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Abstract

In a joint of different materials thermal stresses occur after a change of the temperature in relation to the joining temperature. Here graded material joints are studied. Graded material joints possess a continuous transition of the material parameters between the layers.
In a graded joint the thermal stresses, caused by a homogeneous change of temperature are calculated analytically based on the plate theory. The influence of the layer thicknesses of the different layers on the thermal stresses is examined, as well the influence of the transition function which describes the distribution of the material parameters in the graded layer.
A method for stress optimization in a three-layer joint is presented.

Thermospannungen in einem gradierten Mehrschichtverbund

Zusammenfassung

In einem Verbund von verschiedenen Materialien treten bei einer Änderung der Temperatur gegenüber der Fügetemperatur thermische Spannungen auf. Hier werden im Besonderen gradierte Werkstoffverbunde untersucht. Gradierte Werkstoffverbunde weisen einen kontinuierlichen Übergang der Materialparameter zwischen den einzelnen Schichten auf.
Die thermischen Spannungen, die in einem Gradientenwerkstoff durch eine homogene Tempraturänderung entstehen, werden anhand der Plattentheorie analytisch berechnet.
Der Einfluss der Schichtdicken der verschiedenen Schichten auf die Thermospannungen wird untersucht, des weiteren der Einfluss der Übergangsfunktion, die den Verlauf der Materialparameter in der gradierten Schicht beschreibt.
Ein Verfahren zur Spannungsoptimierung in einem Dreischichtverbund wird beschrieben.

Contents

1 Introduction

In a dissimilar materials joint, thermal stresses occur after a homogeneous change of temperature due to the different thermal expansion coefficients. One possibility to change or reduce these thermal stresses is the introduction of a graded layer. In a graded layer or a Functionally Graded Material (FGM), the material properties follow a continuous transition function. In a joint, there are two areas where a FGM may be beneficial. At the free edge of the interface the stress singularities disappear [1]-[3]. Outside the free edge region, a redistribution of the stresses occurs, which was shown in several investigations [4]-[6]. In this paper, the effect of the thickness of the graded layer as well as of the transition function in the functionally graded material on the stress in a three layers joint will be studied. Some ideas for the optimization of the stresses in the joint are presented.
Four different combinations of materials in the joint will be investigated to see the effect of the materials' data on the results. The material data of the four combinations are given in Table 1. The Poisson's ratio is assumed to be constant ($\nu = 0.3$) in all layers. The effect of ν on the stresses will be discussed in the next section.

comb.	E_1, GPa	E_2, GPa	$\alpha_1, 10^{-6}/K$	$\alpha_2, 10^{-6}/K$
1	200	100	5	10
2	1000	200	1	10
3	100	200	5	10
4	10	200	5	10

Table 1.: The four materials combinations.

The first materials combination is assumed to be a combination of a ceramic layer with a metallic layer. The second joint is a combination of diamond with steel. The materials combinations of the third and fourth joint are chosen arbitrarily to see the effect of the ratios E_1/E_2. In these joints, the Young's modulus of the first layer is smaller than that in the second layer. The ratio in the third combination is $E_1/E_2 = 0.5$ and in the fourth combination $E_1/E_2 = 0.05$.

2 Calculation of the stresses in FGM

For the calculation of the stresses in a joint, the following assumptions were made:

- perfect bonding between the layers,
- material properties independent of the temperature,
- homogeneous change in temperature,

- Young's modulus and thermal expansion coefficient in the graded layer follow the same transition function.

The stresses are calculated for an infinite plate in $x-$ and $z-$ directions, which correspond to the stresses in the center of a finite joint. The thickness of the plate is H. Figure 1 shows the geometry and coordinate system for a three layers joint.

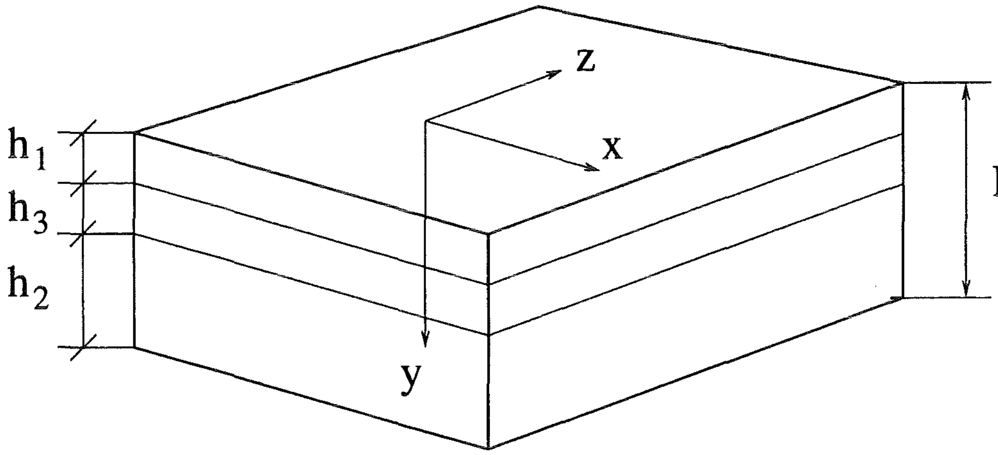

Figure 1: Geometry and coordinate system of a three layers joint.

It is assumed that in y- direction $\sigma_y = 0$. The problem is symmetric for the x- and z- direction, so $\sigma_x = \sigma_z$ and $\varepsilon_x = \varepsilon_z$. For a circular curvature of the joint, the strain $\varepsilon_x(y)$ is given as:

$$\varepsilon_x(y) = \varepsilon_0 + \frac{y}{R} \quad , \tag{1}$$

where ε_0 is a constant which describes the constant strain of the joint, R is the radius of the curvature.
Following the Hooke's law and Eq.(1), we have:

$$\sigma_x = \frac{E(y)}{1-\nu(y)}\left((\varepsilon_0 + \frac{y}{R}) - \alpha(y)\Delta T\right), \tag{2}$$

where ΔT is the change of temperature from a reference temperature T_0, where all stresses are zero.
To determine the unknown parameters R and ε_0 in Eq.(2), the equilibrium equations of the system should be used. For this problem there is no external load and constraint. Therefore, the equilibriums of forces and momentums at the plane x =const require:

$$\int_0^H \sigma_x(y)dy = 0 \tag{3}$$

$$\int_0^H \sigma_x(y)ydy = 0, \tag{4}$$

where $H = h_1 + h_2 + h_3$. Insertion of Eq.(2) into Eqs.(3) and (4) yields:

$$\int_0^H \frac{E(y)}{1-\nu(y)} \left((\varepsilon_0 + \frac{y}{R}) - \alpha(y)\Delta T\right) dy = 0, \tag{5}$$

$$\int_0^H \left[\frac{E(y)}{1-\nu(y)} \left((\varepsilon_0 + \frac{y}{R}) - \alpha(y)\Delta T\right)\right] ydy = 0. \tag{6}$$

There are now two equations for R and ε_0. If ε_0 is eliminated from these two equations, we get:

$$R = \frac{\left(\int_0^H \frac{E(y)}{1-\nu} ydy\right)^2 - \int_0^H \frac{E(y)}{1-\nu} y^2 dy \int_0^H \frac{E(y)}{1-\nu} dy}{\int_0^H \frac{E(y)\alpha(y)\Delta T dy}{1-\nu} \int_0^H \frac{E(y)ydy}{1-\nu} - \int_0^H \frac{E(y)\alpha(y)\Delta T ydy}{1-\nu} \int_0^H \frac{E(y)dy}{1-\nu}}. \tag{7}$$

With this R the value of ε_0 can be determined from:

$$\varepsilon_0 = \frac{\int_0^H \frac{E(y)}{1-\nu(y)} \left[\alpha(y)\Delta T - \frac{y}{R}\right] dy}{\int_0^H \frac{E(y)}{1-\nu(y)} dy}. \tag{8}$$

The stress in the range far away from the free edges can now be calculated analytically from Eqs.(2), (7) and (8).

