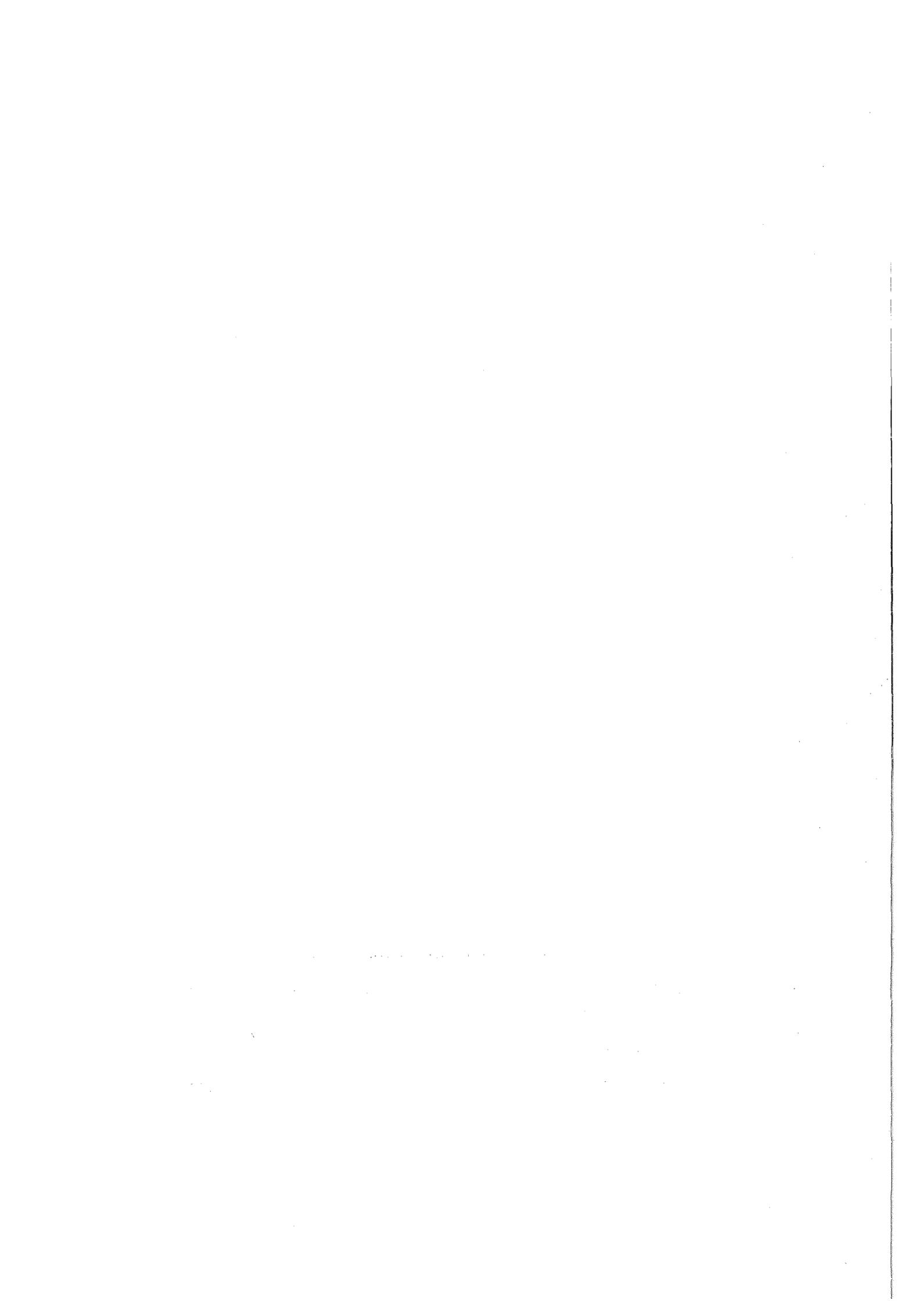


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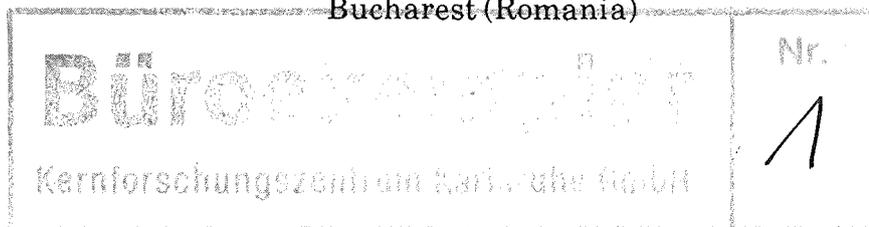
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THE EXTENDED SUM - RULE MODEL VIEW OF LIGHT AND  
INTERMEDIATE MASS FRAGMENT EMISSION IN  
NUCLEAR REACTIONS AT INTERMEDIATE ENERGIES

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## ABSTRACT

The original sum-rule model worked out by Wilczyński et al. and successfully used for a global description of complete and incomplete fusion reactions has been extended by a term accounting for dissipative processes of the dinuclear system on its way to fusion. When applying to light and heavy ion collisions with various targets at energies in the transitional region, the new term proves to be rather essential for reproducing the element distributions of the fragments emitted from rather asymmetric systems.

## DIE EMISSION LEICHTER UND MITTELSCHWERER FRAGMENTE IN KERNREAKTIONEN BEI MITTLEREN ENERGIEN AUS DER SICHT DES ERWEITERTEN SUMMENREGEL - MODELLS

Das ursprüngliche Summenregel-Modell von Wilczyński et al. zur Beschreibung der Emission von leichten und mittelschweren Fragmenten bei Stößen leichter und schwerer Ionen mittlerer Energie wurde erweitert durch einen Term, der das dynamische Verhalten des kurzlebigen dinuklearen Systems in Rechnung stellt und abhängig ist vom kritischen Bahndrehimpuls für eine Fusion unter Dissipation. Dieser Term erweist sich als wichtig, um die Z-Verteilungen der Fragmente aus asymmetrischen Stoßsystemen zu beschreiben. Die Charakteristika des Modells werden mit der Anwendung auf verschiedene experimentell untersuchte Reaktionen demonstriert.

## 1. Introduction

At intermediate energies both equilibrium and nonequilibrium reaction mechanisms appear to coexist for complex-fragment emission in light and heavy ion reactions. Their relative importance depends as much on the mass asymmetry of the entrance channel as on the bombarding energy. Additionally to fast quasifree and deep inelastic processes which are responsible for the fragment production, in particular in the vicinity of the target and projectile masses, near - equilibrium emission of heavy clusters from fusion - like processes has been found to be a most important source<sup>1-5</sup> which is considered as an interesting phenomenon with signatures of the properties of excited nuclear matter. However, the origin and detailed mechanisms of intermediate mass fragment (IMF) emission are still a matter of debate. A most interesting aspect arises from the question to which extent IMF emission is associated with the decay of a fully equilibrated compound nucleus, or whether the system prefers to re-separate into fragments before equilibration by some kind of dissipative binary reaction modes. Recently the sum-rule model for complete and incomplete fusion reactions as worked out by Wilczyński et al.<sup>6</sup> has been generalized<sup>7</sup> in order to account for additional competing processes as sources of complex ejectiles from nuclear collisions. The extended sum-rule model (ESM) adopts the view that the near-equilibrated component may arise with the dynamical evolution of the dinuclear system via partially equilibrated states on the way to fusion and through some type of a rather asymmetric *fast* or *quasi-fission* process: "*dissipative fragmentation*".

The present paper briefly describes the basis and the formalism of the extended sum-rule model and applies it to analyses of IMF emission in nuclear reactions, in particular of various asymmetric colliding systems like the case of collisions of 156 MeV  ${}^6\text{Li}$ <sup>8,9</sup>. We show that the sum-rule model leads to a consistent description of the element distributions and of the localization of the reaction in the angular momentum space.

