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Stress intensity factors and weight functions for special crack problems

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Abstract:

The failure of cracked components is governed by the stresses in the vicinity of the crack tip. The singular stress contribution is characterised by the stress intensity factor *K*. Stress intensity factors depend on the geometry of the component and on the special loading conditions (tension, bending, thermal stresses,...). A procedure for their determination is the weight function technique where the weight functions are only dependent on the crack geometry.

Stress intensity factors and weight functions are reported for many practical problems in handbooks. In this report new solutions for stress intensity factors and weight functions are compiled in form of tables or approximate relations.

Spannungsintensitätsfaktoren und Gewichtsfunktionen für spezielle Rißprobleme

Kurzfassung:

Das Versagen von Bauteilen mit Rissen wird durch die unmittelbar an der Rißspitze auftretenden Spannungen verursacht. Der singuläre Anteil diese Spannungen wird durch den Spannungsintensitätsfaktor *K* charakterisiert. Spannungsintensitätsfaktoren hängen von der Riß- und Bauteilgeometrie sowie von der speziellen Belastung (Zug, Biegung, Thermospannungen,...) ab. Eine Methode zur Bestimmung von Spannungs-intensitätsfaktoren ist die Methode der Gewichtsfunktionen. Diese sind nur von den Geometriedaten abhängig.

Lösungen für Spannungsintensitätsfaktoren und Gewichtsfunktionen werden für viele praktisch relevante Fälle in Handbüchern angegeben. Im vorliegenden Bericht werden neue Ergebnisse in Form von Tabellen und Näherungsformeln mitgeteilt.

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1 Introduction

The fracture behaviour of cracked structures is dominated by the near-tip stress field. In fracture mechanics most interest is focussed on stress intensity factors, which describe the singular stress field ahead of a crack tip and govern fracture of a specimen when a critical stress intensity factor is reached.

While stress intensity factor solutions are reported in handbooks [1-4] for many crack geometries and loading cases, weight functions are seldomly available [5-7]. In [7] a large number of stress intensity factor solutions were given, methods for the determination of weight functions were reported and numerical results for a number of crack geometries were compiled.

In the meantime, further crack problems have been evaluated which will be addressed in this report. For the published results the reference is given. All other data are unpublished results. Section 2 deals with cracks in rectangular plates and Section 3 provides data for circular disks.

1.1 Stress intensity factor

For the determination of stress intensity factors the Boundary Collocation Method (BCM) was used and for the determination of the weight function the direct adjustment method [7] was applied.

The stress intensity factor K is a measure for the singular stress term occurring near the tip of a crack and defined by

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi a}} f_{ij}(\varphi)$$

where r and φ are polar coordinates with the origin at the crack tip. The angular functions are for mode I:

$$f_{xx} = \cos\left(\frac{\varphi}{2}\right)\left[1 - \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right]$$

$$f_{yy} = \cos\left(\frac{\varphi}{2}\right)\left[1 + \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right]$$

$$f_{xy} = \cos\left(\frac{\varphi}{2}\right)\sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{3\varphi}{2}\right)$$

and for mode II:

$$f_{xx} = \sin\left(\frac{\varphi}{2}\right)\left[2 + \sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{3\varphi}{2}\right)\right]$$

$$f_{yy} = \sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{\varphi}{2}\right)\cos\left(\frac{3\varphi}{2}\right)$$

$$f_{xy} = \cos\left(\frac{\varphi}{2}\right)\left[1 - \sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{3\varphi}{2}\right)\right]$$

K is the stress intensity factor. For the loading modes considered in this report the stress intensity factors K_{I} and K_{II} are expressed as

$$K_I = \sigma^* \sqrt{\pi a}\, F_I(a/W)$$

$$K_{II} = \tau^* \sqrt{\pi a}\, F_{II}(a/W)$$

where *a* is the crack length, *W* is the width of the component and σ^*, τ^* are characteristic stresses in the component, e.g. the outer fibre stress in a bending bar. F_{I} and F_{II} are functions of the ratio of the crack length to the specimens width as well as of the type of load applied.

1.2 Weight function

Most of the numerical methods require separate calculation of the stress intensity factor for each given stress distribution and each crack length. The weight function procedure developed by Bückner [8] simplifies the determination of stress intensity factors. If the weight function is known for a crack in a component, the stress intensity factor can be obtained by multiplying this function by the stress distribution and integrating it along the crack length.

If $\sigma(x)$ is the normal stress distribution and $\tau(x)$ are the shear stresses in the uncracked component along the prospective crack line of an edge crack, the stress intensity factors are given by

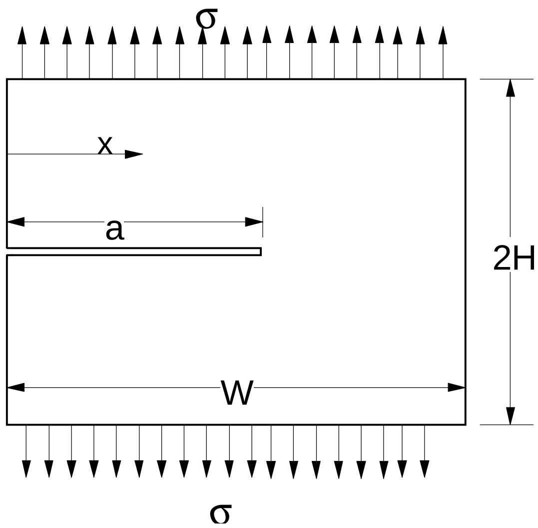
$$K_I = \int_0^a \sigma(x)\, h_I(x,a)\, dx$$

$$K_{II} = \int_0^a \tau(x)\, h_{II}(x,a)\, dx$$

The integration has to be performed over the crack length. The weight function h(x,a) does not depend on the special stress distribution, but only on the geometry of the component.

2 Cracks in rectangular plates or bars

2.1 The rectangular plate with an edge crack



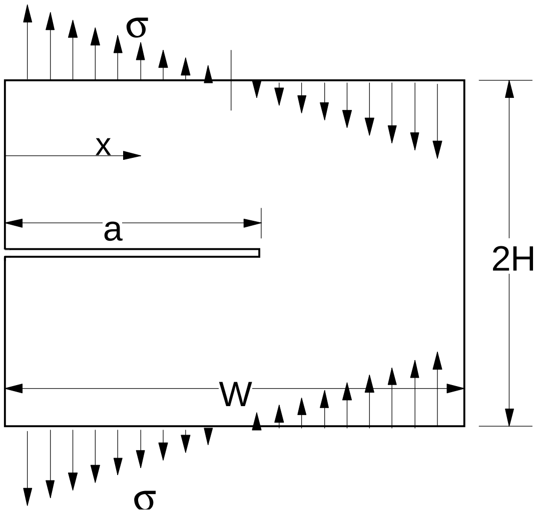
Stress intensity factor for pure tension

$$K = \sigma\sqrt{\pi a}F_t$$

	H/*W*=1.5	1.25	1.00	0.75	0.5	0.4	0.3	0.25
α=0	1.1215	1.1215	1.1215	1.1215	1.1215	1.1215	1.1215	1.1215
0.1	1.0170	1.0172	1.0174	1.0182	1.0352	1.0649	1.1455	1.2431
0.2	0.9800	0.9799	0.9798	0.9877	1.0649	1.1625	1.3619	1.5358
0.3	0.9722	0.9723	0.9729	0.9840	1.0821	1.2134	1.4892	1.7225
0.4	0.9813	0.9813	0.9819	0.9915	1.0819	1.2106	1.5061	1.7819
0.5	0.9985	0.9986	0.9989	1.0055	1.0649	1.1667	1.4298	1.7013
0.6	1.0203	1.0203	1.0204	1.0221	1.0496	1.1073	1.2898	1.5061
0.7	1.0440	1.0441	1.0441	1.0442	1.0522	1.0691	1.1498	1.2685
0.8	1.0683	1.0683	1.0683	1.0690	1.0691	1.0734	1.0861	1.1201
1.0	1.1215	1.1215	1.1215	1.1215	1.1215	1.1215	1.1215	1.1215

Table 1 Geometric function for tension $F_t \cdot (1-a/W)^{3/2}$ [9].

Stress intensity factor for pure bending



$$\sigma(x) = \sigma_0(1-2x/W) \quad , \qquad K = \sigma_0\sqrt{\pi a}F_b$$

	H/W=1.5	1.25	1.00	0.75	0.5	0.4
α=0	1.1215	1.1215	1.1215	1.1215	1.1215	1.1215
0.2	0.7561	0.7561	0.7562	0.7628	0.8279	0.9130
0.3	0.6583	0.6583	0.6589	0.6677	0.7444	0.8475
0.4	0.5861	0.5861	0.5865	0.5930	0.6567	0.7505
0.5	0.5293	0.5293	0.5296	0.5332	0.5717	0.6388
0.6	0.4842	0.4842	0.4842	0.4852	0.5022	0.5367
0.7	0.4481	0.4479	0.4478	0.4478	0.4514	0.4621
0.8	0.4203	0.4188	0.4191	0.4185	0.4180	0.4185
1.0	0.374	0.374	0.374	0.374	0.374	0.374

Table 2 Geometric function for bending $F_b \cdot (1-a/W)^{3/2}$.

2-terms weight function:

$$h = \sqrt{\frac{2}{\pi a}}\left[\frac{1}{\sqrt{1-\rho}} + D_0\sqrt{1-\rho} + D_1(1-\rho)^{3/2}\right] \ , \quad \rho = x/a$$

with coefficients

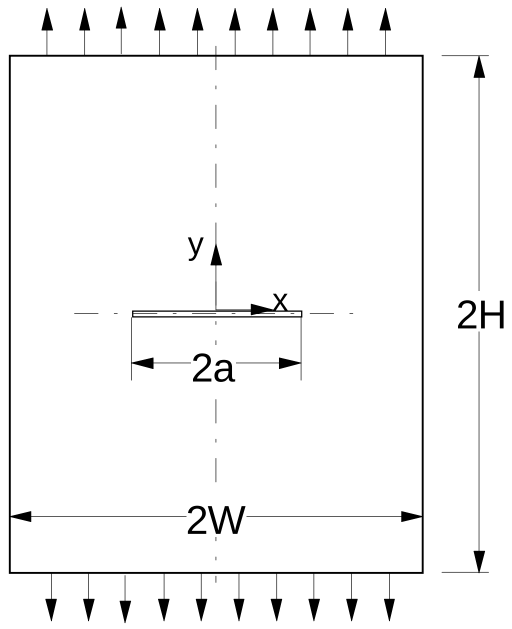
	H/*W*=1.5	1.25	1.00	0.75	0.5	0.4
0.2	1.001	1.001	1.003	1.010	1.249	1.347
0.3	1.298	1.302	1.326	1.317	1.539	1.816
0.4	1.581	1.581	1.598	1.616	1.836	2.036
0.5	1.827	1.829	1.835	1.859	1.973	2.122
0.6	1.996	1.996	1.998	2.001	2.027	2.110
0.7	2.070	2.071	2.071	2.079	2.104	2.094
0.8	2.015	2.015	2.017	2.054	2.064	2.094

Table 3 Coefficient D_0 for weight function.