If we assume $\nu = const$, Eq.(7) can be simplified as:

$$R = \frac{\left(\int_0^H E(y)ydy\right)^2 - \int_0^H E(y)y^2dy \int_0^H E(y)dy}{\int_0^H E(y)\alpha(y)\Delta T dy \int_0^H E(y)ydy - \int_0^H E(y)\alpha(y)\Delta T ydy \int_0^H E(y)dy}. \tag{9}$$

For a linear transition function through the whole layer (for $0 \le y \le H$), $E(y)$ and $\alpha(y)\Delta T$ can be written as:

$$E(y) = ay + b, \tag{10}$$

$$\alpha(y)\Delta T = cy + d. \tag{11}$$

Corresponding to Eqs.(10) and (11), the integrals in Eq.(9) read:

$$\int_0^H E(y)dy = \frac{a}{2}H^2 + bH, \tag{12}$$

$$\int_0^H E(y)ydy = \frac{a}{3}H^3 + \frac{b}{2}H^2, \tag{13}$$

$$\int_0^H E(y)y^2dy = \frac{a}{4}H^4 + \frac{b}{3}H^3, \tag{14}$$

$$\int_0^H E(y)\alpha(y)\Delta T dy = \frac{ac}{3}H^3 + \frac{ad+bc}{2}H^2 + bdH, \tag{15}$$

$$\int_0^H E(y)\alpha(y)\Delta T y dy = \frac{ac}{4}H^4 + \frac{ad+bc}{3}H^3 + \frac{bd}{2}H^2. \tag{16}$$

Inserting the Eqs.(12) - (16) into Eq.(9) yields:

$$R = \left\{ \left(\frac{a}{3}H^3 + \frac{b}{2}H^2\right)^2 - \left(\frac{a}{4}H^4 + \frac{b}{3}H^3\right)\left(\frac{a}{2}H^2 + bH\right) \right\} /$$
$$/ \left\{ \left(\frac{ac}{3}H^3 + \frac{ad+bc}{2}H^2 + bdH\right)\left(\frac{a}{3}H^3 + \frac{b}{2}H^2\right) - \right.$$
$$\left. \left(\frac{ac}{4}H^4 + \frac{ad+bc}{3}H^3 + \frac{bd}{2}H^2\right)\left(\frac{a}{2}H^2 + bH\right) \right\}. \tag{17}$$

Simplifying Eq.(17) gives:

$$R = \frac{1}{c}. \tag{18}$$

From Eqs.(18) and (8):

$$\varepsilon_0 = d \tag{19}$$

is obtained.

With the known R and ε_0, it follows from Eq.(2):

$$\sigma_x = \frac{E(y)}{1-\nu}\left((d+cy) - (cy+d)\right) \equiv 0. \tag{20}$$

This means that in a linear graded material with the joint containing one graded layer only, the stress in the whole joint is always zero. In fact, it is only essential that α follows a linear transition function, the distribution of E may be arbitrary. This can be proved easily with an arbitrary function for E and a linear function for α in Eq.(9). The integrals in Eq.(9) can be written as:

$$\int_0^H E(y)dy = A \tag{21}$$

$$\int_0^H E(y)ydy = B \tag{22}$$

$$\int_0^H E(y)y^2dy = C \tag{23}$$

$$\int_0^H E(y)\alpha(y)\Delta T dy = cB + dA \tag{24}$$

$$\int_0^H E(y)\alpha(y)\Delta T y dy = cC + dB \tag{25}$$

Inserting the Eqs.(21) - (25) into Eq.(9) yields:

$$R = \frac{B^2 - CA}{(cB + dA)B - (cC + dB)A} \tag{26}$$

This also leads to:

$$R = \frac{1}{c},$$

$$\varepsilon_0 = d,$$

and

$$\sigma_x \equiv 0.$$

3 Transition functions

The distribution of the material properties in the graded layer can be described with a transition function. In this paper, four different transition functions are considered:

fct. 1 : $M(y) = M_1$

fct. 2 : $M(y) = M_2 - (M_2 - M_1)\dfrac{h_1 + h_3 - y}{h_3}$

fct. 3 : $M(y) = M_2 - (M_2 - M_1)\left(\dfrac{h_1 + h_3 - y}{h_3}\right)^2$

fct. 4 : $M(y) = M_1 - (M_1 - M_2)\left(\dfrac{y - h_1}{h_3}\right)^2$

where:
$M(y)$: material properties, Young's modulus or thermal expansion coefficient in the graded layer,
M_1 : material properties of the first material,
M_2 : material properties of the second material.

Function 1 represents the case of a non-graded joint with the material properties of material 1 in the 'graded' layer. The distribution of the Young's modulus for the different transition functions is shown in Fig. 2.

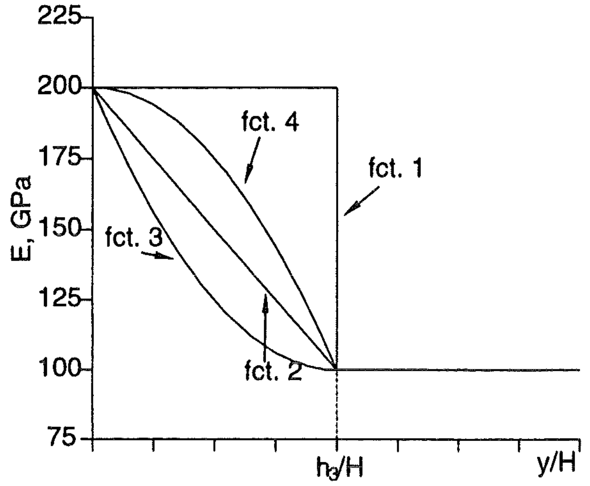

Figure 2: Distribution of Young's modulus for the different transition functions.

The Poisson's ratio ν is assumed to be constant = 0.3 in all layers. In [7] it is mentioned that the influence of ν on the stresses is small.
For a linear transition function in the FGM and materials combination 1, Fig. 3 shows the stress at $y = 0, y = h_1, y = h_1 + h_3$ and $y = H$ for a joint with $h_1 = h_3 = 0.1H$, $0.2 < \nu_1 < 0.4$ and $\nu_2 = 0.3$. Fig. 4 shows the same stresses for materials combination 3, Fig. 5 shows the stresses for another combination, with the Young's modulus as in combination 1, but $\alpha_1 = 10 * 10^{-6}/K$ and $\alpha_2 = 5 * 10^{-6}/K$, i.e. $\alpha_1 > \alpha_2$. It can be seen that in the range of $0.25 \leq \nu \leq 0.35$ the difference of the stresses with that of ν = constant = 0.3 is less than 5 %.

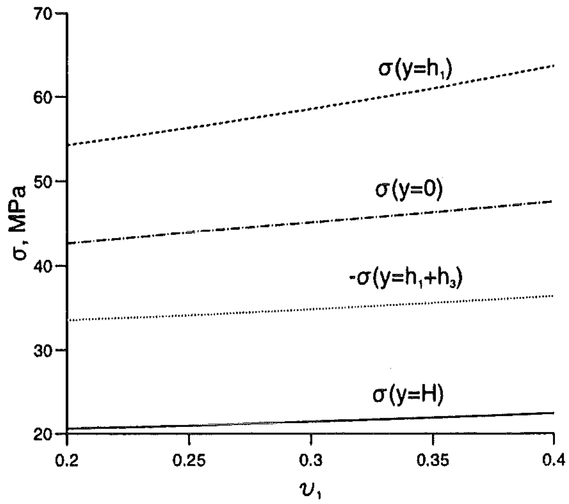

Figure 3: Influence of ν on the thermal stresses for materials combination 1.

