Coherent Electron Transport in Superconducting-Normal Metallic Films

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We study the transport properties of a *quasi-two-dimensional* diffusive normal metal film attached to a superconductor. We demonstrate that the properties of such films can essentially differ from those of quasi-one-dimensional systems: in the presence of the proximity induced superconductivity in a sufficiently wide film its conductance may not only increase but also *decrease* with temperature. We develop a quantitative theory and discuss the physical nature of this effect. Our theory provides a natural explanation for recent experimental findings referred to as the "anomalous proximity effect".

A normal metal attached to a superconductor also acquires superconducting properties [1]: at sufficiently low temperatures "superconducting" electrons penetrating into a normal metal (N) remain coherent even far from a superconductor (S). This proximity effect can strongly influence transport properties of the system and becomes particularly pronounced in the case of transparent inter-metallic interfaces.

Recent theoretical and experimental studies of diffusive mesoscopic NS proximity structures [2–8] (see also Refs. therein) revealed various interesting features of long-range coherent states in such systems. One of such features is a non-monotonic dependence of the system conductance on temperature [3,4,9,10]: as the temperature T decreases below the transition temperature T_C its linear conductance G increases above the normal state value G_N, reaches its maximum at T of the order of the Thouless energy E_d of the normal metal and then decreases down to $G = G_N$ at $T = 0$. This non-monotonic behavior has been detected in recent experiments [7].

The high temperature behavior of $G(T)$ can be easily understood: as the temperature is lowered superconductivity expands in the normal metal and its conductance increases. The decrease of G with temperature at $T \lesssim E_d$ is due to the presence of a proximity induced (pseudo)gap in the density of states of the N-metal at energies $E \lesssim E_d$ [4]. It is also important to emphasize that at any $0 < T < T_C$ the conductance G was found to be *larger* than G_N [3,4].

Surprisingly, in several experiments with proximity NS structures [5,6,8] a *decrease* of the conductance *below* its normal state value already *at the onset* of superconductivity was observed. In some cases [5] a negative correction to G was as large as 30 % of G_N. Even more puzzling was the sample dependence of this result: in [6] a *decrease* of $G(T)$ with temperature was reported if Sb was chosen as a normal conductor, whereas if Sb was substituted by Ag the conductance *increased* with decreasing T.

It appears that the explanation of the above effects based on the assumption of low transparent NS boundaries should be ruled out: in [5,6] the current does not flow directly through NS interfaces and, on top of that, the NS boundaries in these experiments were believed to be highly transparent. One can also recall that in the presence of proximity induced correlations the electric field penetrating into the normal metal can "overshoot" its normal state value [4]. This effect, although in principle could be interpreted as a suppression of the local conductivity inside a part of the N-metal, can hardly explain the results [5,6]: at sufficiently high T the "overshooting" effect is weak [4] and is unlikely to be detected in the experimental setup [5,6]. Thus it was not completely clear whether the above observations are consistent with the existing theory of the proximity effect.

In this Letter we will develop a theory of coherent charge transport in two-dimensional (2D) proximity metallic films. We will demonstrate that kinetic properties of such systems can substantially differ from those of quasi-1D proximity structures [3,4] due to nonuniform distribution of the current in the film. We will show that this effect might cause a substantial *decrease* of the system conductance in four-point measurements [5,6] where the width of the samples was of the same order as their length. We will provide a transparent physical interpretation of the effect within a standard picture of the proximity effect for quasi-1D normal conductors combined with the Kirchhoff's laws. We will also discuss possible new experiments with proximity metallic films.

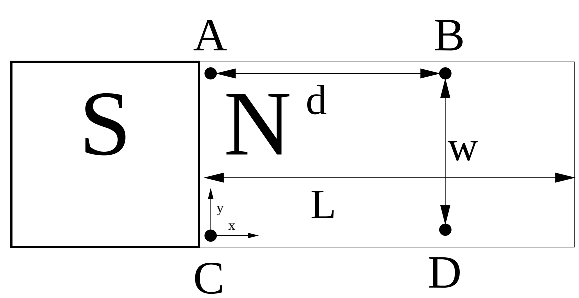

FIG. 1. A quasi-2D proximity film. The contacts A and B are used as voltage and C and D as current probes. An alternative setup: A and C are voltage probes, while the current flows through B and D.

