340
	1.02975160	1.01494740	1.00263350	0.99645863	0.99479710	0.99430155	0.99418068
	0.99430254	0.99462789	0.99503631	0.99574945	0.99660273	1.00000000	
1300.	1.29558880	1.29970980	1.29050390	1.23850110	1.18617400	1.12674730	1.08078950
	1.04977320	1.02590110	1.01157430	1.00551770	1.00199420	0.99973174	0.99784208
	0.99650852	0.99560397	0.99536441	0.99578932	0.99660354	1.00000000	
1350.	1.30826760	1.31156760	1.30159260	1.25131370	1.20007040	1.14010400	1.09075460
	1.05495070	1.02813110	1.01495070	1.00854530	1.00441820	1.00150630	0.99899276
	0.99714451	0.99580259	0.99533821	0.99568703	0.99652117	1.00000000	
1380.	1.31553750	1.31833340	1.30810580	1.25811500	1.20696900	1.14621870	1.09467760
	1.05624650	1.02872350	1.01651780	1.00989700	1.00551650	1.00230980	0.99951524
	0.99743391	0.99589271	0.99532745	0.99564978	0.99649531	1.00000000	

Table 25. $E(T^x, \tau)$, 1400 - 1530

$E(T^x, \tau)$, $T^x = 1400 - 1530K$							
$T^x \setminus \tau$	0.00	0.02	0.04	0.07	0.10	0.15	0.22
	0.30	0.38	0.45	0.52	0.59	0.66	0.74
	0.82	0.90	0.95	0.98	0.99	1.00	
1400.	1.32037690	1.32146610	1.31185840	1.26169760	1.21045390	1.14913690	1.09622830
	1.05627800	1.02887190	1.01732360	1.01061260	1.00611480	1.00276250	0.99982704
	0.99762804	0.99598495	0.99536837	0.99567823	0.99651890	1.00000000	
1430.	1.32612240	1.32693330	1.31571450	1.26513160	1.21375010	1.15159110	1.09679160
	1.05502950	1.02874030	1.01802100	1.01126370	1.00664970	1.00314250	1.00005490
	0.99772721	0.99597201	0.99530269	0.99561661	0.99647896	1.00000000	
1450.	1.32820800	1.32799710	1.31606640	1.26541180	1.21408800	1.15157500	1.09580230
	1.05335750	1.02839730	1.01811840	1.01141720	1.00678070	1.00322420	1.00008570
	0.99771256	0.99592478	0.99523872	0.99556716	0.99644833	1.00000000	
1470.	1.32872340	1.32742090	1.31474620	1.26408450	1.21292080	1.15022970	1.09373860
	1.05113200	1.02785330	1.01796410	1.01138270	1.00676530	1.00319860	1.00004540
	0.99765678	0.99584506	0.99515678	0.99550912	0.99641258	1.00000000	
1500.	1.32579030	1.32171550	1.30868530	1.25844000	1.20786970	1.14541900	1.08846870
	1.04692020	1.02656350	1.01723470	1.01095100	1.00643770	1.00293130	0.99982664
	0.99747040	0.99568318	0.99501726	0.99542115	0.99636106	1.00000000	
1530.	1.31808520	1.31336170	1.29807020	1.24860910	1.19907740	1.13744180	1.08088050
	1.04213580	1.02466520	1.01594120	1.01004840	1.00571370	1.00233700	0.99934569
	0.99707327	0.99535645	0.99474041	0.99522430	0.99623583	1.00000000	

Table 26. $E(T^x, \tau)$, 1550 - 1800

$E(T^x, \tau)$, $T^x = 1550 - 1800$							
$T^x \setminus \tau$	0.00	0.02	0.04	0.07	0.10	0.15	0.22
	0.30	0.38	0.45	0.52	0.59	0.66	0.74
	0.82	0.90	0.95	0.98	0.99	1.00	
1550.	1.31098990	1.30489320	1.28943420	1.24006760	1.19121070	1.13042910	1.07482430
	1.03893080	1.02320530	1.01495930	1.00937800	1.00521380	1.00196620	0.99908988
	0.99690560	0.99526192	0.99469085	0.99522203	0.99624263	1.00000000	
1600.	1.27948480	1.27135710	1.25477250	1.20815080	1.16276240	1.10614800	1.05689920
	1.03056310	1.01832230	1.01151690	1.00681710	1.00322710	1.00042010	0.99794131
	0.99606444	0.99468285	0.99427386	0.99498014	0.99609978	1.00000000	
1650.	1.23244520	1.22211990	1.20458110	1.16300500	1.12326680	1.07529640	1.03894450
	1.02184990	1.01248920	1.00729940	1.00355110	1.00066190	0.99840338	0.99642330
	0.99492847	0.99388446	0.99368569	0.99461660	0.99588176	1.00000000	
1700.	1.16954510	1.15883150	1.14275840	1.10843260	1.07811080	1.04515040	1.02442600
	1.01334680	1.00676480	1.00307020	1.00028380	0.99812679	0.99644783	0.99499697
	0.99391113	0.99323123	0.99325857	0.99441275	0.99576962	1.00000000	
1750.	1.09915070	1.08989500	1.07764530	1.05647260	1.03973200	1.02398410	1.01315920
	1.00587120	1.00162240	0.99905830	0.99709295	0.99557079	0.99440009	0.99341433
	0.99268739	0.99233893	0.99259122	0.99400696	0.99552747	1.00000000	
1800.	1.04334170	1.03794530	1.03221910	1.02403690	1.01795250	1.01093870	1.00438250
	1.00002730	0.99744241	0.99576472	0.99447316	0.99348313	0.99274340	0.99215183
	0.99173121	0.99167434	0.99213650	0.99379107	0.99540993	1.00000000	

Table 27. $E(T^x, T)$, 1850 - 2200

$E(T^x, T)$, $T^x = 1850 - 2200$							
$T^x \setminus T$	0.00	0.02	0.04	0.07	0.10	0.15	0.22
	0.30	0.38	0.45	0.52	0.59	0.66	0.74
	0.82	0.90	0.95	0.98	0.99	1.00	
1850.	1.01427750	1.01150950	1.00893620	1.00556390	1.00275590	0.99922593	0.99624596
	0.99448765	0.99322801	0.99238417	0.99173586	0.99125875	0.99093555	0.99071502
	0.99060371	0.99083694	0.99153074	0.99346393	0.99522349	1.00000000	
1900.	0.99777327	0.99652328	0.99541875	0.99409275	0.99311811	0.99211382	0.99147583
	0.99088566	0.99035570	0.98998767	0.98971813	0.98954941	0.98947767	0.98947075
	0.98956048	0.98998935	0.99088965	0.99310890	0.99501878	1.00000000	
1950.	0.99030181	0.99003317	0.98983445	0.98966735	0.98960827	0.98958893	0.98933594
	0.98896284	0.98864811	0.98844811	0.98832692	0.98828602	0.98831725	0.98837942
	0.98857817	0.98912294	0.99021891	0.99273769	0.99480396	1.00000000	
2000.	0.98865809	0.98866667	0.98878733	0.98862481	0.98844335	0.98813174	0.98776058
	0.98743424	0.98721168	0.98709987	0.98706294	0.98709483	0.98717127	0.98727225
	0.98755915	0.98820666	0.98952968	0.99237484	0.99460227	1.00000000	
2100.	0.98394366	0.98388965	0.98383991	0.98377321	0.98371602	0.98364152	0.98357997
	0.98356929	0.98362035	0.98371221	0.98383964	0.98397029	0.98409181	0.98434905
	0.98482530	0.98578096	0.98777548	0.99147078	0.99409644	1.00000000	
2200.	0.97828379	0.97829912	0.97831638	0.97834626	0.97838091	0.97844901	0.97856480
	0.97872183	0.97889452	0.97902019	0.97912313	0.97927116	0.97952906	0.97997986
	0.98065577	0.98225941	0.98539743	0.99029865	0.99345363	1.00000000	

Table 28. $E(T^x, T)$, 2300 - 2470

$E(T^x, T)$, $T^x = 2300 - 2470$							
$T^x \setminus T$	0.00	0.02	0.04	0.07	0.10	0.15	0.22
	0.30	0.38	0.45	0.52	0.59	0.66	0.74
	0.82	0.90	0.95	0.98	0.99	1.00	
2300.	0.96937403	0.96913979	0.96910292	0.96930607	0.96942074	0.96954775	0.96964186
	0.96973155	0.96988892	0.97011819	0.97043593	0.97083104	0.97130436	0.97202872
	0.97334395	0.97666210	0.98191383	0.98866636	0.99257484	1.00000000	
2350.	0.96073438	0.96075421	0.96077772	0.96082221	0.96087863	0.96100070	0.96123109
	0.96157280	0.96198255	0.96238463	0.96282699	0.96333545	0.96398862	0.96515689
	0.96741646	0.97251512	0.97949481	0.98758775	0.99201170	1.00000000	
2400.	0.94639408	0.94643071	0.94650252	0.94664062	0.94677740	0.94701101	0.94735002
	0.94776522	0.94823807	0.94874288	0.94939893	0.95031627	0.95167452	0.95416510
	0.95849958	0.96667956	0.97622575	0.98614920	0.99125384	1.00000000	
2430.	0.93052629	0.93059769	0.93069232	0.93083806	0.93099274	0.93127458	0.93173567
	0.93239779	0.93327366	0.93429756	0.93566825	0.93753475	0.94012015	0.94441435
	0.95105325	0.96211777	0.97380312	0.98516255	0.99076652	1.00000000	
2450.	0.91272227	0.91289892	0.91307931	0.91336809	0.91368041	0.91426038	0.91522107
	0.91658783	0.91833159	0.92026700	0.92270802	0.92582067	0.92984572	0.93604540
	0.94489339	0.95849284	0.97193961	0.98441881	0.99040389	1.00000000	
2470.	0.88664412	0.88703914	0.88749457	0.88820459	0.88895725	0.89031524	0.89246251
	0.89533354	0.89875107	0.90229917	0.90649951	0.91152559	0.91762499	0.92639942
	0.93804046	0.95463852	0.97004232	0.98369560	0.99005908	1.00000000	

Table 29. $E(T^x, \tau)$, 2490 - 2507

$E(T^x, \tau)$, $T^x = 2490 - 2507$							
$T^x \setminus \tau$	0.00	0.02	0.04	0.07	0.10	0.15	0.22
	0.30	0.38	0.45	0.52	0.59	0.66	0.74
	0.82	0.90	0.95	0.98	0.99	1.00	
2490.	0.85403506	0.85462036	0.85559715	0.85713387	0.85872587	0.86151493	0.86573634
	0.87107948	0.87707769	0.88297199	0.88959841	0.89712404	0.90578822	0.91755834
	0.93222792	0.95180207	0.96892874	0.98345233	0.99002688	1.00000000	
2500.	0.84078611	0.84210621	0.84343112	0.84546492	0.84755707	0.85118215	0.85657497
	0.86325437	0.87057988	0.87762133	0.88537335	0.89399417	0.90370813	0.91658928
	0.93221668	0.95246539	0.96971412	0.98404520	0.99045497	1.00000000	
2502.	0.84017258	0.84154950	0.84292762	0.84504247	0.84721477	0.85097104	0.85654188
	0.86341515	0.87092196	0.87810916	0.88599183	0.89472421	0.90452503	0.91746428
	0.93308487	0.95321202	0.97027318	0.98440136	0.99069760	1.00000000	
2504.	0.84135738	0.84272810	0.84414851	0.84631950	0.84854624	0.85238985	0.85807462
	0.86506352	0.87266642	0.87991761	0.88784096	0.89658510	0.90636141	0.91921474
	0.93465919	0.95445696	0.97114311	0.98491151	0.99103114	1.00000000	
2506.	0.84680346	0.84783954	0.84924806	0.85136870	0.85354964	0.85731965	0.86290615
	0.86978744	0.87728315	0.88443439	0.89224185	0.90083852	0.91041328	0.92292224
	0.93782070	0.95675960	0.97270741	0.98583633	0.99161005	1.00000000	
2507.	0.85371996	0.85446912	0.85583688	0.85811263	0.86040003	0.86430864	0.86999694
	0.87685431	0.88416695	0.89102154	0.89840309	0.90644832	0.91535924	0.92700918
	0.94101671	0.95911015	0.97436841	0.98674482	0.99217072	1.00000000	

Appendix D. The code ZUNGE

D.1 ZUNGE.