## 2. Formalism of the sum-rule model

Certainly part of the observed cross section of light and intermediate fragment emission has to be attributed to incomplete fusion processes in the sense of massive transfers predominantly from the projectile to the target, signalled by fast projectile-like remnants of break up-fusion reactions in various partitions. Considering complete and incomplete fusion channels on equal footing the original sum-rule model has been worked out as a global description of the contributions of the different competing channels. Following the assumption of partial statistical equilibrium<sup>10</sup> of the strongly interacting dinuclear system the different channel (i) reaction probabilities are governed by the available phase space, as determined by the groundstate Q-values  $Q_{gg}$ , i.e by the scaling factor

$$P(i) \propto \exp \{ [ Q_{gg}(i) - Q_c(i) ] / T \} \quad (2.1)$$

with T being the effective (apparent) temperature.  $Q_c(i)$  is the change in the Coulomb interaction energy due to charge transfer (assumed to happen at a relative distance  $R_c = r_{oc} (A_1^{1/3} + A_2^{1/3})$  where the system is supposed to separate). Whether for a given partial wave a reaction channel is closed or open depends on the critical angular momentum ( $\ell_{crit}(i)$ ) above which a particular fragment cannot be captured. The entrance channel angular momentum limitation  $\ell_{lim}(i)$  follows the concept of the generalized angular momentum<sup>11</sup>. With the plausible assumption that the entrance channel angular momentum is shared between the ejectile and the remainder in the ratio of their reduced masses the critical angular momentum value  $\ell_{crit}(i)$  is related to  $\ell_{lim}(i)$  by

$$\ell_{lim}(i) = \frac{A_1}{a} \ell_{crit}(i) \text{ if the target } A_2 \text{ picks up the cluster } a \quad (2.2 a)$$

or

$$\ell_{lim}(i) = \frac{A_2}{b} \ell_{crit}(i) \text{ if the projectile } A_1 \text{ picks up the cluster } b \quad (2.2 b)$$

Actually, the limitation is expressed by a smooth transition of the channel transmission coefficients  $T_\ell(i)$  parametrized as

$$T_{\ell}^{(i)} = \left[ 1 + \exp\left( \frac{\ell - \ell_{lim}^{(i)}}{\Delta\ell} \right) \right]^{-1} \quad (2.3)$$

The original sum-rule model explicitly assumes that the total reaction cross section is fully exhausted by complete ( $i = 1$ ) and incomplete ( $i > 1$ ) fusion channels for entrance channel angular momenta up to a particular value  $\ell_{max}$ . Thus, using the unitarity condition

$$N_{\ell} \sum_i^n T_{\ell}^{(i)} P(i) = 1 \quad (2.4)$$

the angle integrated cross section for the channel  $i$  is given by

$$\sigma(i) = \pi\kappa^2 \sum_{\ell=0}^{\ell_{max}} (2\ell + 1) \frac{T_{\ell}^{(i)} P(i)}{\sum_j T_{\ell}^{(j)} P(j)} \quad (2.5)$$

The  $\ell$ -value which corresponds to the partial wave with its classical turning at the critical distance is adopted for  $\ell_{max}$ . Though the expression eq. 2.5 resembles strikingly the Hauser-Feschbach formula, it should be noted that the  $T_{\ell}^{(i)}$  are entrance channel transmission coefficients applying to the captured fragment rather than to the ejectile in the exit channel. Specifying the ingredients of the model, in particular the apparent temperature  $T$  and the critical angular momenta  $\ell_{crit}^{(i)}$  through an estimate based on the liquid-drop model, the model has been remarkably successful in predicting absolute cross sections as well as their localization in the  $\ell$ -space for reaction of 140 MeV  $^{14}\text{N}$  with  $^{159}\text{Tb}$ <sup>6</sup>.

With increasing projectile energies when complete and incomplete fusion modes appear to be reduced, IMF emission gets generally more pronounced. For such a situation Fig. 1 displays the result of an application of the original sum-rule model to collisions of 156 MeV  $^6\text{Li}$  ions with  $^{nat}\text{Ag}$ . Typically (see also ref. 7) the best fit to the measured data leads to an unreasonable value of the apparent temperature; it fails also to reproduce the observed  $Z$ -distribution, in particular by underestimating the emission of heavier products.

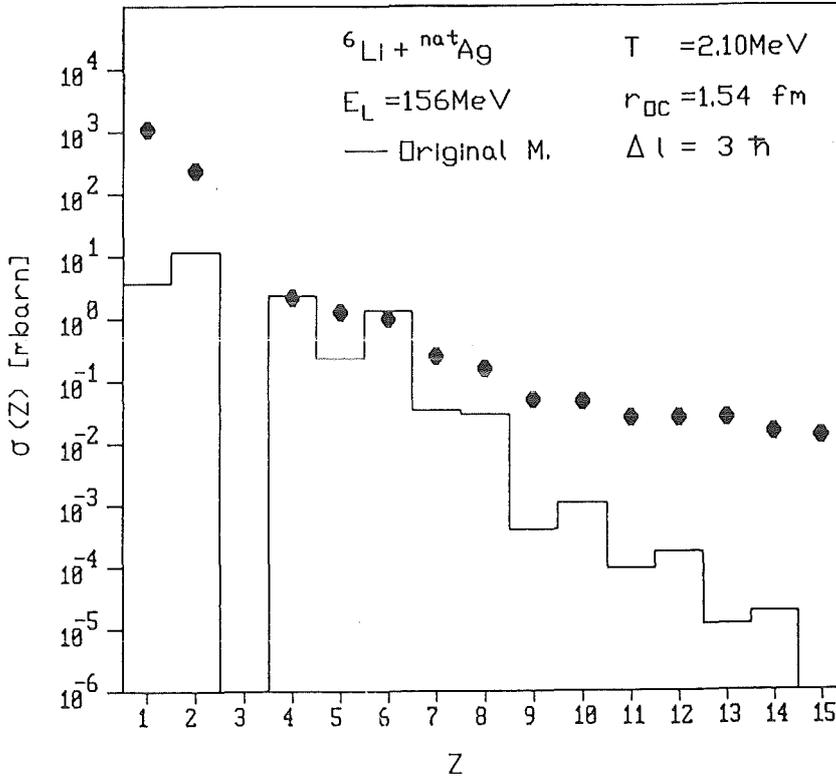


Fig. 1 Element distribution of light and intermediate mass fragment emission from collisions of 156 MeV  ${}^6\text{Li}$  ions with  $\text{natAg}$ <sup>9,12</sup> as compared with results of the analysis based on the original sum-rule model<sup>16</sup>.

As obvious in Fig. 1 already the original sum-rule predicts at higher energies the onset of a reverse mass flow as the phase space factors  $P(i)$  do not make any distinction between the mass flow in one or the other direction. However, in contrast to deep inelastic processes with dissipation of kinetic energy and orbital angular momentum, this reverse mass flow has signatures of quasi-elastic processes for which the sum-rule model predicts only minor contributions due to the large  $Q$ -values of "multinucleon-pickup" reactions. Nevertheless the localization around the grazing angular momentum does no more tolerate the simplification of a sharp cut-off at  $\ell = \ell_{\text{max}}$  in eq. 2.5.

