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Stick–slip instability of decelerative sliding

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Abstract

Considering different friction laws the stability of decelerative sliding motions of a driven mechanical system is investigated. Both sudden and permanent disturbances are applied. The resulting stick–slip phenomena mainly depend on the properties of the mechanical system, especially on the drive, and less on different characteristics of the friction laws. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Stick–slip; Stability; Deceleration; Friction laws

1. Introduction

The existence of dry friction in mechanical systems involves basic difficulties arising from the fact that the defining friction laws are not exactly known [1]. Investigations are focused either on microscopic approaches [2] to explain the physical nature of friction or on macroscopic assumptions. The latter are phenomenological in their character, like Coulomb's law, or various modifications of this traditional law. They are used to analyse the motions of bodies in friction contact. In most cases, the selection of a particular friction law is made heuristically without any explanation. Hence, the main objects of these investigations are not the friction laws as such, but primarily certain properties of motions, such as stick–slip, self-excitation, limit cycles or general stability problems. As far as stick–slip is concerned, these investigations restrict on a drive with constant velocity where the resulting motion is governed by the chosen friction law. In the following a linearly decreasing velocity of the drive will be considered and the influence of different laws in addition with internal damping will be discussed. These kinds of problems occur very often in the engineering practice. Automobile or train brakes can become noisy for low velocities near to standstill. Contact problems with dry friction like metal forming are unstationary processes. Here, Coulomb's law is mainly used, which does not show stick–slip phenomena in the stationary case, i.e. constant velocity of the drive.

All friction models need certain parameters to match experimental observations and numerical simulations. From this viewpoint the number of parameters should be as small as possible. In the following, we therefore restrict ourselves to four well-known macroscopic laws [3] with a maximum of three parameters to compare their influence on the motion of a given mechanical system, especially on stick–slip phenomena.

A necessary condition to observe stick–slip motions [4] is a strict separation between static

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friction ("sticking") and dynamic friction ("sliding"). The classical paradigm is a block lying on a horizontal plane with a constant, time-invariant, normal force and pulled with constant velocity via the action of a spring. In the following, a decelerated motion of an extended mechanical system with internal damping will be considered. The question arises how different friction laws influence the stick–slip response in this rather general case.

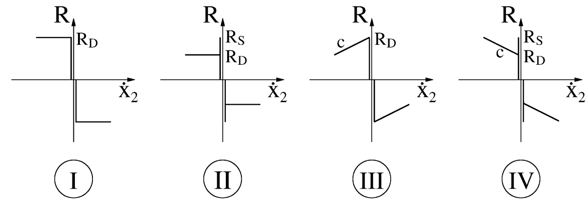


Fig. 1. Friction laws: (I) one-parametric, (II), (III) two-parametric, (IV) three-parametric.

2. Friction laws

Four types of dynamic friction laws will be considered. They can be formulated by only one equation

$$R = -R_D \operatorname{sgn} \dot{x}_2 + C\dot{x}_2, \quad |\dot{x}_2| \neq 0. \tag{1}$$

Later on, the velocity $\dot{x}_2$ will be the velocity of a mass m. Please note that a positive force R acts in the positive direction of a coordinate x_2.

For a temporary state "sliding" the validity of this active force is limited by the necessary condition of non-vanishing velocity $\dot{x}_2$. The parameters R_D and C are constant due to the implicit assumption of time-invariant normal forces acting at the contact surfaces. In a temporary state "sticking" with vanishing velocity $\dot{x}_2 \equiv 0$ the active force turns over to a passive one to be calculated from the solution of the corresponding motion equation. This intermittent behaviour of the contact force leads to a strict separation of portions of "sticking" and "sliding" during the course of time. A state "sticking" is terminated when the absolute value of the passive contact force reaches a certain threshold R_S. This gives a maximum number of three parameters R_S, R_D and C to comprehend the friction properties.

Eq. (1) allows splitting up into four friction laws. The influence of each law on the motion of the stick–slip paradigm is known from literature.

I. *One-parametric law* (Fig. 1(I)):

$R_S = R_D$,

$C = 0$.

In the case of the traditional Coulomb law the trivial solution (permanent sliding) of the stick–slip paradigm is globally stable. No stick–slip phenomenon can occur.

II. *Two-parametric law* (Fig. 1(II)):

$R_S > R_D$,

$C = 0$.

This law is a simple approximation of a clockwise loop of a curved characteristic experimentally investigated in [5]. The stick–slip paradigm shows a limit cycle which is only reached for sufficiently large disturbances. The trivial solution is stable with respect to disturbances lying inside the limit cycle [6, p. 130].

III. *Two-parametric law* (Fig. 1(III)):

$R_S = R_D$,

$C > 0$.

This law is the approximation by the tangent of a curved characteristic at the origin $\dot{x}_2 = 0$. Its applicability is therefore restricted to low velocities. There exists a limit cycle for the stick–slip paradigm. The trivial solution is unstable even for arbitrary small disturbances [6, p. 131].