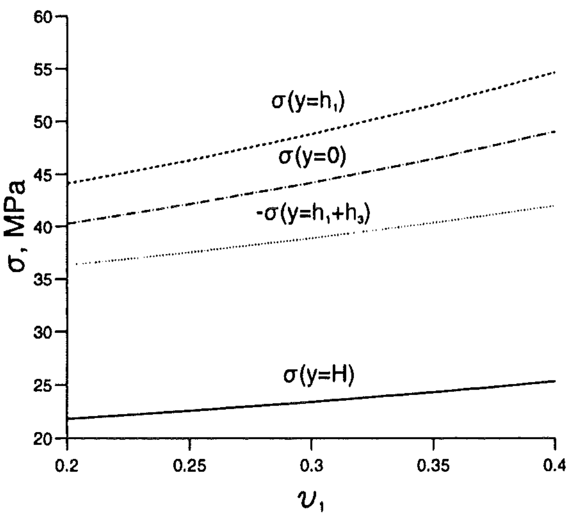

Figure 4: Influence of ν on the thermal stresses for materials combination 3.

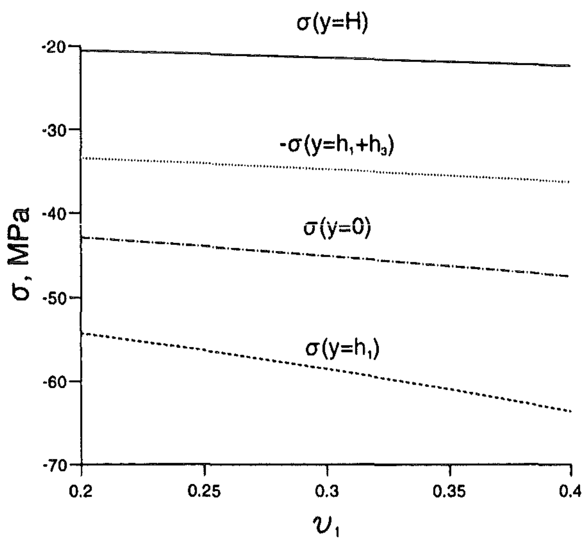

Figure 5: Influence of ν on the thermal stresses for $\alpha_1 > \alpha_2$.

4 Effect of the layer thicknesses on the thermal stresses

A three layer joint with the material properties of combination 1 (see Table 1) is considered. Due to a homogeneous change of temperature, thermal stresses occur. In the three layers joint the following geometry is chosen:

- the top layer consists of material 1 with the height h_1,
- the middle layer is a FGM with the height h_3,
- the bottom layer is made of material 2 with the height h_2,

and $h_1 + h_2 + h_3 = H$. In this three layers joint the stress σ_x depends on the material properties, the transition function in the FGM and the thickness ratios $h_1/H, h_2/H$ and h_3/H.

As an example, Figure 6 shows the distribution of σ_x for a joint with $h_1/H = 0.1$ and $h_3/H = 0.1$ for $\Delta T = 100K$. It can be seen that at the upper surface ($y = 0$) the stresses corresponding to the functions 2 - 4 are higher than for the non-graded function 1. However, at the interface ($y = 0.2$) the stresses are lower for the graded cases. Another effect of the gradiation is the smooth transition of the stresses at the interface. According to Mortensen and Suresh [8], this has a beneficial effect. The possible reason of such an effect has been discussed in [9].

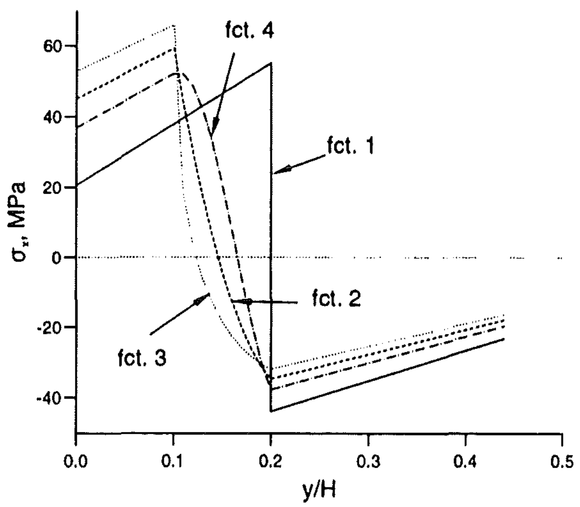

Figure 6: Stresses σ_x for the different transition functions.

Now, the effect of the thicknesses of the three layers and the transition functions on the stresses will be studied. The stress σ_x at different positions ($y = 0, y = h_1, y = h_1 + h_3$) is shown in Figures 7 - 9 for varying thicknesses h_1, h_2, h_3 and for different transition functions. In these figures the x-axis represents h_1/H and the y-axis is h_2/H. Along the diagonal between $h_1/H = 1$ and $h_2/H = 1$ there is $h_3 = 0$ and for lines parallel to the diagonal there is $h_3 =$ constant. Therefore, all possibilities of the geometry can be found in this $h_1 - h_2$ plane. The stress σ_x is shown in the form of isolines. Function 1 is not shown here, because it is equivalent to a two layers joint.
The Figures 7 - 9 can be used for the optimization of the stresses in a three layers joint, if the geometry of the joint can be varied.
A favorable transition function to minimize the stress at a given position is dependent on the materials combination and geometry. If the thickness of the layers can be changed, it is possible to find the best combination of thicknesses and the transition function for a given optimization criterion and materials combination. As an example, it is assumed that the optimization criterion is compressive stress on the upper surface of material 1 ($y = 0$). From Figure 7a it can be seen that for function 2 with $h_2/H < 0.3$ and h_1 arbitrary, or $h_1/H > 0.3$ and h_2 arbitrary, the stresses at the upper surface are compressive; for function 3 (see Figure 7b) with $h_1/H > 0.25$, h_2 arbitrary, the stresses at the upper surface are compressive; for function 4 (see Figure 7c) with $h_2/H < 0.5$, h_1 arbitrary, or $h_1/H > 0.25$, h_2 arbitrary, the stresses at the upper surface are compressive.
The ranges of h_1 and h_2, in which the stresses at the upper surface are compressive, are dependent on the transition function and materials combination, which can be determined easily from analytical calculations. For this criterion, function 4 is the best one, which provides the largest range of h_1/H and h_2/H where surface stress is compressive. Another optimization criterion may be that at the interface the absolute values of the stresses should be small, for which Figures 8 and 9 can be used. For the interface $y = h_1$, function 2 is the best one and for the interface $y = h_1 + h_3$, the result is dependent on the ratio of h_1/H and h_2/H. There is no general result for the optimization of the stress in a three layers joint with FGM.

