DISSIPATION AND CHARGING EFFECTS IN JOSEPHSON JUNCTION ARRAYS

Karl-Heinz Wagenblast[1], Anne van Otterlo[2] and Gerd Schön[1]
[1] *Institut für Theoretische Festkörperphysik, Universität Karlsruhe D-76128 Karlsruhe, Germany*

[2] *Institut für Theoretische Physik, Eidgenössische Technische Hochschule CH-8093 Zürich, Switzerland*

ABSTRACT

We study the dynamics of interacting bosons on a lattice which are coupled to the environment. In the imaginary time formalism we derive a coarse-grained description for an order parameter which characterizes the phase coherence in the system. We include ohmic damping on each lattice site. This translates into a non-ohmic dynamics of the order parameter field,depending on the strength of the ohmic bath. This remarkable fact influences the dynamical response of the system. The low frequency behavior of the conductivity is qualitatively changed as the dynamics of the order parameter field changes from sub-ohmic to super-ohmic. The conductivity at the transition was argued to be universal. In our model its value is of the order of the conductance quantum, but it depends on the dynamics of the order parameter field.

1. Introduction

Interacting bosons at low temperature show interesting collective phenomena. Repulsive interactions between the bosons may prevent the formation of long range phase coherence and superconductivity. A sufficiently strong interaction leads to localization of the bosons,and Mott-insulating behavior. In a description in terms of the phase of the boson field, the interaction introduces quantum effects. Strong quantum fluctuations may destroy long range phase coherence. An experimental realization are Josephson junction arrays (JJA). The Cooper pairs, which form composite bosons, are the relevant objects at low temperature. The environment will be taken into account by coupling the phase of the boson field to a bath of harmonic oscillators. In the physical realization of a JJA this means that we introduce resistive shunts to the ground and between neighboring islands. The coupling to the environment suppresses the quantum fluctuations which are induced by the interaction. This leads to a phase transition as a function of the strength of the coupling. The question how the environment influences the dynamical behavior of these systems will be the contents of the present paper. The environment influences the response of the system introducing a damping mechanism. We will see that the dynamics of the order parameter will generally be non-ohmic, although we started from a description which included ohmic shunts only. At the transition we find a finite value for the d.c. conductivity which depends on the dynamics of the order parameter, but not on its strength.

2. Effective Action

The starting point of our description is a phase model for JJA. At temperatures well below the bulk transition temperature of the islands a superconducting order paramenter is well defined on each island. The relevant dynamical variable is the phase of the order parameter. Situations where the superconducting order parameter of the islands itself is suppressed are not described in this paper. This phase model is well suited to describe JJA and other bosonic systems where amplitude fluctuations may be neglected. We couple the phases and the phase difference across a junction to an external bath. In JJA this means that we introduce resistors to the ground and between the islands. The spectral density of the bath will be chosen such, that the shunts are ohmic. The charges are then allowed to flow continuously and the action is not periodic in the phase variables. The Euclidean action for the JJA reads

$$S = S_C + S_J + S_D \tag{1}$$

with the charging part

$$S_C[\varphi(i,\tau)] = \frac{1}{8e^2}\int_0^\beta d\tau \sum_{ij} \partial_\tau\varphi(i,\tau) C_{ij} \partial_\tau\varphi(j,\tau) \; . \tag{2}$$

The capacitance matrix contains diagonal elements C_0 and off-diagonal elements C_1, a convenient energy scale is $U = (2e)^2/C_0$. The Josephson coupling part is

$$S_J[\varphi(i,\tau)] = -J\int_0^\beta d\tau \sum_{<ij>} \cos(\varphi(i,\tau) - \varphi(j,\tau)) \tag{3}$$

the sum runs over nearest neighbors pairs only. After integrating out the bath degrees of freedom the dissipative part of the action is given by

$$S_D[\varphi(i,\tau)] = \frac{1}{2}\int_0^\beta d\tau d\tau' \sum_{ij} \varphi(i,\tau)\alpha_{ij}(\tau-\tau')\varphi(j,\tau') \; . \tag{4}$$

For an ohmic bath the fourier transform of $\alpha_{ij}(\tau-\tau')$ is given by $|\omega_\mu|\left(\alpha_0 + \alpha_1 k^2\right)/2\pi$ We includes shunts to the ground ($\alpha_0 = R_Q/R_0$) and shunts between the islands ($\alpha_1 = R_Q/R_1$), where $R_Q = h/4e^2$.

