Coherent Charge Transport in Superconducting/Normal Proximity Structures

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We use a detailed microscopic analysis to study electron transport in normal diffusive conductors in the presence of proximity induced superconducting correlation. In the case of transparent barriers, superconducting correlations and electrical field can penetrate causing a whole range of novel nonequilibrium effects. These effects are explained by the introduction of a second, correlated density of states. Our results are fully consistent with recent experimental findings. PACS: 73.23.Pg, 74.40.+k, 74.50.+r, 74.80.Fp.

1. Introduction

Recent progress in nanolithographic technology revived the experimental[1–3] and theoretical[4,5] interest on the inestigation of electron transport in superconductor/normal metal heterostructures governed by the proximity effect, which is basically already understood for a long time.[6]

In this paper we study the influence of the proximity effect on transport properties of a diffusive conductor. We will show that if the system contains no tunnel barriers there are two different physical regimes which determine the system conductance in different temperature intervals. It is well known that proximity induced superconducting correlation between electrons in a diffusive normal metal survives at a distance of order $\xi_N \sim \sqrt{\mathcal{D}/T}$, where $\mathcal{D} = v_F l_{imp}/3$ is the diffusion coefficient. As T is lowered the proximity induced superconductivity expands into the normal metal. This results in an increased conductance of a normal metal. At sufficiently low temperature

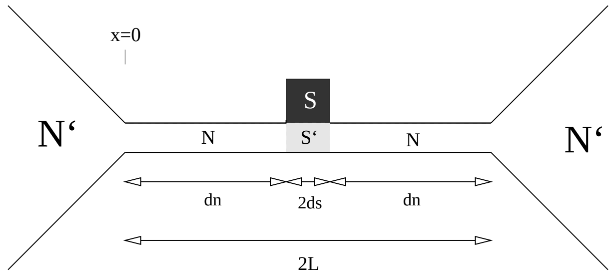

Fig. 1. The system under consideration

the length ξ_N becomes of order of the size of the normal layer and the system behavior becomes sensitive to a physical choice of the boundary condition at the edge of the normal wire opposite to that attached to a superconductor.

We will demonstrate that, as long as the electrical field can penetrate the sample, the system is in a non-equilibrium state and exhibits novel features. These effets will be explained by introducing a "correlation DOS" familiar from the theory of nonequilibrium superconductivity.

2. Kinetic approach

Let us consider a quasi-one-dimensional normal conductor like in Fig. 1. We assume, that $l_{el} \ll L \ll l_{inel}$. This geometrical realization has a direct relation to experiments.[1–3] Two big normal reservoirs N' are assumed to be in thermodynamic equilibrium at the potentials $2V$ and 0 respectively. The general approach to calculate the conductance of these structures in the formalism of quasiclassical Green's functions is outlined in.[4,5]

The effective transparency of the structure reads

$$D(\varepsilon) = \frac{1+r}{\frac{r}{\nu(x=0)} + \frac{1}{L}\int_0^L dx\,(\nu^2(x,\varepsilon) + \eta^2(x,\varepsilon))^{-1}}, \tag{1}$$

$R = R_b + R_N$ and $r = R_b/R_N \equiv \gamma_B \xi_N^*/L$, R_N and R_b are the resistance of the N-metal and the tunnelning barrier respectively, ν and η are normal and correlated densities of states, see below. For the differential conductance of the N-part $0 \le x \le d$ normalized to its normal value in the zero bias, we find

$$\bar{G}_N = \left(\frac{RdI}{dV}\right)_{V=0} = \frac{1}{2T}\int_0^\infty d\varepsilon D(\varepsilon)\mathrm{sech}^2(\varepsilon/2T). \tag{2}$$

In addition to the normal density of states ν the "correlation DOS" η plays a role. η belongs to the set of generalized densities of states familiar from the standard theory of nonequilibrium superconductivity.[8] It reflects the presence of superconducting correlations at low energies. E.g. in a BCS