The unitarity condition for the partial reaction cross section given by eq. 2.4 has to be modified to

$$N_\ell \sum_i T_\ell(i) P(i) = 1 - |S_\ell|^2 = K_\ell \quad (2.6)$$

where  $S_\ell$  are the scattering amplitudes which may be independently deduced from elastic scattering analysis. The general behaviour of  $S_\ell$  in cases of strong absorption guarantees a smooth transition of the transmission factor  $K_\ell$  from unity to zero (see also the formulation given in ref.13). Thus, eq. 2.5 is rewritten

$$\sigma(i) = \pi\lambda^2 \sum_{\ell=0}^{\infty} (2\ell + 1) K_\ell \frac{T_\ell(i) \cdot P(i)}{\sum_j T_\ell(j) \cdot P(j)} \quad (2.7)$$

While incomplete fusion apparently contributes to "nonequilibrium" components, the *extended* sum-rule model <sup>7,14</sup> regards the near-equilibrated IMF component to originate from cluster emission during the dissipative evolution of the dinuclear system before the partners have completely given up their individualities and collapse to a mononucleus without memory. Without further specification we associate IMF emission predominantly to a reaction mode intermediate between deep inelastic reactions and compound nucleus formation, say to rather asymmetric fast or quasi fission modes proceeding through partially equilibrated states : "*dissipative fragmentation*". Introducing corresponding transmission coefficients  $T'_\ell$  alters the normalization (eq. 2.4) to

$$N_\ell \left\{ \sum_{i=1}^n T_\ell(i) P(i) + \sum_{i=2}^n T'_\ell P(i) \right\} = K_\ell \quad (2.8)$$

For the dissipative processes under consideration it appears quite natural to assume that the corresponding transmission coefficients  $T'_\ell$  are limited by a critical  $\ell$ -value  $\ell_{cr}^{dyn}$  which includes the angular momentum dissipation<sup>15</sup> during the dynamical evolution of the system.

$$T'_\ell = \left\{ 1 + \exp \left[ (\ell - \ell_{cr}^{dyn}) / \Delta\ell \right] \right\}^{-1} \quad (2.9)$$

Thus, the cross section is expressed by a sum of two contributions

$$\sigma^{tot}(i) = \sigma(i) + \sigma'(i) \quad (2.10)$$

where

$$\sigma(i) = \pi\lambda^2 \sum_{\ell=0}^{\infty} (2\ell + 1) K_{\ell} \frac{T_{\ell}(i) P(i)}{\sum_{j=1}^n T_{\ell}(j) P(j) + \sum_{j=2}^n T'_{\ell} P(j)} \quad (2.11)$$

gives the complete fusion and the incomplete fusion ( $i = 2 \dots n$ ) contributions while

$$\sigma'(i) = \pi\lambda^2 \sum_{\ell=0}^{\infty} (2\ell + 1) K_{\ell} \frac{T'_{\ell} P(i)}{\sum_{j=1}^n T_{\ell}(j) P(j) + \sum_{j=2}^n T'_{\ell} P(j)} \quad (2.12)$$

represents the intermediate fragments emission by dissipative fragmentation of the dinuclear system feeding the exit channels  $i = 2, \dots n$ . For angular momenta less than  $\ell_{cr}^{dyn}$  dissipative fragmentation can be associated to phenomena similar to fast fission or quasi-fission processes while for  $\ell > \ell_{cr}^{dyn}$  contributions from deep inelastic collisions are expected to show up.

### 3. Application to analyses of Z-distributions

The phenomenological application of the model prescriptions implies the adjustment of three parameters : the apparent temperature  $T$ , the effective relative distance  $R_c = r_{0c} (A_1^{1/3} + A_2^{1/3})$  where the charge transfer takes place and which determines  $Q_c(i) = (Z_1^f Z_2^f - Z_1^i Z_2^i) e^2 / R_c$ , and the "diffuseness"  $\Delta\ell$  in the angular momentum space of the contributions around  $\ell_{lim}(i)$ . In addition the critical angular momenta  $\ell_{crit}(i)$  and  $\ell_{cr}^{dyn}$ , as well as the entrance transmission factor  $K_{\ell}$  or  $\ell_{max}$ , respectively, have to be specified on the basis of independent considerations.

- a. A reasonable estimate of the apparent temperature is provided by the well known relation

$$T = \sqrt{\frac{E^* c}{A}} \quad (3.1)$$

where  $E^*$  is the excitation energy and  $8 \leq c \leq 13$  (see ref. 16). As far as experimental Z-distributions are available, the phenomenological sum-rule analysis infers T from the parameter adjustments, but it is expected that the result does not significantly differ from the estimate of eq. 3.1.

- b. Through the exponential factors (eq. 2.1) the results can be considerably influenced by the particular choice of  $Q_c$  or  $R_c$ , respectively, and there appears also for the best-fit results a correlation between  $R_c$  and T (see ref. 7). Within some limits smaller values of  $R_c$  can be compensated by larger values of T, which is obvious from the structure of P(i). It is also possible that  $R_c$ , the distance where charge transfer takes place, is different for different types of processes. The cluster emission during the evolution of the dinuclear system may happen from rather deformed intermediate shapes. Some attempts following the suggestion <sup>17</sup> to use

$$R_c = 1.225 (A_1^{1/3} + A_2^{1/3}) + d$$

with d roughly simulating deformation effects and treated as free parameter did not lead to distinct differences from the choice

$$R_c = r_{0c} (A_1^{1/3} + A_2^{1/3}).$$

- c. The value of the critical angular momenta  $\ell_{\text{crit}}(i)$  limiting the formation of a compound nucleus in complete and incomplete fusion channels are calculated with a statical condition assuming that a given fragment can be captured only if it penetrates the region of attraction of the total nucleus-fragment potential <sup>18</sup>. The cluster emission from the dinuclear system on its way to fusion is supposed to depend on the critical angular momentum value  $\ell_{\text{cr}^{\text{dyn}}}$ , for fusion, which takes into account the angular momentum dissipation. The specification of  $\ell_{\text{cr}^{\text{dyn}}}$  is based on a dynamical model of fusion and follows the procedure of Ngô et al. <sup>15,19</sup>. The computer routines necessary for sum-rule analyses are compiled by the program LIMES <sup>20</sup>.

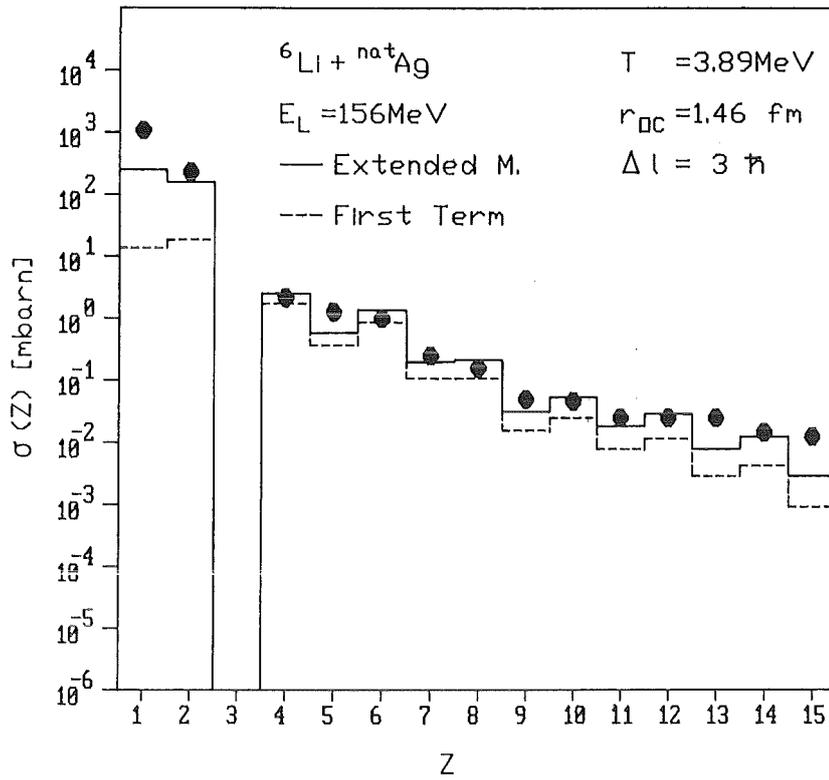


Fig. 2a Extended sum-rule analysis of IMF emission from the  ${}^6\text{Li} + \text{natAg}$  reaction at 156 MeV<sup>9</sup>. The dashed curve represents the contribution of the first term (eq. 2.10).