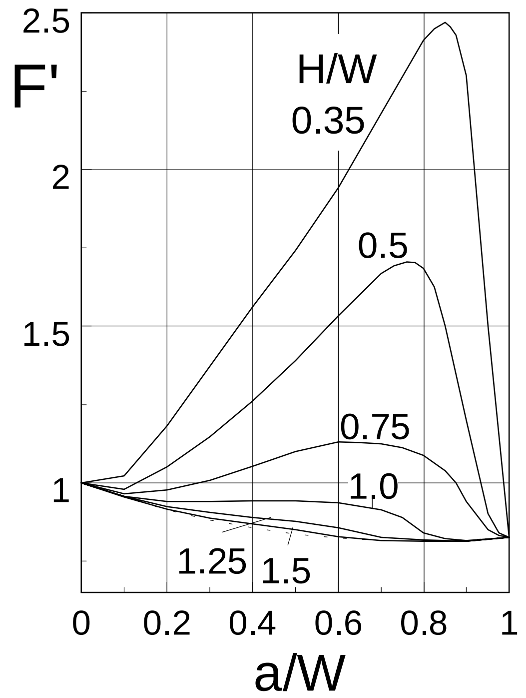
	H/*W*=1.5	1.25	1.00	0.75	0.5	0.4
0.2	0.1963	0.200	0.2100	0.2245	0.255	0.634
0.3	0.3072	0.301	0.2641	0.3422	0.516	0.784
0.4	0.4909	0.4909	0.4661	0.4887	0.624	1.006
0.5	0.7329	0.7300	0.7213	0.7183	0.857	1.170
0.6	1.074	1.074	1.072	1.077	1.186	1.368
0.7	1.526	1.525	1.525	1.513	1.516	1.629
0.8	2.128	2.128	2.128	2.066	2.050	2.018

Table 4 Coefficient D_1 for weight function.

2.2 Rectangular plate with an internal crack



$$K_I = \sigma F \sqrt{\pi a}$$

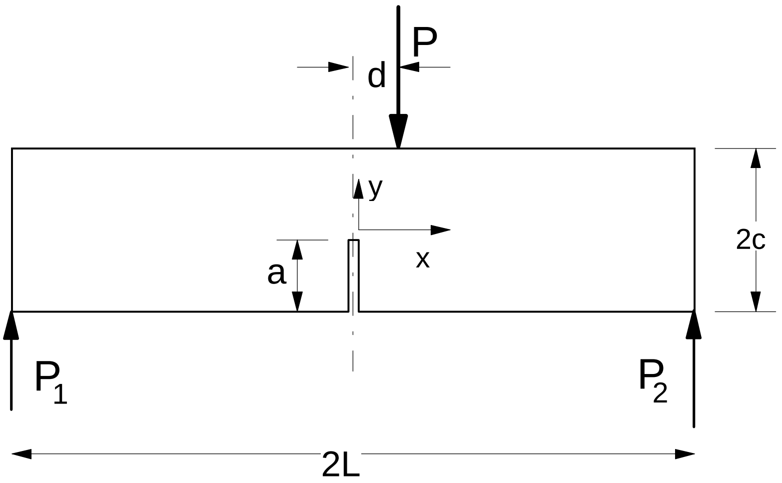


Stress intensity factor for tensile loading.

	H/W=1.5	1.25	1.00	0.75	0.5	0.35
α=0	1.00	1.00	1.00	1.00	1.00	1.00
0.2	0.916	0.924	0.940	0.977	1.051	1.182
0.3	0.888	0.905	0.940	1.008	1.147	1.373
0.4	0.869	0.890	0.942	1.053	1.262	1.562
0.5	0.851	0.877	0.943	1.099	1.391	1.742
0.6	0.827	0.856	0.937	1.130	1.533	1.938
0.7	0.816	0.826	0.914	1.125	1.668	2.197
0.8	0.814	0.818	0.840	1.088	1.689	2.41
1.0	0.826	0.826	0.826	0.826	0.826	0.826

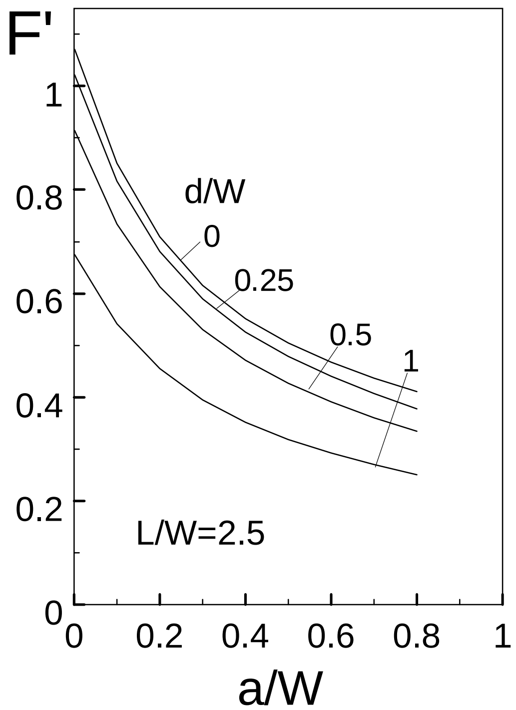
Table 5 Geometric function for tension $F' = F \cdot (1-a/W)^{1/2}$.

2.3 3-Point bending test with eccentric load



Mode-I stress intensity factor

$$K_I = \sigma_0 F \sqrt{\pi a}\,, \quad \sigma_0 = \frac{3PL}{W^2}\,, \quad F' = F(1 - a/W)^{3/2} \quad (W{=}2c)$$



Geometric function for eccentric 3-point bending tests.

a/W	d/W=0	0.0625	0.125	0.50	1.0	1.25	1.50
0.05	0.9506	0.9475	0.9374	0.7809	0.5079	0.3790	0.2519
0.1	0.8504	0.8475	0.8388	0.7016	0.4583	0.3420	0.2273
0.2	0.7046	0.7022	0.6950	0.5828	0.3832	0.2862	0.1904
0.3	0.6091	0.6069	0.6003	0.5015	0.3310	0.2476	0.1648
0.4	0.5447	0.5425	0.5360	0.4441	0.2938	0.2200	0.1465
0.5	0.4985	0.4961	0.4893	0.4010	0.2658	0.1993	0.1328
0.6	0.4626	0.4599	0.4523	0.3665	0.2434	0.1826	0.1217
0.7	0.4328	0.4295	0.4207	0.3375	0.2246	0.1685	0.1124
0.8	0.4076	0.4031	0.3924	0.3132	0.2088	0.1567	0.1044

Table 6 Geometric function F' for $L/W = 2.0$.

a/W	d/W=0	0.125	0.25	0.50	1.0	1.25	1.5	1.75
0.05	0.9609	0.9503	0.9205	0.8249	0.6069	0.5034	0.4028	0.3020
0.1	0.8616	0.8523	0.8263	0.7425	0.5480	0.4548	0.3636	0.2726
0.2	0.7156	0.7079	0.6865	0.6181	0.4585	0.3808	0.3045	0.2283
0.3	0.6188	0.6118	0.5925	0.5327	0.3964	0.3296	0.2636	0.1976
0.4	0.5527	0.5458	0.5271	0.4722	0.3521	0.2930	0.2344	0.1758
0.5	0.5048	0.4974	0.4785	0.4268	0.3187	0.2654	0.2124	0.1593
0.6	0.4672	0.4590	0.4393	0.3903	0.2919	0.2432	0.1947	0.1460
0.7	0.4359	0.4262	0.4055	0.3597	0.2694	0.2245	0.1797	0.1348
0.8	0.4094	0.3973	0.3762	0.3339	0.2504	0.2086	0.1670	0.1253

Table 7 Geometric function F' for $L/W = 2.5$.

a/W	d/W=0	0.125	0.25	0.50	1.0	1.25	1.5	1.75	2.0
0.05	0.9684	0.9595	0.9347	0.8551	0.6733	0.5871	0.5026	0.4192	0.3355
0.1	0.8694	0.8617	0.8400	0.7701	0.6081	0.5303	0.4541	0.3785	0.3028
0.2	0.7231	0.7167	0.6989	0.6419	0.5089	0.4441	0.3803	0.3170	0.2536
0.3	0.6255	0.6196	0.6035	0.5537	0.4401	0.3844	0.3293	0.2744	0.2196
0.4	0.5582	0.5524	0.5368	0.4911	0.3910	0.3418	0.2928	0.2441	0.1953
0.5	0.5091	0.5030	0.4872	0.4441	0.3540	0.3096	0.2653	0.2212	0.1769
0.6	0.4704	0.4636	0.4471	0.4063	0.3243	0.2837	0.2432	0.2027	0.1622
0.7	0.4381	0.4300	0.4128	0.3746	0.2994	0.2619	0.2245	0.1872	0.1497
0.8	0.4108	0.4006	0.3831	0.3478	0.2782	0.2434	0.2087	0.1739	0.1392

Table 8 Geometric function F' for $L/W = 3.0$.

a/*W*	*d*/*W*=0	0.125	0.25	0.50	1.0	1.25	1.5	1.75	2.0	2.25
0.05	0.9737	0.9661	0.9448	0.8766	0.7208	0.6469	0.5745	0.5026	0.4308	0.3590
0.1	0.8749	0.8683	0.8497	0.7899	0.6509	0.5843	0.5189	0.4540	0.3891	0.3243
0.2	0.7285	0.7230	0.7077	0.6589	0.5448	0.4893	0.4346	0.3802	0.3259	0.2716
0.3	0.6304	0.6252	0.6114	0.5687	0.4713	0.4236	0.3763	0.3292	0.2822	0.2351
0.4	0.5621	0.5572	0.5438	0.5046	0.4188	0.3766	0.3346	0.2928	0.2510	0.2091
0.5	0.5122	0.5069	0.4934	0.4565	0.3793	0.3412	0.3032	0.2653	0.2274	0.1895
0.6	0.4727	0.4669	0.4527	0.4178	0.3475	0.3127	0.2779	0.2432	0.2085	0.1737
0.7	0.4408	0.4328	0.4179	0.3852	0.3207	0.2887	0.2566	0.2245	0.1924	0.1604
0.8	0.4124	0.4030	0.3880	0.3578	0.2981	0.2683	0.2385	0.2087	0.1789	0.1490

Table 9 Geometric function F' for L/W=3.5.

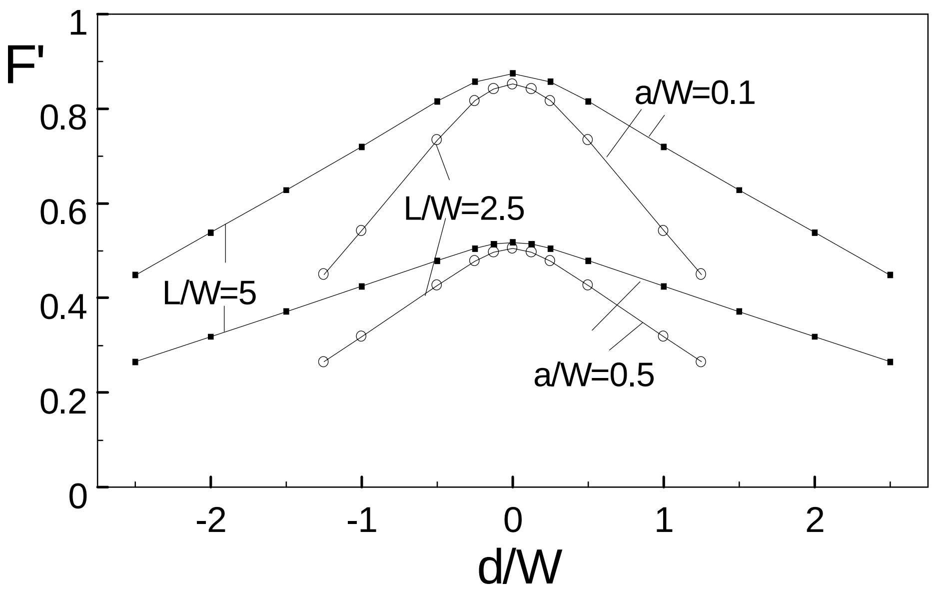
a/*W*	*d*/*W*=0	0.125	0.25	0.50	1.0	1.25	1.5	2.0	2.25	2.50
0.05	0.9777	0.9710	0.9524	0.8810	0.7465	0.6827	0.6201	0.4961	0.4340	0.3772
0.1	0.8791	0.8733	0.8570	0.7951	0.6750	0.6174	0.5608	0.4486	0.3925	0.3406
0.2	0.7325	0.7277	0.7143	0.6656	0.5667	0.5186	0.4712	0.3769	0.3298	0.2852
0.3	0.6337	0.6293	0.6143	0.5767	0.4919	0.4504	0.4093	0.3274	0.2864	0.2469
0.4	0.5651	0.5607	0.5490	0.5135	0.4386	0.4017	0.3651	0.2921	0.2555	0.2196
0.5	0.5145	0.5099	0.4981	0.4658	0.3982	0.3649	0.3317	0.2653	0.2322	0.1990
0.6	0.4744	0.4693	0.4569	0.4271	0.3655	0.3350	0.3045	0.2436	0.2131	0.1824
0.7	0.4408	0.4348	0.4218	0.3941	0.3376	0.3094	0.2813	0.2250	0.1969	0.1684
0.8	0.4124	0.4048	0.3910	0.3660	0.3136	0.2875	0.2614	0.2091	0.1830	0.1565

