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Accelerator Driven Systems Subcritical Level Monitoring: Evaluation of Spatial Correction Factors for Source-Jerk and Area Methods

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Accelerator Driven Systems subcritical level monitoring: evaluation of spatial correction factors for Source-Jerk and Area methods

Abstract

One of the operative problems in the Accelerator Driven Systems (ADS) is developing a strategy for inferring the subcritical reactivity level of these systems. In fact because of the presence of the external source, which is necessary because of the very low reactivity level ($k_{\text{eff}} \sim 0.95-0.97$), ADS systems show neutron fluxes profiles, especially near the external source, very different respect to the fundamental mode which characterizes the neutron population in the critical systems. In order to study the neutronics of ADS, the zero-power MUSE experimental program was launched by CEA in 1995 at the CEA-Cadarache MASURCA facility. One of the main target of the program was investigating several experimental methods (such as the source jerk, pulsed neutron source and noise techniques) to infer the reactivity levels in different subcritical configurations without the need of a reference measurement in a critical configuration. In particular, MUSE-4 experimental phase aimed to analyze the system response to neutron pulses provided by the GENEPI accelerator, in order to investigate the possibility to infer the subcritical level of a source driven system by using the Pulsed Neutron Source (PNS) methods based on the point kinetics assumption. Results indicated that PNS area method is a very reliable technique at large subcriticalities for what concerns the definition of the reactivity spatial dependence. Even if it does not allow an on-line subcritical level monitoring, PNS area method can represent a valid calibration technique with regards to some selected positions in the system to be analysed by alternative methods, like Source Jerk/Prompt Jump (which can work also on-line). Depending on the subcriticality level and on the presence of spatial effects, the reactivity level of a system may not be inferred by the detectors responses on the basis of a pure point kinetics approach and then corrective spatial factors have to be applied to the experimental results in order to infer the subcriticality level of the system. Moreover, depending on the used method, these corrective factors may have different amplitudes and may give information about the effectiveness of the used point kinetics based method. In this report a MUSE-4 experimental configuration at a meaning full subcritical level with a D-T external source located at the reactor center was analyzed by means of the ERANOS deterministic codes system, by assessing two static calculation procedures reproducing the application of the 'explicit' Source-Jerk method and PNS area method in order to evaluate the corrective spatial factors to be applied to the experimental data. Results coming from the application of the two calculation procedures were firstly compared and then a comparison with the experimental results was performed. For what concerns the source jerk method, it has to be underlined that an 'explicit' application of the source jerk method, by means of the GENEPI accelerator, cannot be allowed in the MUSE-4 experiment because of the pulsed regime of the external source together with the characteristics of the GENEPI accelerator (frequency, pulse duration).

Kontrolle der Unterkritikalität eines Beschleuniger-getriebenen Systems:

Bestimmung der örtlichen Korrekturfaktoren

der ‚Source-Jerk‘- und - ‚Area‘ - Methoden

Kurzfassung

Die laufende Bestimmung der Unterkritikalität ist eines der Probleme beim Betrieb eines Beschleuniger getriebenen Systems. Wegen der bei einer Unterkritikalität ($k_{\text{eff}} \sim 0.95-0.97$) benötigten externen Quelle weichen die Neutronenflußprofile ganz wesentlich vom sog. Fundamentalmode eines kritischen Systems ab. Um das neutronische Verhalten eines ADS zu untersuchen, wurde von CEA im Jahre 1995 in der CEA-Cadarache-Anlage MASURCA das Nullleistungsexperiment MUSE durchgeführt. Eines der Ziele des Programms war der Test verschiedener experimenteller Methoden (Source Jerk, Pulsed Neutron Source, Rauschmethoden), die Unterkritikalität sicher zu bestimmen ohne eine begleitende Referenzmessung in einem kritischen System. Besonders das MUSE-4 Experiment zielte auf die Bestimmung der Systemantwort mit Hilfe der Pulsed Neutron Source (PNS) Methode basierend auf Punktkinetik. Die Neutronenpulse wurden dabei mit Hilfe des GENEPI Beschleunigers ausgelöst. Die Ergebnisse zeigen, dass die PNS eine sehr zuverlässige Methode in Hinblick auf die örtliche Reaktivitätsverteilung bei starker Unterkritikalität darstellt. Sie erlaubt zwar keine on-line Überwachung, stellt aber eine hervorragende Kalibrierungstechnik an bestimmten Positionen im Systems für die Source Jerk/Prompt Jump Methoden dar, die auch on-line genutzt werden können. Abhängig von der Unterkritikalität und dem Vorhandensein örtlicher Effekte kann das Unterkritikalitätsniveau nicht direkt aus den Detektorantworten auf der Basis reiner Punktkinetik gewonnen werden. Örtliche Korrekturfaktoren müssen berechnet werden, um die experimentellen Ergebnisse nachzuvollziehen. Die Korrekturfaktoren haben dabei verschiedene Amplituden und geben Auskunft über die Effektivität der verwendeten Punktkinetikmethode.

In der Arbeit wird mit Hilfe des Code Systems ERANOS eines der MUSE-4 Experimente mit typischem Unterkritikalitätsniveau mit einer zentralen externen D-T Quelle analysiert. Zwei Prozeduren, basierend auf statischen Neutronenflussberechnungen, werden getestet in einer Simulation der expliziten Source-Jerk und PNS Methoden, um die örtlichen Korrekturfaktoren zur Interpretation des Experimentes zu evaluieren. Die Ergebnisse der beiden Vorgehensweisen wurden zuerst untereinander und dann mit den experimentellen Ergebnissen verglichen. Die Anwendung des Source-Jerk Methode ist allerdings nicht kompatibel mit den Randbedingungen des MUSE-4 Experimentes wegen der gepulsten Betriebsweise des GENEPI Beschleunigers und dessen charakteristischer Eigenschaften (Frequenz and Pulsdauer).

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1. INTRODUCTION

One of the problems of the Accelerator Driven Systems [1, 2] operation is developing a strategy for inferring the subcritical reactivity level. In fact, the presence of the external source in these systems, due to the very low reactivity level ($k_{\text{eff}} \sim 0.95-0.97$), excites several neutron modes in the fuel region. As result, ADS systems show neutron fluxes profiles, especially near the external source, very different respect to the fundamental mode characterizing the neutron population in the critical systems [3].

In the frame of the experimental programs supporting the definition of the European Transmutation Demonstrator (ETD), the zero-power MUSE (Multiplication avec Source Externe) experimental program [4] was launched by CEA in 1995 at the CEA-Cadarache MASURCA facility in order to study the neutronics of Accelerator Driven Systems (ADS). The aim was the investigation of several subcritical configurations (the value k_{eff} is included in the interval 0.95-1) driven by an external neutron source by (d,d) and (d,t) reactions located at the reactor center, the incidents deuterons being provided by the GENEPI deuteron pulsed accelerator.

One of the main target of the program was the investigation of the experimental technique to control and, possibly, to monitor on-line the reactivity of an ADS. With this objective many experimental methods (such as the source jerk techniques, pulsed neutron source experiments and noise techniques) were used to assess reactivity levels in various subcritical configurations without the need of a reference measurement in a critical configuration [5].

In particular, the MUSE-4 experimental phase aimed to analyze the system response to neutron pulses provided by the GENEPI accelerator (with frequencies from 50 Hz to 4.5 kHz, and less than 1 μ s wide), in order to investigate the possibility to infer the subcritical level of a source driven system using the Pulsed Neutron Source (PNS) methods [6, 7, 8].

MUSE results indicated PNS area method to be very reliable at large subcriticalities for what concerns the definition of the reactivity spatial dependence [9]. Even if PNS area method does not allow an on-line subcritical level monitoring, it can represent a valid calibration technique in the ETD situation with regards to some selected positions in the system to be analysed by alternative methods, like Source Jerk/Prompt Jump (which can work also on-line).

All the methods to measure the reactivity level of a system are initially based on the point kinetics assumption. But, depending on the subcriticality level and on the presence of spatial effects, inferring the subcriticality level of the real systems generally needs at least corrective spatial factors, evaluated by means of calculations. Moreover, depending on the used method, these corrective factors may have different amplitudes; thus, in principle, the effectiveness of a point kinetics based method will be characterized by the application of close to the unity corrective factors.

In the frame of the analysis of time-dependent MUSE responses in the prompt time domain, ENEA concentrated its efforts on the assessment of calculation procedures by means of CEA ERANOS deterministic codes system [10] in order to contribute to the analysis of the experimental results coming from PNS α -fitting and area methods in the MUSE-4 SC0 1108 fuel cells configuration with 3 SR up, SR1 down and PR down with the D-T external source ($3.3 \cdot 10^6$ n/pulse of 14 MeV) [9, 11]. Results coming from the application of the static calculation procedure reproducing the PNS area method showed a very good agreement with the experimental data, being the maximum difference of about +5% [9, 11].

As further improvement, ENEA assessed a calculation procedure reproducing the application of the 'explicit' source jerk method by means of the ERANOS deterministic codes system.

In this report the assessed source jerk calculation procedure is described and compared with the area method one, whose an improvement is also shown. The procedure has been firstly tested in a simplified subcritical configuration (being the calculated subcritical level about -2200 pcm) with only one fissile isotope composing the fuel and with a D-T external source located at the reactor center. Then the MUSE-4 SC0 1108 fuel cells experimental configuration with 3 SR up, SR1 down

and PR down (at a very low subcritical level ~ -4200 pcm) with a D-T external source has been analyzed. Results have been successively compared both with the experimental data and with the results coming from the application of the assessed area method calculation procedure [11].

It has to be underlined that an ‘explicit’ application of the source jerk method, by means of the GENEPI accelerator, cannot be allowed in the MUSE-4 experiment because of the pulsed regime of the external source together with the characteristics of the GENEPI accelerator (frequency, pulse duration).

In the following, a theoretical background is shown for the source jerk and PNS area reactivity measurement methods and the calculation procedures assessed by ENEA by means of ERANOS deterministic codes system are described; then calculated results are shown and compared with the experimental ones.

2. Simplified subcritical configuration

Preliminary analyses were performed in a RZ simplified configuration (Fig. 1) with a D-T external source ($3.3 \cdot 10^6$ n/pulse of 4 MeV) located at the reactor center. Only one fissile isotope (^{239}Pu) was used in the fuel region.