3. Coarse-grained Description

In the coarse graining approximation we introduce a complex order parameter field ψ via a Hubbard-Stratonovich transformation. The field ψ is chosen such that its expectation value is proportional to that of $\exp(i\varphi)$. An expansion in powers of ψ yields the Ginzburg-Landau-Wilson (GLW) free energy functional

$$F[\psi,\bar\psi] = \frac{1}{\beta N}\sum_{k,\mu} \bar\psi(k,\omega_\mu)\left\{\frac{1}{2J}\left(1+\frac{k^2}{4}\right) - g(\omega_\mu)\right\}\psi(k,\omega_\mu) + \kappa|\psi|^4 \; . \tag{5}$$

The coefficients in this free energy are determined by the phase-phase correlator g, it is given by an expectation value with the gaussian action $S_C + S_D$.

$$\begin{aligned} g(\tau) &= \langle \exp\{i\varphi(i,\tau) - i\varphi(j,\tau')\}\rangle \\ &= \delta_{ij} \exp\left\{ -\frac{1}{\beta N} \sum_{k,\mu\neq 0} \frac{1-\cos(\omega_\mu \tau)}{\omega_\mu^2 C(k)/(2e)^2 + |\omega_\mu|\alpha(k)/(2\pi)} \right\} \end{aligned} \tag{6}$$

The k-dependence may be treated exactly. We are, however, interested in the slow modes and use $C(k) = C_0 + C_1 k^2$, $\alpha(k) = \alpha_0 + \alpha_1 k^2$, with a cutoff at the Debye wave-number. For low temperature and sufficiently large damping the sum can be approximated by neglecting the ω_μ^2 contribution in the denominator and introducing a cutoff $1/\tau_c = (2e)^2\alpha(k)/[2\pi C(k)]$ in the frequency sum. In order to do this we need a finite α_0, a finite value of C_0 is also necessary for stability reasons. The correlator is then approximated by

$$g(\tau) = \left| \frac{\beta}{\pi\tau_c} \sin\left(\frac{\pi\tau}{\beta}\right) \right|^{-\frac{2}{\alpha}} \qquad \text{for} \quad \tau > \tau_c \tag{7}$$

In the limit of zero temperature the correlator and its fourier transformed is given by

$$g(\tau) = |\tau/\tau_c|^{-\frac{2}{\alpha}} \quad , \quad g(\omega_\mu) = |\omega_\mu/\omega_c|^{\frac{2}{\alpha}-1} \tag{8}$$

Here we notice, that the GLW free energy contains a dissipative part ($\propto |\omega_\mu|^s$), with $s = 2/\alpha - 1$. In this way an *ohmic* bath produces a *non-ohmic* damping of the order parameter. In two dimensions the value of α is expressed by the parameters of the original model through

$$\alpha = \frac{4\pi\alpha_1}{\log(1 + 4\pi\alpha_1/\alpha_0)} = \alpha_0 + 2\pi\alpha_1 + \mathcal{O}(\alpha_1^2) \tag{9}$$

3.1. Static Properties

The influence of the dissipation on the static properties was intensively studied in the discussion about the dissipative phase transition by many authors [1,2,3]. The saddle point solutions of Eq. (5) provide us with information about the mean field phase diagram. The phase boundaries for various temperatures are shown in Fig. 1. We use $C_1 = \alpha_1 = 0$, finite values of C_1 and α_1 will not change the picture qualitatively. All lines are second order transitions. At zero temperature the correlator $g(\omega_\mu = 0)$ diverges for $\alpha \geq 2$. The dissipation suppresses quantum fluctuations of the phase and the system is superfluid for arbitrary small Josephson coupling J. At finite temperature this critical value of α does not exist, since $g(\omega_\mu = 0) \leq \beta$ is finite. The temperature dependence of the phase boundary shows reentrant behavior for small values of the dissipation α. This is due to the fact that we allow continuous

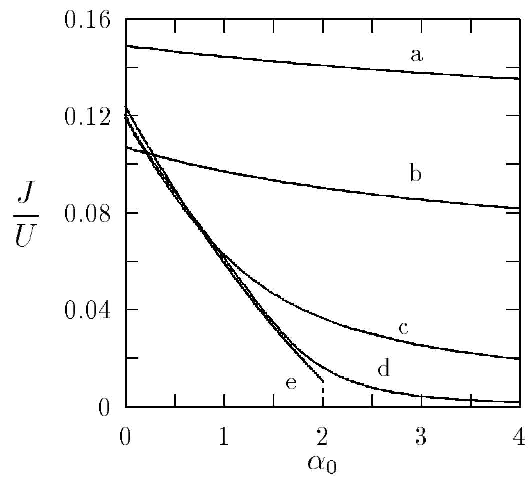

Figure 1: Phase boundaries as a function of the dissipation α_0 for different temperatures. a: T=U/5, b: T=U/10, c: T=U/100 d: T=U/10000, e: T=0. With $C_1 = \alpha_1 = 0$. The superconducting phase is above the phase boundary.

charge fluctuations, these charge fluctuations stabilize the superfluid phase at finite temperature.

