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# **Application of Near Real Time Accountancy to Nuclear Material Balance Data**

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## **ABSTRACT**

The application of near real time accountancy to nuclear material balance data can be performed effectively only with the help of computerised nuclear material accounting and information systems. In this context, two computer programmes will be introduced in this report: DIDI, a programme for computing the MUF series and the measurement model of a reprocessing plant which is assumed to be a one-block model from data resulting from the routine operation of the facility, and PROSA, a programme for statistical analysis of NRTA data, which evaluates the MUF series on the basis of the measurement model. After the presentation of the two computer programmes two examples with realistic balance data will demonstrate the application of NRTA measures. Furthermore, some new remarks on the precision of Monte-Carlo simulations are mentioned which provide a substantial better estimation.

## **Anwendung von Near Real Time Accountancy auf Bilanzdaten von Kernmaterial**

### **ZUSAMMENFASSUNG**

Die effektive Anwendung von Near Real Time Accountancy auf Bilanzdaten von Kernmaterial kann nur mit Hilfe rechnergestützter Kernmaterialbuchhaltungs- und Informationssysteme erfolgen. In diesem Zusammenhang werden in diesem Bericht zwei Computer Programme vorgestellt: DIDI, ein Programm zur Berechnung der MUF Reihe und des Meßmodells einer "Ein-Block"-Wiederaufarbeitungsanlage mittels Daten aus dem Routinebetrieb der Anlage, und PROSA, ein Programm zur statistischen Auswertung von NRTA-Daten, das die MUF Reihe auf der Basis des Meßmodells bewertet. Nach der Vorstellung der beiden Computerprogramme demonstrieren zwei Beispiele mit realistischen Bilanzdaten die Anwendung der NRTA-Verfahren. Darüberhinaus werden einige Überlegungen zur Genauigkeit von Monte-Carlo-Simulationen dargelegt, die eine wesentlich bessere Abschätzung ermöglichen.

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## INTRODUCTION

Industrial facilities that handle valuable or hazardous materials are often forced to apply material control procedures that effectively detect losses of the special material under consideration. In many cases the material under consideration can only be measured with random errors that means statistical control procedures have to be considered. The statistical test procedures that are presented in this report have been developed to control Plutonium in nuclear facilities. The basic principle of material control is accountancy /1/, where material input, output and inventories are measured and the book inventory is compared with the measured inventory /2/. The requirement of early detection of possible material losses leads to a sequential evaluation of material balance data, where the boundary condition of a final decision at the end of a given reference time has to be observed. Truncated sequential test procedures based on the material accountancy statistic have been developed and applied to detect losses of material with sufficient high probability and in a timely manner /3,4/. Furthermore, the statistical tests are intended to be used by the plant operator for process control purposes.

The application of near real time accountancy to nuclear material balance data can be performed effectively only with the help of computerised nuclear material accounting and information systems. This is true for two reasons: Firstly, there is a big set of data which has to be worked up. Secondly, to perform the sequential test procedures we have to do a lot of mathematical computations, for example Monte-Carlo simulations to determine the thresholds of the tests. In this context, two computer programmes will be introduced in this report: DIDI /5/, a programme for computing the MUF series and the measurement model of a reprocessing plant which is assumed to be a one-block model from data resulting from the routine operation of the facility, and PROSA /6,7/, a programme for statistical analysis of NRTA data, which evaluates the MUF series on the basis of the measurement model.

After the presentation of the two computer programmes some remarks on the precision of Monte-Carlo simulations are mentioned. A new estimate is introduced which provides a substantial better estimation applicable in practice. Finally, two examples with realistic balance data will demonstrate the application of NRTA measures. But first of all, the materials balance concept and the multiple balance model will be introduced in a general manner.

## THE MATERIAL BALANCE CONCEPT

Let us consider a well-defined box /2/ that contains at a given time  $t_0$  some material into which material enters and from which material leaves during a given interval of time  $(t_0, t_N)$ . This box, which is also called material balance area, may represent, for example a nuclear material reprocessing plant. The material contained in the material balance area at time  $t_0$  is called the physical inventory  $I_0$ . The algebraic sum of the amounts of material that enter or leave the material balance area in the interval of time  $(t_0, t_N)$  - which in the case of an industrial plant are called receipts and shipments - is called the net transfer  $T$ . The physical inventory at  $t_0$  plus the net transfer in  $(t_0, t_N)$  gives the book inventory at  $t_N$ , i.e. the amount of material that should be contained in the balance area at time  $t_N$ :

$$B := I_0 + T .$$

The amount of material actually contained in the material balance area at time  $t_N$  is called physical inventory  $I_N$ .

If all material contained in, and passing through, the material balance area in the interval of time  $(t_0, t_N)$  is exactly accounted for, and if no material has disappeared or has been diverted, then the difference between book inventory  $B$  at time  $t_N$  and the physical inventory  $I_N$  must be zero. This difference is called MUF (*material unaccounted for*):

$$\text{MUF} := I_0 + T - I_N .$$

One application of materials accounting is to detect an unauthorized loss timely. However, the disposition of the material-balance concept as usual gives no possibility to detect an occurred loss soon, because the decision can be made even at the end of the reference time. For that reason one switches over to the concept of sequential balancing (Near-real-time accountancy, NRTA). In order to follow that idea, the reference-time period  $(t_0, t_N)$  is subdivided into  $N$  balance periods.

With the definitions

$$\begin{aligned} I_i &: \text{physical inventory at time } t_i, \quad i=0, \dots, N \\ T_i &: \text{net transfer during } (t_{i-1}, t_i), \quad i=1, \dots, N \end{aligned}$$

and

$$\text{MUF}_i := I_{i-1} + T_i - I_i$$

we obtain a sequence of MUF values  $(\text{MUF}_1, \dots, \text{MUF}_N)$ . At the end of the  $i$ -th balance period, statements can be made based on all  $i$  balance results  $\text{MUF}(j)$ ,  $j \leq i$ , whether there exists an eventual loss or not.



Thus, trends can be recognized at an early stage and losses may be detected timely related to their occurrence /8/, see Tab.1.

Tab.I: Material balance as usual compared to sequential balancing			
Material balance	Scheme	Decision is possible	Decision is based on
conventional	-----  $t_0$ $t_N$	only at the end of the reference time	only 1 balance (total balance)
sequential (NRTA)	- - .... - -  $t_0$ $t_i$ $t_N$	at the end of each balance period $t_i$	all balances up to the associated point of time

## MULTIPLE BALANCE MODEL

The concept of multiple balances is primarily used for detection of possible materials losses. One application is the international safeguards in nuclear facilities. The detection has to be timely and with sufficient high probability. The true MUF values are zero in the ideal situation of no losses and no measurement errors. In actual practice, however, nonzero MUF values may occur for a number of reasons, e.g. (a) measurement uncertainties and (b) loss of material. Measurement uncertainties will be included in our model by using the concept of random variables in determining the materials balances.

We assume that  $I(k)$  and  $T(k)$  are random variables that can be written as

$$I(k) = E(I(k)) + ZI(k) + SI(k) , k=0, \dots, N$$

with  $E(I(k))$  : true value of the inventory  
 $ZI(k)$  : random error of the measurement  
 $SI(k)$  : systematic error of the measurement

and analogous

$$T(k) = E(T(k)) + ZT(k) + ST(k) , k=1, \dots, N .$$

Another assumption is that all measurement errors are distributed normally with mean zero and with known variances. Furthermore, we assume that the inventory measurement and the transfer measurement are stochastically independent.