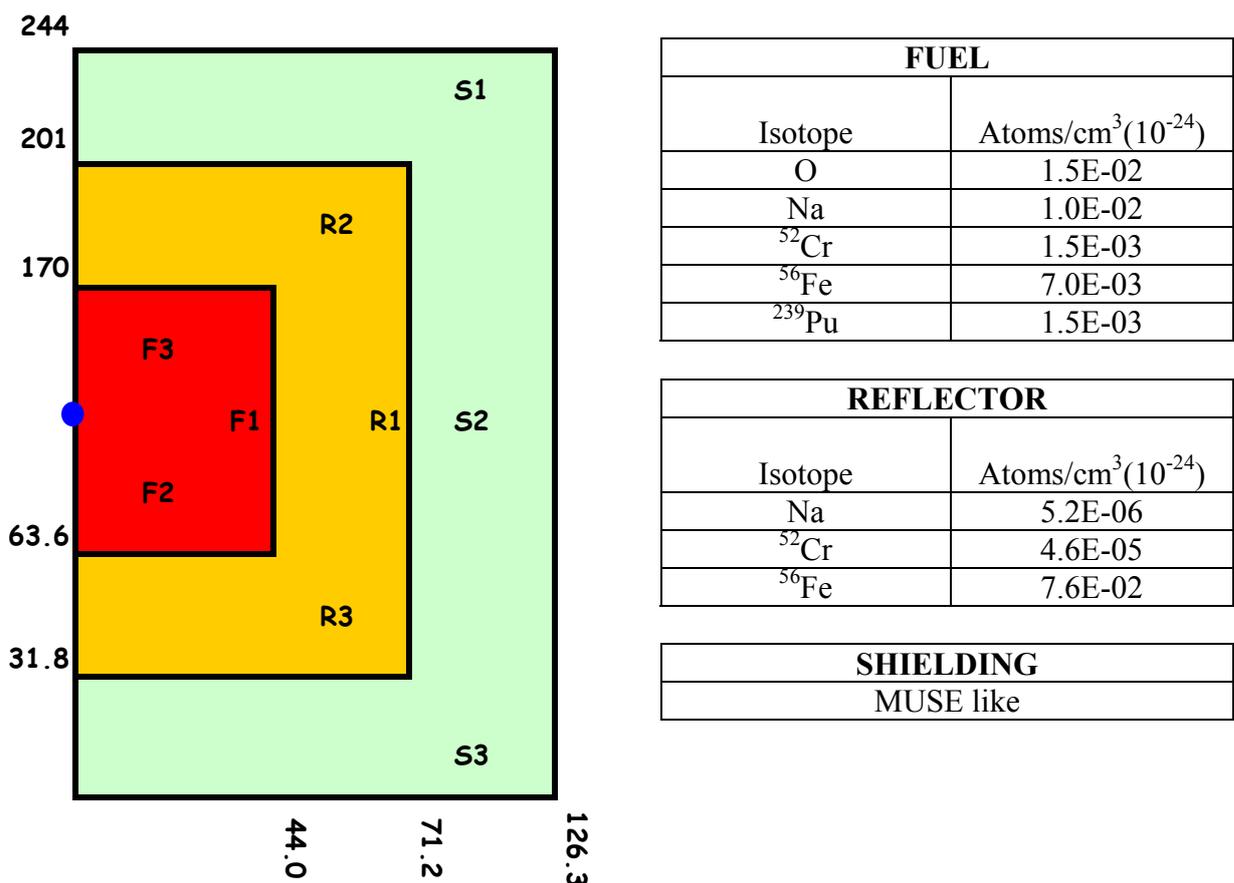


Figure 1. RZ simplified configuration for tests

In this assessment, the configuration has been assumed to have homogeneous compositions, then, in order to take into account the core-reflector coupling, cross section processing was performed by means of a macrocell calculation using the ECCO ERANOS cell code [12]. The macrocell

calculation option allows to adequately consider the spatial effects at the core/reflector interface [13], by weighting the cross sections for each region with the corresponding cell flux.

3. MUSE-4 core configurations: geometry and material description

The MASURCA facility is an arrangement of tubes of $10.6 \times 10.6 \times 164.16 \text{ cm}^3$, building a parallel-piped assembly of 17×16 tubes in the MUSE-4 configurations, with a total dimension of $180.2 \times 169.6 \times 164.16 \text{ cm}^3$. Core is based on a U/Pu MOX fuel: the fuel subassembly is made up of MOX fuel pellets and solid sodium loaded in a ring steel wrapper (Fig. 2).

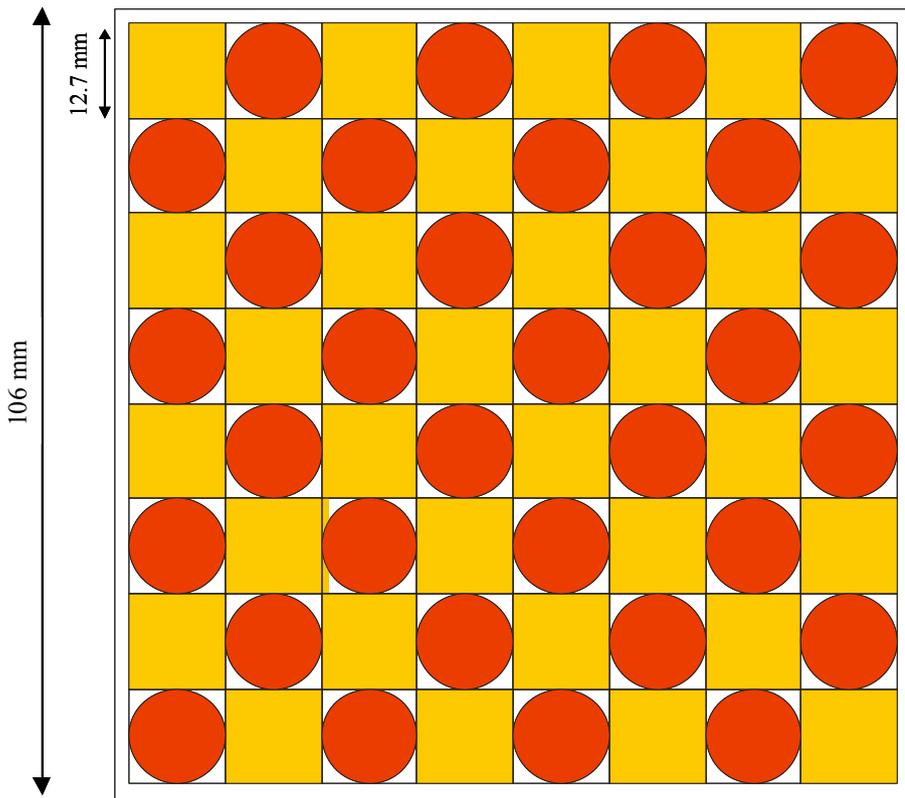


Figure 2. XY loading of the MUSE-4 U/Pu MOX fuel subassembly

Reflector, surrounding the core, is composed by a mixture of sodium and steel (Fig. 2) and the axial and radial shielding are composed by steel only. Two other regions are present: one modeling the central lead zone and the target (containing tritium or deuterium) and one modeling the accelerator beam pipe (vacuum + layer made of a mixture of aluminum and lead) [14].

In the MUSE-4 configurations three kinds of fuel subassemblies are loaded, the difference mainly due to the different density of the Pu isotopes: MOX1, MOX2 and MOX3 in Fig. 3.

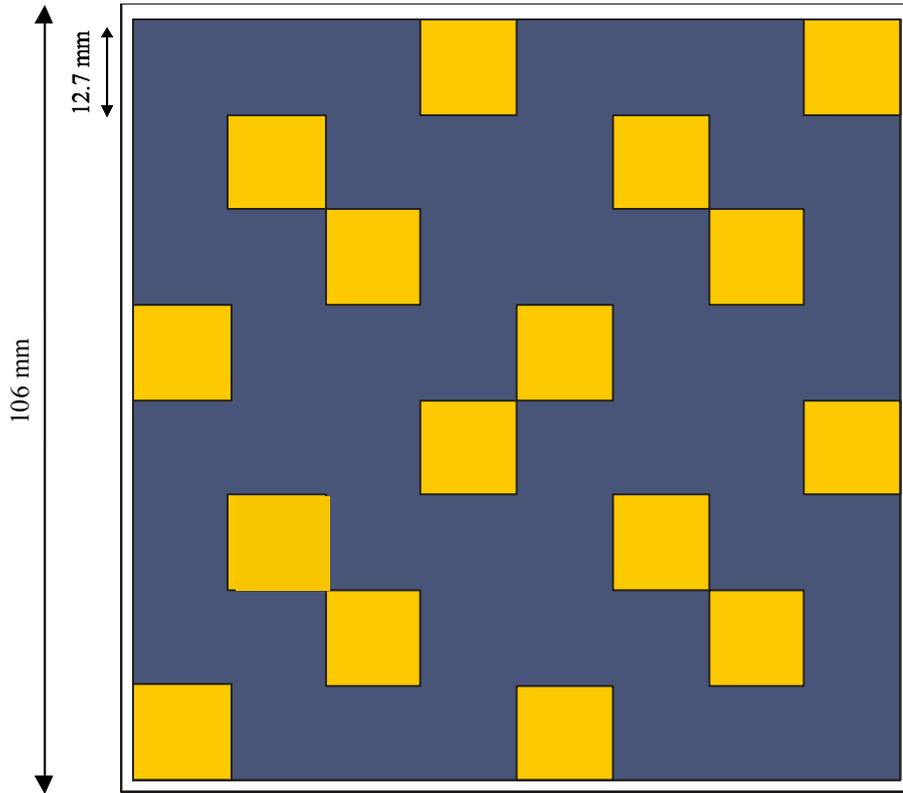


Figure 3. XY loading of the MUSE-4 sodium and stainless steel reflector subassembly

Tab. 1 shows the cell calculation results performed by means of ECCO ERANOS cell code [12] using the heterogeneous fuel subassembly geometry (Fig. 2) and the volume percentage of each fuel region in the two considered MUSE-4 subcritical configurations.

	k_{∞}	Volume of each fuel region respect to the total (%)	
		MUSE-4 976 fuel cells	MUSE-4 1108 fuel cells *
MOX1	1.64684	~ 80.3	~ 82.4
MOX2	1.64441	~ 11.5	~ 10.2
MOX3	1.57438	~ 8.2	~ 7.4

* with 3 SR up, SR1 down and PR down.

Table 1. Fuel ECCO cell calculation results

It can be noted that the MOX1 and MOX2 k_{∞} values are very close and that the MOX3 volume fraction is always much less than the others.

3.1 MUSE-4 SC0 1108 fuel cells configuration

In this report, the experimental data relevant to the MUSE-4 SC0 1108 fuel cells configuration (Fig. 4) with 3 SR up, SR1 down and PR down have been compared with the calculation results. This configuration, with a D-T external source ($3.3 \cdot 10^6$ n/pulse of 14 MeV) located in the reactor center, is characterized by a significant subcriticality level (~ -4200 pcm) and by a large availability of experimental data [5, 15]; in fact the statistics of the time-depending experimental results using a D-T external source, collected in the latest campaign by the MASURCA team, is largely better than in the case with a D-D source.