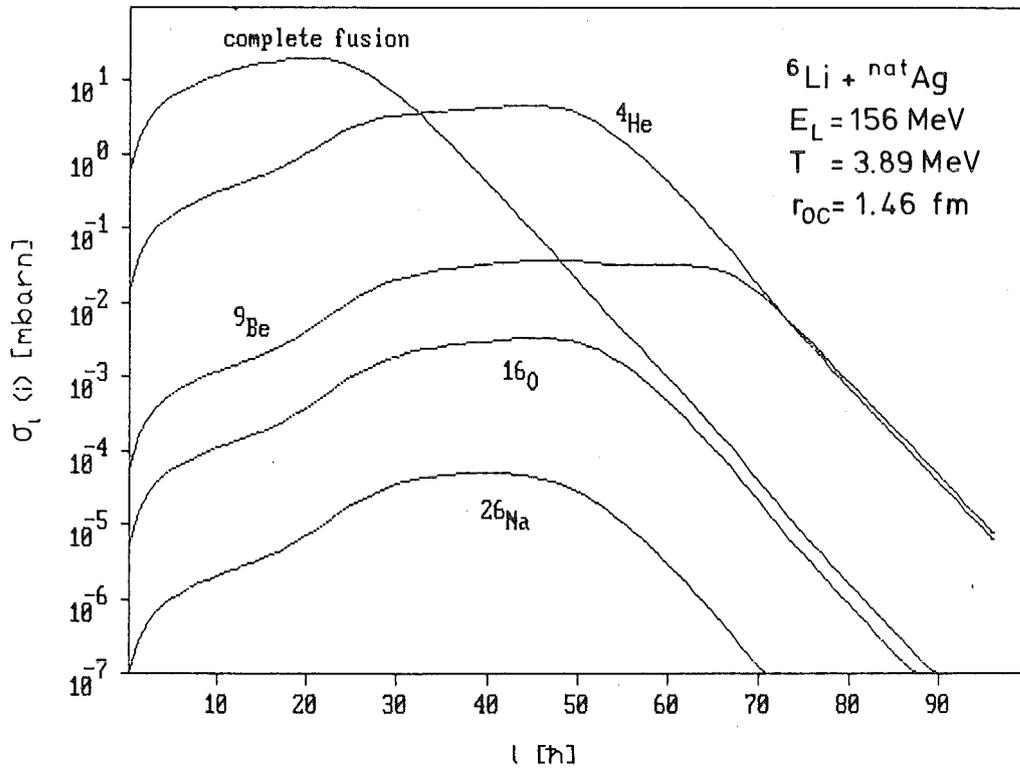


Fig. 2b The partial cross sections  $\sigma_l$  for the  ${}^6\text{Li} + \text{natAg}$  reaction at 156 MeV ( $\ell_{\text{cr}}^{\text{dyn}} = 51 \hbar$ )

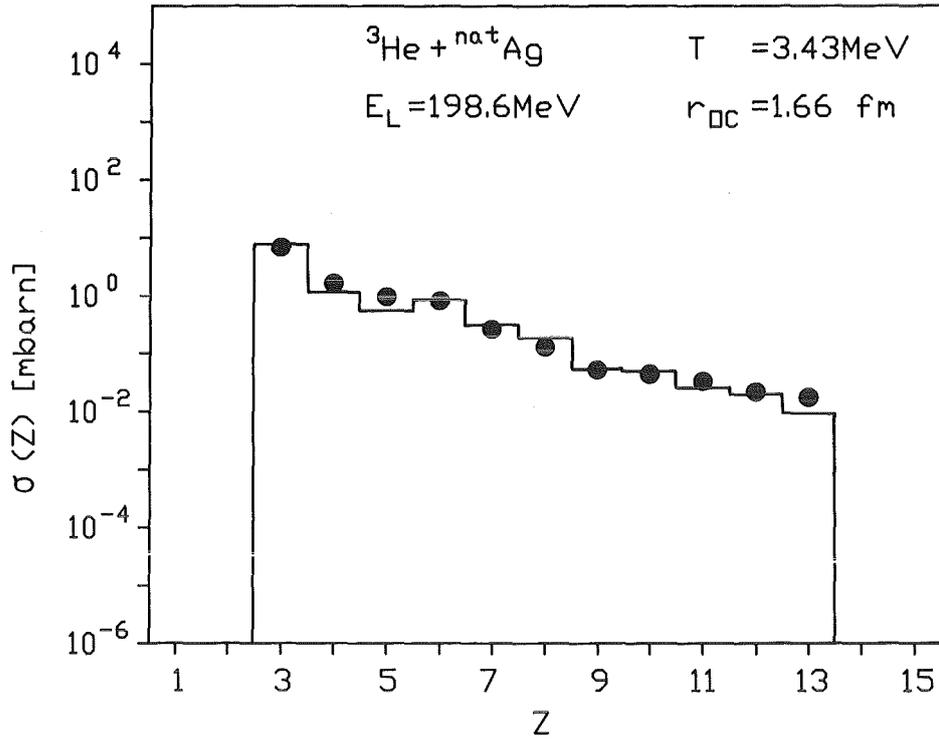


Fig. 3 Extended sum-rule analysis of IMF emission for collisions of 198.6 MeV  ${}^3\text{He}$  with  ${}^{\text{nat}}\text{Ag}$  <sup>21</sup>.

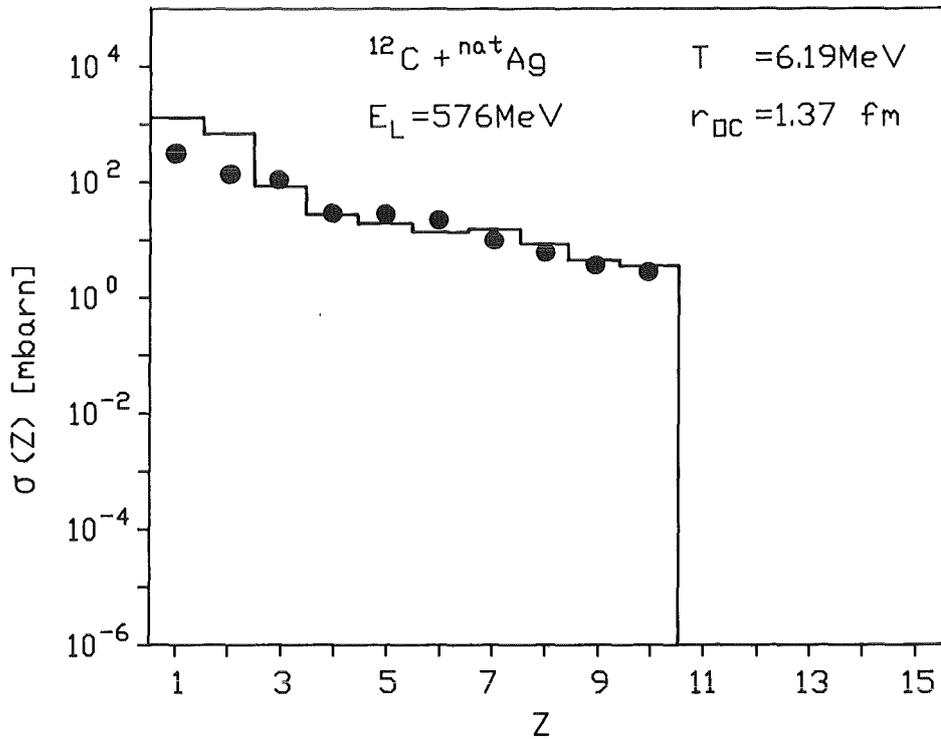


Fig. 4 Extended sum-rule analysis of IMF emission for the reaction  ${}^{12}\text{C} + {}^{\text{nat}}\text{Ag}$  at  $E/\text{amu} = 48\text{ MeV}$  <sup>22</sup>.

- d. The entrance transmission coefficient  $K_\ell = 1 - |S_\ell|^2$  may be derived by optical model or parametrized phase shift analyses of elastic scattering data, or more simply by introducing a smooth cut-off factor around  $\ell_{\max} \simeq \ell_{\text{grazing}}$  with an reasonable estimate of the transition width  $\Delta L$ .

Fig. 2a shows the result of the analysis of the experimental Z-distribution of the fragments emitted in collisions of 156 MeV  ${}^6\text{Li}$  ion with  $\text{natAg}$ <sup>9,12</sup>. In contrast to the result shown in Fig. 1 the calculations reproduce fairly well the experimental data, and the apparent temperature is consistent with the value estimated on the basis of eq. 3.1, as used for a multistep-evaporation analysis of the same data<sup>9</sup>. The corresponding partial cross sections  $\sigma_\ell$  calculated by a smooth cut-off entrance transmission factor  $K_\ell$  deduced from elastic scattering are given in Fig. 2b. The contribution at large  $\ell$ -values is due to the second term  $\sigma'$  (i) of eq. 2.10 which obviously explains the experimentally observed enhancement in the production of light fragments in forward direction and small energy dissipation (see also Fig. 7).