3.2. Dynamic Properties

In this section we focus our intention on the response of the system. We saw in the derivation for the GLW free energy that the ohmic dissipation in the original model leads to non-ohmic damping on the coarse-grained scale. From the gaussian part of the free energy we can derive the fluctuation conductivity. Only in the absence of ohmic shunts between the islands ($\alpha_1 = 0$) it coincides with the total conductivity. As a model we study the following GLW free energy

$$F[\psi] = \frac{1}{\beta N} \sum_{k,\mu} \bar{\psi} \left\{ \epsilon + k^2 + \zeta \omega_\mu^2 + \eta |\omega_\mu|^s \right\} \psi \ . \tag{10}$$

The conductivity as a function of Matsubara frequencies is readily derived [4]. In two spatial dimensions it is

$$\sigma(i\omega_\nu) = \frac{1}{R_Q \omega_\nu} \frac{1}{\beta} \sum_\mu \int dk k^3 G(k, \omega_\mu) \left[G(k, \omega_\mu) - G(k, \omega_\mu + \omega_\nu) \right]$$

where

$$G(k, \omega_\mu) = \frac{1}{\epsilon + k^2 + \zeta \omega_\mu^2 + \eta |\omega_\mu|^s} \ . \tag{11}$$

In order to extract measurable quantities we have to analytically continue it to real frequencies by identifying $i\omega_\nu = \omega + i\delta$. Therefore write the Matsubara-sum into a

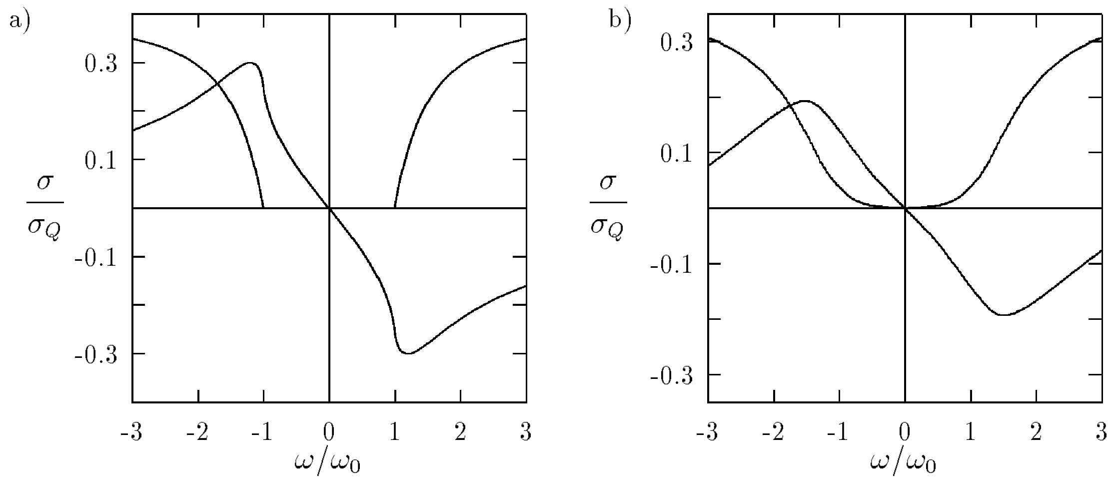

Figure 2: Frequency dependence of the conductivity: a) without damping; b) with damping $\eta\omega_0^s/\epsilon = 1$, $s = 1/2$; $\sigma_Q = 4e^2/h$.

contour integral. Due to the external frequency in Eq. (11) we have to introduce two cuts. The deformation of the contours leads us to

$$\sigma(\omega) = \frac{1}{2\pi R_Q \omega} \int_{-\infty}^{\infty} \frac{dx}{1 - e^{-\beta x}} \int_0^{\infty} dk k^3 \left[G^R(k,x) - G^A(k,x)\right] \times$$

$$\times \left[G^R(k,x) + G^A(k,x) - G^R(k,x+\omega) - G^A(k,x-\omega)\right]. \qquad (12)$$

The advanced and retarded Green's functions on the real frequency axis are given by

$$G^{A/R}(k,\omega) = \frac{1}{\epsilon + k^2 - \zeta\omega^2 + \eta|\omega|^s \cos(\pi s/2) \pm i\eta|\omega|^s \mathrm{sign}(\omega)\sin(\pi s/2)} \qquad (13)$$

A numerical integration leads to the results shown in Fig. 2 and Fig. 3. We focus on T=0. For small amount of the damping the real part shows a smeared excitation gap ($\omega_0^2 = 4\epsilon/\zeta$). The imaginary part behaves capacitively, see Fig. 2. The d.c. conductivity vanishes. The low frequency behavior depends qualitatively on the value of s, the real part is shown in Fig. 3a. It shows a power law behavior with an exponent smaller than two in the subohmic case ($0 < s < 1$), the exponent is larger than two in the ohmic and super-ohmic case ($s \geq 1$). At this stage we can extract information about the universal conductivity at the transition [5]. The d.c. conductivity at the transition ($\epsilon \to 0$) is finite, i.e. the system behaves metallic at the transition. Its value, however, depends on the strength of the dissipation, it is a function of $s = 2/\alpha - 1$ as shown in Fig. 3b. In the points $s = 1$ and $s = 2$ it coincides with former results [4].