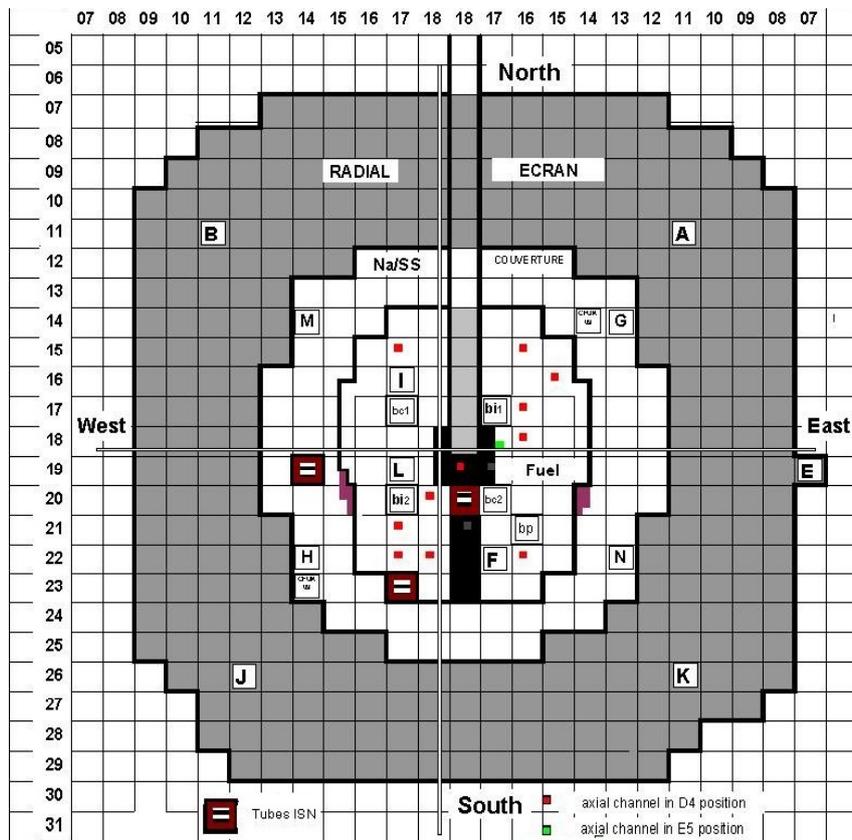


Figure 4. XY view of the MUSE-4 SC0 1108 fuel cells configuration and ^{235}U detectors position

During a previous experimental campaign, the MASURCA team measured the MSM subcriticality level [8] of the MUSE-4 SC0 1086 fuel cells configuration with a D-D external source (Fig. 5); this configuration is very close to the MUSE-4 SC0 1108 fuel cells configuration analyzed in this report, the difference mainly being represented by the presence of electronics devices instead of 22 fuel cells in the standard fuel sub-assembly. It has to be mentioned that, because of some heterogeneities in the core (represented by GENEPI beam pipe, the lead buffer and the tubes allocating the data acquisition system), the fuel cells number cannot be deduced by the layouts of the configurations, but a detailed analysis of the composition of each channel has to be performed [5].

Concerning the MUSE-4 SC0 1086 fuel cells, MSM subcriticality levels of $-1.74 \$$ and $-12.53 \$$ ($\beta_{\text{eff}}=335 \text{ pcm}$) have been obtained respectively with PR down and with 3 SR up and SR1 and PR down [10], while the measured subcriticality in the MUSE-4 SC0 1108 fuel cells configuration with PR down was $-1.95 \$$ [5]; because of the difference of only $0.2 \$$ between the two configurations with PR down, the MSM reactivity measured in the MUSE-4 SC0 1086 fuel cells configuration with PR down and with 3 SR up and SR1 and PR down ($\sim -4200 \text{ pcm}$) was assumed as reference reactivity level for the MUSE-4 SC0 1108 fuel cells configuration here investigated.

A XY calculation model of the MUSE-4 SC0 1108 fuel cells configuration was assessed, i.e. reactor at half height (Fig. 4), where MOX1 fuel was considered.

Cross sections processing were performed by means of heterogeneous calculations using the ECCO ERANOS cell code; neutron leakage from MOX1 fuel cell calculation was computed and used as source term for the heterogeneous cell calculations in the no-multiplying regions (as instance, the heterogeneous description of reflector subassembly is shown in Fig. 3).

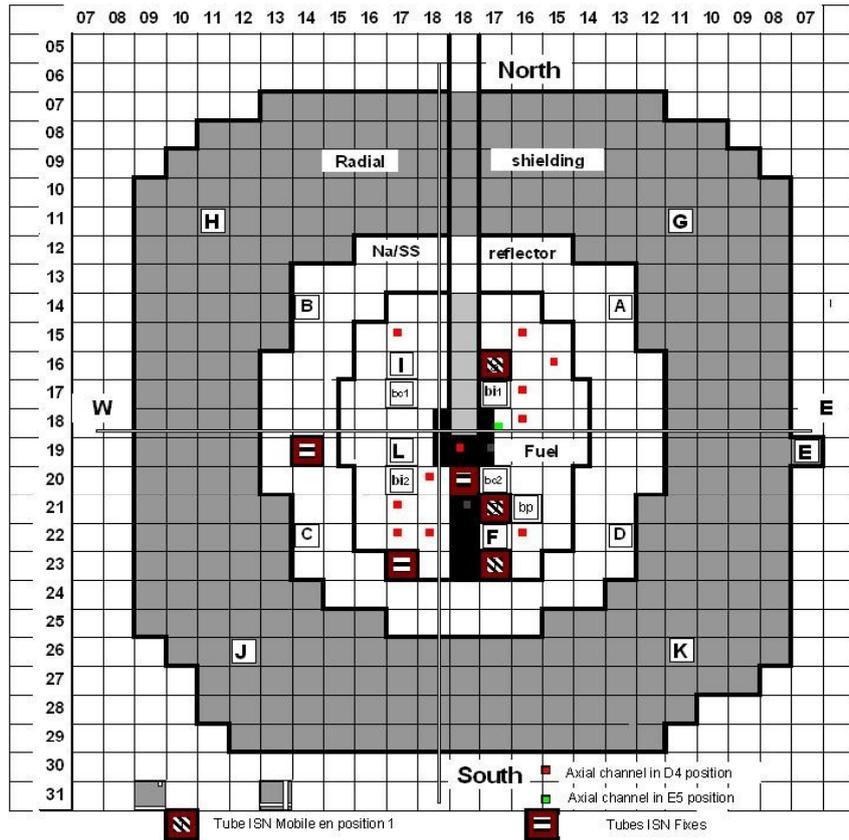


Figure 5. XY view of the MUSE-4 SC0 1086 fuel cells configuration

4. SOURCE-JERK METHOD

The source-jerk method [16] is a dynamic method based on the utilization of a time dependent external source for reactivity determination by means of a source perturbation measurement technique. In such technique, the external source is suddenly removed from the initial steady state condition, where there is the equilibrium of both the delayed neutron precursors and the prompt neutron concentrations.

If the system is initially at the equilibrium on the level P_0 (Fig. 6) and the external source is cut-off at the time t_0 , the flux level after the source-jerk will evolve with the following law, obtained from the point-reactor kinetics equations:

$$P(t) = P_0 e^{\alpha(t-t_0)} + \frac{\beta_{\text{eff}}}{\rho - \beta_{\text{eff}}} \cdot P_0 [e^{\alpha(t-t_0)} - 1] \quad (1)$$

the first term on the right is the main feature until the instant t_1 (red line) when the delayed contribution begins to dominate the shape (blue line) and the concavity of the curve changes; then, the flux shape will show an asymptotic behavior and the asymptotic level P_1 will be reached. From Eq. 1, the subcriticality level (in units of dollars) of the system can be related to the change in the flux level after and before the source jerk:

$$\rho(\$) = \frac{P_1 - P_0}{P_1} \quad (2)$$

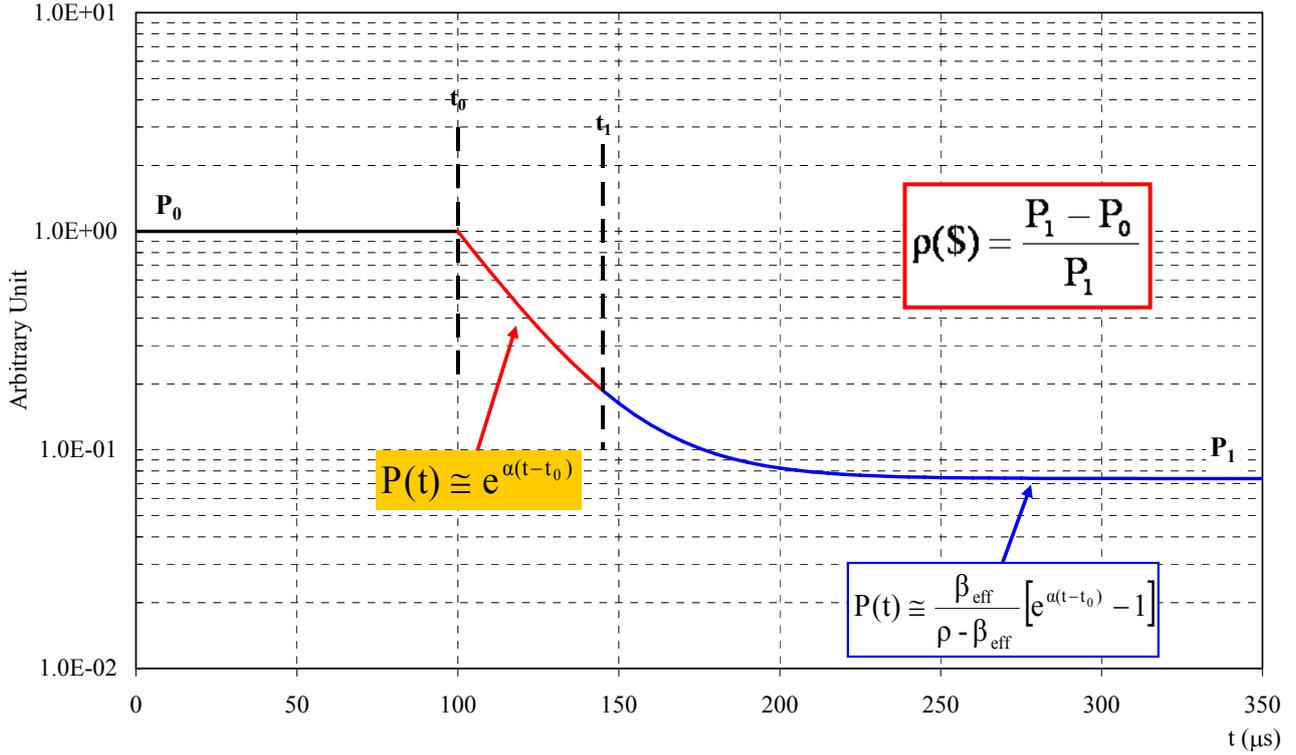


Figure 6. Source-jerk method

4.1 Source jerk calculation procedure by means of ERANOS code

A static approach was used in order to assess a calculation procedure reproducing a source jerk by means of the ERANOS deterministic codes system. The strategy consists on solving two inhomogeneous multigroup transport problems describing the system before and immediately after the external source removal (Fig. 7).