The case $C < 0$ shows no stick–slip. It can be interpreted as law (I), having additional viscous damping during sliding.

IV. *Three-parametric law* (Fig. 1(IV)):

$R_S > R_D$,

$C < 0$.

The friction characteristic first decreases by a jump and then increases when raising the velocity $\dot{x}_2$. This property is similar to the well-known Stribeck curve. The stick–slip paradigm shows a limit cycle. The trivial solution is stable with respect to disturbances lying inside the limit cycle. The case $C > 0$ will be discussed later.

3. Mechanical problem

Consider a mechanical system with two masses (M, m) connected by a linear spring (k) and a viscous damper (d) (Fig. 2).

The displacements of both masses are given by the absolute coordinates x_1 and x_2. The mass M is driven via a second linear spring (K). The drive reads

$$y(t) = \begin{cases} Vt + At^2/2 + U\sin\Omega t, \\ \quad V > 0, \quad A < 0, \quad 0 \leqslant t < -V/A, \\ 0, \\ \quad t \geqslant -V/A. \end{cases} \tag{2}$$

Starting with a positive initial velocity $V > 0$ at time $t = 0$ there exists a constant negative deceleration $A < 0$ superimposed by a harmonic permanent disturbance with amplitude U and natural frequency Ω. If the mean driving velocity $\dot{y} = V + At$ becomes zero, the total drive is switched off. A frictional device is attached to the mass m, the properties of which are defined in Eq. (1).

In a temporary state "sliding" (2DOF) the behaviour of the system is described by

$$M\ddot{x}_1 + d(\dot{x}_1 - \dot{x}_2) + (K + k)x_1 - kx_2 = Ky,$$

$$m\ddot{x}_2 - d(\dot{x}_1 - \dot{x}_2) - kx_1 + kx_2 = R \tag{3}$$

and in a state "sticking" (1DOF) by

$$M\ddot{x}_1 + d\dot{x}_1 + (K + k)x_1 - kx_2^* = Ky. \tag{4}$$

Here, the constant displacement x_2^* of the mass m is known from the previous state "sliding". Similarly, the initial conditions of a new state follow from the known displacements and velocities at the end of the previous state. The total motion generally consists of an a priori unknown sequence of both intermittent states. The points of transition from one state to the next are controlled by switching conditions. The friction law (1) generally describes sliding phases in two directions with $\dot{x}_2 > 0$ and $\dot{x}_2 < 0$. To allow a comparison with the well-known results of the stick–slip paradigm, where only unidirectional sliding phases with $\dot{x}_2 > 0$ can exist, because of the positive velocity of the drive, the following considerations will also be restricted to a class of motions with variable positive sliding velocities $\dot{x}_2 > 0$. This assumption allows to consider only necessary conditions for a transition from one state to the next. On the other hand, the result of a calculation must be proved, if the contact force is always smaller than the positive value of the static threshold R_S.

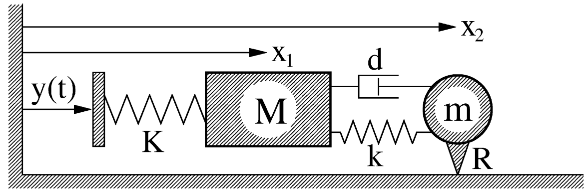


Fig. 2. Mechanical system.

A change from "sliding" to "sticking" occurs if

$$\dot{x}_2 = 0 \tag{5}$$

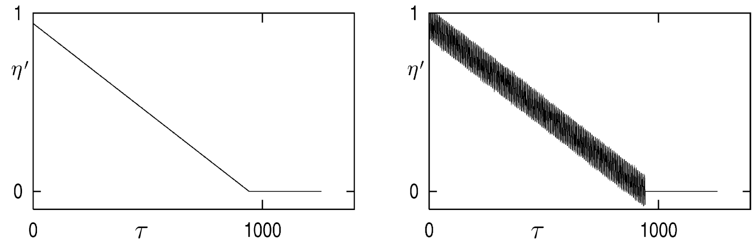
and in the opposite direction if

$$|k(x_2^* - x_1) - d\dot{x}_1| = R_S. \tag{6}$$

A numerical calculation needs non-dimensional quantities. The ratio $\omega_0^2 = k/m$ is used to introduce a non-dimensional time $\tau = \omega_0 t$, an exciting frequency ratio $\alpha = \Omega/\omega_0$, a damping factor $D = d/(2m\omega_0)$ and the slope of the frictional characteristics in the cases (II) and (IV) $D^* = C/(2m\omega_0)$. The static spring compression $X_0 = R_S/k$ defines the properties of the drive $\eta = Y/X_0$ by the new parameters $v = V/(X_0\omega_0)$, $a = A/(X_0\omega_0^2)$ and $u = U/X_0$ and the coordinates by $\xi_1 = x_1/X_0$ and $\xi_2 = x_2/X_0$. The system parameters are given by the ratios $\mu = M/m$ and $\sigma = K/k$. The value of the dynamic friction at the origin is named $\rho = R_D/R_S \leqslant 1$. In all frictional laws considered the threshold value of the non-dimensional passive contact force is normalised to 1.