ZUNGE is a system of routines, developed and used to calculate and test the thermal properties of the sodium. ZUNGE is written in the programming language SPEAKEASY - version IV Delta+ (for a reference on SPEAKEASY s. e.g. /7/). Since ZUNGE was intended to be used by the author solely, there are only few comments and hints to help the user.

Caveat : during the development of the SODIUM-code a number of the ZUNGE-routines suffered adjustments without a thorough checking of the code-system for internal consistency. So a guarantee for flawless operation in the whole region of validity cannot be given.

D.2 GLOSSARY.

CBUS	$\frac{\partial U}{\partial T} (\rho = \rho_L)$
CS	velocity of the sound in the saturated liquid in 1000 cm/sec units
CV	heat capacity at constant volume in RGAS units
CVIS	heat capacity along an isotherm
CVL	$C_V(\rho_L)$
CVLM	$C_V(\rho_L - 0)$
CVV	$C_V(\rho_V)$
CVVP	$C_V(\rho_V + 0)$
LIQ	indices of liquid points of an isotherm
LMX	indices of mixed-state points of an isotherm
LVP	indices of vapor points of an isotherm
DPVAP	$d(PVAP)/dT$
D8DPVAP	$\frac{T}{DPVAP} \frac{d(DPVAP)}{dT}$
P	pressure in bars
PIS	pressure along an isotherm
PRIS	P_ρ along an isotherm
PRL	$\frac{\partial P}{\partial \rho} (\rho_L) = P_\rho(\rho_L)$
PRLR	$\frac{P_\rho(\rho_L)}{RGAS \cdot T}$
PRV	$\frac{\partial P}{\partial \rho} (\rho_V) = P_\rho(\rho_V)$

PRVR	$\frac{P_\rho(\rho_V)}{RGAS \cdot T}$
PSR	$\frac{1}{RGAS \cdot T} \cdot \frac{\partial P}{\partial \rho} \Big _S$ in the saturated liquid
PTIS	P_T along an isotherm
PTL	$\frac{\partial P}{\partial T}(\rho_L) = P_T(\rho_L)$
PTLR	$\frac{P_T(\rho_L)}{RGAS \cdot \rho}$
PTV	$\frac{\partial P}{\partial T}(\rho_V) = P_T(\rho_V)$
PTVR	$\frac{P_T(\rho_V)}{RGAS \cdot \rho}$
PVAP	vapor-pressure
QKL	$\frac{T}{\rho'} \frac{d\rho'}{dT}$ in the saturated liquid
QKV	$\frac{T}{\rho'} \frac{d\rho'}{dT}$ in the saturated vapor
QT	Q_T , thermal conductivity , CGS-units
QTIS	thermal conductivity along an isotherm
RKL	$\frac{T}{\rho} \frac{d\rho}{dT}$ in the saturated liquid
RKV	$\frac{T}{\rho} \frac{d\rho}{dT}$ in the saturated vapor
RGAS	the gas-law constant of the Sodium in 0.1J/(g•K) units
RH	density , ρ in g/cm ³
RHC	ρ_c , the critical density
RHL	density of the saturated liquid
RHV	density of the saturated vapor
S	$\ln(\rho) / \ln(\rho_c)$
SIG	σ , $DPVAP \equiv \frac{PVAP}{T} (SIG + 1)$
T	temperature in K
TC	the critical temperature
TISO	temperature of an isotherm
TS	saturation temperature ;
	$\rho = \rho_L(T^x)$ for $\rho > \rho_c$ resp. $\rho = \rho_V(T^x)$ for $\rho < \rho_c$
U	density of the internal energy in K•RGAS units
UCR	density of the internal energy at the critical point

UIS	density of the internal energy along an isotherm
UIZ	density of the internal energy along an isotherm , corrected
ULM	$U(\rho_L)$
URL	$\frac{\partial U}{\partial \rho}(\rho_L)$
UVP	$U(\rho_V)$
UVPC	$U(\rho = \rho_V, T_c)$
UVV	$\frac{\partial U}{\partial V}(\rho_V)$
V	$1/\rho$
VQ	LOG10(V)
W	ρ/ρ_c
XK	$1 - \frac{T}{T_c}$ for $T \leq T_c$, $\frac{T}{T_c} - 1$ for $T > T_c$
YK	T_c/T
YT	Y_T , dynamic viscosity , CGS-units
YTIS	viscosity along an isotherm
ZIS	factor of reality along an isotherm
ZRIS	reduced $P_\rho = PRIS/(RGAS \cdot RH)$ on the isotherm TISO
ZTIS	reduced $P_T = PTIS/(RGAS \cdot TISO)$ on the isotherm TISO
ZV	factor of reality , $\frac{P}{\rho \cdot T \cdot RGAS}$ in the saturated vapor

D.3 CALLING RELATIONS

INTERNAL MODULS.					
Module	is called by				
ARRHOW	SATIS	SUPER			
CORCIM	ISVAP				
CRIPER	DERLIQ	DERVAP			
CRISEL	SATLIQ	SATVAP	QTLIQ	QTVAP	YTLIQ
	YTVAP				
CVCO	SUPER				
CVCU	DERLIQ	DERVAP			
CVINT	ISVAP				
DERLIQ	SATIS	ISLIQ			
DERVAP	SATIS	ISVAP			
GEVONS	CORCIM	CVINT			
ISLIQ	SATIS				
ISMIX	SATIS				
ISVAP	SATIS				
HAMU	CORCIM	CVINT			
QTLIQ	TRANIS				
QTVAP	TRANIS				
QVONWY	SUPER				
SATLIQ	CVCU				
SATVAP	CVCU	CVINT			
SUPDCV	SUPER				
TSATUR	SATIS				
REDUZ	SATIS				
UBASE	CVCU				
VAPRES	SATVAP				
YTLIQ	TRANIS				
YTVAP	TRANIS				
ZUDER	SUPER				

Table 30. Calling relations for internal moduls.

EXTERNAL MODULS.					
Module	is called by				
DISTRI	CRISEL	CVCU	UBASE		
FINOM	ISVAP				
GRIF	ARRHOW				
POLYST	SATLIQ	SATVAP	TSATUR	DERLIQ	DERVAP
	QTLIIQ	QTVAP	GEVONS	YTVAP	QVONWY
VAG	SATLIQ	SATVAP	TSATUR	DERLIQ	DERVAP
	QTLIIQ	QTVAP	YTLIQ	YTVAP	

Table 31. Calling relations for external modulS.

D.4 MODULE TREES

SATIS	ARRHOW	GRIF	-				
	TSATUR	VAG	-				
		POLYST	-				
	DERLIQ	CVCU	DISTRI				
			SATLIQ	VAG	-		
				POLYST	-		
				CRISEL	DISTRI	-	
			SATVAP	VAG	-		
				VAPRES	-		
				POLYST	-		
				CRISEL	DISTRI	-	
			UBASE	DISTRI	-		
		VAG	-				
		POLYST	-				
		CRIPER	-				
	DERVAP	CVCU	DISTRI				
			SATLIQ	VAG	-		
				POLYST	-		
				CRISEL	DISTRI	-	
			SATVAP	VAG	-		
				VAPRES	-		
				POLYST	-		
				CRISEL	DISTRI	-	
			UBASE	DISTRI	-		
		VAG	-				
		POLYST	-				
		CRIPER	-				
	ISLIQ	DERLIQ	-				
	ISMIX	-					
	ISVAP	DERVAP	CVCU	DISTRI	-		
				SATLIQ	VAG	-	
					POLYST	-	
					CRISEL	DISTRI	-
				SATVAP	VAG	-	
					VAPRES	-	
					POLYST	-	
					CRISEL	DISTRI	-
			UBASE	DISTRI	-		
		CORCIM	GEVONS	POLYST	-		
			HAMU	-			
		FINOM	-				
		CVINT	HAMU	-			
			SATVAP	VAG	-		
			GEVONS	POLYST	-		
	REDUZ	-					

Figure 44. MODULE TREE FOR A SUBCRITICAL ISOTHERM

SUPER	ARRHOW	GRIF	-
	QVONWY	POLYST	-
	CVCO	-	
	SUPDCV	-	
	ZUDER	-	

Figure 45. MODULE TREE FOR A SUPERCRITICAL ISOTHERM

D.5 SATURATION PROPERTIES

D.5.1 SATURATION LINE

PROGRAM SATLIQ

```
1 PROGRAM
2 VAG(T,1700,JU,JO,NINCS)
3 RHL = T-T;RKL = RHL;QKL = RHL
4 IF(NINCS .GT. 0) GOTO LO
5 POLYST(T(JU),GPOLYRL,FY0,FY1,FY2)
6 RHL(JU) = FY0;RKL(JU) = FY1/FY0;QKL(JU) = FY2/FY1
7 FREE FY0 FY1 FY2
8 LO: IF(NINCS .LT. 0) GOTO LE
9 CRISEL(T(JO),2,FK,GK,HK,CHI,TC,BET,POLYXK(1))
10 RHL(JO) = RHC*FK
11 RKL(JO) = -CHI*GK/FK
12 QKL(JO) = -CHI*HK/GK
13 FREE FK GK HK CHI
14 LE: DRKL = 1-RKL + QKL
15 END
```