The model and the formalism. Consider a planar diffusive NS-system with four probes directly attached to the normal metal (Fig. 1). In what follows we will assume that the NS interface as well as contacts between probes and the N-metal are perfectly transparent. We will also assume that the contact area between the probes and the N-metal is small and neglect the influence of the probes on the proximity effect. Below we will mainly consider the following experimental arrangement: the voltage V is applied to the probes A and B, and the current I flowing in the probes C and D is measured. A systematic description of proximity-induced coherent phenomena in mesoscopic diffusive NS metallic structures was obtained in [2–4] within the quasiclassical Green functions formalism of nonequilibrium superconductivity theory (see e.g. [11]). The proximity effect can be described in a standard way by means of the Usadel equation [12]. In the absence of inelastic scattering and interaction in the N-metal it reads $\mathcal{D}\partial_x^2\alpha_E = -2iE\sinh\alpha_E(x)$, where $G_E^R = \cosh\alpha_E(x)$ and $F_E^R = \sinh\alpha_E(x)$ are the retarded normal and anomalous Green functions and $\mathcal{D}$ is the diffusion coefficient for the N-metal. In the geometry of Fig. 1 these functions depend only on one coordinate x normal to the NS interface. For $E \ll E_L = \mathcal{D}/L^2 \ll \Delta$ and assuming that no current is flowing across the metal interface at $x = L$ one readily finds

$$\alpha_E(x) = \frac{E}{E_L}\frac{x}{L}\left(2 - \frac{x}{L}\right) - i\pi/2. \tag{1}$$

For $E \gg E_L$ superconducting correlations decay exponentially in the normal metal and we have [2,4]

$$\tanh(\alpha_E/4) = \tanh(\alpha_s/4)\exp(-\sqrt{-2iE/\mathcal{D}}x), \tag{2}$$

$$\alpha_s = \frac{1}{2}\tanh\left|\frac{\Delta + E}{\Delta - E}\right| - i\frac{\pi}{2}\theta(\Delta - E) \tag{3}$$

In the absence of a supercurrent in the system the total current can be defined as

$$j = \int dE\ M_E(r)\nabla f_t(r), \tag{4}$$

where f_t is the transverse component of the distribution function describing deviation from equilibrium. It satisfies the diffusion-type kinetic equation

$$\nabla(M_E(r)\nabla f_t) = I_E(\delta(r - r_C) - \delta(r - r_D)), \tag{5}$$

where I_E is the spectral component of the current I at the energy E. The voltage probes A and B are assumed to be in thermal equilibrium. Then we get [2]: $f_{tA} = 0$ and

$$f_{tB} = \left(\tanh\left(\frac{E+V}{2T}\right) - \tanh\left(\frac{E-V}{2T}\right)\right). \tag{6}$$

A "no current flow" condition at the N-metal edges yields

$$\partial_n f_t = 0. \tag{7}$$

The problem (5) is analogous to that of finding the potential distribution in a classical inhomogenous conductor with a local (spectral) conductivity $M_E(r)$. Here this quantity is fully determined by the proximity effect

$$M_E = \sigma_N \cosh^2(Re\alpha_E(x)). \tag{8}$$

where σ_N is the normal-state conductivity. It is important to emphasize that although the physical picture of the proximity effect in our system is effectively one-dimensional (and thus M_E depends only on x), the kinetic problem (5) is essentially *two-dimensional.* This is the main difference of our model as compared to that studied in [2–4]. We will demonstrate that this feature is crucially important leading to new physical effects.