The new formulation gives the drive

$$\eta = \begin{cases} v\tau + a\tau^2/2 + u\sin\alpha\tau, \\ \quad v > 0, \quad a < 0, \quad 0 \leqslant \tau < -v/a, \\ 0, \\ \quad \tau \geqslant -v/a. \end{cases} \tag{7}$$



(a) (b)

Fig. 3. Velocity of the drive, (a) without disturbance, (b) harmonic disturbance is added.

Corresponding to the restriction to motions with unidirectional sliding velocities $\dot{x}_2 > 0$ the dimensionless velocity ξ_2' is always positive. This yields $\operatorname{sgn}\xi_2' = 1$. The equations of motion (3) and (4) get the simpler form

$$\left.\begin{aligned} &\mu\xi_1'' + 2D\xi_1' - 2D\xi_2' + (1+\sigma)\xi_1 - \xi_2 \\ &\quad = \sigma\eta \\ &\xi_2'' - 2D\xi_1' + 2(D - D^*)\xi_2' - \xi_1 + \xi_2 \\ &\quad = -\rho \end{aligned}\right\} \quad \xi_2' > 0 \tag{8}$$

and

$$\mu\xi_1'' + 2D\xi_1' + (1+\sigma)\xi_1 - \xi_2^* = \sigma\eta,$$
$$\xi_2' \equiv 0, \quad \xi_2^* - \xi_1 - 2D\xi_1' > -1. \tag{9}$$

4. Choice of parameter values and integration procedures

The mechanical system is described by the group of parameters μ, σ, D, the drive by the group v, a, u, α and the friction by the group $1, \rho, D^*$. This large number allows a great variety of examples. At first, we therefore restrict ourselves to a distinct set of parameters and then we discuss the influence of a change on the system's response.

The mechanical system can be interpreted as a heavy, stiff superstructure connected with a frictional device as substructure. This leads to the heuristical choice $\mu = 5$ and $\sigma = 10$. A small internal damping $D = 0.01$ seems to be a realistic value. A deceleration $a = -0.001$ allows a slow approach to stop the drive. This choice gives raise to long sliding paths despite comparatively low initial velocities v. The amplitude $u = 0.1$ of the permanent disturbances is considerably smaller than the maximum displacement $\eta(-v/a)$ in the interesting time interval $0 < \tau < -v/a$. The frequency ratio $\alpha = 0.85$ is sufficiently far away from the lowest natural frequency 0.92 of the free mechanical system without friction. Therefore, resonance effects can be excluded. As a result, only the velocity distribution $\eta'(\tau)$ is affected by the existence of permanent disturbances (Fig. 3).

The introduction of non-dimensional quantities automatically gives $\rho = 1$ for the laws (I) and (III). In both other cases $\rho = 0.75$ is chosen. The slope of the friction characteristic D^* will be correlated later on with the internal damping D. The values $D^* = +0.02$ in the case (III) and $D^* = -0.02$ in the case (IV) have the same order of magnitude as D.

Procedures for integrating motion equations of non-smooth systems have been discussed in Ref. [7]. They consist of two tasks. Firstly, the integration of the smooth problems (8) and (9) within two successive separation points. This is done numerically by a method given in Ref. [8]. Secondly, the determination of the separation points. This is performed by numerical means in a way described in Ref. [7]. Both tasks can be carried out with limited accuracy only. Numerical experiments are necessary to obtain orbitally stable results [9].

The aim is to find both the velocity distribution $\xi_2'(\tau)$ and the contact force $F(\tau)$

$$F(\tau) = \begin{cases} -\rho + 2D^*\xi_2'(\tau), & \xi_2' > 0, \\ \xi_2^* - \xi_1(\tau) - 2D\xi_1'(\tau), & \xi_2' \equiv 0. \end{cases} \tag{10}$$

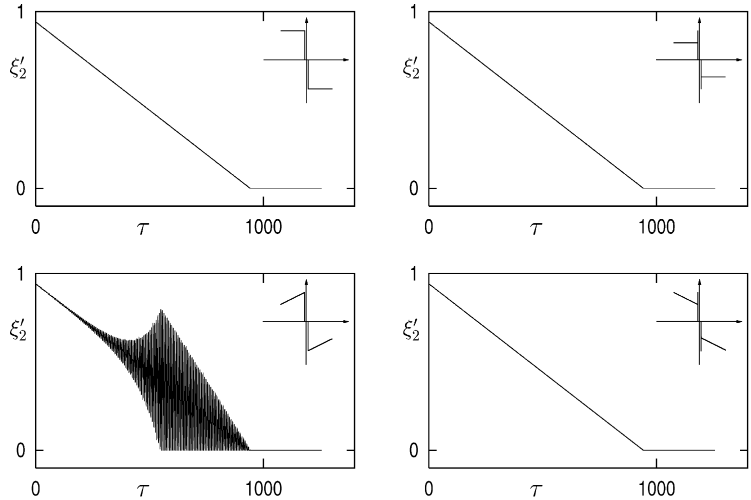


Fig. 4. Velocities versus time for the undisturbed system.

at the friction device in the vicinity of the stop of the drive, which does not coincide with the stop of the whole system.