PROGRAM SATVAP

```
1 PROGRAM
2 VAG(T,1700,JU,JO,NINCS); VAPRES
3 RHV = T-T;RKV = RHV;QKV = RHV;ZV = RHV + 1
4 IF(NINCS .GT. 0) GOTO LO
5 POLYST(T(JU),GPOLYZV,ZEX,TAU,TTDDFY)
6 ZVU = EXP(ZEX);ZV(JU) = ZVU
7 RHV(JU) = PVAP(JU)/(RGAS*T(JU)*ZV(JU))
8 RKV(JU) = SIG(JU)-TAU
9 QKV(JU) = (S6(JU)-TAU*(2*RKV(JU) + TAU)-TTDDFY)/RKV(JU)
10 FREE ZEX ZVU TAU TTDDFY S6
11 LO: IF(NINCS .LT. 0) GOTO LE
12 CRISEL(T(JO),-2,FK,GK,HK,CHI,TC,BET,POLYXK(2))
13 RHV(JO) = RHC*FK
14 RKV(JO) = -CHI*GK/FK
15 QKV(JO) = -CHI*HK/GK
16 ZV(JO) = PVAP(JO)/(RGAS*T(JO)*RHV(JO))
17 FREE FK GK HK CHI
18 LE: DRKV = 1-RKV + QKV
19 END
```


PROGRAM VAPRES

```
1 PROGRAM
2 $ P IN BARS
3 S1 = 11.919;S2 = -12153.;S3 = -.195;S4 = S3-1.
4 SIG = S4-S2/T; S6 = SIG*SIG-2*SIG + S4
5 LNP = S1 + S2/T + S3*LOG(T)
6 PVAP = EXP(LNP);DPVAP = (PVAP/T)*(SIG + 1)
7 D8DPVAP = (SIG*SIG + S4)/(SIG + 1)
8 FREE LNP
9 END
```

SUBROUTINE CRISEL

```
1 SUBROUTINE CRISEL(Y,B,F,G,H,DXT,TC,BET,POL)
2 DISTRI(Y,TC-1.E-14,MY,JAM)
3 XK = MY;WHERE(MY .LE. 0)  XK = TC-Y
4 DXT = -Y/XK; XK = XK/TC
5 BKL = Y-Y;G = MY;H = BKL;F = MY
6 K = INDEXER(POL)
7 DPOL(K) = POL(K)*(K-1)
8 DDPOL(K) = DPOL(K)*(K-2)
9 WHERE(MY .LE. 0)  BKL = B*XK**BET
10 WHERE(MY .LE. 0)  F = 1 + BKL + POLYVAL(POL,XK)
11 WHERE(MY .LE. 0)  G = -BET*BKL-POLYVAL(DPOL,XK)
12 WHERE(MY .LE. 0)  H = BET*(BET-1)*BKL + POLYVAL(DDPOL,XK)
13 END
```

PROGRAM TSATUR

```
1 PROGRAM
2 VAG(RH,RHC,JU,JO,NINCS)
3 TS = RH-RH
4 IF(NINCS .GT. 0)      GOTO LO
5 S = LOG(RH(JU))/LOG(RHC)
6 POLYST(S,GPOLYTRV,TSU,DFX,DDFX)
7 TS(JU) = TSU
8 FREE S TSU DFX DDFX
9 LO: IF(NINCS .LT. 0)  GOTO LE
10 POLYST(RH(JO),GPOLYTRL,TSO,DFX,DDFX)
11 TS(JO) = TSO
12 FREE TSO DFX DDFX
13 LE: WHERE(TS-TC .GT. 0)  TS = TC
14 END
```

D.5.2 PRESSURE & INTERNAL ENERGY

PROGRAM CVCU

```
1 PROGRAM
2 IF(CLASS(T) .LE. 0) T = A1D(:T)
3 DISTRI(T,TC-1.E-14,MYC,JAM)
4 ULM = T-T; XK = MYC*30
5 WHERE(MYC .LE. 0) XK = 1-T/TC
6 CVLM = XK;DCVLM = XK
7 WHERE(MYC .LE. 0) CVLM = POLYVAL(POLYCF(1),T) +
8 & 5.53*(XK**(-ALF)-POLYVAL(POLYCF(2),T))
9 WHERE(MYC .LE. 0) DCVLM = POLYVAL(POLYCF(3),T) +
10 & 5.53*(ALF*XK**(-ALF-1)/TC-POLYVAL(POLYCF(4),T))
11 SATLIQ; SATVAP; VL = 1/RHL; VV = 1/RHV
12 CVVP = CVLM + (VV-VL)*D8DPVAP*DPVAP/RGAS
13 CBUS = CVLM-RKL*SIG*PVAP/(T*RHL*RGAS)
14 WHERE(MYC .GT. 0) XK = 0.
15 UBASE(XK,GPOLYUF,DELT,ULM)
16 UVP = ULM + (VV-VL)*SIG*PVAP/RGAS
17 END
```

SUBROUTINE UBASE

```
1 SUBROUTINE UBASE(X,PY,DELT,FX)
2 $ X : 1-T/TC ; FX : ENERGY-DENSITY ON THE BASELINE
3 $ 0 = FX(TMELT)
4 Y = REFLECT(X)
5 IF(CLASS(X) .EQ. 5) GOTO L0
6 X; "ERROR : X MUST BE AN 1D ARRAY"; RETURN
7 L0: IF(CLASS(PY) .EQ. 6) GOTO L1
8 PY; "ERROR : PY MUST BE AN 2D ARRAY"; RETURN
9 L1: YG = PY(:,1);PYPY = ELIMCOLS(PY,1)
10 JM = NOROWS(PYPY)
11 DISTRI(Y,YG,MY,JAM)
12 IF(KIND(JAM) .EQ. 0) RETURN
13 PYC = PYPY(:,1); PYN = ELIMCOLS(PYPY,1)
14 FY = Y-Y
15 FOR J = 1,JAM
16 L = MIN(J,JM)
17 WHERE(MY .EQ. J-1) FY = PYC(L) + (Y**(DELT + 1))*POLYVAL(PYN(L),Y)
18 NEXT J
19 FX = REFLECT(FY)
20 END
```

PROGRAM DERLIQ

```
1 PROGRAM
2 CVCU;ZETA = DPVAP/(RGAS*RHL)
3 TDZETA = ZETA*(D8DPVAP-RKL)
4 PRLR = T-T;DLPRLR = PRLR
5 VAG(T,1700,JU,JO,NINCS)
6 IF(NINCS .GT. 0) GO TO LO
7 TU = T(JU); POLYST(TU,GPOLYCS,CS,TDCS,TTDD)
8 PSR = CS*CS/(RGAS*TU); TDPSR = PSR*(2*TDCS/CS-1)
9 NENNER = CVLM(JU)
10 & -2*ZETA(JU)*RKL(JU) + RKL(JU)*RKL(JU)*PSR
11 ZAHLER = CVLM(JU)*PSR-ZETA(JU)*ZETA(JU)
12 PR1 = ZAHLER/NENNER
13 TDRKL = RKL*DRKL
14 DPR1 = TDPSR*(CVLM(JU)-RKL(JU)*RKL(JU)*PR1) +
15 & TU*DCVLM(JU)*(PSR-PR1) +
16 & 2*TDRKL(JU)*(ZETA(JU)-RKL(JU)*PSR)*PR1 -
17 & 2*TDZETA(JU)*(ZETA(JU)-RKL(JU)*PR1)
18 PRLR(JU) = PR1;DLPRLR(JU) = DPR1/ZAHLER
19 FREE ZAHLER,NENNER,TTDD,TDPSR,TDRKL,TDCS,PSR,TU
20 LO: IF(NINCS.LT. 0) GOTO LE
21 S0 = 71508.9;S0R = S0/(RHC*RGAS)
22 CRIPER(T(JO),POLYPRL,PR1,DPR1,TC,GAM)
23 PRLR(JO) = PR1*S0R*RHL(JO)
24 DLPRLR(JO) = RKL(JO)-DPR1
25 LE: PRL = T*RGAS*PRLR;DPRL = (DLPRLR + 1)*(PRL/T)
26 PTLR = ZETA-RKL*PRLR;PTL = RGAS*RHL*PTLR
27 TDPTLR = TDZETA-RKL*PRLR*(DLPRLR + DRKL)
28 CVL = CVLM-RKL*RKL*PRLR
29 URL = (PVAP-T*PTL)/(RHL*RHL*RGAS)
30 FREE PR1 DPR1 ZETA TDZETA
31 END
```

PROGRAM DERVAP

```
1 PROGRAM
2 CVCU;ZETA = DPVAP/(RGAS*RHV)
3 TDZETA = ZETA*(D8DPVAP-RKV)
4 PRVR = T-T;PTVR = PRVR;DLPRVR = PRVR
5 VAG(T,2320,JU,JO,NINCS)
6 IF(NINCS .GT. 0) GO TO LO
7 TU = T(JU)
8 POLYST(TU,GPOLYPTV,PT,TDPT,TTDD)
9 RPR = ZETA(JU)-PT
10 PRVR(JU) = RPR/RKV(JU)
11 DLPRVR(JU) = (TDZETA(JU)-TDPT)/RPR-DRKV(JU)
12 FREE TTDD TDPT RPR PT
13 LO: IF(NINCS .LT. 0) GOTO LE
14 S0 = 71508.9;S0R = S0/(RHC*RGAS)
15 TO = T(JO);CRIPER(TO,POLYPRV,PR1,DPR1,TC,GAM)
16 PRVR(JO) = PR1*S0R*RHV(JO)
17 DLPRVR(JO) = RKV(JO)-DPR1;FREE PR1 DPR1
18 LE: PRV = T*RGAS*PRVR;DPRV = (DLPRVR + 1)*(PRV/T)
19 PTVR = ZETA-RKV*PRVR;PTV = RGAS*RHV*PTVR
20 TDPTVR = TDZETA-RKV*PRVR*(DLPRVR + DRKV)
21 CVV = CVVP-RKV*RKV*PRVR
22 UVV = ZV-PTVR
23 POLYST(T,GPOLYDU,DUCV,F1,F2)
24 UVPC = UVP + DUCV
25 FREE F1 F2
26 END
```

SUBROUTINE CRIPER

```
1 SUBROUTINE CRIPER(Y,PYPY,FY,DFY,TC,GAM)
2 Z1 = TC-Y;WHERE(Z1 .LE. 0) Z1 = 1
3 XK = Z1/TC;FY = Y-Y;DFY = FY
4 K = INDEXER(PYPY);DPYP(K) = PYPY(K)*(K-1)
5 F1 = POLYVAL(PYPY,XK)
6 F2 = POLYVAL(DPYP,XK)/F1
7 WHERE(Y .LT. TC) FY = F1*XK**GAM/Y
8 WHERE(Y .LT. TC) DFY = 1 + (GAM + F2)*Y/Z1
9 END
```

