Translating Inheritance Nets to Default Logic

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Abstract. We give a translation of inheritance nets to normal default theories. To avoid discussions about sceptical and credulous reasoning we only regard unambiguous nets but allow explicit exception links restricting the validity of direct links. Contrary to former translations every link will be translated to a "hard" fact of the corresponding default theory while the defaults only represent the implicit assumptions made when computing the extension of the net. The translation is sound and complete if we restrict the deduction mechanism of default logic appropriately.

1 Introduction

Taxonomic hierarchies are widely used in natural language processing (cp. [K94, Le83, EG90]). As the concepts employed may turn out inadequate or incomplete later on, we want to regard hierarchies that could be altered subsequently.

Inheritance nets are a means for representing taxonomic knowledge. They allow overriding information in the presence of exceptions. On the other hand, approaches to belief revision where consistent sets of formulas are changed after adding contradictory information have been treated extensively in the literature. In order to be able to apply those results, we need a representation of inheritance nets as a set of first-order formulas. Our translation to normal default theories yields such a set; only the implicit assumptions underlying the conclusions of the net remain to the defaults. Thus, we present the first step for applying the results of belief revision on inheritance nets and leave the rest for future investigations.

Previous investigations (e.g. [T86]) generally asked how to define the set of conclusions supported by a net resp. how to define a valid path in it. In the presence of ambiguities the most specific information should be chosen. If none of the contradictory information is most specific, it is either allowed to arbitrarily choose between the competing information (credulous reasoning), or all of it has to be blocked (sceptical reasoning). Unfortunately the intuitions about the valid conclusions of a net often depend on the labeling of the nodes.

For that reason we do not discuss this problem, but regard such nets as *underspecified* and restrict ourselves to the investigation of unambiguous nets. Paths are always valid if there are no exceptions present. Inadequate representations may be altered later on. We can model sceptical, as well as credulous reasoning by modifying the net appropriately, or mixing both if required.

We will begin with Touretzkys definitions of inheritance nets (cp. [T86]). Contrary to him, we do not allow nets that have valid paths contradicting each other. We only regard unambiguous nets but preserve enough expressive capabilities to model interesting situations since we extend the formalism by explicit exception links. These start at a node and point to a link that is to be blocked. We require to eliminate ambiguities by blocking appropriate links. Exception links may only start at nodes "deeper" in the hierarchy than the link to be blocked.

This makes Touretzkys definition that subclasses override superclasses (inferential distance) explicit. In ambiguous situations we decide which path shall be believed. Credulous or sceptical reasoning will be expressed by links appropriately marked with exceptions.

Etherington and Reiter already regarded inheritance nets with exceptions in [ER83]. They also gave a semantics by translating the nets to default logic. But they translated strict links to facts and defeasible links to semi-normal defaults. We do not distinguish between strict and defeasible links, since we represent all links uniformly as facts of the corresponding default theory. We assume that all links can be restricted, so are defeasible. Distinguishing strict and defeasible links could be done by defining priorities among the facts, thus allowing even more than two priorities (it would correspond to strict/defeasible links).

Our translation yields a consistent set of formulas that allows belief revision. Revision of that set shall correspond to a revision of the net. The defaults needed only express the underlying assumptions that there exists no information more specific than already provided. Contradictions shall be removed by restricting links rather then by erasing them.

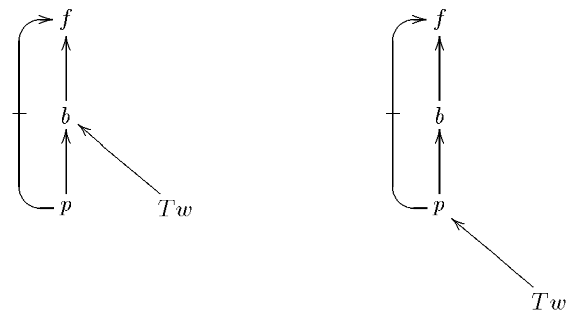
2 Tweety and Nixon

Example 1. (1) If we know that birds can fly, pinguins are birds, pinguins cannot fly, and that Tweety is a bird, we can conclude that Tweety can fly.
(2) If we come to know afterwards that Tweety is a pinguin we want to conclude instead that Tweety cannot fly. The usual final representation as inheritance nets looks like fig. 1.