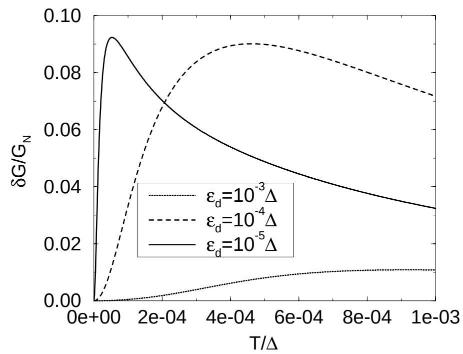

Fig. 2. Conductance for transparent interfaces

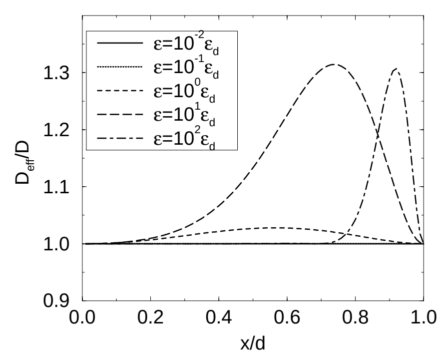

Fig. 3. Local effective diffusion constant

superconductor this function reads $\eta = \frac{\Delta\Theta(\Delta-\epsilon)}{\sqrt{\Delta^2-\epsilon^2}}$. In our case this function is of couse not only energy- but also space-dependent. However, the physical meaning of it remains the same as in standard nonequilibrium superconductivity theory:[8] η plays a role whenever the quasiparticle distribution function of a superconductor is driven out of equilibrium. Here, this happens due to a simultaneous presence of the electric field and the proximity induced superconducting correlation in the normal metal.

3. Conductance

The analysis of the problem can be significantly simplified in the case of perfectly transparent interfaces ($\gamma_B = 0$).

For $T = 0$ we get $\bar{G}_N = 1$. This does not depend on $\mathcal{D}$, so the correlations are destroyed by the influence of the boundary conditions but not by thermal excitation or by impurity scattering. This result, however, by no means implies the destruction of the proximity induced superconductivity in the N-layer, which still influences the DOS and the electrical field (see below).

For $T \ll \epsilon_d$ we then have $\bar{G}_N - 1 \propto (T/\epsilon_d)^2$, in the limit $T \gg \epsilon_d$ we find $\bar{G}_N - 1 \propto \sqrt{\epsilon_d/T}$, see[5] for more details. The latter result has a simple physical interpretation: Superconductivity penetrates into the normal part up to $\xi_N = \sqrt{\frac{D}{2\pi T}}$, whereas the rest stays normal, so the total voltage drops over a reduced distance $d - \xi_N$. Thus the resistance of the structure is reduced according to Ohms law. Let us point out, that at both edges of the N-metal the local effective diffusion constant $\mathcal{D}_{\text{eff}} = \cosh^2\theta_1\mathcal{D}$ is not enhanced (see Fig. 2) in comparison to its normal state value, because either the Cooper pair amplitude (at the NN' boundary) or the electric field (at

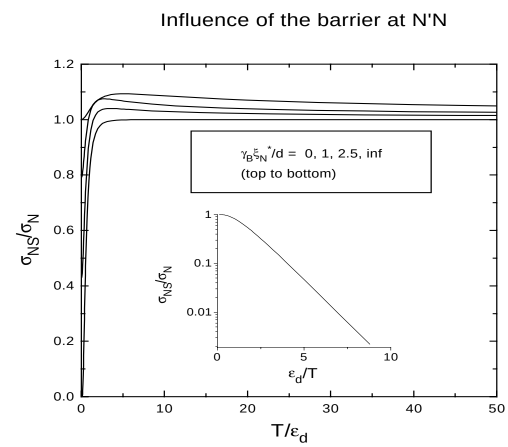

Fig. 4. Conductance in the presence of tunneling barriers. Top: Transparent, bottom: Intransparent

the NS boundary) is equal to zero due to the imposed boundary conditions. Inside the N-metal the value $\mathcal{D}_{\rm eff}$ becomes higher due to nonequilibrium effects in the presence of superconducting correlations.

For temperatures comparable to ε_d the problem was treated numerically. The results (Fig. 3) show excellent agreement with our analytical expressions obtained in the corresponding limits and demonstrate the universal scaling with T/ϵ_d for $\epsilon_d \ll \Delta$.

Let us now assume that a tunnel barrier is present at the N'-N interface. If one lowers the transparency of this barrier the crossover takes place to the behavior demonstrating monotonously decreasing conductance with T (Fig. 4), which is typical for two serial NIS tunnel junctions. The inset shows the Arrhenius plot for the case of a strong barrier.

Formally this is due to the fact, that the expression (1) reduces to the standard tunnel formula in the small transparency limit $r \gg 1$. This has an obvious physical interpretation: For $r \ll 1$ the presence of a tunnel barrier is not important, the electric field penetrates inside the normal metal causing nonequilibrium like in the case of transparent boundaries, however, for $r \gg 1$ the electric field is concentrated at the barrier like in standard tunneling situations.