D.5.3 TRANSPORT PROPERTIES

PROGRAM QTLIQ

```
1 PROGRAM
2 VAG(T,1700,JU,JO,NINCS)
3 QTL = T-T;DQTL = QTL
4 IF(NINCS .GT. 0) GOTO LO
5 POLYST(T(JU),GPOLYQTL,FY0,FY1,FY2)
6 QTL(JU) = FY0;DQTL(JU) = FY1/FY0
7 FREE FY0 FY1 FY2
8 LO: IF(NINCS .LT. 0) GOTO LE
9 CRISEL(T(JO),2,FK,GK,HK,CHI,TC,BET,POLYXQ(1))
10 QTL(JO) = FK
11 DQTL(JO) = -CHI*GK/FK
12 FREE FK GK HK CHI
13 LE: QTL = QTC*QTL; QTL2 = QTL*DQTL/T
14 END
```

PROGRAM QTVAP

```
1 PROGRAM
2 VAG(T,1700,JU,JO,NINCS)
3 QTV = T-T;DQTV = QTV
4 IF(NINCS .GT. 0) GOTO LO
5 POLYST(T(JU),GPOLYQTV,FY0,FY1,FY2)
6 QTV(JU) = FY0;DQTV(JU) = FY1/FY0
7 FREE FY0 FY1 FY2
8 LO: IF(NINCS .LT. 0) GOTO LE
9 CRISEL(T(JO),-2,FK,GK,HK,CHI,TC,BET,POLYXQ(2))
10 QTV(JO) = FK
11 DQTV(JO) = -CHI*GK/FK
12 FREE FK GK HK CHI
13 LE: QTV = QTC*QTV; QTV2 = QTV*DQTV/T
14 END
```

PROGRAM YTLIQ

```
1 PROGRAM
2 VAG(T,2250,JU,JO,NINCS)
3 YTL = T-T;DYTL = YTL
4 IF(NINCS .GT. 0) GOTO LO
5 TU = T(JU);VIG = POLYYTL(2)/T(JU)
6 DYTL(JU) = POLYYTL(3)-VIG
7 YTL(JU) =
8 & 10**(POLYYTL(1) + VIG + POLYYTL(3)*LOG10(TU))
9 FREE VIG TU
10 LO: IF(NINCS .LT. 0) GOTO LE
11 CRISEL(T(JO),1.5,FK,GK,HK,CHI,TC,BET,POLYXY(1))
12 YTL(JO) = FK
13 DYTL(JO) = -CHI*GK/FK
14 FREE FK GK HK CHI
15 LE: YTL = YTC*YTL; YTL2 = YTL*DYTL/T
16 END
```

PROGRAM YTVAP

```
1 PROGRAM
2 VAG(T,1950,JU,JO,NINCS)
3 YTV = T-T;DYTV = YTV
4 IF(NINCS .GT. 0) GOTO LO
5 POLYST(T(JU),GPOLYYTV,FY0,FY1,FY2)
6 YTV(JU) = FY0;DYTV(JU) = FY1/FY0
7 FREE FY0 FY1 FY2
8 LO: IF(NINCS .LT. 0) GOTO LE
9 CRISEL(T(JO),-1.5,FK,GK,HK,CHI,TC,BET,POLYXY(2))
10 YTV(JO) = FK
11 DYTV(JO) = -CHI*GK/FK
12 FREE FK GK HK CHI
13 LE: YTV = YTC*YTV; YTV2 = YTV*DYTV/T
14 END
```

D.6 PROPERTIES ON ISOTHERMS

D.6.1 SUBCRITICAL ISOTHERM

PROGRAM SATIS

```
1 PROGRAM
2 LT: REQUEST TISO
3 IF(TC-TISO .LE. 0) TISO " IS SUPERCRITICAL !"
4 IF(TC-TISO .LE. 0) GOTO LT
5 ARRHOW(RH); LMX = INDEXER(RH)
6 TSATUR; WHERE(TS .GT. TISO) TS = TISO
7 DTISO = TISO-TS
8 T = TISO; DERLIQ
9 RHLS = RHL(1);CVLS = CVL(1);ULMS = ULM(1)
10 PVAPS = PVAP(1); DPVAPS = DPVAP(1)
11 CVLMS = CVLM(1); VLS = VL(1)
12 DCV = D8DPVAP*DPVAP/RGAS; DCVS = DCV(1)
13 DU = PVAP*SIG/RGAS; DUS = DU(1)
14 DRL = RH-RHLS; LIQ = LOCS(DRL .GE. 0:ALL)
15 NIQ = MIN(NOELS(LIQ),SUM(LIQ))
16 T = TISO; DERVAP
17 RHVS = RHV(1);CVVS = CVV(1);UVPS = UVP(1)
18 DRV = RHVS-RH; LVP = LOCS(DRV .GE. 0:ALL)
19 NVP = MIN(NOELS(LVP),SUM(LVP))
20 LMX = RELCOMP(LMX,UNION(LIQ,LVP))
21 NMX = MIN(NOELS(LMX),SUM(LMX))
22 PTIS = RH-RH;PIS = PTIS;PRIS = PTIS
23 CVIS = PTIS;UIS = CVIS
24 IF(NIQ .GE. 1) ISLIQ
25 IF(NMX .GE. 1) ISMIX
26 IF(NVP .GE. 1) ISVAP
27 FREE DU DCV DRL DTISO
28 REDUZ
29 END
```

PROGRAM REDUZ

```
1 PROGRAM
2 ZIS = PIS/(RGAS*RH*TISO)
3 ZTIS = PTIS/(RGAS*RH)
4 ZRIS = PRIS/(RGAS*TISO)
5 VQ = -LOG10(RH)
6 END
```

SUBROUTINE ARRHOW

```
1 SUBROUTINE ARRHOW(RH)
2 L0: IF(KIND(RH) .EQ. 0) "RH IS NOT DEFINED !"
3 "RH-SETTING ( 0 = OLD | 1 = LIMITS ) "
4 ASK("RHMOD","RHMOD =", "RHMOD = 0")
5 IF(RHMOD .NE. 0) GOTO L1
6 IF(KIND(RH) .EQ. 0) GOTO L0
7 IF(RH(NOELS(RH)) .GT. RH(1)) GOTO LE
8 RH = REFLECT(RH); "RH REFLECTED !"
9 LE: RETURN
10 L1: " RHMI , RHMA , RHFK = : "
11 REQUEST RHMI , RHMA ,RHFK
12 GRIF(RHMI,RHMA,RHFK,RH)
13 IF(SUM(RH) .LE. 0) GOTO L1
14 MAKEGLOBAL RHMI RHMA RHFK
15 END
```

PROGRAM ISLIQ

```
1 PROGRAM
2 TSL = TS(LIQ);DTISL = DTISO(LIQ)
3 T = TSL; DERLIQ
4 PTIS(LIQ) = PTL
5 PIS(LIQ) = PVAP + DTISO(LIQ)*PTL
6 DEPRIS = DTISL*(PTLR + TDPTLR/RKL)
7 PRIS(LIQ) = RGAS*(TSL*PRLR + DEPRIS)
8 CVIS(LIQ) = CVLS
9 DULR = INTEGRAL(URL:RH(LIQ))
10 UIS(LIQ) = ULMS + DULR
11 FREE DEPRIS TSL DTISL T
12 END
```

PROGRAM ISMIX

```
1 PROGRAM
3 PIS(LMX) = PVAPS
4 PTIS(LMX) = DPVAPS
5 PRIS(LMX) = RH(LMX)-RH(LMX)
6 CVIS(LMX) = CVLMS + (1/RH(LMX)-VLS)*DCVS
7 UIS(LMX) = ULMS + (1/RH(LMX)-VLS)*DUS
8 END
```


PROGRAM ISVAP

```
1 PROGRAM
2 TSV = TS(LVP); DTISV = DTISO(LVP)
3 S = LOG(RHC)/LOG(RH(LVP))
4 T = TSV; DERVAP
5 CORCIM; VOFA = RGAS*GVS*RH(LVP)
6 DTRR = 1/RKV
7 FTS1 = PRVR-GVS*MUE*ZWEI*DTRR
8 FDTS = PTVR + TDPTVR*DTRR + GVS*MUE*EINS
9 PIS(LVP) = PVAP + DTISV*(PTV + VOFA*MUE)
10 PTIS(LVP) = PTV + VOFA*HVV
11 PRIS(LVP) = RGAS*(TSV*FTS1 + DTISV*FDTS)
12 FREE DTRR EINS ZWEI VOFA FTS1 FDTS
13 DURC = -GVS*MUE*(1 + (TISO/TSV)*(UMU-1))
14 URTS = (UVV + DURC)*RKV
15 J = REFLECT(LVP); TSVR = TSV(J)
16 IF(NVP .GT. 1) GOTO L1
17 TSVR = 0.75*TSVR(1),TSVR
18 L1: FINOM(TSVR,Y,VIP,10)
19 IF(NVP .LE. 1) VIP = INTS(10,10)
20 CVINT; DCV = INTEGRAL(FY:Y)
21 DCV = DCV(VIP); DCV = DCV(J)
22 CVIS(LVP) = CVVS + DCV
23 DUVR = INTEGRAL(URTS(J):TSV(J))
24 DUVR = DUVR(J); UIS(LVP) = UVPS + DUVR
25 UIZ = UIS; FROE(TSV,TISO,TC,UFR)
26 UIZ(LVP) = UVP + UFR*DUCV
27 FREE TSVR Y FY J VIP TSV
28 END
```

PROGRAM CORCIM

```
1 PROGRAM
2 GEVONS(S,GPOLYET,ETA,GVS,DSRGVS)
3 EINS = 1-S*DSRGVS/LOG(RHC)
4 HAMU(TSV,TISO,TC,U,UMU,MUE,HVV,JVU)
5 ZWEI = UMU-0.2*(UMU-1)/U
6 FREE JVU ETA U DSRGVS
7 END
```

SUBROUTINE FROE

```
1 SUBROUTINE FROE(TS,TISO,TC,UFR)
2 NEN = TC-TS
3 ZAH = TISO-TS
4 UFR = ZAH/NEN
5 END
```

PROGRAM CVINT

```
1 PROGRAM
2 TISO = TISO - 1.E-10; WHERE(Y - TISO .GT. 0) Y = TISO
3 FY = TISO / (0.2 * (TC - Y))
4 HAMU(Y, TISO, TC, U, UMU, MUE, HVONU, UJVONU)
5 FY = FY * UJVONU
6 FREE U UMU MUE HVONU UJVONU
7 T = Y; SATVAP; FY = FY * (RKV / Y)
8 S = LOG(RHC) / LOG(RHV)
9 FREE PVAP DPVAP S6 SIG D8DPVAP
10 FREE RHV RKV QKV ZV DRKV JU JO
11 GEVONS(S, GPOLYET, ETA, GVS, DSRGVS)
12 FY = -FY * GVS
13 FREE TISO T S ETA DSRGVS GVS
14 END
```