We want to stress two aspects:

- The conclusion that Tweety *can* fly is possible in the first net if we assume that Tweety *is not* a pinguin. This is a common *implicit assumption* when interpreting inheritance nets. It will appear as a default when translated to default logic.
- The nets contain two contradicting paths about pinguins' ability to fly. Generally the contradiction will be solved by preferring a direct link over a compound path. Thus in the second net we get: Tweety cannot fly.

The representation of this example as a net assumes that all relevant information about the concepts involved is present. We think this assumption is



(1) (2)

Fig. 1. Famous Tweety

inadequate, and allow nets to be generated *iteratively* instead. The conclusions about Tweety's ability to fly depend on the links already present in the net. They can be refined by inserting further links.

Counterexamples can be inserted provided that the net will be altered so that it does not allow contradicting conclusions. This will be done by classifying more specific concepts as exceptions of less specific ones, like pinguins being exceptions of birds.

Example 2. Fig. 2 shows an example of a net allowing different contradicting conclusions:
Nixon is quaker and republican. Quakers are pacifists, republicans are not.

We demand to define exactly whether a conclusion from nixon to pacifists is allowed and if so which one. So, if the last link of the Nixon-diamond is added, a contradiction appears that has to be resolved by blocking at least one of the two conflicting paths. For this use, we allow exception links starting from a node and pointing to a link, blocking the link pointed at. The starting point of such a link is called *exception*.

If the Nixon-diamond is drawn with exception links we can model credulous (fig. 3) or sceptical reasoning (fig. 4). By this means we always get nets with exactly one extension thus avoid the discussion about the advantages of sceptical vs. credulous inheritance reasoners.

Supporting uniformity we also add an exception link if a path is overridden by a direct link. To obtain the same conclusions it has to point from the most specific concept participating in the contradiction to the last link of the path to be contradicted. "Tweety" would then look like fig. 5.

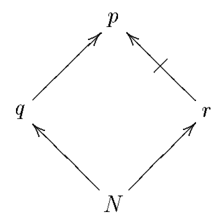


Fig. 2. The Nixon-diamond

Fig. 3. The Nixon-diamond with exception links: the credulous extensions

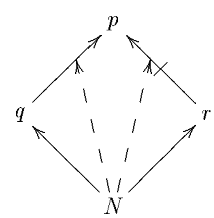


Fig. 4. The Nixon-diamond with exception links: the sceptical extension

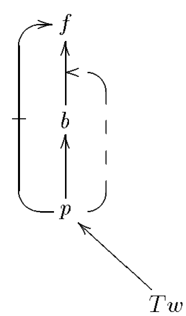


Fig. 5. Famous Tweety with exception links

3 Inheritance Nets

Our language will contain

- Individuals $a, b, \ldots$
- Concepts $X, Y, \ldots$ with P being the set of all concepts
- Signed concepts $+X$ or $-X$ called positive or negative concepts, respectively.
- Links (x, y, Z) where x is an individual or a positive concept, y is a positive or negative concept and $Z \subseteq P$ the set of the exceptions of that link.

We introduce the orderings $\prec$, $\prec_{tr}$, $\prec_{is-a}$ and $\prec_{is-a_{tr}}$. If (x, y, Z) is a link, then $x \prec y$. If y is positive, then also $x \prec_{is-a} y$. $\prec_{tr}$ and $\prec_{is-a_{tr}}$ are the transitive closures of $\prec$ and $\prec_{is-a}$.

If y is positive we write $x \rightarrow_Z y$. If it is negative, there is a positive concept V with $y = -V$. We then write $x \not\rightarrow V$. The links are called positive or negative links, respectively.

In the following x, y, z will denote individuals or positive or negative concepts, if not declared otherwise. For a signed concept x, $|x|$ will denote the corresponding unsigned object.

Definition 1. An *inheritance net* N is a finite set of links satisfying the following conditions:
1. If $(x, y, Z) \in N$, then $x \notin Z$ and $y \notin Z$.
2. (x, y, Z) and $(x, y, Z') \in N$ implies $Z = Z'$.
3. If $(x, y, Z) \in N$ then $z \prec_{is-a_{tr}} x$ for all $z \in Z$ and all z are positive concepts.

Definition 2. A *path* of a net N (from x_1 to x_n) is an n-tuple $(x_1, \ldots, x_n)$ so that
- $n = 2$ and there is a link $(x_1, x_2, Z) \in N$ or
- $n \geq 3$ and there is a path $(x_1, \ldots, x_{n-1})$ and a link $(x_{n-1}, x_n, C) \in N$, $x_1, \ldots, x_n$ pairwise disjoint, $x_1, \ldots, x_{n-1}$ positive.