Conductance. A formal solution of Eq. (5) reads

$$f_t(E,r) = I_E(\mathcal{G}_E(r,r_C) - \mathcal{G}_E(r,r_D)), \tag{9}$$

where $\mathcal{G}_E = (\nabla M_E(r)\nabla + M_E(r)\nabla^2)^{-1}$ is the Green function of the operator (5). Making use of (6, 7) and (9), and integrating I_E over energies we obtain the total current I and arrive at the expression for the differential four-point-conductance $G = dI/dV$:

$$G(V,T) = \int_0^\infty \frac{g(E)}{2T\cosh^2((E-V)/2T)}dE, \tag{10}$$

where

$$g(E) = G_N\frac{\mathcal{G}_0^{BC} - \mathcal{G}_0^{BD} - \mathcal{G}_0^{AC} + \mathcal{G}_0^{AD}}{\mathcal{G}_E^{BC} - \mathcal{G}_E^{BD} - \mathcal{G}_E^{AC} + \mathcal{G}_E^{AD}} \tag{11}$$

is the spectral conductance. We introduced the notation $\mathcal{G}^{ij} = \mathcal{G}(r_i, r_j)$ and $\mathcal{G}_0$ is the Green's function of (5) in the normal state ($M_E(r) = \sigma_N$). The spectral conductance (11) was calculated numerically from eqs. (5), (7) and (8). The results are presented in Fig. 2.

For narrow films the well known results of quasi-1D calculations [3,4] are qualitatively reproduced: the linear conductance $G(T)$ exceeds G_N at all T showing a non-monotonic feature at $T \lesssim E_d$ (for simplicity we put $L = d$ here and below). The only quantitative difference with [3,4] occurs at low energies due to different boundary conditions at $x = d$: here no contact with a big normal reservoir is assumed and the maximum conductance $G_{max} \approx 1.12G_N$ is reached at $T \sim E_d/4$, i.e. at roughly by a factor 20 lower temperature than $G_{max} \approx 1.09G_N$. In [3,4], the proximity induced superconductivity was slightly weaker due to the contact with a normal reservoir at $x = d$.

For broader films $G(T)$ *decreases* below the normal state value at high temperatures and reaches the minimum at $T \sim 10E_d$. At lower T the conductance grows with decreasing T, becomes larger than G_N and then decreases again down to $G(T = 0) = G_N$ similarly to the

1D case (see the left inset in Fig. 2). The behavior of $g(E) \equiv G(E, T=0)$ as a function of energy (voltage) is qualitatively identical to that of $G(T)$, the negative peak at $E \sim 10E_d$ turns out to be even somewhat deeper.