- Before the source cut-off, the external source Q is switched-on and the system is actually at equilibrium on both the delayed neutron precursors and the prompt neutron concentrations. The reactor system obeys the ordinary inhomogeneous steady-state transport equation, the solution being the neutron flux Φ_0 :

$$\Omega \cdot \nabla \Phi_0 + \Sigma_t \Phi_0 = \langle \Sigma_{\text{in scat}} \Phi_0 \rangle + \chi \langle \nu \Sigma_f \Phi_0 \rangle + Q \quad (3)$$

where χ is the total spectrum.

- After the source cut-off, the system is in a quasi-equilibrium condition driven by the delayed neutron source Q_d . Under the hypothesis of complete separability between the prompt and the delayed neutrons contributions, the system obeys the following relationship [16], the solution being the neutron flux Φ_1 :

$$\Omega \cdot \nabla \Phi_1 + \Sigma_t \Phi_1 = \langle \Sigma_{\text{in scat}} \Phi_1 \rangle + \left(\chi - \sum_i \beta_i \chi_d^i \right) \langle \nu \Sigma_f \Phi_1 \rangle + Q_d \quad (4)$$

where β_i and χ_d^i are respectively the nuclear β and the delayed spectrum of each isotope.

Finally, using the neutron fluxes Φ_0 and Φ_1 , the reactivity in each position of the system is evaluated by means of Eq. 2.

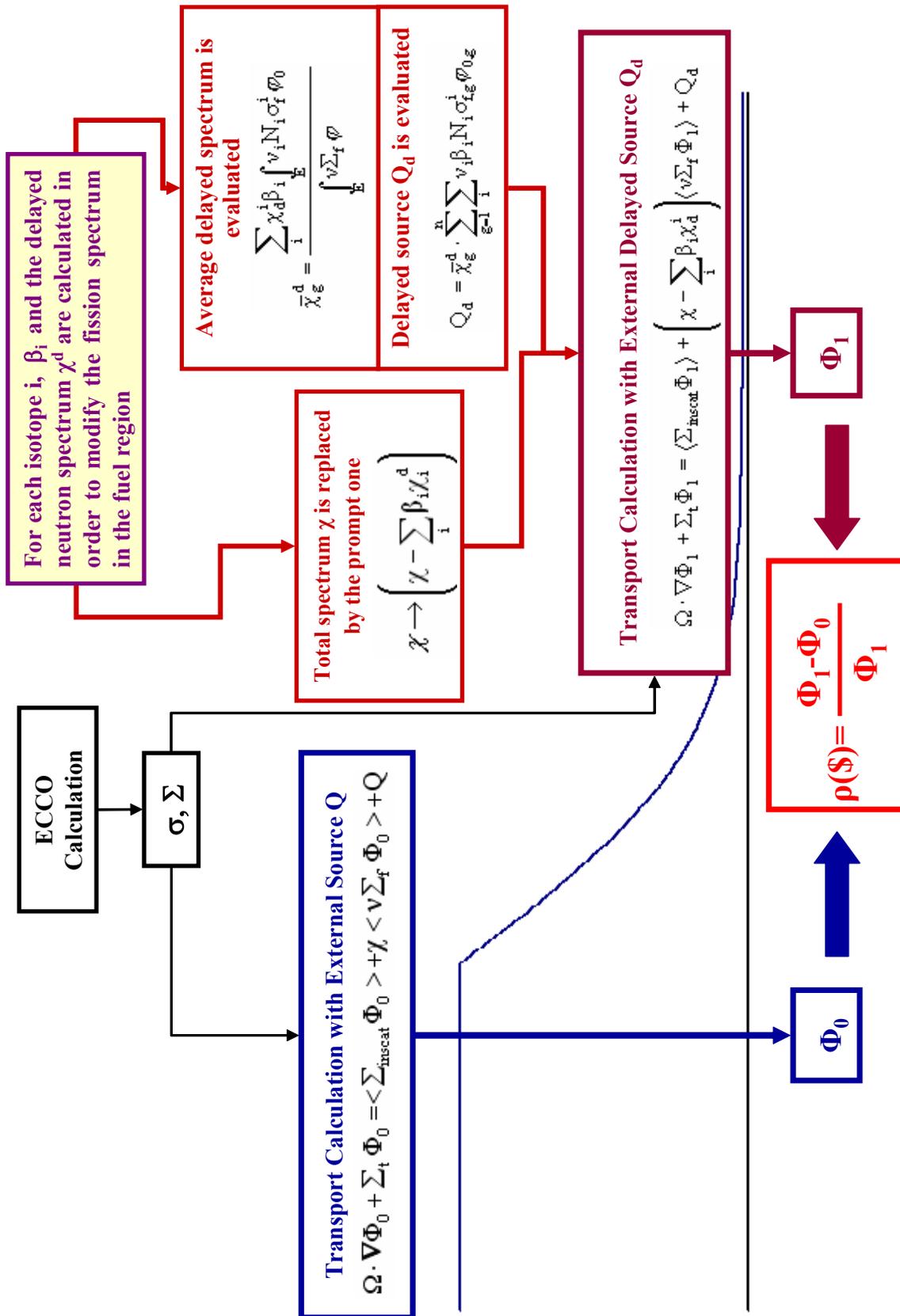


Figure 7. Flow chart of the assessed ERANOS calculation procedure simulating the ‘explicit’ source-jerk method

A particular attention has to be devoted to the calculation of the total prompt spectrum $\chi - \sum_i \beta_i \chi_i^d$ and of the delayed source Q_d . In fact, because the delayed neutrons contribution is a little part of the total neutron one (less than 10%), a little change of this value involves meaningful deviation in the final result, because the delayed contribution directly affects the denominator in Eq. 2.

In order to calculate the total prompt spectrum a careful evaluation of the nuclear beta β_i and of the delayed spectrum χ_i^d has to be carried out for each isotope i .

β_i calculation is performed by means of the classical formulas [16], where ϕ is the flux coming from a homogeneous calculation:

- the average number of neutrons \bar{v}_{p+d}^i per fission for each isotope i is calculated:

$$\bar{v}_{p+d}^i = \frac{\int v^i \sigma_f^i \phi dE}{\int \sigma_f^i \phi dE} \quad (5)$$

- β_i for each isotope i is evaluated using the total delayed neutron yields recommended data [17]:

$$\beta_i \equiv \frac{\bar{v}_d^i}{\bar{v}_{p+d}^i} \quad (6)$$

The multigroup delayed spectrum χ_i^d for each fissile isotope i and for each energy E is evaluated as sum at the energy E of the 8 families fine delayed spectrum (10 keV steps from 0.05 MeV to 11.365 MeV) [18], the spectrum for each family i being weighted with the 8 groups relative abundances α_i recommended data [17].

The n groups delayed neutron source Q_d driving the system after the external source cut-off is evaluated for each group g and in each calculation mesh as product between the total delayed productions in the mesh and the average delayed neutron spectrum $\bar{\chi}_g^d$ in each group g :

$$Q_d = \bar{\chi}_g^d \cdot \sum_{g=1}^n \sum_i v_i \beta_i N_i \sigma_{f,g}^i \phi_{0,g} \quad (7)$$

The average delayed neutron spectrum $\bar{\chi}_g^d$ is obtained in each group g as sum of the delayed productions fractions of all the isotopes, each one with its multigroup delayed spectra χ_i^d [16]:

$$\bar{\chi}_g^d = \frac{\sum_i \chi_d^i \beta_i \int_E v_i N_i \sigma_f^i \phi_0}{\int_E v \Sigma_f \phi} \quad (8)$$

Finally, in order to check the coherence of the procedure, before the jerk the system has to obey both to Eq. 3 and to the equivalent following relationship, where the total production term $\chi \langle v \Sigma_f \Phi_0 \rangle$ is separated into the prompt one ($\chi - \sum_i \beta_i \chi_i^d$) and in the delayed one Q_d :

$$\Omega \cdot \nabla \Phi_0' + \Sigma_t \Phi_0' = \langle \Sigma_{\text{in scat}} \Phi_0' \rangle + \left(\chi - \sum_i \beta_i \chi_i^d \right) \langle v \Sigma_f \Phi_0' \rangle + Q_d + Q \quad (9)$$

As result, the flux shapes Φ_0 and Φ_0' have to be exactly the same.

5. PNS AREA METHOD: A STATIC APPROACH

Pulsed Neutron Source (PNS) methods are based on the analysis of the responses shown by a subcritical system to an external source pulse [6, 7, 8, 16]. Concerning the MUSE-4 SC0 1108 fuel cells experimental configuration with 3 SR up, SR1 down and PR down, a typical ^{235}U detector response to a D-T external source in three different positions in the core is shown in Fig. 8 where the contributions of the prompt and delayed neutrons and of the intrinsic source are evidenced.