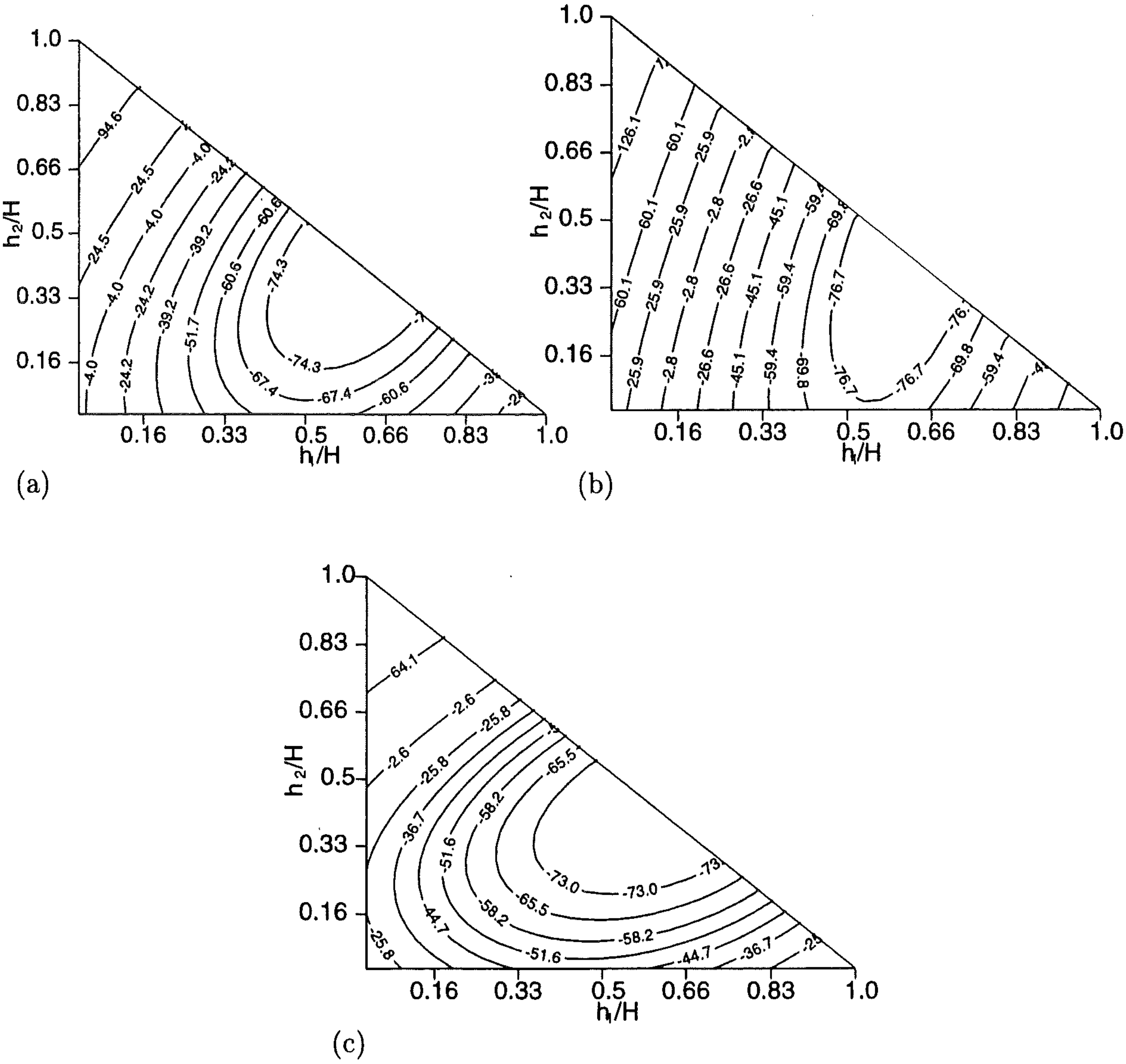

Figure 7: σ_x at the upper surface (y=0) for (a) function 2, (b) function 3, (c) function 4.

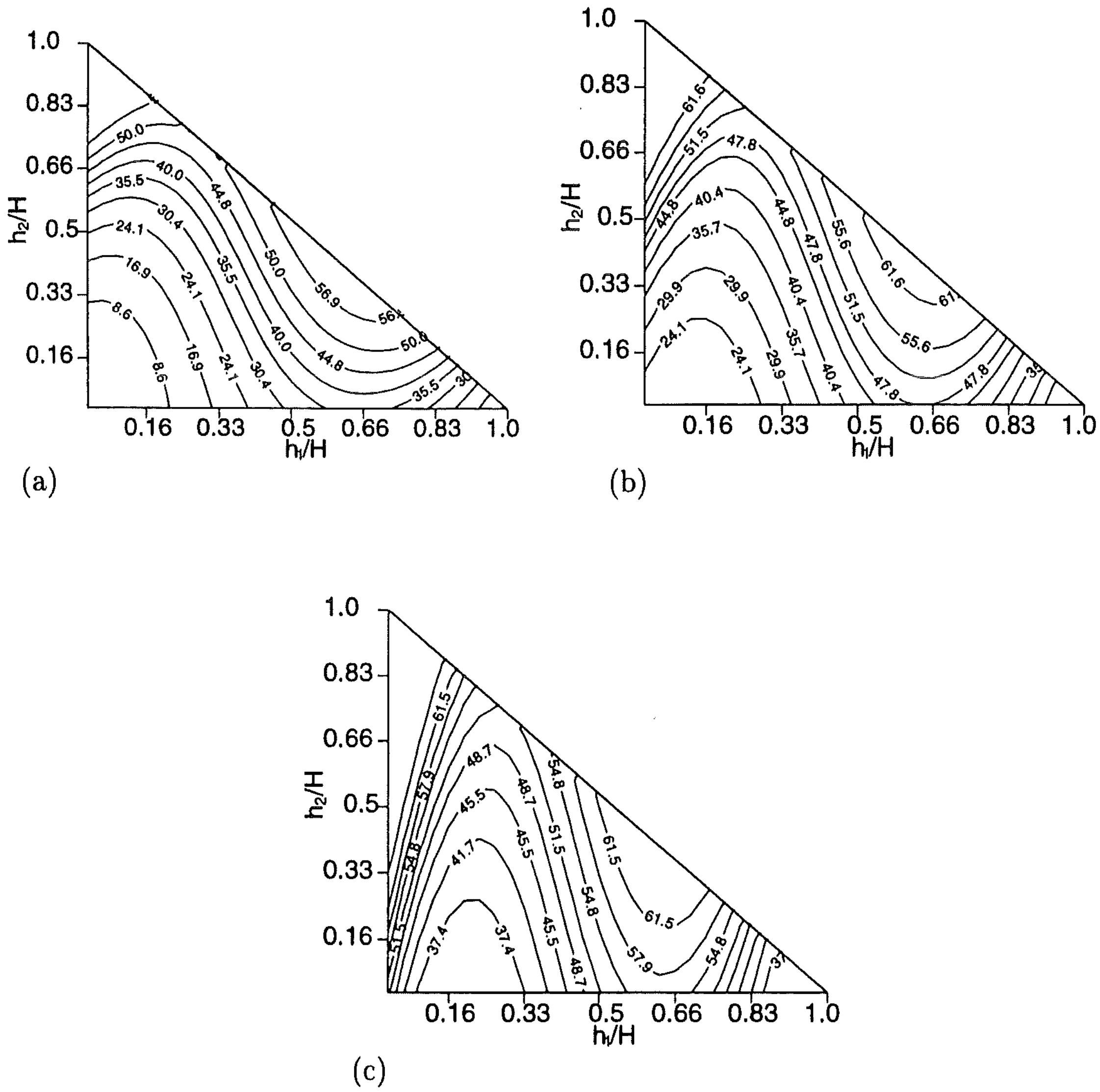

Figure 8: σ_x at the first interface (y=h_1) for (a) function 2, (b) function 3, (c) function 4.