5. Trivial solution and its stability in the small

The non-smooth problem (8) and (9) has a trivial solution which describes permanent decelerative sliding with $\xi_1'' = \xi_2'' = a$. Because of $\xi_2' \neq 0$ only the smooth linear problem (8) is of interest. Its particular solution

$$\xi_{1P} = a\frac{\tau^2}{2} + \left(v + \frac{2D^*a}{\sigma}\right)\tau - \frac{1}{\sigma}\left\{(1+\mu)a + \rho - 2D^*\left[v + 2D^*a\frac{1+\sigma}{\sigma}\right]\right\},$$

$$\xi_{2P} = a\frac{\tau^2}{2} + \left(v + 2D^*a\frac{1+\sigma}{\sigma}\right)\tau - \frac{1+\sigma}{\sigma}\left\{\frac{(1+\sigma+\mu)a}{1+\sigma} + \rho - 2D^* \times\left[v + 2a\left(D^*\frac{1+\sigma}{\sigma} - D\frac{\sigma}{1+\sigma}\right)\right]\right\} \tag{11}$$

is valid for all friction laws considered. The velocity of the friction device and the drive differ by a small constant

$$\xi_{2P}' - \eta' = 2(1+\sigma)D^*a/\sigma, \tag{12}$$

which becomes zero for laws (I) and (II).

Stick–slip effects must be avoided in the engineering practice. Rapid and sudden changes from stick to slip and vice versa induce a broadband excitation spectrum into the technical system, leading to squeal, rattling or even damage. In the mechanical system considered, the trivial motion (11) is smooth without any oscillating part. All following considerations therefore start with this trivial solution in a state of permanent sliding to find out the reasons of turning over into stick–slip.

Solution (11) yields the initial conditions

$$\xi_1(0) = \xi_{1P}(0),$$
$$\xi_2(0) = \xi_{2P}(0),$$
$$\xi_1'(0) = \xi_{1P}'(0),$$
$$\xi_2'(0) = \xi_{2P}'(0), \tag{13}$$

for numerical integrations. In the following, the interesting time range before the stop of the drive is chosen to be 300π. Because of $a = -0.001$, this gives an initial driving velocity in all plots of $v = 0.3\pi$.

The trivial solution is stable in the small for the laws (I), (II) and (IV) (see Fig. 4). The velocity is

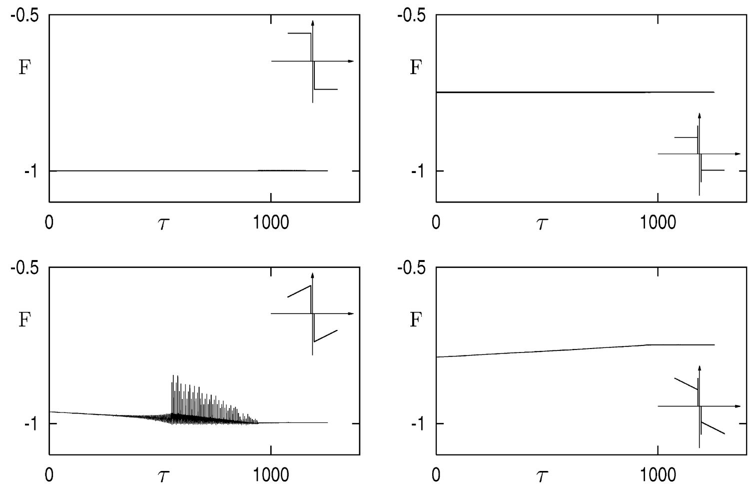


Fig. 5. Contact forces versus time for the undisturbed system.

identical to the drive (see Fig. 3a). Law (III) shows an exceptional behaviour. The trivial solution is unstable because of unavoidable numerical disturbances. Displacement and velocity of the frictional device grow exponentially. When the velocity becomes zero, stick–slip is initiated. This result will be modified later for different values of the internal damping D.

Except for law III the corresponding contact forces clearly exhibit the state "sliding" up to the stop of the drive. At this point, they change from active to passive with a jump (Fig. 5).