A path is called *positive*, if x_n is a positive concept and *negative*, if x_n is a negative concept.

Definition 3. An *explicit path* of a net N (from x_1 to x_n) is a n+1-tuple $(x_1, \ldots, x_n, Z)$ so that
- $n = 2$ and $(x_1, x_2, Z) \in N$ or
- $n \geq 3$ and there exists an explicit path $(x_1, \ldots, x_{n-1}, B)$ and a link (x_{n-1}, x_n, C) in N, $Z = B \cup C$, $x_1, ..., x_n$ pairwise disjoint and not in Z, $x_1, \ldots, x_{n-1}$ positive.

A path $(x_1, \ldots, x_n)$ is *explicit* if there is an explicit path $(x_1, \ldots, x_n, Z)$ in N.

Lemma 4. *Let $(a, x_1, \ldots, x_n, Z)$ be an explicit path in N. Then for every $i \in \{1, \ldots, n\}$ there exists a set Z_i so that $(a, x_1, ..., x_i, Z_i)$ is an explicit path in N.*

Proof
See definition of explicit path. ∎

Remarks 3.1

- Contrary to the definitions in [T86] we allow links being blocked by exceptions (cp. [ER83]). This extension is rational as it allows the explicit blocking of paths if contradictions are present.
- In condition 3 of definition 1 we could also allow individuals as exceptions blocking a link. This would complicate the presentation of our results and proofs. Nets containing individuals as exceptions can be transformed by introducing a new concept being true only of the exceptional individual and pointing to the link to be blocked.
- A link may only be blocked by more specific concepts. In terms of paths this means: If $(x_1, \ldots, x_n, Z)$ is a path in $N, z \in Z$, then there is an path from z to an x_i. This is a very natural restriction, because exceptions of concepts are only exceptions if they really belong to that concept. This condition reflects the idea that inheritance nets are generated iteratively from links without exceptions and exceptions are only added to remove contradictions.
- Contrary to Touretzky we define paths by forward chaining rather than by double chaining. There will be no differences here because we only consider unambiguous nets. Thus, the problems like decoupling e.g. (cp. [Lo89]) do not occur.

Definition 5. An inheritance net is called *acyclic*, if $\prec_{tr}$ contains no pairs (x, x).

Definition 6. The *extension* of a net N is $\text{Ext}(N) := \{ (x, y) |$ there is an explicit path $(x, \ldots, y, Z)$ in $N\}$.

Definition 7. A net N is called *ambiguous* if $(x, y) \in Ext(N)$ and $(x, -y) \in Ext(N)$. N is *unambiguous* if it is not ambiguous.

Definition 8. An inheritance net N is called *clear*, if all positive paths from x to y in N are explicit or all are not explicit and the same is true of all negative paths (cp. fig. 6).

Remarks 3.2

- If a net is unambiguous there are no conflicting paths. A (possibly ambiguous) inheritance net constructed as described by Touretzky can be transformed to an unambiguous one agreeing with our definitions by inserting explicit exception links if a path shall be overridden by a more specific one (inferential distance). As a direct link can only be blocked by more specific concepts it always wins against a longer path.
- If a net has more than one extension we select one of them by inserting exceptions. Credulous reasoning means blocking one of the conflicting paths, sceptical reasoning blocking all of them.
- We only want to consider acyclic nets in order to get a good translation.
- We could omit the condition of a net being clear if we adopted a strategy like Touretzkys “inferential distance”. We had to guarantee that an explicit path is not considered for the extension if there is a more specific path being blocked by an exception. We omitted this concept for simplicity.

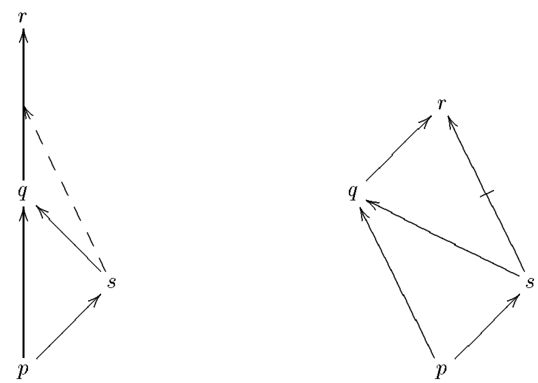


Fig. 6. The left net is not clear, the right one is ambiguous.