That means:

$$\begin{aligned} \text{cov}(I(k), T(k)) &= 0 \\ \text{cov}(ZI(i), SI(j)) &= 0 \\ \text{cov}(ZI(i), ST(j)) &= 0 \\ \text{cov}(ZT(i), ST(j)) &= 0 \\ \text{cov}(ZT(i), SI(j)) &= 0 \end{aligned}$$

The variance for period  $k$  can be calculated as

$$\begin{aligned} \text{var}(I(k)) &= \text{var}(ZI(k)) + \text{var}(SI(k)) && \text{and} \\ \text{var}(T(k)) &= \text{var}(ZT(k)) + \text{var}(ST(k)) . \end{aligned}$$

For two periods  $i$  and  $j$  we define the covariance of  $MUF(i)$  and  $MUF(j)$  as

$$\sigma_{ij} = \text{cov}(MUF(i), MUF(j)) .$$

All the variance and covariance calculations may be summarized into the variance-covariance matrix  $\Sigma$  also called dispersion matrix of the sequence MUF(1),MUF(2),...,MUF(N):

$$\Sigma = \begin{pmatrix} \sigma_{11} & \cdot & \cdot & \cdot & \sigma_{1N} \\ & & & & \\ & & & & \\ & & & & \\ \sigma_{N1} & \cdot & \cdot & \cdot & \sigma_{NN} \end{pmatrix}.$$

The matrix  $\Sigma$  is the condensed form of the measurement model of the facility considered. It is an essential part for the statistical analysis of the MUF sequence. Given a sequence of nonzero MUF values we have to decide on the basis of the measurement model whether the reason for nonzero values is due to measurement errors or loss. In our case we use the theory of mathematical statistical hypotheses testing to decide at the hand of a given sequence of MUF values whether the situation of no loss of material is given.

Loss of material may occur in a variety of pattern and we have to take into account that one has no knowledge of the actual loss situation.

We assume two hypotheses for the mean values of the random variables MUF(1),...,MUF(N). If there is no loss of material all materials balances have mean zero. This situation is described by the null hypothesis:

$$H_0: E(\text{MUF}(k)) = 0 \quad \text{for all periods } k=1,\dots,N \quad .$$

A loss of material can take place in one or more balance periods. Taking this into account, we formulate the alternative hypothesis:

$$H_1: E(\text{MUF}(k)) = M(k) \quad \text{and} \quad M := \sum M(k) > 0 \quad .$$

Hypothesis  $H_1$  means that we have a loss of material in at least one balance period  $k$ . For process control purposes we would replace the sign '>' in the alternative hypothesis by the sign  $\neq$ .

The problem now is to find (sequential) test procedures which enable a decision between  $H_0$  and  $H_1$ . A further problem is to find test procedures with a given probability of type I error  $\alpha$  (false alarm probability) and with a small probability of type II error (decision for  $H_0$  if  $H_1$  is true, i.e. we have a loss and we do not detect it). An even further problem is to find test procedures which indicate a loss almost immediately after it has happened.

## COMPUTER PROGRAM PROSA

In the Karlsruhe Nuclear Research Center (KfK) a computer program called PROSA (Program for Statistical Analysis of Near-Real-Time Accountancy Data) has been developed as a tool to apply truncated sequential statistical tests to a sequence of materials balance results the origin of which is a model facility or an existing plant /6,7/. PROSA is a decision tool to decide on the basis of statistical considerations if in a given sequence of materials balance periods a loss of material might have happened or not.

PROSA is written in Fortran 77 and is in use on IBM mainframes and on personal computer with operating system DOS.

The essential input of PROSA consists of:

- (1) the desired false alarm probability  $\alpha$  for a given number of balance periods
- (2) the variance/covariance matrix  $\Sigma$  which is the condensed form of the measurement model of the facility considered
- (3) the series of material balance results  $(MUF_1, \dots, MUF_N)$  which is assumed to be multivariate normally distributed with known dispersion matrix  $\Sigma$ .

A few remarks to the input components:

- (i) the desired false alarm probability  $\alpha$  enables some sensitivity studies for given data sets. In the most cases a fixed false alarm probability of 5 percent is chosen.
- (ii) The determination of the measurement model for real process data is an essential step in the application of NRTA. /9/ The matrix as well as the MUF series can be computed with a computer program called DIDI.

The essential output of PROSA is:

an information if there is a significant test result that means an alarm. The balance period where the first alarm happens is indicated.

The evaluation of the materials balance data is based on statistical test procedures. One essential part for designing statistical tests for materials accounting data is their expected performance in detecting losses of material. Performance measures include the concepts of loss-detection probability and loss-detection time. The performance of a special test has to be studied under a variety of loss patterns, which have to be selected according to reasonable assumptions.



## NRTA TEST PROCEDURES

In the following the three tests will be described.

### a) CUMUF test

CUMUF is used for the cumulative sum of the material balance results MUF(k):

$$\text{CUMUF}(i) = \text{MUF}(1) + \dots + \text{MUF}(i) \quad \text{for all } i=1, \dots, N \quad .$$

The test is performed as follows /10/:

For  $i=1, \dots, N-1$  :

$$\text{CUMUF}(i) \quad \begin{cases} > s(i) & \text{reject } H_0 \\ \leq s(i) & \text{no decision, go to the next period} \end{cases}$$

for  $i=N$  :

$$\text{CUMUF}(N) \quad \begin{cases} > s(N) & \text{reject } H_0 \\ \leq s(N) & \text{reject } H_1 \end{cases} .$$

The test thresholds  $s(1), s(2), \dots, s(N)$  are determined by Monte Carlo simulations to fulfill a given false alarm probability  $\alpha$ .

### b) GEMUF test

Testing the single hypothesis  $H_0$  against a simple alternative

$$H_2: E(\text{MUF}(k)) = M(k) \quad \text{and} \quad M = \sum M(k) > 0$$

there exists exactly one best test. To use this test one has to know the loss pattern  $M(1), \dots, M(N)$ . However, there exist various possibilities to distribute a loss over the balance periods. So these tests (for each loss pattern one best test) are not applicable in practice. The idea leading to the GEMUF test is the following: the (in general unknown) loss pattern  $M(1), \dots, M(N)$  will be estimated as  $\text{MUF}(1), \dots, \text{MUF}(N)$  for  $\text{MUF}(i)$  is an unbiased estimation for  $M(i)$  /4/.

We define :

$\Sigma(i)$  = the first  $i$  rows and columns of  $\Sigma$

$\text{MUF}_i = (\text{MUF}(1), \dots, \text{MUF}(i))^T$  and

$\text{ZG}(i) = \text{MUF}_i^T * (\Sigma(i))^{-1} * \text{MUF}_i$  .

The test is performed as follows:

For  $i=1, \dots, N-1$  :

$$ZG(i) \begin{cases} > s(i) & \text{reject } H_0 \\ \leq s(i) & \text{no decision, go to the next period} \end{cases}$$

for  $i=N$  :

$$ZG(N) \begin{cases} > s(N) & \text{reject } H_0 \\ \leq s(N) & \text{reject } H_1 . \end{cases}$$

The test thresholds  $s(1), s(2), \dots, s(N)$  are determined by Monte Carlo simulations to fulfill a given false alarm probability  $\alpha$ .

c) Pages test

The materials balance results  $MUF(i)$  are random variables which are stochastically dependent. With a linear transformation it is possible to transform the sequence  $MUF(1), \dots, MUF(N)$  to a sequence of stochastically independent random variables  $MUFR(1), \dots, MUFR(N)$ .

The pages test uses the following statistics:

$$S_0 = 0$$

$$T_0 = 0$$

$$S_i = \max(0, S_{i-1} + MUFR(i) - k)$$

$$T_i = \min(0, T_{i-1} + MUFR(i) + k)$$

for  $i=1, \dots, N$  where  $k$  is a fixed number /12/.

The test is performed as follows:

For  $i=1, \dots, N-1$  :

$$S_i > h \text{ or } T_i < -h \quad \text{reject } H_0$$

$$S_i \leq h \text{ or } T_i \geq -h \quad \text{no decision, next period}$$

for  $i=N$  :

$$S_N > h \text{ or } T_N < -h \quad \text{reject } H_0$$

$$S_N \leq h \text{ or } T_N \geq -h \quad \text{reject } H_1 .$$

The parameters  $h$  and  $k$  are determined by simulations to guarantee a false alarm probability  $\alpha$  for the  $N$  balance periods. In our case we have selected  $k=0$ .