SUBROUTINE GEVONS

```
1 SUBROUTINE GEVONS(S, PYPY, ETA, FY, DSRFY)
2 DIS = S - 0.32; ICO = S .LT. 0.32
3 POLYST(S, PYPY, ETA, DF, DDF)
4 DF = DF / (ETA + (ETA.EQ.0))
5 ETA = ETA * ICO
6 DSRFY = DF + 2 * (S + DIS) / (DIS + (DIS.EQ.0))
7 DSRFY = DSRFY * ICO
8 FY = S * S * DIS * DIS * ETA
9 END
```

SUBROUTINE HAMU

```
1 SUBROUTINE HAMU(TS, TISO, TC, U, UMU, MUE, HVONU, JVONU)
2 V = TISO - TS
3 WHERE(V .LE. 0) V = 1.E-12
4 U = 0.2 * (1 + (TC - TISO) / V)
5 WHERE(U .GT. 50) U = 50
6 MUE = U - U + 1
7 WHERE(U .LT. 0.001) MUE = 1 - 0.5 * U * (1 - (U/6) * (1 - U * U/60))
8 WHERE(U .GE. 0.001) MUE = U / (EXP(U) - 1)
9 UMU = U + MUE
10 HVONU = MUE * UMU
11 JVONU = HVONU * (UMU + MUE - 2) * U
12 END
```

D.6.2 SUPERCRITICAL ISOTHERM

PROGRAM SUPER

```
1 PROGRAM
2 LT: REQUEST TISO
3 IF(TISO-TC .LT. 0) TISO " IS SUBCRITICAL !"
4 IF(TISO-TC .LT. 0) GOTO LT
5 ARRHOW(RH); W = RH/RHC; YK = TC/TISO
6 AG = 0.814159413; BG = 1.297860659
7 AK = YK*AG; DW = W-1
8 S = 1 + W*(W*BG-AK); S1 = 1 + BG-ÄK
9 QVONWY(W,YK,GPOLYZW,GPOLYZY,GIPOLYZW,GIPOLYZY,
10 & QIS,QIW,QIY,QIC,QIU)
11 KERN = AG/S
12 ZIS = 1 + W*YK*(KERN-QIS)
13 KER2 = KERN/S
14 WZW = W*YK*(KER2*(1-W*W*BG)-QIW)
15 ZRIS = ZIS + WZW
16 YZY = W*YK*(KER2*(1 + W*W*BG)-QIY)
17 ZTIS = ZIS-YZY
18 FREE KERN KER2 YZY WZW QIS QIW QIY
19 T = TISO; CVCO
20 SUPDCV(AK,BG,S,S1,DW,DELI,DELIU)
21 CVIS = CVBS + YK*YK*(2*QIC-AG*AG*DELI)
22 UIS = UBS + TC*(AG*DELIU-QIU)
23 FREE AK S S1 QIC QIU DELI DELIU DW
24 ZUDER
25 END
```

PROGRAM CVCO

```
1 PROGRAM
2 XK = T-TC;WHERE(T .EQ. TC)   XK = 1.21307E-7; XK = XK/TC
3 CVBS = T-T;UBS = CVBS + UCR
4 WHERE(T .GE. TC)   CVBS = 2.2*XK**(-ALF)
5 WHERE(T .GT. TC)   UBS = UBS + (2.2*TC/(1-ALF))*XK**(1-ALF)
6 END
```

PROGRAM ZUDER

```
1 PROGRAM
2 PIS = ZIS*(RGAS*RH*TISO)
3 PTIS = ZTIS*(RGAS*RH)
4 PRIS = ZRIS*(RGAS*TISO)
5 VQ = -LOG10(RH)
6 END
```

SUBROUTINE QVONWY

```
1 SUBROUTINE QVONWY(W,YK,PYW,PYY,IPYW,IPYY,F,FW,FY,FC,FU)
2 J = INTS(2,NOELS(PYW(1)))
3 PYF = PYW + (YK-1)*PYY
4 PYF(,1) = PYW(,1)
5 POLYST(W,PYF,F,F2,F3)
6 PYR(,1) = PYW(,1)
7 PYR(,J) = PYF(,J)*(J-1)
8 POLYST(W,PYR,FW,F2,F3)
9 PYT = PYF + YK*PYY
10 PYT(,1) = PYW(,1)
11 POLYST(W,PYT,FY,F2,F3)
12 POLYST(W,IPYY,FC,IF2,IF3)
13 IPYU = IPYW + (2*YK-1)*IPYY
14 IPYU(,1) = PYW(,1)
15 Z = A1D(1:1); POLYST(Z,IPYU,FZ,IF2,IF3)
16 POLYST(W,IPYU,FU,IF2,IF3)
17 FU = FU-FZ
18 END
```

SUBROUTINE SUPDCV

```
1 SUBROUTINE SUPDCV(AK,BG,S,S1,DW,DELIC,DELIU)
2 SIG = 4*BG-AK*AK;WSIG = SQRT(SIG)
3 T1 = 1/S1; T = 1/S
4 BSI = BG/WSIG; XI = 6*AK*BSI
5 U = 2*S1 + (2*BG-AK)*DW
6 V = 2*BG-AK + DW*BG
7 DLF1 = XI*(SIG-BG*U)
8 DLF2 = (SIG-2*BG)*AK*S1*T
9 DLF3 = V*(SIG + (SIG-2*BG + AK)*AK*(T + T1))
10 DELFI = DW*(T*T1/BG)*(DLF1-DLF2 + DLF3)
11 ATAWU = ATAN(DW*WSIG/U)
12 DELIC = ((4*XI/WSIG)*ATAWU + DELFI)/SIG
13 DU2 = BG*(2-AK)
14 DU3 = 4*BG-AK*(BG + 1)
15 DULF = (DU2*DW + DU3)/(S*S1)
16 DELIU = (8*BSI*ATAWU + AK*DULF*DW)/SIG
17 END
```

D.6.3 TRANSPORT PROPERTIES ALONG AN ISOTHERM

PROGRAM TRANIS

```
1 PROGRAM
2 AUA = KIND(RH,TISO,TS);IF(MIN(AUA) .GT. 0) GOTO L0
3 " SOME PROPERTIES ARE MISSING !"
4 TYPE("RH , TISO , TS = > ", AUA)
5 RETURN
6 L0: QTIS = TS-TS; YTIS = QTIS
7 IF(NIQ .LT. 1) GOTO L2
8 T = TS(LIQ)
9 YTLIQ;YTIS(LIQ) = YTL
10 QTLIQ;QTIS(LIQ) = QTL
11 L2: IF(NMX .LT. 1) GOTO L3
12 TAU = (RH(LMX)-RHVS)/(RHLS-RHVS)
13 TA1 = TAU**(2/3)
14 T = TISO
15 YTTLIQ;YTLS = YTL; QTLIQ;QTLS = QTL
16 YTVAP;YTVS = YTV; QTVAP;QTVS = QTV
17 QW1 = QTVS/(QTLS-QTVS)
18 YW1 = YTVS/(YTLS-YTVS)
19 QTIS(LMX) = QTVS/(1-TAU/(QW1 + TA1))
20 YTIS(LMX) = YTVS/(1-TAU/(YW1 + TA1))
21 L3: IF(NVP .LT. 1) GOTO L4
22 T = TS(LVP)
23 YTVAP;YTIS(LVP) = YTV
24 QTVAP;QTIS(LVP) = QTV
25 L4: FREE QW1 YW1 TA1 TAU
26 END
```

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D.7 EXTERNAL MODULS

SUBROUTINE GRIF

```
1 SUBROUTINE GRIF(RMI,RMA,RFAK,R)
2 $ R = A LOGARITMIC GRID ; RFAK = R(I + 1)/R(I)
3 IF((RMI .GT. 0) .AND. (RMA .GT. 0)
4 & .AND. (RMA .GT. RMI)) GOTO H1
5 "ERROR : ", RMI RMA; R = 0,0; RETURN
6 H1: IF(RFAK .LE. 1) V = GRID(LN(RMI),LN(RMA))
7 IF(RFAK .GT. 1) V = GRID(LN(RMI),LN(RMA),LN(RFAK))
8 R = RMI,EXP(V),RMA; R = UNIQUE(R)
9 END
```

SUBROUTINE FINOM

```
1 SUBROUTINE FINOM(Y,YU,MU,GR)
2 $ YU = PARTITION ( Y ) ; MU = INDEX( Y IN YU )
3 $ ( ) = > 1 ( ) ... GR ( )
4 IF(CLASS(Y) .EQ. 5) GOTO L0
5 Y; "ERROR : Y MUST BE AN 1D ARRAY"; RETURN
6 L0: IF(GR .GT. 1) GOTO L1
7 YU = Y; MU = INDEXER(YU); RETURN
9 L1: NY = NOELS(Y); GL = GR - 1
10 IF(NY .GT. 1) GOTO L2
11 NY; "ERROR : THERE MUST BE BE AT LEAST 2 POINTS"; RETURN
12 L2: R = ELIMELS(Y,NY); S = ELIMELS(Y,1)
13 DR = (S - R)/GR; YU = Y
14 FOR I = 1, GL
15 YU = YU, (R + I * DR)
16 NEXT I
17 YU = RANKED(YU)
18 IF(Y(1) .GT. Y(NY)) YU = REFLECT(YU)
19 MU = INDEXER(R)
20 MU = 1, GR * MU + 1
21 END
```

SUBROUTINE VAG

```
1 SUBROUTINE VAG(T,TGR,JU,JO,NINCS)
2 $ T = TU, TO = T(JU), T(JO) ; TGR = MAX(TU) = MIN(TO)
3 $ NINCS = -1 : T = TU; NINCS = + 1 : T = TO
4 IF(CLASS(T) .LE. 0) T = A1D(1:T)
5 TIN = T - TGR
6 OT = TIN .GT. 0; UT = .NOT. OT
7 JO = LOCS(OT); JU = LOCS(UT)
8 NINCS = 0
9 IF(SUM(OT) .LT. 1) NINCS = -1
10 IF(SUM(UT) .LT. 1) NINCS = 1
11 END
```

SUBROUTINE DISTRI

```
1 SUBROUTINE DISTRI(Y,YG,MY,JAM)
2 $ MY ( Y ) = STEP-FUNCTION TO THE PARTITION YG
3 IF(CLASS(Y) .EQ. 5) GOTO L0
4 Y; "ERROR : Y MUST BE AN 1D ARRAY"; RETURN
5 L0: IF(LOCMAX(Y) .GT. LOCMIN(Y)) GOTO L1
6 Y(1);Y(NOELS(Y)); "ERROR : Y MUST BE RANKED "; RETURN
7 L1: IF(CLASS(YG) .NE. 5) YG = A1D(2:YG,MAX(Y))
8 YG = UNIQUE(YG);YG = RANKED(YG)
9 IF(MIN(Y) .EQ. MIN(YG)) YG = ELIMELS(YG,1)
10 IF(MAX(Y) .GT. MAX(YG)) YG = YG,MAX(Y)
11 JAM = NOELS(YG);MY = Y-Y
12 FOR J = 1,JAM
13 WHERE(Y-YG(J) .GT. 0) MY = J
14 NEXT J
15 END
```