Fig. 3 displays the result for the data<sup>21</sup> of another very asymmetric case : 198.6 MeV  ${}^3\text{He} + \text{natAg}$ . The value of the apparent temperature is in reasonable agreement with that found by a multistep-evaporation model analysis<sup>9</sup>. The analysis of the element distribution observed<sup>22</sup> for  ${}^{12}\text{C}$  collisions with  $\text{natAg}$  at  $E/\text{amu} = 48$  MeV reproduces the increased apparent temperature expected for this incident energy (Fig. 4).

Fig. 5 shows additionally predictions of the Z-distributions from reactions of 104 MeV  $\alpha$ -particles with  $\text{natAg}$  and  ${}^{58}\text{Ni}$ . A value  $r_{0c} = 1.5$  fm and  $T$  corresponding to eq. 3.1 ( $c = 10$ ) have been adopted for the calculations.

#### 4. Entrance channel angular momentum windows

With the calculation of the element distribution  $\sigma(Z)$  the model predicts the partial cross sections  $\sigma_\ell(i)$ , i.e. the angular momentum localization of the various

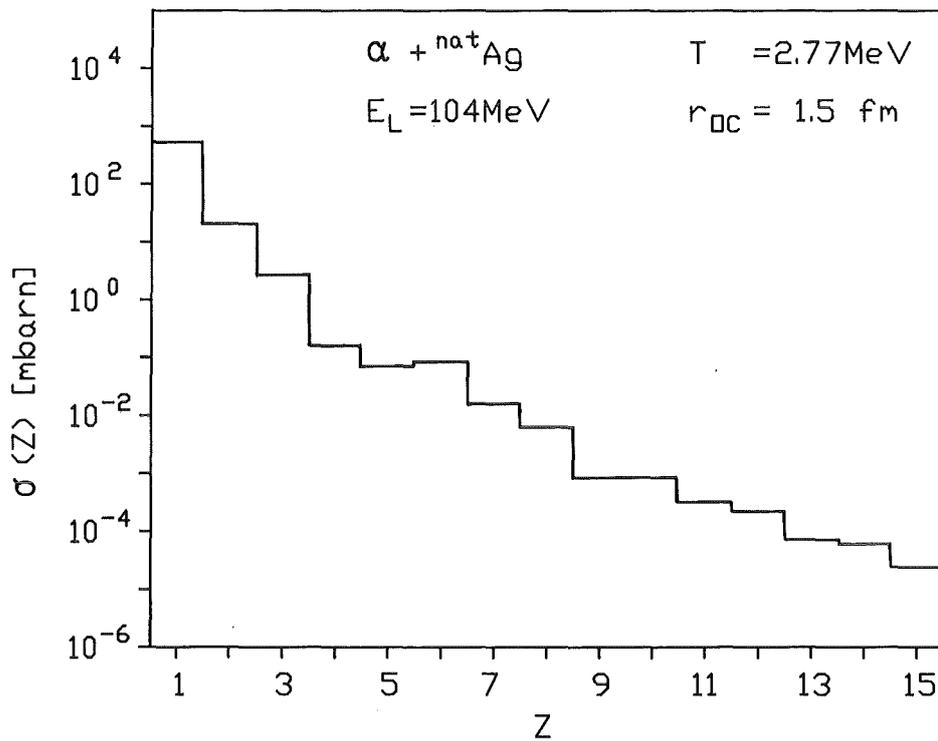
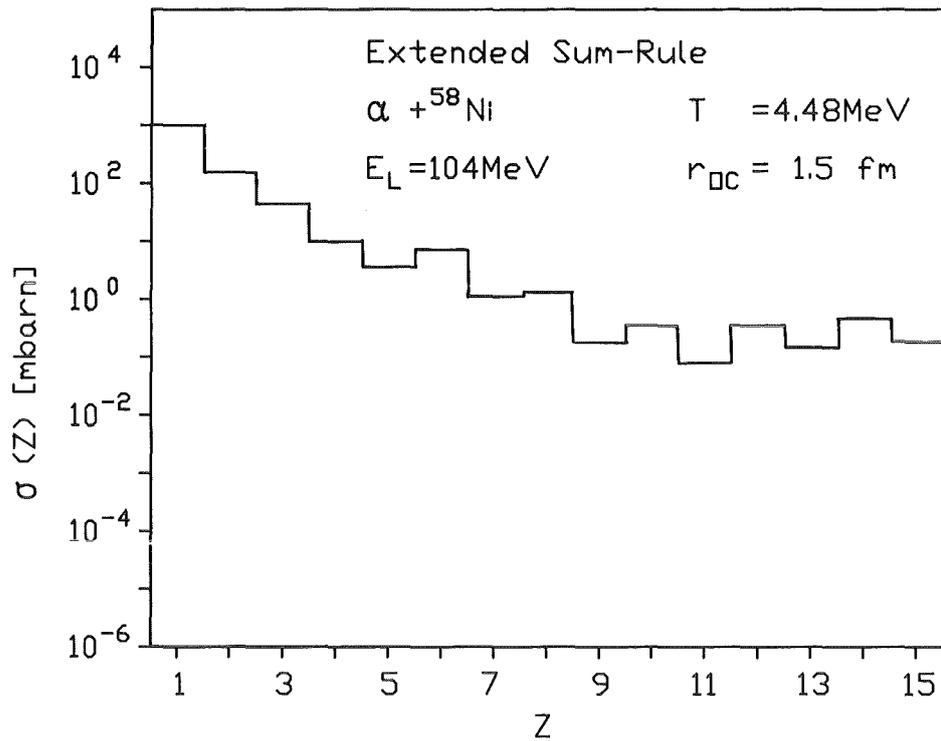


Fig. 5 Sum-rule predictions of the element distributions of IMF emission for  $\alpha$ -particle induced reactions at  $E_\alpha = 104\text{ MeV}$ .

reaction channels (Fig. 2b). When applying the ESM to IMF ( $3 \leq Z \leq 9$ ) emission to data measured<sup>23</sup> for the emission in the backward hemisphere in the 336 MeV  $^{40}\text{Ar} + \text{natAg}$  reactions (see Fig. 6), we may compare with independent information about the angular momentum windows, available from recent coincidence studies<sup>24</sup> of the same nuclear system at the same incident energy.

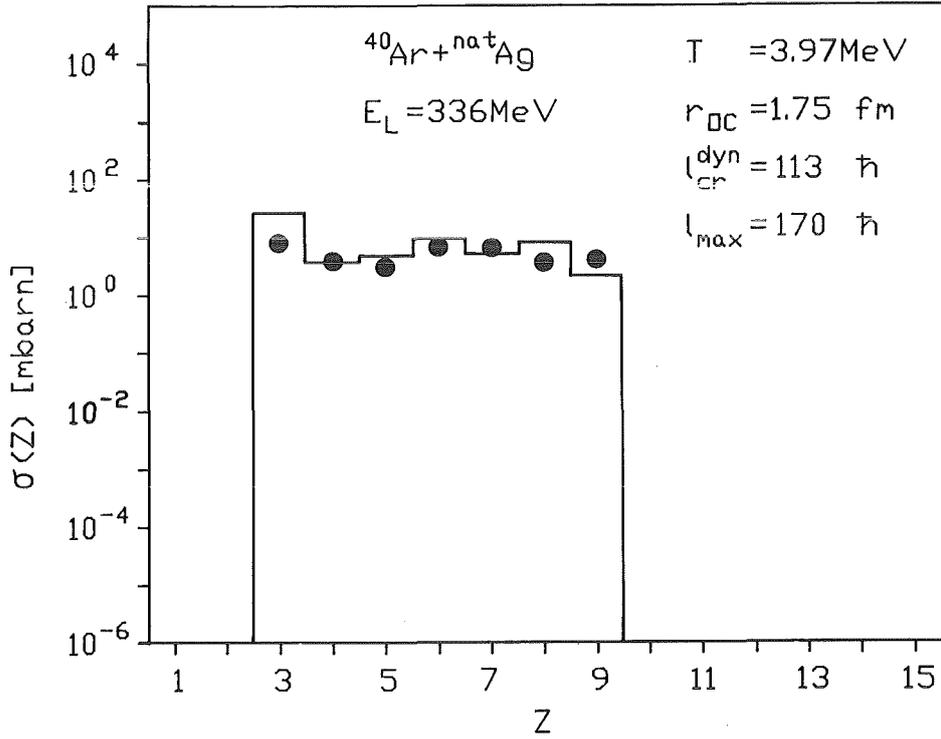


Fig. 6 Extended sum-rule description of  $\sigma(Z)$  of IMF emission from collision of 336 MeV  $^{40}\text{Ar}$  ions with  $\text{natAg}$ <sup>23</sup>.