## SOME NEW REMARKS ON THE PRECISION OF MONTE-CARLO SIMULATIONS

### a) Mathematical model

The application of Monte-Carlo simulations is an essential part of PROSA. The calculation of the thresholds for the test procedures is based on the desired false-alarm probability  $\alpha$  and the measurement model. Simulations are necessary because the analytical calculation is impossible. That means "empirical probabilities" like "empirical false alarm rates" or "empirical detection probabilities" based on Monte-Carlo simulations will be determined. The question arises how "good" are these determinations or how close is the empirical probability to the true, theoretical probability. The following mathematical model answers this question.

We call MC the number of the Monte-Carlo simulation runs.

For each run  $i$  we define a random variable  $Y(i)$ :

$$(1) \quad Y(i) = \begin{cases} 1 & \text{if there is an alarm} \\ 0 & \text{if there is no alarm} \end{cases}, i=1, \dots, MC$$

We call  $\alpha$  the true (theoretical) detection probability that means

$$(2) \quad E(Y(i)) = \alpha \quad i=1, \dots, MC$$

The  $Y(i)$  are stochastically independent, Bernoulli-distributed random variables with parameter  $\alpha$ .

$$(3) \quad Y(i) \sim B(1, \alpha)$$

The empirical detection probability  $\alpha(e)$  based on MC simulation runs is defined in the following manner:

$$(4) \quad \alpha(e) = \frac{1}{MC} * \sum_{i=1}^{MC} Y(i)$$

Consequently, the random variable  $\alpha(e)$  is also Bernoulli-distributed with:

$$E(\alpha(e)) = \alpha$$

$$(5) \quad \text{var}(\alpha(e)) = \alpha*(1-\alpha)/MC$$

The "Strong Law of Large Numbers" /13/ says that  $\alpha(e)$  converges "almost surely" (e.g. with probability 1) against  $\alpha$ :

$$(6) \quad P\{ \lim_{MC \rightarrow \infty} \alpha(e) = \alpha \} = 1$$

However, the "Strong Law of Large Numbers" only says that  $\alpha(e)$  converges against  $\alpha$ , but nothing about the rate of convergence. Therefore, this law does not provide any error estimate.

b) Chebychev-estimate

A first error estimate provides "Chebychev's inequility" /14/:

$$(7) \quad \begin{aligned} p(|\alpha(e) - \alpha| \geq \varepsilon) &\leq \text{var}(\alpha(e))/\varepsilon^2 \\ &= \frac{\alpha*(1-\alpha)}{MC * \varepsilon^2} \end{aligned}$$

On the wright hand of (7) the unknown value  $\alpha$  occurs. For  $\alpha \in [0,1]$  it can be easily shown:

$$(8) \quad \alpha*(1-\alpha) \leq 1/4$$

and further:

$$(9) \quad p(|\alpha(e) - \alpha| \geq \varepsilon) \leq 1/(4*\varepsilon^2*MC).$$

c) Direct estimate

Because inequility (9) is very gross, we looked for another way to yield better estimations. The "Central Limit Theorem" /14/ says that for  $MC > 30$  the random variable  $\alpha(e)$  is normally distributed with  $E(\alpha(e)) = \alpha$  and

$\text{var}(\alpha(e)) = \alpha^*(1-\alpha)/MC$ . Defining  $X := \alpha(e) - \alpha$  and  $Y := X/\sigma$  with  $\sigma^2 = \alpha^*(1-\alpha)/MC$  it is easily to check that  $Y$  is  $N(0,1)$ -distributed.

So we get:

$$\begin{aligned} (10) \quad p(|X| > \sigma^*k) &= p(|Y| > k) \\ &= 2(1 - \Phi(k)). \end{aligned}$$

With  $\varepsilon = k^*\sigma$  we get:

$$(11) \quad p(|\alpha(e) - \alpha| \geq \varepsilon) \leq 2^*(1 - \Phi(\varepsilon/\sigma)),$$

rsp. with (8):

$$(12) \quad p(|\alpha(e) - \alpha| \geq \varepsilon) \leq 2^*(1 - \Phi(2^*\varepsilon^*\sqrt{MC})).$$

Inequility (12) provides a substantial better estimation than inequility (9). This inequility (12) is applicable in practice. The following tables demonstrate the improvements of the new estimate.

Number of Simulation runs MC

EPS	100		200		300		400		500	
0.01	25.000	0.842	12.500	0.777	8.333	0.729	6.250	0.689	5.000	0.655
0.02	6.250	0.689	3.125	0.572	2.083	0.488	1.563	0.424	1.250	0.371
0.03	2.778	0.549	1.389	0.396	0.926	0.299	0.694	0.230	0.556	0.180
0.04	1.563	0.424	0.781	0.258	0.521	0.166	0.391	0.110	0.313	0.074
0.05	1.000	0.317	0.500	0.157	0.333	0.083	0.250	0.046	0.200	0.025
0.06	0.694	0.230	0.347	0.090	0.231	0.038	0.174	0.016	0.139	0.007
0.07	0.510	0.162	0.255	0.048	0.170	0.015	0.128	0.005	0.102	0.002
0.08	0.391	0.110	0.195	0.024	0.130	0.006	0.098	0.001	0.078	0.000
0.09	0.309	0.072	0.154	0.011	0.103	0.002	0.077	0.000	0.062	0.000
0.10	0.250	0.046	0.125	0.005	0.083	0.001	0.063	0.000	0.050	0.000

Number of Simulation runs MC

EPS	600		700		800		900		1000	
0.01	4.167	0.624	3.571	0.597	3.125	0.572	2.778	0.549	2.500	0.527
0.02	1.042	0.327	0.893	0.290	0.781	0.258	0.694	0.230	0.625	0.206
0.03	0.463	0.142	0.397	0.112	0.347	0.090	0.309	0.072	0.278	0.058
0.04	0.260	0.050	0.223	0.034	0.195	0.024	0.174	0.016	0.156	0.011
0.05	0.167	0.014	0.143	0.008	0.125	0.005	0.111	0.003	0.100	0.002
0.06	0.116	0.003	0.099	0.001	0.087	0.001	0.077	0.000	0.069	0.000
0.07	0.085	0.001	0.073	0.000	0.064	0.000	0.057	0.000	0.051	0.000
0.08	0.065	0.000	0.056	0.000	0.049	0.000	0.043	0.000	0.039	0.000
0.09	0.051	0.000	0.044	0.000	0.039	0.000	0.034	0.000	0.031	0.000
0.10	0.042	0.000	0.036	0.000	0.031	0.000	0.028	0.000	0.025	0.000

Tab.2: Probability of a Deviation of  $\alpha(e)$  from  $\alpha$  of more then EPS  
 Based on MC Simulation Runs,  
 1.Row Using Chebychev's Inequility  
 2.Row Using the Direct Estimate

Number of Simulation runs MC

EPS	1000		2000		3000		4000		5000	
0.01	2.500	0.527	1.250	0.371	0.833	0.273	0.625	0.206	0.500	0.157
0.02	0.625	0.206	0.313	0.074	0.208	0.028	0.156	0.011	0.125	0.005
0.03	0.278	0.058	0.139	0.007	0.093	0.001	0.069	0.000	0.056	0.000
0.04	0.156	0.011	0.078	0.000	0.052	0.000	0.039	0.000	0.031	0.000
0.05	0.100	0.002	0.050	0.000	0.033	0.000	0.025	0.000	0.020	0.000
0.06	0.069	0.000	0.035	0.000	0.023	0.000	0.017	0.000	0.014	0.000
0.07	0.051	0.000	0.026	0.000	0.017	0.000	0.013	0.000	0.010	0.000
0.08	0.039	0.000	0.020	0.000	0.013	0.000	0.010	0.000	0.008	0.000
0.09	0.031	0.000	0.015	0.000	0.010	0.000	0.008	0.000	0.006	0.000
0.10	0.025	0.000	0.013	0.000	0.008	0.000	0.006	0.000	0.005	0.000