Table 10 Geometric function F' for L/W=4.0.

a/*W*	*d*/*W*=0	0.125	0.25	0.50	1.0	1.5
0.05	0.9704	0.9651	0.9504	0.9032	0.7956	0.6946
0.1	0.8744	0.8698	0.8570	0.8155	0.7194	0.6281
0.2	0.7316	0.7278	0.7172	0.6833	0.6041	0.5277
0.3	0.6351	0.6316	0.6220	0.5923	0.5245	0.4583
0.4	0.5677	0.5643	0.5550	0.5276	0.4677	0.4089
0.5	0.5177	0.5140	0.5046	0.4788	0.4247	0.3715
0.6	0.4775	0.4734	0.4636	0.4391	0.3898	0.3410
0.7	0.4434	0.4386	0.4283	0.4053	0.3601	0.3151
0.8	0.4142	0.4081	0.3976	0.3764	0.3345	0.2927

a/W	2.0	2.5	3.0	3.5	3.75
0.05	0.5953	0.4961	0.3969	0.2977	0.2515
0.1	0.5383	0.4486	0.3589	0.2692	0.2271
0.2	0.4523	0.3769	0.3015	0.2261	0.1902
0.3	0.3928	0.3274	0.2619	0.1964	0.1646
0.4	0.3505	0.2921	0.2337	0.1752	0.1464
0.5	0.3184	0.2653	0.2123	0.1592	0.1327
0.6	0.2923	0.2436	0.1949	0.1462	0.1216
0.7	0.2700	0.2250	0.1800	0.1350	0.1123
0.8	0.2509	0.2091	0.1673	0.1255	0.1043

Table 11 Geometric function F' for $L/W = 5.0$.



Geometric function F' as a function of eccentricity and crack depth.

Mode-II stress intensity factor

The mode-II stress intensity factor solutions are represented in the following tables by F_{II} defined with the same bending stress as used for the mode-I solution

$$K_{II} = \sigma_0 F_{II} \sqrt{\pi a}\,, \quad \sigma_0 = \frac{3PL}{W^2}$$

a/W	*d/W*=0	0.0625	0.125	0.5	1.0	1.25	1.5
0.05	0	0.0031	0.0059	0.0138	0.0090	0.0064	0.0042
0.1	0	0.0057	0.0110	0.0258	0.0172	0.0123	0.0080
0.2	0	0.0101	0.0196	0.046	0.0314	0.0226	0.0147
0.3	0	0.0141	0.0272	0.0621	0.0426	0.0309	0.0202
0.4	0	0.0183	0.0351	0.0756	0.0516	0.0377	0.0248
0.5	0	0.0237	0.0450	0.0876	0.0589	0.0434	0.0286
0.6	0	0.0320	0.0594	0.0994	0.0658	0.0487	0.0323
0.7	0	0.0469	0.0833	0.1124	0.0738	0.0550	0.0365
0.8	0	0.0798	0.1275	0.1309	0.0862	0.0645	0.0430

Table 12 Geometric function F_{II} for $L/W = 2.0$.

a/W	*d/W*=0	0.025	0.04	0.05	0.10	0.125	0.15	0.20	0.25
0.05	0	0.0010	0.0016	0.0020	0.0040	0.0049	0.0058	0.0074	0.0087
0.1	0	0.0019	0.0030	0.0038	0.0074	0.0091	0.0108	0.0138	0.0163
0.2	0	0.0034	0.0054	0.0067	0.0132	0.0163	0.0192	0.0245	0.0290
0.3	0	0.0047	0.0075	0.0094	0.0183	0.0226	0.0266	0.0338	0.0398
0.4	0	0.0061	0.0098	0.0121	0.0237	0.0291	0.0341	0.0431	0.0503
0.5	0	0.0079	0.0126	0.0157	0.0304	0.0371	0.0433	0.0539	0.0620
0.6	0	0.0107	0.0170	0.0212	0.0404	0.0488	0.0562	0.0683	0.0766
0.7	0	0.0159	0.0251	0.0311	0.0575	0.0681	0.0768	0.0892	0.0961
0.8	0	0.0279	0.0436	0.0534	0.0915	0.1037	0.1120	0.1206	0.1229

a/W	0.40	0.50	0.75	0.875	1.0	1.125	1.25	1.50	1.75
0.05	0.0112	0.0117	0.0105	0.0095	0.0085	0.0076	0.0068	0.0053	0.0039
0.1	0.0210	0.0219	0.0200	0.0182	0.0163	0.0146	0.0130	0.0102	0.0076
0.2	0.0373	0.0391	0.0360	0.0329	0.0297	0.0267	0.0239	0.0187	0.0139
0.3	0.0507	0.0529	0.0487	0.0447	0.0405	0.0365	0.0327	0.0258	0.0192
0.4	0.0625	0.0644	0.0587	0.0539	0.0491	0.0444	0.0399	0.0316	0.0236
0.5	0.0739	0.0747	0.0669	0.0616	0.0562	0.0510	0.0461	0.0365	0.0273
0.6	0.0858	0.0846	0.0745	0.0687	0.0629	0.0573	0.0518	0.0412	0.0309
0.7	0.0994	0.0958	0.0833	0.0769	0.0707	0.0646	0.0586	0.0467	0.0350
0.8	0.1175	0.1116	0.0969	0.0898	0.0827	0.0757	0.0688	0.0550	0.0412

Table 13 Geometric function F_{II} for $L/W = 2.5$.

a/W	d/W=0	0.125	0.25	0.50	1.0	1.25	1.50	1.75	2.0
0.05	0	0.0042	0.0075	0.0101	0.0078	0.0066	0.0055	0.0046	0.0037
0.1	0	0.0078	0.0139	0.0190	0.0150	0.0126	0.0106	0.0088	0.0070
0.2	0	0.0139	0.0248	0.0339	0.0274	0.0231	0.0195	0.0161	0.0129
0.3	0	0.0192	0.0341	0.0458	0.0373	0.0317	0.0268	0.0222	0.0178
0.4	0	0.0248	0.0430	0.0558	0.0453	0.0387	0.0328	0.0273	0.0218
0.5	0	0.0316	0.0529	0.0647	0.0519	0.0447	0.0380	0.0316	0.0255
0.6	0	0.0414	0.0653	0.0734	0.0581	0.0503	0.0429	0.0357	0.0286
0.7	0	0.0576	0.0817	0.0831	0.0654	0.0569	0.0487	0.0405	0.0324
0.8	0	0.0874	0.1043	0.0969	0.0766	0.0669	0.0573	0.0477	0.0382

Table 14 Geometric function F_{II} for $L/W = 3.0$.

a/W	0	0.125	0.25	0.50	1.0	1.25	1.5	1.75	2.00	2.25
0.05	0	0.0036	0.0065	0.0089	0.0072	0.0062	0.0054	0.0047	0.0040	0.0034
0.1	0	0.0068	0.0122	0.0167	0.0137	0.0119	0.0104	0.0090	0.0077	0.0064
0.2	0	0.0121	0.0216	0.0298	0.0250	0.0218	0.0191	0.0166	0.0142	0.0118
0.3	0	0.0168	0.0297	0.0404	0.0342	0.0299	0.0262	0.0229	0.0196	0.0163
0.4	0	0.0216	0.0376	0.0492	0.0415	0.0365	0.0322	0.0280	0.0240	0.0200
0.5	0	0.0274	0.0462	0.0570	0.0476	0.0422	0.0372	0.0325	0.0278	0.0232
0.6	0	0.0359	0.0568	0.0647	0.0533	0.0475	0.0421	0.0367	0.0315	0.0262
0.7	0	0.0498	0.0711	0.0732	0.0600	0.0537	0.0477	0.0417	0.0357	0.0298
0.8	0	0.0755	0.0906	0.0854	0.0703	0.0632	0.0561	0.0491	0.0421	0.0351

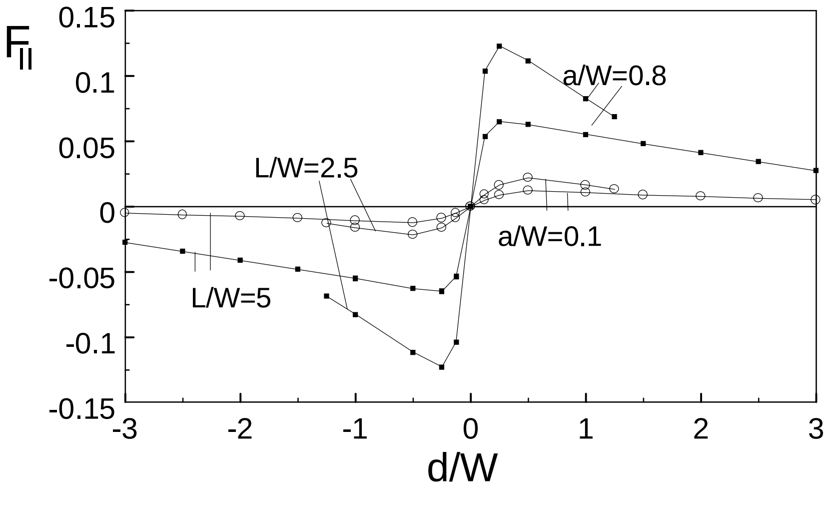
Table 15 Geometric function F_{II} for $L/W = 3.5$.

a/*W*	0	0.125	0.25	0.50	1.0	1.25	1.5	2.0	2.25	2.50
0.05	0	0.0032	0.0058	0.0079	0.0066	0.0058	0.0052	0.0041	0.0036	0.0031
0.1	0	0.0060	0.0108	0.0149	0.0126	0.0111	0.0099	0.0079	0.0069	0.0059
0.2	0	0.0107	0.0192	0.0266	0.0229	0.0204	0.0182	0.0145	0.0127	0.0109
0.3	0	0.0148	0.0264	0.0360	0.0313	0.0280	0.0251	0.0200	0.0175	0.0150
0.4	0	0.0191	0.0333	0.0439	0.0380	0.0342	0.0308	0.0245	0.0214	0.0184
0.5	0	0.0243	0.0409	0.0509	0.0437	0.0394	0.0356	0.0284	0.0249	0.0213
0.6	0	0.0317	0.0503	0.0577	0.0490	0.0445	0.0402	0.0321	0.0281	0.0241
0.7	0	0.0439	0.0628	0.0653	0.0551	0.0503	0.0456	0.0365	0.0319	0.0274
0.8	0	0.0664	0.0800	0.0762	0.0646	0.0591	0.0537	0.0430	0.0376	0.0322

Table 16 Geometric function F_{II} for $L/W = 4.0$.