SUBROUTINE POLYST

```
1 SUBROUTINE POLYST(Y,PY,FY,YDFY,YYDDFY)
2 $ PY = (YG,PYPY)
3 $ FY ( Y ) = SET OF POLYNOMIALS PYPY
4 $ TO THE PARTITION YG OF Y
5 $ YDFY = Y*FY' , YYDDFY = Y*Y*FY''
6 IF(CLASS(Y) .EQ. 5) GOTO L0
7 Y; "ERROR : Y MUST BE AN 1D ARRAY"; RETURN
8 L0: IF(CLASS(PY) .EQ. 6) GOTO L1
9 PY; "ERROR : PY MUST BE AN 2D ARRAY"; RETURN
10 L1: YG = PY(:,1);PYPY = ELIMCOLS(PY,1)
11 DISTRI(Y,YG,MY,JAM)
12 IF(KIND(JAM) .EQ. 0) RETURN
13 JM = NOROWS(PYPY)
14 J = INTEGERS(JM);K = INDEXER(PYPY(1))
15 DPYPY(J,K) = PYPY(J,K)*(K-1)
16 DDPYPY(J,K) = DPYPY(J,K)*(K-2)
17 FY = Y-Y;YDFY = FY;YYDDFY = FY
18 FOR J = 1,JAM
19 L = MIN(J,JM)
20 WHERE(MY .EQ. J-1) FY = POLYVAL(PYPY(L),Y)
21 WHERE(MY .EQ. J-1) YDFY = POLYVAL(DPYPY(L),Y)
22 WHERE(MY .EQ. J-1) YYDDFY = POLYVAL(DDPYPY(L),Y)
23 NEXT J
24 END
```

D.8 DATA

D.8.1 CRITICAL DATA

$$RGAS = 3.6165 \quad 0.1 * J / (G * K)$$

$$\begin{aligned} TC &= 2508.00 && K \\ RHC &= 0.23000 && G / CM^{**3} \\ QTC &= 0.05000 && W / (CM * K) \\ YTC &= 0.65E-3 && G / (CM * S) \end{aligned}$$

$$\begin{aligned} ALF &= .110 \\ BET &= .325 \\ GAM &= 1.24 \\ DELT &= -.68 \end{aligned}$$

$$UCR = 10229.5 \quad K * RGAS$$

D.8.2 SATURATION PROPERTIES

SATURATION LINE

GPOLYRL (A 2 BY 5 ARRAY)

1700	1.01165	-2.20523E-4	-1.92252E-8	5.63797E-12
2508	1.01165	-2.20523E-4	-1.92252E-8	5.63797E-12

GPOLYZV (A 4 BY 6 ARRAY)

334.5	0.000000000	0.00000E-0	0.0000E-0	0.00000000E-00	0.0000000E-00
900.0	-.0450205874	3.11010E-4	-6.6703E-7	4.62373333E-10	-1.3446975E-13
1400.	-.0901803003	4.48651E-4	-7.9053E-7	4.74670000E-10	-1.1564100E-13
1700.	-1.903059930	.581003E-2	-6.7302E-6	3.39640000E-09	-6.5409500E-13

POLYXK (A 2 BY 5 ARRAY)

0.0	.569387000	.578911000	0.0	1.83755000
0.0	1.38357611	-.332589774	0.0	-.183511941

GPOLYTRL (A 7 BY 7 ARRAY)

.301455	2563.13595	662.140362	-17005.9531	101550.625	-234815.183	176699.101
.411945	2859.68111	-4223.49780	15366.6252	-6280.30208	-54281.0898	55186.0031
.511440	3455.75201	-11557.5490	51554.7343	-95791.6016	56706.4040	0.00000000
.562477	1739.94994	1723.39144	12986.6914	-45988.5547	32577.7246	0.00000000
.608899	-3762.78167	40771.5911	-90956.7305	77025.0833	-22033.7812	0.00000000
.858158	5349.14987	-9979.99560	12080.4962	-12960.9666	7607.69965	-2043.63682
1.00000	3998.76770	-2600.66431	-4007.10352	4523.36198	-1861.71680	0.00000000

GPOLYTRV (A 9 BY 7 ARRAY)

1.26902	5323.08321	-11256.8151	17677.4948	-13640.0485	5199.460230	-795.174780E-0
1.53120	3428.29668	-3706.99393	5620.65881	-3994.07170	1333.503540	-174.257748E-0
2.01090	2128.00063	527.589404	89.5214844	-372.104848	144.5267260	-17.7426773E-0
3.10486	1519.53837	2081.02670	-1502.95190	447.035905	-66.83475010	4.13665509E-0
4.01990	5143.27072	-3479.16030	1906.49344	-597.342204	92.98185350	-5.63748455E-0
5.34602	3240.06391	-73.0786658	-326.292465	97.8306290	-11.66503380	.516808534E-0
8.69186	4493.34350	-1441.45467	262.336910	-27.3442416	1.528652850	-.356319302E-1
14.3515	3127.98872	-644.436160	73.9352739	-4.81667613	.1674014380	-.241660272E-2
25.0000	2023.48317	-251.198836	17.1329955	-.659952561	.0135161901	-1.14688956E-4

PRESSURE & PRESSURE DERIVATIVES

GPOLYCS (A 2 BY 4 ARRAY)

1132	272.354127	-.05360700	1.44001767E-8
2508	247.576000	-.00982939	-1.93220000E-5

GPOLYPTV (A 4 BY 5 ARRAY)

1100	.904096	2.87341E-4	-7.28024E-8	2.02252E-11
1820	1.25073	-4.71509E-4	4.50434E-7	-8.87277E-11
2050	3.43495	-.769492E-2	6.39912E-6	-1.53883E-09
2508	-25.6077	.358686E-1	-1.53757E-5	2.08806E-09

POLYPRL (A 1 BY 5 ARRAY)

1.0	-2.49586	4.09230	-9.91952	13.3423
-----	----------	---------	----------	---------

POLYPRV (A 1 BY 5 ARRAY)

1.0	-1.38740	117.942	-905.726	3852.37
-----	----------	---------	----------	---------

CORRECTURES IN THE OVERHEATED VAPOR

GPOLYET (A 3 BY 5 ARRAY)

.071	0.00000	-294.391	-13180.0	0.00000
.150	196.585	-6344.31	38950.6	-83405.9
.320	-301.692	1272.06	2943.64	-34227.1

HEAT CAPACITY ON THE BASELINE

POLYCF (A 4 BY 3 ARRAY)

4.50751E-0	-.230472E-2	1.27973E-6
1.00626E-0	2.01370E-5	3.11600E-8
-.230472E-2	2.55946E-6	0.00000E-0
2.01370E-5	6.23200E-8	0.00000E-0

INTERNAL ENERGY ON THE BASELINE

GPOLYUF (A 5 BY 7 ARRAY)

.0191387560	10229.5008	-2042.22400	-78065.9744	2511177.29	-81006320.0	1178874458
.0929027113	10341.6536	-2757.89452	-38538.2315	215007.718	-1081769.23	2663207.91
.2224880380	10671.3230	-4067.16847	-23542.9909	53789.1415	-89762.0622	72579.1811
.4816586920	11329.9662	-5991.86879	-15291.0697	21486.9458	-18323.5565	5469.52667
.8520733650	12817.7740	-9397.15227	-8580.38304	9658.02015	-6751.58703	723.761257

INTERNAL ENERGY IN THE COMPRESSED LIQUID

GPOLYURL (A 6 BY 6 ARRAY)

.286917407	7303.54338	-91015.9670	442159.710	-992034.292	825340.942
.389617480	3546.06960	-37671.6942	157398.581	-314788.431	219976.266
.498812327	-813.467390	6372.82607	-10001.2425	-31088.4033	39071.9634
.599429782	-10999.8692	87965.4140	-255580.101	298088.192	-126724.093
.724234849	3813.43278	-13662.7562	5166.72253	1515.89299	-533.043530
.927710140	8025.26857	-35849.9885	49028.2505	-37057.6243	12202.9296

INTERNAL ENERGY IN THE OVERHEATED VAPOR

GPOLYDU (A 6 BY 6 ARRAY)

760	5620.92287	-5.92925212	.00621749218	-2.54397356E-6	0
1310	5768.99650	-6.13159176	.00597711442	-2.21305212E-6	0
1700	-6470.66401	21.5484023	-.01494472670	3.07277200E-6	0
2390	8747.11044	-5.93019371	.00160164707	-2.49691696E-7	0
2502	316806.230	-386.257521	.15808858100	-2.17078315E-5	0
2508	-1886953.82	1510.57451	-.30231276100	0.00000000E-0	0

TRANSPORT PROPERTIES

THERMAL CONDUCTIVITY

GPOLYQTL (A 3 BY 5 ARRAY)

1280.	21.9594000	-.01289934000	2.34558000E-6	0.00000000E-00
1490.	15.5088839	-.00120754936	-4.11105759E-6	9.83993945E-10
1700.	16.3302297	-.00285554124	-3.00788635E-6	7.37621266E-10

GPOLYQTV (A 4 BY 5 ARRAY)

1280.	.004098520	5.39050000E-6	0.00000000E-0	0.00000000E-00
1440.	-.250479043	6.01102017E-4	-4.64673101E-7	1.20824384E-10
1590.	-.317471512	7.39894286E-4	-5.60566367E-7	1.42919535E-10
1700.	-.460201694	.100943007E-2	-7.30280263E-7	1.78549779E-10

POLYXQ (A 2 BY 5 ARRAY)

0.0	8.97741456	11.853935500	0.0	-9.25661142
0.0	1.28153149	-.00866046586	0.0	-.498362427

VISCOSITY

POLYYTL (A 3 COMPONENT ARRAY)

1.50564664	234.655	-.42961
------------	---------	---------

GPOLYYTV (A 2 BY 5 ARRAY)

1950	.829964508E-1	2.02065996E-4	0.0	0.0
2508	.829964508E-1	2.02065996E-4	0.0	0.0

POLYXY (A 2 BY 5 ARRAY)

0.0	-3.13975436	7.55526637	0.0	46.4387195
0.0	3.06452902	-6.48697910	0.0	14.9746177

D.8.3 SUPERCRITICAL STATE

GPOLYZW (A 9 BY 6 ARRAY)

.050	6.0334199	-266.45944	9335.6518	-137699.05	774431.000
.200	1.9237665	57.225193	-538.34256	2037.0960	-2865.18970
.525	5.0795444	-10.290233	15.905257	-14.822874	5.64031320
.850	4.4720136	-6.2228329	5.7459901	-3.6235670	1.05586720
1.10	3.9783644	-3.7809708	1.2089292	.12780685	-.108332270
1.80	4.2712852	-4.7468967	2.3970671	-.51780325	.022297722
2.50	4.5044220	-5.7703931	3.7061770	-1.1864015	.142987750
3.20	-2.4280828	5.6205268	-3.4498913	.84693365	-.076921413
10.0	13.563572	-14.603827	6.1541777	-1.1831529	.084273184