4 Reiter's Default Theories

We work with a finite signature $\Sigma := (Const, P)$, $Const$ is a set of constants $a, b, \ldots$ and P contains unary predicate symbols $x, y, \ldots$ A default theory is a pair (F, D), with F being a set of first-order formulas, D a set of (here: normal) defaults without prerequisites, written $: \neg x(v)/\neg x(v)$ with constant or variable v. This means that $\neg x(v)$ may be inferred in the default theory if it is consistent (cp. [R80]).

Definition 9. The *extension* of a default theory is defined as a fixpoint of an operator Γ satisfying:
Let (F, D) be a closed default theory, S a set of closed first-order formulas. Then $\Gamma(S)$ is the smallest set satisfying the following three properties:
D1) $F \subseteq \Gamma(S)$.
D2) The theorems of $\Gamma(S)$ are contained in $\Gamma(S)$.
D3) If $: \neg x(v)/\neg x(v) \in D$ and $x(v) \notin S$, then $\neg x(v) \in \Gamma(S)$.

Every closed normal default theory has an extension ([R80]).

Definition 10. Let E be an extension of (F, D). Then $GD(E)$ are called the *generating defaults* of E if $GD(E)$ is the smallest subset of D so that E is also an extension of $(F, GD(E))$.

5 Translation

We identify individuals of inheritance nets with constants of default theories. Positive concepts are identified with predicate symbols, negative concepts with negated predicate symbols.

Links (x, y, Z) of a net N will be translated to

- $y(x)$, if x is an individual. Then always $Z = \emptyset$.
- $(\forall v)[\bigvee_{z \in Z} z(v) \vee \neg x(v) \vee y(v)]$ with $z_i \prec_{is-a_{tr}} x \prec_{tr} y$.

Here z and x are predicate symbols, y is a (possibly negated) predicate symbol.

Definition 11. F_N and (F_N, D)
Let N be an inheritance net. Then F_N is the set of all formulas that are the translations of all links of N. We build the default theory (F_N, D) with $D := \{: \neg x(a)/\neg x(a) | x \in P, a \in Const\}$. Thus we assume that a has not property x if it is consistent to do so (closed world assumption).

Remarks 5.1

- We would like to get a result that the default theory corresponding to an inheritance net always has one extension yielding the same conclusions as the net. Unfortunately this is not true:

  Example 3. The translation of the net in fig. 7 yields a default theory with two different extensions, one containing $\{b(Tw), f(Tw), \neg p(Tw)\}$, the other $\{b(Tw), \neg f(Tw), p(Tw)\}$. The first extension needs the default $:\neg p(Tw)/\neg p(Tw)$, the second needs $:\neg f(Tw)/\neg f(Tw)$. Thus we can deduce that an exception is present even if there would be another possibility. This is due to the fact that default logic, contrary to inheritance nets, may use contraposition of formulas.

- It can easily be shown that the extensions of the default theories described above are always complete, i.e. for every constant a and every predicate y we get either $y(a)$ or $\neg y(a)$. Obviously that is different from the conclusions in the extension of the original inheritance net. To tackle these problems, we restrict the deduction mechanism.
- If nets are not clear, $(a, p) \in Ext(N)$ does not imply that (F_N, D) has an extension that contains $p(a)$. The left net of fig. 8 is not clear, $r(a) \in Ext(N)$, but $r(a)$ is not deducible from (F_N, D), because $s(a)$ is deducible from F_N. The right net of fig. 8 is neither clear nor unambiguous, but F_N is consistent. Defining I by $I(p) = I(q) = I(s) = \{a\}$ and $I(r) = \emptyset$ we get a model of F_N.
- *Example 4.* The nets of fig. 9 are ambiguous as their extensions contain (a, s) as well as $(a, \neg s)$. But F_N is consistent, yielding $p(a), q(a), r(a), \neg s(a)$ and $t(a)$.

Theorem 12. *Let N be an acyclic, unambiguous and clear inheritance net. Then the set of formulas F_N resulting from translating N to default logic is consistent.*

Proof
We show the assertion by constructing a model of F_N.
We classify the predicates corresponding to the relation $\prec$ on them.
$P_0 := \{y \in P | \text{there is no } x \in P \text{ with } x \prec y\}$
$P_i := \{y \in P | \text{there is no } x \in P \setminus P_{i-1} \text{ with } x \prec y\}$

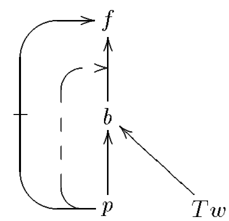


Fig. 7.