Number of Simulation runs MC

EPS	6000		7000		8000		9000		10000	
0.01	0.417	0.121	0.357	0.094	0.313	0.074	0.278	0.058	0.250	0.046
0.02	0.104	0.002	0.089	0.001	0.078	0.000	0.069	0.000	0.063	0.000
0.03	0.046	0.000	0.040	0.000	0.035	0.000	0.031	0.000	0.028	0.000
0.04	0.026	0.000	0.022	0.000	0.020	0.000	0.017	0.000	0.016	0.000
0.05	0.017	0.000	0.014	0.000	0.012	0.000	0.011	0.000	0.010	0.000
0.06	0.012	0.000	0.010	0.000	0.009	0.000	0.008	0.000	0.007	0.000
0.07	0.009	0.000	0.007	0.000	0.006	0.000	0.006	0.000	0.005	0.000
0.08	0.007	0.000	0.006	0.000	0.005	0.000	0.004	0.000	0.004	0.000
0.09	0.005	0.000	0.004	0.000	0.004	0.000	0.003	0.000	0.003	0.000
0.10	0.004	0.000	0.004	0.000	0.003	0.000	0.003	0.000	0.003	0.000

Tab.3: Probability of a Deviation of  $\alpha(e)$  from  $\alpha$  of more then EPS  
 Based on MC Simulation Runs,  
 1.Row Using Chebychev's Inequility  
 2.Row Using the Direct Estimate



## DETERMINATION OF THE MEASUREMENT MODEL

As mentioned earlier, the determination of the measurement model for real process data is an essential step in the application of the NRTA procedures. In the following, two concepts of determination of realistic measurement models are described. The measurement models analyzed in this paper are "one-block models". This simplification means that all the nuclear inventory of the facility is assumed to be contained in one component, but the plant is considered to be in a non-steady operating state. The main problem in this context is to determine the variance of the inventory measurement. Two approaches are introduced to meet this problem:

- the "deductive way" and
- the "inductive way" .

In the following, the two methods are briefly discussed.

### a) "Deductive way"

The "deductive way" of determination the inventory variance can be performed if one has a few balance periods without any transfer. In this case the inventory of the facility is constant. So one gets at least two measurements for the same inventory. This situation allows to calculate a point estimate of the variance of the measurement. The method has two disadvantages. Firstly the estimation is very gross and secondly the distribution of the inventory among the plant components is not taken into account. Nevertheless, the deductive method may be a rather realistic approach and provides a first idea about the measurement uncertainties.

### b) "Inductive way"

For the "inductive way", operational informations as well as target values are needed for the measurement uncertainties /16,17/. The determination of Plutonium (Pu) in each process component of the facility is the product of volume (VOL) and concentration (CON) measurement

$$(1) \quad \text{Pu} = \text{VOL} * \text{CON}.$$

Both measurements are assumed to be random variables with random and systematic components. The standard deviations of the two types of errors are assumed to be known. The means are assumed to be zero for both errors so that each measurement is an unbiased estimate for the true value. Using truncated Taylor series expansion in the mean values  $E(\text{VOL})$  and  $E(\text{CON})$  one gets:



$$(2) \quad \text{VOL} * \text{CON} = \text{E}(\text{VOL}) * \text{E}(\text{CON}) + (\text{CON} - \text{E}(\text{CON})) * \text{E}(\text{VOL}) + (\text{VOL} - \text{E}(\text{VOL})) * \text{E}(\text{CON})$$

which results in the reasonable equation:

$$(3) \quad \text{E}(\text{Pu}) = \text{E}(\text{VOL}) * \text{E}(\text{CON}).$$

For the variances the following expression holds:

$$(4) \quad \text{var}(\text{Pu}) = (\text{E}(\text{VOL}))^2 * \text{var}(\text{CON}) + (\text{E}(\text{CON}))^2 * \text{var}(\text{VOL}).$$

Based on this procedure, the (average) relative variance of the inventory of the whole facility can be estimated if flow-sheet information supplied by the operator is available. Furthermore, the variance of the inventory of the whole facility can be split into a random component and into a systematic component. Both components are of random origin. They differ only in the way of error propagation. In the present case systematic errors are assumed to be constant for the whole reprocessing campaign.

## THE COMPUTER PROGRAMME DIDI

The programme DIDI /5/ (Programme for Computing the Measurement Model of a Reprocessing Facility - *DI*scontinuous *DI*spersion Matrix) is able to compute the measurement model that means the variance/covariance matrix of a one-block model of a reprocessing plant. Therefore DIDI is the connection piece between the data resulting from the working of the reprocessing facility and PROSA (see Tab.4).

Input are the inventory data, all transfers and the relative standard deviations, respectively, random as well as systematical.

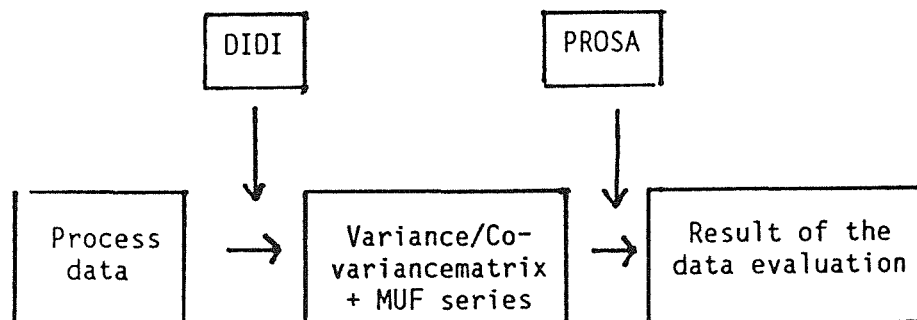
Output is primarily the variance/covariance matrix (measurement model) of the reprocessing plant. But also the absolute standard deviations, random as well as systematical, the MUF and CUMUF series, the variance of CUMUF and the Neyman-Pearson optimum loss-pattern will be computed.

The programme DIDI is written in FORTRAN 77. It is tested on a host computer (IBM3090) and on a personal computer with operating system DOS.

Tab.4: The Computer Programme DIDI

### THE COMPUTER PROGRAMME DIDI

Programme for Computing the Measurement Model of  
a Reprocessing Facility - DIscontinuous DIspersion Matrix





## REALISTIC DATA SET

The data base which the NRTA procedures are applied to results of a reprocessing campaign of the Wiederaufarbeitungsanlage Karlsruhe (WAK). In fig. 1 the Pu inventory is plotted. It is obvious that the data represent by no means a steady-state operation of the facility. There was an increasing inventory, and it has to be mentioned that the transfers differed considerably in each period and there were only small waste outputs during the campaign. The MUF and CUMUF series is plotted on fig. 2. Furthermore, two measurement models have been developed according to the two above described approaches. It has to be pointed out that the two models assume a simplified "one-block model". Nevertheless, this should give a realistic first rough estimate of the true measurement situation. In Tab.5 and Tab.6 the relative standard deviations of the four material balance components inventory, input, product output and waste output are shown. The inventory data are the result of the two different approaches of measurement model construction. It can be realized that they differ considerably. The whole data sets are the basis for the NRTA analysis performed by PROSA.