The results shown in Figs. 7a and 7b demonstrate that the major part of IMF emission (in the backward angular region) has to be attributed to the second term of eq. 2.10. Obviously the fast fragments originating from incomplete fusion are fairly well concentrated in the angular momentum range with 60-100  $\hbar$  while dissipative fragmentation is found at larger  $\ell \approx 90 - 140 \hbar$ , i.e. in the region around  $\ell_{\text{cr}}^{\text{dyn}}$ . This finding is in reasonable agreement with the results of ref. 24 attributing the quasi-fission channel to  $\ell = 103 - 133 \hbar$ , e.g. The example may demonstrate the predictive power of the ESM though, of course, such a global model cannot be invoked for predictions of further details of the reaction

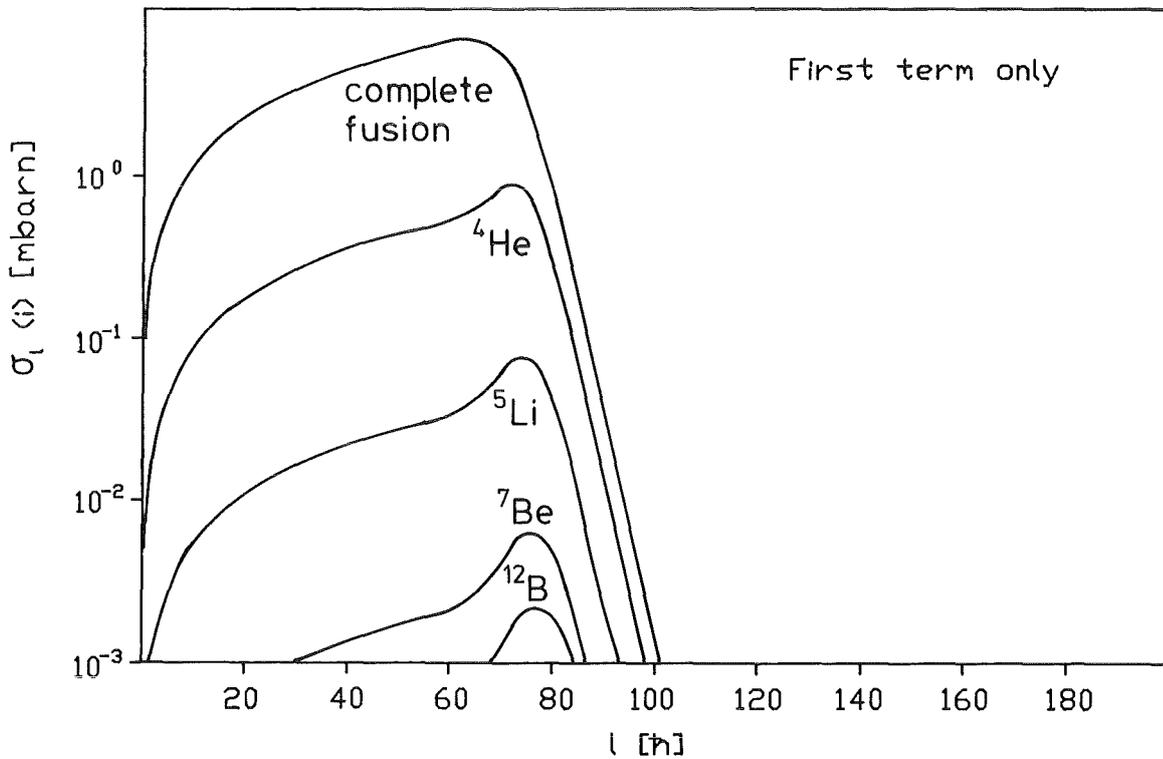
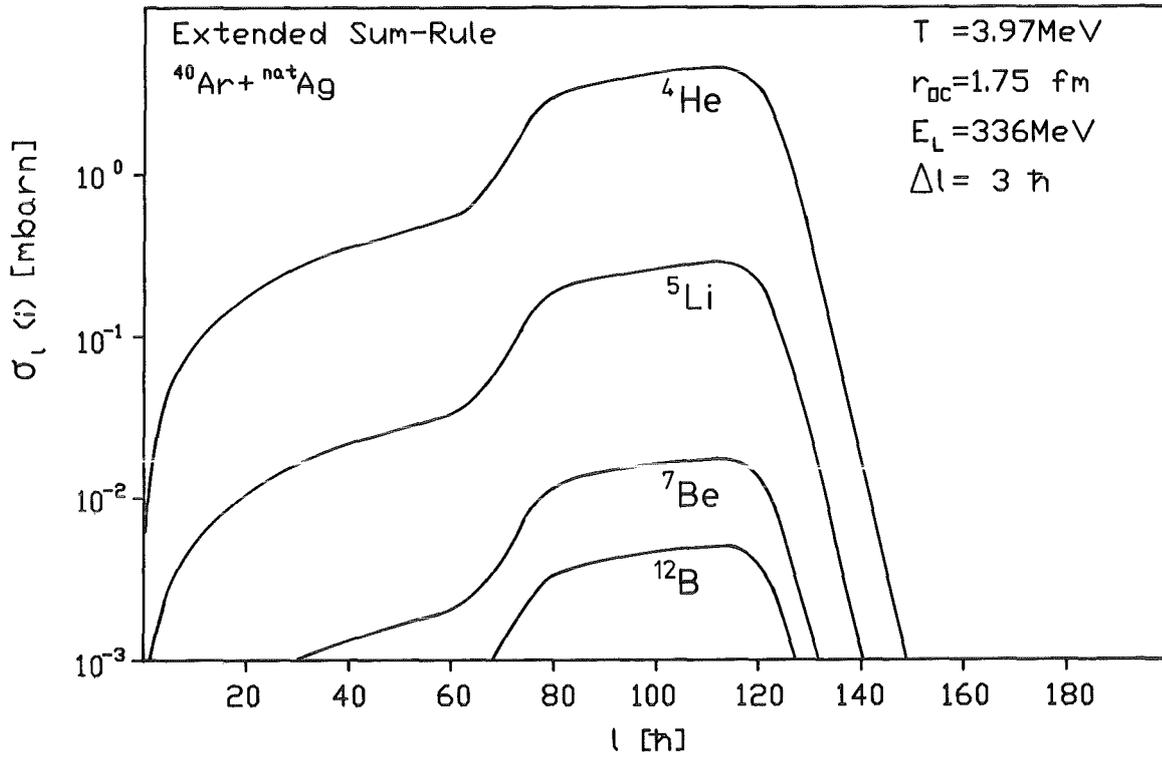


Fig. 7 Partial cross section  $\sigma_\ell$  for the emission of various complex fragments in 336 MeV  $^{40}\text{Ar} + \text{natAg}$  collisions: Prediction of the extended sum-rule model.

mechanism. Nevertheless the result suggests that the emission of IMF may be understood as arising during the dynamical evolution of the dinuclear system via partially equilibrated states, in a mode which is similar to a rather asymmetric fast or quasi-fission process.