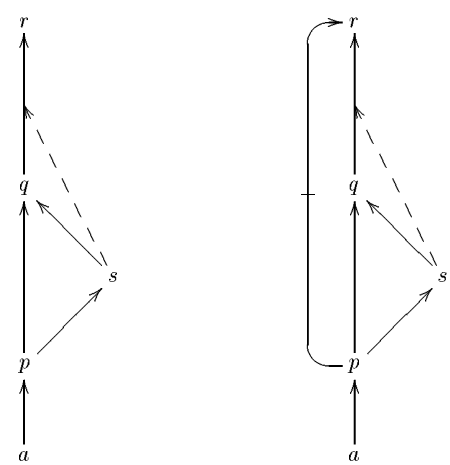


Fig. 8. These nets are not clear. Their extensions differ from the conclusions of the corresponding default theories.

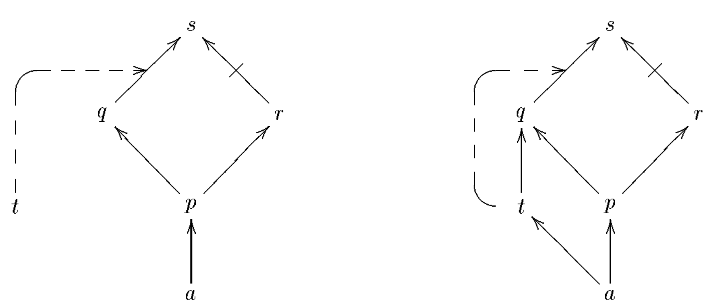


Fig. 9. The default theories of these ambiguous nets are consistent.

Now the formulas are classified accordingly:
$F_N^0 := \{(\neg)y(a) \in F_N | y \in P_0, a \in Const\}$
$F_N^i := \{\bigvee_{z \in Z} z(v) \vee \neg x(v) \vee (\neg)y(v) \in F_N | y \in P_i,\ x \prec y, z \prec_{tr} x$ *for all* $z \in Z\} \cup \{(\neg)y(a) \in F_N | y \in P_i, a \in Const\}$
We iteratively construct a Herbrand-model:
For $y \in P$ *we define*
$y_w^0 := \{a \in Const | y(a) \in F_N^0\}$,
$y_f^0 := \{a \in Const | \neg y(a) \in F_N^0\}$.
$y_w^i := \{a \in Const | [\bigvee_{z \in Z} z(v) \vee \neg x(v) \vee y(v) \in F_N^i, a \notin z_w^{i-1}$ *for all* $z \in Z, a \in x_w^{i-1}]$ *or* $y(a) \in F_N^i\} \cup y_w^{i-1}$
$y_f^i := \{a \in Const | [\bigvee_{z \in Z} z(v) \vee \neg x(v) \vee \neg y(v) \in F_N^i\ , a \notin z_w^{i-1}$ *for all* $z \in Z, a \in x_w^{i-1}]$ *or* $\neg y(a) \in F_N^i\} \cup y_f^{i-1}$
We get that for $y \in P_i$, $y_w^j = y_f^j = \emptyset$ *for* $j < i$ *as well as* $y_w^j = y_w^{j+1}$ *and* $y_f^j = y_f^{j+1}$ *for* $j \geq i$. *Let n be minimal with* $P_n = P$. *We show that* $M := (Const, I)$ *with* $I(y) = y_w^n$ *for all* $y \in P$ *is a Herbrand-model of* F_N. *Thus on level i all predicates of* P_i *are defined based on the definitions of the predicates of the levels 0 to i-1.*

In order to proceed with the proof we show

Lemma 13. *Let* N *be an acyclic, unambiguous and clear inheritance net, n minimal with* $P_n = P$. *Then for all* $a \in Const, y \in P_n : [a \in y_w^n$ *iff* $(a, y) \in Ext(N)]$ *and* $[a \in y_f^n$ *iff* $(a, -y) \in Ext(N)]$.

Proof
We proof the assertion by induction on the classification of the predicates:
1. $i = 0$*:*
For $i = 0$ *all links in the net pointing to a node* $y \in P_0$ *cannot have any exceptions, they look like* $(a, (-)y, \emptyset)$ *with* $a \in Const$. *Then* $y(a) \in F_N^0$ *resp.* $\neg y(a) \in F_N^0$ *iff* $(a, y, \emptyset)$ *resp.* $(a, -y, \emptyset) \in N$ *iff* $(a, y) \in Ext(N)$ *resp.* $(a, -y) \in Ext(N)$.