6. Influence of a finite disturbance on the trivial solution

Up to here, all considerations are based on idealistic assumptions. In reality, a drive will never have a perfectly constant deceleration and, moreover, all friction laws always have stochastic deviations from the assumed deterministic characteristics. Various unpredictable disturbances can act on the system during motion. Except law (III), the trivial solution is stable in the small, or in other words, stable with respect to infinite small disturbances. This property changes with a change to finite disturbances. The simplest procedure to investigate the stability of the undisturbed solution is the traditional approach of adding a sudden disturbance of displacement and/or velocity and to consider the resulting system behaviour [10]. For reasons of clarity, ξ_2' is set equal to zero when the drive reaches a certain velocity η_0' less than the initial velocity. This large disturbance can be interpreted as the result of a single removable obstacle on the sliding surface. A first stick phase is initiated. The consequences can be seen for an arbitrarily chosen value $\eta_0' = 0.15\pi$ in Fig. 6. For practical applications it is important to decide about two possibilities. Either the system reacts on a large disturbance with transient damped vibrations tending to the smooth trivial motion and containing only the natural frequencies or the disturbance initiates a permanent stick–slip motion with properties already discussed before. Unstationary stick–slip is induced for laws (II) and (III). Both the other cases show permanent sliding with damped vibrations following the disturbance. This can be clearly seen from the contact forces corresponding to laws (I) and (IV).

A smaller velocity $\eta_0' = 0.08\pi$ also creates stick–slip for law (IV) (Fig. 7).

On the other hand, if η_0' is enlarged, stick–slip also vanishes for law (II). Except Coulomb's law (case I), stick–slip can be induced in all other cases by finite disturbances, if the velocity of the drive is sufficiently low. The exceptional behaviour of

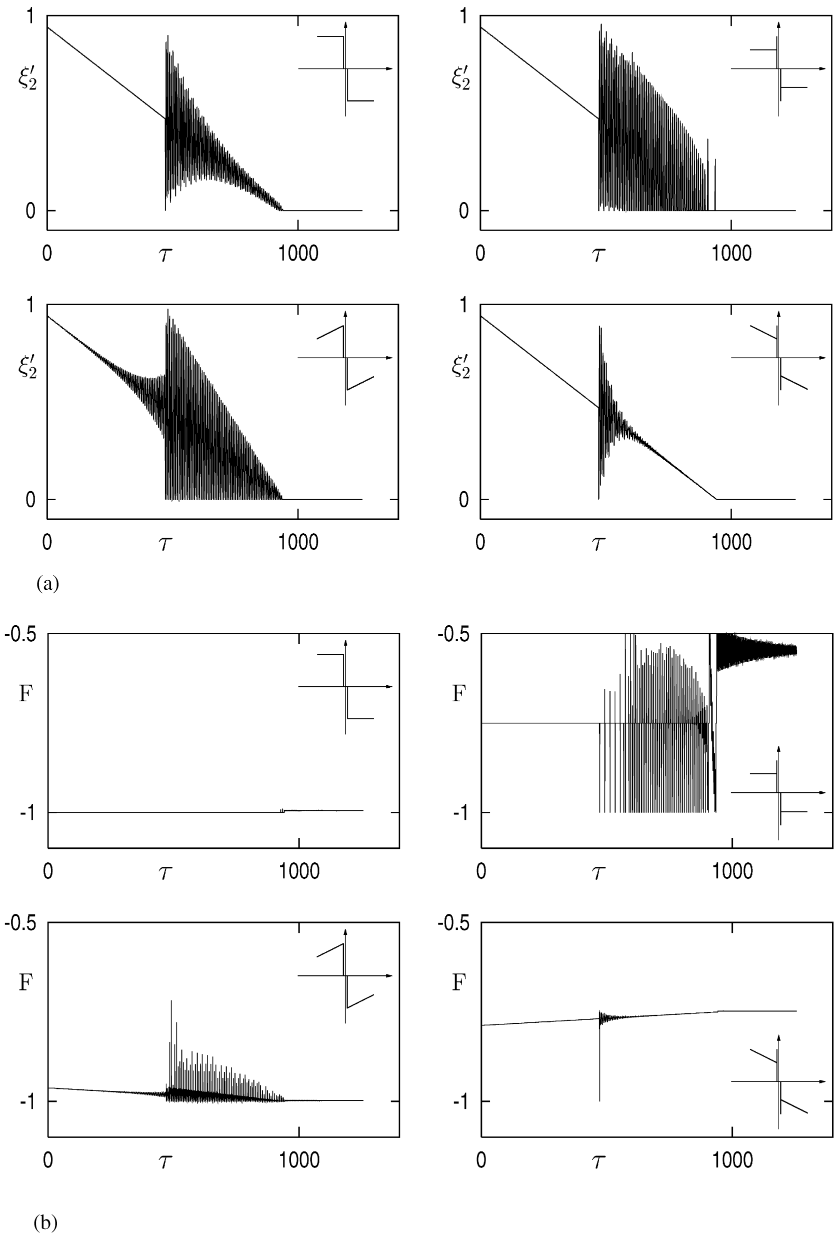


Fig. 6. Sudden disturbance at $\eta'_0 = 0.15\pi$, (a) velocity, (b) contact force.