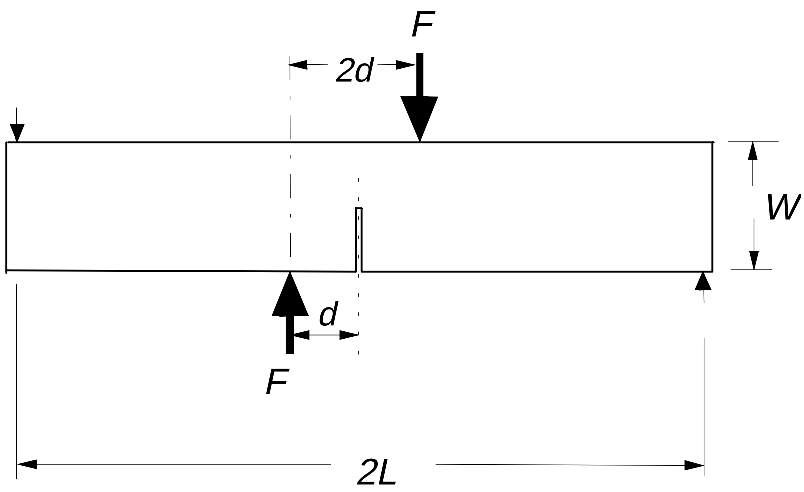
a/*W*	*d*/*W*=0	0.125	0.25	0.50	1.0	1.5	2.0	2.5	3.0	3.5	3.75
0.05	0	0.0026	0.0047	0.0065	0.0056	0.0046	0.0039	0.0033	0.0026	0.0020	0.0016
0.1	0	0.0049	0.0088	0.0122	0.0107	0.0089	0.0076	0.0063	0.0051	0.0038	0.0032
0.2	0	0.0087	0.0157	0.0219	0.0195	0.0163	0.0139	0.0116	0.0093	0.0070	0.0058
0.3	0	0.0121	0.0215	0.0296	0.0266	0.0225	0.0192	0.0160	0.0128	0.0096	0.0080
0.4	0	0.0155	0.0271	0.0361	0.0324	0.0276	0.0235	0.0196	0.0157	0.0118	0.0098
0.5	0	0.0197	0.0333	0.0419	0.0372	0.0319	0.0273	0.0227	0.0182	0.0136	0.0114
0.6	0	0.0257	0.0409	0.0475	0.0413	0.0361	0.0309	0.0257	0.0206	0.0154	0.0129
0.7	0	0.0355	0.0510	0.0537	0.0470	0.0409	0.0350	0.0292	0.0233	0.0175	0.0146
0.8	0	0.0536	0.0649	0.0627	0.0551	0.0481	0.0412	0.0344	0.0275	0.0206	0.0172

Table 17 Geometric function F_{II} for $L/W = 5.0$.



Geometric function F_{II} as a function of eccentricity and crack depth

2.4 The asymmetric 4-point bending test



The geometric functions F_I and F_{II} are defined by

$$K_I = \frac{F}{BW}\left(1-\frac{d}{L}\right)F_I\sqrt{\pi a} \quad , K_{II} = \frac{F}{BW}\left(1-\frac{d}{L}\right)F_{II}\sqrt{\pi a}$$

a/*W*	*d*/*W*=0.025	0.05	0.1	0.125	0.15	0.20	0.30
0.1	3.4311	2.6223	1.3401	0.9719	0.7334	0.4835	0.3426
0.2	1.9547	1.8070	1.4662	1.3023	1.1571	0.9367	0.7238
0.3	1.4029	1.3739	1.2957	1.2501	1.2036	1.1173	1.0011
0.4	1.1443	1.1601	1.1837	1.1908	1.1949	1.1967	1.1882
0.5	1.0237	1.0677	1.1507	1.1878	1.2213	1.2763	1.3422
0.6	0.9943	1.0661	1.2019	1.2627	1.3173	1.4065	1.5114
0.7	1.0551	1.1677	1.3701	1.4543	1.5258	1.6322	1.7361
0.8	1.2632	1.4574	1.7587	1.8603	1.9339	2.0203	2.0735

a/*W*	*d*/*W*=0.4	0.5	0.75	0.875	1.00	1.125	1.25
0.1	0.3308	0.3431	0.3695	0.3748	0.3773	0.3783	0.3786
0.2	0.6673	0.6633	0.6853	0.6915	0.6946	0.6960	0.6965
0.3	0.9542	0.9432	0.9519	0.9557	0.9578	0.9587	0.9591
0.4	1.1799	1.1761	1.1755	1.1760	1.1763	1.1765	1.1765
0.50	1.3675	1.3733	1.3680	1.3658	1.3646	1.3640	1.3638
0.60	1.5502	1.5583	1.5495	1.5461	1.5442	1.5434	1.5431
0.70	1.7647	1.7670	1.7560	1.7529	1.7514	1.7507	1.7504
0.80	2.0778	2.0735	2.0649	2.0633	2.0625	2.0623	2.0621

Table 18 Geometric function F_{II} for L/W=2.5.

a/*W*	*d*/*W*=0.025	0.05	0.1	0.125	0.15	0.20	0.30
0.1	-1.9336	-1.7418	-0.9978	-0.6406	-0.3487	0.0507	0.3949
0.2	-0.8259	-0.7661	-0.6023	-0.4986	-0.3887	-0.1776	0.1198
0.3	-0.5437	-0.4805	-0.3607	-0.3020	-0.2437	-0.1318	0.0473
0.4	-0.4459	-0.3734	-0.2472	-0.1932	-0.1446	-0.0620	0.0514
0.5	-0.4086	-0.3224	-0.1797	-0.1232	-0.0756	-0.0036	0.0718
0.6	-0.3928	-0.2856	-0.1200	-0.0605	-0.0142	0.0463	0.0862
0.7	-0.3803	-0.2385	-0.0459	0.0113	0.0494	0.0851	0.0817
0.8	-0.3483	-0.1474	0.0525	0.0858	0.0968	0.0872	0.0484

a/*W*	*d*/*W*=0.4	0.5	0.75	0.875	1.00	1.125	1.25
0.1	0.4663	0.4330	0.2433	0.1604	0.0984	0.0557	0.0284
0.2	0.2492	0.2753	0.1818	0.1252	0.0799	0.0473	0.0256
0.3	0.1455	0.1772	0.1281	0.0902	0.0586	0.0353	0.0196
0.4	0.1074	0.1227	0.0852	0.0595	0.0384	0.0230	0.0126
0.50	0.0934	0.0899	0.0522	0.0347	0.0214	0.0123	0.0063
0.60	0.0800	0.0629	0.0270	0.0160	0.0088	0.0044	0.0018
0.70	0.0563	0.0354	0.0091	0.0039	0.0011	-0.0002	-0.0007
0.80	0.0244	0.0118	0.0008	-0.0006	-0.0011	-0.0011	-0.0008

Table 19 Geometric function F_I for L/W=2.5.

a/*W*	*d*/*W*=0.025	0.05	0.1	0.15	0.25	0.50
0.1	3.4250	2.6140		0.7295	0.3817	0.3444
0.2	1.9521	1.8029	1.4607	1.1521	0.7985	0.6646
0.3	1.4018	1.3722	1.2932	1.2009	1.0473	0.9438
0.4	1.1440	1.1600	1.1835	1.1947	1.1930	1.1761
0.50	1.0240	1.0685	1.1520	1.2227	1.3167	1.3729
0.60	0.9950	1.0676	1.2042	1.3196	1.4712	1.5577
0.70	1.0561	1.1695	1.3726	1.5281	1.6992	1.7663
0.80	1.2643	1.4592	1.7606	1.9351	2.0587	2.0731

a/*W*	*d*/*W*=0.75	1.0	1.25	1.50
0.1	0.3700	0.3774	0.3786	0.3787
0.2	0.6860	0.6948	0.6965	0.6967
0.3	0.9524	0.9579	0.9591	0.9593
0.4	1.1756	1.1763	1.1765	1.1766
0.50	1.3678	1.3645	1.3638	1.3637
0.60	1.5491	1.5441	1.5430	1.5429
0.70	1.7557	1.7513	1.7504	1.7503
0.80	2.0648	2.0625	2.0621	2.0621

Table 20 Geometric function F_{II} for L/W=3.0.

a/W	d/W=0.025	0.05	0.1	0.15	0.25	0.50
0.1	-1.9304	-1.7354		-0.3451	0.2705	0.4157
0.2	-0.8246	-0.7632	-0.5982	-0.3846	-0.0049	0.2643
0.3	-0.5428	-0.4786	-0.3583	-0.2412	-0.0321	0.1701
0.4	-0.4452	-0.3718	-0.2455	-0.1431	0.0027	0.1178
0.50	-0.4080	-0.3210	-0.1785	-0.0748	0.0426	0.0864
0.60	-0.3922	-0.2842	-0.1192	-0.0140	0.0744	0.0604
0.70	-0.3798	-0.2370	-0.0457	0.0488	0.0884	0.0340
0.80	-0.3479	-0.1457	0.0519	0.0958	0.0656	0.0114

a/W	d/W=0.75	1.0	1.25	1.50
0.1	0.2271	0.0886	0.0244	0.0024
0.2	0.1697	0.0719	0.0219	0.0035
0.3	0.1196	0.0527	0.0168	0.0030
0.4	0.0796	0.0345	0.0108	0.0018
0.50	0.0487	0.0193	0.0055	0.006
0.60	0.0252	0.0080	0.0015	-0.0002
0.70	0.0086	0.0011	-0.0005	-0.0005
0.80	0.0008	-0.0009	-0.0007	-0.0003

Table 21 Geometric function F_I for L/W=3.0.

a/W	d/W=0.025	0.05	0.1	0.20	0.25	0.50
0.1	3.4267	2.6020	1.3236	0.4794	0.3811	0.3458
0.2	1.9536	1.7969	1.4535	0.9287	0.7960	0.6660
0.3	1.4037	1.3696	1.2899	1.1121	1.0452	0.9445
0.4	1.1463	1.1594	1.1831	1.1958	1.1925	1.1760
0.50	1.0266	1.0691	1.1535	1.2789	1.3177	1.3724
0.60	0.9980	1.0690	1.2068	1.4106	1.4726	1.5569
0.70	1.0594	1.1714	1.3755	1.6357	1.7003	1.7655
0.80	1.2681	1.4611	1.7629	2.0214	2.0586	2.0724