2. i-1 $\rightarrow$ *i:*
Let $a \in y_w^i$ *for* $y \in P_i$. *Then* F_N^i *contains the formula* $y(a)$ *or the formula* $\bigvee_{z \in Z} z(v) \vee \neg x(v) \vee y(v)$ *with* $a \in x_w^{i-1}$, $a \notin z_w^{i-1}$ *for all* $z \in Z$.
2.1. If $y(a) \in F_N^i$ *we know that* $(a, y, \emptyset) \in N$, *so* $(a, y) \in Ext(N)$.
2.2. If $[\bigvee_{z \in Z} z(v) \vee \neg x(v) \vee y(v)] \in F_N^i$ *we know that* $x \in P_j$ *for a* $j < i$. *Using the induction hypothesis we get that in* N *there exists an explicit path* $(a, x_1, \ldots, x_n, x, C)$ *and a link* (x, y, Z). *We assume for contradiction that the path* $(a, x_1, \ldots, x_n, x, y, C \cup Z)$ *is not explicit. Thus w.l.o.g. there is an k with* $x_k \in Z, x_k \in P_{i-1}$ *(for uniformity of the representation we have excluded the possibility of a being an element of Z, see remark in the definitions). We know there is an explicit path* $(a, x_1, \ldots, x_k, D)$ *in* N *(Lemma 3.4), so* $(a, x_k) \in Ext(N)$. *Using the induction hypothesis we get* $a \in x_{k_w}^{i-1}$ *with* $x_k \in Z$. *Contradiction.*
The proof is analogous if $a \in y_f^i$.

Now let $(a, y) \in Ext(N)$. *So there exists an explicit path* $(a, \ldots, x, y, C) \in N$. *Then* N *contains an explicit path* $(a, \ldots, x, C')$ *and a link* (x, y, Z) *with* $C =$

$Z \cup C'$. Using the induction hypothesis, we get $a \in x_w^{i-1}$ and $a \notin z_w^{i-1}$ for all $z \in C'$. We show $a \notin z_w^{i-1}$ for all $z \in Z$.

Assume, $a \in z_w^{i-1}$ for a $z \in Z$. Then $(a, z) \in Ext(N)$ and $z \prec_{is-a_{tr}} x$. As N is clear and there exists an explicit path from a to x, there is also an explicit path $(a, \ldots, z, \ldots, x, Z_z)$ from a via z to x. Adding the link (x, y, Z) yields the path $(a, \ldots, z, \ldots, x, y, Z_z \cup Z)$ being not explicit for $z \in Z$. Contradiction to the assumption of N being clear as $(a, \ldots, x, y, C)$ is an explicit path.

For negative paths and links the proof is analogous. ▮

We now show that the model constructed above always exists. Assume that F_N^i contains the two formulas $\bigvee_{z \in Z} z(v) \vee \neg x(v) \vee \neg y(v)$ and $\bigvee_{z \in Z'} z(v) \vee \neg x'(v) \vee y(v)$ and there is a constant a with $a \in x_w^n \cup {x'_w}^n$. We know by Lemma 13 that then (a, x) and $(a, x') \in Ext(N)$. As N is unambiguous $(a, y) \notin Ext(N)$ or $(a, \neg y) \notin Ext(N)$ but (x, y, Z) and $(x', \neg y, Z')$ are in N. Thus one of the paths from a to $(\neg)y$ is blocked by an exception $z \in Z \cup Z'$ and $(a, z) \in Ext(N)$. By Lemma 13 we can conclude $a \in z_w^n$ for a $z \in Z \cup Z'$. Thus at most one of the above formulas is applicable and $y_w \cap y_f = \emptyset$ is guaranteed. ▮

Corollary 14. *Let N be an acyclic, unambiguous and clear inheritance net. Then the resulting default theory (F_N, D) has a consistent extension.*

Proof
See theorem 12 and [R80], page 91, Corollary 2.2. ▮

We now show how to restrict the deduction mechanism. We start out from Reiters description of top down default proofs used to determine whether a default theory has an extension containing a formula β. As pointed out before we do not want to get all formulas of an extension. So we use an appropriate restriction. Recall the definition of a linear resolution proof of a formula β ([Lo70, Lu70]) :

Definition 15. A *linear resolution proof of a formula β from a set of clauses S* has the form of fig. 10 where
(1) the top clause, R_0, is a clause of $\neg\beta$.
(2) For $1 \leq i \leq n, R_i$ is a resolvent of R_{i-1} and C_{i-1}.
(3) For $0 \leq i \leq n-1, C_i \in S$ or C_i is a clause of $\neg\beta$ or C_i is R_j for some $j < i$.
(4) R_n is the empty clause.