4. Density of states

Let us first study the spatially averaged normal (N_N) and correlated (N_S) densities of states. Our numerical data (see Fig. 5) and analytical results[5] demonstrate the presence of a soft (no sharp edge) pseudo (0 <

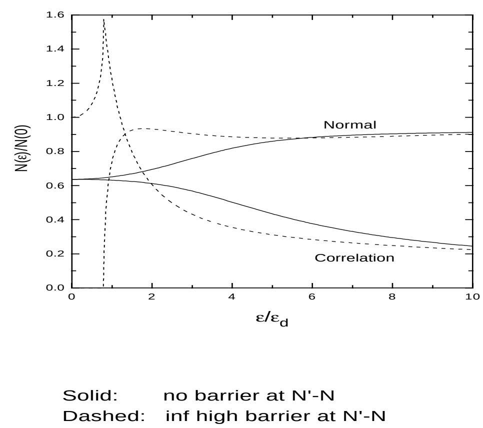

Fig. 5. Spatially averaged DOS

$N_N(0) < N(0)$) gap in the density of states below the energy ϵ_d.

Let us also point out that one can also extract information about the correlation DOS by making two kinds of measurements with the same sample. Indeed, by measuring the conductance of the system (or a part of it) with no tunnel barriers one obtains information about the combination of N_N and N_S entering the expression for the system conductance G, whereas performing local tunnel experiments[9] one probes only the normal DOS N_N. Then the correlation DOS can be easily recovered.

5. Extension to systems containing a loop

Let us now consider systems containing a mesoscopic loop, see Fig. 6. If the wire was a real superconductor, the magnetic flux would induce a supercurrent into the ring. As a function of Φ, this current has a period of the *superconducting* flux quantum $\Phi_0 = h/2e$.

To describe these type of systems, our kinetic method has to be extended in several points.[5]

For convenience,[5] we have chosen $d_1 = d_2 = d_3 = d_4$ and $A_1 = 2A_2 = 2A_3 = A_4$. The Thouless energy of just one branch will be labeled as $\epsilon_d = \frac{D}{d_i^2}$.

5.1. T-dependent Amplitude of $h/2e$-Oscillations

For $T = 0$ the conductance is again equal to its normal state value, being independent of Φ, so conductance oscillations are absent.

At $T \ll \epsilon_d$, we find analytically, that the amplitude of the conductance

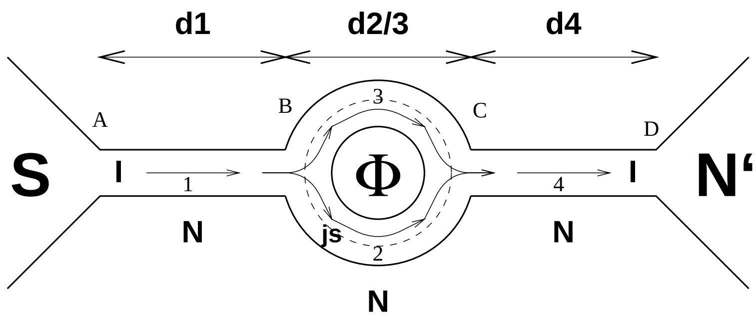

Fig. 6. A proximity wire with a loop

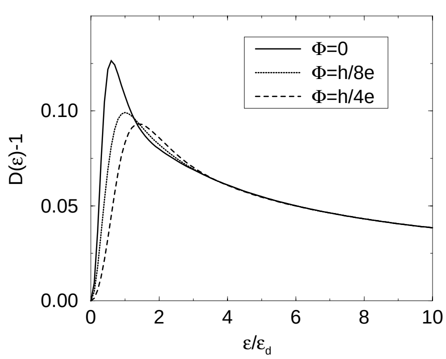

Fig. 7. Transparency at different fluxes

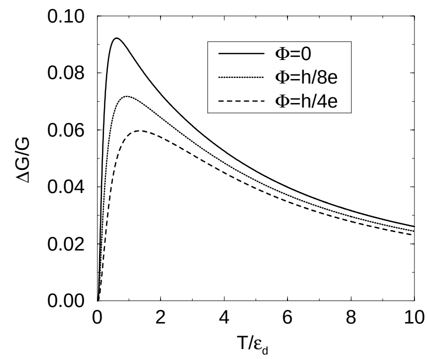

Fig. 8. Conductance at different fluxes

oscillations increases as T^2.[5] In order to establish the temperature dependence of this amplitude at higher $T \gg \epsilon_d$ we make use of the fact that for electrons with sufficiently large energies $\epsilon > \epsilon_d$ superconducting correlation is destroyed already before they reach the loop. Therefore, to calculate the flux-dependent part of the system conductance we only have to take into account the contribution of low energy quasiparticles which remain correlated in the loop area. However, due to the peculiar form of the thermal distribution (2), this low energy range enters with a weight of $1/T$ into the total result.