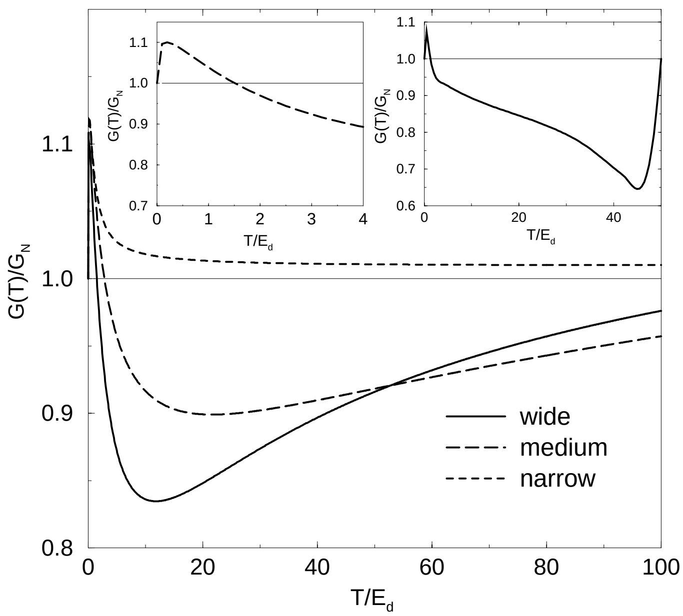

FIG. 2. The linear conductance $G(T)$ for films of different widths $w/d = 0.05, 0.5, 1.0$ calculated for $d = L$ and $T_C = 5.7\,10^5 E_d$. Left inset: The same curve at $w = 0.5d$. The T-axis is zoomed to demonstrate the presence of a usual 1D-type non-monotonic behavior at $T \sim E_d$. Right inset: $G(T)$ for a wide film and $T_C = 50E_d$. The amplitude of the negative conductance peak is increased due to the effect of a superconducting gap $\Delta(T)$.

Thus we conclude that although at $T \lesssim E_d$ the behavior of 2D samples essentially resembles that of a 1D system, at higher temperatures an additional structure with the negative conductance peak is present in the 2D case. For sufficiently wide films the amplitude of this peak can exceed that of the positive peak at lower T. This effect becomes even more pronounced if E_d is not too small as compared to T_C and the peak of the density of states around the superconducting gap should be taken into account. For typical parameters (see e.g. the right inset in Fig. 2) the minimum conductance can be by more than 35% smaller than G_N.

The network model and current flow. In order to provide a transparent physical interpretation of the above effect let us consider a simplified model of our system: the network of quasi-1D diffusive normal wires is attached to a superconductor as well as to current and voltage probes, see Fig. 3. Similar equivalent cirquit model was previously used for qualitative description of inhomogenous superconducting films [13].

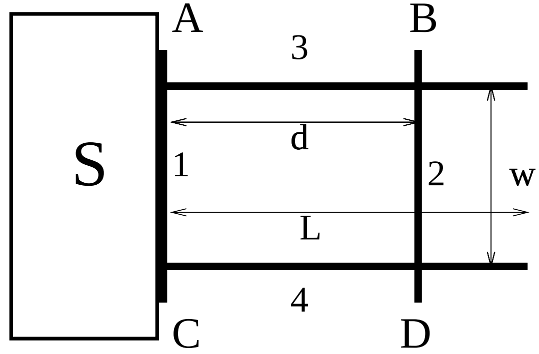

FIG. 3. An equivalent circuit with the probe configuration as in Fig. 1.

Exploiting the analogy between f_T and the electrical potential in a conventional circuit, Kirchhoff's laws for the spectral conductances can be derived [4,14]. For the present circuit, we find (c.f. [13])

$$g_{Net} = g_3 g_4 \sum_{i=1}^{4} g_i^{-1} \qquad (12)$$

where the g_i are the spectral conductances [2,4] of the wires 1–4

$$g_i = \left(\int_{\text{wire i}} \frac{ds}{M(s)} \right)^{-1} . \qquad (13)$$

At $T \gg E_d$ only the wire 1 directly attached to a superconductor acquires superconducting properties, whereas the proximity effect in the wires 2, 3 and 4 is suppressed. Thus only g_1 increases, and $g_{2,3,4}$ remain unaffected. According to eq. (12) g_{Net} decreases below G_N. At $T \lesssim E_d$ the proximity induced superconducting correlation penetrates into all four wires, $g_{2,3,4}$ increase leading to the increase of g_{Net} above G_N.

These simple arguments also suggest that the distribution of the current in our 2D proximity system should depend on T: more current will flow through "more conducting" parts of the N-metal. And indeed our numerical analysis clearly demonstrates this redistribution effect in 2D proximity films (see Fig. 4).

At low energies (where $M_E \simeq \sigma_N$) the current lines are symmetric because the effective (spectral) conductivity $M_E \simeq \sigma_N$ is the same everywhere in the system. At higher energies $E > E_d$ more current is flowing near the superconductor, where M_E is larger due to the proximity effect. This distorsion of the current lines is clearly seen in Fig. 4. At very high energies M_E is increased only in a very narrow region near the superconductor, and most current lines become symmetric again. This illustrates the importance of the geometry in the measuring process.