GIPOLYZW (A 9 BY 7 ARRAY)

.050	-2.57418548	6.0334199	-133.229720	3111.88393	-34424.76250	154886.2000
.200	-2.53164888	1.9237665	28.6125965	-179.447520	509.2740000	-573.0379400
.525	-2.65345569	5.0795444	-5.14511650	5.30175233	-3.705718500	1.128062640
.850	-2.58114612	4.4720136	-3.11141645	1.91533003	-.9058917500	.2111734400
1.10	-2.50114066	3.9783644	-1.89048540	.402976400	.0319517125	-.0216664540
1.80	-2.57187230	4.2712852	-2.37344835	.799022367	-.1294508130	.0044595444
2.50	-2.57979880	4.5044220	-2.88519655	1.23539233	-.2966003750	.0285975500
3.20	.864339845	-2.4280828	2.81026340	-1.14996377	.2117334130	-.0153842826
10.0	-9.26239382	13.563572	-7.30191350	2.05139257	-.2957882250	.0168546368

GPOLYZY (A 9 BY 6 ARRAY)

.050	20.8058160	-927.02882	33402.567	-601791.79	3921787.20
.200	17.3320300	-283.31725	1932.0445	-6386.2301	8330.10560
.525	5.05625640	-46.151186	144.48637	-198.40566	102.658370
.850	-.020526223	-4.0296012	12.068890	-12.109581	3.97221040
1.10	7.98087010	-43.630698	83.885431	-68.887919	20.5251740
1.80	5.38963280	-14.130624	12.810811	-4.8987166	.697983620
2.50	3.81427660	-9.6062484	8.0301248	-2.6875239	.319326340
3.20	11.3124600	-15.374629	8.0516051	-1.8560865	.160536950
10.0	-10.1029610	12.780187	-5.8143280	1.1767982	-.088127741

GIPOLYZY (A 9 BY 7 ARRAY)

.050	-.775571008	20.8058160	-463.514410	11134.1890	-150447.948	784357.4400
.200	-.780979519	17.3320300	-141.658625	644.014833	-1596.55753	1666.021120
.525	-.250897552	5.05625640	-23.0755930	48.1621233	-49.6014150	20.53167400
.850	.245614338	-.020526223	-2.01480060	4.02296333	-3.02739525	.7944420800
1.10	-1.01038648	7.98087010	-21.8153490	27.9618103	-17.2219798	4.105034800
1.80	-1.50940062	5.38963280	-7.06531200	4.27027033	-1.22467915	.1395967240
2.50	-1.08165072	3.81427660	-4.80312420	2.67670827	-.671880975	.0638652680
3.20	-6.93094699	11.3124600	-7.68731450	2.68386837	-.464021625	.0321073900
10.0	6.08105981	-10.1029610	6.39009350	-1.93810933	.294199550	-.0176255482



Appendix E. Results of the examples

presented in “Using the code SODIUM.” on page 29.

Figure 46. Example 1 / 1.

```
MVS/XA DFP VER 2 LINKAGE EDITOR          18:53:03 SAT NOV 13, 1988
JOB INR105B1 STEP SODTES      PROCEDURE L
INVOCATION PARAMETERS - LIST
ACTUAL SIZE=(317440,86016)
      SYSPRINT DEFAULT BLOCKING USED  1 - 1
OUTPUT DATA SET SYS00317.T185304.RA000.INR105B1.G0SET IS ON VOLUME
** MAIN      DID NOT PREVIOUSLY EXIST BUT WAS ADDED AND HAS RMODE 31
** LOAD MODULE HAS RMODE ANY
** AUTHORIZATION CODE IS          0.
```


Figure 47. Example 1 / 2.

KRITISCHE DATEN :

TC = 2508.0 K
RHC = 230.00 KG/M**3
QTC = 5.0000 W/(M*K)
YTC = 0.65000E-04 M/(KG*S)
RGAS = 361.65 J/(KG*K)

EINHEITEN :

RH IN KG/M**3 ,
P IN J/M**3 ,
PT IN J/(M**3*K)
PR IN J/KG ,
CV IN J/(KG*K) ,
U IN J/KG ,
QT IN W/(M*K) ,
YT IN M/(KG*S) .

Figure 48. Example 1 / 3.

ISOTHERM-TEMPERATUR = 1500.000 K

RH *	P *	PT *	PR *	CV *	U *	QT *	YT *
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.37415	658355.	569.173	443914.	986.841	0.469760E+07	0.588790E-01	0.236914E-04
1.58233	749763.	668.825	434209.	1040.70	0.467652E+07	0.602768E-01	0.240255E-04
1.82206	852458.	793.767	422442.	1124.75	0.465236E+07	0.620658E-01	0.243719E-04
2.09810	967145.	945.523	408698.	1221.31	0.462398E+07	0.643649E-01	0.247310E-04
2.41596	0.109236E+07	5758.17	0.000000E+00	15387.3	0.458477E+07	0.675490E-01	0.250972E-04
2.78198	0.109236E+07	5758.17	0.000000E+00	13542.7	0.417390E+07	0.720104E-01	0.251538E-04
3.20345	0.109236E+07	5758.17	0.000000E+00	11940.9	0.381708E+07	0.741039E-01	0.252172E-04
3.68877	0.109236E+07	5758.17	0.000000E+00	10549.7	0.350720E+07	0.757746E-01	0.252884E-04
4.24762	0.109236E+07	5758.17	0.000000E+00	9341.65	0.323810E+07	0.772756E-01	0.253688E-04
4.89114	0.109236E+07	5758.17	0.000000E+00	8292.50	0.300440E+07	0.787023E-01	0.254594E-04
5.63214	0.109236E+07	5758.17	0.000000E+00	7381.38	0.280145E+07	0.801044E-01	0.255616E-04
6.48541	0.109236E+07	5758.17	0.000000E+00	6590.14	0.262520E+07	0.815132E-01	0.256770E-04
7.46795	0.109236E+07	5758.17	0.000000E+00	5903.00	0.247214E+07	0.829514E-01	0.258071E-04
8.59935	0.109236E+07	5758.17	0.000000E+00	5306.27	0.233921E+07	0.844374E-01	0.259538E-04
9.90215	0.109236E+07	5758.17	0.000000E+00	4788.04	0.222378E+07	0.859877E-01	0.261193E-04
11.4023	0.109236E+07	5758.17	0.000000E+00	4338.00	0.212353E+07	0.876179E-01	0.263057E-04
13.1298	0.109236E+07	5758.17	0.000000E+00	3947.17	0.203647E+07	0.893440E-01	0.265157E-04
15.1189	0.109236E+07	5758.17	0.000000E+00	3607.76	0.196087E+07	0.911826E-01	0.267521E-04
17.4095	0.109236E+07	5758.17	0.000000E+00	3313.00	0.189521E+07	0.931518E-01	0.270183E-04
20.0470	0.109236E+07	5758.17	0.000000E+00	3057.02	0.183819E+07	0.952716E-01	0.273179E-04
23.0841	0.109236E+07	5758.17	0.000000E+00	2834.73	0.178868E+07	0.975649E-01	0.276550E-04
26.5814	0.109236E+07	5758.17	0.000000E+00	2641.68	0.174567E+07	0.100058	0.280345E-04
30.6084	0.109236E+07	5758.17	0.000000E+00	2474.03	0.170833E+07	0.102781	0.284617E-04
35.2456	0.109236E+07	5758.17	0.000000E+00	2328.43	0.167590E+07	0.105769	0.289428E-04
40.5853	0.109236E+07	5758.17	0.000000E+00	2201.99	0.164774E+07	0.109067	0.294850E-04
46.7340	0.109236E+07	5758.17	0.000000E+00	2092.19	0.162328E+07	0.112725	0.300967E-04
53.8142	0.109236E+07	5758.17	0.000000E+00	1996.83	0.160204E+07	0.116806	0.307877E-04
61.9670	0.109236E+07	5758.17	0.000000E+00	1914.02	0.158359E+07	0.121388	0.315698E-04
71.3551	0.109236E+07	5758.17	0.000000E+00	1842.11	0.156757E+07	0.126568	0.324569E-04
82.1653	0.109236E+07	5758.17	0.000000E+00	1779.65	0.155366E+07	0.132469	0.334659E-04
94.6134	0.109236E+07	5758.17	0.000000E+00	1725.42	0.154158E+07	0.139250	0.346177E-04
108.947	0.109236E+07	5758.17	0.000000E+00	1678.32	0.153109E+07	0.147121	0.359381E-04
125.453	0.109236E+07	5758.17	0.000000E+00	1637.41	0.152197E+07	0.156358	0.374595E-04
144.459	0.109236E+07	5758.17	0.000000E+00	1601.89	0.151408E+07	0.167347	0.392235E-04
166.344	0.109236E+07	5758.17	0.000000E+00	1571.04	0.150719E+07	0.180624	0.412843E-04
191.546	0.109236E+07	5758.17	0.000000E+00	1544.25	0.150122E+07	0.196975	0.437137E-04
220.565	0.109236E+07	5758.17	0.000000E+00	1520.99	0.149604E+07	0.217589	0.466092E-04
253.980	0.109236E+07	5758.17	0.000000E+00	1500.78	0.149154E+07	0.244356	0.501068E-04
292.458	0.109236E+07	5758.17	0.000000E+00	1483.23	0.148763E+07	0.280475	0.544018E-04
336.766	0.109236E+07	5758.17	0.000000E+00	1468.00	0.148424E+07	0.331833	0.597854E-04
387.786	0.109236E+07	5758.17	0.000000E+00	1454.76	0.148129E+07	0.410555	0.667117E-04
446.535	0.109236E+07	5758.17	0.000000E+00	1443.27	0.147873E+07	0.546280	0.759300E-04
514.186	0.109236E+07	5758.17	0.000000E+00	1433.29	0.147651E+07	0.835617	0.88715E-04
592.085	0.109236E+07	5758.17	0.000000E+00	1424.63	0.147458E+07	1.87694	0.107850E-03
681.786	0.658288E+08	624332.	0.27861E+07	943.791	0.142467E+07	42.4586	0.136677E-03
785.076	0.460649E+09	867250.	0.504555E+07	943.791	0.125746E+07	58.3178	0.189597E-03
904.015	0.128841E+10	0.125258E+07	0.905130E+07	943.791	0.113212E+07	81.9997	0.465410E-03