Tab. 5: Measurement Uncertainties ( $1\sigma$ -values)  
related to the "deductive" measurement model

Balance Term	Uncertainty random (%)	Uncertainty systematic (%)
INVENTORY	3.3	---
INPUT	0.9	1.3
PRODUCT	0.5	0.5
WASTE	20.0	---

Tab. 6: Measurement Uncertainties ( $1\sigma$ -values)  
related to the "inductive" measurement model

Balance Term	Uncertainty random (%)	Uncertainty systematic (%)
INVENTORY	1.56	0.86
INPUT	0.9	1.3
PRODUCT	0.5	0.5
WASTE	20.0	---

Fig. 1: Plant inventory in the course of the campaign

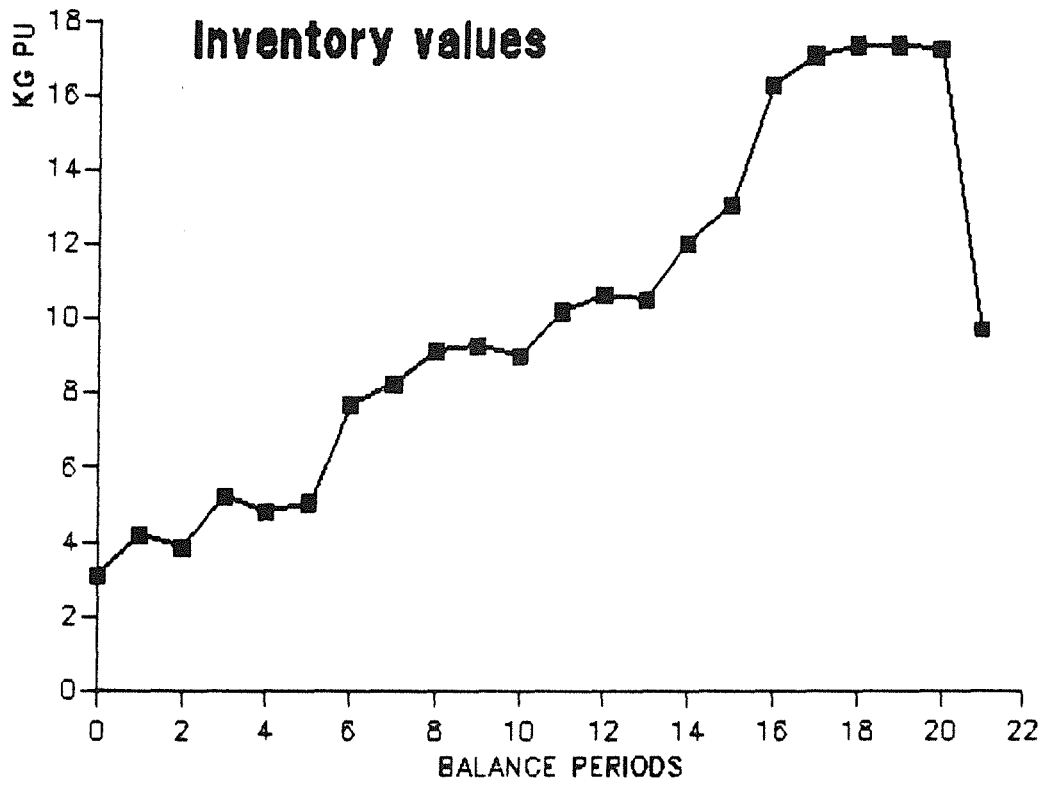
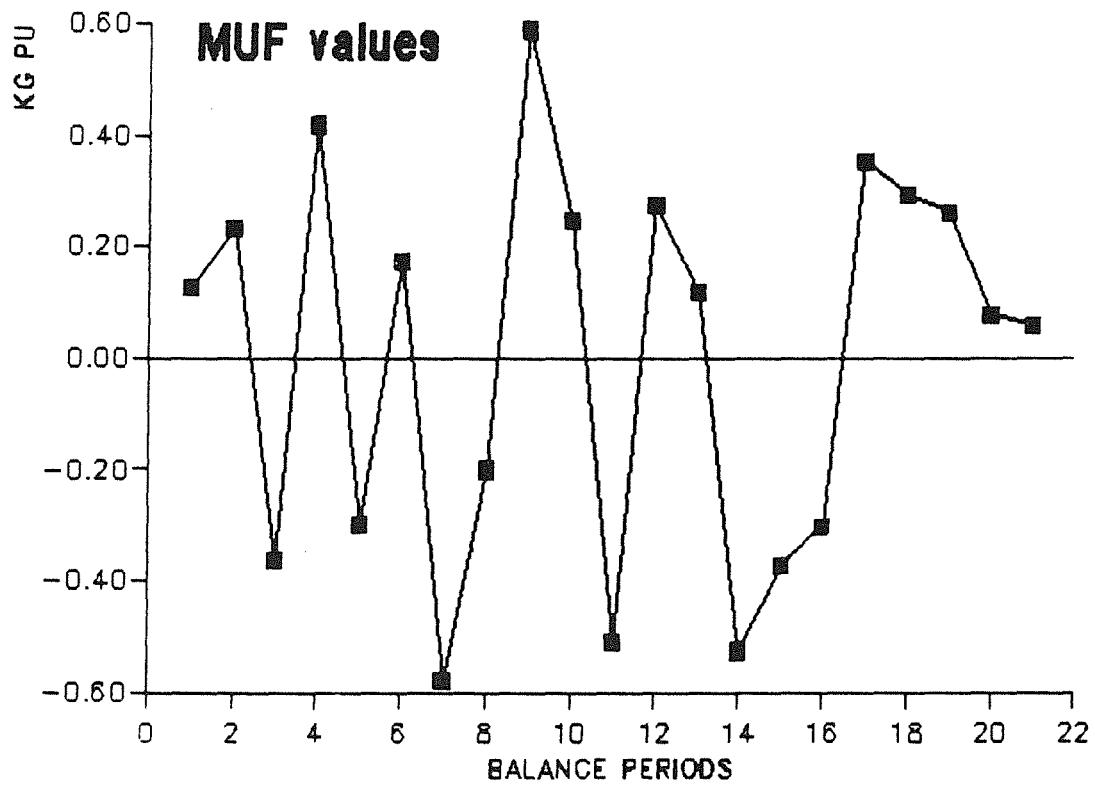


Fig. 2: Results of the material balances in the course of the campaign





## DATA EVALUATION

With the tool PROSA, the balance data of Fig.2 have been evaluated based on the measurement models given in Tab.4 and 5. Together with the inventory, input and output data, the variance/covariance matrices  $\Sigma(i)$  can be computed with the tool DIDI. For all test procedures a 5% false alarm probability for the 20 balance periods has been assumed. The next figures show the results on the basis of the "deductive model". Fig. 3 shows that the CUMUF test statistic stays all the time below the threshold. That means we have no alarm, the balance data can be explained by the assumed measurement model. The same is true for the PAGES test (fig. 4). The upper statistic stays below the upper threshold and the lower statistic stays above the lower threshold. Also the GEMUF test gives no alarm (fig. 5).

Now we turn over to the evaluation on the basis of the inductive measurement model. There are quite different results. The CUMUF test gives an alarm in the second and fourth period (fig. 6), the PAGES test in the 15th and following periods (fig. 7). The GEMUF test alarms in every period without the first (fig. 8). This example shows that for the same MUF series we have no alarms if the evaluation is based on the deductive model and we have alarms if the evaluation is based on the inductive model. This demonstrates the great importance of an exact determination of the measurement model. But what could be the reason for the alarms in the second case? It seems that the "inductive model" results in our case in a too optimistic measurement uncertainty for the inventory measurements. This might to a great deal be caused by the simplification of the "one-block model" because in this case the distribution of the inventory among the facility components is not at all taken into account.

Let's have a look again at the determination of the measurement model. Formula 4 gives the equation for the variance of the plutonium in one component. On the right hand, the terms "mean of VOL" and "mean of CON" appear. Here we have to put in the true values. But we don't know them. Due to this reason we replace them by the measurement results because they are an unbiased estimate for them. But in our first approach we didn't know even these data, we knew only the inventory data of the whole facility. But we had informations from the operator about the average volume and concentration in each component. So we replaced "mean of VOL" and "mean of CON" by these average values and we were able to compute the average standard standard deviation of the whole inventory. If we have knowlegde of all measurement data that means the volume and concentration measurement of each component, we will be able to determine a detailed measurement model which is no longer a one-block-model. These detailed data will be provided from another computer programme called PRODES (Process data acquisition and information system) /18/.



First calculation with this detailed measurement model have shown that the balance data are in accordance with this measurement model.