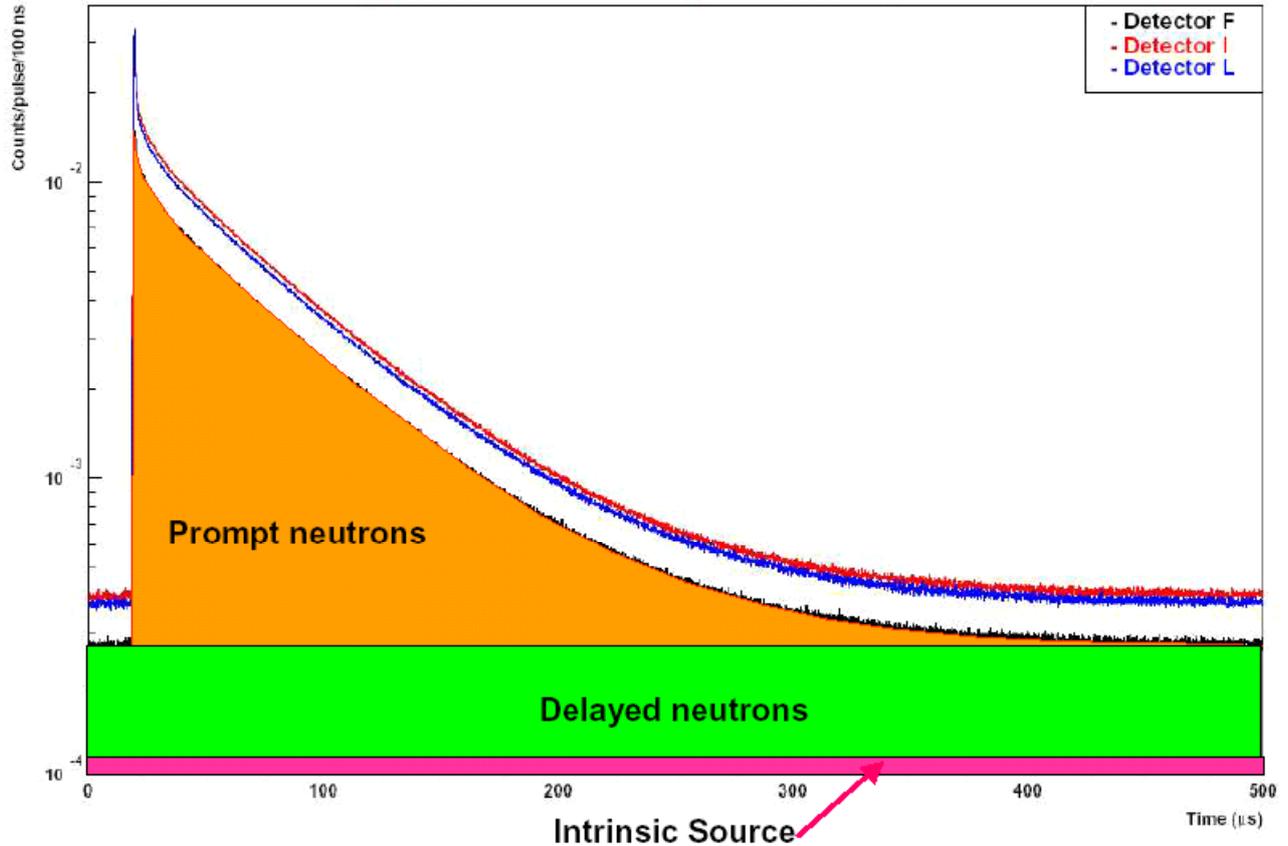


Figure 8. ^{235}U detector response to a D-T external neutron pulse

The area method consists of the evaluation of the absolute level of reactivity (in dollars) by the injection of a neutron pulse in a subcritical system by evaluating the ratio between the area under the prompt peak and the delayed one [7]:

$$\frac{-\rho}{\beta_{\text{eff}}} = \frac{\text{Prompt neutron area}}{\text{Delayed neutron area}} = \frac{I_p}{I_t - I_p} \quad (10)$$

where prompt area I_p is proportional to the detector response without delayed neutron precursors and delayed area I_d is equal to the difference between the total area I_t (corrected for any possible intrinsic source response) and the prompt one. If the system response cannot be approximated by point kinetics, the reactivity value will depend on the detector position. As reported in [4], these spatial effects can be taken into account by solving inhomogeneous transport time-independent problems.

Consider the neutron source represented by $Q(r,E,\Omega,t)=Q(r,E,\Omega)\delta_+(t)$ and the signal due only to the prompt neutrons; the prompt neutron flux $\Phi_p(r,E,\Omega,t)$ will obey the prompt time-dependent ordinary transport equation, with the usual free-surface boundary conditions and the initial condition $\Phi_p(r,E,\Omega,t)=0$. Integrating from $t=0$ to $t=\infty$ and defining the time integrated prompt neutron flux $\tilde{\Phi}_p(r,\Omega,E)=\int_0^\infty \Phi_p(r,\Omega,E,t)dt$, $\tilde{\Phi}_p(r,\Omega,E)$ will satisfy the inhomogeneous (with external source) time-independent prompt transport equation with the initial condition and the condition $\lim_{t \rightarrow \infty} \Phi_p = 0$ (because the system is subcritical):

$$\Omega \cdot \nabla \tilde{\Phi}_p + \Sigma_t \tilde{\Phi}_p = \langle \Sigma_{\text{in-scatt}} \tilde{\Phi}_p \rangle + \left(\chi - \sum_i \beta_i \chi_i^d \right) \langle v \Sigma_f \tilde{\Phi}_p \rangle + Q(r, E, \Omega) \quad (11)$$

where β_i and χ_i^d are respectively the nuclear β and the delayed spectrum of each isotope and χ is the total spectrum. In an analogous manner the total time-integrated flux $\tilde{\Phi}(r,\Omega,E)$ can be defined, by integrating the transport equation over all time; $\tilde{\Phi}(r,\Omega,E)$ will satisfy Eq. 11 with $\chi - \sum_i \beta_i \chi_i^d$ replaced by χ . Therefore $\tilde{\Phi}_p(r,\Omega,E)$ and the reactivity level of the system using the area method equation (Eq. 10) can be evaluated by means of a standard multigroup method:

- the prompt neutron area I_p is calculated by integrating over time the detector output response using the time-integrated prompt neutron flux $\tilde{\Phi}_p(r,\Omega,E)$ by solving Eq. 11:

$$I_p = \iiint \sigma_d(r,E) \tilde{\Phi}_p dV d\Omega dE ;$$
- total area I_t is evaluated integrating over time the detector output response using the time-integrated neutron flux $\tilde{\Phi}(r,\Omega,E)$, by solving the inhomogeneous ordinary time-independent transport equation: $I_t = \iiint \sigma_d(r,E) \tilde{\Phi} dV d\Omega dE ;$
- the delayed neutron area is obtained as difference between the total and the prompt area:

$$I_d = \iiint \sigma_d(r,E) (\tilde{\Phi} - \tilde{\Phi}_p) dV d\Omega dE ;$$
- finally, reactivity level can be obtained by applying Eq. 10.

Note that I_p and I_d are functions of detector position by means of the spatial dependence of the detector microscopic cross section σ_d .

5.1 Area method calculation procedure by means of ERANOS code

The static approach to the area method was used in order to assess a calculation procedure using the ERANOS deterministic codes system (Fig. 9):

- n energy groups cross sections processing is performed by means of ECCO ERANOS cell code [8];
- an inhomogeneous core transport calculation is carried out by means of the ERANOS spatial module: as result the time-integrated neutron flux $\tilde{\Phi}$ is obtained;
- an inhomogeneous core prompt transport calculation is carried out by means of the ERANOS spatial module, by replacing the total fission spectrum χ with the prompt one $\chi - \sum_i \beta_i \chi_i^d$: as result the time-integrated prompt neutron flux $\tilde{\Phi}_p$ is obtained.

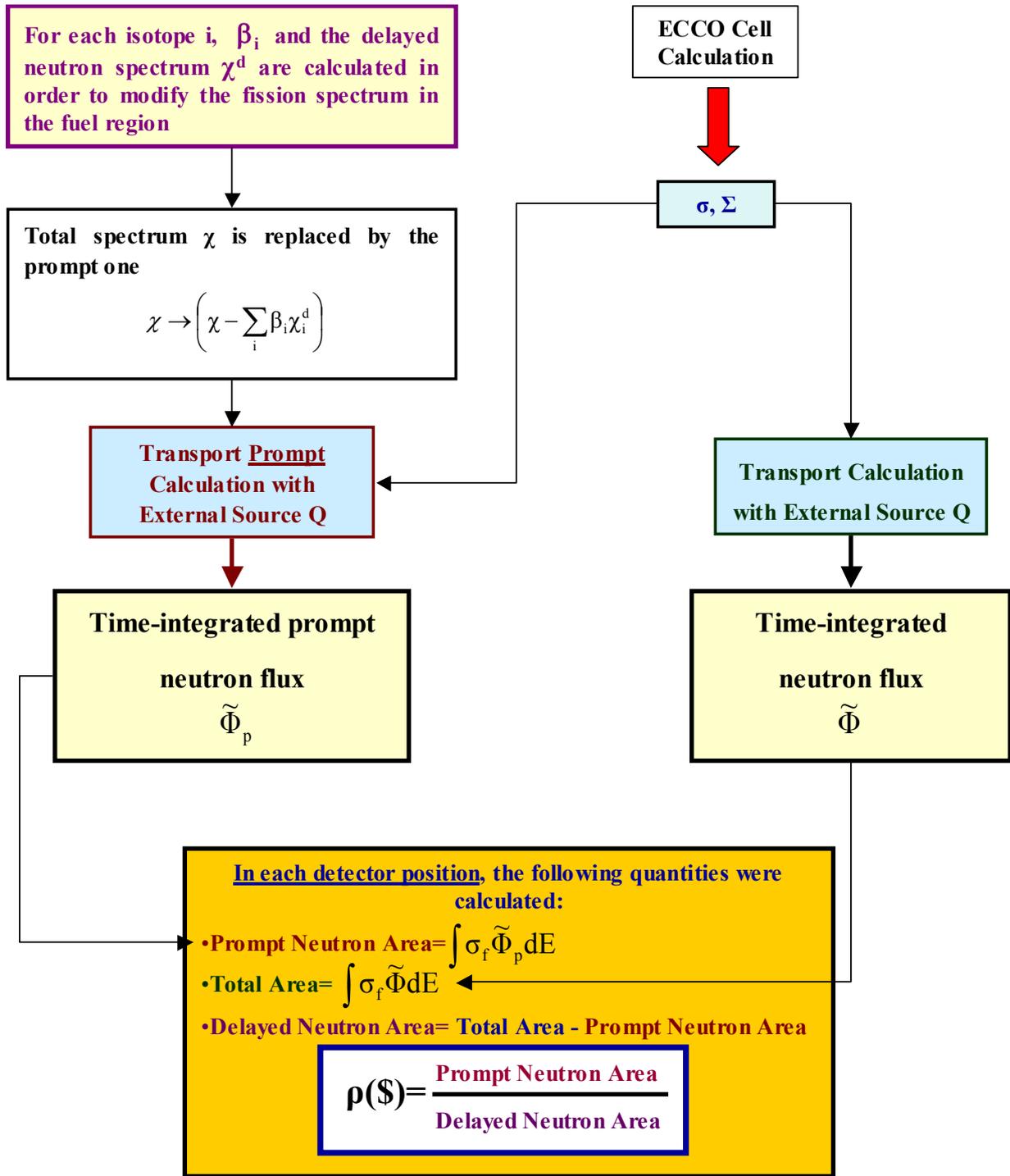


Figure 9. Flow chart of the assessed ERANOS calculation procedure simulating the PNS area method

Then, in each detector position, characterized by the microscopic cross section σ_d , the following quantities are calculated:

- the total area, i.e. the total reaction rate in the detector position using the flux $\tilde{\Phi}$;
- the prompt area, i.e. the total reaction rate in the detector position using the flux $\tilde{\Phi}_p$;
- the delayed area, as difference between the total and the prompt areas;
- the reactivity in dollars (Eq. 10), as ratio between the prompt and the delayed areas.