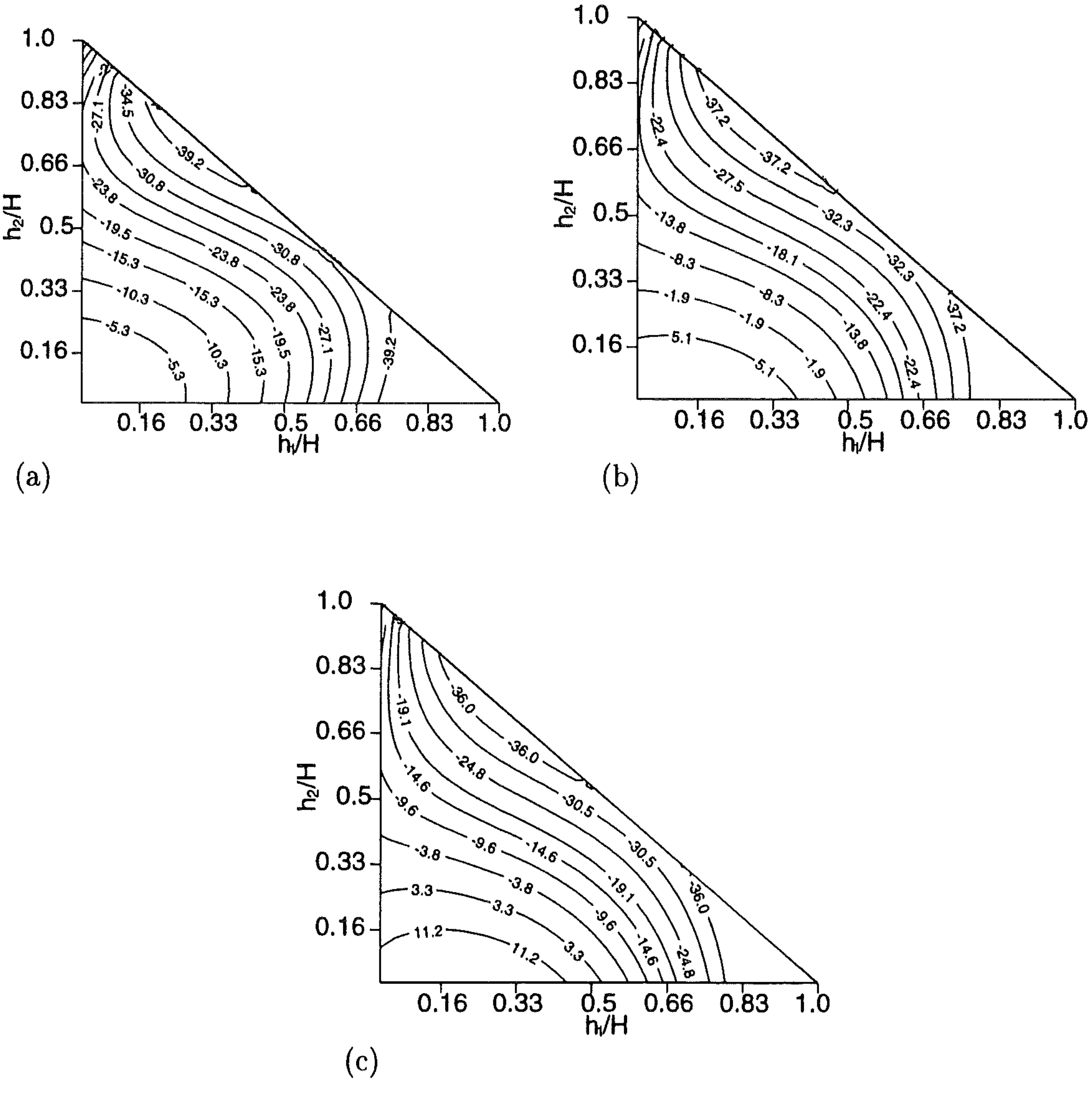

Figure 9: σ_x at the second interface (y=$h_1 + h_3$) for (a) function 2, (b) function 3, (c) function 4.

5 Stress-free surface with a tailored transition function

Another optimization criterion would be the requirement of a stress-free surface (i.e. $\sigma_x = 0$ at $y = 0$). For this purpose, a new type of transition function is chosen:

$$M(y) = M_2 - (M_2 - M_1)\left(\frac{h_1 + h_3 - y}{h_3}\right)^n, \tag{27}$$

where n is a variable. With the exponent n, it is possible to obtain different shapes of the transition function (see Figure 10).

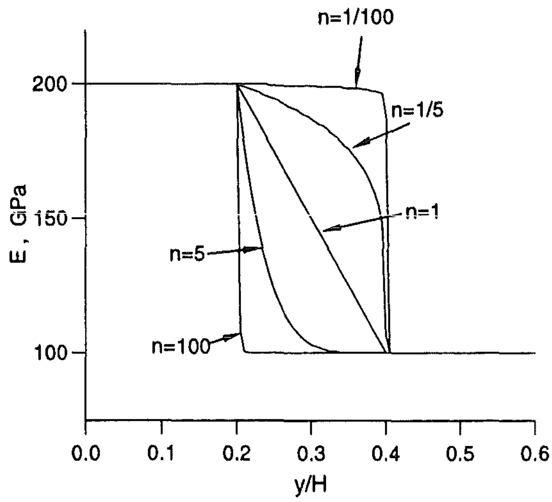

Figure 10: The distribution of Young's modulus for different n in Eq.(27).

By varying n, it is possible to satisfy the above requirement.

5.1 Stress optimization for the first materials combination

Figure 11a shows the values of n found for all possible joint geometries, with which σ_x is equal to zero at $y = 0$ for materials combination 1. In areas, in which no isolines are shown, it is not possible to obtain a stress-free surface by using the transition function given in Eq.(27). However, using another type of transition function, this may be possible. The point on the diagonal, where all isolines intersect, is the point, at which the stress at the upper surface is zero for a two layers system (because h_3 equals zero on the diagonal). At the point $h_1 = h_2 = 0$, a value of $n = 1$ is always found (see also Figures 11b - 17). At this point, the joint is a single layer with $h_3 = H$. In such a graded layer with a linear transition function (i.e. $n = 1$), the stress in the whole layer is always zero (see Eq.(20)). Figure 11b shows the n, which fulfills the requirement of $\sigma_x(y = h_1) = 0$. Figure 12 shows the n for $\sigma_x(y = h_1 + h_2) = 0$ (Fig. 12a) and $\sigma_x(y = H) = 0$ (Fig. 12b). In general, the favorable transition function in the FGM can be found for a given requirement. Of course, this transition function is not universal.

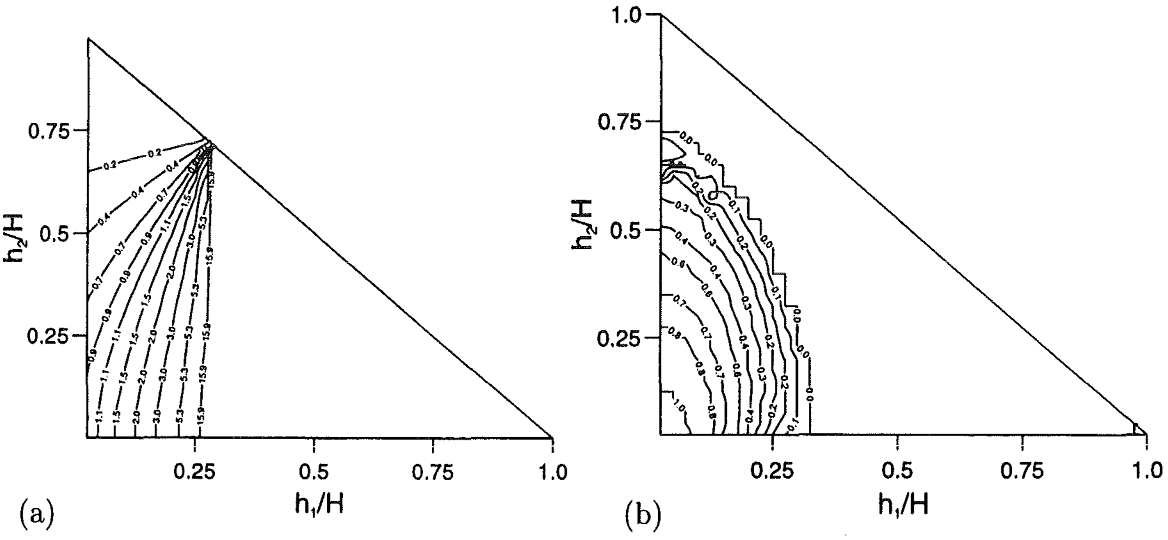

Figure 11: Exponent n for the cases of (a) $\sigma_x(y = 0) = 0$ and (b) $\sigma_x(y = h_1) = 0$ (combination 1).