Fig. 3: Evaluation of the balance data with the deductive measurement model using the CUMUF test

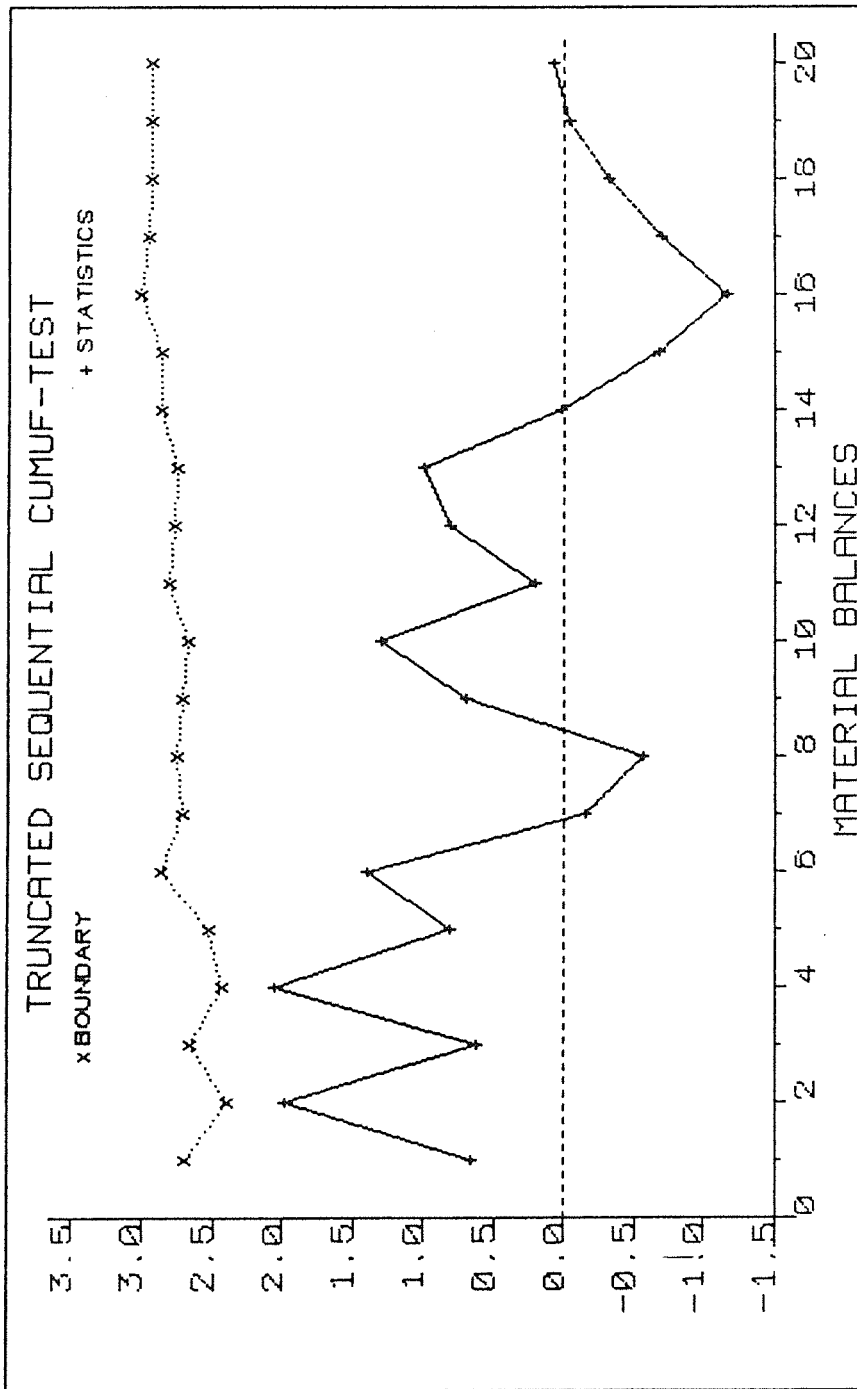


Fig. 4: Evaluation of the balance data with the deductive measurement model using the Page's test

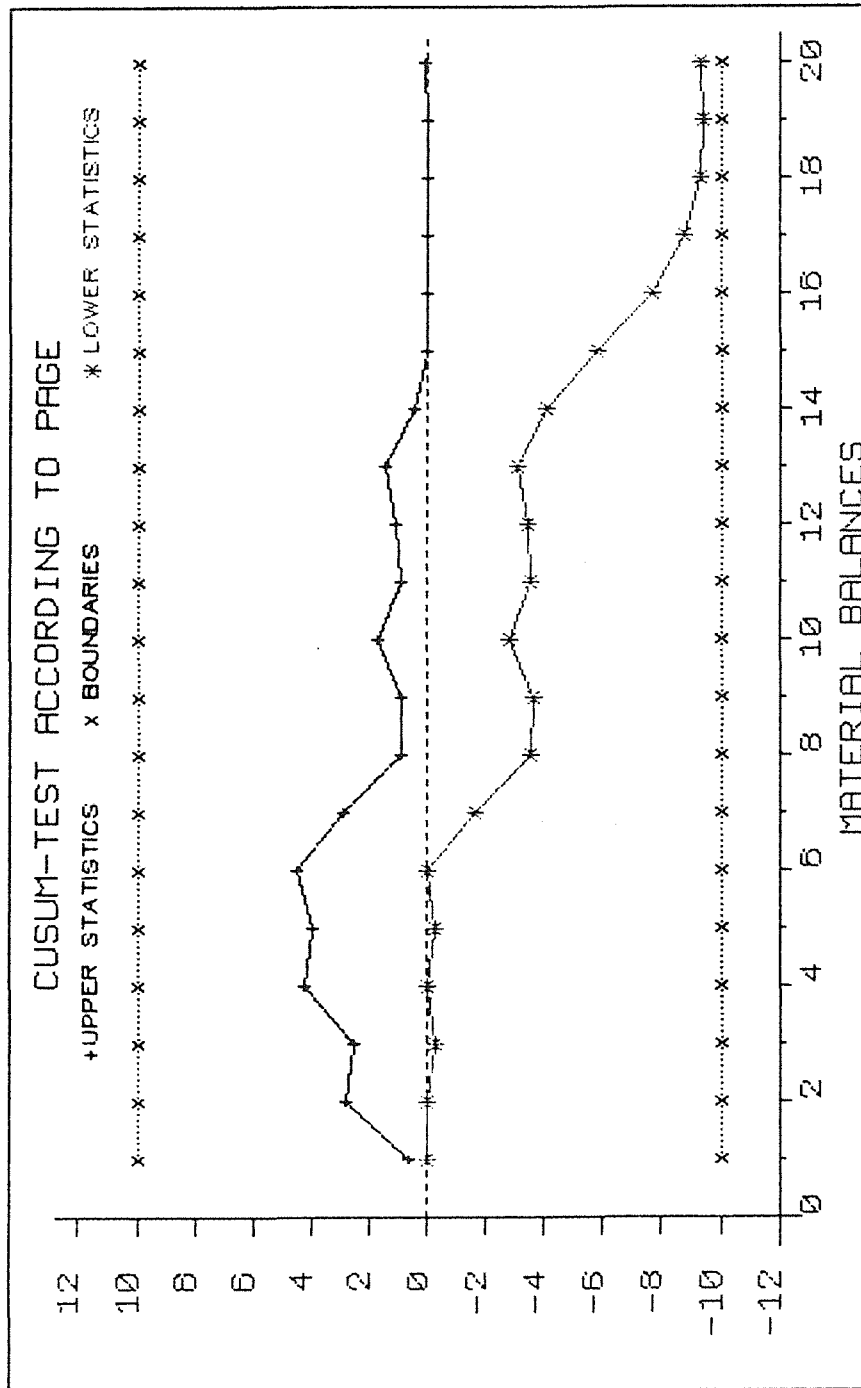


Fig. 5: Evaluation of the balance data with the deductive measurement model using the GEMUF test