6. EQUIVALENCE BETWEEN THE METHODS

In view of the utilization in the ETD, PNS area and source jerk methods show large differences from the experimental point of view. Source jerk method would allow to infer an on-line reactivity of the subcritical system because of the operative use of the external source in the ETD. In fact, a continuous beam will be used as external source and beam interruptions are expected.

On the contrary, area method does not allow an on-line reactivity measurement, but can represent a good calibration technique with regard to other methods (i.e. source jerk).

Even if these differences exist, from a theoretical point of view no discrepancies are expected on reactivity results by performing the two above assessed calculation procedures. In fact, by using a static approach, the reactivity is evaluated using by means of the same relationship coming from the source jerk and area method approaches (Eq. 2 and 10). In the source jerk procedure the total and the delayed contributions are directly evaluated, while in the area calculation procedure the total and the prompt ones are calculated, the delayed one being obtained as difference between the total and the prompt contribution. Little discrepancies will be due only to the systematic inside the calculations, in particular when actual systems are considered because of the presence of several fissile isotopes.

In the following the source jerk method calculation procedure will be tested in the RZ subcritical test configuration (Fig. 1) and results will be compared with that ones coming from the application of the area method calculation procedure.

6.1 Preliminary test

The assessed source jerk calculation procedure was performed in the RZ subcritical test configuration (Fig. 1) where only one fissile isotope (^{239}Pu) is present in the fuel. 4 energy groups cross sections calculation (Tab. 2) has been carried out by means of the ERANOS ECCO cell code [12] by a fine energy group structure collapsing in conjunction with the JEF2.2 nuclear data library. Core transport calculations were performed using the BISTRO spatial module [19].

	Group 1	Group 2	Group 3	Group 4
Upper Energy Limit (MeV)	1.9640E+01	1.3534E+00	6.7380E-02	2.0347E-03

Table 2. 4 energy groups structure

Homogeneous transport calculation was performed and the reference reactivity (-2238 pcm) was obtained.

In order to describe the system before the jerk (Eq. 3), a D-T external source ($3.3 \cdot 10^6$ n/pulse of 14 MeV) was located at the reactor center and an inhomogeneous transport calculation was carried out. As result the neutron flux Φ_0 was obtained. Then, in order to describe the system after the jerk (Eq. 4), the ^{239}Pu prompt neutron spectrum and the ^{239}Pu delayed source in each calculation mesh were calculated.

After performing the calculation of both the ^{239}Pu nuclear beta (Tab. 3) and the 4 energy groups delayed spectrum (Fig. 10), the prompt neutron spectrum ($\chi_{^{239}\text{Pu}} - \beta_{^{239}\text{Pu}} \chi_{^{239}\text{Pu}}^d$) for each group was evaluated.

$\bar{V}_d^{239}\text{Pu}$ [17]	$\bar{V}_{p+d}^{239}\text{Pu}$	$\beta_{^{239}\text{Pu}}$
0.00651	2.9618	0.00220

Table 3. ^{239}Pu nuclear beta calculation (Eq. 5 and 6)

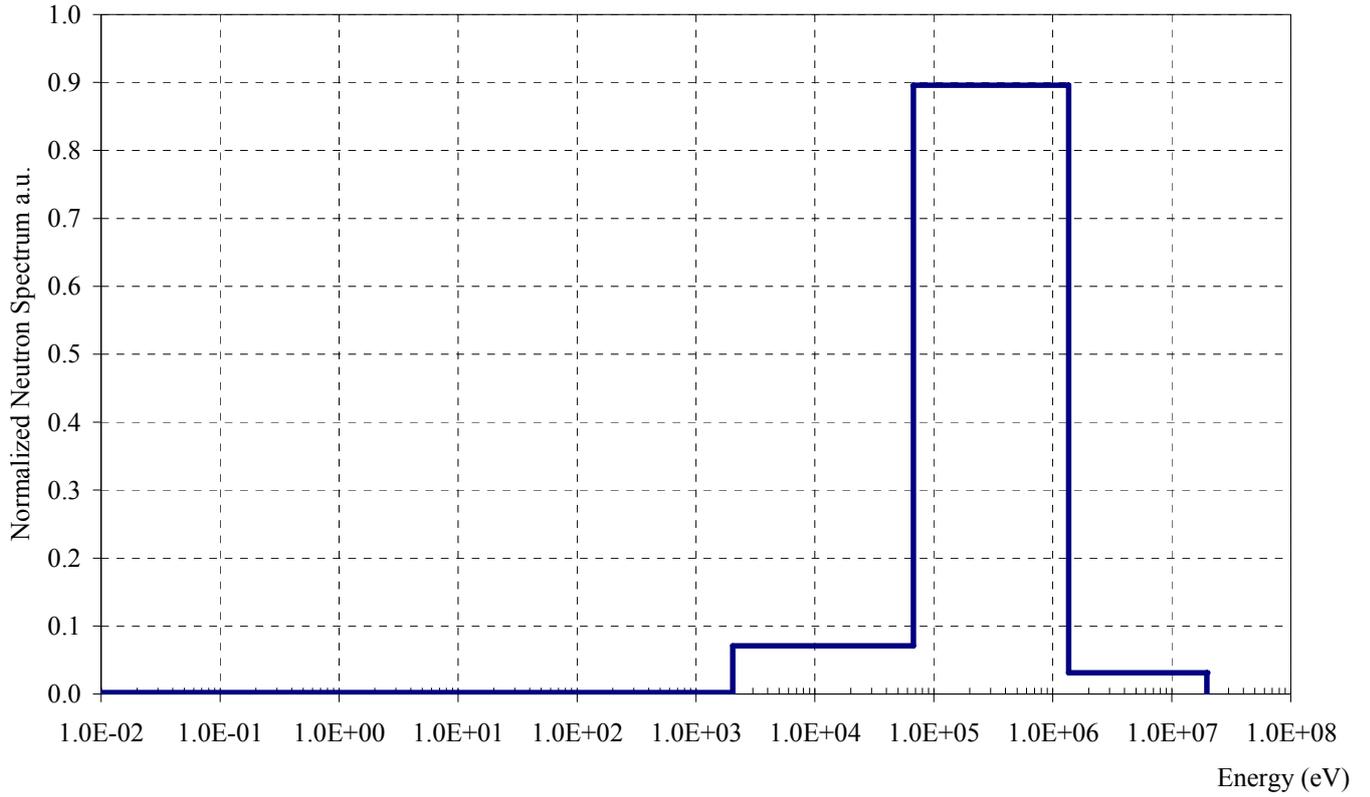


Figure 10. 4 energy groups ^{239}Pu delayed spectrum

Concerning the external delayed source Q_d , Eq. 7 becomes:

$$Q_d = \chi_{239\text{Pu},g}^d \beta_{239\text{Pu}} \cdot \sum_{g=1}^n v_{239\text{Pu}} N_{239\text{Pu}} \sigma_{f,g}^{239\text{Pu}} \varphi_{0,g}$$

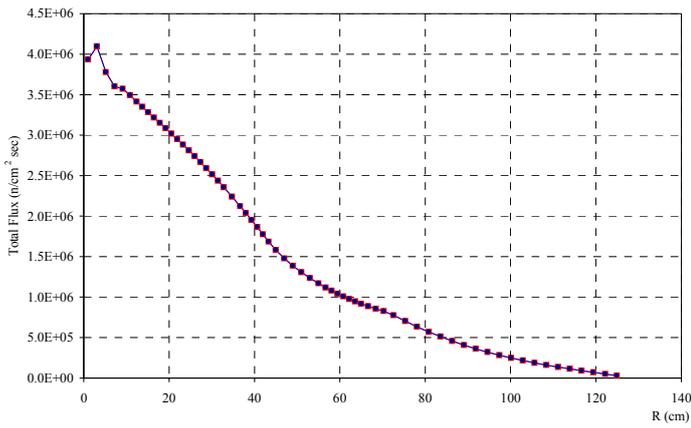
where Q_d calculation is performed in each calculation mesh and for each group g .

Finally an inhomogeneous transport calculation was carried out and the neutron flux Φ_1 was obtained (Eq. 4).

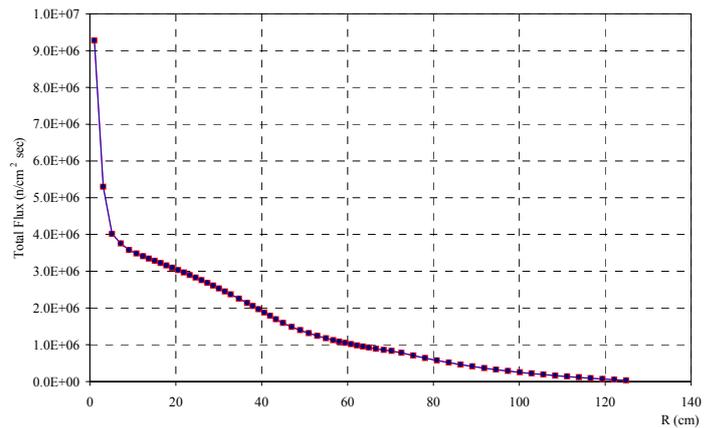
The correct evaluation of the delayed source Q_d represents the main stage in the calculation procedure assessment. Before the jerk, the system obeys the inhomogeneous steady-state transport equation with the external source Q (Eq. 3), the solution being the flux Φ_0 . At the same time, the system obeys the following equivalent relationship where the total neutron productions were separated into the prompt and the delayed ones, the solution being the flux Φ_0' :

$$\Omega \cdot \nabla \Phi_0' + \Sigma_t \Phi_0' = \langle \Sigma_{\text{in scat}} \Phi_0' \rangle + \left(\chi - \sum_i \beta_i \chi_d^i \right) \langle v \Sigma_f \Phi_0' \rangle + Q_d + Q \quad (12)$$

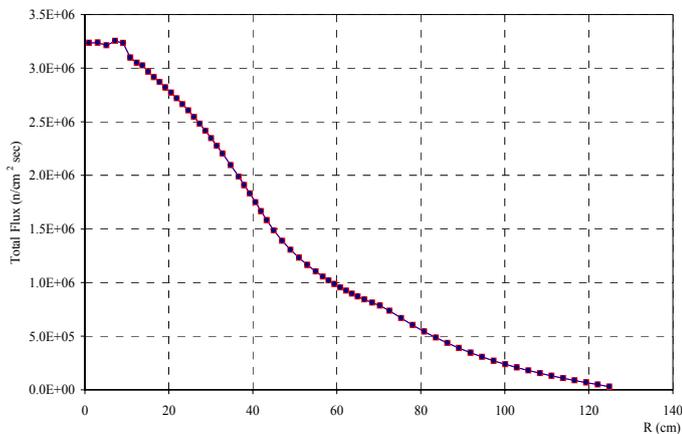
If the delayed source is well evaluated, the Φ_0 and Φ_0' flux shapes have to be equal everywhere in the system. In order to check this condition two inhomogeneous transport calculations were performed by means of ERANOS BISTRO spatial module [19] and the total flux radial traverses at different heights were compared (Fig. 11).