## 5. Concluding remarks

Light and intermediate mass fragment emission is a quite general phenomenon in nuclear reactions. Though the details may depend in a rather complicated way on the specific properties of the particular system under consideration, the general features and overall tendencies, evident in results of inclusive experiments, are conspicuously similar and point to a common basic process and origin which should be accessible to a simple phenomenological description of the most prominent global observations. Generalizing the original sum-rule model <sup>6</sup> for complete and incomplete fusion processes, the extended sum-rule model, illustrated in the present paper, adopts the view that IMF emission preferentially originates from cluster emission during the dissipative evolution of the dinuclear system *before* complete equilibration. The ESM describes the nearly equilibrated component of IMF emission with entrance channel transmission coefficients limited by the critical value of the angular momentum for fusion with angular momentum dissipation taken into account. This view seems to be supported by a successful description of the element distributions (including light particle emission) and of the angular momentum localization, also implying identical shapes of the angular distributions of the heavier fragments. The sum-rule model is based on the very general assumption of partial statistical equilibrium and does not further specify the dynamics of the underlying process. Nevertheless we may envisage one of the variants of various dissipative processes <sup>25-28</sup>, say some type of rather asymmetric fast fission or (complete or incomplete <sup>27</sup>) deep inelastic processes <sup>28</sup>. A recent extension <sup>29</sup> of the random walk model for mass exchange reactions is guided by similar ideas.

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#### References

1. L.G. Sobotka, M.L. Padgett, G.J. Wozniak, G. Guariono, A.J. Pacheco, L.G. Moretto, Y. Chan, R.G. Stokstadt, I. Tserruya, and S. Wald, Phys. Rev. Lett. **51** (1983) 2187  
M.A. McMahan, L.G. Moretto, M.L. Padgett, G.J. Wozniak, L.G. Sobotka, and M.G. Mustafa, Phys. Lett. **54** (1985) 1995
2. B. Borderie, J. Phys. (Paris) **47** Colloq. (1986) C4-251
3. R. Charity, Nucl. Phys. **A471** (1987) 225c
4. L.G. Moretto and G.J. Wozniak, Nucl. Phys. **A488** (1988) 337c
5. R.J. Charity, D.R. Bowmann, Z.H. Liu, R.J. McDonald, M.A. McMahan, G.J. Wozniak, L.G. Moretto, S. Bradley, W.L. Kehoe, and A.C. Mignery, Nucl. Phys. **A476** (1988) 516

6. J. Wilczyński, K. Siwek-Wilczyńska, J. van Driel, S. Gonggrijp, D.C.J.M. Hageman, R.V.F. Janssens, J. Lukasiak, R.H. Siemssen, and S.Y. van der Werf, *Phys. Rev. Lett.* **45** (1980) 606; *Nucl. Phys.* **A373** (1982) 109
7. I.M. Brâncuș and H. Rebel, *Rev. Roum. Physique* **34** (1989) no. 10  
I.M. Brâncuș, KfK-Report 4453 (Oct. 1988) ISSN 0303-4003
8. T. Kozik, J. Buschmann, K. Grotowski, H.J. Gils, N. Heide, J. Kiener, H. Klewe-Nebenius, H. Rebel, S. Zagromski, A.J. Cole, and S. Micek, *Z. Phys.* **A326** (1987) 421
9. K. Grotowski, J. Ilnicki, T. Kozik, J. Lukasik, S. Micek, Z. Sosin A. Wieloch, N. Heide, H. Jelitto, J. Kiener, H. Rebel, S. Zagromski, and A.J. Cole, *Phys. Lett.* **B223** (1989) 287
10. J.P. Bondorf, F. Dickmann, D.M.E. Gross, and J.P. Siemens  
*J. Phy. (Paris) Colloq.* (1971) No. 6, 145
11. J.K. Siwek-Wilczyńska, E.M. du Marchie van Voorthuysen, J. van Popta, R.H. Siemssen, and J. Wilczyński, *Phys. Rev. Lett.* **42** (1979) 1599;  
*Nucl. Phys.* **A330** (1979) 150
12. J. Wentz, H. Rebel, V. Corcalciuc, H.J. Gils, N. Heide, H. Jelitto, J. Kiener, and I.M. Brâncuș, to be published
13. R.H. Siemssen, *Nucl. Phys.* **A400** (1983) 245c
14. H. Rebel, I.M. Brâncuș, A.J. Cole, K. Grotowski, and T. Kozik  
*Proc. Symp. Nucl. Phys.*, Dec. 27-31, 1988, Vol. **31a** (1988) 209
15. C. Ngô, *Prog. Part. Nucl. Phys.* **16** (1986) 139
16. G. Nebia, K. Hagel, D. Fabris, Z. Majka, J.B. Natowitz, R.P. Schmitt, B. Sterling, G. Mouchaty, G. Berkowitz, K. Strozewski, G. Vieste,

- P.L. Gouthier, B. Wilkin,, M.N. Namboodisi, and H. Ho,  
J. Phys. (Paris ) 47 Colloq. (1986) C4-385
17. M. Lefort, Prog. Part. Nucl. Phys. 4 (1980) 197
  18. J. Wilczyński, Nucl. Phys. A216 (1973) 386
  19. T. Suomijarvi, R. Lucas, C. Ngô, E. Tomasi, D. Dalili, and J. Matuszek,  
Nuov. Cim. 82A (1984) 51
  20. I.M. Brâncuș, J. Wentz, and H.U. Hohn, KfK-Report 4610 B (Sept. 1989)  
ISSN 0303-4003
  21. K. Kwiatowski, J. Bashkin, H. Karworski, M. Fatyga, and  
P.E. Viola, Phys. Lett. B171 (1986) 41  
K. Kwiatowski, Nucl. Phys. A471 (1987) 271c
  22. R. Trockel, GSI-87-17 Report (Sept. 1987) ISSN 0171-4546
  23. L.C. Vaz, D. Logan, J.M. Alexander, E. Dudek, D. Guerreau, L. Kowalski,  
M.F. Rivet, and M.S. Zisman, Z. Phys. A311 (1983) 89
  24. R. Lacey, N.N. Ajitanand, J.M. Alexander, D.M. de Castro Rizzo,  
G.F. Peaslee, L.C. Vaz, M. Kaplan, M. Kildir, G. La Rana, D.J. Moses,  
W.E. Parker, D. Logan, M.S. Zisman, P. De Young, and L. Kowalski,  
Phys. Rev. C37 (1988) 2540
  25. A. Olmi, Nucl. Phys. A471, 97c (1987)
  26. V.V. Volkov, 2nd Int. Conf. Nucleus Collision, Visby, Sweden,  
June 10-14, 1985, Vol. 1 p. 52, eds. B. Jacobson, and K. Aleklett 1985  
5th Int. Conf. on Clustering Aspects Kyoto, Japan, July 25-29, 1988
  27. T. Gazman Martinez and R. Reif, Nucl. Phys. A436 (1985) 294
  28. B. Borderie, M. Montoya, M.F. Rivet, D. Jouan, C. Cabot, H. Fuchs,  
D. Gardes, H. Gauvin, D. Jacquet, and F. Monet, Phys. Lett. B205 (1988)

29. Z. Sosin and H. Wilschut (to be published); KVI annual report 1988, p. 58;  
Z. Sosin, private communication (1989)
30. H.M. Xu, W.G. Lynch, C.K. Gelbke, M.B. Tsang, D.J. Fields, M.R. Maier,  
D.J. Monissey, T.K. Nayak, J. Pochodzalla, D.G. Sarantites, L.G. Sobotka,  
M.L. Halbert, and D.C. Hensley, Phys. Rev. C40 (1989) 186
31. R. Planeta, H. Klewe-Nebenius, J. Buschmann, H.J. Gils, H. Rebel, S.  
Zagromski, T. Kozik, L. Freindl, and K. Grotowski,  
Nucl. Phys. A448 (1986) 110
32. J. Brzychczyk, K. Grotowski, A. Panasiewicz, Z. Sosin, A. Wieloch,  
H.J. Gils, N. Heide, S. Münzel, and H. Rebel (in preparation)