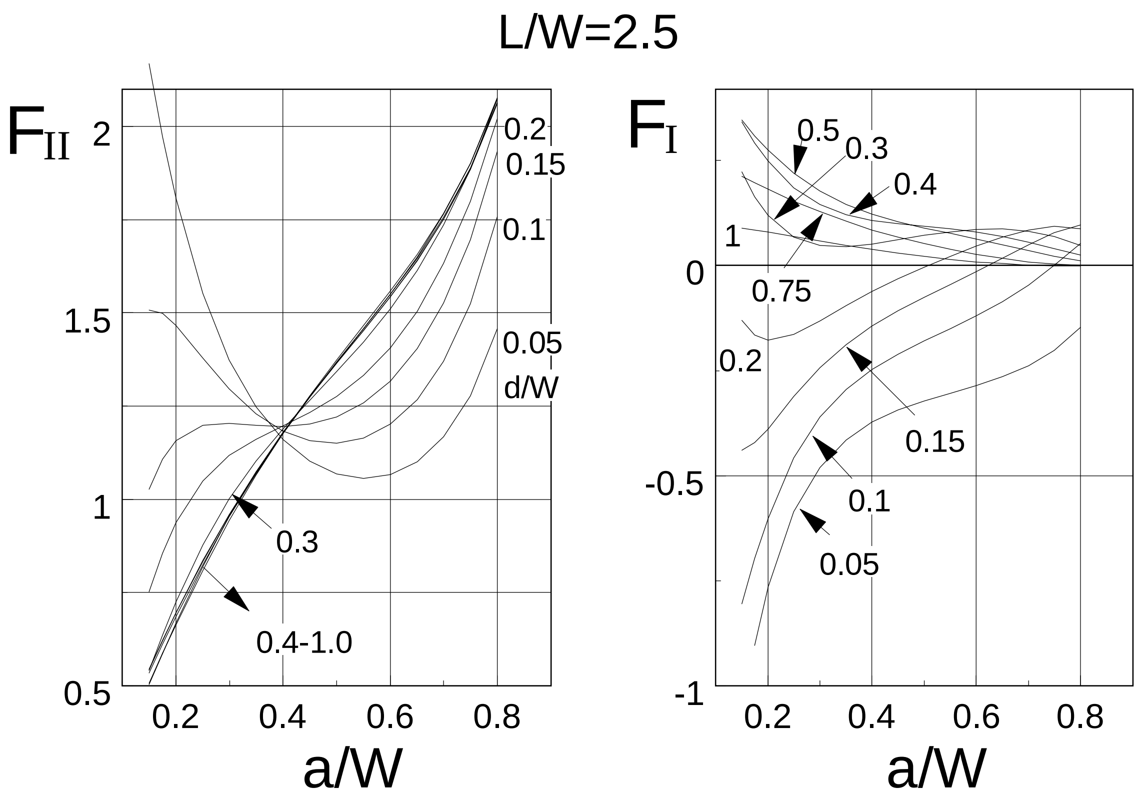
a/W	0.75	0.85	1.0	1.25	1.50
0.1	0.3705	0.3746	0.3774	0.3785	0.3786
0.2	0.6868	0.6915	0.6949	0.6965	0.6966
0.3	0.9528	0.9557	0.9580	0.9591	0.9592
0.4	1.1757	1.176	1.1763	1.1765	1.1766
0.50	1.3673	1.3657	1.3644	1.3637	1.3637
0.60	1.5486	1.5459	1.5439	1.5430	1.5429
0.70	1.7553	1.7528	1.7511	1.7504	1.7503
0.80	2.0645	2.0632	2.0624	2.0621	2.0621

Table 22 Geometric function F_{II} for L/W=4.0.

a/*W*	*d*/*W*=0.025	0.05	0.1	0.20	0.25	0.50
0.1	-1.9262	-1.7281	-0.9824	0.0491	0.2646	0.3959
0.2	-0.8228	-0.7600	-0.5930	-0.1719	-0.0048	0.2518
0.3	-0.5416	-0.4766	-0.3552	-0.1276	-0.0314	0.1620
0.4	-0.4442	-0.3703	-0.2434	-0.0600	0.0027	0.1122
0.50	-0.4070	-0.3196	-0.1769	-0.0035	0.0417	0.0823
0.60	-0.3913	-0.2830	-0.1181	0.0449	0.0728	0.0576
0.70	-0.3788	-0.2361	-0.0452	0.0825	0.0865	0.0325
0.80	-0.3467	-0.1451	0.0516	0.0846	0.0643	0.0110

a/*W*	0.75	0.85	1.0	1.25	1.50
0.1	0.2096	0.1470	0.0788	0.0207	0.0020
0.2	0.1567	0.1139	0.0640	0.0187	0.0028
0.3	0.1104	0.0817	0.0469	0.0143	0.0024
0.4	0.0735	0.0540	0.0307	0.0092	0.0015
0.50	0.0450	0.0318	0.0172	0.0047	0.0006
0.60	0.0233	0.0151	0.0072	0.0014	-0.0001
0.70	0.0080	0.0040	0.0010	-0.0004	-0.0003
0.80	0.0009	-0.0002	-0.0007	-0.0004	0.0000

Table 23 Geometric function F_I for L/W=5.0.

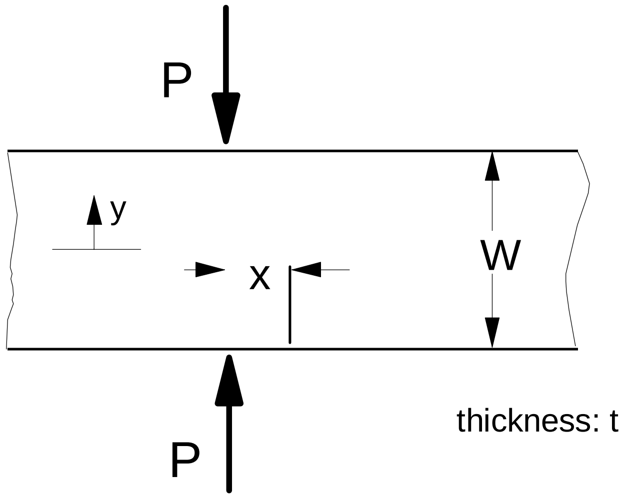


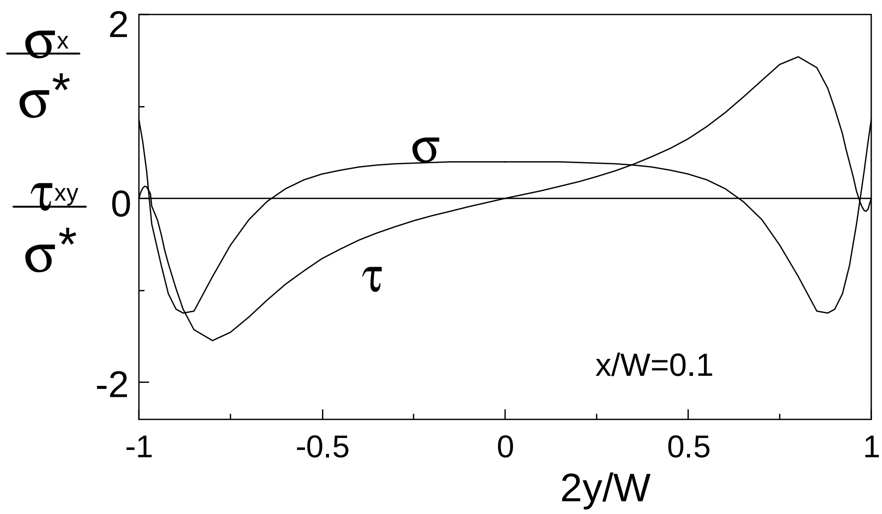
Geometric functions for the asymmetric 4-point bending test (L/W=2.5).

2.5 Crack in the neighbourhood of opposite concentrated forces

The stress intensity factors K_I and K_{II} with the geometric functions F_I and F_{II} are defined by

$$K_I = \sigma^* \sqrt{\pi a} F_I \;, \qquad K_{II} = \sigma^* \sqrt{\pi a} F_{II} \;, \quad \sigma^* = \frac{P}{Wt}$$





Stresses across the bar.

a/*W*	*x*/*W*=0.025	0.05	0.1	0.125	0.15	0.20	0.25	0.30
0.1	-3.369	-2.513	-1.175	-0.786	-0.528	-0.238	-0.099	-0.026
0.2	-1.885	-1.670	-1.210	-0.993	-0.800	-0.495	-0.286	-0.148
0.3	-1.319	-1.206	-0.969	-0.849	-0.733	-0.522	-0.347	-0.212
0.4	-1.041	-0.955	-0.781	-0.695	-0.612	-0.455	-0.319	-0.207
0.5	-0.895	-0.811	-0.649	-0.572	-0.499	-0.366	-0.254	-0.163
0.6	-0.824	-0.728	-0.549	-0.468	-0.395	-0.270	-0.174	-0.102
0.7	-0.807	-0.678	-0.453	-0.361	-0.282	-0.163	-0.086	-0.038
0.8	-0.832	-0.627	-0.316	-0.212	-0.138	-0.050	-0.010	0.008

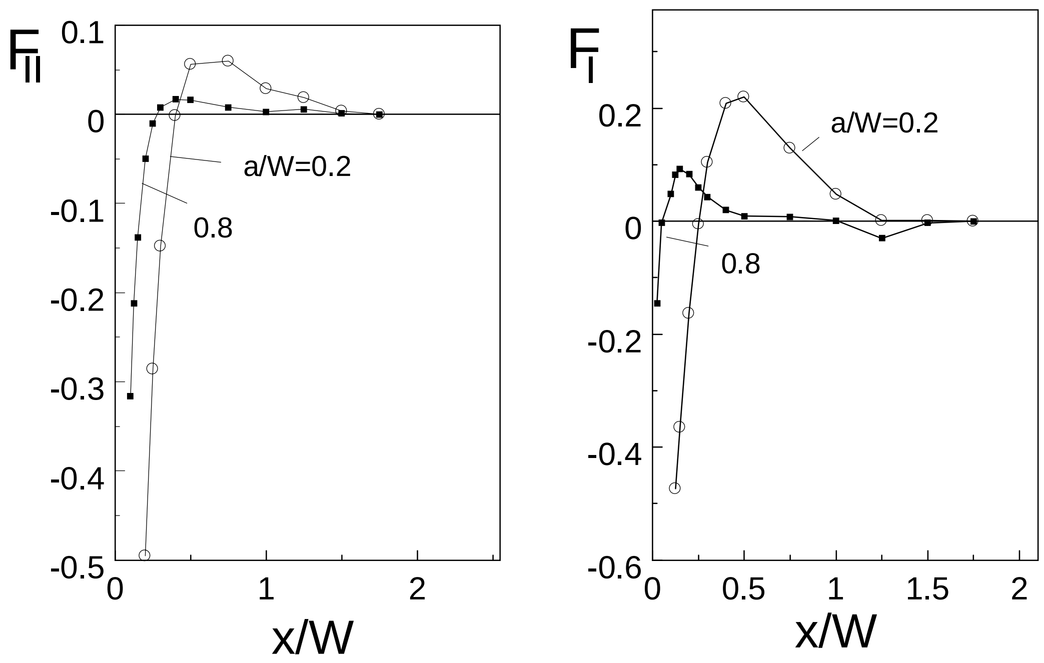
a/*W*	0.4	0.50	0.75	1.0	1.25	1.5
0.1	0.037	0.054	0.040	0.017	0.024	0.005
0.2	-0.001	0.056	0.060	0.029	0.019	0.004
0.3	-0.041	0.038	0.064	0.032	0.018	0.004
0.4	-0.053	0.025	0.057	0.030	0.016	0.004
0.5	-0.041	0.021	0.046	0.024	0.013	0.003
0.6	-0.016	0.023	0.033	0.017	0.010	0.002
0.7	0.008	0.023	0.020	0.009	0.007	0.001
0.8	0.017	0.016	0.008	0.003	0.006	0.001

Table 24 Geometric function F_{II}.

a/*W*	*x*/*W*=0.025	0.04	0.05	0.1	0.125	0.15	0.20	0.25
0.1	-1.906	-1.832	-1.704	-0.959	-0.624	-0.328	0.048	0.248
0.2	-0.812	-0.791	-0.749	-0.579	-0.484	-0.365	-0.163	-0.005
0.3	-0.533	-0.509	-0.469	-0.347	-0.296	-0.229	-0.121	-0.029
0.4	-0.436	-0.408	-0.364	-0.238	-0.193	-0.136	-0.056	0.003
0.5	-0.398	-0.366	-0.313	-0.173	-0.129	-0.071	-0.003	0.039
0.6	-0.380	-0.342	-0.276	-0.116	-0.073	-0.013	0.044	0.068
0.7	-0.364	-0.317	-0.229	-0.046	-0.013	0.047	0.080	0.081
0.8	-0.322	-0.269	-0.135	0.047	0.040	0.092	0.083	0.060