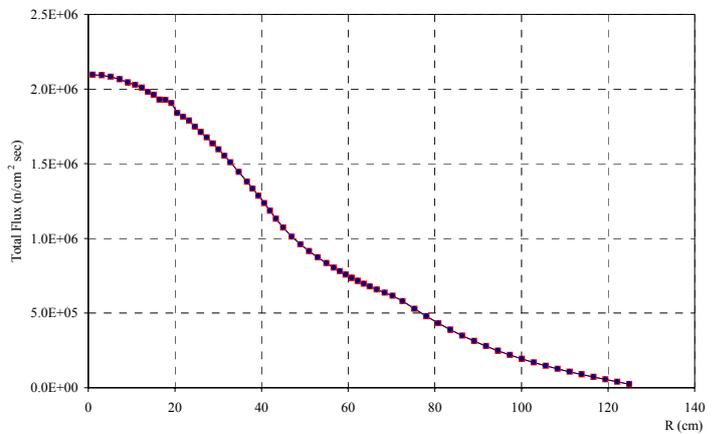
Z=111.3 cm



Z=118.3 cm



Z=136.0 cm



Z=160.8 cm

Figure 11. Comparison between the Φ_0 (red squares) and Φ'_0 (blue bullets) traverses total fluxes at different height in the RZ test configuration

As expected, results show a very good agreement being the average percent difference between the total fluxes in each calculation mesh in the whole reactor of about 0.004%.

Then, for some selected positions in the fuel, reflector and shielding regions (Fig. 1), the reactivity was evaluated (Eq. 2) and compared to the reference one (-10.2 \$, labelled with R in Tab. 4).

Detector	Calculated reactivity (\$)	(R-C)/C (%)
F1	-10.3	-0.9
F2	-10.1	+0.8
F3	-10.1	+0.4
R1	-10.2	-0.4
R2	-10.2	0.0
R3	-10.1	+0.4
S1	-10.2	-0.1
S2	-10.2	-0.4
S3	-10.1	+0.3

Table 4. RZ test configuration: ERANOS source jerk calculation procedure results

Results show the very good coherence among the calculations, being the maximum difference between the homogeneous calculation result and that one coming from the application of the calculation procedure of about 1%.

As further test about the coherence of calculations, the assessed area method procedure was performed in the RZ test configuration using the ^{239}Pu prompt neutron spectrum in Eq. 11. In this case, while the prompt contribution is directly evaluated, the delayed one is calculated as difference between the total and the prompt contributions (Eq. 10).

Tab. 5 shows the neutron contributions in the RZ test configuration from the application of the source jerk and area method calculation procedures in each detector position. In the last column the percent difference between the delayed contributions coming from the application of the source jerk (SJ) and area method (A) approaches is shown.

	Source Jerk Approach		Area Method Approach			(SJ-A)/A (%)
	Before Jerk	After Jerk	Total	Prompt	Delayed=(Total-Prompt)	
F1	2.135E+06	1.895E+05	2.135E+06	1.945E+06	1.896E+05	-0.08
F2	1.431E+06	1.290E+05	1.431E+06	1.302E+06	1.292E+05	-0.11
F3	1.447E+06	1.299E+05	1.447E+06	1.317E+06	1.300E+05	-0.08
R1	8.910E+05	7.943E+04	8.910E+05	8.115E+05	7.949E+04	-0.08
R2	5.539E+05	4.956E+04	5.539E+05	5.043E+05	4.959E+04	-0.06
R3	6.664E+05	5.988E+04	6.664E+05	6.065E+05	5.992E+04	-0.07
S1	6.498E+04	5.809E+03	6.498E+04	5.961E+04	5.813E+03	-0.06
S2	2.838E+05	2.532E+04	2.838E+05	2.585E+05	2.534E+04	-0.08
S3	5.417E+04	4.862E+03	5.417E+04	4.930E+04	4.865E+03	-0.06

Table 5. Comparison between the neutron total fluxes coming from the application of the ERANOS source jerk and area method calculation procedures

It looks evident the very good agreement between the delayed contributions calculated using the two different approaches. As result, the two assessed calculation procedures will exactly supply the same reactivity value in each detector position.

6.2 Application to the MUSE-4 and comparison with the experimental data

Source jerk calculations were carried out in the the MUSE-4 1108 fuel cells configuration with 3 SR up, SR1 down and PR down and with the D-T external source ($3.3 \cdot 10^6$ n/pulse of 14 MeV) located at the reactor centre.

In order to carry out the calculations, a XY model was assessed (Fig. 4) and 33 energy groups cross sections processing (Tab. 6) was performed by means of ECCO cell code [12] by a fine energy group structure collapsing in conjunction with the JEF2.2 nuclear data.

Core transport calculations were carried out by means of BISTRO spatial module [19] and the reference reactivity in the initial steady state (-12.53 \$, -4200 pcm) was obtained by tuning the leakage component.

g	Upper Energy Limit (MeV)	g	Upper Energy Limit (MeV)	g	Upper Energy Limit (MeV)
1	1.9640E+01	12	6.7380E-02	23	3.0433E-04
2	1.0000E+01	13	4.0868E-02	24	1.4863E-04
3	6.0653E+00	14	2.4788E-02	25	9.1661E-05
4	3.6788E+00	15	1.5034E-02	26	6.7904E-05
5	2.2312E+00	16	9.1188E-03	27	4.0169E-05
6	1.3534E+00	17	5.5308E-03	28	2.2603E-05
7	8.2085E-01	18	3.3546E-03	29	1.3710E-05
8	4.9787E-01	19	2.0347E-03	30	8.3153E-06
9	3.0197E-01	20	1.2341E-03	31	4.0000E-06
10	1.8316E-01	21	7.4852E-04	32	5.4000E-07
11	1.1109E-01	22	4.5400E-04	33	1.0000E-07

Table 6. 33 energy groups structure

In order to evaluate both the prompt spectrum and the delayed neutron source (Eq. 4 and 7), the nuclear beta for each isotope i composing the MOX fuel was evaluated (Tab. 7) using the recommended data [17].

Isotope	$\bar{\nu}_d^i$ [17]	$\bar{\nu}_{p+d}^i$	β_i
²³⁵ U	0.01630	2.4783	0.00658
²³⁸ U	0.04650	2.7582	0.01686
²³⁸ Pu	0.00400	3.0323	0.00132
²³⁹ Pu	0.00651	2.9608	0.00220
²⁴⁰ Pu	0.00960	3.0196	0.00318
²⁴¹ Pu	0.01490	2.9894	0.00498
²⁴² Pu	0.02290	3.0599	0.00748
²⁴¹ Am	0.00460	3.6535	0.00126

Table 7. Nuclear beta values (Eq. 5 and 6)

Then, the delayed spectra at 33 energy groups were calculated for each isotope i (Fig. 12) using the recommended data [18]. The average neutron delayed spectrum calculation was performed by using Eq. 8 (red bold line in Fig. 12).

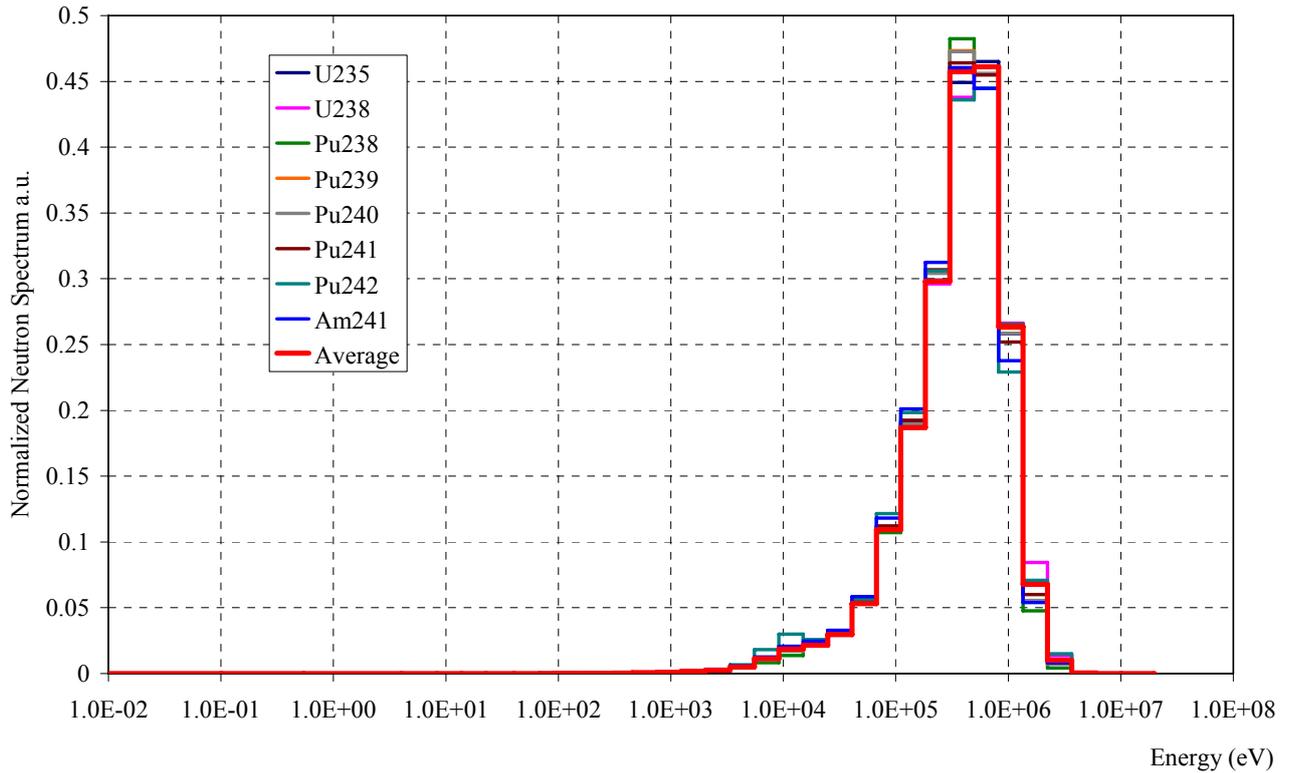


Figure 12.33 energy groups delayed spectra and average delayed spectrum

Finally the source-jerk calculation procedure was performed in the XY MUSE-4 1108 fuel cells model with 3 SR up, SR1 down and PR down (Fig. 4) and the D-T external source located at the reactor center.

Inhomogeneous transport calculations were performed by means of BISTRO spatial module [19]. For each ^{235}U detectors position (Fig. 4), the reactivity in dollars was calculated (Eq. 2) and compared with the experimental data [5, 15] in Tab. 8, where in the ‘dispersion’ column the ratio between the experimental and calculated reactivity and the reference reactivity ($\rho(\$)=-12.53$) is shown; moreover the percentage difference between the experimental and calculated data is also reported.