The results of our numerical analysis fully support those arguments. The system transparency $D(\epsilon)$ is depicted in Fig. 7 for different values of the flux Φ. The value $D(\epsilon)$ depends on Φ only at low energies, whereas for $\epsilon > \epsilon_d$ all curves merge giving the $1/T$ law for the conductance (see Figs. 8 and 9). Also the T^2 behavior of ΔG in the low temperature limit is recovered (Fig. 9).

This behavior has been also found in recent experiments.[2,3] We would like to point out that a slow power-law decay of the conductance due to

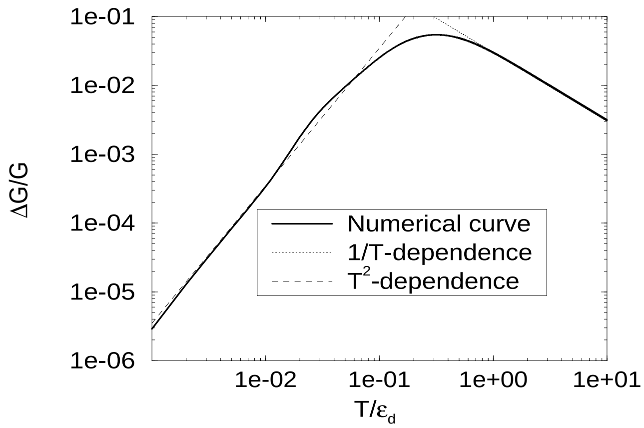

Fig. 9. Oscillation Amplitude

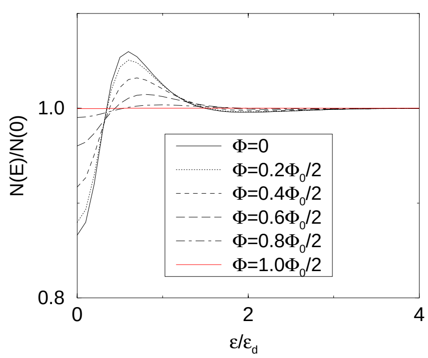

Fig. 10. Normal DOS in C

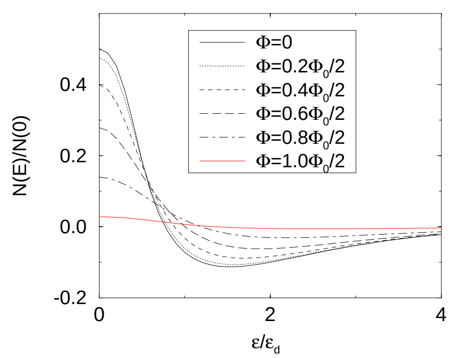

Fig. 11. Correlated DOS in C

a dominating contribution of low energy quasiparticles just emphasizes the physical difference between kinetic and thermodynamic quantities like supercurrent which decays exponentially with increasing T.[7]

5.2. Flux-dependent DOS

As one might expect for the region between the superconductor and the loop (between the points A and B) the dependence of the two densities of states ν and η is quite weak and both DOS practically coincide with those calculated above for a wire without the loop. On the other hand, in the region between the loop and the normal reservoir N' (between the points C and D) the quantities $\nu(x)$ and $\eta(x)$ are very sensitive to the flux Φ, see figs. 10, 11. We see that with increasing the value of the magnetic flux the proximity induced pseudogap decreases and vanishes completely as the flux approaches the value $\Phi_0/2$. For such value of Φ the proximity effect in the region "after" the loop is completely destroyed, the pseudogap is fully suppressed and the normal DOS coincides with $N(0)$ at all energies. Accordingly the correlation DOS vanishes at $\Phi = \Phi_0/2$. Thus in this case the resistance of the region between the points C and D is equal to its normal state value at all T.

These results demonstrate that "the strength" of the proximity effect in our system can be regulated by the external magnetic flux. This might serve as an additional experimental tool for investigation of proximity induced superconductivity in normal metalic structures. The measurement of the local DOS in such a loop system may provide a new experimental test for this theory.

6. Summary and outlook

We have used a microscopic kinetic analysis to describe the transport properties of superconductor-normal metal proximity structures. We demonstrated the nontrivial behavior of transport quantities in the nonequilibrium case as well as the crossover to the standard equilibrium situation in the presence of tunneling barriers. This was explained by the use of a correlated DOS peculiar to nonequilibrium superconductivity. To verify our arguments, new experiments for probing our arguments have been proposed.

We acknowledge useful discussions with C.Bruder, W.Belzig, H.Courtois, D.Esteve, B.Pannetier, V.T.Petrashov, G.Schön, B.Spivak, A.F.Volkov and B.J. van Wees.

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