Coulomb's law will be eliminated later, when considering the influence of a change in the driving deceleration.

These results contradict a statement about decelerative motions given in Ref. [11]. The authors only considered law (IV) with the modification $D^* > 0$. A plot for $D^* = 0.02$, $\eta'_0 = 0.15\pi$, keeping all other parameters unchanged, can be seen in Fig. 8. The stick–slip region of the motion resembles the one in Fig. 7 with $D^* = -0.02$. Moreover, all laws show the same properties in a phenomenological sense. The differences in the

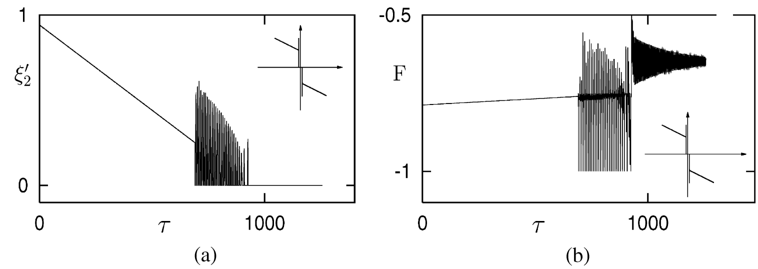


Fig. 7. Stick–slip for law (IV) for a disturbance at $\eta_0' = 0.08\pi$; (a) velocity, (b) contact force.

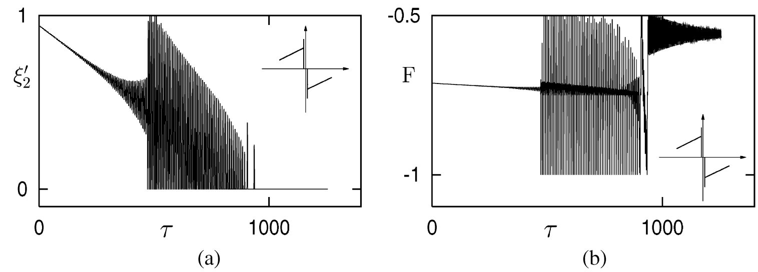


Fig. 8. Stick–slip for a modified law (IV) with $D^* = 0.02$ at $\eta_0' = 0.15\pi$; (a) velocity, (b) contact force.

shape of the velocity distribution and the contact forces are marginal. It seems rather impossible to identify a distinct law by experimental means if decelerative motion is present.

7. Permanent disturbances of the trivial solution

The latter statement is improved for a non-ideal drive. The simplest permanent disturbances are harmonic ones, as defined in Eq. (2). To exclude transient effects at the beginning, the time of integration is increased to 2400π. All results in the interval $0 \leqslant \tau \leqslant 2000\pi$ are omitted. As before, the plots start with the same distance 300π from the stop of the drive.

Fig. 9 shows the velocity ξ_2' versus time for the four laws considered. The number of stick phases are approximately the same in all cases. Even the global shape of the velocity distribution looks similar. Coulomb's law is no longer an exceptional case.

The transition from permanent sliding (trivial solution) to stick–slip can be seen clearly in Fig. 10. All laws exhibit a similar contact force F rapidly varying from active to passive and vice versa in the stick–slip region.

8. Influence of parameters

The drive is given by the group v, a, u, α. The initial driving velocity v depends on a and the arbitrarily chosen time of integration. Stick–slip is not influenced by the start of the motion. Increasing the deceleration diminishes the number of stick phases and vice versa. On the other hand, a large deceleration gives rise to stick–slip after a finite disturbance even for Coulomb's law. Fig. 11 shows velocity and contact force versus time. The disturbance occurs at a driving velocity $\eta_0' = 0.5\pi$, but the deceleration $a = -0.01$ is ten times larger than before. (Note the different scale for Fig. 11a.)

Increasing the amplitude u of the permanent disturbances increases the number of stick phases. The exciting frequency α enlarges the stick–slip region when approaching to one of the resonance frequencies of the mechanical system without friction.