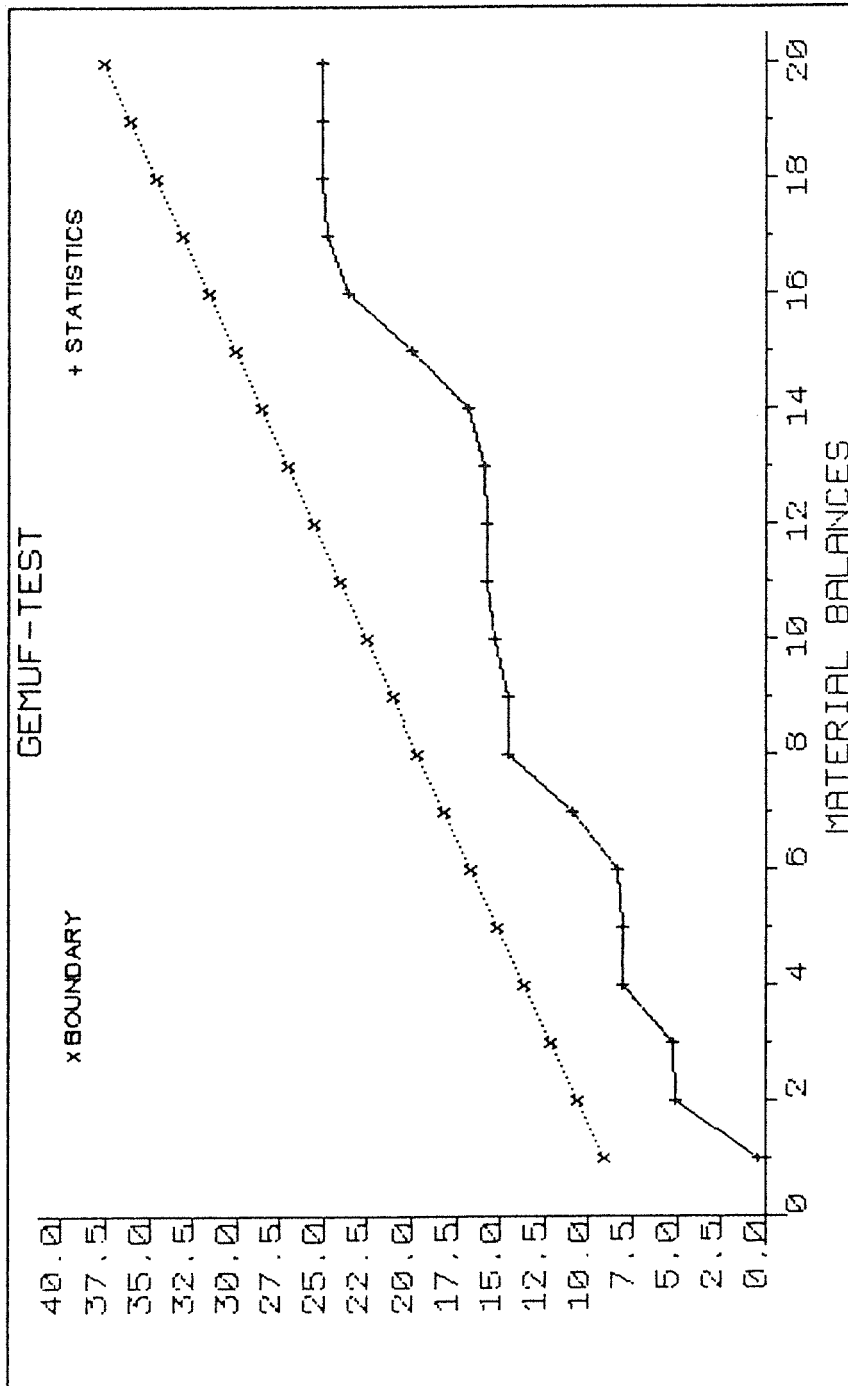


Fig. 6: Evaluation of the balance data with the inductive measurement model using the CUMUF test

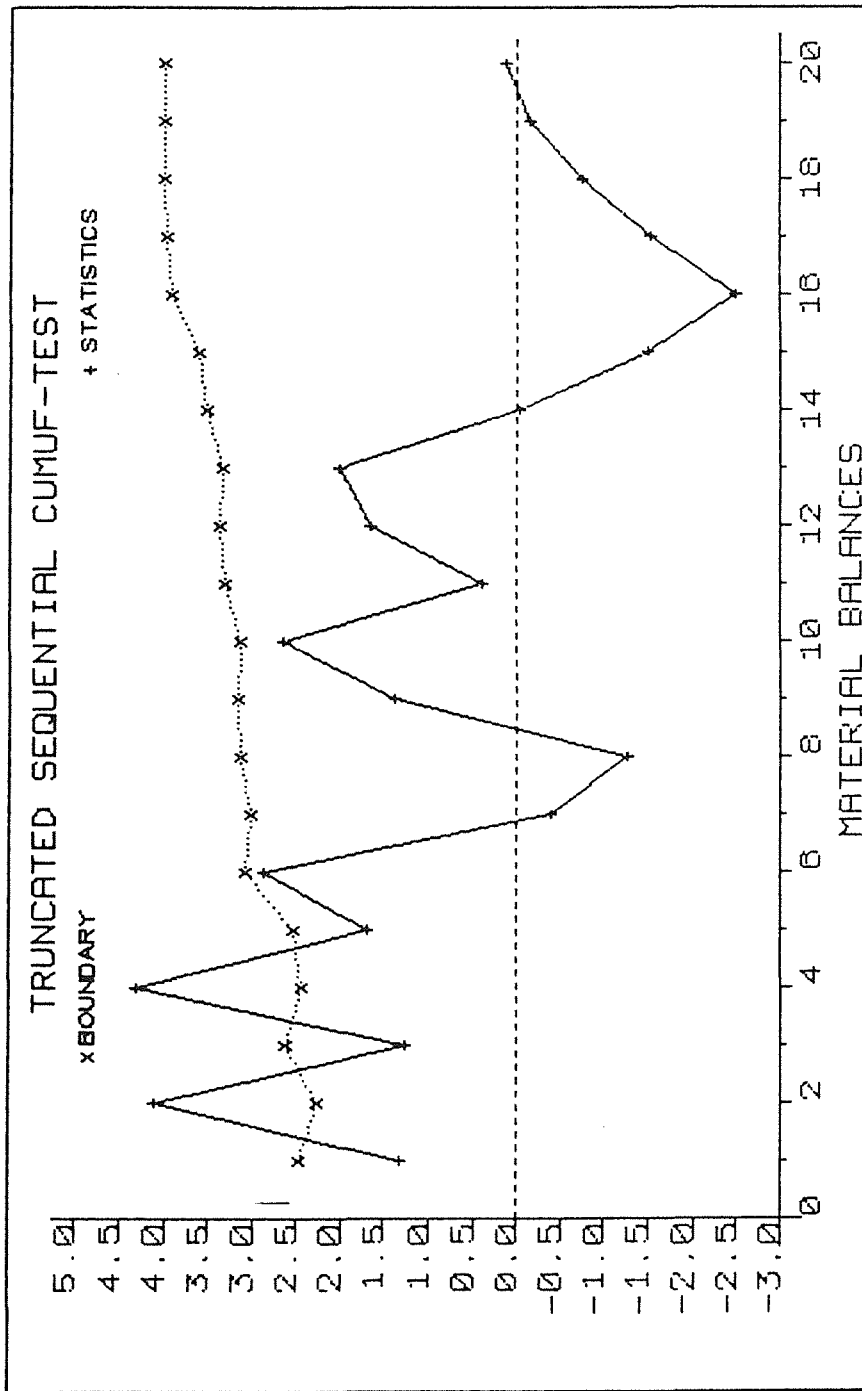


Fig. 7: Evaluation of the balance data with the inductive measurement model using the Page's test