Definition 16. *$Res_{\prec} - proof$*

- A *$res_{\prec} - proof$* of a literal β from a set of clauses S is a linear resolution proof of β from S respecting the order $\prec$ on the predicates of P: If $R_{i-1} = \bigvee_{i \in I} L_i(a) \vee L(a)$ and $C_{i-1} = \bigvee_{j \in J} L_j(a) \vee \neg L(a)$, then $R_i = \bigvee_{i \in I \cup J} L_i(a)$ is the resolvent of R_{i-1} and C_{i-1} and $L \not\prec_{tr} L_i$ for all $i \in I \cup J$.

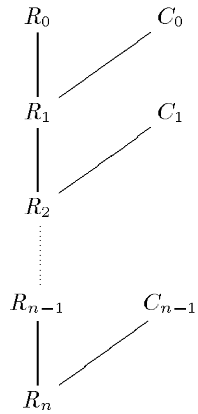


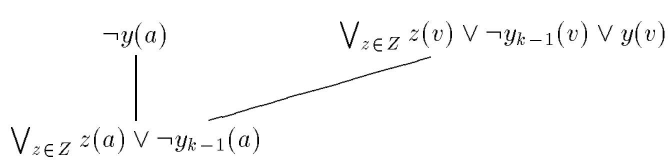
Fig. 10. Structure of a linear resolution proof

- A $res_{\prec} - proof$ of β from a *default theory* (F_N, D) is a $res_{\prec} - proof$ of β from F_N with $C_0 \in F_N$ and for $0 < i \leq n-1, C_i \in F_N \cup \{\neg p(a)|$ there exists no $res_{\prec} - proof$ of $p(a)$ from (F_N, D).

Lemma 17. *Let N be an acyclic, unambiguous and clear inheritance net, (F_N, D) the corresponding default theory, y_w^n and y_f^n as in theorem 12. Then [there exists a $res_{\prec} - proof$ of $y(a)$ from (F_N, D) iff $a \in y_w^n$] and [there exists a $res_{\prec} - proof$ of $\neg y(a)$ from (F_N, D) iff $a \in y_f^n$].*

Proof
Induction on the classification of the predicates:
1. Let $y \in P_0$. There is a $res_{\prec} - proof$ of $(\neg)y(a)$ is equivalent to $(\neg)y(a) \in F_N$ being equivalent to $(\neg)y(a) \in F_N^0$ (cp. theorem 12).
$(\neg)y(a) \in F_N^0$ iff $a \in y_w^n$ resp. $a \in y_f^n$.

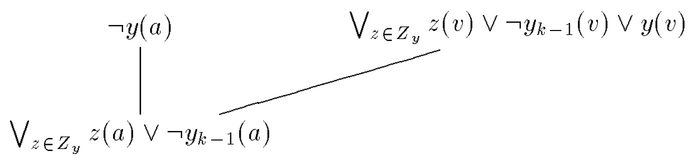
2. For all $p \prec_{tr} y$ we assume: There exists a $res_{\prec} - proof$ of $(\neg)p(a)$ iff $a \in y_w^n$ resp. $a \in y_f^n$.
2.1: $(\neg)y(a) \in F_N$. Analogous to 1.
2.2: There is a formula $\bigvee_{z \in Z} z(v) \vee \neg y_{n-1}(v) \vee (\neg)y(v)$ in F_N. Then the first step in the proof will be



(resp. y and $\neg y$ exchanged)
For all $z \in Z$ there is no $res_{\prec} - proof$ of $z(a)$ (as F_N is consistent and the $res_{\prec} - proof$ of $(\neg)y(a)$ is also a proof of $\bigwedge_{z \in Z} \neg z(a) \wedge y_{k-1}(a)$.

For inheritance nets we know $z \prec_{tr} y_{k-1}$ *for all* $z \in Z$. *With the induction hypothesis we get* $a \in y^n_{k-1_w}, a \notin z^n_w$ *for all* $z \in Z$. *But then* $a \in y^n_w$ *resp.* $a \in y^n_f$.