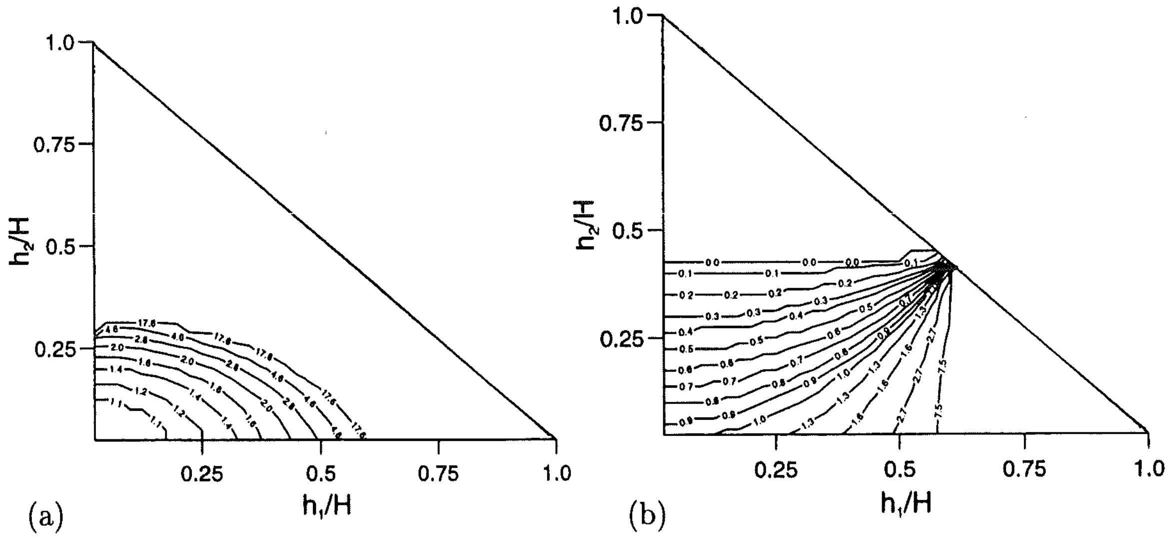

Figure 12: Exponent n for the cases of (a) $\sigma_x(y = h_1 + h_3) = 0$ and (b) $\sigma_x(y = H) = 0$ (combination 1).

5.2 Optimization for the second materials combination

As another example, stress optimization for the second materials combination (see Table 1) is considered. This joint is a combination of diamond and steel. Here also transition functions are searched, which fulfill the requirement of zero stress at some points of the joint for a variation of the layer thicknesses h_1, h_2 and h_3. Figure 13a shows the values of n, which fulfill the requirement of $\sigma_x(y = 0) = 0$ and Figure 13b shows n for $\sigma_x(y = h_1) = 0$. Figure 14 shows the n for $\sigma_x(y = h_1 + h_3) = 0$ (Fig. 14a) and $\sigma_x(y = H) = 0$ (Fig. 14b).
In Table 1, it can be seen that the ratio of the thermal expansions coefficient is different for materials combination 2 ($\alpha_1/\alpha_2 = 0.1$) and the other combinations ($\alpha_1/\alpha_2 = 0.5$). In the following it will be proved that this difference has no influence on the area, where a fitting transition function can be found, nor does it influence the values of n. As transition function for α:

$$\alpha(y)\Delta T = \alpha_1 \Delta T - (\alpha_1 - \alpha_2)\Delta T f(y), \tag{28}$$

is used, with $f(y = h_1) = 0$ and $f(y = h_1 + h_3) = 1$.

With Eq.(28), the integrals in Eq.(9) read:

$$\int_0^H E(y)dy = A, \tag{29}$$

$$\int_0^H E(y)ydy = B, \tag{30}$$

$$\int_0^H E(y)y^2dy = C, \tag{31}$$

$$\int_0^H E(y)\alpha(y)\Delta T dy = \alpha_1 \Delta T A - (\alpha_1 - \alpha_2)\Delta T F, \tag{32}$$

with:

$$F = \int_0^H E(y)f(y)dy,$$

$$\int_0^H E(y)\alpha(y)\Delta T y dy = \alpha_1 \Delta T B - (\alpha_1 - \alpha_2)\Delta T G, \tag{33}$$

with:

$$G = \int_0^H E(y)f(y)ydy.$$

Inserting the Eqs.(29) - (33) into Eq.(9) yields:

$$R = \frac{1}{\Delta T(\alpha_1 - \alpha_2)} \frac{B^2 - CA}{AG - FB} = \frac{1}{\Delta T(\alpha_1 - \alpha_2)} R', \tag{34}$$

with:

$$R' = \frac{B^2 - CA}{AG - FB}.$$

From Eqs.(34) and (8):

$$\varepsilon_0 = \alpha_1 \Delta T - (\alpha_1 - \alpha_2)\Delta T \left(\frac{F}{A} + \frac{B}{AR'}\right) \tag{35}$$

is obtained. From Eqs.(2), (34) and (35) the stress at any point y can be calculated by

$$\sigma_x = \frac{E(y)}{1 - \nu(y)} \left(\frac{G(-B + Ay) + F(C - By)}{B^2 - CA} + f(y)\right) (\alpha_1 - \alpha_2)\Delta T. \tag{36}$$

If $\sigma(y) = 0$ is assumed at an arbitrary point of $y = \hat{y}$, from Eq.(36):

$$\frac{F(C - B\hat{y}) + G(-B + A\hat{y})}{B^2 - CA} = f(\hat{y}). \tag{37}$$

can be obtained. This means that the searched function $f(y)$ is independent of α_1 and α_2.

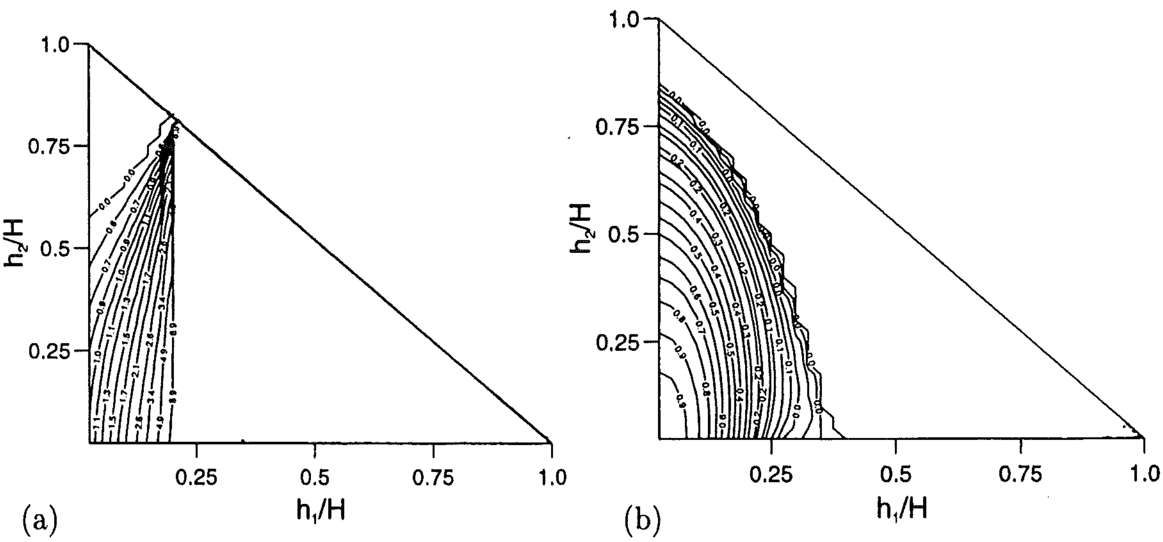

Figure 13: Exponent n for the cases of (a) $\sigma_x(y = 0) = 0$ and (b) $\sigma_x(y = h_1) = 0$ (combination 2).