## Appendix : Alternative and refined formulation of the ESM

The presented formulation of the ESM considers the formation of fully equilibrated compound nuclei and dissipative fragmentation of the dinuclear target-projectile system as two competing dissipative processes with the partial cross sections

$$\sigma_{\ell}^c(1) + \sum_{i=2}^n \sigma_{\ell}'(1 \rightarrow i) \quad . \quad (A1)$$

For sake of clarity, the notation has been slightly altered. Here  $\sigma_{\ell}^c(1)$  indicates the compound nucleus formation (production of evaporation residua) through *complete* fusion (assumed to be limited by  $\ell_{cr}$ ) while  $\sigma_{\ell}'(1 \rightarrow i)$  with  $i > 1$  are contributions of cluster emission from dissipative fragmentation of the completely fusing system. Obviously, the incomplete fusion channels are taken into account only by their "fast" products with the partial cross sections  $\sigma_{\ell}(i)$ , but ignoring the possibility<sup>30</sup> that partially fused systems may additionally feed the exit channels  $i > 1$  through dissipative fragmentation. The contribution to complex particle emission from a *particular incomplete fusion* channel  $k$  can be taken into account by

$$\sigma_{\ell}^c(k) + \sum_{i=2}^n \sigma_{\ell}'(k \rightarrow i) \quad . \quad (A2)$$

Since this contribution is sequential to the massive transfers accounted for by  $\sigma_{\ell}$ , the unitarity conditions (eq. 2.4, e.g.) stay to be correct.

Actually the production of completely equilibrated compound nuclei ( $\sigma_{\ell}^c(k)$ ) and dissipative fragmentation ( $\sigma_{\ell}'(k \rightarrow i)$ ) in incomplete fusion channels could provide interesting additional information which can be inferred from coincidence experiments<sup>31,32</sup>. The observed experimental element distributions  $\sigma(Z)$  possibly include such contributions.

Again invoking the concept of partial statistical equilibrium in the subsystems, the cross sections  $\sigma_{\ell}^c(k)$  and  $\sigma_{\ell}'(k \rightarrow i)$  are governed by the probability factors  $P_{ki}$  expressed in the form of eq. 2.1. In general, the  $P_{ki}$  values explicitly depend on the subsystem  $k$  and differ from  $P_{1i}$  ( $\equiv P(i)$  in eq. 2.1) for the complete system. Approximately, we may assume that the initially available thermal energy is shared between the binary reaction products in the ratio of their masses ("equal temperature"), but due to different  $Q_r$ -values and corrections  $Q_c$  when separating a particular fragment from the total or the partial system, respectively, we have to introduce an explicit dependence of the probability factors  $P$  on the subsystem  $k$ .

Specific studies of IMF emission from incompletely fusing systems would inform about the question whether the widely used "equal temperature" assumption is correct. In view of current experimental efforts<sup>32</sup> we give here a formulation of ESM which explicitly includes complex cluster emission from incomplete fusion channels.

In addition, we take a slightly changed view. We consider the formation of equilibrated compound nuclei and dissipative fragmentation as two competing processes of the dynamical evolution, of the system *after* an initial reaction step which we call in a rather general sense *complete* or *incomplete fusion*, respectively, as the case may be that the full system or only a part of it starts a further dissipative evolution. Thus, the first step is governed by ( $K_\ell = 1$ , for sake of simplicity)

$$N_\ell \sum_{i=1}^n T_\ell^{in}(i) P_{1i} = 1 \quad (\text{A3})$$

assuming that complete or incomplete fusion (understood in the generalized sense of an entry state) exhaust the partial reaction cross section up to the *dynamical* critical angular momentum. Though eq. A3 resembles to the unitarity condition of the original sum-rule model, the limitations of  $T_\ell^{in}$  in the angular momentum space are significantly different. The incomplete processes lead to particle emission into the exit channels  $i > 1$  through the first step with

$$\sigma_\ell(i) = \pi \lambda^2 (2\ell + 1) \frac{T_\ell^{in}(i) P_{1i}}{\sum_{j=1}^n T_\ell^{in}(j) P_{1j}} \quad (\text{A4})$$

Due to the sequential mechanism the normalization appears to be formally different from the denominator of eq. 2.11. But, in particular, as a consequence of the different meaning of  $T_\ell^{in}(1)$  (absorbing the formation of compound nuclei and dissipative fragmentation of the fusion path as well) it turns out that the  $\sigma_\ell(i)$  do not change significantly.

In the equilibration phase (second step) compound nucleus formation and dissipative fragmentation processes compete in all channels, and dissipative fragmentation of an incomplete fusion channel  $k$  may additionally feed all exit channels ( $i > 1$ ). This implies the relations

$$N_\ell^{(1)} \left[ T_\ell^{fin}(1) P_{11} + \sum_{i=2}^n T_\ell^{in}(i) P_{1i} \right] = N_\ell T_\ell^{in}(1) P_{11} \quad (\text{A5})$$

in the complete fusion channel, e.g. and

$$N_\ell^{(k)} \left[ T_\ell^{fin}(k) P_{kk} + \sum_{i=2}^n T_\ell'(k) P_{ki} \right] = N_\ell T_\ell^{in}(k) P_{1k} \quad (\text{A6})$$

for the incomplete fusion contribution with the normalization factors

$$N_\ell^{(k)} = \frac{T_\ell^{in}(k) P_{1k}}{\sum_j^n T_\ell^{in}(j) P_{1j}} \frac{1}{T_\ell^{fin}(k) P_{kk} + \sum_{j=2}^n T_\ell'(k) P_{kj}} \quad (\text{A7})$$

We note that the  $T_\ell'(k)$  depend on the particular channel due to different values of  $\ell_{\text{cr}}^{\text{dyn}}(k)$  of the various subsystems.

As the formation of fully equilibrated residua, accounted by the cross section

$$\sigma_\ell^c(k) = \pi \lambda^2 (2\ell + 1) N_\ell^{(k)} T_\ell^{fin}(k) P_{kk} \quad (k = 1, \dots, n) \quad (\text{A8})$$

may be differently limited (using a static value for  $\ell_{\text{cr}}$ ) in the angular momentum space, there is also a distinction between  $T_\ell^{\text{fin}}$  and  $T_\ell^{\text{in}}$ .

The contribution of the channel  $k$  to IMF emission into the exit channel ( $i > 1$ ) through dissipative fragmentations is represented by the cross sections

$$\sigma_\ell'(k \rightarrow i) = \pi \lambda^2 (2\ell + 1) N_\ell^{(k)} T_\ell'(k) P_{ki} \quad (\text{A9})$$

leading to a summed-up cross section

$$\sigma_\ell'(i) = \sigma_\ell'(1 \rightarrow i) + \sum_{k=2}^n \sigma_\ell'(k \rightarrow i) \quad (\text{A10})$$

The first term of eq. A10 just corresponds to the dissipative-fragmentation term in the ESM, but *renormalized for a sequential process*. The second term represents the contribution from dissipative fragmentation of the partially fused systems (via the first step correlated with the cluster emission given by  $\sigma_\ell$  (eq. A4)).

We note also that the formulation may include the extreme limit of the formation of excited systems with subsequent decay in various final channels. Some exploratory studies have been performed applying the two-step procedure to the case of 156 MeV  ${}^6\text{Li}$  collisions with  ${}^{\text{nat}}\text{Ag}$ , including additionally to  $\sigma_\ell(i)$  only  $\sigma_\ell'(1 \rightarrow i)$  i.e. the first term of eq. A10. If just omitting the  $T_\ell'$  in the normalization factor  $N_\ell$  corresponding to eq. 2.8, the fit to the experimental data tends to compensate this neglect by an unreasonably low value of the temperature. However, the proposed extension of the angular momentum limitation of the  $T_\ell(i)$  ( $\rightarrow T_\ell^{\text{in}}(i)$  in eq. A4) restores the ESM result given in Fig. 2. Here, the term with  $i = 1$  appears to be most efficient since the larger values of  $\ell_{\ell_{\text{im}}}(i)$  are anyway

cut-off by the limitation with  $\ell_{\max}$ . These findings may indicate a near-equivalence of the sequential formulation with the procedure given in sect. 2 as far as the contributions from incomplete fusion channels to IMF emission can be neglected.