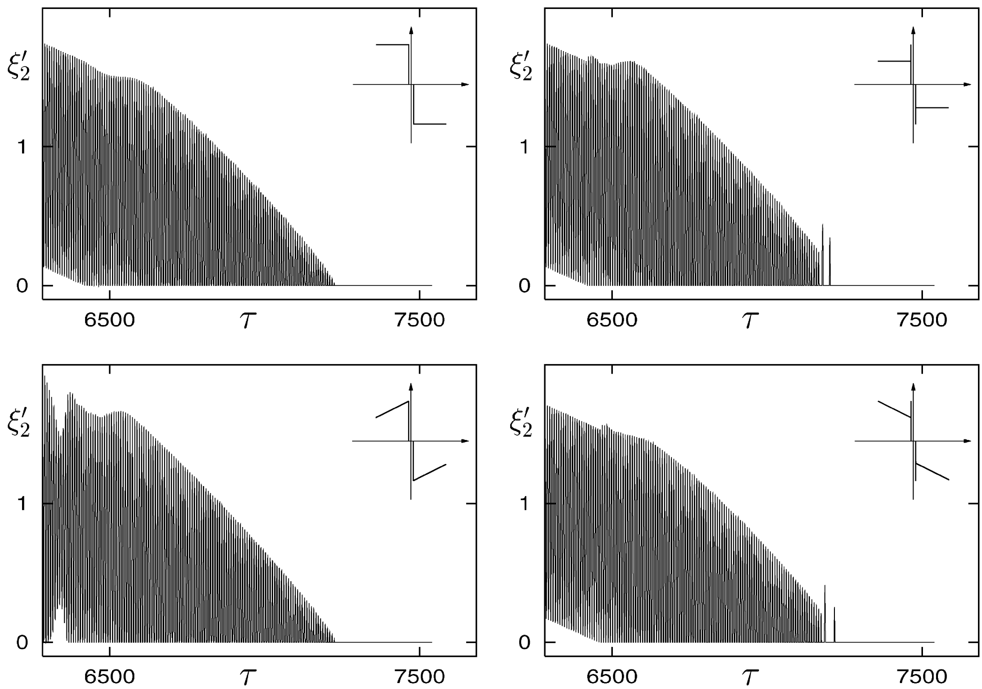


Fig. 9. Velocity versus time for permanent disturbances.

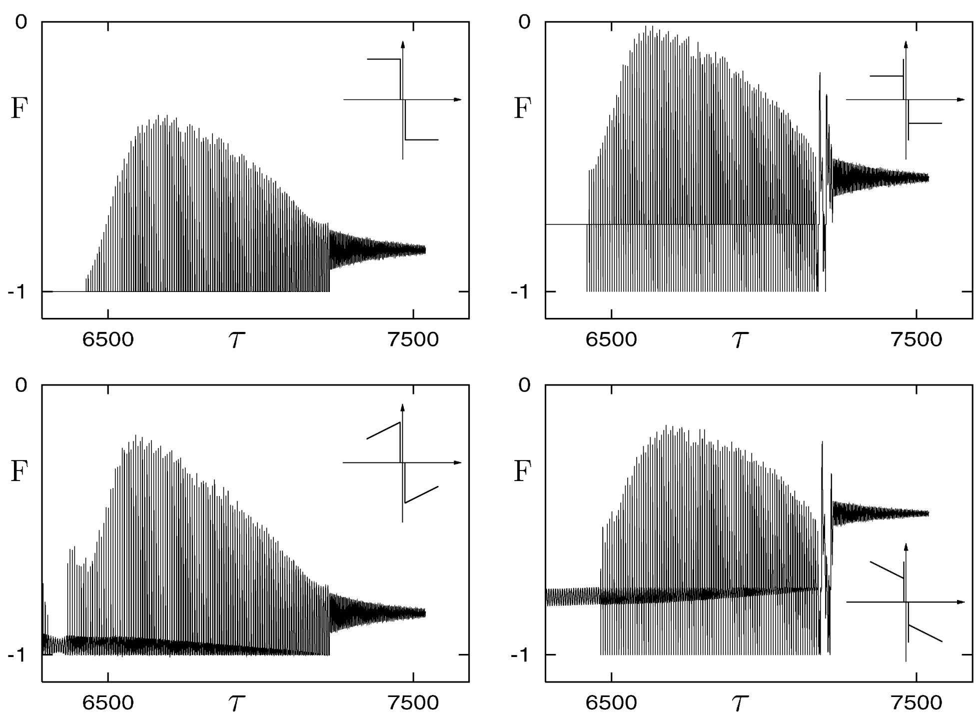
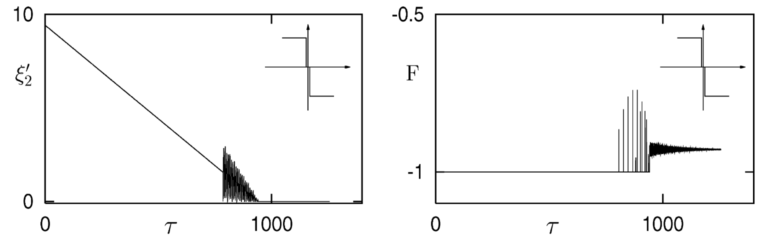
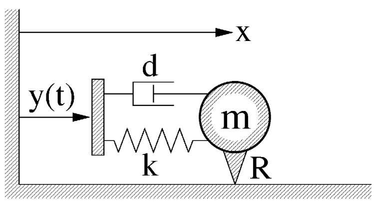


Fig. 10. Contact force versus time for permanent disturbances.