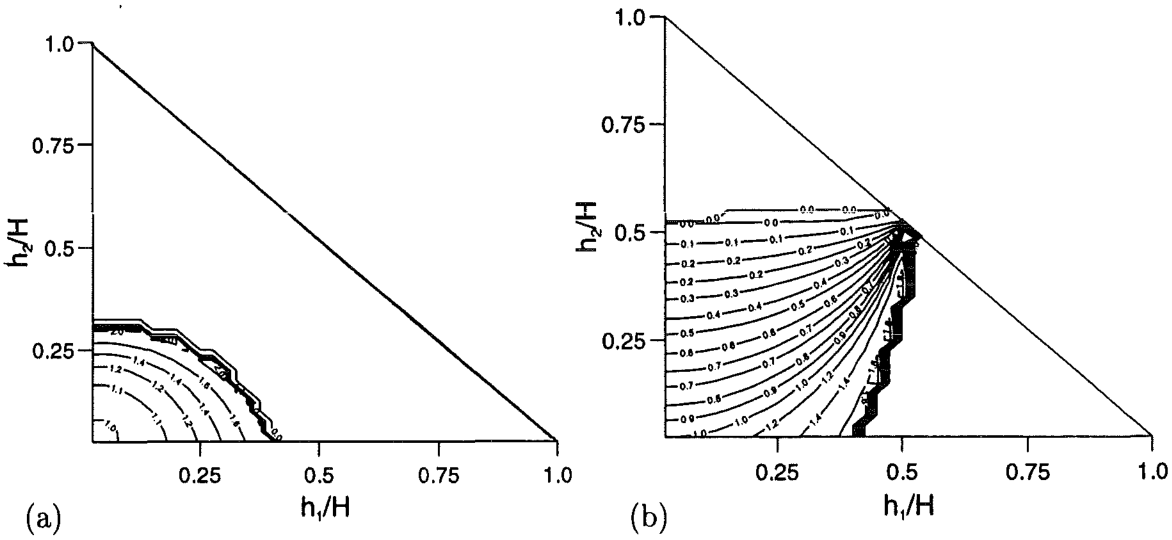

Figure 14: Exponent n for the cases of (a) $\sigma_x(y = h_1 + h_3) = 0$ and (b) $\sigma_x(y = H) = 0$ (combination 2).

5.3 Optimization for the third materials combination

In the third combination (see Table 1), the Young's modulus of the top layer is smaller than that of the bottom layer. The ratio is $E_1/E_2 = 0.5$. Here also transition functions are searched, which fulfill the demand of zero stress at some points of the joint for a variation of the layer thicknesses h_1, h_2 and h_3. Figure 15a shows the values of n, which fulfill the requirement of $\sigma_x(y=0)=0$ and Figure 15b shows n for $\sigma_x(y=h_1)=0$. Figure 16 shows the n for $\sigma_x(y=h_1+h_3)=0$ (Fig. 16a) and $\sigma_x(y=H)=0$ (Fig. 16b).

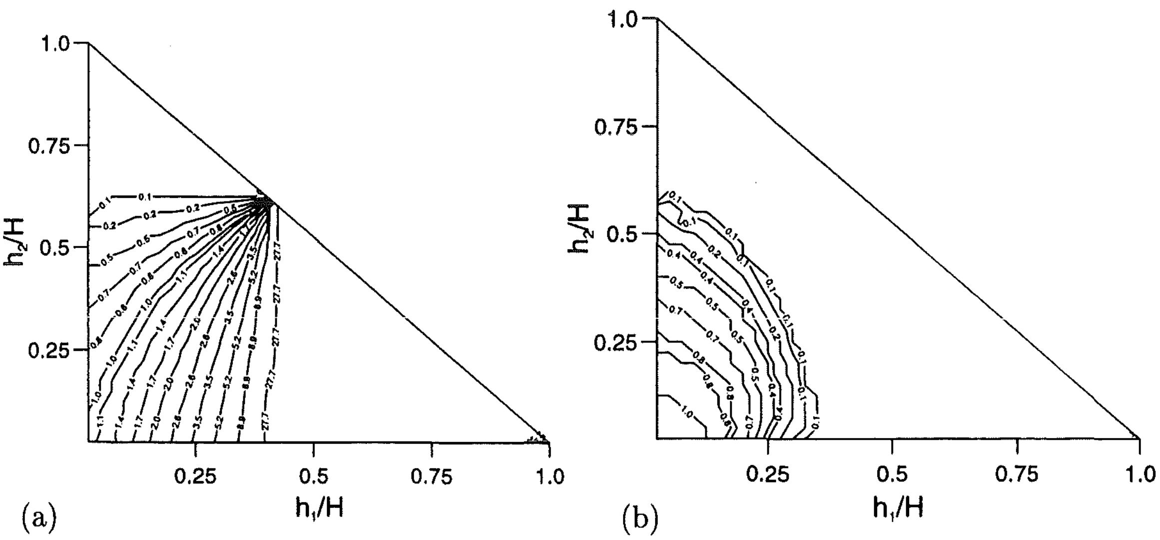

Figure 15: Exponent n for the cases of (a) $\sigma_x(y=0)=0$ and (b) $\sigma_x(y=h_1)=0$ (combination 3).

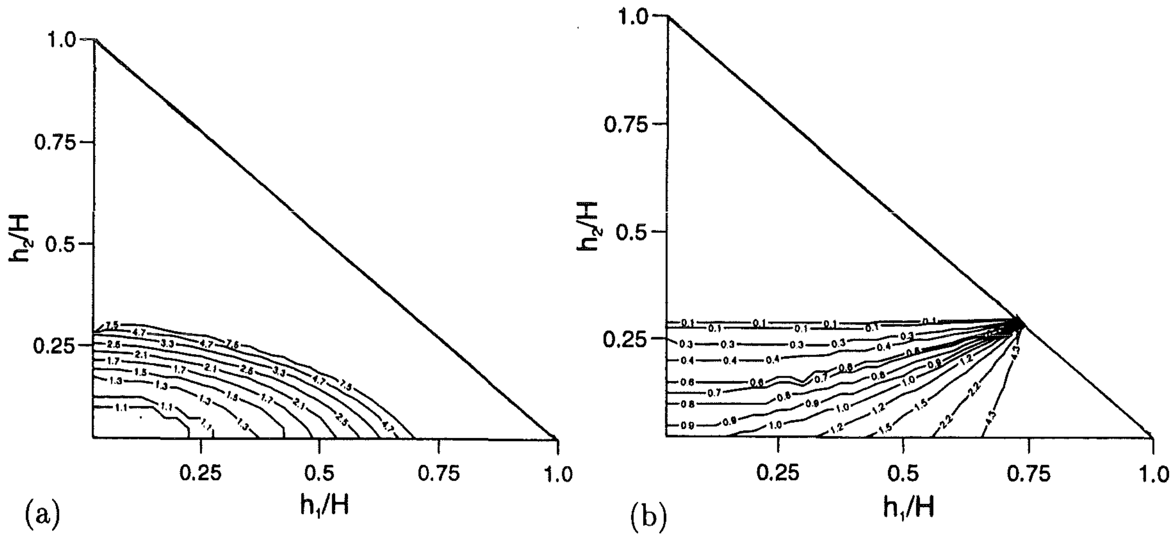

Figure 16: Exponent n for the cases of (a) $\sigma_x(y = h_1 + h_3) = 0$ and (b) $\sigma_x(y = H) = 0$ (combination 3).