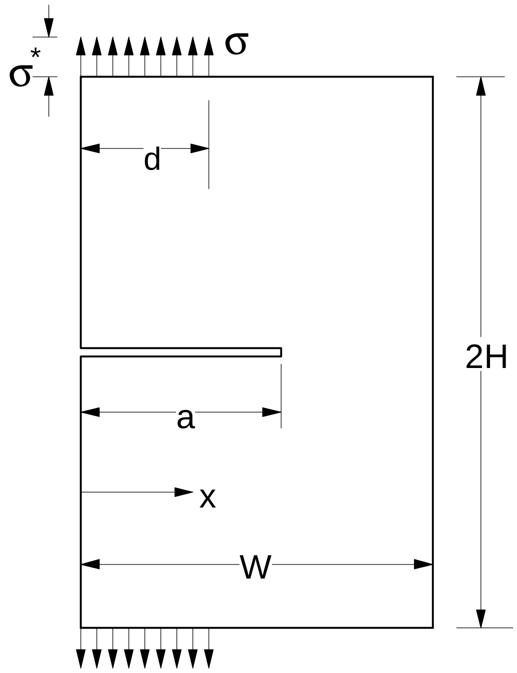
a/W	x/W=0.30	0.4	0.50	0.75	1.0	1.25	1.5
0.1	0.347	0.392	0.346	0.173	0.060	-0.005	0.000
0.2	0.105	0.209	0.220	0.129	0.048	0.001	0.001
0.3	0.041	0.122	0.142	0.091	0.036	-0.001	0.001
0.4	0.045	0.090	0.098	0.061	0.024	-0.006	0.000
0.5	0.063	0.079	0.072	0.039	0.013	-0.011	-0.001
0.6	0.076	0.067	0.050	0.022	0.006	-0.019	-0.001
0.7	0.072	0.047	0.028	0.011	0.002	-0.026	-0.002
0.8	0.043	0.020	0.009	0.008	0.001	-0.030	-0.003

Table 25 Geometric function F_I.



Geometric function for an edge crack near concentrated forces.

2.6 Partially loaded rectangular plate with edge crack



Partially loaded edge-cracked rectangular plate.

Geometric function for stress intensity factor defined by

$$K_I = \sigma^* F\sqrt{\pi a}$$

α=*a*/*W*	*d*/*W*=0	0.25	0.5	0.75	1.0
0.3	0	1.049	1.643	1.859	1.637
0.4	0	1.245	1.990	2.318	2.103
0.5	0	1.546	2.538	2.968	2.825
0.6	0	2.054	3.472	4.080	4.034
0.7	0	3.138	5.274	6.191	6.327

Table 26 Geometric function *F* for *H*/*W*=1.25.

α=*a*/*W*	*d*/*W*=0	0.25	0.5	0.75	1.0
0.3	0	1.056	1.668	1.871	1.656
0.4	0	1.280	2.009	2.296	2.112
0.5	0	1.568	2.599	2.982	2.824
0.6	0	2.139	3.483	4.101	4.035
0.7	0	3.207	5.229	6.280	6.353

Table 27 Geometric function *F* for *H*/*W*=1.00.

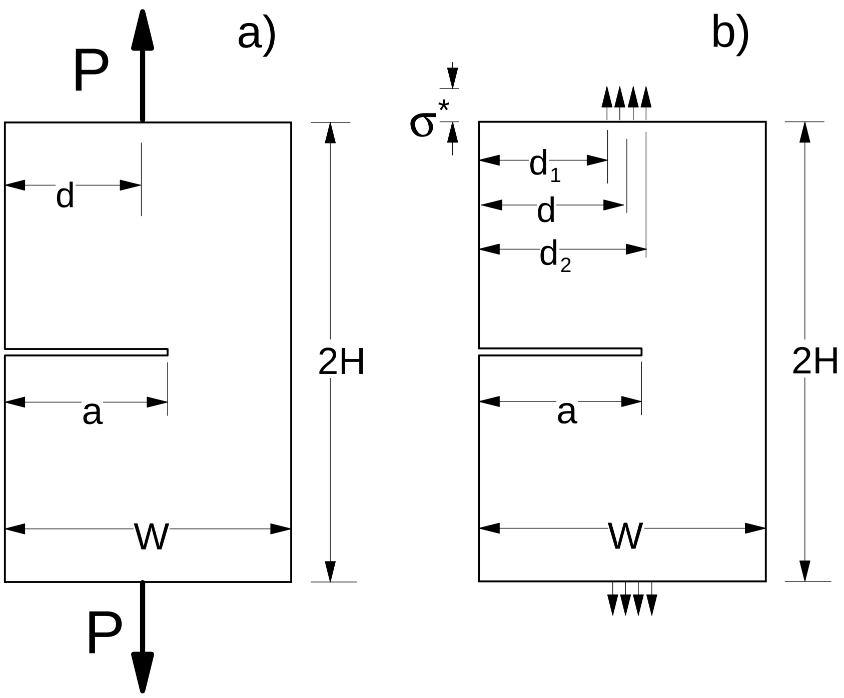
α = *a*/*W*	*d*/*W*=0	0.25	0.5	0.75	1.0
0.3	0	1.100	1.697	1.864	1.681
0.4	0	1.302	2.038	2.295	2.135
0.5	0	1.614	2.612	3.012	2.842
0.6	0	2.129	3.435	4.099	4.043
0.7	0	3.174	5.209	6.284	6.357

Table 28 Geometric function *F* for *H*/*W*=0.75.

α = *a*/*W*	*d*/*W*=0	0.25	0.5	0.75	1.0
0.3	0	1.296	1.862	1.961	1.847
0.4	0	1.479	2.242	2.422	2.323
0.5	0	1.676	2.752	3.126	3.007
0.6	0	2.193	3.575	4.249	4.146
0.7	0	3.190	5.240	6.307	6.386

Table 29 Geometric function *F* for *H*/*W*=0.50.

An example of application of this loading case may be demonstrated for a plate with *H*/*W* = 1.25 loaded by a couple of point forces *P* at several locations *d*/*W* as illustrated below.



Computation of stress intensity factors in plates loaded by a couple of point forces.

First, we determine the stress intensity factor -values for two values d_1 and d_2 with $d_1 = d$-ε and $d_2 = d$+ε (ε « d) by interpolation of the tabulated results applying cubic splines. The normal force P is given by

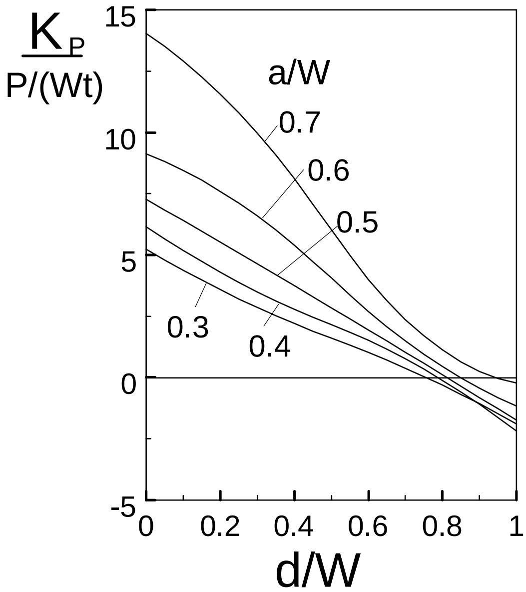
$$P = \sigma^*(d_2 - d_1)t$$

(t = thickness). The stress intensity factor for this case is

$$K_P = \left(\frac{K_2}{\sigma^*} - \frac{K_1}{\sigma^*}\right)\sigma^* = \left(\frac{K_2}{\sigma^*} - \frac{K_1}{\sigma^*}\right)\frac{P}{t(d_2 - d_1)}$$

and for the case of d_1, $d_2 \rightarrow d$ ($\varepsilon \rightarrow 0$)

$$K_P = \frac{\partial(K/\sigma^*)}{\partial(d/W)}\frac{P}{Wt}\,.$$

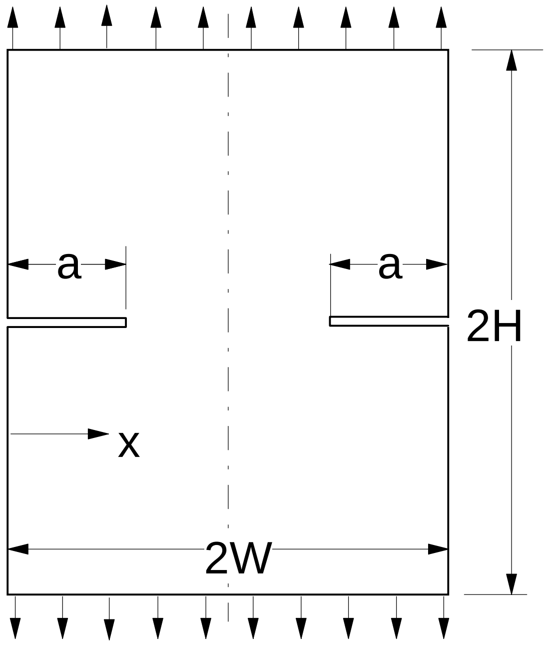


Stress intensity factor caused by a couple of forces acting at location d (H/W= 1.25).

If a smooth distribution of normal tractions acts at the ends of the plate it is of advantage to evaluate

$$K = \frac{K_d}{\sigma^*}\sigma_n\Big|_{x=d=W} - \int_0^W \frac{K_d}{\sigma^*}\frac{\mathrm{d}\sigma}{\mathrm{d}x}\,\mathrm{d}x\,.$$

2.7 Double-edge-cracked plate



Stress intensity factor for pure tension

$$K_I = \sigma F \sqrt{\pi a}\ , \quad F' = F\,(1 - a/W)^{1/2}$$

a/*W*	*L*/*W*=1.5	1.25	1.0	0.75	0.50	0.35
0	1.1215	1.1215	1.1215	1.1215	1.1215	1.1215
0.3	0.94	0.96	1.029	1.18	1.496	1.891
0.4	0.8891	0.9197	0.9946	1.1926	1.646	2.196
0.5	0.8389	0.8659	0.9427	1.1537	1.719	2.437
0.6	0.7900	0.8135	0.8760	1.0597	1.6529	2.535
0.7	0.7420	0.7492	0.8029	0.9297	1.4142	2.46
1.0	0.6366	0.6366	0.6366	0.6366	0.6366	0.6366

Table 30 Geometric function F_I' [10].