For the other direction:
Let $a \in y^n_w$ *resp.* $a \in y^n_f$, $y \in P_i$. *For* $i = 0$ *see 1.*
Let $i > 0$. *Then there is a formula* $\bigvee_{z \in Z_y} z(v) \vee \neg y_{k-1}(v) \vee (\neg) y(v)$ *in* F^i_N, $a \in y^n_{k-1_w}$, $a \notin z^n_w$ *for all* $z \in Z_y$. *We construct a* $res_{\prec} - proof$ *of* $(\neg)y(a)$*:*



(resp. y *and* $\neg y$ *exchanged)*
will be the first step, $z \prec_{tr} y_{k-1}$. *With the induction hypothesis we get that there exists a* $res_{\prec} - proof$ *of* $y_{k-1}(a)$ *and no* $res_{\prec} - proof$ *of* $z(a)$ *for all* $z \in Z$. *Therefore we may use the defaults* $: \neg z(a) / \neg z(a)$ *in the proof. Putting the first step together with the proof of* $y_{k-1}(a)$ *and using the defaults* $: \neg z(a) / \neg z(a)$ *if necessary yields a* $res_{\prec} - proof$ *of* $(\neg)y(a)$. ∎

Theorem 18. *Let* N *be an unambiguous, clear inheritance net,* (F_N, D) *the corresponding default theory. Then* (F_N, D) *has an extension* E *so that:* $[(a, y) \in Ext(N)$ *iff* $y(a) \in E]$ *and* $[(a, -y) \in Ext(N)$ *iff* $\neg y(a) \in E$ *and* $: \neg y(a)/\neg y(a) \notin GD(E)]$.

Proof
We constructed a model of F_N *(see theorem 12). As all extensions of* (F_N, D) *are complete, every extension has exactly one model and every model of* F_N *is the model of one extension of* (F_N, D). *Thus* (F_N, D) *has an extension with the model constructed in theorem 12. Lemma 17 shows that we get all ground literals valid in that model by* $res_{\prec} - proofs$ *or by defaults.* ∎

6 Summary

We provided a translation from inheritance nets with exceptions to normal default theories having the following properties:

- Every link in the net corresponds to a fact in the default theory. The implicit assumptions underlying the conclusions of a net are translated to defaults. This differs from former approaches ([ER83, S86]) that represented defeasible links as defaults.
- Our translation is sound for unambiguous and clear acyclic nets.

– Inheritance nets implicitly define an order on the concepts used. For default theories we define a deduction mechanism respecting that order. Thus, the translation becomes complete.

Our translation is very natural because ist respects the difference between explicit information (= links) and implicit assumptions (= defaults) underlying the conclusions of a net. Furthermore we can then apply results of approaches to belief revision. This will be subject to future investigations.

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References

[E88] Etherington, D.: Reasoning with incomplete information. Los Altos, Morgan Kaufmann (1988).

[ER83] Etherington, D. W. and Reiter, R.: On inheritance hierarchies with exceptions. Proc. AAAI-83, Washington, DC (1983) 104-108.

[EG90] Evans, R. and Gazdar, G. (eds.): The DATR papers, Vol. 1. CSRP 139. School of Cognitive and Computing Sciences, The University of Sussex, Brighton BN19QH (1990).

[K94] Kilbury, J.: Strict inheritance and the taxonomy of lexical types in DATR. This volume.

[Le83] Levinson, S. C.: Pragmatics. Cambridge University Press (1983).

[Lo89] Lorenz, S.: Nichtmonotones Schließen mit ordnungssortierten Defaults. Master thesis, University of Karlsruhe (1989).

[Lo70] Loveland, D. W.: A linear format for resolution. Proc. IRIA Symp. Automatic Demonstration, Versailles, France, 1968 (Springer-Verlag, New York, 1970) 147-162.

[Lu70] Luckham, D.: Refinements in resolution theory, Proc. IRIA Symp. Automatic Demonstration, Versailles, France, 1968 (Springer-Verlag, New York, 1970) 163-190.

[R80] Reiter, R.: A logic for default reasoning. Artificial Intelligence **13** (1980) 81-132.

[S86] Sandewall, E.: Nonmonotonic inference rules for multiple inheritance with exceptions. Proc. IEEE **74** (1986) 81-132.

[SL93] Selman, B. and Levesque, H.J.: The complexity of path-based defeasible inheritance. Artificial Intelligence **62** (1993) 303-339.

[T86] Touretzky, D.: The mathematics of inheritance systems. Los Altos, Morgan Kaufmann (1986).