*** S O D I U M *** PROGRAM RANGE EXCEEDED *** T = 1500.00 RH = 931.000 *****

DEFINITIONSBEREICH UEBERSCHRITTEN I
T = 1500.0 K RH = 931.00 KG/M**3

Figure 49. Example 2 / 1.

```

MVS/XA DFP VER 2 LINKAGE EDITOR          18:49:33 SAT NOV 13, 1988
JOB INR105B2 STEP S00TES          PROCEDURE L
INVOCATION PARAMETERS - LIST,MAP
ACTUAL SIZE=(317440,86016)
      SYSPRINT DEFAULT BLOCKING USED  1 - 1
OUTPUT DATA SET SYS88317.T184928.RA000.INR105B2.G0SET IS ON VOLUME
IEW0000 INCLUDE BIKT(SODIUM,CALDRA,DPCLIQ,DPCYAP,SODINV)
IEW0000 ENTRY S00TES
  
```

MODULE MAP

CONTROL SECTION			ENTRY							
NAME	ORIGIN	LENGTH	NAME	LOCATION	NAME	LOCATION	NAME	LOCATION	NAME	LOCATION
S00TES	00	9C0								
SODIUM	9C0	27A0								
CALDRA	3160	1822								
SATLIN	4C88	22D0								
TYDNRH	6F58	12B0								
ZUBER	8208	15EE								
PINT	97FB	6DE								
VISCON	9E08	1A06								
DPCLIQ	88E0	1A5C								
DU1DRAH	D340	71E								
DPCYAP	DA60	18CC								
HAMU	F330	CC6								
GEVONS	FFF8	37E								
DELCEY	10378	1734								
YALSE	11A80	1298								
DU2DT	12D48	D40								
SURFAC	13A88	7C54								
BORDUR	186E0	AF2								
DERAN	1C108	CA0								
SYXDUR	1CE78	F9C								
SVYDUR	1DE18	F66								
MAPROD	1ED80	36E								
FINTER	1F0F0	236								
SODINV	1F328	34EC								
JZLFXPD*	22B18	518								
			F*ADX0	22B18						
JZLDXPDF*	22D30	4								
JZLLEXP *	22D38	CA0								
			F*DEXP	22D38	DEXP	22D38	F*A2XD	22E28	F*DEXP2	22E28
			DEXP2	22E28	F*RTXD	22E66	F*DEXP10	22E66	DEXP10	22E66
JZLEXPL *	23908	4								
JZLLATN2*	239E0	410								
			F*DATAN	239E0	DATAN	239E0	DATAN2	23A88	F*DATAN2	23A88
JZLATN2L*	23DF0	4								
JZLLLOG *	23DF8	990								
			F*DLOG	23DF8	DLOG	23DF8	DLOG2	23EC2	F*DLOG2	23EC2
			F*DLOG10	23EDC	DLOG10	23EDC				
JZLLOGL *	24788	4								
JZLLSQRT*	24790	688								
			F*DSQRT	24790	DSQRT	24790				
JZLSQRTL*	24E18	4								
JZLACLS**	24E20	32C								

Figure 51. Example 2 / 3.

NAME	ORIGIN	LENGTH	NAME	LOCATION	NAME	LOCATION	NAME	LOCATION	NAME	LOCATION
JZLFCTI**	20500	20								
JZLFPSI**	205F0	FA4								
JZLEDTI**	2E598	374								
JZLEDTI *	2E910	12F4								
JZLGSVA**	2FC08	27C	JZLFSVA*	2F0A6						
JZLGSVA *	2FE88	4								
JZLGTLCR*	2FE90	160								
JZLGMTBL*	2FFFO	4								
JZLILCA**	2FFF8	108								
JZLILCA *	30100	280								
JZLMDSW**	30380	1C0								
JZLMDSW *	30570	4								
JZLXINIT*	30578	18								
JZLXINI *	30590	4								
JZLLCA* *	30598	13A8	JZLALCA*	30598						
JZLELCA**	31940	400								
JZLLCA *	31040	8								
JZLGLCA**	31048	8								
JZLGLCA *	31050	4								
DATDUR	31058	8								

ENTRY ADDRESS DD

TOTAL LENGTH 31060

- ** MAIN DID NOT PREVIOUSLY EXIST BUT WAS ADDED AND HAS RMODE 31
- ** LOAD MODULE HAS RMODE ANY
- ** AUTHORIZATION CODE IS 0.

Figure 52. Example 2 / 4.

KRITISCHE DATEN :

TC = 2508.0 K
RHC = 230.00 KG/M³
QTC = 5.0000 W/(M*K)
YTC = 0.65000E-04 M/(KG*S)
RGAS = 361.65 J/(KG*K)

EINHEITEN :

RH IN KG/M³ ,
P IN J/M³ ,
PT IN J/(M³*K)
PR IN J/KG ,
CV IN J/(KG*K) ,
U IN J/KG ,
QT IN W/(M*K) ,
YT IN M/(KG*S) .

Figure 53. Example 2 / 5.

ISOTHERM-TEMPERATUR = 3000.000 K

RH • 1.00000	P • 1.00000	PT • 1.00000	PR • 1.00000	CV • 1.00000	U • 1.00000	QT • 1.00000	YT • 1.00000
0.900000	971938.	341.198	D.107612E+07	771.501	D.608007E+07	D.561642E-01	D.227571E-04
1.03635	D.111861E+07	395.156	D.107532E+07	776.731	D.607148E+07	D.568949E-01	D.230577E-04
1.19336	D.128738E+07	457.886	D.107453E+07	782.607	D.606184E+07	D.577779E-01	D.233689E-04
1.37415	D.148158E+07	530.853	D.107377E+07	789.191	D.605105E+07	D.588790E-01	D.236914E-04
1.58233	D.170504E+07	615.773	D.107303E+07	796.544	D.603902E+07	D.602769E-01	D.240255E-04
1.82206	D.196219E+07	714.655	D.107233E+07	804.732	D.602565E+07	D.620660E-01	D.243719E-04
2.09810	D.225810E+07	829.856	D.107163E+07	813.818	D.601084E+07	D.643651E-01	D.247310E-04
2.41596	D.259861E+07	964.155	D.107088E+07	823.871	D.599449E+07	D.673079E-01	D.251034E-04
2.78198	D.299039E+07	1120.84	D.106990E+07	834.962	D.597649E+07	D.710633E-01	D.254895E-04
3.20345	D.344104E+07	1303.85	D.106845E+07	847.171	D.595670E+07	D.758255E-01	D.258895E-04
3.68877	D.395903E+07	1517.91	D.106605E+07	860.593	D.593499E+07	D.818206E-01	D.263038E-04
4.24762	D.455373E+07	1768.75	D.106199E+07	875.337	D.591114E+07	D.893218E-01	D.267324E-04
4.89114	D.523508E+07	2063.27	D.105517E+07	891.539	D.588491E+07	D.986359E-01	D.271751E-04
5.63214	D.601307E+07	2409.57	D.104405E+07	909.351	D.585595E+07	D.110114	D.276313E-04
6.48541	D.689682E+07	2816.55	D.102653E+07	928.923	D.582387E+07	D.124148	D.281002E-04
7.46795	D.789295E+07	3292.66	D.100000E+07	950.353	D.578827E+07	D.141179	D.285806E-04
8.59935	D.900336E+07	3843.28	961591.	973.587	D.574880E+07	D.161701	D.290719E-04
9.90215	D.102226E+08	4467.74	908730.	998.302	D.570540E+07	D.186269	D.295726E-04
11.4023	D.115352E+08	5162.72	840359.	1023.85	D.565828E+07	D.215513	D.300818E-04
13.1298	D.129541E+08	5944.66	791956.	1049.59	D.560760E+07	D.250152	D.305983E-04
15.1189	D.144637E+08	6811.95	726531.	1075.04	D.555350E+07	D.280995	D.311209E-04
17.4095	D.160471E+08	7763.45	657373.	1099.51	D.549626E+07	D.338963	D.316446E-04
20.0470	D.176870E+08	8797.39	588022.	1122.33	D.543626E+07	D.395099	D.321582E-04
23.0841	D.193696E+08	9913.61	522503.	1142.80	D.537390E+07	D.460592	D.326525E-04
26.5814	D.210898E+08	11116.0	463918.	1160.33	D.530965E+07	D.536786	D.331232E-04
30.6084	D.228497E+08	12413.0	412200.	1174.45	D.524393E+07	D.625215	D.335719E-04
35.2456	D.246435E+08	13811.6	362311.	1184.81	D.517714E+07	D.727621	D.340077E-04
40.5853	D.264321E+08	15305.4	308213.	1191.10	D.510964E+07	D.845989	D.344485E-04
46.7340	D.281749E+08	16886.7	278748.	1192.97	D.504182E+07	D.982582	D.349232E-04
53.8142	D.299771E+08	18658.7	232871.	1190.40	D.497376E+07	1.13998	D.354722E-04
61.9670	D.317178E+08	20535.0	196765.	1183.35	D.490565E+07	1.32115	D.361495E-04
71.3551	D.334347E+08	22589.6	171318.	1171.67	D.483770E+07	1.52946	D.370231E-04
82.1653	D.351890E+08	24943.2	154814.	1155.51	D.476978E+07	1.76882	D.381753E-04
94.6134	D.370373E+08	27733.6	142729.	1135.33	D.470139E+07	2.04369	D.397021E-04
108.947	D.389997E+08	31066.5	131731.	1111.61	D.463193E+07	2.35926	D.417118E-04
125.453	D.411450E+08	35124.8	129818.	1084.84	D.456081E+07	2.72151	D.443214E-04
144.459	D.436258E+08	40166.0	130959.	1055.83	D.448722E+07	3.13738	D.476512E-04
166.344	D.464734E+08	46269.2	128232.	1025.43	D.441054E+07	3.61498	D.518164E-04
191.546	D.495566E+08	53138.3	113635.	994.235	D.433082E+07	4.16381	D.569138E-04
220.565	D.522431E+08	59571.6	62151.3	961.889	D.424952E+07	4.79545	D.630065E-04
253.980	D.546214E+08	65651.7	115479.	926.589	D.416999E+07	5.52604	D.700566E-04
292.458	D.614632E+08	79378.0	246690.	892.131	D.408860E+07	6.44708	D.777497E-04
336.786	D.766657E+08	105596.	450813.	866.601	D.399646E+07	7.77829	D.851968E-04
387.786	D.108202E+09	149261.	820879.	858.386	D.388486E+07	9.92166	D.911129E-04
446.535	D.175489E+09	214082.	D.150266E+07	875.008	D.374892E+07	13.5753	D.950015E-04
514.186	D.310235E+09	295962.	D.250171E+07	920.008	D.359424E+07	19.8288	D.101203E-03
592.085	D.550061E+09	387353.	D.375072E+07	990.375	D.344022E+07	29.6788	D.113654E-03
681.786	D.972061E+09	511333.	D.570121E+07	1084.82	D.330782E+07	42.4586	D.136677E-03
785.076	D.171485E+10	692271.	D.896834E+07	1211.18	D.321485E+07	58.3178	D.189597E-03
900.000	D.303689E+10	964131.	D.140522E+08	1377.43	D.319020E+07	81.0904	D.440342E-03