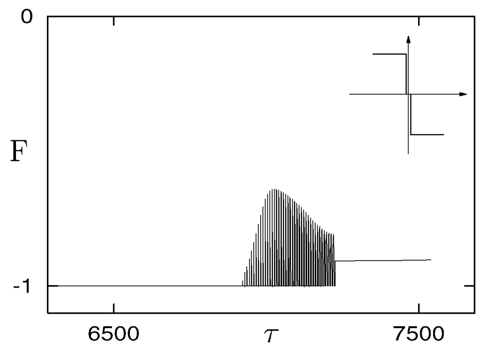


(a) (b)

Fig. 11. Stick–slip for Coulomb's law for a disturbance at $\eta_0 = 0.5\pi$ and a deceleration $a = -0.01$; (a) velocity, (b) contact force.



(a)



(b)

Fig. 12. Simplified system with Coulomb's law; (a) model, (b) contact force for permanent disturbances.

The properties of the mechanical system are described by the group μ, σ and D. A discussion about the influence of a change in masses and stiffnesses cannot be given generally, because of the nonlinearity of the problem. But it seems sufficient to consider the extreme assumption $M \to 0$ and $K \to \infty$. Then, the mechanical system is simplified to the stick–slip paradigm (see Fig. 12a).

All preceding statements keep their validity in a qualitative sense. As an example only the contact force for permanent disturbances and Coulomb's law are plotted (Fig. 11b). A qualitative difference lies in the fact of constant passive contact forces after the stop of the drive, because the damped vibration of mass M does no longer exist.

Internal damping D mainly influences the law (III). As can be seen from the motion equations (8) the damping matrix is asymmetric. It contains the element $2(D - D^*)$. All calculations concerning law (III) had been carried out with $D = 0.01$ and $D^* = 0.02$ which leads to a negative number $2(D - D^*) = -0.01$. In a state of permanent sliding only the linear system (8) is of interest. The stability of its solution depends on the sign of real parts of the roots of the characteristic equation. So far, the values of both parameters give positive real parts. This corresponds to instability due to the so-called negative damping. Keeping the property D^* of the friction law (III) fixed and increasing internal damping D there will be a change from positive to negative. This transition occurs for a constant $D^* = 0.02$ at a value $D = 0.0277$ which is still very small. For $D = 0.03$ the trivial solution for law (III) becomes stable in the small (Fig. 13a). A sudden finite disturbance (Fig. 13b) and permanent disturbances (Fig. 13c) lead to results similar to those of all other laws.

9. Conclusions

Considered is a mechanical system with one frictional device. The system is driven with constant

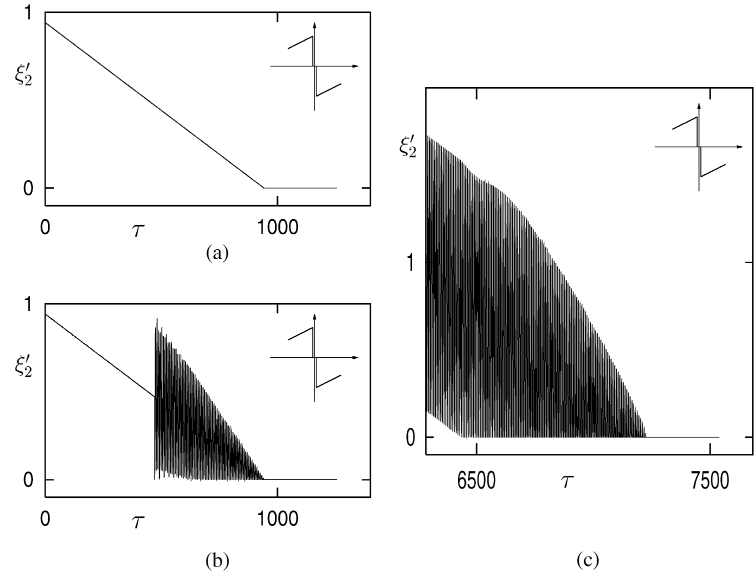


Fig. 13. Velocity versus time for the law (III) with $D > D^*$; (a) undisturbed, (b) sudden disturbance, (c) permanent disturbance.

deceleration until the driving velocity reaches zero. The interest lies in the stick–slip behaviour of the frictional device in the vicinity of the state of rest of the total system. The influence of four different frictional laws on the system's response is investigated. This leads to the question for the stability of the trivial motion, which corresponds to permanent sliding without stick phases. The concept of the disturbed motion is used with two modifications. Firstly, a finite disturbance is applied at the undisturbed motion. All laws can react with stick–slip phenomena depending on the deceleration, the driving velocity at the time when the disturbance occurs and on the value of internal viscous damping. Secondly, permanent disturbances can be present. Then the system always behaves the same, independent of the chosen friction law. From the point, when the velocity of the friction device becomes zero the first time, permanent stick–slip remains until the stop of the drive.

This leads to the conclusion that the existence of stick–slip at decelerative motion mainly depends on the properties of the mechanical system, especially the drive, and less on the characteristic of a distinct frictional force.

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