4. Conclusions

We showed how the coupling of the phases to an ohmic bath leads to a GLW free energy in which the damping of the order parameter is generally non-ohmic. This free energy is a useful tool to study static and dynamic properties of these systems.

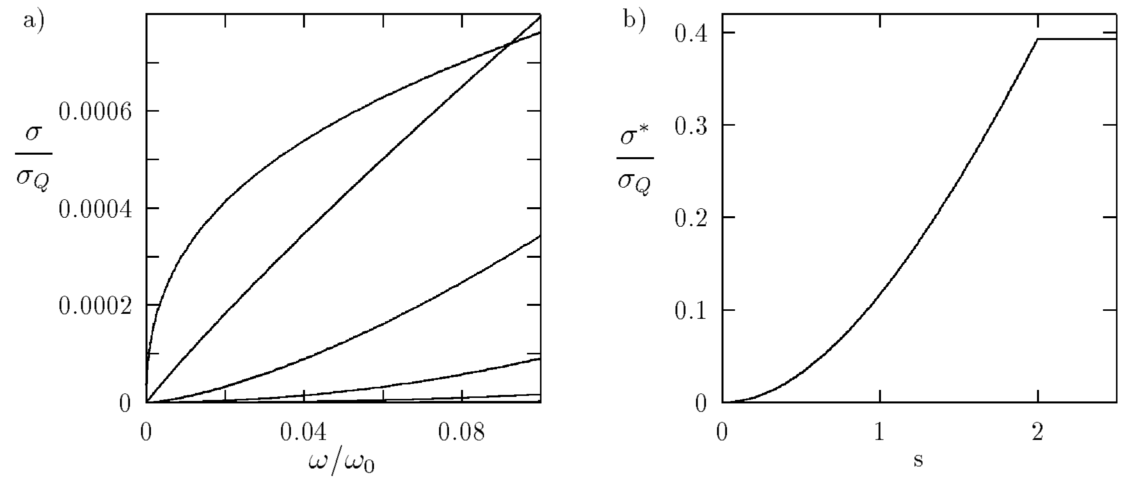

Figure 3: a) Low frequency behavior of the real part of the conductivity for different exponents s from 0.25(upper curve) , 0.5, 0.75, 1.0, 1.25, 1.5(lower curve) with the value of the damping $\eta\omega_0^s/\epsilon = 1$. b) D.C. conductivity at the transition as a function of s.

Concerning the static properties, i.e. the phase diagram, we see a transition tuned by the strength of the coupling to the environment. Sufficiently large damping suppresses phase fluctuations and favors global superconductivity. At zero temperature there exists a critical value of the dissipation beyond which the system is superconducting at arbitrarily weak Josephson coupling. The surprising fact that we arrive at non-ohmic damping of the order parameter, although we coupled to ohmic baths has nice consequences for the dynamic behavior of the model. The low frequency behavior of the conductivity is qualitatively changed by changing the magnitude of the ohmic shunts. And the value of the conductivity at the transition, which was argued to be universal [5], is a function of the parameter s.

Acknowledgements

We would like to thank Rosario Fazio and Arno Kampf for valuable discussions.

References

1. S. Chakravarty, G.-L. Ingold, S. Kivelson and A. Luther, *Phys. Rev. Lett.* **56**, (1986) 2303. S. Chakravarty, G.-L. Ingold, S. Kivelson and G. Zimanyi, *Phys. Rev. B* **37**, (1988) 3283.
2. M.P.A. Fisher, *Phys. Rev. B* **36**, (1987) 1917.
3. A. Kampf and G. Schön, *Phys. Rev. B* **36**, (1987) 3651.
4. A. van Otterlo, K.-H. Wagenblast, R. Fazio and G. Schön, *Phys. Rev. B* **48** (1993) 3316.
5. M.P.A. Fisher, G. Grinstein and S.M. Girvin, *Phys. Rev. Lett.* **64**, (1990) 587; M.-C. Cha, M.P.A. Fisher, S.M. Girvin, M. Wallin, A.P. Young, *Phys. Rev. B* **44**, (1991) 6883.