Weight function for symmetric loading, represented in the form of

$$h' = h\sqrt{W}\,\sqrt{1 - x/a}$$

x/*a*	*L*/*W*=0.35	0.50	0.75	1.00	1.50
0	10.70	7.21	4.27	3.04	2.39
0.1	9.45	6.49	3.92	2.84	2.26
0.2	8.28	5.80	3.58	2.64	2.14
0.3	7.18	5.12	3.24	2.44	2.01
0.4	6.14	4.47	2.91	2.24	1.89
0.5	5.16	3.84	2.60	2.06	1.78
0.6	4.25	3.25	2.29	1.88	1.66
0.7	3.41	2.69	2.01	1.71	1.55
0.8	2.63	2.16	1.73	1.54	1.45
0.9	1.91	1.68	1.49	1.40	1.35
1.0	1.262	1.262	1.262	1.262	1.262

Table 31 Weight function *h*' for *a*/*W*=0.4.

x/*a*	*L*/*W*=0.35	0.50	0.75	1.00	1.50
0	12.2	7.84	4.20	2.89	2.26
0.1	10.8	7.01	3.85	2.70	2.15
0.2	9.40	6.20	3.49	2.51	2.03
0.3	8.10	5.43	3.14	2.31	1.91
0.4	6.90	4.68	2.80	2.12	1.79
0.5	5.70	3.97	2.48	1.93	1.67
0.6	4.65	3.30	2.17	1.75	1.55
0.7	3.65	2.68	1.87	1.58	1.44
0.8	2.75	2.10	1.60	1.41	1.32
0.9	1.90	1.56	1.35	1.26	1.22
1.0	1.128	1.128	1.128	1.128	1.128

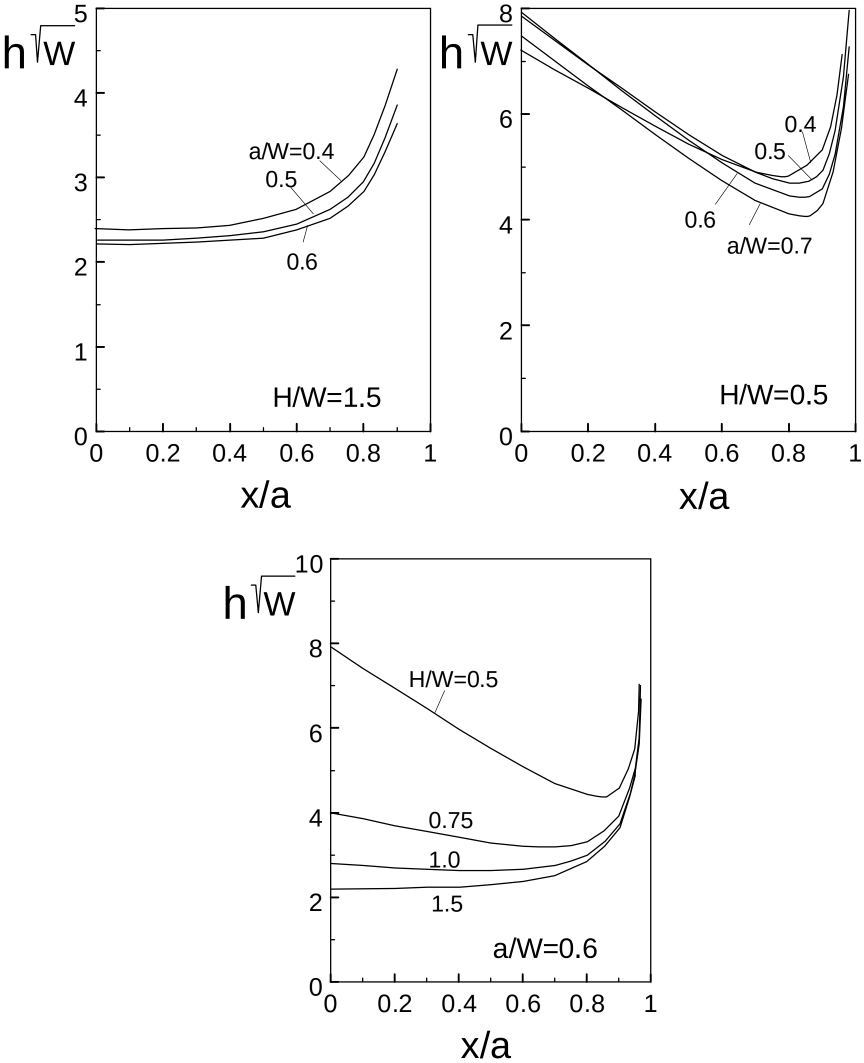
Table 32 Weight function *h*' for *a*/*W*=0.5.

x/*a*	*L*/*W*=0.35	0.50	0.75	1.00	1.50
0	13.2	7.91	4.00	2.79	2.21
0.1	11.65	7.04	3.66	2.61	2.09
0.2	10.20	6.21	3.31	2.42	1.98
0.3	8.75	5.40	2.98	2.23	1.87
0.4	7.45	4.63	2.65	2.04	1.75
0.5	6.20	3.90	2.33	1.86	1.62
0.6	5.00	3.21	2.03	1.68	1.51
0.7	3.90	2.57	1.75	1.51	1.38
0.8	2.85	1.99	1.48	1.34	1.27
0.9	1.91	1.45	1.24	1.18	1.15
1.0	1.030	1.030	1.030	1.030	1.030

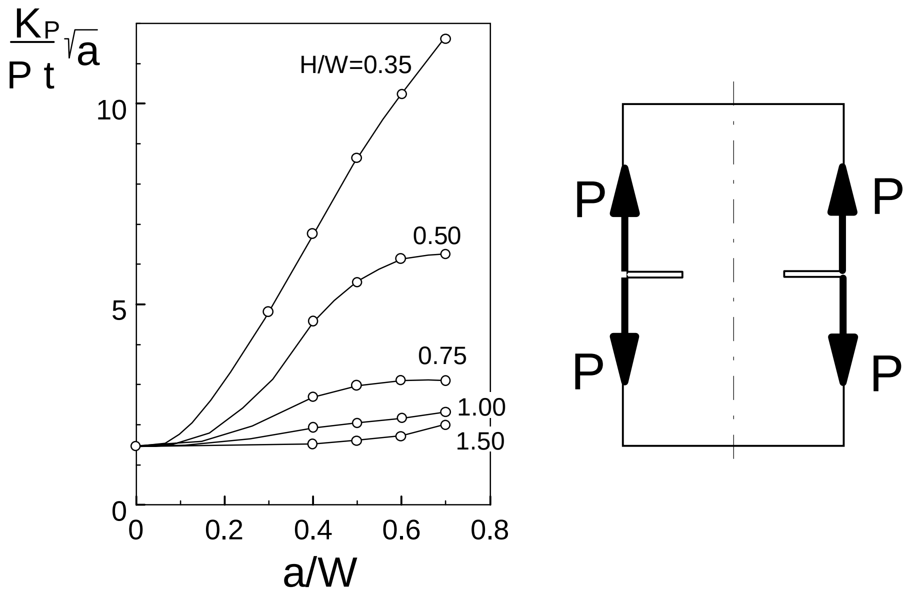
Table 33 Weight function *h*' for *a*/*W*=0.6.

x/*a*	*L*/*W*=0.35	0.50	0.75	1.00	1.50
0	13.90	7.47	3.69	2.77	2.40
0.1	12.25	6.65	3.39	2.59	2.27
0.2	10.70	5.85	3.09	2.41	2.13
0.3	9.20	5.09	2.78	2.23	1.96
0.4	7.80	4.35	2.49	2.05	1.82
0.5	6.48	3.65	2.20	1.86	1.67
0.6	5.23	3.00	1.93	1.68	1.53
0.7	4.05	2.39	1.66	1.50	1.38
0.8	2.95	1.84	1.41	1.32	1.24
0.9	1.91	1.36	1.18	1.13	1.10
1.0	0.954	0.954	0.954	0.954	0.954

Table 34 Weight function *h*' for *a*/*W*=0.7.



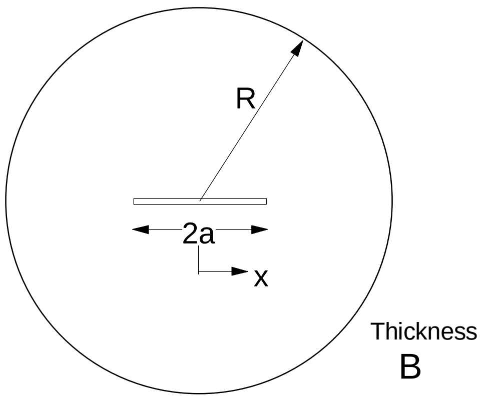
Influence of the plate height and crack length on the weight function.



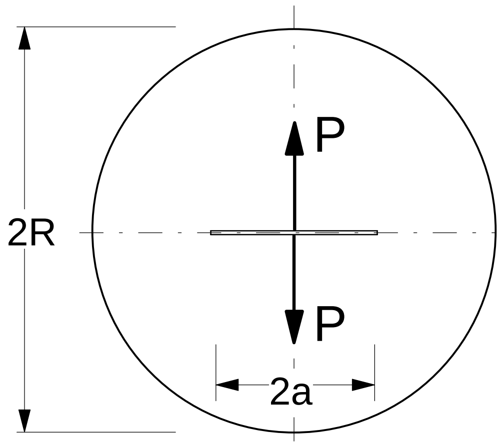
Stress intensity factor K_P for pairs of concentrated forces at the crack mouth (x = 0).

3 Internally cracked circular disk

3.1 Mode-I loading



Internal radial crack in a circular disk (geometric data).

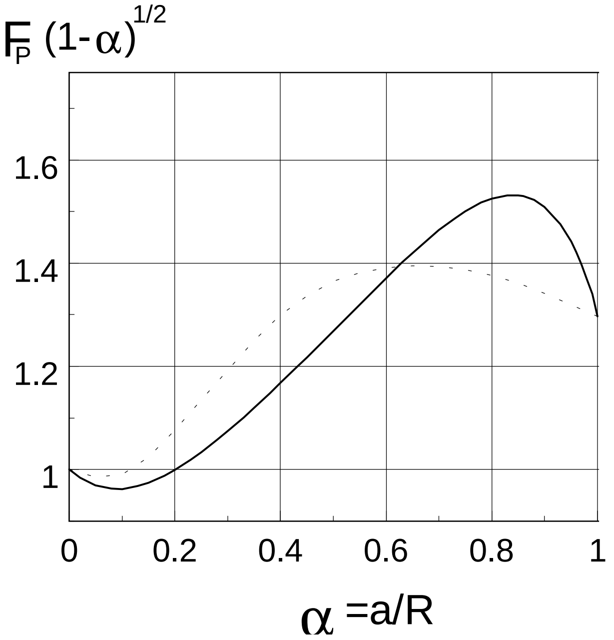


Circular disk with a couple of forces acting on the crack faces.

Stress intensity factor for central point forces [11]

$$K_I = \frac{P}{\sqrt{\pi a}} F_P$$

$$F_P = \frac{1 - 1.07884\alpha + 8.24956\alpha^2 - 17.9026\alpha^3 + 20.3339\alpha^4 - 9.305\alpha^5}{\sqrt{1-\alpha}}$$



Stress intensity factor for a couple of forces *P* at the crack center, represented by the geometric function F_P. Solid curve: derived in [11,12], dashed curve: Tada et al. [1].

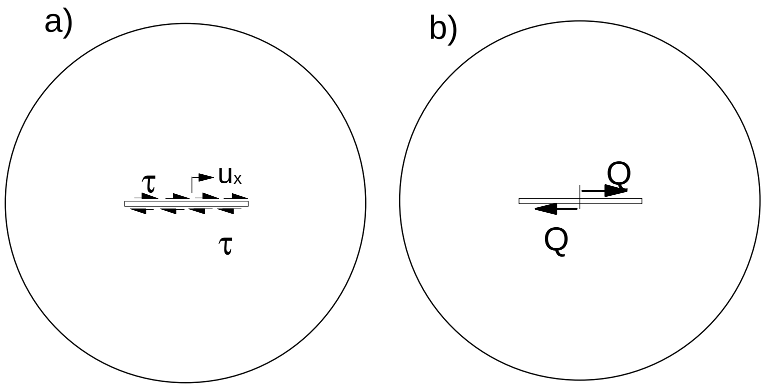
Mode-I weight function [7]

$$h_I = \frac{2}{\sqrt{\pi a}}\left[\frac{1}{\sqrt{1-\rho^2}} + C_0\sqrt{1-\rho^2} + C_1(1-\rho^2)^{3/2}\right] \quad , \rho = x/a$$

$$C_0 = \frac{8-4\alpha+3.8612\alpha^2-15.9344\alpha^3+24.6076\alpha^4-13.234\alpha^5}{\sqrt{1-\alpha}} - 8$$

$$C_1 = -\frac{8-4\alpha+0.6488\alpha^2-14.1232\alpha^3+24.2696\alpha^4-12.596\alpha^5}{\sqrt{1-\alpha}} + 8$$

3.2 Mode-II loading



Internal radial crack loaded by shear tractions, a) constant shear stress τ, b) pair of concentrated shear forces *Q*.