Let us finally point out that with the aid of the above network model and the results [2,4] one can estimate the energy E_{cr}, at which the crossover between the quasi-1D ($g > G_N$) and the quasi-2D ($g < G_N$) regimes occurs. We find that $E_{cr} \approx \mathcal{D}/w^2$ for narrow and $E_{cr} \approx \mathcal{D}/d^2$

for wide films. This estimate is in a good agreement with our numerical results for 2D films.

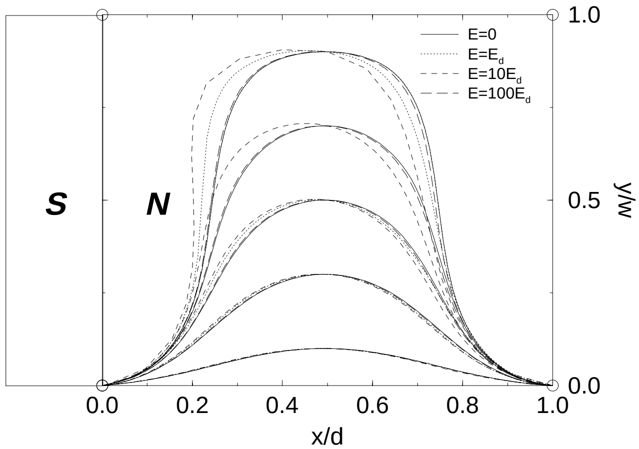

FIG. 4. Spectral current lines in a 2D proximity film for various energies.

Discussion. Our analysis clearly demonstrates that both the temperature dependence of G and the amplitude of the effect [5,6] can be explained within the standard quasiclassical theory of superconductivity applied to 2D proximity metallic films. This is consistent with the fact, that in other experiments, where contacts were placed in line [7], *no* resistance increase below T_C was observed. Furthermore, it also allows to understand the sample dependence of the conductance of NS structures observed in [6].

Indeed, for the parameters of this experiment one has $E_{cr} \approx 10E_d \approx 40\mu$V and $V \simeq R_N I \approx 7\mu$V and 100 μV respectively for Ag and Sb samples. Thus for the Ag sample $V < E_{cr}$, the effective 1D picture applies and the conductance *increases* due to the proximity effect. On the contrary, for the Sb sample $V > E_{cr}$ and the conductance *decreases* due to 2D effects. This is exactly what has been found in [6]. We believe that at very low voltages and temperatures it should be possible to observe the excess conductance effect also for Sb samples.

Finally let us briefly discuss another possible four-point conductance measurement with different arrangement of voltage (A and C) and current (B and D) probes (Fig. 1). In this case the spectral properties, i.e. the spread of correlations into the normal metal, which determine $M_E(r)$, remain the same, however the kinetics changes. Again applying the Kirchhoff analysis we now find

$$g_{Net} = g_1 g_2 \sum_{i=1}^{4} g_i^{-1} \tag{14}$$

If the voltage and current probes are close to each other, the local conductivity is recovered. 2D effects are weak in this case since $g_1 \approx g_2$ at all energies and $g_{3,4} \ll g_{1,2}$ for $w \gg d$. If, however, the current and voltage probes are sufficiently far from each other one recovers two *positive* conductance peaks: one at low $T \lesssim E_d$ and the second at higher T. The position of this second *positive* conductance peak roughly coincides with that of the *negative* peak ($T \sim 10E_d$) in Fig. 2 for a different contact arrangement. The physical reason for this second peak can be again understood within the network model analysis (14): at high enough energies only g_1 is increased by the proximity effect. These predictions agree with the results of our 2D numerical analysis.

In conclusion, we studied kinetic properties of a 2D diffusive normal metal film attached to a superconductor and demonstrated that the proximity effect can lead to both increase and decrease of the film conductance depending on the type of measurement and the energies involved. Our results are fully consistent with experimental findings [5,6,8]. We also propose new experiments for further study of the phenomena discussed here.

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