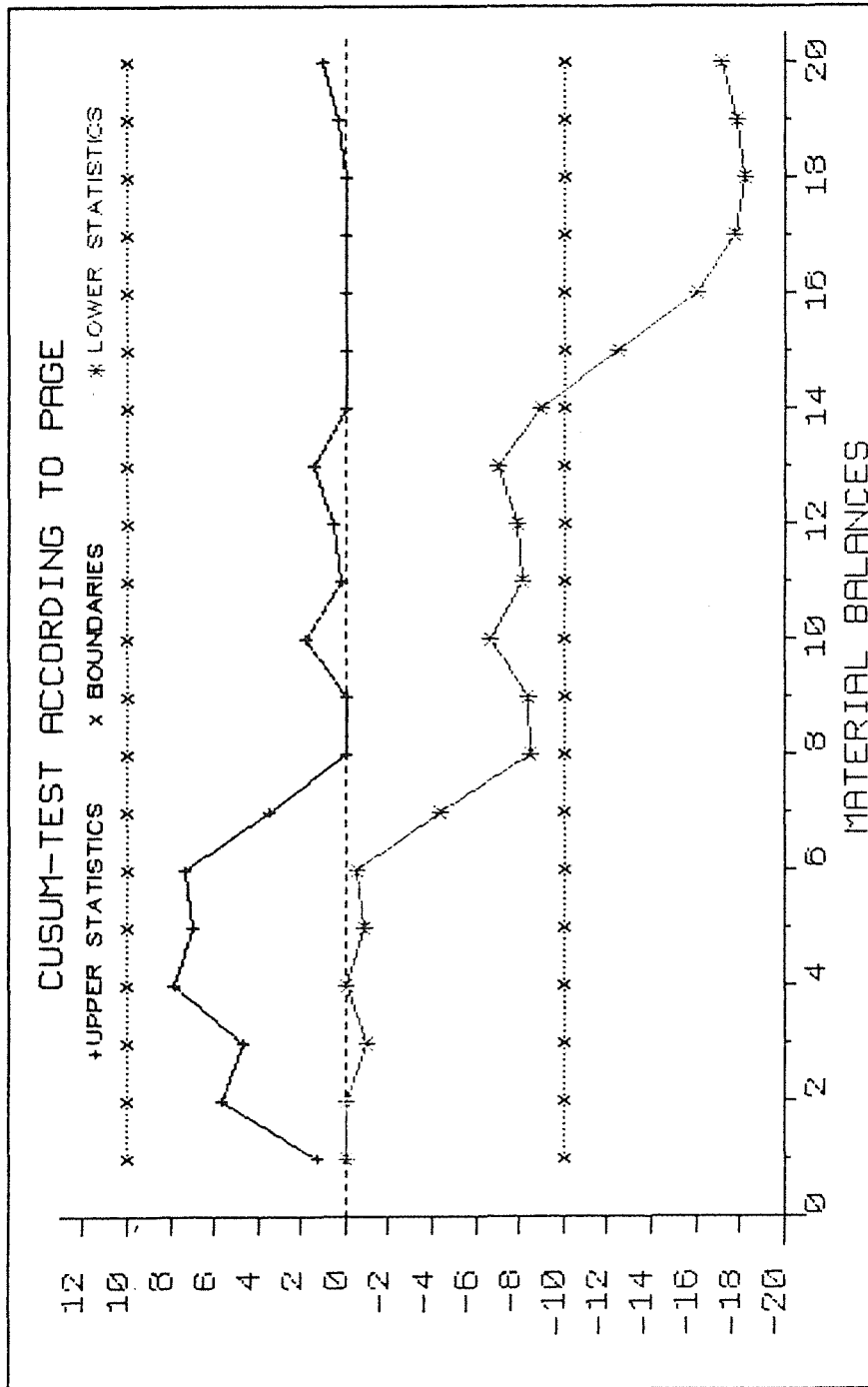
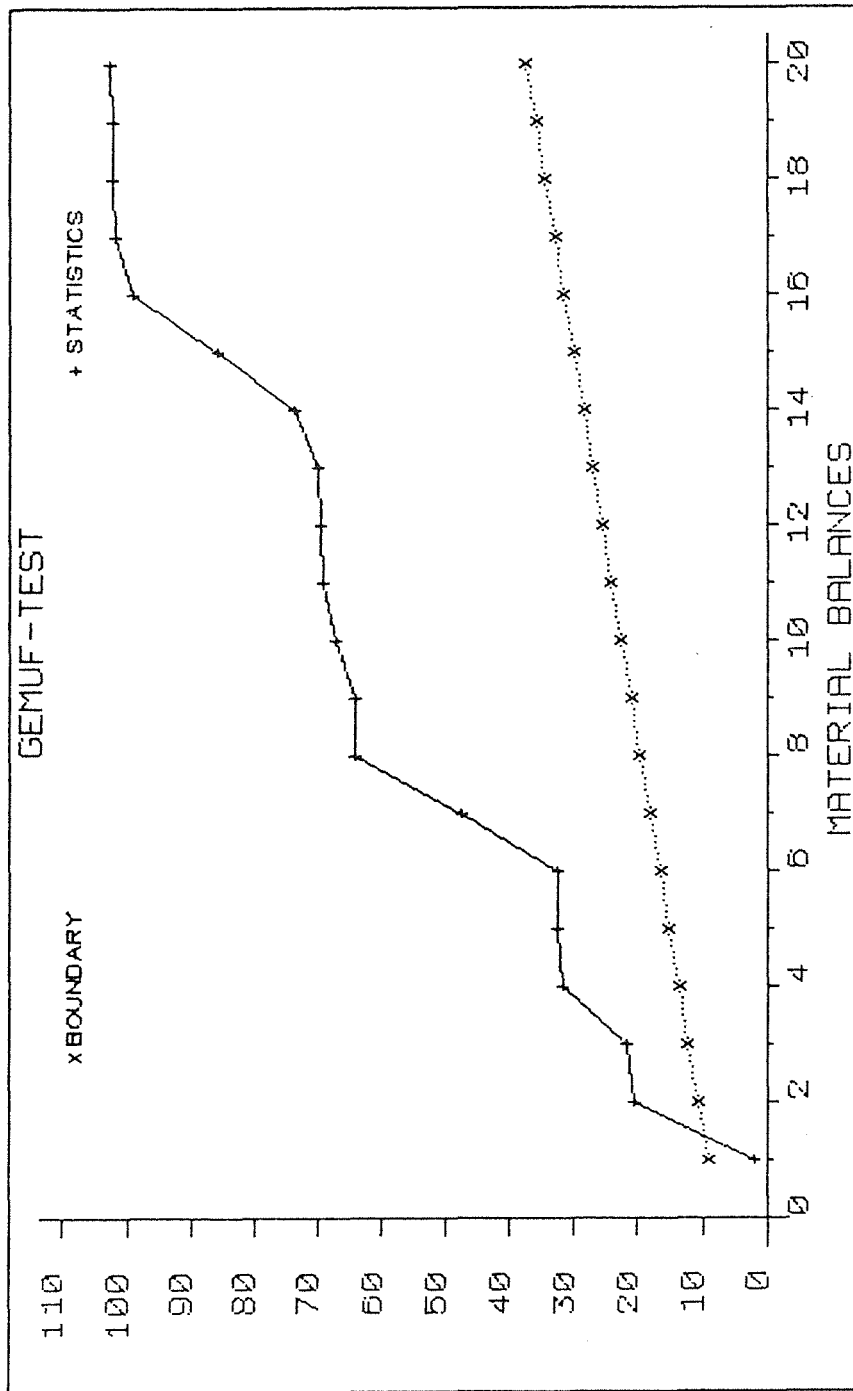


Fig. 8: Evaluation of the balance data with the inductive measurement model using the GEMUF test



## CONCLUSIONS

- The results of the data evaluation demonstrate that the NRTA tests included in PROSA are not only able to detect timely a possible loss of nuclear material but even to test the underlying measurement model if a non-loss situation can be assumed.

- The determination of the measurement model is an essential part in the data evaluation.





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