Results show a very good agreement between the calculated values and the experimental ones, being the main difference relevant to the detector I, very close to the SR1 control rod.

Detector	Reactivity (\$)		Dispersion		
	Exp.	Cal.	Exp.	Cal.	(E-C)/C (%)
I	-14.3	-13.1	1.14	1.05	-8.1
L	-12.9	-12.9	1.03	1.03	+0.1
F	-11.9	-11.7	0.95	0.93	-1.9
M	-12.7	-12.7	1.01	1.02	+0.3
G	-13.0	-12.2	1.04	0.97	-6.1
N	-12.1	-11.7	0.96	0.93	-3.3
H	-12.6	-12.0	1.00	0.96	-4.9
A	-12.7	-12.3	1.01	0.98	-3.1
B	-13.0	-12.8	1.04	1.02	-1.7

Table 8. MUSE-4 1108 fuel cells configuration: comparison between the ERANOS source jerk calculation procedure and the experimental results

In particular the comparison between the calculated values and the experimental ones shows that:

- respect to the reference reactivity (-12.53\$), the maximum spatial ‘dispersion’ of the calculated results is about -6% (detector G);
- the maximum percentage difference E/C is about -6% (detectors G last column in Tab. 8).

Then, the source jerk calculation procedure seems to well reproduce the spatial distribution of the experimental reactivity. Only the highest reactivity result relevant to the detector I, very close to the SR1 control rod, is not well reproduced.

It has to be remarked here that the calculation procedure was applied to the MUSE-4 1108 fuel cells configuration with 3 SR up, SR1 down and PR down and with the D-T external source located at the reactor centre with the only aim to test the procedure; in fact, as mentioned above, MUSE-4 experiment does not allow for an ‘explicit’ application of the source jerk method, because of the pulsed regime of the external source together with the characteristics of the GENEPI accelerator (frequency, pulse duration);

Then the area method calculation procedure was carried out in this MUSE-4 configuration and results were compared to the source jerk ones (Tab. 9).

Detector	Reactivity (\$)		Dispersion		
	Source Jerk	Area	Source Jerk	Area	(SJ-A)/A (%)
I	-13.1	-13.2	1.05	1.05	-0.15
L	-12.9	-12.9	1.03	1.03	+0.01
F	-11.7	-11.7	0.93	0.93	+0.01
M	-12.7	-12.7	1.02	1.02	-0.10
G	-12.2	-12.2	0.97	0.98	-0.06
N	-11.7	-11.7	0.93	0.93	-0.01
H	-12.0	-12.0	0.96	0.96	+0.02
A	-12.3	-12.3	0.98	0.98	-0.04
B	-12.8	-12.8	1.02	1.02	-0.06

Table 9. MUSE-4 1108 fuel cells configuration: comparison between the results coming from the ERANOS source jerk and area method calculation procedures

Results show a very good agreement between the calculation results, being negligible the difference between the reactivities calculated in each detector position.

7. CONCLUSIONS

The calculation procedure simulating the ‘explicit’ source jerk method was assessed by means of CEA ERANOS deterministic codes system in conjunction with JEF2.2 nuclear data reference libraries by solving time-independent problems. Moreover a calculation procedure simulating the PNS area method was assessed using a static approach.

Source jerk procedure was initially performed in a RZ test configuration. Results showed a good internal coherence of the calculation method, being the reactivity of the system well reproduced by calculations.

Moreover the PNS area method calculation procedure was carried out, results showing an excellent agreement with that ones coming from the application of the source jerk approach and then constituting a further proof of the coherence of the method. In fact, while the delayed contribution is directly evaluated in the source jerk calculation procedure, in the area method static approach this contribution is calculated as difference between the total and the prompt one.

The calculation procedures were performed in the MUSE-4 SC0 1108 fuel cells with 3 SR up, SR1 down and PR down driven by a D-T external source provided by the GENEPI pulsed deuteron accelerator and calculated results were compared with the experimental measurements with the aim to contribute to the experimental data analysis by calculating the spatial dependence of the area method experimental results.

Concerning the source jerk method, it has to be remarked that an ‘explicit’ source-jerk method cannot be explored in MUSE configurations driven by GENEPI accelerator because of the pulsed nature of the external source; in fact in these conditions the equilibrium of both the delayed neutron precursor and the prompt neutron concentrations just before the source cut-off cannot be reached.

In any case, the spatial dispersion of the calculated results show a very good agreement with the experimental data in the examined MUSE-4 configuration with the D-T external source, being the maximum difference of about -6%; because the static calculation procedure seems to be capable to predict the same spatial reactivity spread of the experimental data, it is possible to evaluate by calculations the corrective factors to be applied to the experimental measurements in order to infer the actual subcriticality level of the system.

Results from the application of the source jerk procedure also indicated a very good agreement with that ones coming from the area method static approach.

At present, the possibility to perform subcriticality level measurements by means of the source jerk technique has been investigating is some MUSE-4 configurations at different subcriticality level with a ^{252}Cf external source.

Further investigations have to be performed in systems where the condition to apply the source-jerk method can be reached (i.e. RACE experiment).

Bibliography

- [1] C. Rubbia, J.A. Rubio, S. Buono, F. Carminati, N. Fietier, J. Galvez, C. Geles, Y. Kadi, R. Klapisch, P. Mandrillon, J. P. Revol, C. Roche, 'Conceptual design of a fast neutron operated high power energy amplifier', CERN/AT/95-44 (ET).
- [2] C. D. Bowman, E. D. Arthur, P. W. Lisowski, G. P. Lawrence, R. J. Jensen, J. L. Anderson, B. Blind, M. Cappiello, J. W. Davidson, T. R. England, L. N. Engel, R. C. Haight, H. G. Hugens III, J. R. Ireland, R. A. Krakowski, R. J. Labauve, B. C. Letellier, R. T. Perry, G. J. Russel, K. P. Staudhammer, G. Versamis, W. B. Wilson, 'Nuclear Energy Generation and Waste Transmutation Using an Accelerator-Driven Intense Thermal Neutron Source', *Nucl. Instr. and Meth.*, **A320**, 336, 1992.
- [3] M. Carta, A. D'Angelo, G. Bianchini, P. Bosio, P. Ravetto, 'Monitoring of Subcriticality Level in Accelerator Driven Systems: Harmonic Modulated Source - Spatial Source Jerk Intercomparison', International Conference on Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications, Madrid, Spain, September 27 - 30, 1999.
- [4] M. Salvatores et al., 'MUSE-1: A First Experiment at MASURCA to Validate the Physics of Sub-Critical Multiplying Systems Relevant to ADS', 2nd International Conference on Accelerator-Driven Transmutation Technologies and Applications, Kalmar, Sweden, Vol. 1, p 513 (1996)
- [5] F. Mellier, 'The MUSE Experiment for the subcritical neutronics validation', 5th European Framework Program MUSE-4 Deliverable 6, Contract n° FIKW-CT-2000-00063, CEA, June 2005.
- [6] B.E. Simmons and J.S King: 'A Pulsed Neutron Technique for Reactivity Determination', *Nucl. Sci. and Eng.*, **3**, 595 (1958).
- [7] N.J. Sjöstrand, *Arckiv. Fis.* 11, 233 (1956), see also in G.I. Bell and S. Glasstone, 'Nuclear Reactor Theory', (pp.546-549), 1970, Van Nostrand Reinhold Company.
- [8] E. Garelis and J. Russel, *Nucl. Sci. and Eng.*, **6**, 263 (1963), see also in G. I. Bell and S. Glasstone, 'Nuclear Reactor Theory', (pp.550), *ibidem*.
- [9] M. Carta, A. D'Angelo, V. Peluso, G. Aliberti, G. Imel, G. Palmiotti, V. Kulik, J. F. Lebrat, Y. Rugama, C. Destouches, E. González-Romero, D. Villamarín, S. Dulla, F. Gabrielli, P. Ravetto, M. Salvatores, et al., 'Reactivity assessment and spatial time-effects from the MUSE kinetics experiment', PHYSOR 2004 – The Physics of Fuel Cycles and Advanced Nuclear Systems: Global Developments, Chicago, Illinois, April 25-29, 2004.
- [10] G. Rimpault et al., 'The ERANOS code and data system for fast reactor neutronic analyses', PHYSOR 2002, Seoul, Korea, October 7-10, 2002.
- [11] F. Gabrielli, M. Carta, A. D'Angelo, V. Peluso, 'Interpretation of the MUSE-4 kinetics experiments: analysis of a selected configuration', ENEA Technical Report, RT/2004/28/FIS, June 2004.
- [12] G. Rimpault, 'Algorithmic Features of the ECCO Cell Code for Treating heterogeneous Reactor Subassemblies', International Conference On Mathematics and Computations, Reactor Physics and Environmental Analyses, Portland, OR, April 30 – May 4, 1995.
- [13] J. F. Lebrat, R. Jacqmin, F. Gabrielli, M. Carta, V. Peluso, G. Buzzi, G. Bianchini, A. D'Angelo, G. Aliberti, G. Palmiotti, 'Fast Reactor Core-Reflector interface effects revisited', PHYSOR 2002, Seoul, Korea, October 7-10, 2002.

- [14] R. Soule, W. Assal, D. Villamarin, 'Geometrical and physical data associated to the MUSE 4 Reference configuration', CEA, Internal Report 01-015.
- [15] E. González-Romero, D. Villamarín, M. Embid, G. Aliberti, G. Imel, G. Palmiotti, V. Kulik, C. Destouches, F. Mellier, C. Jammes, G. Perret, A. Billebaud, R. Brissot, D. Heuer, C. Lebrun, E. Liatard, J. M. Loiseaux, O. Meplan, E. Merle, F. Perdu, J. Vollaire, A. D'Angelo, M. Carta, V. Peluso, J. L. Kloosterman, Y. Rugama, P. Beaten, F. Gabrielli, 'Pulsed Neutron Source measurements of kinetic parameters in the source-driven fast subcritical core MASURCA', International Workshop on P&T and ADS Development, organized on behalf of the "ADvanced Option for Partitioning & Transmutation Thematic Network (ADOPT)", SCK-CEN, Mol, Belgium, October 6-8, 2003.
- [16] G. R. Keepin, 'Physics of Nuclear Kinetics', (pp. 251-252), 1965, Addison Wesley Pb. Company, Inc., Reading, Massachusetts, USA.
- [17] 'Progress In Nuclear Energy ',ISSN 0149-1970, 41 (2002).
- [18] G. Spriggs, Private Communication.
- [19] G. Palmiotti, J. M. Rieunier, C. Gho, M. Salvatores, 'BISTRO Optimized Two Dimensional Sn Transport Code', Nucl. Sci. and Eng., 104, 26 (1990).