Stress intensity factor under constant shear tractions τ [12]

$$K_{II} = \tau F_{II} \sqrt{\pi a}\ , \quad F_{II} = \frac{1 - 0.5\alpha + 0.9274\alpha^2 - 0.88414\alpha^3 + 0.28226\alpha^4}{\sqrt{1-\alpha}}$$

with $\alpha = a/R$.

Stress intensity factor for a point load *Q* in the crack center [12]

$$K_{II,Q} = \frac{2Q}{B\sqrt{\pi a}} F_{II,Q}\ , \quad F_{II} = \frac{1 - 0.5\alpha + 1.977\alpha^2 - 1.5655\alpha^3 + 0.3851\alpha^4}{\sqrt{1-\alpha}}\ .$$

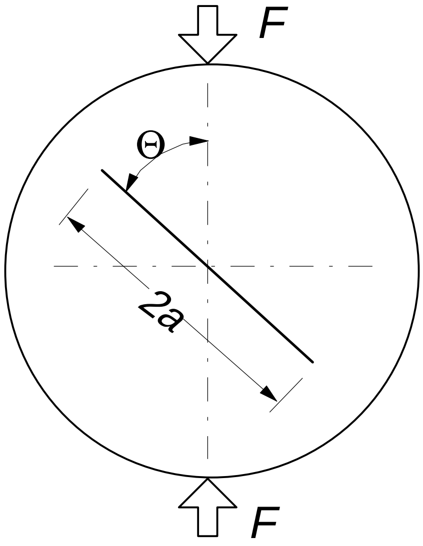
Mode-II weight function

$$h_{II} = \frac{2}{\sqrt{\pi a}} \left[\frac{1}{\sqrt{1-\rho^2}} + D_0\sqrt{1-\rho^2} + D_1(1-\rho^2)^{3/2} \right]$$

$$D_0 = \frac{5 - 2.5\alpha + 1.4882\alpha^2 - 2.3766\alpha^3 + 1.1028\alpha^4}{\sqrt{1-\alpha}} - 5$$

$$D_1 = \frac{-4 + 2\alpha + 0.4888\alpha^2 + 0.81112\alpha^3 - 0.7177\alpha^4}{\sqrt{1-\alpha}} + 4$$

3.3 Mixed-mode loading (Brazilian disk test)



Diametral compression test with internal crack.

Stress intensity factors K_I, K_{II} and related geometric functions F_I, F_{II}

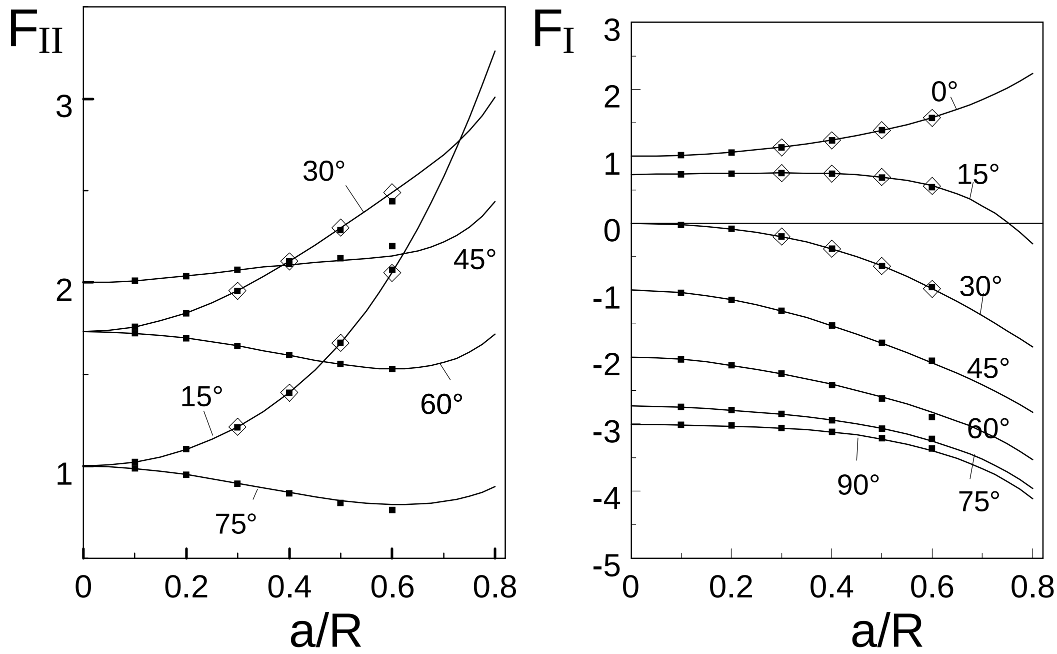
$$K_I = \sigma_0 F_I \sqrt{\pi a} = \int_0^a \sigma(x) h_I(x,a)\, dx$$

$$K_{II} = \sigma_0 F_{II} \sqrt{\pi a} = \int_0^a \tau(x) h_{II}(x,a)\, dx$$

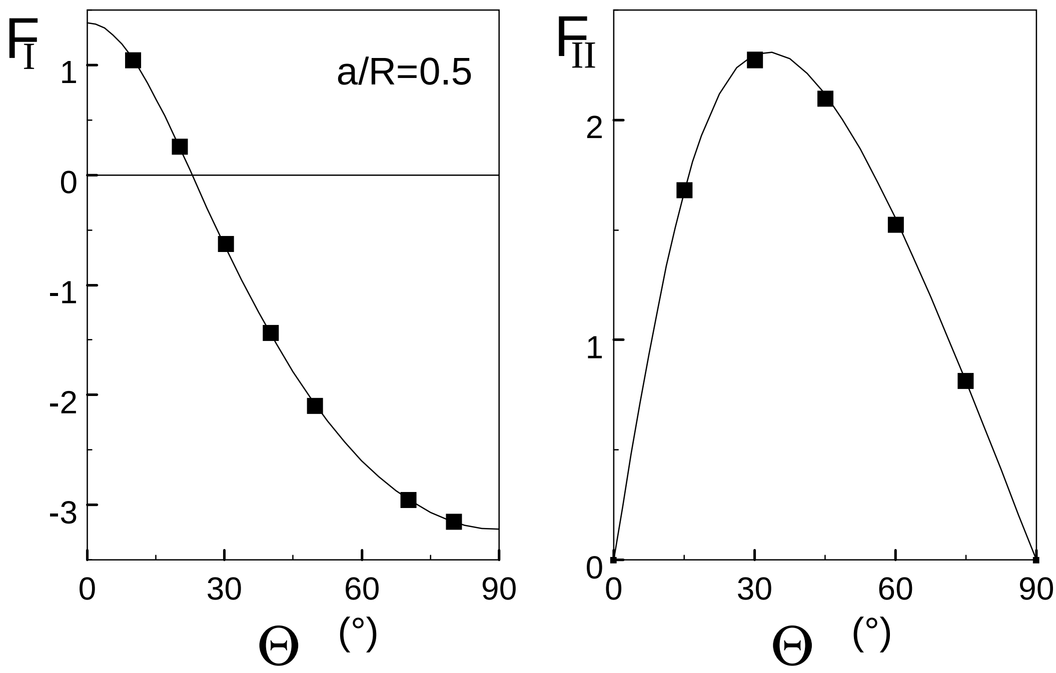
Characteristic stress:

$$\sigma_0 = \frac{F}{\pi a B} \quad ,$$

(identical with the maximum tensile stress in the center of the disk).



Geometric functions for mode-II and mode-I stress intensity factors. Curves: obtained with weight functions [12]; Solid squares: Atkinson et al. [13]; Open squares: Sato and Kawamata [14].



Geometric functions for a/R=0.5 as a function of the angle Θ. Curves: obtained with the weight function procedure; squares: Results from Atkinson et al. [13] and Awaji and Sato [15].

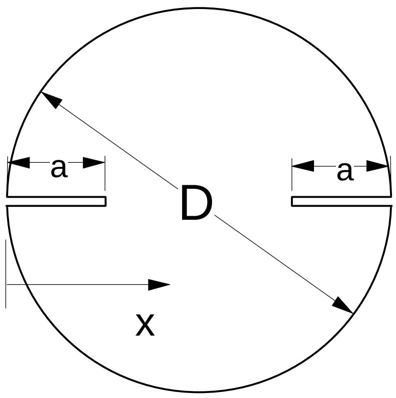
a/*R*	Θ=0°	15°	30°	45°	60°	75°	90°
0	0.	1.000	1.732	2.000	1.732	1.000	0.
0.1	0.	1.023	1.758	2.010	1.724	0.988	0.
0.2	0.	1.092	1.835	2.036	1.698	0.955	0.
0.3	0.	1.214	1.957	2.069	1.656	0.907	0.
0.4	0.	1.400	2.116	2.097	1.603	0.856	0.
0.5	0.	1.670	2.299	2.119	1.554	0.813	0.
0.6	0.	2.053	2.491	2.146	1.530	0.792	0.
0.7	0.	2.578	2.697	2.220	1.564	0.808	0.
0.8	0.	3.260	3.009	2.441	1.720	0.889	0.

Table 35 Geometric function F_{II} for the Brazilian disk tests.

a/*R*	Θ=0°	15°	30°	45°	60°	75°	90°
0	1.000	0.732	0	-1.000	-2.000	-2.732	-3.000
0.1	1.017	0.737	-0.020	-1.037	-2.033	-2.750	-3.016
0.2	1.063	0.746	-0.084	-1.141	-2.120	-2.793	-3.031
0.3	1.137	0.752	-0.200	-1.308	-2.248	-2.854	-3.062
0.4	1.241	0.742	-0.379	-1.527	-2.406	-2.940	-3.118
0.5	1.384	0.693	-0.635	-1.789	-2.594	-3.065	-3.220
0.6	1.578	0.562	-0.973	-2.083	-2.819	-3.250	-3.393
0.7	1.846	0.263	-1.381	-2.413	-3.108	-3.525	-3.665
0.8	2.244	-0.302	-1.843	-2.824	-3.530	-3.965	-4.112

Table 36 Geometric function F_{I} for the Brazilian disk tests.

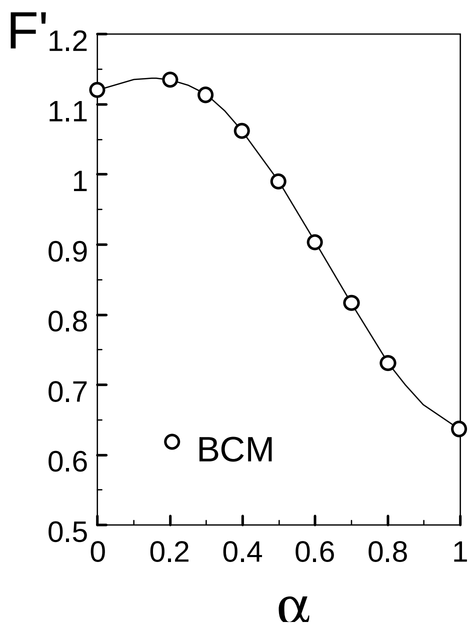
4 Double-edge-cracked circular disk



Double-edge-notched disk.

Disk loaded by constant circumferential normal tractions σ_n. Geometric function for stress intensity factor:

$$K = \sigma_n F \sqrt{\pi a} \quad , \quad F' = F\sqrt{1-\alpha} \ , \quad \alpha = 2a / D$$



Geometric function *F*' for the Double-edge-cracked disk.

$$F = \frac{1.1215 + 0.2746\,\alpha - 0.7959\,\alpha^2 - 1.1411\,\alpha^3 + 1.1776\,\alpha^4}{\sqrt{1-\alpha}}$$

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