5.4 Optimization for the fourth materials combination

The stress optimization for the fourth materials combination (see Table 1) is considered. In this combination, the Young's modulus of the top layer is much smaller than that of the bottom layer. The ratio is $E_1/E_2 = 0.05$. Here also transition functions are searched, which fulfill the demand of zero stress at some points of the joint for a variation of the layer thicknesses h_1, h_2 and h_3. Figure 17a shows the values of n, which fulfill the requirement of $\sigma_x(y=0)=0$ and Figure 17b shows n for $\sigma_x(y=h_1)=0$. Figure 18 shows the n for $\sigma_x(y=h_1+h_3)=0$ (Fig. 18a) and $\sigma_x(y=H)=0$ (Fig. 18b).

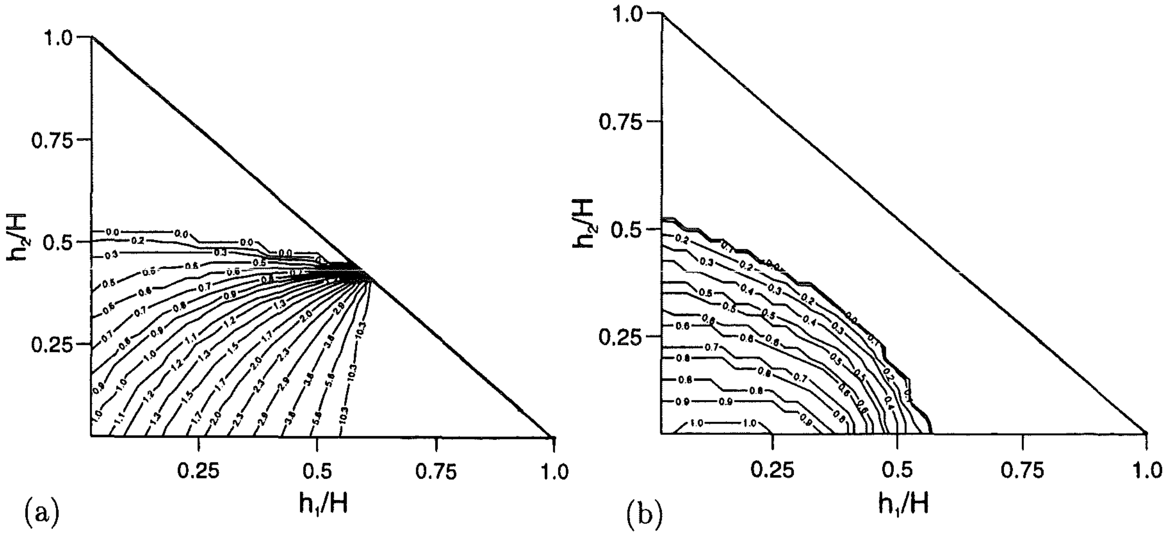

Figure 17: Exponent n for the cases of (a) $\sigma_x(y=0)=0$ and (b) $\sigma_x(y=h_1)=0$ (combination 4).

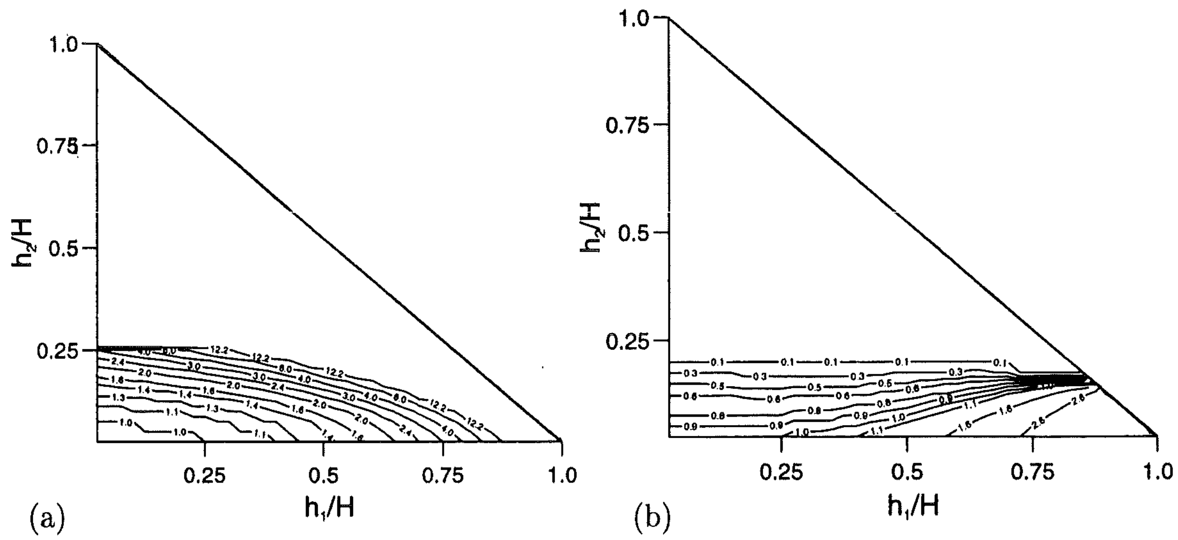

Figure 18: Exponent n for the cases of (a) $\sigma_x(y = h_1 + h_3) = 0$ and (b) $\sigma_x(y = H) = 0$ (combination 4).

6 Conclusions

The results have shown that the possibility to redistribute the stresses strongly depends on the ratio of the thickness of the graded layer to the other layers. For very thin graded layers, a redistribution of the stresses is nearly impossible, for thicker graded layers it is possible to optimize stresses for some combinations of materials and layer thicknesses.
The method to reduce the stresses to zero at some special points in the joint can be applied in a wide range of material and geometry variations. Comparing the results for different materials combinations, it becomes evident that the areas where a favorable transition function can be found are very similar. If we compare the Figures 11a, 13a, 15a and 17a (surface stress is equal to zero), it can be seen that with an increasing ratio E_1/E_2 the maximum value of h_1, where a fitting function can be found, decreases. In the same relation, the maximum h_2 grows with increasing ratio E_1/E_2. In Figure 19, this correlation is shown.

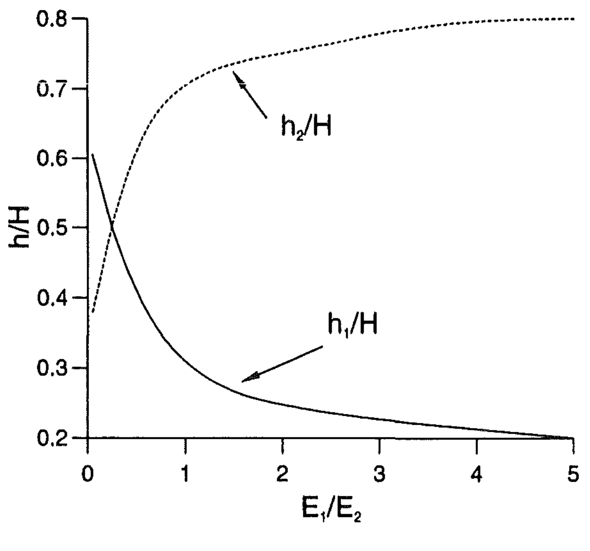

Figure 19: Maximum h_1/H and h_2/H for which a stress-free surface can be obtained in dependence of E_1/E_2.

The areas of n for the other optimization points show a similar behavior. This can be explained by the equilibrium of forces: The higher the Young's modulus in the first layer is, the larger is the influence of the first layer on the equilibrium of forces. Therefore, the stresses at the surface can also be reduced to zero with a smaller h_1.
In a graded joint the stress value is proportional to the difference of the thermal expansion coefficient of the top layer and the bottom layer. It should be noted that this is also valid for a multi-layered joint.

In a pure graded material, i.e. the joint has one graded layer only, if the transition function for the thermal expansion coefficient α is a linear function, the stress at any point is always zero, independent of the transition function of the Young's modulus E.

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